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ABSTRACT

Strikes as the 'Tip of the Iceberg' in a Theory of Firm-Union Cooperation*

We model cooperation between an employer and a workers' union as an equilibrium in an infinitely repeated game with discounting and imperfect monitoring. The employer has private information about firm profitability. The model explains the incidence and duration of strikes, as well as the employer's outsourcing (or partial lock-out) decisions. By means of an effort variable, it also extends the theory to account for worker resistance phenomena, taking the form of low effort on the part of employees. Strikes appear as random equilibrium phenomena, during finite-duration, but recurrent phases of play, triggered by the occurrence of a low-profitability state. We show that high-effort and high-pay cooperative agreements between the union and the employer can be supported as perfect public equilibria of the repeated game, if players are patient enough, but only at the cost of random reversions to noncooperative equilibrium in which strikes, low effort, low pay, and outsourcing take place.

JEL Classification: C73, D2, J45 and J5
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1 Introduction

Labor disputes typically occur in ongoing, long-run relationships, between an employer and the workers' union. Many strikes seem to be the result of a conflict situation that has been built up by past responses of the employer to changes in the environment. From the point of view of empirical economics, it is very likely that strikes are only the "tip of the iceberg": that is, the visible part of the firm-union cooperation enforcement problem. Many internal problems and negotiation spells never result in a strike that would be recorded by outside observers; workers can use other strategies to obtain improvements of working conditions and pay, using various forms of resistance and slowdown. It is also widely recognized that informational asymmetries are essential for the understanding of strikes, because fully informed and rational parties should typically avoid the social costs of work stoppages. The observed strikes are relatively rare events; they are likely to be random symptoms of the tension at work within a hidden machinery. It therefore seems appealing to embed the theory of strikes in a dynamic model, with no predictable horizon, in which cooperation and strikes can potentially intervene in a recurrent and alternating way.

We propose a model of cooperation between an employer and a workers' union, based on an infinitely repeated game with discounting and imperfect monitoring. The model explains the incidence and duration of strikes, as well as the employer's outsourcing (or partial lock-out) decisions. By means of an additional effort variable, it also extends the theory to account for worker resistance phenomena, such as "slowdown", which take the stylized form of low effort on the part of employees. These resistance problems are less easily observable, but are likely to cause inefficiencies as severe as strikes. Strikes appear as random equilibrium phenomena, during finite-duration, recurrent phases of play, in which effort and (real) pay are low, while outsourcing takes place. We show that high-effort and high-pay cooperative agreements between the union and the employer can be supported as Nash equilibria of the repeated game, if players are patient enough (as in the classic Folk Theorems), but only at the cost of random reversions to noncooperative equilibrium, in which strikes, low effort, low pay and outsourcing take place. We view the ongoing relationship between the employer and the union as having no finite horizon, and disputes can intervene at any moment. In such a context, inflation is a source of real-wage erosion, creating the need for nominal wage adjustments in each period. The union cannot observe the real state of the firm, which can be one of high profitability (low cost) or low profitability (high cost). When the proposed pay raise is low, a period of noncooperation is triggered, because the union cannot trust the employer, if the bad state of nature has been drawn. The periods of reversion to noncooperation are socially costly, but necessary to generate the employer's incentives to honestly reveal the state of nature in the good states, and thus to sustain cooperation. The occurrence of "bad times" being random, reversions to noncooperation, and therefore strikes, are also random. In addition, we show a number of comparative statics properties of the model, reproducing a number of empirical facts about strikes. In particular, for some non negligible sets of parameters, the model exhibits counter-cyclical strike durations and pro-cyclical strike incidence.

Our repeated game model essentially provides a theory of self-enforcing cooperation between employer and employees, in which work stoppages are a symptom, or epiphenomenon. In other words, strikes are just the "tip of the iceberg", the visible part of the more encompassing problem of contractual enforcement, and they appear for a given set of values of the model's parameters. Strikes can have a positive, or zero probability of occurrence, but the underlying contract enforcement problem is always present. The fundamental underpinning of the theory is the tension between cooperative and noncooperative equilibria, and the fact that repeated play, added to a sufficient degree of patience, allows players to reap the benefits of cooperation.

In our model, strikes and low effort are static best responses of the union, in the stage-game's Nash equilibrium. In addition to their role as a "screening" instrument, strikes lower the profitability of outsourcing, and thus protect the level of employment. It follows that striking activity is not necessarily followed by wage settlements, and that there are no necessary connections between (duration of) strikes and (the magnitude of) pay rises in equilibrium. These predictions are different from that of the bargaining models, but are in accordance with some well-known facts (see references below). Yet, increased real wages are a feature of cooperative phases of play, which always start after some noncooperative spell. Periods of conflict — during which work stoppages appear with some probability—, do not directly cause wage settlements, but "support" pay raises as parts of a broader cooperative agreement.

Important contributions to the strategic theory of strikes have been made possible, in the mid-eighties, with the development of incomplete-information and bargaining games. Incomplete information (of the worker's unions) is now widely understood as an essential ingredient to rationalize inefficiencies in the bargaining processes — delays and strikes. Bargaining inefficiencies (delays) have been explained by informational asymmetries in a number of contributions (see the surveys of Kennan and Wilson (1993), and Ausubel et al. (2001)). Specific applications to firm-union negotiations and strikes have first been developed with the help of Contract Theory, in a static form, see Hayes (1984), and Card (1990). Game-theoretic extensions of these ideas have been proposed by Cramton and Tracy (1992), and others. The ability of incomplete-information and game-theoretic bargaining models to reproduce (and thus provide an interpretation of) a number of empirical facts about striking activity is discussed in the papers of Kennan and Wilson (1989), and Card (1990).

When applied to firm-union negotiation, bargaining theory models are essentially static (even if defined as multi-stage games). A finite-length time interval describes a contract-period (3 years say). Bargaining over the wage rate takes place at the beginning of the interval, and strikes are a form of inefficient delay, before an agreement is signed. More realistic descriptions would certainly take dynamic aspects into consideration: the fact that players are engaged in a long-run relationship, in which memory can play a role, with a sequence of contract negotiations, etc. Contracting periods do not always have fixed durations; changing circumstances in the input and output markets, unexpected inflation, and all sorts of business cycle phenomena can randomly force players to renegotiate, at almost any moment.

The informational asymmetry between negotiating parties is fuelled by random shocks, to the extent that the unions make less precise observations of these shocks than the employers. To the best of our knowledge, James Robinson (1999) is the — apparently isolated— forerunner of a theory of labor disputes that is cast in an infinite-horizon, repeated-game, asymmetric information framework. Principal-Agent theory has been extended to repeated interaction settings to study worker moral hazard and efficiency wages (e.g., Malcolmson and

Spinnewyn (1988), McLeod and Malcolmson (1989), (1998)), but this literature does not encompass striking activity. Robinson (1999) has modified the ideas at the heart of Green and Porter's (1984) oligopoly model under incomplete information, and adapted them to allow for an analysis of the repeated employer-union interaction[1].

Our model has relationships with the subsequent literature on repeated games with imperfect monitoring, although not in a completely obvious way. Fudenberg, Maskin and Levine (1994) show that a repeated game in which (i), the players' types are private information, (ii) new types are drawn independently at each stage, and (iii), the players rely on type reports at each repetition of the constituent game, can formally be reformulated as a repeated game with imperfect monitoring, if a player's payoff does not directly depend on the other players' types. In other words, *repeated mechanism design* problems under private values can be written in the form of repeated games with imperfect monitoring, which are typically multi-person moral-hazard situations. This is why our model can also be described as a mechanism design problem under pure adverse selection, in which future payoffs are used to provide incentives. For instance, Athey and Bagwell (2001) analyze a repeated price-competition (i.e., Bertrand) duopoly in which each firm's cost is private information. Both duopolists can report their type at the beginning of each repetition of the constituent game, and a repeated-mechanism-design approach can therefore be used to explore the conditions under which perfect-equilibrium strategies support a cooperative agreement. The players' imperfect information may give rise to allocative inefficiencies, even if players are very patient. The game and to a certain extent, the solution method used below, have much

[1] In Green and Porter's (1984) oligopoly model, firms imperfectly observe market demand and their rivals decisions, and the oligopolists want to enforce some cartel agreement. A price decrease can be interpreted, either as the result of low economic activity or as a signal of a firm's attempt to free-ride the cartel agreement, by selling more than the agreed-upon production quota. Each oligopolist faces a signal-extraction, or statistical decision problem: which is the posterior probability of a rival's deviation, given that random shocks can affect demand? In equilibrium, cooperation is randomly interrupted by punishment phases, during which firms revert to non-cooperative equilibrium, to punish a likely deviation. The analogy with our theory is clear: strikes are an aspect of reversion to noncooperative equilibrium, used as a punishment by the union in the event of low profitability, to enforce truthful revelation on the part of the employer when profitability is high.

in common with this recent work of Athey and Bagwell (2001). The general philosophy of this literature is, that for fixed values of the players' patience (i.e., discount factors strictly smaller than one) the Pareto frontier cannot in general be reached in the space of average discounted payoffs, because of imperfect information. This is true, even if Pareto-efficient payoffs can be approached in the limit, when players become extremely patient, i.e., when the discount factor goes to 1.

Viewed as a theory of strikes, our repeated-game approach has been anticipated by Robinson (1999), but there are many differences in our respective ways of modelling the problem. First, an essential difference is the presence of the workers' effort variable. There is no effort (or slowdown) and no outsourcing (or partial lock-out) in Robinson (1999), who makes the simpler assumption that, provided a fixed cost is incurred, the employer can replace the work force entirely. Second, another important difference is the mode of operation of strikes. In Robinson's model, striking periods are complete interruptions, with zero payoffs for the firm and workers. Third, in Robinson's paper, strikes are retrospective punishments: in each period t, the union receives a noisy signal of period t-1's true state of nature, and the union strikes for T periods if it is sufficiently likely that the firm has lied about the state of nature in the past period. Fourth, Robinson (1999) also assumed that the employer is risk-averse and that the union is risk-neutral. This latter assumption might have obscured the main message of his pioneering paper.

In contrast, in our model: first, workers can increase effort as part of a cooperative contract, in exchange for less outsourcing and higher wages. Second, strikes typically last for only a fraction of each period, during noncooperative phases of play. The advantage of our way of modelling being that it draws a distinction between the duration and the incidence of strikes, both notions having empirical counterparts. Third, in our model, the true state of nature is never disclosed to the union. The union simply reacts to the employer's pay-raise offers: disappointing offers do trigger a reversion to non-cooperative play. In addition, the stage game's Nash equilibrium exhibits a positive duration of strikes (thus determined as a static best response). Fourth, we do not need to assume that employers are risk-averse. Finally, we also need fewer assumptions than Robinson to state our main results. For instance, we don't need an upper bound on the player's discount factor to construct cooperative equilibria of the repeated game.

In the following, Section 2 is devoted to a description of the model. Section 3 presents the construction of the equilibrium and discusses its properties. For illustrative purposes, Section 4 sketches a case study of the striking activity of municipal dustmen, and discusses some empirical evidence. Proofs are gathered in the Appendix.

2 A Repeated Game Model of Strikes

We consider a repeated game with 2 players: the Employer and the Union. To fix ideas, the Employer can be viewed as a public entity, the "Town Hall", and the union is that of dustmen and sanitation workers. The Employer can formally be viewed as a Principal, whith a given "task": a given constant number of tons of waste, say, has to be collected during each time period t. Time is discrete and the horizon is infinite.

