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ABSTRACT

Optimal Informed Trading in the Foreign Exchange Market*

We formulate a market microstructure model of exchange determination we employ to investigate the impact of informed trading on exchange rates and on foreign exchange (FX) market conditions. With our formulation we show how strategic informed agents influence exchange rates via both portfolio-balance and information effects. We outline the connection which exists between the private value of information, market efficiency, liquidity and exchange rate volatility. Our model is also consistent with recent empirical research on the microstructure of FX markets.

JEL Classification: D82, G14 and G15
Keywords: exchange rate dynamics, foreign exchange micro structure, order flow and private information

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Introduction

A recent strand of research in exchange rate economics investigates the microstructure of FX markets and the impact of trading activity on exchange rate dynamics. The principal result of this new strand is that *order flow* is an important determinant of exchange rate dynamics in the short-term and possibly even in the medium-term.[1] Theoretical underpinnings of this empirical result link the explanatory power of order flow to two different channels of transmission, due respectively to portfolio-balance and information effects. With respect to the former channel, it has simply been suggested that trade innovations perturb the inventories of FX investors which need to be compensated with a shift in expected returns.

With respect to the latter, it has been claimed that the empirical failure of the traditional models of exchange rate determination lies with the particular forward looking nature of the exchange rate and with the impact that news arrivals have on the value of currencies. Thus, when news arrivals condition market expectations of future values of exchange rate fundamental variables, currency values immediately react anticipating the effect of these fundamental shifts. Since news is hard to observe, it is difficult to control for news effects in the empirical investigation of exchange rate dynamics and hence it is difficult to conduct any meaningful analysis of the traditional models of exchange rate determination.

However, the analysis of the relation between fundamental variables and exchange rates can be bypassed by analyzing buying/selling pressure in FX markets. Indeed, it has been argued that the imbalance between buyer-initiated and seller-initiated trades in FX markets, i.e. *signed* order flow, represents the transmission link between information and exchange rates, in that it conveys information on deeper determinants of exchange rates, which FX markets need to aggregate and impound in currency values. More specifically, according to this thesis, FX traders collect from various sources information on the fundamental value of foreign currencies and trade accordingly. A general consensus and equilibrium exchange rates are then reached via the trading process, in that information contained in order flow is progressively shared among market participants and incorporated into exchange rates.

Empirical studies of the impact of order flow on exchange rates, notably Lyons (1995), Evans and Lyons (2002), Payne (2003), Berger *et al.* (2005) and Biønnes and Rime (2005) suggest that in FX markets order flow possesses an information content. Indeed, the impact of

[1]See Lyons (2001) and Vitale (2007) for presentations of this literature.

trade innovations on exchange rates is large, significant and persistent. There is also evidence that order flow anticipates shifts in exchange rate fundamental variables.

Recently several attempts to model the microstructure of FX markets have been made. Thus, Hau and Rey (2006) consider a model in which exchange rate dynamics is linked to equity returns and portfolio flows. In particular, in the face of constant risk-free interest rates, dividend innovations influence the portfolio holdings of international investors and hence affect capital flows and exchange returns. Carlson and Osler (2005), on the other hand, develop a model where exchange rate dynamics is linked to shifts in interest rates and current account flows. Specifically, according to their formulation the demand for foreign currency of risk averse speculators meets the supply of non-speculative traders. While the former reflects shifts in the interest rate differential, which modify expected currency returns, the latter is price sensitive and reflects current account transactions.

Importantly, in these models there exists *no* asymmetric information between FX traders, so that order flow cannot have any information content. On the contrary, Evans and Lyons (2004) and Bacchetta and van Wincoop (2006) propose market microstructure models of exchange rate determination which assign an informative role to order flow. The former contribution combines a general equilibrium set up, derived from the recent new open macroeconomics literature, with a microstructure framework, originally proposed by Lyons (1997), in which spot contracts are traded on a decentralized dealership market. Bacchetta and van Wincoop instead combine a standard monetary model of exchange rate determination with a microstructure framework in which trading takes place on a centralized platform. Given the recent increasing consolidation of FX markets in our formulation we follow Bacchetta and van Wincoop's lead.

In both models there is no asymmetric information between FX dealers and their customers. Nevertheless, Evans and Lyons (2005) find that it is *customer* order flow which possesses an information content. Consequently, in the model we formulate private information on exchange rate fundamentals reach FX markets via the trading activity of FX customers. We are then able to isolate the portfolio-balance and information effects of customer order flow on exchange rates, identifying a clear *link* between trade imbalance, FX dealers' expectations and exchange dynamics. In addition, we derive testable implications which are consistent with a large body of empirical evidence concerning the statistical relations between trade innovations, exchange rates and macroeconomic innovations.

This paper is organized as follows. In Section 1 we develop our analytical framework: we first describe how spot FX contracts are traded in FX markets and set out the corresponding equilibrium conditions; we then discuss the properties of the fundamental process which governs exchange rate dynamics. In section 2 we introduce privately informed traders and characterize the rational expectations (RE) equilibria of the model.

In Section 3 we analyze the impact of private information on FX markets and consider an extensive comparative statics exercise. In Sections 4 to 6 we investigate extensions of our benchmark formulation, considering alternative assumptions on the nature of private information. In the last Section we propose some final remarks and suggest further lines of research.

1 A Market Microstructure Model

We present a basic market microstructure model of exchange rate determination which replicates the specific features of the spot FX markets. This model, based on the formulation put forward by Breedon and Vitale (2004), modifies the analytical framework originally proposed by Bacchetta and van Wincoop (2006), giving a prominent role to customer order flow.

1.1 The FX Market

We assume that a single foreign currency is traded for the currency of a large domestic economy in the spot FX market by a population of FX dealers and their clients. Trading takes place according to a sequence of Walrasian auctions. Clearly, this sequence of auctions intends to represent the trading activity over the two centralized electronic trading platforms, Reuters D2 and EBS, which dominate FX markets.

In any day t a single auction is completed. When an auction is called customers enter market orders while FX dealers submit limit orders into the centralized platform. Hence, a clearing price (exchange rate) for the foreign currency is established.

1.2 FX Dealers' Trading

At the beginning of trading day t, before trading starts on the centralized platform, a generic FX dealer d owns w_t^d units of the foreign currency and g_t^d units of a domestic bond. By trading in the bond and FX markets, he can liquidate his initial endowment and invest into a portfolio of domestic bonds, b_t^d, foreign currency, f_t^d, and real balances, $\underline{M}_t^d \equiv (M_t^d/P_t)$.

Domestic bonds pay day-by-day interest rate i_t, foreign currency is held in foreign bonds which pay day-by-day interest rate i_t^*, whereas real balances return total output $h(\underline{M}_t^d)$ at the end of the day, as they are employed into a given production technology. This last assumption is introduced for tractability, as it is the simplest way to obtain demand functions for domestic real balances.

Therefore, dealer d's budget constraint is

$$g_t^d + S_t w_t^d = b_t^d + S_t f_t^d + \underline{M}_t^d,$$

where S_t is the spot rate, ie. the number of units of domestic currency required to purchase one unit of the foreign one. A log-linearization of the end-of-day wealth for dealer d allows us to write his end-of-day wealth as follows

$$\tilde{W}_{t+1}^d = (1 + i_t)\, g_t^d - (1 + i_t)\, \underline{M}_t^d + h(\underline{M}_t^d) + (1 + i_t^* + \tilde{s}_{t+1})\, w_t^d + (i_t^* - i_t + \tilde{s}_{t+1} - s_t)\, x_t^d,$$

where $s_t = \ln(S_t)$, ie. the log of the spot rate, and $x_t^d = f_t^d - w_t^d$, ie. the quantity of the foreign currency dealer d will purchase (sell) in the FX market.

Our dealers are supposed to be short-sighted in that their investment horizon is just one day long. This assumption is introduced for tractability but also captures a quite well known feature of the behavior of FX dealers, which usually attempt to unwind their FX exposure by the end of any trading day.[2] Thus, dealer d selects his optimal portfolio of domestic and foreign assets in order to maximize the expected utility of his end-of-day wealth, where his utility is given by a CARA utility function with coefficient of absolute risk-aversion γ_d (and coefficient of risk-tolerance $\tau_d = 1/\gamma_d$).

Assuming that dealer d is price-taker, the optimal quantity of foreign currency he will trade corresponds to a linear excess demand function, ie. a *limit order*, in the log of the spot rate,

$$x_t^d\,(s_t) = \nu_d \left(E_t^d\left[\,\tilde{s}_{t+1}\,\right] - s_t + (i_t^* - i_t) \right) - w_t^d,$$

where $E_t^d[\tilde{s}_{t+1}]$ denotes the conditional expectation of next day spot rate given the information dealer d possesses in day t,

$$E_t^d\left[\,\tilde{s}_{t+1}\,\right] \equiv E\left[\,\tilde{s}_{t+1} \mid \Omega_t^d\,\right],$$

[2] See Lyons (1995) and Biønnes and Rime (2005).

while ν_d is dealer's d *trading intensity* given by $\nu_d = \tau_d \pi^d_{s+,t}$. Here, $\pi^d_{s+,t}$ represents the conditional precision of next day spot rate given the information dealer d possesses in day t,

$$\pi^d_{s+,t} \equiv \left(\mathrm{Var}\left[\, \tilde{s}_{t+1} \mid \Omega^d_t \,\right] \right)^{-1}.$$

1.3 The FX Market Equilibrium

Given the demands of the individual FX dealers, through aggregation we can obtain the total demand for the foreign currency on the part of the population of FX dealers. In particular, we assume that the FX dealers form a continuum of agents of mass 1, uniformly distributed in the interval $[0, 1]$.[3] Thus, in day t

$$x_t \equiv \int_0^1 x^{d'}_t \, dd' = \nu_t \left(\bar{E}^1_t \left[\, \tilde{s}_{t+1} \,\right] - s_t + (i^*_t - i_t) \right) - w_t, \tag{1.1}$$

where $\nu_t \equiv \int_0^1 \tau'_d \, \pi^{d'}_{s+,t} \, dd'$ is the *aggregate* trading intensity of the population of FX dealers, $w_t \equiv \int_0^1 w^{d'}_t \, dd'$ is the corresponding *aggregate* initial endowment of the foreign bond they hold, and $\bar{E}^1_t[\tilde{s}_{t+1}]$ is the weighted *average* of the expected value of next day spot rate across all FX dealers, where the individual FX dealers' weights are given by their trading intensities,

$$\bar{E}^1_t \left[\, \tilde{s}_{t+1} \,\right] = \frac{1}{\nu_t} \int_0^1 \nu^{d'}_t \, E^{d'}_t \left[\, \tilde{s}_{t+1} \,\right] dd'.$$

In equilibrium the total demand for foreign currency on the part of the population of FX dealers, x_t, equals the total amount of foreign currency supplied by their customers, o_t.

$$x_t = o_t. \tag{1.2}$$

The supply of of foreign currency, o_t, corresponds to order flow, ie. the imbalance between buy and sell orders submitted by customers in the FX market.[4]

[3] This assumption is reasonable given that according to the Bank of International Settlements (BIS (2004)) there are more than 2000 dealers operating in FX markets. Clearly this implies that all FX dealers are price-takers.