2.1 Basic Assumptions

The task is normalized to 1 without any loss of generality. A fraction x_t , $x_t \in [0, 1]$ of the task can be outsourced (i.e., privatized) by delegation to a private sector company, and $1-x_t$ is the fraction of the waste collected by the union's employees (i.e. public sector employees, say). At each period, t, the Union and the Employer play a stage game G with two abstract sub-periods. In the first sub-period, the Union chooses a level of effort e_t in the set $\{0, \varepsilon\}$, with $\varepsilon > 0$ and a "length," or probability of strike s_t in the interval $[0, 1]$. We assume that the Union has enough command on its members to implement the effort policy. More precisely, we suppose that in the absence of incentives, effort zero is the "natural" behavior of workers, but, following a good agreement, the worker's "morale" can be raised by Union's officials, and high effort $\varepsilon > 0$ is then implemented. We will see that if high effort is not exerted, the employer can credibly use outsourcing to punish the Union.

In addition, there is a certain positive inflation rate which erodes nominal wages. The Employer chooses a pay raise in each period t, and simultaneously, chooses the extent of outsourcing x_t. Let w_t denote the workers' real wage rate. We assume that the Employer observes (e_t, s_t), and then chooses (x_t, w_t). The Union is thus a Stackelberg leader in each repetition of the stage game G. We also assume that outside options (if the employer is a private firm) and (or) rules concerning the workers (if public sector employees are considered) impose that the real wage rate w_t cannot fall below a minimum $\overline{w}$, which is assumed constant for simplicity.

Now, the Employer faces changing circumstances, a changing state of the world, that we model as a random cost parameter a_t— interpreted as a "cost of public funds" in the case of a public employer. This cost parameter is observed by the Employer (i.e., the Mayor's cabinet if we have the Town Hall example in mind) but cannot be observed by the Union. It takes to possible values: 0, with probability π, and a value $\alpha > 0$, with probability $1 - \pi$. The drawings a_t are assumed independent through time. The cost of public funds summarizes a number of random phenomena: business cycle fluctuations, price changes in product and input markets. In the case of a public employer, cycles cause changes in tax revenues, changes in the interest rate on public debt, but there are also unexpected costs faced by the Town Hall, and changes in the priorities or in the political agenda of the ruling Mayor's team, which change the tradeoffs that they face, in view of reelection. A Political Economy interpretation of the model is possible, in view of applications to the public sector. All these factors are subsumed in parameter a_t, which modifies the Employer's resistance to the Union's demands.

We can now specify the players' payoffs in the stage game. The Union's utility function in period t is denoted U_t, and defined as follows,

$$U_t = (1 - s_t)(w_t - e_t)(1 - x_t). \tag{1}$$

The interpretation of this specification is easy. The worker's surplus $(w_t - e_t)$ is multiplied by quantity produced (the number of tons of waste), $(1 - x_t)$. We assume that the workers receive a zero wage during strikes, and that $(1 - s_t)$ is the proportion of non-strike days. The Union is assumed to maximize the total surplus of its members, which is exactly U_t. The Employer maximizes the expected value of the task v, minus the internal production costs and the price of outsourced services. Effort increases productivity; for simplicity, assume that the unit cost of production is $(w_t - e_t)$. Assuming in addition that the cost of public funds is expressed as a percentage, we find that the total cost of production is simply

$$(1+a_t)(1-s_t)(1-x_t)(w_t-e_t).$$

For various reasons, the marginal cost of outsourcing is increasing — in the short run at least. For simplicity again, we assume that the cost of outsourcing x_t is $(1+a_t)(x_t^2/2)$. When the workers are not striking, the fraction of the task performed by the Union and the external providers taken together is $(1-s_t)$. When the workers are on strike, the external provider produces a fraction $s_t x_t$ of the task. It follows that the total fraction of the task produced by both the Union and external firms is $(1-s_t)+s_t x_t$, while the remaining part $s_t(1-x_t)$ of the task is not accomplished. In the waste collection example, $s_t(1-x_t)$ is the fraction of the waste that is collected neither by the public, nor by the private sector dustmen, and stays in the streets (forcing citizens to do the job themselves, say). The Employer's utility in period t and state a is denoted V_{at}, and defined as follows,

$$V_{at} = v(1-s_t) + v s_t x_t - (1+a_t)\left((1-s_t)(1-x_t)(w_t-e_t) + \frac{x_t^2}{2}\right) \quad (2)$$

This expression can be rewritten $V_{at} = v - v s_t(1-x_t) - (1+a_t)C$, where $C = (1-s_t)(1-x_t)(w_t-e_t)+x_t^2/2$. In the waste collection example, the term $v s_t(1-x_t)$ is the social disutility of the garbage left around on the sidewalks.

We could have used a more general formulation for outsourcing costs; for instance, $\beta_0 x_t + \beta_1 x_t^2$; we could also have chosen to define the unit cost of public production as, say, $w_t - \gamma e_t$; this would only lead to more complex computations, without adding any interesting insights.

The following technical assumptions make the problem interesting:

Assumption 1. The impact of effort is bounded: $\overline{w} > \varepsilon$

Assumption 2. Internal production is inefficient if workers do not exert effort: $0 < v \leq \overline{w} \leq 1$.

Define the expectation

$$\mu = E\left[\frac{1}{1+a}\right] = \frac{1+\alpha\pi}{1+\alpha}. \quad (3)$$

Then, we assume the following.

Assumption 3. The discounted value of production is higher than the minimum unit cost of union workers, provided that effort is high: $\overline{w} - \varepsilon \leq v\mu$

In the infinitely repeated game, denoted $G(\delta, \infty)$, the Union's and Employer's payoffs are respectively

$$U = \sum_{t=0}^{\infty} \delta^t U_t, \qquad V_a = \sum_{t=0}^{\infty} \delta^t V_{at},$$

where δ, belonging to the open interval $(0, 1)$, is the players' common discount factor.

2.2 Noncooperative equilibrium of the stage game

To solve the stage game G, we first analyze the Employer's best reply to a given (e_t, s_t). To simplify notation, we drop index t everywhere. The trade union chooses (e, s) ex ante, i.e., without knowing the realized value of the cost parameter a. The Employer's response is to offer a wage w, and to outsource (or privatize) a fraction x, knowing the value of a.

2.2.1 The Employer's best reply

The Employer's best reply is a pair of functions, $(x_a^*(s, e), w_a^*(s, e))$. Given that V_a is a decreasing function of w whatever happens, the Employer always chooses $w_a^*(s, e) \equiv \overline{w}$ in a noncooperative equilibrium. The best reply $x_a^*(s, e)$ is the value of x which maximizes V_a. The first-order condition for an interior maximum can be written,

$$vs + (1 + a)(1 - s)(\overline{w} - e) - (1 + a)x_a^*(s, e) = 0,$$

from which we immediately derive,

$$x_a^*(s, e) = \frac{sv}{1 + a} + (1 - s)(\overline{w} - e). \tag{4}$$

Under Assumptions 1 and 2, if $w_a^* = \overline{w}$, then, $x_a^*(s, e)$ lies in the interval $[0, 1]$, because it is a convex combination of numbers in $[0, 1]$. Since V_a is a concave function of x, the above necessary solution is also sufficient. Denote $V_a^*(s, e)$ the value of the employer's objective, when the best reply $(x_a^*(s, e), w_a^*(s, e))$ is chosen.

2.2.2 The Union's optimal strategy

The Union chooses the pair (s^*, e^*) so as to maximize the expected utility $E\left[U_a^*(s,e)\right]$, where by definition,

$$U_a^*(s,e) = (\overline{w} - e)(1-s)(1 - x_a^*(s,e)). \tag{5}$$

Maximisation of $E\left[U_a^*(s,e)\right]$ with respect to s is equivalent to maximizing $(1-s)(1 - Ex_a^*(s,e))$ with respect to s. Let $s^*(e)$ denote the maximizer of this latter function. We can state the following Lemma.

Lemma 1.

$$s^*(\varepsilon) = 0, \quad \textit{and} \quad s^*(0) = \max\left\{0, \frac{1}{2}\left[1 + \frac{1-\overline{w}}{v\mu - \overline{w}}\right]\right\} \leq \frac{1}{2} \tag{6}$$

We now determine the Union's non-cooperative effort level e^*. Given the results of Lemma 1, we easily find that,

Lemma 2.

$$E\left[U_a^*(s^*(\varepsilon),\varepsilon)\right] = E\left[U_a^*(0,\varepsilon)\right] = (\overline{w} - \varepsilon)(1 - \overline{w} + \varepsilon),$$

and,

$$E\left[U_a^*(s^*(0),0)\right] = \frac{\overline{w}}{4}\frac{(1-v\mu)^2}{\overline{w} - v\mu}.$$

A zero effort level is the Union's noncooperative equilibrium strategy if and only if

$$E\left[U_a^*(s^*(0),0)\right] \geq E\left[U_a^*(s^*(\varepsilon),\varepsilon)\right],$$

that is, by Lemma 2, if and only if the following condition holds:

$$\frac{\overline{w}}{4}\frac{(1-v\mu)^2}{\overline{w} - v\mu} \geq (\overline{w} - \varepsilon)(1 - \overline{w} + \varepsilon). \tag{A}$$

Condition **A** is always satisfied when effort is "powerful" enough, that is, if ε is close enough to $\overline{w}$. We hereafter assume,

Assumption 4: Condition **A** holds.

We can then summarize our results as follows:

Proposition 1. *Under Assumptions 1-4, the noncooperative equilibrium of the stage game G is unique, and characterized by the properties,*

(i) $e^* = 0$, *and* $s^* = s^*(0)$, *given by Lemma 1 above;*

(ii) $s^* > 0$ *if and only if* $\overline{w} > \frac{1+v\mu}{2}$*;*

(iii) x_a^* *is given by (4) above,* $x_a^*(s^*, 0) = s^*v/(1+a) + (1-s^*)\overline{w}$, *and* $w_a^* = \overline{w}$.

We conclude that the stage game's noncooperative equilibrium exhibits a positive probability of strike $s^*(0)$, a certain amount of outsourcing $x_a^*(s^*, 0)$, and a low effort level on the part of employees. Strikes reduce the expected amount of outsourcing when effort is low.

To simplify notation, denote, $V_a^* = V_a(s^*(0), 0)$, with a in $\{0, \alpha\}$, and $EV_a^* = \pi V_0^* + (1-\pi)V_\alpha^*$. Similarly, denote $U_a^* = U_a(s^*(0), 0)$, and $EU_a^* = \pi U_0^* + (1-\pi)U_\alpha^*$. To lighten notation, we also denote x_0^* for $x_0^*(s^*(0), 0)$, and x_α^* pour $x_\alpha^*(s^*(0), 0)$.

2.3 Cooperative solutions

Define the total surplus,

$$W_a = V_a + U_a. \tag{7}$$

A cooperative solution maximizes the expected value of W, taking into account that the Employer knows the state of nature a when choosing x and w. Our cooperative solution can be described as follows: the Union first commits to a policy (s, e) ex ante, and the Employer, in exchange, chooses (x, w) once the cost of funds a is known. Let (s^c, e^c, x_a^c, w_a^c) denote the cooperative solution. Then, (x_a^c, w_a^c) maximizes W_a for given (s, e) and (s^c, e^c) maximizes the expected value $E\left[\max_{(x_a, w_a)} W_a\right]$, where,

$$W_a = v(1-s) + vsx - a(w-e)(1-s)(1-x) - (1+a)\frac{x^2}{2}. \tag{8}$$

We compute cooperative solutions by considering the two states of nature separately. First, if $a = 0$, then $W_0 = v(1-s) + vsx - \frac{x^2}{2}$ is independent of w. Thus, there is a degree of indeterminacy of the cooperative wage. Denote by w_0^c the cooperative wage in state $a = 0$; it can be chosen so that $w_0^c > \overline{w}$, which is a benefit of cooperation for the Union, as compared to the noncooperative equilibrium studied above. The cooperative pay rises $w_0^c - \overline{w}$ are costless transfers between the Employer and the Union in this state.

Second, if $a = \alpha$, then, $W_\alpha = v(1-s) + vsx - \alpha(w-e)(1-s)(1-x) - (1+\alpha)\frac{x^2}{2}$ is a decreasing function of w. Efficiency then requires $w_\alpha^c = \overline{w}$. A pay raise is too costly and the Union should accept austerity.

We now search for the cooperative amount of outsourcing $x_a^c(s,e)$. Note that W_a is concave with respect to x, so that necessary conditions for optimality are also sufficient. The first-order condition for surplus maximization in state a can be written,

$$vs + a(w-e)(1-s) - (1+a)x_a^c(s,e) = 0,$$

which immediately yields,

$$x_a^c(s,e) = s\frac{v}{1+a} + (1-s)\frac{a}{1+a}(w_a^c - e) \tag{9}$$

It is easy to check that $0 \leq x_a^c(s,e) \leq 1$, under Assumptions 1 and 2.

Define $W_a^c(s,e) = W_a(x_a^c(s,e), s, e)$. We now maximize $EW_a^c(s,e)$ with respect to s, for given effort e. Denote also, for simplicity, $x_a^c(s,e) = x_a^c$. After some easy computations, we obtain:

Lemma 3.

$$W_a^c(s,e) = v - (1+a)x_a^c + (1+a)\frac{(x_a^c)^2}{2}, \tag{10}$$

and $EW_a^c(s,e)$ is a conxex function of s on the interval $[0,1]$.

Lemma 3 shows that s^c is equal to 0 or 1. We now compute the values of $EW_a^c(s,e)$ at the endpoints $s = 0$ and 1.

Lemma 4.

$$EW_a^c(0,e) = v + (1-\pi)\alpha(\overline{w}-e)\left\{\frac{\alpha(\overline{w}-e)}{2(1+\alpha)} - 1\right\}, \tag{11}$$

and,

$$EW_a^c(1,e) = \frac{v^2}{2}\mu. \tag{12}$$

Now, given Lemma 3, $(s^c, e^c) = (0, \varepsilon)$ is the cooperative solution iff $EW_a^c(0,\varepsilon) \geq EW_a^c(s,e)$, for all e in $\{0,\varepsilon\}$ and all s in $\{0,1\}$. Given Lemma 4, $EW_a^c(0,\varepsilon) \geq EW_a^c(1,\varepsilon)$ is equivalent to the condition

$$v\left(1-\frac{v}{2}\mu\right) \geq (1-\pi)\alpha(\overline{w}-\varepsilon)\left(1-\frac{\alpha(\overline{w}-\varepsilon)}{2(1+\alpha)}\right). \tag{B}$$

Under our assumptions, we have $v\,(1-v\mu/2)>0$. Thus, Condition **B** will be satisfied[2] if ε is close enough to $\overline{w}$ (exactly like condition **A** above).

Remark that $EW_a^c(0,\varepsilon) \geq EW_a^c(1,0)$ is also equivalent to condition **B**, given Lemma 4. High effort and no strikes are always more efficient than strikes. This conclusion is natural. Then, the cooperative effort level will be high (i.e., $e^c=\varepsilon$), if and only if high effort is always more efficient than low effort when there are no strikes. But the answer is given by the following Lemma.

Lemma 5. *Under Assumptions 1 and 2, $EW_a^c(0,\varepsilon) \geq EW_a^c(0,0)$.*

We therefore assume,

Assumption 5. Condition **B** holds.

And we can summarize our findings as Proposition 2.

Proposition 2.
Under Assumptions 1, 2, 3, and 5, we have $s^c=0$, $e^c=\varepsilon$, $x_0^c=x_0^c(0,\varepsilon)=0$ $x_\alpha^c=x_\alpha^c(0,\varepsilon)=\frac{\alpha}{1+\alpha}(\overline{w}-\varepsilon)$, $w_\alpha^c=\overline{w}$, and w_0^c is undetermined.

We conclude that efficient cooperative agreements involve no strikes, high effort, no outsourcing in the good state $a=0$, some outsourcing (but not much) in the bad state, a minimal wage in the bad state, and a high compensation in the good state. Intuitively, the wage w_0^c must be high enough in "good times" when $a=0$, to compensate workers for the effort, and for austerity in "bad times".

Note that the wage w_0^c can be chosen in such a way that the Union would be just indifferent between cooperation and noncooperative equilibrium, i.e., $EU_a^*=EU_a^c$ and the

[2] Remark that Condition **B** is always satisfied if $(1-\pi)\alpha$ is small enough. In other words, Condition **B** holds under reasonable conditions, because π is the probability of "normal" times, with $a=0$, and hence $(1-\pi)$ can be viewed as a small probability.

Employer strictly prefers the cooperative allocation. To see this clearly, note that,

$$EU_a^c = \pi U_0^c + (1-\pi)U_\alpha^c = \pi\left(w_0^c - \varepsilon\right) + (1-\pi)(\overline{w} - \varepsilon)(1 - x_\alpha^c).$$

Set the cooperative wage at the value $w_0^c = \pi^{-1}EU_a^* + \varepsilon - (1-\pi)\pi^{-1}(\overline{w} - \varepsilon)(1 - x_\alpha^c)$, so that $EU_a^* = EU_a^c$. We know that $EW_a^c > EV_a^* + EU_a^*$, because the cooperative solution is the only global maximum of EW_a, which is different from the noncooperative solution. Thus, $EV_a^c + EU_a^c > EV_a^* + EU_a^*$, and $EU_a^* = EU_a^c$ implies $EV_a^c > EV_a^*$. The value w_0^c can be raised slightly so as to insure that the Union strictly prefers cooperation.

3 Construction of a cooperative equilibrium in the repeated game $G(\delta, \infty)$

To sustain cooperation in a repeated interaction context, the main difficulty stems from asymmetric information: the Union has no reason to believe the Employer when they say that the state of nature is bad (i.e., the cost parameter is high). So, reversion to noncooperative equilibrium during a certain length of time T with a positive probability will be a way to insure incentive compatibility (i.e., truthful revelation of the state by the Employer), and a positive probability of strike results when $s^* > 0$.

Consider the following way of playing the repeated game $G(\delta, \infty)$. At the beginning, i.e., $t = 0$, the Union cooperates and plays $s_0^c = 0$ and $e_0^c = \varepsilon$. At the beginning of each stage-game play, in each period t, the Employer announces a state of nature $\widehat{a}_t \in \{0, \alpha\}$. If $\widehat{a}_t = 0$, the employer also cooperates and plays (x_0^c, w_0^c). If so, the Union continues to cooperate in period $t+1$. The Employer can of course deviate by choosing another outsourcing level and (or) another pay rise (x', w'), or by announcing that times are bad, i.e., $\widehat{a}_t = \alpha$. The former type of deviation will be called “off-schedule", while the latter is called “on-schedule" (using a terminology introduced by Athey and Bagwell (2001)). The on-schedule deviation is not observable, as a deviation, to the Union.

If $\widehat{a}_t = \alpha$, the Employer also cooperates and chooses (x_α^c, w_α^c). But then, in the next period $t+1$, the union reverts to noncooperative equilibrium during T periods, and then starts to cooperate again in period $t+T+1$. If the Employer announces $\widehat{a}_t = 0$ and deviates by choosing any off-schedule $(x', w') \neq (x_0^c, w_0^c)$, the Union reverts to noncooperative play in the stage game during T' periods, with $T' > T$. In these cases, the Employer replies optimally by playing the noncooperative equilibrium strategies (x_a^*, w_a^*) during the entire "punishment" (i.e., noncooperative) sequences.

If $\widehat{a}_t = \alpha$ and if the Employer deviates and plays any unexpected, off-schedule (x', w'), the Union also reverts to noncooperation during T' periods, and returns to cooperation in period $t + T' + 1$.

We will show that this pair of strategies constitute a perfect public equilibrium[3] in the infinitely repeated game $G(\delta, \infty)$. To check this, we must show that it is possible to choose T, T', and the cooperative allocation, in such a way that the Employer has no incentive to lie about the state of nature (on-schedule deviation), and no incentive to deviate from the cooperative agreement, even if they declare the state of nature truthfuly (off-schedule deviation).

3.1 Value functions

Let $\overline{V}_a$ be the equilibrium value of the Employer's expected, discounted utility when the period 0 state is a. If $a_0 = 0$, then $\overline{V}_0$ satisfies the equation,

$$\overline{V}_0 = V_0^c + \delta \left[\pi \overline{V}_0 + (1 - \pi) \overline{V}_\alpha \right]. \tag{13}$$

The interpretation of this expression is elementary: the true value of the state is $a = 0$, the employer says the truth, $\widehat{a} = 0$, and abides by the terms of the agreement, that is, plays $x_0^c = 0$, and $w_0^c > \overline{w}$,while the Union's workers exert high effort high and do not strike. The immediate benefit of this is

$$V_0^c = v - (w_0^c - \varepsilon).$$

In the next period, nature draws a new state and cooperation continues. The Employer's (discounted) continuation payoff is $\delta \left[\pi \overline{V}_0 + (1 - \pi) \overline{V}_\alpha \right]$.

[3]In a perfect public equilibrium, players condition only on the history of realized announcements, wages, outsourcing and effort decisions and not on the history of their own private type. In the model under study, a_t is the Employer's type. The Union has no type (or a fixed type).

Next, $\overline{V}_\alpha$ satisfies the equation,

$$\overline{V}_\alpha = V_\alpha^c + \delta\frac{(1-\delta^T)}{1-\delta}EV_a^* + \delta^{T+1}\left[\pi\overline{V}_0 + (1-\pi)\overline{V}_\alpha\right]. \tag{14}$$

The true value of the state is α and the Employer declares the truth $\widehat{a} = \alpha$, and abides by the terms of the agreement, i.e., plays $x_\alpha^c > 0$ and $w_\alpha^c = \overline{w}$. The first-period gain of this is

$$V_\alpha^c = v - (1+\alpha)(1-x_\alpha^c)(\overline{w}-\varepsilon) - (1+\alpha)\frac{x_\alpha^{c2}}{2},$$

but a punishment phase starts in the period immediately following and lasts for T periods. The noncooperative equilibrium of the stage game is played T times; the per-period expected utility of the employer is EV_a^*, and this latter term must be weighted by the annuity factor

$$\delta + \delta^2 + ... + \delta^T = \delta\frac{(1-\delta^T)}{1-\delta}.$$

Finally, the players return to cooperation at the end of the punishment phase. This has an expected, discounted value $\delta^{T+1}\left[\pi\overline{V}_0 + (1-\pi)\overline{V}_\alpha\right]$. Equations (13) and (14) form a system of two linear equations in two unknowns $(\overline{V}_0, \overline{V}_\alpha)$. The solutions are as follows:

$$\overline{V}_0 = \frac{\left(1-(1-\pi)\delta^{T+1}\right)V_0^c + \delta(1-\pi)\left(V_\alpha^c + \delta\frac{(1-\delta^T)}{1-\delta}EV_a^*\right)}{\Delta}; \tag{15}$$

$$\overline{V}_\alpha = \frac{\pi\delta^{T+1}V_0^c + (1-\delta\pi)\left(V_\alpha^c + \delta\frac{(1-\delta^T)}{1-\delta}EV_a^*\right)}{\Delta}, \tag{16}$$

where to simplify notation, we denote the system's determinant as

$$\Delta = 1 - \delta\pi - (1-\pi)\delta^{T+1}. \tag{17}$$

Since $(1-\pi)\delta^{T+1} < 1-\pi < 1-\delta\pi$, we easily conclude that $\Delta > 0$.