[4] Differently from the usual convention a positive o_t indicates a net sale of foreign currency. If instead o_t is negative, FX customers collectively place an order to purchase the foreign currency.

1.4 The Uncovered Interest Rate Parity

Considering equations (1.1) and (1.2) and the definition of order flow, one finds that

$$i_t - i_t^* = \left(\bar{E}_t^1 \left[\tilde{s}_{t+1} \right] - s_t \right) - \frac{1}{\nu_t} z_t, \tag{1.3}$$

where z_t corresponds to *cumulative* order flow,

$$z_t = z_{t-1} + o_t.$$

Equation (1.3) implies that, thanks to the FX-dealers' risk-aversion, the *uncovered interest rate parity* does not hold. Indeed, the interest rate differential, $i_t - i_t^*$, is equal to the difference between the average expected devaluation of the domestic currency in day t and a risk-premium on the foreign currency the FX dealers collectively require to hold foreign assets. This is a *time-varying* risk-premium, given by the product of the total supply of foreign assets the FX dealers have to share and the inverse of their aggregate trading intensity, ν_t. This aggregate trading intensity is a measure of the FX dealers' capacity to hold risky assets in their portfolios. It is not difficult to show that this value is an increasing function of the weighted average of the FX dealers' risk-tolerances, $\bar{\tau}_t$, and of the weighted average of the FX dealers' conditional precisions, $\bar{\pi}_{s+,t}$. In fact,

$$\nu_t = \int_0^1 \tau^{d'} \pi_{s+,t}^{d'} \, dd' = \bar{\tau}_t \int_0^1 \pi_{s+,t}^{d'} \, dd' = \bar{\pi}_{s+,t} \int_0^1 \tau^{d'} \, dd',$$

where

$$\bar{\tau}_t = \frac{\int_0^1 \tau^{d'} \pi_{s+,t}^{d'} \, dd'}{\int_0^1 \pi_{s+,t}^{d'} \, dd'}, \qquad \bar{\pi}_{s+,t} = \frac{\int_0^1 \tau^{d'} \pi_{s+,t}^{d'} \, dd'}{\int_0^1 \tau^{d'} \, dd'}.$$

In other words, the larger the average risk-tolerance of our population of FX dealers, $\bar{\tau}$, the smaller the risk-premium imposed on the foreign currency. Likewise, the smaller the perceived uncertainty on the currency return, measured by the inverse of the average precision $1/\bar{\pi}_{s+,t}$, the smaller the risk-ness of the foreign currency and the imposed risk-premium.

Re-inserting the modified uncovered interest rate parity into the excess demand function of dealer d, we find that

$$f_t^d \; = \; \nu_t^d \left(E_t^d \left[\tilde{s}_{t+1} \right] \; - \; \bar{E}_t^1 \left[\tilde{s}_{t+1} \right] \right) \; + \; \frac{\nu_t^d}{\nu_t} \, z_t.$$

Thus, dealer d's holding of foreign currency in day t can be decomposed in two parts: the former due to a speculative motive for investment, proportional to the difference between his individual expectation and the average expectation of the future spot rate, $\bar{E}_t^d[\tilde{s}_{t+1}] - \bar{E}_t^1[\tilde{s}_{t+1}]$; the latter associated with risk-sharing. In particular, one should notice that if dealer d expects a larger value for next period spot rate than the rest of the population of FX dealers, he will be willing to bet on his *belief* and *ceteris paribus* purchase the foreign currency. If instead all FX dealers possess symmetric information, then the amount of foreign currency absorbed by dealer d for risk-sharing is proportional to his risk-tolerance.

This indicates that *ceteris paribus* investors with a larger degree of risk-tolerance will hold a larger proportion of the foreign currency. In fact, if $\forall d \; E_t^d[\tilde{s}_{t+1}] = \bar{E}_t^1[\tilde{s}_{t+1}]$ and $\pi_{s+,t}^d = \pi_{s+,t}$, then

$$\frac{f_t^d}{f_t} \; = \; \frac{\nu_t^d}{\nu_t} \; = \; \frac{\tau_d}{\int_0^1 \, \tau_{d'} \, dd'}.$$

This is consistent with the common presumption that moderately risk-averse end-users, such as currency and hedge funds, absorb most of the trade imbalance in spot FX markets.

1.5 The Monetary Market

Assuming that the production function $h(\underline{M}_t^d)$ respects the following formulation

$$h(\underline{M}_t^d) \; = \; \underline{M}_t^d \; - \; \frac{1}{\alpha} \, \underline{M}_t^d \left(\ln \underline{M}_t^d \; - \; 1 \right),$$

the optimal holding of domestic real balances on the part of dealer d turns out to be

$$\underline{M}_t^d \; = \; \exp\left(- \, \alpha \, i_t \right) \; \Longleftrightarrow \; m_t^d \; = \; p_t \; - \; \alpha \, i_t,$$

where $p_t \; = \ln(P_t)$, ie. the log of the price level, $m_t^d \; = \ln(M_t^d)$, ie. the log of the demand for money of dealer d, and the positive coefficient α corresponds to the semi-elasticity of money demand to the interest rate.

Given the demands of the individual FX dealers, through aggregation we can obtain the total demand of domestic money on the part of the population of FX dealers. In particular, as we assume that the FX dealers form a continuum of agents of mass 1, uniformly distributed in the interval $[0, 1]$, we have that in day t

$$m_t \equiv \int_0^1 m_t^{d'} \, dd' = p_t - \alpha \, i_t. \tag{1.4}$$

Finally, we suppose that a demand function for foreign real balances, M_t^*/P_t^*, exists and presents an identical formulation, so that

$$m_t^* = p_t^* - \alpha \, i_t^*. \tag{1.5}$$

1.6 The Spot Rate Fundamental Equation

By definition we can write the spot rate as follows

$$s_t \equiv p_t - p_t^* + q_t, \tag{1.6}$$

where q_t is the log of the *real* exchange rate, $q_t \equiv \ln(S_t P_t^*/P_t)$. Combining (1.6) with (1.4) and (1.5) we conclude that

$$s_t \equiv p_t - p_t^* + q_t = \alpha \, (i_t - i_t^*) + m_t - m_t^* + q_t,$$

an expression which we can also write as

$$s_t = \alpha \, (i_t - i_t^*) + v_t, \tag{1.7}$$

where v_t denotes the exchange rate "*fundamental*" variable,

$$v_t \equiv m_t - m_t^* + q_t.$$

Hence, substituting the expression for the interest rate differential presented in equation (1.3) into equation (1.7), we obtain the following forward-looking equilibrium condition for the spot rate

$$s_t = v_t + \alpha \left(\bar{E}_t^1 \left[\, \tilde{s}_{t+1} \, \right] - s_t \right) - \frac{\alpha}{\nu_t} \, z_t. \tag{1.8}$$

In a stationary equilibrium the conditional precisions of $\tilde{s}_{t+1}$ of our dealers are constant over time ($\forall t\ \pi^d_{s,t} = \pi^d_s$) and so is the aggregate trading intensity of the FX dealers ($\forall t\ \nu_t = \nu = \int_0^1 \tau_{d'}\, \pi^d_s\, dd'$). Then, proceeding via iterated substitutions, we find from (1.8) that

$$s_t \;=\; \frac{1}{1+\alpha}\left(v_t \;-\; \frac{\alpha}{\nu}\, z_t \;+\; \sum_{k=1}^{\infty}\left(\frac{\alpha}{1+\alpha}\right)^k \left(\bar{E}^k_t\left[\tilde{v}_{t+k}\right] \;-\; \frac{\alpha}{\nu}\,\bar{E}^k_t\left[\tilde{z}_{t+k}\right]\right)\right), \quad (1.9)$$

where $\bar{E}^k_t[\tilde{v}_{t+k}]$ is the *order* k weighted average expectation across all FX dealers of period $t+k$ "fundamental" variable, $\tilde{v}_{t+k}$,

$$\bar{E}^k_t[\tilde{v}_{t+k}] \;=\; \bar{E}^1_t\, \bar{E}^1_{t+1} \ldots \bar{E}^1_{t+k-2}\, \bar{E}^1_{t+k-1}\left[\tilde{v}_{t+k}\right].$$

Similarly, $\bar{E}^k_t[\tilde{z}_{t+k}]$ is the *order* k weighted average expectation across all FX dealers of period $t+k$ supply of foreign currency, $\tilde{z}_{t+k}$.

Equation (1.9) clarifies why we have put the term fundamental in quotes. Indeed, differently from traditional monetary models of exchange rate determination, this variable does not exhaust all the factors which underpin the equilibrium value of the spot rate. Thus, the term v_t denotes the "traditional" fundamental variable of the monetary approach, while the total supply z_t indicates a "new" fundamental value. The former pertains to traditional nominal, $m_t - m^*_t$, and real, q_t, macroeconomic factors, while the latter captures the impact of the portfolio-balance effect related to risk-aversion and exchange rate risk.

To determine a closed form solution for the equilibrium spot rate we need to investigate the processes governing the dynamics of the fundamental variable $\tilde{v}_t$ and the total supply $\tilde{z}_t$, alongside the information the FX dealers possess on such processes.

1.7 The Fundamental Process

Employing the definition of the fundamental variable, $v_t \equiv m_t - m^*_t + q_t$, we can decompose the fundamental variable into two components, v^m_t and v^q_t, respectively function of the difference in the money supply and the real exchange rate. The former, v^m_t, reflects domestic and foreign monetary policies, while the latter, v^q_t, is subject to real perturbations associated with demand and supply shocks, such as technological innovations, fiscal stimuli, taste changes and so on.[5]

[5]Studies of the impact of macro announcements on exchange rates (notably Hardouvelis (1985), Ito and Roley (1987), Goodhart (1992) and Andersen, Bollerslev, Diebold, and Vega (2003)) indicate that news releases on several macro aggregates, such as money supply, industrial production, trade balance and employment level,

In the formulation we study we assume that the *monetary* fundamental component v_t^m follows a white noise process ($\tilde{v}_t^m \sim \text{NID}(0, \sigma_m^2)$), whereas the (*real*) fundamental component v_t^q is generated by a random walk process ($v_t^q = v_{t-1}^q + \epsilon_t^q$, with $\tilde{\epsilon}_t^q \sim \text{NID}(0, \sigma_q^2)$).[6]

1.8 Customer Order Flow

Customers provide all order flow in the FX market. Customers are mostly formed by the financial arms of industrial corporations and by other unsophisticated commercial and financial traders, whose FX transactions are due to liquidity needs and are not motivated by movements in exchange rates. Their order flow may be associated with current account transactions, such as trade in goods and services, transfers of capital income, public and private unilateral transfers of funds, or with capital movements, such as foreign direct and portfolio investment. We suppose that these customers correspond to *liquidity* traders and that their orders are neither price-sensitive, nor linked to innovations in the fundamental variable v_t. In synthesis, we assume that their aggregate market order in day t will amount to pure *noise trading*, u_t (where $\tilde{u}_t \sim \text{NID}(0, \sigma_u^2)$).

Beside this population of unsophisticated customers, we assume that some sophisticated traders, *hedge funds*, may have access in day t to some fundamental information, i.e. some signal on future realizations of the fundamental variable, v_{t+k}, or of its components, v_{t+k}^m and v_{t+k}^q. These informed traders will act rationally and trade in the FX market on their private information in order to gain speculative profits.[7] This means that in day t order flow, o_t, is the sum of noise, u_t, and informed trading, I_t.

2 Private Information on Real Fundamental Innovations

As Evans and Lyons (2005) have noticed, order flow anticipates news surprises in macroeconomic variables such as output growth, inflation and money supply growth. We then suppose that in day t, before an auction is called, a single trader (*hedge fund*), observes next

condition exchange rates, confirming that among the determinants of currency values we need to list both real and monetary factors.

[6] A broader specification would require that the monetary component v_t^m follows an AR(1) process.

[7] Indeed, recently some financial institutions, mainly hedge and currency funds, have been allowed to complete transactions directly on the EBS trading platform. In addition, these financial institutions may access these centralized trading platforms via FX brokers.

day innovation, ϵ^q_{t+1}, in the *real* fundamental component, v^q_t.[8] We assume that this trader acts strategically, can only submit market orders to the centralized trading platform and trades in order to maximize the expected profits she obtains from her trading activity.[9] Since the economic value of her informational advantage is limited to one day of trading, the horizon of her optimal trading strategy corresponds to one period (day).

2.1 RE Equilibria with Private Information on Fundamental Innovations

We can now characterize stationary, linear equilibria.

Proposition 1 *In a stationary, linear RE equilibrium in day t the spot rate satisfies the following relation*

$$s_t \;=\; \lambda_m \, v^m_t \;+\; \lambda_o \, o_t \;+\; \lambda_q \, v^q_t \;+\; \lambda_z \, z_t, \tag{2.1}$$

where

$$\lambda_m \;=\; \frac{\alpha - 1}{\alpha(1+\alpha)}\,, \quad \lambda_o \;=\; -\left(\frac{\alpha-1}{4(1+\alpha)}\right)^{1/2}\left(\frac{\sigma^2_q}{\sigma^2_u}\right)^{1/2}, \quad \lambda_q \;=\; 1\,, \quad \lambda_z \;=\; -\frac{\alpha}{\nu}\,,$$

while the market order of the informed trader, I_t, is equal to

$$I_t \;=\; -\,\theta_m \, v^m_t \;-\; \theta_q \, \epsilon^q_{t+1}, \tag{2.2}$$

where

$$\theta_m \;=\; \frac{2}{\alpha}\,\frac{1}{1+\alpha}\left(\frac{1+\alpha}{\alpha-1}\right)^{1/2}\left(\frac{\sigma^2_u}{\sigma^2_q}\right)^{1/2}, \quad \theta_q \;=\; \left(\frac{1+\alpha}{\alpha-1}\right)^{1/2}\left(\frac{\sigma^2_u}{\sigma^2_q}\right)^{1/2}.$$

Proof. See Appendix.

An important Corollary of Proposition 1 is the following.

Corollary 1 *Stationary, linear RE equilibria exist* only if $\alpha > 1$.

[8] At a later stage, Section 6, we will analyze a different specification in which private information pertains to the *monetary* fundamental component.

[9] It would somewhat complicate things to assume that the informed trader is allowed to submit limit orders.

Corollary 1 indicates that stationary linear equilibria may exist *only* when the semi-elasticity of money demand to the interest rate is large enough. Technically this ensures that the second order condition of the maximization problem of the informed trader is met. The semi-elasticity of money demand, α, determines the sensibility of the spot rate to order flow. This means that for α small the informed trader can place large market orders in the centralized trading platform without moving much the spot rate against which she trades. Then, If $\alpha \leq 1$ the hedge fund finds it optimal to place infinite market orders destabilizing the FX market, which consequently crashes. One should however notice that the condition $\alpha > 1$, while *necessary*, is not *sufficient* to ensure the existence of stationary, linear RE equilibria. To see this we now turn to the analysis of the conditional variance of the spot rate.

2.2 The Conditional Variance of the Spot Rate

The coefficients λ's in the linear RE equilibria we characterized depend on the average trading intensity of the population of FX dealers, ν. This on turn depends on the conditional variance of the spot rate of our population of FX dealers. In a stationary equilibrium the conditional variance of next day spot rate of FX dealer d, $\sigma^2_{s+,t}$, is time invariant, so that we can write $\text{Var}\left[\tilde{s}_{t+1} \mid \Omega^d_t\right] = \sigma^2_{s+}$, $\forall t$.

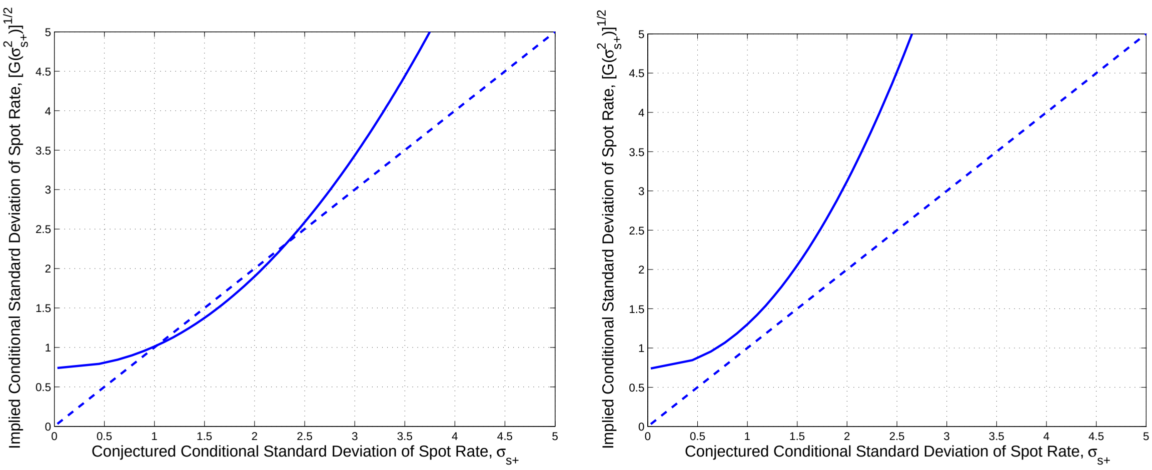
2.2.1 The Fixed Point Problem

The conditional variance of the spot rate, σ^2_{s+}, depend on the coefficients λ's of the stationary linear RE equilibria. These coefficients depend on the average trading intensity ν, which in turn depends on the conditional variance σ^2_{s+}. In other words, proving the existence of linear RE equilibria entails solving a fixed point problem.

To solve such a fixed point problem we define a mapping. Thus, one can *conjecture* that a given conditional variance for the spot rate, σ^2_{s+}, applies to all FX dealers, since they share symmetric information. Given this value one can derive the corresponding average trading intensity ν and the coefficients λ's. These allow to derive the *actual* conditional variance of the average dealer, ${\sigma^2_{s+}}'$. In synthesis, we have the following mapping

$$\sigma^2_{s+} \ \longrightarrow \ {\sigma^2_{s+}}' \ = \ G(\sigma^2_{s+}).$$

Figure 1. Steady state values of the conditional standard deviation



(a) Low Risk-aversion, $\gamma = 5$

(b) High Risk-aversion, $\gamma = 10$

Notes: The dashed line represents the conjectured conditional standard deviation for the generic FX dealer, σ_{s+}. The continuous line represents the implied conditional standard deviation, σ_{s+}'. This implied conditional standard deviation is obtained from the mapping $\sigma_{s+}' = [G(\sigma^2_{s+})]^{1/2}$ described in (2.4). In the plots an intersection between the continuous and dashed lines identifies a fixed point for the conditional standard deviation of the spot rate. Parameter values are $\alpha = 1.5$, $\sigma^2_m = 0.5$, $\sigma^2_q = 1$, $\sigma^2_u = 0.000256$.

Fixed points for this mapping,

$$\sigma^{2\ \star}_{s+} \quad = \quad G(\sigma^{2\ \star}_{s+}), \tag{2.3}$$

identify RE equilibria for the FX market.