The Union's expected value functions have parallel definitions. If the Employer declares $\widehat{a} = 0$ and both players abide to the terms of the agreement, the Union gets

$$\overline{U}_0 = U_0^c + \delta\overline{U},$$

where $U_0^c = w_0^c - \varepsilon$, and $\overline{U} = \pi\overline{U}_0 + (1-\pi)\overline{U}_\alpha$. Similarly, if $\widehat{a} = \alpha$, and none of the players deviates, the Union obtains,

$$\overline{U}_\alpha = U_\alpha^c + \delta\frac{(1-\delta^T)}{1-\delta}EU_a^* + \delta^{T+1}\overline{U},$$

where,

$$U_\alpha^c = (\overline{w} - \varepsilon)\left(1 - \frac{\alpha(\overline{w} - \varepsilon)}{1 + \alpha}\right).$$

This is again a linear system in $\overline{U}_0$ et $\overline{U}_\alpha$ (and $\overline{U}$). To solve this system, it is sufficient to find the value of $\overline{U}$, as follows:

$$\overline{U} = \pi\left(U_0^c + \delta\overline{U}\right) + (1 - \pi)\left(U_\alpha^c + \delta\frac{(1 - \delta^T)}{1 - \delta}EU_a^* + \delta^{T+1}\overline{U}\right),$$

which easily yields

$$\overline{U} = \frac{EU_a^c + (1 - \pi)\delta\frac{(1-\delta^T)}{1-\delta}EU_a^*}{\Delta}. \tag{18}$$

3.2 Incentive constraints

We consider now possible deviations of the Employer. There are several types of deviations. Firstly, the on-schedule deviations by which the Employer lies about the state but chooses a pair $(x_{\widehat{a}}^c, w_{\widehat{a}}^c)$ consistent with the state announcement $\widehat{a}$. Secondly, there are "type A" off-schedule deviations, characterized by the fact that the employer doesn't lie about the state, but chooses an unexpected (x', w') instead of the agreed upon (x_a^c, w_a^c). Thirdly, there are "type B" off-schedule deviations in which the Employer lies about the state and plays an out-of-equilibrium (x', w'). And finally, we shall also discuss deviations by the Union, but they do not pose any difficult technical problem.

3.2.1 Truthful revelation of the state of nature : on-schedule deviations

Our goal is to show the existence of an equilibrium with the above described cooperative features, and such that the Employer has no incentive to falsely declare $\widehat{a} = \alpha$ when the true state is 0. This imposes,

$$\overline{V}_0 \geq \widehat{V}_\alpha^c + \delta\frac{(1 - \delta^T)}{1 - \delta}EV_a^* + \delta^{T+1}\overline{V}, \tag{IC$_1$}$$

where by definition,

$$\widehat{V}_\alpha^c = v - (1 - x_\alpha^c)(\overline{w} - \varepsilon) - \frac{x_\alpha^{c2}}{2} \tag{19}$$

is the temporary value of this deviation, obtained when the Employer plays (x_α^c, w_α^c) but does not face a high cost. A punishment phase starts immediately after this deviation. Using

$\overline{V}_0 = V_0^c + \delta\overline{V}$, constraint IC_1 can be rewritten,

$$\delta(1-\delta^T)\left(\overline{V} - \frac{EV_a^*}{1-\delta}\right) \geq \widehat{V}_\alpha^c - V_0^c.$$

Now, using the expressions (15) and (16) for $\overline{V}_0$ and $\overline{V}_\alpha$, simple computations show that

$$\overline{V} - \frac{EV_a^*}{1-\delta} = \frac{E(V_a^c - V_a^*)}{\Delta}. \tag{20}$$

It follows that IC_1 has the equivalent form

$$\frac{\delta(1-\delta^T)}{\Delta} E(V_a^c - V_a^*) \geq \widehat{V}_\alpha^c - V_0^c, \tag{IC$_1$}$$

i.e., the appropriately discounted value of the expected incremental benefits from cooperation must be greater than or equal to the temporary benefits of declaring a high cost when the cost is in fact low.

The Employer must also prefer to tell the truth instead of hiding the bad news that the cost is high to avoid the punishment phase. Formally, we must have,

$$\overline{V}_\alpha \geq \widehat{V}_0^c + \delta\overline{V}, \tag{IC$_2$}$$

where,

$$\widehat{V}_0^c = v - (1+\alpha)(w_0^c - \varepsilon), \tag{21}$$

is the employer's first-period "cost" of this deviation — because the Union receives a high wage. Using the expression $\overline{V}_\alpha = V_\alpha^c + \delta(1-\delta^T)(1-\delta)^{-1}EV_a^* + \delta^{T+1}\overline{V}$, it is easy to check that IC_2 is equivalent to

$$V_\alpha^c - \widehat{V}_0^c \geq \delta(1-\delta^T)\frac{E(V_a^c - V_a^*)}{\Delta}. \tag{IC$_2$}$$

To simplify notation, define

$$\varphi(\delta,\pi) = \frac{\delta(1-\delta^T)}{1 - \delta\pi - (1-\pi)\delta^{T+1}}. \tag{22}$$

The incentive constraints IC_1 and IC_2 can be rewritten as

$$V_\alpha^c - \widehat{V}_0^c \geq \varphi(\delta,\pi)E(V_a^c - V_a^*) \geq \widehat{V}_\alpha^c - V_0^c. \tag{IC$_{12}$}$$

Using the above expressions for V_α^c, $\widehat{V}_0^c$, $\widehat{V}_\alpha^c$, and V_0^c, we easily get the following result.

Lemma 6.

IC_{12} is equivalent to the following string of inequalities,

$$k \leq \varphi(\delta, \pi) E(V_a^c - V_a^*) \leq (1+\alpha)k, \tag{IC$_{12}$}$$

where,

$$k = w_0^c - \overline{w} + \frac{(x_\alpha^c)^2}{2\alpha}(2+\alpha). \tag{23}$$

It remains to show that the set of parameter values for which IC_{12} holds is non-empty (and non-negligible). We have a good chance to find that this is true, intuitively, if the punishment phase is long enough (i.e., $T \to +\infty$) and the players are sufficiently patient (i.e., $\delta \to 1$). Remark that using l'Hôpital's rule, we obtain

$$\lim_{\delta \to 1} \varphi(\delta, \pi) = \frac{T}{1 + T(1-\pi)}$$

and

$$\lim_{T \to +\infty} \varphi(\delta, \pi) = \frac{\delta}{1 - \delta\pi}. \tag{24}$$

And finally,

$$\lim_{T \to +\infty} \lim_{\delta \to 1} \varphi(\delta, \pi) = \frac{1}{1-\pi}.$$

If π is large, then φ can be chosen arbitrarily large, provided that δ and T are large enough. But by choosing T small and δ close to zero, we can also set a value of φ which is close to zero, if needed. The value of k does not vary with parameters (δ, π) and T, but varies with w_0^c, which is endogenous; and $E(V_a^c - V_a^*)$ can also vary with (δ, π) and T, through changes of w_0^c — some work is therefore needed to prove that an equilibrium exists. We can state the following result (proved in the appendix).

Proposition 3.

Under Assumptions 1-5, there exists a real value $\delta_0 < 1$, an integer T_0, and non-empty intervals $[\pi', \pi'']$ and $[w', w'']$ such that, if $1 > \delta > \delta_0$, $T > T_0$, $\pi' < \pi < \pi''$, $w' < w_0^c < w''$, then, IC_1 and IC_2 are simultaneously satisfied as strict inequalities (i.e., IC_{12} is satisfied in a strict sense).

To prove this crucial result, we choose the minimal acceptable value of w_0^c, that is, the value of the "good-times" wage for which the Union is just indifferent between cooperation and noncooperation, i.e., such that $EU_a^* = EU_a^c$. We then substitute this value in the expression for k, for $\varphi(\delta,\pi)E(V_a^c - V_a^*)$, and take limits as $\delta \to 1$ and $T \to \infty$. Then, by an intermediate-value argument, we show that $\varphi(\delta,\pi)E(V_a^c - V_a^*)$ must cross the bound k from below at least once. when π goes from 0 to 1. It follows that there must be a small open interval of values of π at least, over which IC_{12} hold as strict inequality, since $\alpha > 0$. By continuity, the minimal value of w_0^c can be increased, and δ and T can be chosen large enough to ensure that $E(U_a^c - U_a^*) > 0$ without violating IC_{12}.

3.2.2 Further incentives problems: off-schedule deviations

Assume now that the Employer truthfuly reports the state, but decides to choose a level of outsourcing and a wage that are different from the expected (x_a^c, w_a^c). These type-A off-schedule deviations are punished by a reversion to the stage-game's non-cooperative equilibrium during T' periods, with $T' > T$.

The outsourcing level x' that minimizes the Employer's cost is

$$x' = argmin_x \left\{ (1-x)(\overline{w} - \varepsilon) + \frac{x^2}{2} \right\} = (\overline{w} - \varepsilon),$$

and the best possible wage-deviation is obviously the minimal pay $w' = \overline{w}$. It is easy to see that these deviations would always increase outsourcing with respect to the cooperative agreement, since $x' > 0 = x_0^c$ and $x' > \frac{\alpha}{1+\alpha}(\overline{w} - \varepsilon) = x_\alpha^c$. The "spot" value of this deviation is denoted V_a'. Using (2) and the above result, we find the simple expression,

$$V_a' = v - (1+a)\left((\overline{w} - \varepsilon) - \frac{(\overline{w} - \varepsilon)^2}{2} \right). \tag{25}$$

Assume that $\widehat{a} = 0 = a$. Then, the employer chooses to play (x_0^c, w_0^c) instead of (x', w') if and only if

$$\overline{V}_0 \geq V_0' + \delta \frac{(1-\delta^{T'})}{1-\delta} EV_a^* + \delta^{T'+1}\overline{V}, \tag{IC$_3$}$$

because the deviation will be punished, starting from the next period, by a punishment phase of length T'. Using $\overline{V}_0 = V_0^c + \delta \overline{V}$ and (20), we can rewrite IC_3 as,

$$\delta(1-\delta^{T'})\frac{E(V_a^c - V_a^*)}{\Delta} \geq \frac{(\overline{w} - \varepsilon)^2}{2} + (w_0^c - \overline{w}), \tag{IC$_3$}$$

where Δ depends on T (not on T'). It is now easy to see that this constraint holds if T' is large and δ is sufficiently close to 1, because $\lim_{\delta \to 1}(\delta/\Delta) = +\infty$. This is obvious when $T' = +\infty$ and δ is large enough.

Assume now that $\widehat{a} = \alpha = a$. Then, the employer chooses to play (x^c_α, w^c_α) instead of (x', w') if and only if

$$\overline{V}_\alpha \geq V'_\alpha + \delta \frac{(1-\delta^{T'})}{1-\delta} EV^*_a + \delta^{T'+1}\overline{V}. \tag{IC$_4$}$$

Using (14), we transform IC_4 to obtain

$$V^c_\alpha + \delta \frac{(1-\delta^{T})}{1-\delta} EV^*_a + \delta^{T+1}\overline{V} \geq V'_\alpha + \delta \frac{(1-\delta^{T'})}{1-\delta} EV^*_a + \delta^{T'+1}\overline{V}.$$

Rearranging terms yields

$$\frac{\delta}{1-\delta}(\delta^{T'} - \delta^{T})EV^*_a + (\delta^{T+1} - \delta^{T'+1})\overline{V} \geq V'_\alpha - V^c_\alpha,$$

and using (20) again, we finally obtain the equivalent expression:

$$\delta(\delta^T - \delta^{T'})\frac{E(V^c_a - V^*_a)}{\Delta} \geq V'_\alpha - V^c_\alpha = \frac{(\overline{w} - \varepsilon)^2}{2(1+\alpha)}. \tag{IC$_4$}$$

This inequality will be satisfied if $T' - T$ is sufficiently large and δ sufficiently close to 1 because $\delta/\Delta \to +\infty$ when $\delta \to 1$. The constraint obviously holds when $T' = +\infty$ and δ is sufficiently close to 1. We conclude that enough patience and differences in the length of punishment phases take care of these deviations.

There is yet another case, in which the Employer lies about the state and reneges on the agreement: the type B off-schedule deviation. The choice of (x', w'), as defined above, yields the highest possible temporary benefit. It follows that these deviations are not profitable if and only if IC_3 and IC_4 are satisfied. Hence, these deviations are deterred under the same conditions as the type-A off-schedule deviations.

3.2.3 Union deviations

Let us finally consider the Union's deviations. Since the Union is a Stackelberg leader in the stage game, any deviation by the Union will be followed by the Employer's best response, in the second sub-period of the same period t. We assume that any deviation of the Union is followed by a non-cooperative best response of the Employer. The best that the Union can do is therefore to play the stage-game's noncooperative equilibrium strategy, which yields an expected utility equal to EU_a^*. But $EU_a^* < EU_a^c$, meaning that a single-period deviation from the cooperative agreement is never profitable for the Union. During the punishment phases, the Union play their best strategy, which is the Stackelberg leader's optimal strategy in the stage game. There are no profitable deviations in this case too, given that the Employer's behavior is also noncooperative during the punishment phase.

3.3 Main result

These considerations, added to the stationarity of our repeated game, show that the above described strategies of the players form a perfect public equilibrium of $G(\delta, \infty)$. The following Proposition summarizes our result.

Proposition 4. *Under Assumptions 1-5, there exists a real number δ_0 in $(0,1)$, two integers T_0 and $L(\delta_0)$, and non-empty intervals $[\pi', \pi'']$ and $[w', w'']$ such that, if $1 > \delta > \delta_0$, $T > T_0$, $T' - T > L(\delta_0)$, $\pi' < \pi < \pi''$, $w' < w_0^c < w''$ then, the following strategies constitute a perfect public equilibrium of $G(\delta, \infty)$.*

(i) In the initial period, $t = 0$, the union cooperates and plays $(e^c, s^c) = (\varepsilon, 0)$, the employer cooperates and plays (w_0^c, x_0^c), such that $w_0^c > \overline{w}$ and $x_0^c = 0$, in state 0; (w_α^c, x_α^c) such that $w_\alpha^c = \overline{w}$ and $x_\alpha^c > 0$, in state α.

(ii) If (w_α^c, x_α^c) is played in period t, then, starting from period $t+1$, the union plays (e^*, s^*), where $e^* = 0$, and the employer plays (w_a^*, x_a^*), where $w_a^* = \overline{w}$, and $x_a^* > 0$ for all a, during T periods. Both players return to cooperation and play (e^c, s^c), (w_a^c, x_a^c) in period $t + 1 + T$.

(iii) If the union plays $(e', s') \neq (e^c, s^c)$ in period t, the employer plays its noncooperative best response $(\overline{w}, x_a^*(s', e'))$ in period t.

(iv) If the employer plays (w', x') such that $(w', x') \neq (w_\alpha^c, x_\alpha^c)$ and $(w', x') \neq (w_0^c, x_0^c)$, then, starting from period $t + 1$, the union plays (e^*, s^*), where $e^* = 0$, and the employer plays (w_a^*, x_a^*), where $w_a^* = \overline{w}$, and $x_a^* > 0$ for all a, during T' periods. Both players return to cooperation and play again (e^c, s^c), (w_a^c, x_a^c) in period $t + 1 + T'$.

The equilibrium probability of strikes is positive if and only $s^* > 0$, that is, if and *only if* $\overline{w} > \frac{1+v\mu}{2}$.

The proof of this Proposition is a direct consequence of Propositions 1-3, of the above Lemmata, and of the above reasoning about the player's incentives to deviate. The result describes conditions under which there exists a sequential equilibrium characterized by a normal state of affairs in which the Union and Employer cooperate, but such that finite-length noncooperative phases are triggered by the random occurrence of a high-cost state. During these noncooperative conflict phases, the employees' wage is kept at its minimum, the employees' effort is low, there is a positive amount of outsourcing, and the probability of strikes is positive. In contrast, the cooperative phases are characterized by high wages, high effort on the part of employees, zero outsourcing and no strikes. As in theories of bargaining under incomplete information, the occurrence of strikes — as well as the low-effort spells — is entirely due to the presence of informational asymmetries.

3.4 Minimal length of the punishment phase

The expected value of equilibrium surplus, that is, $\overline{W} = \overline{U} + \overline{V}$, can easily be rewritten,

$$\overline{W} = \frac{1}{\Delta}\left[EW_a^c + (1-\pi)\delta\frac{(1-\delta^T)}{1-\delta}EW_a^*\right].$$

We define the optimal length of the punishment phase, denoted T^*, as the value of T which maximizes $\overline{W}$ subject to the constraints IC_1 and IC_2.

The expected surplus $\overline{W}$ is a decreasing function of T. To see this, a simple computation yields,

$$\frac{\partial\overline{W}}{\partial T} = \frac{(1-\pi)\ln(\delta)\delta^{T+1}}{\Delta^2}\left(EW_a^c - EW_a^*\right) < 0,$$

because by definition, $EW_a^c > EW_a^*$ and $\ln(\delta) < 0$.

The optimal value T^* is therefore a minimal punishment length: it is the smallest value which is compatible with IC_1 and IC_2.

It is not difficult to check that, $(\partial/\partial T)\varphi(\delta,\pi) = -\Delta^{-2}\ln(\delta)(1-\delta)\delta^{T+1} > 0$.Given Lemma 6 and IC_{12}, it follows that T^* is such that IC_1 is binding (and IC_2 holds automatically). Therefore, we must have,

$$\frac{\delta(1-\delta^{T^*})}{1-\delta\pi-(1-\pi)\delta^{T^*+1}}E\left(V_a^c-V_a^*\right)=k,$$

which can be solved for T^*as follows,

$$T^*=\frac{1}{\ln(\delta)}\ln\left(\frac{E\left(V_a^c-V_a^*\right)-k(\delta^{-1}-\pi)}{E\left(V_a^c-V_a^*\right)-k(1-\pi)}\right). \tag{26}$$

Remark that T^* is well-defined and positive if $\delta E\left(V_a^c-V_a^*\right)>k(1-\delta\pi)$, which, given (24), is equivalent to

$$\frac{\delta E\left(V_a^c-V_a^*\right)}{1-\delta\pi}=\left(\lim_{T\to+\infty}\varphi\right)E\left(V_a^c-V_a^*\right)>k.$$

But this latter condition holds true for an equilibrium, under IC_1, as shown by Lemma 6 and (24) above.

We now discuss some comparative statics properties of T^*. For convenience, denote $\eta=E\left(V_a^c-V_a^*\right)$. This number doesn't depend on T; it measures the expected "instantaneous" value of cooperation for the Employer. Simple calculus yields the following result:

$$\frac{\partial T^*}{\partial\eta}=\frac{k(1-\delta)}{\ln(\delta)(\delta\eta-k(1-\delta\pi))}<0,$$

since $\ln(\delta)<0$ and $\delta\eta>k(1-\delta\pi)$ under IC_1. An increase of the "instantaneous surplus of cooperation" decreases the minimal punishment phase length — an easily understandable property.

After some computations, we obtain the following "paradoxical" result.

Proposition 5.

$$\frac{\partial T^*}{\partial w_0^c}=\frac{-(1-\delta)(\eta+k\pi)}{\ln(\delta)[\delta\eta-k(1-\delta\pi)][\eta-k(1-\pi)]}>0.$$

Proposition 5 is true because all the factors of this expression are positive in equilibrium except $\ln(\delta)$. It must be that the fraction of the time spent in a situation of noncooperation (i.e. conflict) increases when the cooperative agreement is more generous. There is a tradeoff between the prevalence of conflict and the wage rate obtained by workers in the cooperation phases. Increases in the cooperative wage rate can only be sustained if the length of the minimal punishment phase is increased, because the Employer's rewards from cooperation are reduced.

3.5 Incidence and duration of strikes

We now turn to the comparative statics of strike duration and incidence, and show that our model is able to reproduce a number of empirical observations. The main empirical facts described in the literature are the following: strikes are rare events, with a relatively short median duration of three weeks; strike incidence is procyclical, while strike duration is counter-cyclical (e.g., Card (1990), Kennan and Wilson (1989)). Depending on the way of measuring cycles, the pro or counter-cyclical nature of incidence may change. An increase of the overall unemployment rate seems to be correlated with smaller incidence. But if sector or industry-specific indicators are used, such as relative industry prices, some econometric results yield a counter-cyclical incidence. Strike duration is negatively correlated with macroeconomic output, but seems to be positively correlated with relative industry prices. The evidence on these points is not extremely strong and therefore, these business cycle effects might have an "ambiguous sign".

Incidence is the probability of observing a strike, or the average fraction of time spent on strike in equilibrium. Accordingly, we define incidence I^* as the probability that a period chosen at random belongs to a punishment phase, denoted q^*, multiplied by the probability of strikes during a punishment phase s^*, assuming that the length of punishment phases is minimal, set equal to T^*. We thus define a lower bound, a notion of "minimal incidence". To sum up, $I^* = q^* s^*$: incidence is the probability of non-cooperation in the above described equilibrium, multiplied by the probability of a strike in each period of noncooperation.

We still need to compute the formula for q^*. Given the length of punishment phases, and the probability of "bad times" $(1-\pi)$, we can express the probability of non-cooperation in period t as follows:

$$1 - \Pr(\text{cooperation at } t) = \sum_{\tau=1}^{T^*} (1-\pi) \Pr(\text{cooperation at } t-\tau),$$

because the agents do not cooperate in period t if a punishment phase has started τ periods before, with $\tau \leq T^*$. In equilibrium, a punishment phase starts if the players were cooperating in $t-\tau$ and if a bad state has been drawn (with probability $1-\pi$). Given the stationary nature of the model, $\Pr(\text{cooperation at } t) = \Pr(\text{cooperation at } t-\tau) = 1 - q^*$. From this, we immediately derive, $q^* = (1-\pi)(1-q^*)\ T^*$, that is,

$$q^* = \frac{(1-\pi)T^*}{1+(1-\pi)T^*}. \tag{27}$$

To capture the likely effect of business cycles on the model's equilibrium, we can study the impact of two parameters: π, which is the probability of “good times", and $\overline{w}$, the outside-option wage rate. An increase of π reflects more optimistic beliefs of the union: with higher π, it becomes more difficult to believe that bad times have occured, and intuitively, the duration of punishment phases T^* could increase, while strike duration s^*would decrease. But the sign of the change in strike incidence I^* could then be ambiguous. An increase in the outside option $\overline{w}$ is intuitively associated with a booming economy, but has complex and possibly ambiguous consequences too. We can establish the properties of strike duration, because s^* is a simple mapping. We easily get the following result.

Proposition 6.

$$\frac{\partial s^*}{\partial \pi} < 0, \quad \text{and} \quad \frac{\partial s^*}{\partial \overline{w}} > 0, \tag{28}$$

It thus follows that strike duration is indeed counter-cyclical if π is the relevant cycle parameter. A simultaneous increase of π and $\overline{w}$ could still induce a decline of strike duration: our model is therefore fully compatible with counter-cyclical strike duration.

Now, the sign of the partial derivatives of T^* and I^* with respect to π and $\overline{w}$ is ambiguous (and difficult to study, because these functions are highly nonlinear), so that we used numerical simulations. If we fix $\varepsilon = 0.79$, $v = 0.5$, $\delta = 0.98$, $\alpha = 0.51$, $w_0^c = 1.23$, and let π and $\overline{w}$ vary in the intervals $0.4 \leq \pi \leq 0.9$ and $0.8 \leq \overline{w} \leq 0.95$, we find that all the model's assumptions, as well as conditions A and B, are satisfied as strict inequalities. The minimal length of punishment phases T^* varies between 2 and 10 periods (see Figure 1). And it is particularly interesting to see that incidence increases with the outside option of workers, while it decreases with the probability of good times (see Figure 2). Formally we find,

$$\begin{aligned} \frac{\partial T^*}{\partial \pi} &> 0, \quad \text{and} \quad \frac{\partial T^*}{\partial \overline{w}} < 0, \\ \frac{\partial I^*}{\partial \pi} &< 0, \quad \text{and} \quad \frac{\partial I^*}{\partial \overline{w}} > 0. \end{aligned} \tag{29}$$

The model thus predicts procyclical incidence if $\overline{w}$ is the appropriate cycle parameter, and counter-cyclical incidence if π is used instead. A boom can be more accurately described by means of a particular combination of increments in π and $\overline{w}$ that can produce a variation of incidence in either direction.