Indeed, given a conjectured value σ^2_{s+}, using the expressions for the coefficients λ's, one can prove that

$$\sigma^{2\ \prime}_{s+} \quad = \quad a \; + \; b\,\sigma^2_{s+} \; + \; c\,(\sigma^2_{s+})^2, \tag{2.4}$$

where a, b and c are positive constants which depend on the deep parameters of the model, i.e. the semi-elasticity of money demand to the interest rate, α, the average coefficient of risk aversion, γ, the variance of the monetary fundamental component, σ^2_m, of the real fundamental component, σ^2_q, and of the noise trading component of order flow, σ^2_u.

In Figure 1 we plot, in the $(\sigma_{s+}, \sigma_{s+}')$ space, the mapping between the conjectured and implied conditional standard deviation of next day spot rate, alongside the 45 degree straight line for two different sets of parameter values. The intersections between the two lines identify fixed points for the mapping G.

Thus, the right panel of Figure 1 shows that stationary, linear RE equilibrium may not exist, as no intersection is found between the continuous and dashed lines. Numerical analysis shows that typically RE equilibria do *not* exist when: (i) as shown in the Figure, the FX dealers are particularly risk-averse (γ large); (ii) severe adverse selection conditions prevail in the FX market, in that the hedge fund possesses a large informational advantage (as σ_q^2, the variance of the real fundamental innovation, is large); and (iii) the signal the FX dealers receive from order flow is particulary noisy (as σ_u^2, the volatility of the liquidity component of order flow, is large).

Moreover, the left panel of Figure 1 shows that when intersections between the continuous and the dashed lines exist they usually come in couples, suggesting potential indeterminacy of the equilibrium. However, a graphical investigation indicates that when two different RE equilibria exist only one (that characterized by a smaller value for the conditional variance) is dynamically stable, so that the issue of multiplicity of the equilibria may be less severe that it may appear at *prime facie*.

2.3 The Characteristics of the Stationary, Linear RE Equilibria

Inspection of equation (2.1) proposes interesting empirical implications which are borne out by recent empirical research on the microstructure of FX markets. Thus, imbalances between buy and sell orders possess a large and persistent impact on currency values as confirmed among others by Lyons (1995), Evans and Lyons (2002), Payne (2003) and Berger *et al.* (2005). In addition, equation (2.1) implies that the spot rate and *cumulative* order flow are cointegrated, as suggested empirically by Biønnes and Rime (2005). Finally, the spot rate turns out to be a weak predictor of future fundamental values, as found by Engel and West (2005).

Inspection of the equilibrium coefficients, λ's, also offers some useful insights on the properties of the equilibrium relation. Thus, the fundamental coefficients λ_m and λ_q are positive. This is not surprising. Indeed, an increase in v_t^m corresponds to a rise in the relative money supply, $m_t - m_t^*$, or equivalently in the interest rate differential, $i_t^* - i_t$. Since a rise in the interest rate differential corresponds to an increase in the excess return on the foreign currency, the spot rate, s_t, appreciates when the monetary fundamental variable, v_t^m, augments. Similarly, an increase in v_t^q corresponds to an increase in the real value of the foreign currency, whose nominal value then rises by an equal amount.

The cumulative order flow coefficient, λ_z, is negative because an increase in the supply of the foreign currency depresses its value via the portfolio-balance effect. In fact, the FX dealers will be willing to hold a larger quantity of the foreign currency only if they are compensated for the increased risk they bear. Thus, a larger z_t forces a depreciation of the foreign currency as this corresponds to a larger excess return the FX dealers expect from holding foreign bonds.

The order flow coefficient, λ_o, is also negative in that order flow possesses an information content. In practice, an excess of sell orders might correspond to an impending negative fundamental shock, $\epsilon_{t+1}^q < 0$, and hence induces the FX dealers to expect an exchange rate depreciation. Consequently, they will be willing to hold the same amount of the foreign currency only if a reduction in s_t re-establishes the expected excess return foreign bonds yield.

3 The Impact of Private Information on the FX Market

Having identified the stationary, linear RE equilibria of the model, we are now in the position to study the impact of private information on FX markets.

3.1 The Information Content of Order Flow and the Value of Information

The following Proposition reveals how much of the private information the hedge fund possesses is impounded into the spot rate in equilibrium and what the money value of her informational advantage is.

Proposition 2 *When stationary, linear RE equilibria exist, the conditional precision of the generic FX dealer, d, of next day real fundamental innovation, ϵ_{t+1}^q, is equal to*

$$\pi_{q,1} \;\equiv\; Var\left(\tilde{\epsilon}_{t+1}^q \mid \Omega_t^d\right)^{-1} \;=\; \frac{2\alpha}{\alpha - 1}\,\pi_q\,,$$

while the unconditional expected profits of the informed trader are

$$E\,[\,\tilde{\Pi}_t\,] \;=\; \frac{1}{2}\left[\sigma_q^2 \;+\; \frac{4}{\alpha^2(1+\alpha)^2}\,\sigma_m^2\right]\left(\frac{1+\alpha}{\alpha-1}\right)^{1/2}\left(\frac{\sigma_u^2}{\sigma_q^2}\right)^{1/2}.$$

Proof. See Appendix.

This Proposition clearly indicates that the conditional precision of the generic FX dealer of next day real fundamental innovation more than doubles as consequence of the information he extracts from observing order flow. The information the FX dealers extract from order flow is the result of the informational content of the informed trader's market order given in equation (2.2).

In selecting her optimal trading strategy the hedge fund balances the trade-off between two different forces. On the one hand, a larger market order on her part yields, *ceteris paribus*, larger speculative profits. On the other hand, a larger market order reveals more information on next day fundamental innovation. Thus, the price at which the hedge fund will trade will be less convenient. Since the informed trader acts strategically, she will balance these two forces. As Proposition 2 indicates, she will eventually find it optimal to consume more than 50% of her informational advantage.

Interestingly, her unconditional expected profits depend not only on her informational advantage, given by the variance of the real fundamental innovation σ_q^2, and on the volume of noise trading in the FX market which jams the signal observed by the FX dealers, σ_u^2, but also on the semi-elasticity of money demand to the interest rate, α. Such semi-elasticity determines the slope, $|\lambda_o|$, of the price function against which the hedge fund trades and hence conditions the trade-off she needs to balance.

3.2 Comparative Statics

We now turn to a comparative statics exercise which can shed more light on the link between private information, the trading process and price dynamics in FX markets, investigating how market conditions and agents' behavior depend on the parameters of the model. We start by looking at the profits gained by the hedge fund and the information the FX dealers extract from the trading process. Then, we study the characteristics of the FX market, such as its liquidity and volatility.

3.2.1 Market Efficiency and the Value of Private Information

By exploiting her informational advantage the informed trader gains speculative profits, while at the same time affecting the efficiency of the FX market given that her market order presents

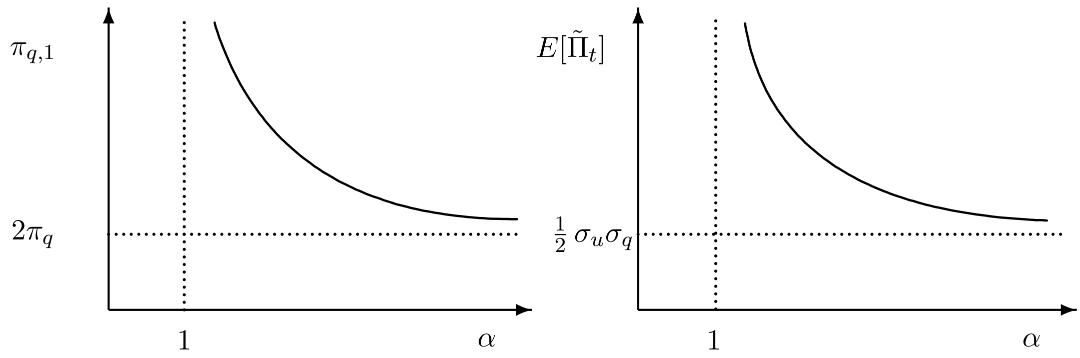


Figure 2. A graphical representation of the dependence of $\pi_{q,1}$ and $E[\tilde{\Pi}_t]$ on α.

an information content. The FX dealers by observing the flow of orders in the FX market extract part of the information contained into the informed trader's market orders. The information they are able to extract allows to reduce their uncertainty over future (real) fundamental innovations and hence increases the overall efficiency of the FX market.

The following Corollary is stark evidence of the interdependence between the private value of information and the efficiency of the FX market.

Corollary 2 *When stationary, linear RE equilibria exist, the conditional precision, $\pi_{q,1}$, of the generic FX dealer of next day real fundamental innovation, ϵ^q_{t+1}, is decreasing in the semi-elasticity of money demand, α, and in the volatility of the real fundamental innovations, σ^2_q. The unconditional expected profits of the informed trader, $E\,[\,\tilde{\Pi}_t\,]$, are decreasing in the semi-elasticity of money demand, α, and increasing in the volume of noise trading in the FX market, σ^2_u, and in the volatility of the real and monetary fundamental innovations, σ^2_m and σ^2_q.*

The intuition for the dependence of the unconditional expected profits of the informed trader on σ^2_u, σ^2_m and σ^2_q is straightforward. In particular, the larger the volume of noise trading in the FX market, σ^2_u, the more aggressively the informed trader can trade, as her market orders will be better hidden in the flow of orders. In addition, the larger the uncertainty of the FX dealers on next period real fundamental innovation, σ^2_q, and hence the larger the informational advantage of the hedge fund, the larger her speculative profits. A larger volatility of the monetary fundamental innovation, σ^2_m, instead generates a more volatile exchange rate and hence larger profits for the informed trader.

Corollary 2 also proposes an intriguing result, as it shows a positive dependence of the conditional precision and the unconditional expected profits on α (also outlined by the graphical representation in Figure 2). In addition, it is immediate to verify that for $\alpha \downarrow 1$, $\pi_{q,1}$ and $E[\tilde{\Pi}_t]$ converge to infinite, so that the informed trader consumes all her informational advantage, as she reveals ϵ_{t+1}^q to the population of FX dealers, while gaining infinite expected profits.[10]

Indeed, as already mentioned, the smaller the semi-elasticity of money demand, α, the less sensitive the spot rate to order flow. The informed trader is then induced to trade more aggressively and reveal more of her informational advantage while gaining larger expected profits. In the limit, as α approaches the area of non-existence of RE equilibria, that is as $\alpha \downarrow 1$, the slope of the price function, $|\lambda_o|$, against which the hedge fund trades is so low as to bring about infinite expected profits for the informed trader and a fully revealing RE equilibrium.

3.2.2 Exchange Rate Volatility and Market Liquidity

We now turn to the properties of two important indicators pertaining to the liquidity and volatility of the FX market. Let us start from the analysis of transaction costs in the FX market.

Corollary 3 *When stationary, linear RE equilibria exist, the absolute value of the order flow coefficient, $|\lambda_o|$, is an increasing function of the semi-elasticity of money demand, α, and the volatility of the real fundamental shock, σ_q^2, and a decreasing function of the volume of noise trading in the FX market, σ_u^2.*

The absolute value of the order flow coefficient, $|\lambda_o|$, determines the size of the price impact of order flow. Its inverse, the market depth, is a very common measure of liquidity. Indeed, the larger the market depth the larger the market order required to move the transaction price by one, and hence the larger the liquidity of the FX market. Then, Corollary 3 provides some interesting insights on the link between the impact of private information and market liquidity.

In particular, it shows how transaction costs are larger when more sever adverse selection conditions prevail. This happens when the hedge fund possesses a larger informational advantage, given by a larger variance of the real fundamental innovation, σ_q^2. In addition, $|\lambda_o|$ is larger when a smaller volume of noise trading is present in the FX market, ie. when σ_u^2 is

[10]On the contrary, for $\alpha \uparrow \infty$ we find that $\pi_{q,1}$ and $E[\tilde{\Pi}_t]$ converge respectively to $2\pi_q$ and $\frac{1}{2}\,\sigma_u\,\sigma_q$.

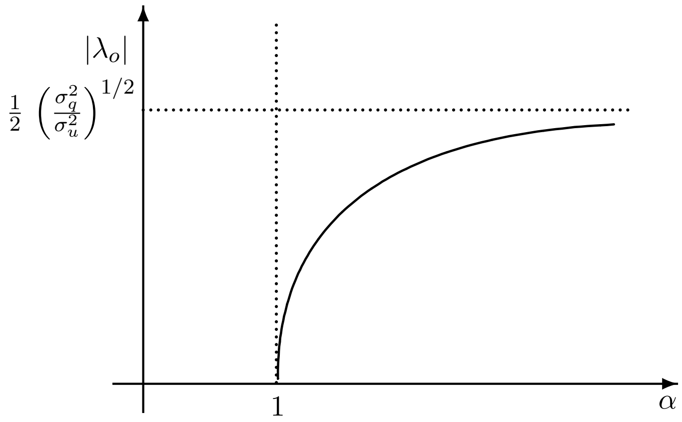


Figure 3. A graphical representation of the dependence of $|\lambda_o|$ on α.

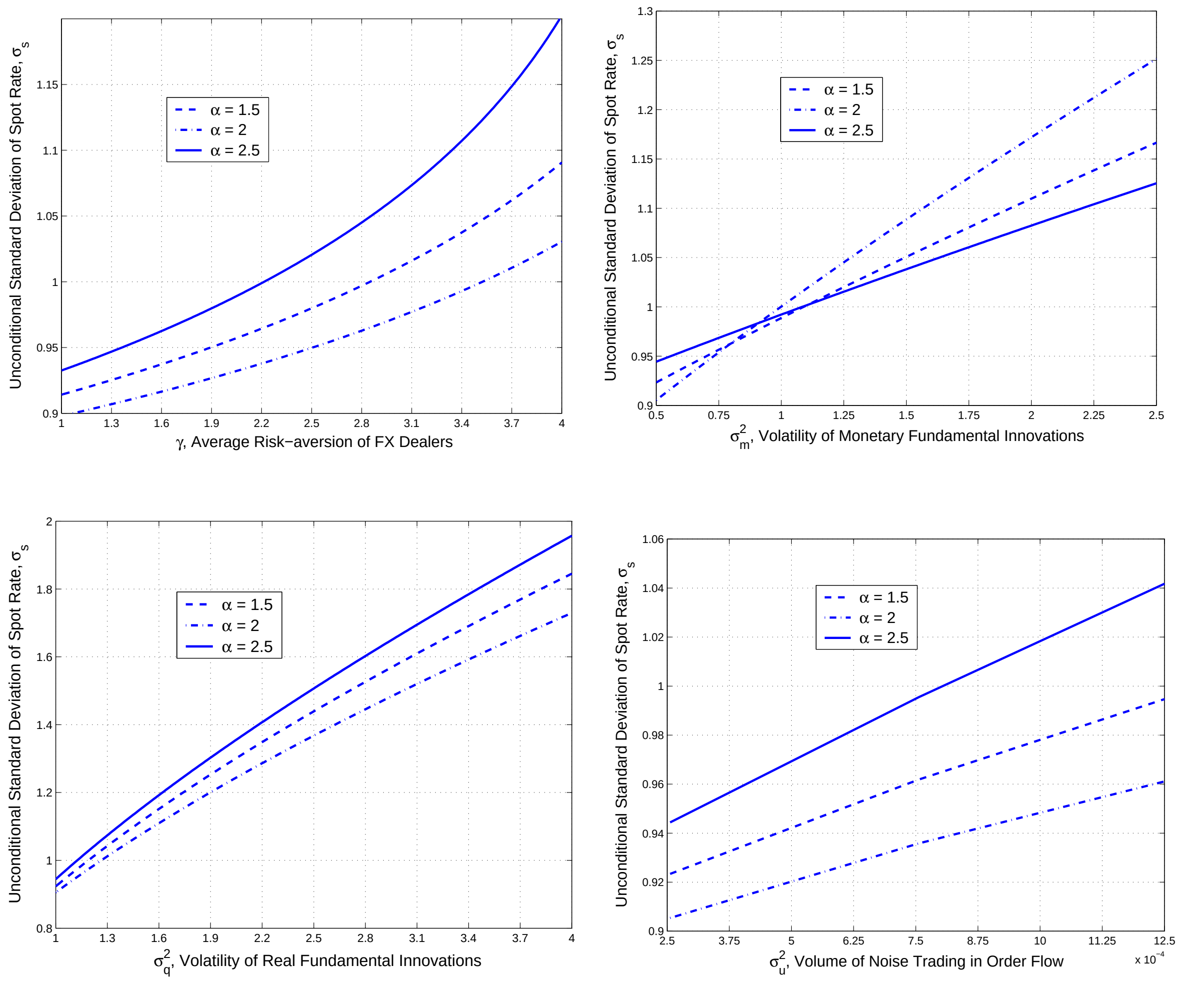
smaller. Under these circumstances, in fact, the losses the FX dealers incur from trading with the hedge fund can be diluted with a smaller population of liquidity traders. This implies that the FX dealers are forced to increase transaction costs.

Finally, transaction costs are larger for a larger value of α, the semi-elasticity of money demand. Notice, that this accompanies smaller efficiency of the FX market and smaller expected profits of the informed trader. This confirms that the equilibrium of the monetary market heavily influences the impact of private information on exchange rate dynamics and the liquidity and efficiency of the FX market. In this respect, the limit behavior of $|\lambda_o|$ is also telling (see Figure 3).

Thus, for $\alpha \downarrow 1$ $|\lambda_o|$ vanishes. Indeed, in the limit the market order selected by the informed trader presents an infinite size ($\theta_q \uparrow \infty$ in (2.2)), so that, as also witnessed by the conditional precision, $\pi_{q,1}$ (for $\alpha \downarrow 1$ $\pi_{q,1} \uparrow \infty$), she consumes all her informational advantage. Then, since in equilibrium asymmetric information on the exchange rate fundamentals vanishes, the order flow coefficient collapses to zero, $\lambda_o \uparrow 0$.

The final piece of comparative statics exercise we conduct concerns exchange rate volatility. In this respect, we notice that an analytical expression for the unconditional variance of the spot rate innovation, $\mathrm{Var}\left[\tilde{s}_{t+1} - \tilde{s}_t\right]$, can be obtained from the equilibrium relation for the spot rate in equation (2.1). This expression is so convoluted that its dependence on the parameters of the model is best analyzed numerically.

Figure 4. Comparative statics: Exchange rate volatility



Notes: In the panels we plot the unconditional standard deviation of the spot rate, σ_s, against the semi-elasticity of money demand, α, for different values of the measure of average risk-aversion of the FX dealers, γ (left-upper panel), the volatility of the monetary shocks, σ_m^2 (right-upper panel), the volatility of the real shocks, σ_q^2 (left-bottom panel), and the volume of noise trading, σ_u^2 (right-bottom panel). Benchmark parameter values are $\gamma = 1.25$, $\sigma_m^2 = 0.5$, $\sigma_q^2 = 1$, $\sigma_u^2 = 0.000256$.

Figure 4 reproduces the dependence of the unconditional standard deviation of the spot rate innovation, $\text{Var}[\tilde{s}_{t+1} - \tilde{s}_t]$, on the risk-aversion of the FX dealers, γ, the volatility of the monetary and real shocks, σ_m^2 and σ_q^2, and the volume of noise trading, σ_u^2, for three different values of the semi-elasticity of money demand, α.

From the panels represented in Figure 4 we clearly conclude that the volatility of the exchange rate is increasing in the risk aversion of the FX dealers, γ (left-upper panel), the volatility of the monetary shocks, σ_m^2 (right-upper panel), the volatility of the real shocks, σ_q^2 (left-bottom panel), and the volume of noise trading, σ_u^2 (right-bottom panel). No clear pattern instead appears to exist with respect the semi-elasticity of money demand, α.

The portfolio-balance effect explains the dependence of exchange rate volatility with respect to γ and σ_u^2. Thus, more risk-averse FX dealers will move more the exchange rate to accommodate a given portfolio shift. Likewise, a larger volume of noise trading in the FX market induces a more volatile exchange rate, in that FX dealers will accommodate ampler portfolio-shifts with larger movements in the price of the foreign currency.