4 Dustmen in the City of P: Sketch of a Case Study

We conclude this paper with the sketch of a case study of municipal dustmen (i.e., garbage collectors) in the city of P, in the country of F. Some original data on the striking activity and wages of these workers, recently collected by the authors, will provide an empirical illustration of the theory, and help measuring the distance between theoretical modelling and reality. The city of P is a major town (3 millions inhabitants, in an urban area of 11 millions), in a large European country (63 millions people in 2005). The garbage collection task is therefore huge, but more or less stationary. The dustmen of P enjoy a civil servant status. A very protective legal charter guarantees life-time employment to all workers: in essence, once hired, dustmen are “tenured", and cannot be fired, except in very serious cases. In the event of a strike, the Town Hall of P has some margin of maneuver on wages, bonuses, duration of work, working conditions, recruitment of new workers, etc. During strikes, they are in a position which is not very far from that of a private firm, given that outright lock-out is prohibited by law, and given that labor markets are heavily regulated in the country. The major difference with the private sector is in the internal provision of effort incentives. Due to seniority rules, and because firing is essentially impossible, the city of P is placed in an uncomfortable position, when it comes to personnel motivation.

There is essentially a single powerful trade union, which negotiates very frequently with the Town Hall. It should be emphasized that strikes are only one visible aspect of the relations between workers and the public employer. The Town Hall cannot lock dustmen out, or use replacement workers, but they can rely on outsourcing to private sector sanitation companies, using public procurement contracts. The only problem is that this process takes time. It takes time and red tape to auction the contract, and several months are needed to build new garbage trucks. It can thus be very costly to outsource a large number of tons of garbage in a short period of time. The army has been used to clean up the streets during some famous strikes of the past.

Figure 3 plots the dustmen's number of strike days, per year, from 1968 to 2004. It seems that the series is non-stationary, with a shift in the early eighties. The recurrent strikes of the late sixties and early seventies had resulted in important pay raises, and many new recruits. A new Mayor, Mr J.C (a prominent politician, who was later to become the President of the Republic), was elected in 1977. Mayor J.C's election was greeted with very tough strikes in 1978 and 1979. Until 1977, the city of P had never had recourse to the private sector: Mayor J.C and his team crossed the Rubicon, and started partial privatization of garbage collection in the early eighties. Figure 3 clearly shows the reduction in strike incidence, starting from this period. Strikes have almost disappeared in the nineties, during Mayor J.C's second and third terms (with the exception of 1990). He stayed in office from 1977 to 1995 and was replaced by a deputy mayor from the same party until 2001. Thus, the outsourcing-privatization policy has lasted long enough to become a well-established, credible mechanism. Its strategic effect on strikes seems obvious. A glance at Figure 4 shows the year to year change in the total number of dustmen, (as voted by the Town Hall, and published in its official record). From 1978 to 2000, with the exception of 1982 and 1983, the total number of dustmen decreased, or remained constant. In the year 2001, a new Socialist mayor took office, with a more public-sector friendly stance. In any case, new recruits had to be hired to compensate for the implementation of the 35-hours week law, which had just been passed by the Socialist government. A resurgence of striking seems to be the result.

A municipal dustman's lifetime career is somewhat rigidly organized. There is a rigid wage-scale divided in 10 grades. Seniority plays a major role in promotions: a dustman stays on each grade (or step) for two or three years (depending on the grade). Those who become foremen (i.e., "head of sanitation team") start to climb up a different ladder. But the future of those who belong to the rank-and-file forever is entirely described by the 10 grades, on the same scale. The pay associated to each grade varies over time, because each negotiation with the union can lead to alterations of the ladder's overall height, and to a lesser extent, of the difference between steps. The value of each grade can also vary (increase) over time, because of pay raises decided by the country's central government and applied to all members of the civil service, including municipal workers. Figure 5 shows the real value of the 10 grades of a dustman's career over time, with the real value of the minimum wage as a point of comparison (at the bottom). Each wage-scale curve is roughly parallel to others. The effects of Mayor J.C's tough externalization policy can easily be measured. In 1977, the dustmen of P seemed to enjoy a comfortable rent; they earned substantially more than the national minimum wage: a first-grade beginner, which is typically an unskilled worker, would earn 25% more than an equivalent private-sector, minimum-wage worker. This rent is the likely result of the numerous strikes of the late sixties and early seventies (in particular of the well-known 1968 events). The wage policy of the State, in the country of F, has been generous in the sixties and seventies, and the workers' wage index was high. Another striking feature is that in the long run, the real values of the first grade and minimum wages did converge. The first grade even dips below the minimum in 1997 and 1998 (this is because the minimum wage legislation doesn't apply to public sector workers in the country of F — a curious, but well-known fact). So, public sector dustmen have lost the wage-premium they enjoyed 30 years ago. This seems to be the combined result of inflation, which eroded the real values, and of Town Hall resistance to the union's claims. The process of return to a form of long-run "equilibrium" wages has been gradual. During some years, particularly the years 1984 to 1990, the real value of the lowest grades has decreased, and the real value of the highest grades has decreased from 1986 to 1989. But none of these curves does in fact describe the evolution of an individual worker's wage, because seniority triggers automatic grade promotions. Figure 6 plots a simulation of a typical career, showing the real wage of a dustman, hired in 1978, starting in the first grade, and promoted using (contemporary) seniority rules. This dustman's wage has an increasing trend, with some fluctuations (such as the visible dip between 1987 and 1989); short periods of stagnation are followed by upward jumps, due to automatic grade promotions. Figure 6 also shows the real minimum wage, and the nation-wide unskilled worker's average wage statistic, as a point of comparison. It is likely that individual, seniority-induced raises have eased the Town Hall's austerity policy, by making the real-value erosion of wages less painful for workers. But there is a partially unobserved downside to this (apparently successful) policy, which is that, since effort incentives are low in the civil service, the privatization and low-pay policy have created the conditions of chronic low effort, and progressively demoralized the workers. We have many reasons to believe that the rate of absenteeism, which is abnormally high among municipal dustmen nowadays, has in fact increased over time. Another serious hidden problem, with potentially negative, long-lasting effects, is the induced adverse selection in the recruitment process. Our inquiry has revealed that, to an extent which is difficult to measure or establish objectively, the “quality" of applicants for vacant public dustmen positions has also decreased in the recent years.

5 Conclusion

We have modeled the long-run relationship between an employer and his workers' union as a repeated mechanism design problem, in a repeated game with imperfect monitoring. The union does not observe firm profitability, drawn at random in every period. Workers can choose not to strike and to exert high effort, and the employer, in exchange, can raise wages and limit potentially profitable outsourcing. In this context, we have shown that cooperation can be sustained, in spite of informational asymmetries, at the cost of random reversions to the non-cooperative equilibrium of the game's stage game. The noncooperative equilibrium is characterized by low effort, low pay, a substantial amount of outsourcing, and a positive probability of work stoppage. Under cooperation, workers enjoy higher wages, less outsourcing (and therefore more employment); the firm avoids strikes completely and productivity is higher, due to high worker effort. The equilibrium path is characterized by alternating phases of cooperative and non-cooperative play, disruptions being caused by random external profitability shocks. Striking does not necessarily produce immediate results in terms of (real) wages: so there is no predicted relationship between strike duration and the magnitude of (real) pay raises. The model is flexible enough to reproduce some empirical observations, such as countercyclical duration and procyclical incidence of strikes. Some empirical facts from the municipal garbage collection industry have been used to illustrate the theory and show the interaction of outsourcing (or privatization), wages, and striking activity in the long run.

6 Appendix: proofs

Proof of Lemma 1.

Clearly, $s^*(e)$ maximizes $(1-s)[1-(sv\mu+(1-s)(\overline{w}-e))]$. This objective function is also equal to $(1-s)[1-(\overline{w}-e)-s(v\mu-\overline{w}+e)]$, which is concave with respect to s if and only if $v\mu-\overline{w}+e\leq 0$. Assume first that $e=0$. Since under Assumption 2, $v\leq\overline{w}$, and since $\mu<1$,we get $v\mu-\overline{w}\leq 0$. So, indeed, if $e=0$, the unique maximum, provided it is interior, is given by

$$s^*(0)=\frac{1}{2}\left[1+\frac{1-\overline{w}}{v\mu-\overline{w}}\right].$$

It is easy to check that $s^*(0)\leq 1/2$, since this is equivalent to $v\mu\leq 1$, which is again true, by Assumption 2. Note that $s^*(0)>0$ if and only if $\overline{w}>\frac{1+v\mu}{2}$, and $s^*(0)=0$ otherwise.

Assume now that $e=\varepsilon$. Remark that $E\left[U_a^*(s,e)\right]$ is a convex function of s under Assumption 3. It follows that the maximizer $s^*(\varepsilon)$ is either 0 or 1. So we compute the value of the objective at each endpoint of the $[0,1]$ interval. We easily find $E\left[U_a^*(0,\varepsilon)\right]=(\overline{w}-\varepsilon)(1-\overline{w}+\varepsilon)$ and $E\left[U_a^*(1,\varepsilon)\right]=0$. The solution is $s^*(\varepsilon)=0$ since $E\left[U_a^*(0,\varepsilon)\right]\geq 0=E\left[U_a^*(1,\varepsilon)\right]$ if and only if $1\geq\overline{w}-\varepsilon$, which is true under Assumptions 1 and 2.

Q.E.D.

Proof of Lemma 2.

The first equality is almost immediate. The second is the result of a simple computation,

$$\begin{aligned}
E\left[U_a^*(s^*(0),0)\right] &= \overline{w}(1-s^*(0))\left[1-Ex_a^*(s^*(0),0)\right]\\
&= \overline{w}\left[1-\frac{1}{2}\left[1+\frac{1-\overline{w}}{v\mu-\overline{w}}\right]\right]\left[1-\overline{w}-(v\mu-\overline{w})\frac{1}{2}\left[1+\frac{1-\overline{w}}{v\mu-\overline{w}}\right]\right]\\
&= \frac{\overline{w}}{4}\frac{(1-v\mu)^2}{\overline{w}-v\mu}
\end{aligned}$$

Q.E.D.

Proof of Lemma 3.

$$\begin{aligned}
W_a^c(s,e) &= v(1-s)+vsx_a^c-a(1-s)(1-x_a^c)(w_a^c-e)-(1+a)\frac{(x_a^c)^2}{2}\\
&= v-(vs+a(1-s)(w_a^c-e))+x_a^c\left(vs+a(1-s)(w_a^c-e)\right)-(1+a)\frac{(x_a^c)^2}{2}\\
&= v-(1+a)x_a^c+(1+a)\frac{(x_a^c)^2}{2}
\end{aligned}$$

Compute the first two derivatives of $EW_a^c(s,e)$ with respect to s. We easily obtain

$$\frac{\partial EW_a^c(s,e)}{\partial s} = E\left((1+a)x_a^c\frac{\partial x_a^c}{\partial s}\right) - E\left((1+a)\frac{\partial x_a^c}{\partial s}\right),$$

and

$$\frac{\partial^2 EW_a^c(s,e)}{\partial s^2} = E\left\{(1+a)\left[x_a^c\frac{\partial^2 x_a^c}{\partial s^2} + \left(\frac{\partial x_a^c}{\partial s}\right)^2 - \frac{\partial^2 x_a^c}{\partial s^2}\right]\right\}.$$

But,

$$\frac{\partial x_a^c}{\partial s} = \frac{v}{1+a} - \frac{a}{1+a}(w_a^c - e) \quad \text{implies} \quad \frac{\partial^2 x_a^c}{\partial s^2} = 0.$$

Therefore,

$$\frac{\partial^2 EW_a^c(s,e)}{\partial s^2} = E\left((1+a)\left(\frac{\partial x_a^c}{\partial s}\right)^2\right) > 0.$$

This shows that $EW_a^c(s,e)$ is convex with respect to s on $[0,1]$.

Q.E.D.