As for the dependence of the volatility of the exchange rate with respect to σ_m^2 one should notice that a larger volatility of the monetary shocks affects the spot rate mainly true a direct dependence of s_t on v_t^m in equation (2.1). The dependence of the volatility of the exchange rate on σ_q^2 hinges on a similar mechanism of transmission.

4 Noisy Private Information

A simple extension of our benchmark formulation entails that the informed trader only observes a noisy signal on the impending real fundamental shock. Therefore, let us assume that before an auction is called in day t the hedge fund observes the signal $\xi_t^q = \epsilon_{t+1}^q + \eta_t^q$ (with $\tilde{\eta}_t^q \sim N(0, \sigma_n^2)$).

4.1 RE Equilibria with Noisy Private Information

With noisy private information the following Proposition applies.

Proposition 3 *With noisy private information, in a stationary, linear RE equilibrium in day t the spot rate satisfies the equilibrium relation in equation (2.1) where*

$$\lambda_m \;=\; \frac{\alpha-1}{\alpha(1+\alpha)}\,, \quad \lambda_o \;=\; -\left(\frac{\alpha-1}{4(1+\alpha)}\right)^{1/2}\left(\frac{\phi_n\,\sigma_q^2}{\sigma_u^2}\right)^{1/2}, \quad \lambda_q \;=\; 1\,, \quad \lambda_z \;=\; -\,\frac{\alpha}{\nu}$$

and $\phi_n = \sigma_q^2/(\sigma_n^2+\sigma_q^2)$, while the market order of the informed trader, I_t, is equal to

$$I_t \;=\; -\,\theta_m\,v_t^m \;-\; \phi_n\,\theta_q\,\xi_t^q, \tag{4.1}$$

where

$$\theta_m \;=\; \frac{2}{\alpha}\,\frac{1}{1+\alpha}\,\left(\frac{1+\alpha}{\alpha-1}\right)^{1/2}\left(\frac{\sigma_u^2}{\phi_n\,\sigma_q^2}\right)^{1/2}, \qquad \theta_q \;=\; \left(\frac{1+\alpha}{\alpha-1}\right)^{1/2}\left(\frac{\sigma_u^2}{\phi_n\,\sigma_q^2}\right)^{1/2}.$$

Proof. See Appendix.

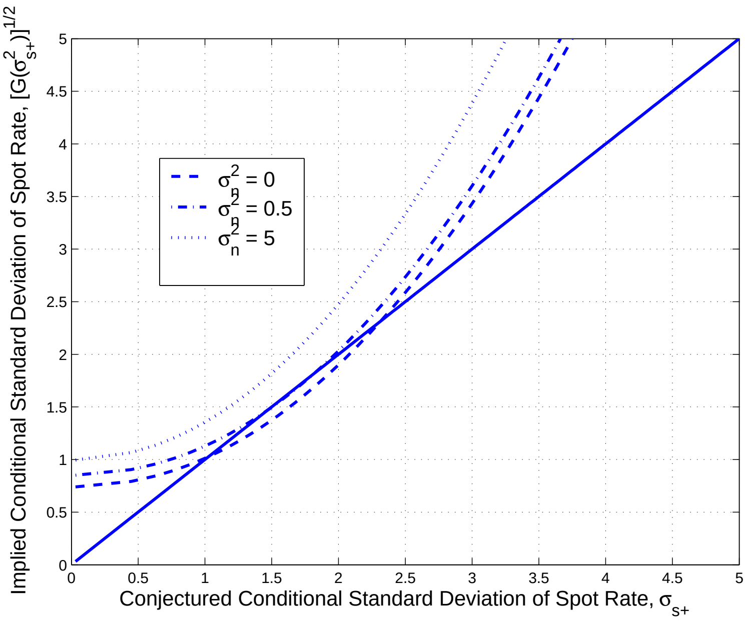
From the statement of Proposition 3 one immediately sees that Corollary 1 still applies, in that stationary linear RE equilibria can exist only if $\alpha > 1$. For $\alpha > 1$, fixed points for a mapping analogous to that reported in equation (2.3) identify stationary, linear RE equilibria for the FX market.

Figure 5 plots, in the $(\sigma_{s+}, \sigma_{s+}')$ space, the mapping between the *conjectured* and *implied* conditional standard deviation of next day spot rate, alongside the 45 degree straight line, for three different values of σ_n^2. From this we see once again that there can be either one, or two or even no stationary, linear RE equilibrium.

If we focus on dynamically stable stationary, linear RE equilibria we are able to outline a clear dependence between the quality of private signals and the conditional variance of the spot rate. In particular, consider that for a given value of σ_n^2 a dynamically stable stationary, linear RE equilibrium corresponds to the *smaller* conditional standard deviation for which an intersection between the continuous and dashed lines occurs in Figure 5. Thus, we see that the poorer the quality of the signal observed by the informed trader (ie. the larger σ_n^2), the larger the conditional standard deviation of next day spot rate, σ_{s+}. This is because FX dealers will be able to extract less precise information from the order flow they observe. Indeed, for σ_n^2 large ($\sigma_n^2 = 5$) we see that actually no stationary, linear RE equilibrium exists.

One can immediately derive results analogous to those reported in Proposition 2.

Figure 5. Steady state values of the conditional standard deviation



Notes: The continuous line represents the conjectured conditional standard deviation for the generic FX dealer, σ_{s+}. The dashed lines represent the implied conditional standard deviation, σ_{s+}', obtained for different values of σ_n^2, the amount of noise in the signal on the impending real fundamental innovation the informed trader observes in day t. In the plot an intersection between the continuous and dashed lines identifies a fixed point for the conditional standard deviation of the spot rate. Parameter values are $\alpha = 1.5$, $\gamma = 5$, $\sigma_m^2 = 0.5$, $\sigma_q^2 = 1$, $\sigma_u^2 = 0.000256$.

Proposition 4 *With noisy private information, when stationary, linear RE equilibria exist, the conditional precision of the generic FX dealer, d, of next day real fundamental innovation,* ϵ_{t+1}^q, *is equal to*

$$\pi_{q,1} \;=\; \frac{2\alpha\,(\pi_n \,+\, \pi_q)}{(\alpha - 1)\,\pi_n \,+\, 2\alpha\,\pi_q}\;\pi_q\,,$$

where $\pi_n = 1/\sigma_n^2$, *while the unconditional expected profits of the informed trader are*

$$E\,[\,\tilde{\Pi}_t\,] \;=\; \frac{1}{2}\left[\phi_n\,\sigma_q^2 \;+\; \frac{4}{\alpha^2(1+\alpha)^2}\;\sigma_m^2\right]\left(\frac{1+\alpha}{\alpha-1}\right)^{1/2}\left(\frac{\sigma_u^2}{\phi_n\,\sigma_q^2}\right)^{1/2}\,.$$

Proof. See Appendix.

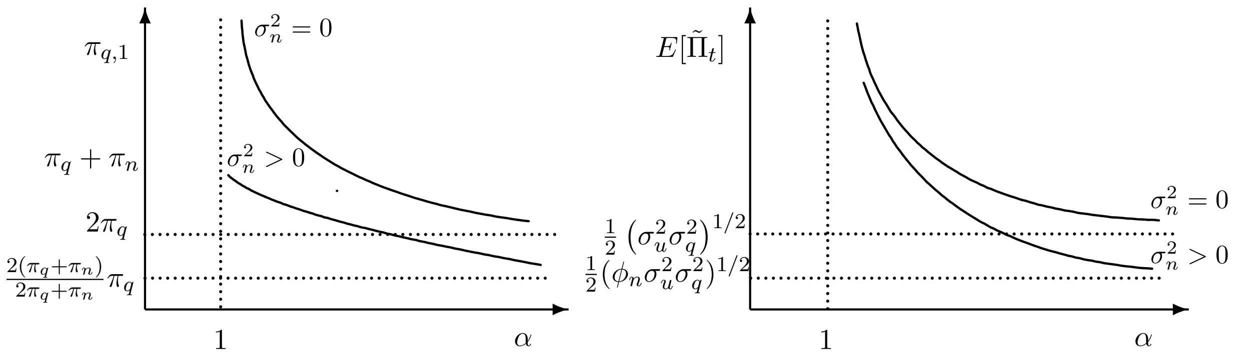


Figure 6. A graphical representation of the dependence of $\pi_{q,1}$ and $E[\tilde{\Pi}_t]$ on σ_n^2.

With respect to the results outlined in Proposition 2, the major difference is that it is no longer the case that for all parameter values the conditional precision of the generic FX dealer of next day real fundamental innovation more than doubles as consequence of the information he extracts from observing order flow.[11] Indeed, the informational content of the informed trader's market order given in equation (4.1) depends crucially on the precision of her signal, π_n. The poorer the quality of her signal, the less aggressive the hedge fund, as shown by the value of ϕ_n in equation (4.1) ($\phi_n = \pi_n/(\pi_n + \pi_q)$), and the less informative her market order.

We now turn to a comparative statics exercise.

4.2 Comparative Statics

With respect to the interdependence between the private value of information and the efficiency of the FX market, Corollary 2 still holds. In addition we have the following result.

Corollary 4 *With noisy private information, when stationary, linear RE equilibria exist, the conditional precision, $\pi_{q,1}$, and the unconditional expected profits, $E[\tilde{\Pi}_t]$, are decreasing in σ_n^2, the amount of noise in the signal observed by the informed trader.*

[11]Only if $\pi_n > \alpha\pi_q$ such a result is maintained. In other words, if the signal observed by the hedge fund is fairly accurate, the uncertainty of the average FX dealer over the impending real fundamental innovation decreases significantly.

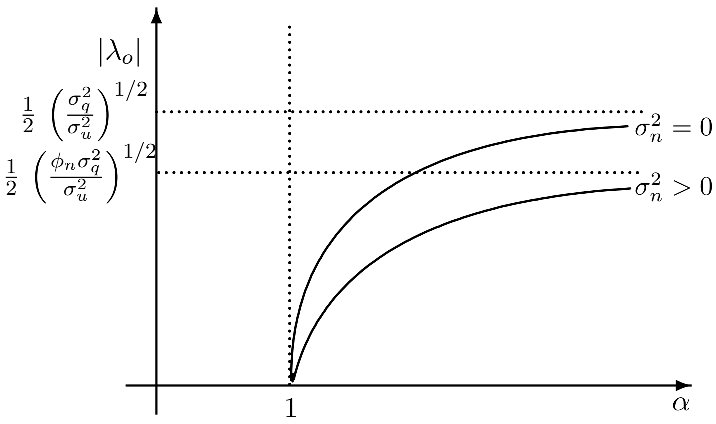


Figure 7. A graphical representation of the dependence of $|\lambda_o|$ on σ_n^2.

Unsurprisingly, more noise in the signal observed by the hedge fund reduces her speculative profits alongside the information extracted by the FX dealers from the order flow they observe, as also graphically shown in Figure 6.[12]

With respect to the liquidity properties of the FX market Corollary 3 is still valid. However, the following Corollary of Proposition 3 integrates its results.

Corollary 5 *With noisy private information, when stationary, linear RE equilibria exist, the absolute value of the order flow coefficient, $|\lambda_o|$, is a decreasing function of the amount of noise in the signal observed by the informed trader, σ_n^2.*

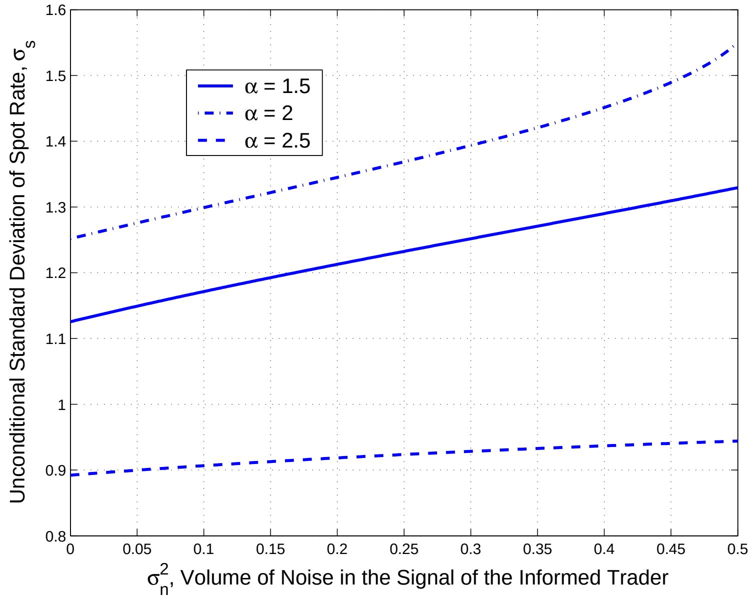
Corollary 5 indicates that more noise in the signals observed by the informed trader increases the liquidity of the FX market. This squares with the conclusion that with a larger σ_n^2 the informed trader will on average gain smaller speculative profits (compare Figure 7 with Figure 6). In other words, a less severe adverse selection problem for the FX dealers reduces transaction costs in the FX market.[13]

Once more, we conclude our comparative statics exercise with the analysis of exchange rate volatility. Thus, in Figure 8 we reproduce the dependence of the unconditional standard deviation of the spot rate innovation, $\text{Var}\left[\tilde{s}_{t+1} - \tilde{s}_t\right]$, on the amount of noise in the signal

[12]This Figure also shows that the limit behavior of the conditional precision and unconditional expected profits changes. Thus, for $\alpha \downarrow 1$ $\pi_{q,1} \uparrow \pi_q + \pi_n$, while $E[\tilde{\Pi}_t]$ converges to infinite. For $\alpha \uparrow \infty$ $\pi_{q,1}$ and $E[\tilde{\Pi}_t]$ converge respectively to $[2(\pi_q + \pi_n)/(2\pi_q + \pi_n)]\,\pi_q$ and $\frac{1}{2}\,(\phi_n\,\sigma_u^2\,\sigma_q^2)^{1/2}$.

[13]Notice that for $\alpha \downarrow 1$ we still find that $|\lambda_o|$ vanishes.

Figure 8. Comparative statics: Exchange rate volatility



Notes: We plot the unconditional standard deviation of the spot rate, σ_s, against the semi-elasticity of money demand, α, for different values of the volume of noise in the signal observed by the informed trader, σ_n^2. Parameter values are $\gamma = 5$, $\sigma_m^2 = 0.5$, $\sigma_q^2 = 1$, $\sigma_u^2 = 0.000256$.

observed by the informed trader, σ_n^2, for three values of the semi-elasticity of money demand, α.

Figure 8 clearly shows that the poorer the signal observed by the informed trader, the more volatile the exchange rate. Notice that a larger σ_n^2 implies that the informational advantage of the hedge fund is smaller. Then, the adverse selection mechanism which makes the spot rate react to trade imbalances is less severe. This should reduce its volatility. On the other hand, a less informative order flow makes future fundamental innovations more uncertainty making the foreign currency riskier and hence more volatile. Figure 8 indicates that the latter effect prevails.

5 Multiple Privately Informed Traders

A second extension of our benchmark formulation we consider entails that in day t $L+1$ (with $L \geq 1$) non-competitive informed traders (hedge funds) observe the impending real fundamental innovation, ϵ^q_{t+1}.

5.1 RE Equilibria with Multiple Privately Information Traders

With $L+1$ informed traders the following Proposition applies.

Proposition 5 *With $L+1$ privately informed traders, in a symmetric and stationary, linear RE equilibrium, in day t the spot rate satisfies the equilibrium relation in equation (2.1) where*

$$\lambda_m = \frac{\alpha - (1+L)}{\alpha(1+\alpha)}, \quad \lambda_o = -\left(\frac{(1+L)[\alpha-(1+L)]}{(2+L)^2(1+\alpha)}\right)^{1/2}\left(\frac{\sigma_q^2}{\sigma_u^2}\right)^{1/2}, \quad \lambda_q = 1, \quad \lambda_z = -\frac{\alpha}{\nu},$$

while the market order of one of the generic informed trader, I_t^l, is equal to

$$I_t^l = -\theta_m^{L+1} v_t^m - \theta_q^{L+1} \epsilon_{t+1}^q, \quad \textit{where} \tag{5.1}$$

$$\theta_m^{L+1} = \frac{2+L}{\alpha(1+\alpha)}\left(\frac{1+\alpha}{(1+L)[\alpha-(1+L)]}\right)^{1/2}\left(\frac{\sigma_u^2}{\sigma_q^2}\right)^{1/2},$$

$$\theta_q^{L+1} = \left(\frac{1+\alpha}{(1+L)[\alpha-(1+L)]}\right)^{1/2}\left(\frac{\sigma_u^2}{\sigma_q^2}\right)^{1/2}.$$

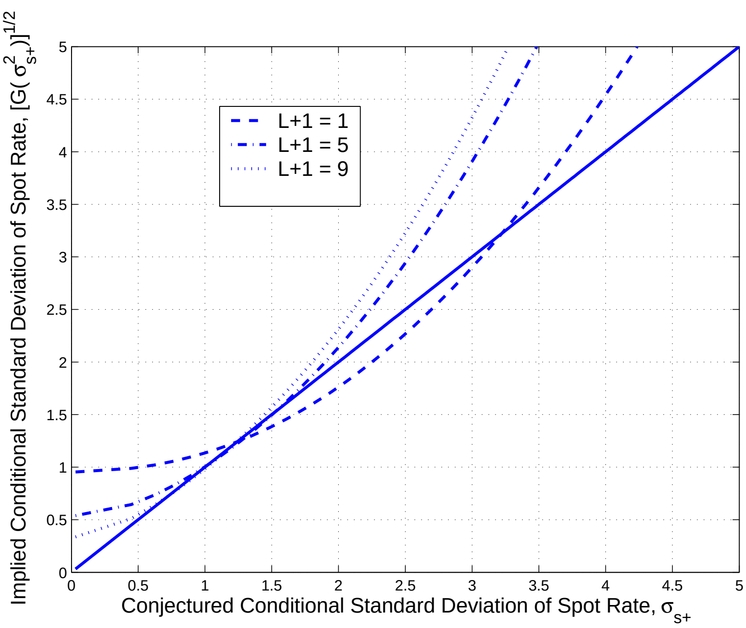
Proof. See Appendix.

An important Corollary of Proposition 5, which adjourns Corollary 1, is the following.

Corollary 6 *When $L+1$ informed traders operate in the FX market, symmetric and stationary, linear RE equilibria exist* only if $\alpha > 1+L$.

With respect to the specification with one informed trader, Corollary 6 imposes a more stringent *necessary* condition for the existence of a RE equilibrium. Indeed, with $L+1$ hedge funds, there is more informed trading in the FX market and hence a larger risk of destabiliza-

Figure 9. Steady state values of the conditional standard deviation



Notes: The continuous line represents the conjectured conditional standard deviation for the generic FX dealer, σ_{s+}. The dashed lines represent the implied conditional standard deviation, ${\sigma_{s+}}'$, obtained for different values of $L+1$, the number of informed traders. In the plot an intersection between the continuous and dashed lines identifies a fixed point for the conditional standard deviation of the spot rate. Parameter values are $\alpha = 10$, $\gamma = 1$, $\sigma_m^2 = 0.5$, $\sigma_q^2 = 1$, $\sigma_u^2 = 0.000256$.

tion. This is not surprising, as when adverse selection increases market activity becomes more precarious.

For $\alpha > 1 + L$, a *sufficient* condition to identify symmetric and stationary, linear RE equilibria for the FX market is to find fixed points for the mapping, analogous to that reported in equation (2.3), between the *conjectured* and *implied* conditional variance of one period ahead spot rate. Figure 9 plots, in the $(\sigma_{s+}, {\sigma_{s+}}')$ space, such a mapping, alongside the 45 degree straight line, for three different values of $L+1$.

Focusing on dynamically stable RE equilibria, a clear dependence between the number of informed traders and the conditional variance of the spot rate appears. Thus, we see that the larger the number of hedge funds (ie. the larger $L+1$), the smaller the conditional standard deviation of next day spot rate. This is because, given the greater competition between the informed traders, the FX dealers will be able to extract more precise information from the order flow they observe.

One can immediately derive results analogous to those reported in Proposition 2.