Proof of Lemma 4.

$$\begin{aligned} EW_a^c(s^c,e) &= \pi\left\{v - x_0^c(s^c,e) + \frac{x_0^c(s^c,e)^2}{2}\right\} \\ &\quad +(1-\pi)\left\{v - (1+\alpha)x_\alpha^c(s^c,e) + (1+\alpha)\frac{x_\alpha^c(s^c,e)^2}{2}\right\} \end{aligned}$$

We also easily obtain, $x_0^c(0,e) = 0$, $x_0^c(1,e) = v$, $x_\alpha^c(0,e) = \frac{\alpha}{1+\alpha}(\overline{w} - e)$, and $x_\alpha^c(1,e) = \frac{v}{1+\alpha}$. Substituting these values in $EW_a^c(s^c,e)$ yields,

$$\begin{aligned} EW_a^c(0,e) &= \pi v + (1-\pi)\left\{v - (1+\alpha)\frac{\alpha}{1+\alpha}(\overline{w}-e) + \frac{(1+\alpha)}{2}\left(\frac{\alpha}{1+\alpha}(\overline{w}-e)\right)^2\right\} \\ &= v + (1-\pi)\left\{-\alpha(\overline{w}-e) + \frac{\alpha^2}{2(1+\alpha)}(\overline{w}-e)^2\right\} \end{aligned}$$

and,

$$\begin{aligned} EW_a^c(1,e) &= \pi\left\{v - x_0^c(1,e) + \frac{x_0^c(1,e)^2}{2}\right\} \\ &\quad +(1-\pi)\left\{v - (1+\alpha)x_\alpha^c(1,e) + (1+\alpha)\frac{x_\alpha^c(1,e)^2}{2}\right\} \\ &= \pi\left\{v - v + \frac{v^2}{2}\right\} + (1-\pi)\left\{v - (1+\alpha)\frac{v}{1+\alpha} + \frac{(1+\alpha)}{2}\left(\frac{v}{1+\alpha}\right)^2\right\} \\ &= \pi\frac{v^2}{2} + (1-\pi)\frac{v^2}{2(1+\alpha)} \\ &= \frac{v^2}{2}\mu \end{aligned}$$

Q.E.D.

Proof of Lemma 5.

$EW_a^c(0,\varepsilon) \geq EW_a^c(0,0)$ if and only if

$$v-(1-\pi)\alpha(\overline{w}-\varepsilon)\left(1-\frac{\alpha(\overline{w}-\varepsilon)}{2(1+\alpha)}\right) \geq v-(1-\pi)\alpha\overline{w}\left(1-\frac{\alpha\overline{w}}{2(1+\alpha)}\right),$$

which is equivalent to

$$(1-\pi)\alpha\overline{w}\left(1-\frac{\alpha\overline{w}}{2(1+\alpha)}\right) \geq (1-\pi)\alpha(\overline{w}-\varepsilon)\left(1-\frac{\alpha(\overline{w}-\varepsilon)}{2(1+\alpha)}\right),$$

in turn equivalent to

$$\overline{w}-\varepsilon/2 \leq 1+1/\alpha.$$

This is true because $\overline{w} \leq 1$.

Q.E.D.

Proof of Lemma 6.

We can rewrite the first inequality as follows.

$$\varphi(\delta,\pi)E(V_a^c - V_a^*) \geq (w_0^c-\varepsilon) - \left((1-x_\alpha^c)(\overline{w}-\varepsilon)+\frac{x_\alpha^{c2}}{2}\right).$$

Uisng the expression for x_α^c, this boils down to,

$$\varphi(\delta,\pi)E(V_a^c - V_a^*) \geq w_0^c - \overline{w} + \frac{\alpha(\overline{w}-\varepsilon)^2}{2(1+\alpha)^2}(2+\alpha).$$

Similarly, the other inequality can be rewritten;

$$(1+\alpha)(w_0^c-\varepsilon) - \left((1+\alpha)(1-x_\alpha^c)(\overline{w}-\varepsilon)+\frac{(1+\alpha)}{2}x_\alpha^{c2}\right) \geq \varphi(\delta,\pi)E(V_a^c - V_a^*).$$

using the expression for x_α^c, simple algebra yields,

$$(1+\alpha)\left(w_0^c - \overline{w} + \frac{\alpha(\overline{w}-\varepsilon)^2}{2(1+\alpha)^2}(2+\alpha)\right) \geq \varphi(\delta,\pi)E(V_a^c - V_a^*).$$

Hence, IC_{12} is equivalent to,

$$(1+\alpha)\,k \geq \varphi(\delta,\pi)E(V_a^c - V_a^*) \geq k$$

Q.E.D.

Proof of Proposition 3.

Choose w_0^c so as to get $EU_a^* = EU_a^c$. This yields,

$$w_0^c = \frac{EU_a^*}{\pi} + \varepsilon - \left(\frac{1-\pi}{\pi}\right)(\overline{w} - \varepsilon)(1 - x_\alpha^c).$$

Substitute this minimal value of w_0^c in the expression for k. This yields a value denoted $k_0(\pi)$, with the particular form,

$$k_0(\pi) = K(\pi) + \frac{(x_\alpha^c)^2}{2\alpha}(2+\alpha),$$

where,

$$K(\pi) = \frac{1}{\pi}\left[EU_a^* - (\overline{w} - \varepsilon)(1 - (1-\pi)x_\alpha^c)\right].$$

by definition. Remark that $K(\pi)$ can be rewritten as,

$$\begin{aligned} K(\pi) &= \frac{1}{\pi}\left[EU_a^* - (\overline{w} - \varepsilon)(1 - Ex_a^c)\right] \\ &= \frac{1}{\pi}\left[EU_a^* - E(\overline{w} - \varepsilon)(1 - x_a^c)\right], \end{aligned}$$

and the fact that $w_0^c > \overline{w}$ implies,

$$K(\pi) > \frac{1}{\pi}\left[EU_a^* - EU_a^c\right] = 0.$$

It is easy to check that U_0^* and U_α^* remain bounded as π varies from 0 to 1, since s^*, x_α^*, and x_0^* are bounded when π varies in $[0,1]$. Thus, $EU_a^* = \pi U_0^* + (1-\pi)U_\alpha^*$ remains bounded when π varies in $[0,1]$. From this we derive the property

$$\lim_{\pi \to 0} K(\pi) = +\infty$$

and it is easy to conclude that $\lim_{\pi \to 0} k_0(\pi) = +\infty$ and that $\lim_{\pi \to 1} k_0(\pi)$ is a finite number.

Second step: we also compute the value of $E(V_a^c - V_a^*)$ with the above defined minimal value of w_0^c. We obtain, using (8) and Proposition 2:

$$EV_a^c = v - EU_a^c - \alpha(1-\pi)U_\alpha^c - (1-\pi)(1+\alpha)\frac{x_\alpha^{c2}}{2}.$$

Since w_0^c is such that $EU_a^c = EU_a^*$, we obtain

$$EV_a^c = v - EU_a^* - \alpha(1-\pi)U_\alpha^c - (1-\pi)(1+\alpha)\frac{x_\alpha^{c2}}{2}.$$

Furthermore, we obtain

$$\begin{aligned} EV_a^* &= \pi\left\{v - vs^* + vs^*x_0^* - (1-s^*)(1-x_0^*) - \frac{x_0^{*2}}{2}\right\} \\ &\quad +(1-\pi)\left\{v - vs^* + vs^*x_\alpha^* - (1+\alpha)(1-s^*)(1-x_\alpha^*) - \frac{(1+\alpha)x_\alpha^{*2}}{2}\right\} \\ &= v(1-s^*) + vs^*Ex_a^* - EU_a^* - (1-\pi)\alpha U_\alpha^* - \frac{1}{2}E\left[(1+a)x_a^{*2}\right]. \end{aligned}$$

From this, simple algebra easily yields

$$\begin{aligned} E\left(V_a^c - V_a^*\right) &= vs^*(1-Ex_a^*) + (1-\pi)\alpha\left(U_\alpha^* - U_\alpha^c\right) \\ &\quad -(1-\pi)(1+\alpha)\frac{x_\alpha^{c2}}{2} + \frac{1}{2}E\left[(1+a)x_a^{*2}\right]. \end{aligned}$$

Note that this expression depends, neither on δ, nor on T.

It follows, using (24), that

$$\lim_{T\to+\infty}\lim_{\delta\to 1}\varphi(\delta,\pi)E\left(V_a^c - V_a^*\right) = \frac{E\left(V_a^c - V_a^*\right)}{1-\pi},$$

and,

$$\begin{aligned} \frac{E\left(V_a^c - V_a^*\right)}{1-\pi} &= -(1+\alpha)\frac{x_\alpha^{c2}}{2} + \alpha\left(U_\alpha^* - U_\alpha^c\right) \\ &\quad + \frac{vs^*(1-Ex_a^*)}{1-\pi} + \frac{E\left[(1+a)x_a^{*2}\right]}{2(1-\pi)}. \end{aligned}$$

Given that U_α^c doesn't depend on π; given that s^*, x_0^*, x_α^*, and therefore U_α^* remain bounded when π varies in $[0,1]$, we conclude that,

$$\lim_{\pi\to 1}\frac{E\left(V_a^c - V_a^*\right)}{1-\pi} = +\infty,$$

and $\lim_{\pi\to 0}(1-\pi)^{-1}E\left(V_a^c - V_a^*\right)$ is finite.

Now, since the functions $k_0(\pi)$ and $(1-\pi)^{-1}E\left(V_a^c - V_a^*\right)$ are continuous on $(0,1)$, by the Intermediate Value Theorem, there exists a value π_0 at which $(1-\pi)^{-1}E\left(V_a^c - V_a^*\right)$ crosses $k_0(\pi)$ from below and since $\alpha > 0$, there exists a value $\pi_1 > \pi_0$ such that $(1-\pi)^{-1}E\left(V_a^c - V_a^*\right)$

lies in the interval $(k_0(\pi), (1+\alpha)k_0(\pi))$ for all π in (π_0, π_1). Using continuity again, for δ close enough to 1 and T large enough, the function $\varphi(\delta, \pi)$ is close enough to $1/(1-\pi)$ so that $\varphi(\delta, \pi)E(V_a^c - V_a^*)$ belongs to the open interval $(k_0(\pi), (1+\alpha)k_0(\pi))$. Finally, we can raise w_0^c slightly above its minimal admissible value. This would decrease EV_a^c and thus $E(V_a^c - V_a^*)$ slightly, without moving $\varphi(\delta, \pi)E(V_a^c - V_a^*)$ out of the interval.
Q.E.D.

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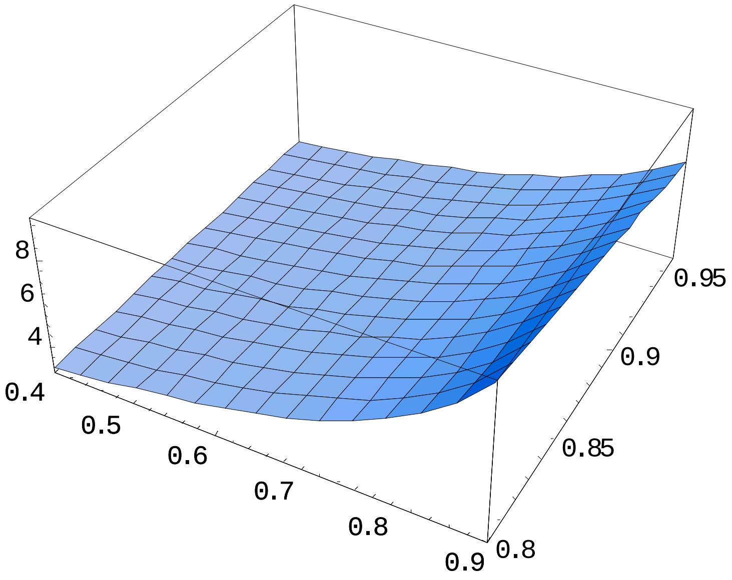

Fig.1. Three-dimensional plot of the punishment phase's minimal length T*. The probability of a good state is between 0.4 and 0.9, the outside-option wage is between 0.8 and 0.95. Other parameters are as indicated in the text. T* varies approximately between 2 and 10 periods.

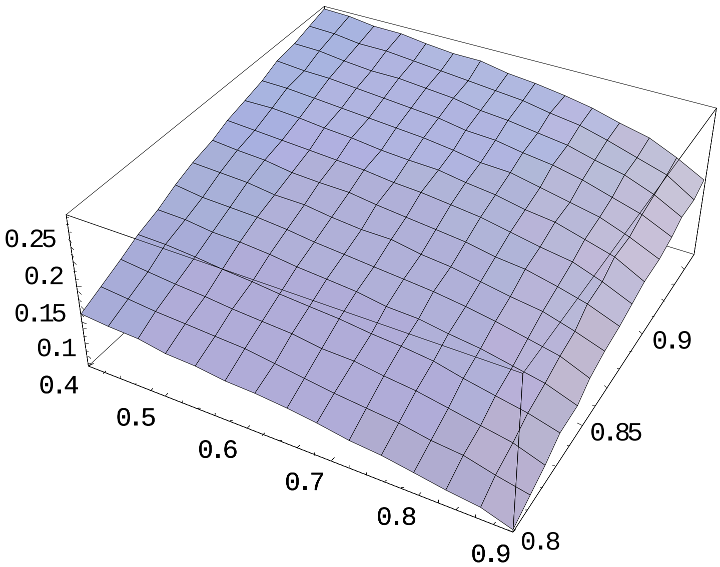

Fig.2. Three-dimensional plot of equilibrium incidence of strikes, I* = s*q*. The probability of a good state π is between 0.4 and 0.9; outside-option wage between 0.8 and 0.95. Other parameters are as indicated in the text. Incidence varies approximately between 10% and 27%. It is clearly an increasing function of outside option wage and a decreasing function of π. Incidence slopes up if a path such that the outside-option wage increases sufficiently quickly is followed on the surface.

Fig 3. Number of days spent on strike, per year

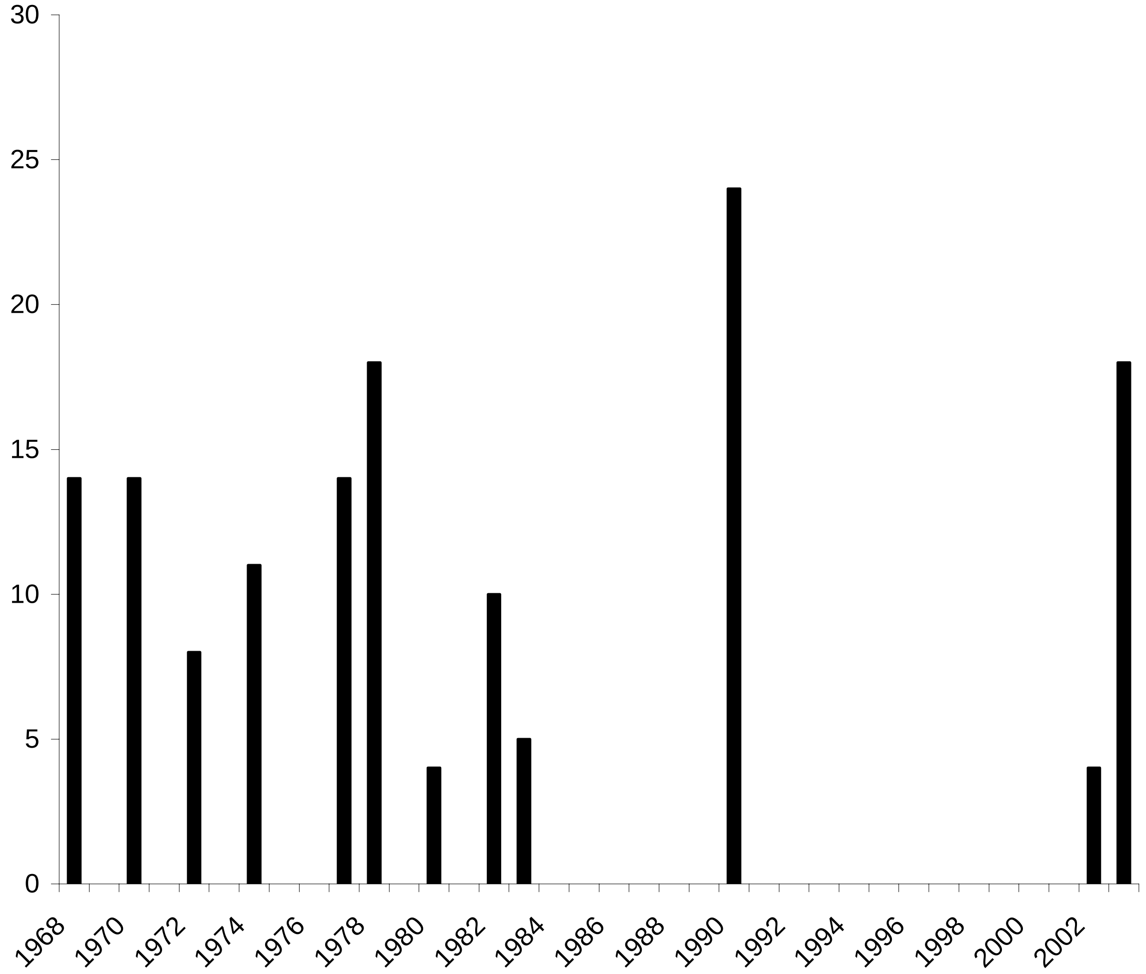

Fig. 4. Change in the total number of dustmen

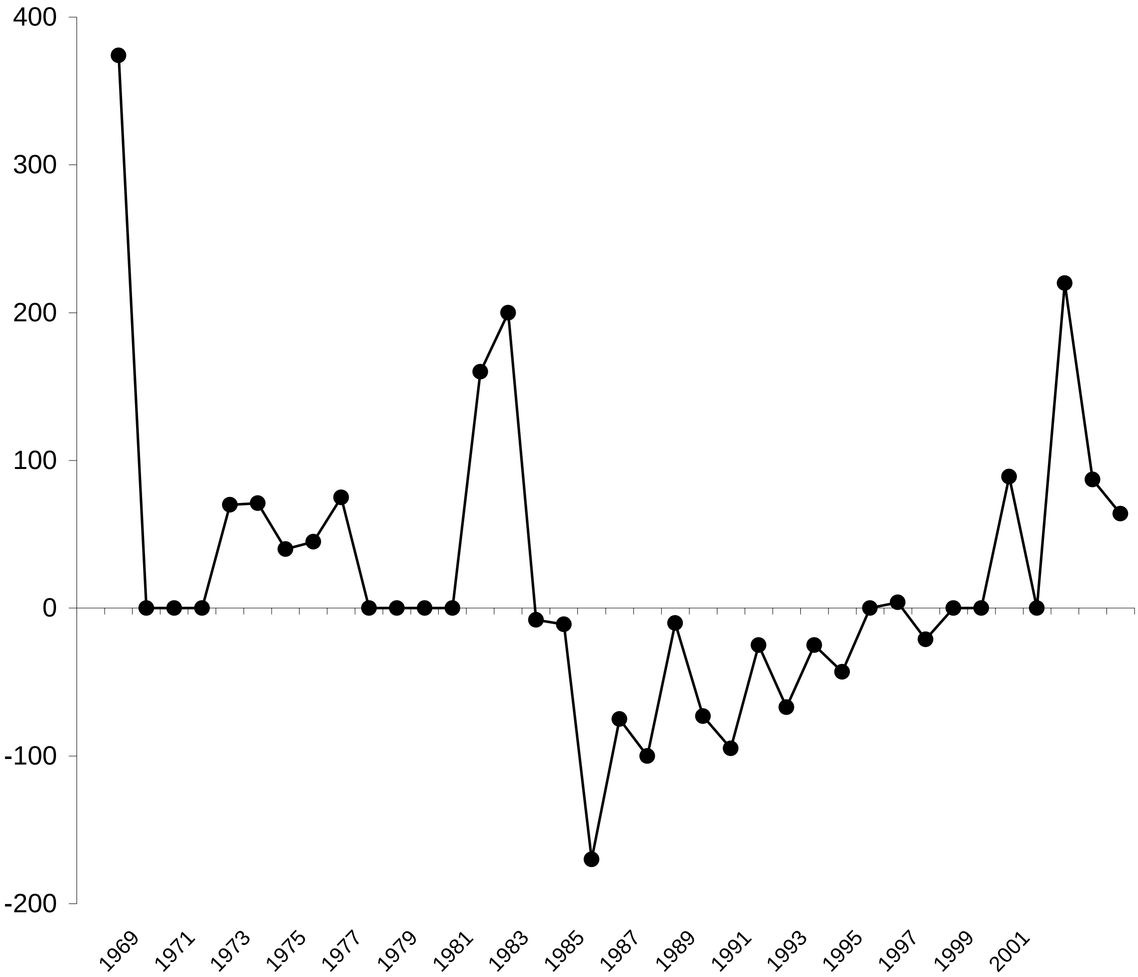

Fig. 5. Monthly Real Gross Earnings (including bonuses) (in 2004 euros)

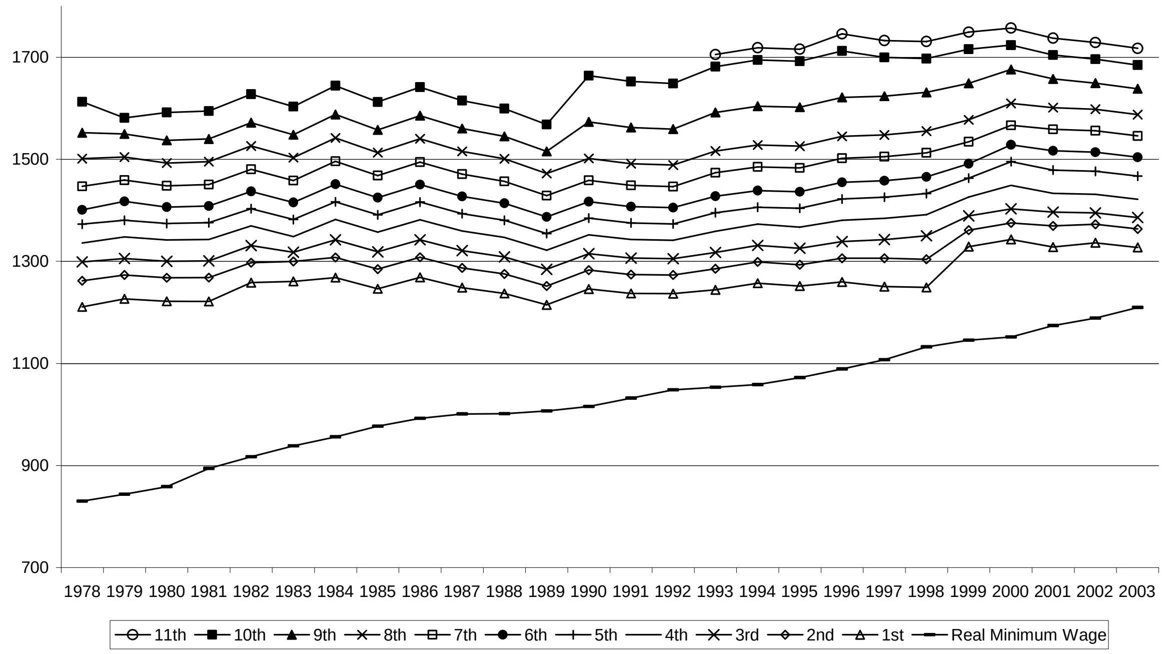

Fig. 6. Career Simulation (dustman starting in 1st grade in 1978), compared with minimum wage and average unskilled workers' wage