Proposition 6 *With $L+1$ informed traders, when symmetric and stationary, linear RE equilibria exist, the conditional precision of the generic FX dealer, d, of next day real fundamental innovation, ϵ^q_{t+1}, is equal to*

$$\pi_{q,1} \;=\; \frac{\alpha\,(2\,+\,L)}{\alpha - (1\,+\,L)}\,\pi_q\,,$$

while the unconditional expected profits of informed trader l are

$$E\,[\,\tilde{\Pi}^l_t\,] \;=\; \left(\frac{1+\alpha}{(1+L)[\alpha-(1+L)]}\right)^{1/2}\left[\left(\frac{1}{2+L}\right)\sigma^2_q + \left(\frac{2+L}{\alpha^2(1+\alpha)^2}\right)\sigma^2_m\right]\left(\frac{\sigma^2_u}{\sigma^2_q}\right)^{1/2}.$$

Proof. See Appendix.

Proposition 6 suggests that the interplay between the information the FX dealers extract from order flow and the profits the hedge funds obtain from their superior information is conditioned by the number of informed traders, $L+1$. To study this dependence in full we now turn to a comparative statics exercise.

5.2 Comparative Statics

Whereas Corollary 2 withholds once again, the following Corollary of Proposition 5 proposes another important result with respect to the interdependence between the private value of information and the efficiency of the FX market.

Corollary 7 *With $L+1$ informed traders, when symmetric and stationary, linear RE equilibria exist, the conditional precision, $\pi_{q,1}$ of the generic FX dealer, d, of next day real fundamental innovation, ϵ^q_{t+1}, is an increasing function of $L+1$, the number of informed trader in the FX market.*

As also shown by the left panel of Figure 10, Corollary 7 indicates that with a greater number of informed traders, when RE equilibria exist, more information is revealed to the FX dealers. Cournot competition among hedge funds induces them to trade more aggressively on

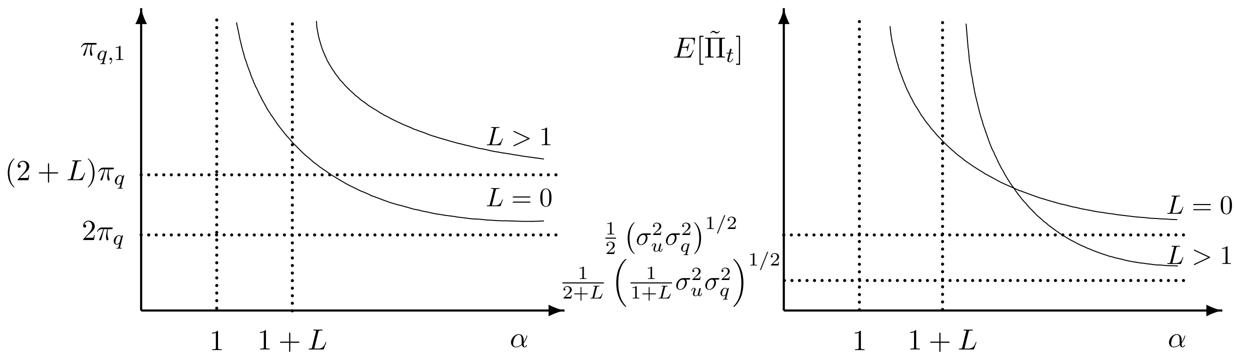


Figure 10. A graphical representation of the dependence of $\pi_{q,1}$ and $E[\tilde{\Pi}_t^l]$ on L.

their private information. This implies that order flow presents a more precise information content and that the conditional precision, $\pi_{q,1}$, of the FX dealers is larger.

On the other hand, as shown by the right panel in Figure 10, a greater number of informed traders does not necessarily implies that individually hedge funds gain smaller unconditional expected profits. We see that as $L+1$ approaches α $E[\tilde{\Pi}_t^l]$ converges, alongside $\pi_{q,1}$, to infinite. This is a striking result, as typically more competition among informed traders reduces the private value of information. However, from Figure 10 we see that for α small a larger number of informed traders, $L+1$, brings about an increase in $E[\tilde{\Pi}_t^l]$. This is coherent with Corollary 6, as when $L+1$ approaches α we reach an area of increasing fragility for the equilibrium of the FX market.

With respect to the liquidity properties of the FX market we see that Corollary still 3 applies. However, the following Corollary of Proposition 5 extends its conclusions.

Corollary 8 *With* $L+1$ *informed traders, when symmetric and stationary, linear RE equilibria exist, the absolute value of the order flow coefficient,* $|\lambda_o|$, *is a decreasing function of the number of informed traders,* $L+1$.

Corollary indicates that with multiple informed traders, when RE equilibria exist, the larger their number the larger the liquidity of the market. Once again, this is consistent with the finding that more information is revealed by order flow, when several hedge funds compete on the same piece of private information.

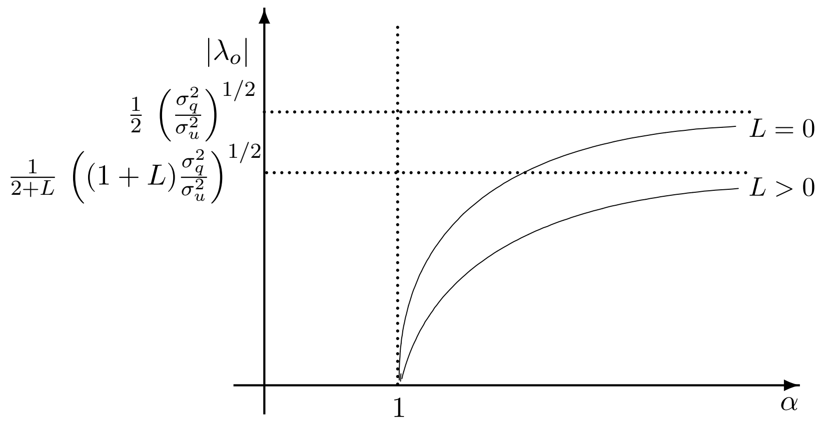


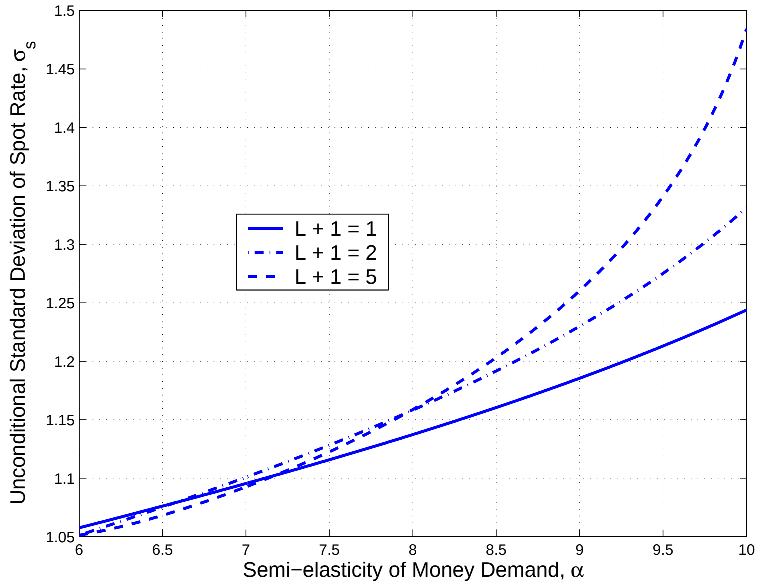
Figure 11. A graphical representation of the dependence of $|\lambda_o|$ on L.

Finally, consider exchange rate volatility. In Figure 12 we reproduce the dependence of the unconditional standard deviation of the spot rate innovation with respect to the semi-elasticity of money demand, α, for different values of the number of hedge funds, $L+1$.

Figure 12 shows that the dependence of the volatility on the number of hedge funds, $L+1$, is not monotonous. Thus, a larger number of informed traders induces a less volatile exchange rate for a small level of the semi-elasticity of money demand, whereas the opposite holds for larger values of α.

In sum, the presence of a larger number of informed traders in the FX market, which compete on the same piece of private information, proposes some striking results. In particular, while the equilibrium of the FX market becomes more fragile (for $\alpha \leq L+1$ no equilibrium exists and the FX market crashes), when a RE equilibrium emerges this is characterized by more efficiency and liquidity (the larger $L+1$ the larger the conditional precision, $\pi_{q,1}$, and the smaller the absolute value of the order flow coefficient, $|\lambda_o|$) and less volatility smaller short-term exchange rate volatility (the larger $L+1$ the larger the conditional variance of next spot rate, σ_{s+}^2). In other words, more informed traders increase the risk the market crashes, but at the same time, when the market operates, competition among hedge funds benefits market performance.

Figure 12. Comparative statics: Exchange rate volatility



Notes: We plot the unconditional standard deviation of the spot rate, σ_s, against the semi-elasticity of money demand, α, for different values of the number of hedge funds, $L+1$. Parameter values are $\gamma = 1$, $\sigma_m^2 = 0.5$, $\sigma_q^2 = 1$, $\sigma_u^2 = 0.000256$.

6 Private Information on Monetary Fundamental Innovations

We now briefly investigate an altogether different formulation of our model, where the informed trader observes a signal on the impending monetary innovation to the fundamental variable, v_{t+1}^m.

6.1 RE Equilibria with Private Information on Monetary Fundamentals

With private information on monetary fundamental innovations the following Proposition applies.

Proposition 7 *With private information on monetary fundamental innovations, in a stationary, linear RE equilibrium in day t the spot rate satisfies the equilibrium relation in equation (2.1) where*

$$\lambda_m \;=\; \frac{\alpha-1}{\alpha(1+\alpha)}\,, \quad \lambda_o \;=\; -\,\frac{1}{2(1+\alpha)}\,\left(\frac{(1+\alpha^2)(\alpha-1)}{\alpha^2(1+\alpha)}\right)^{1/2}\left(\frac{\sigma_m^2}{\sigma_u^2}\right)^{1/2}\,, \quad \lambda_q \;=\; 1\,, \quad \lambda_z \;=\; -\,\frac{\alpha}{\nu}\,,$$

while the market order of the informed trader, I_t, is equal to

$$I_t \;=\; -\,\theta_m\, v_t^m \;-\; \theta_{m+1}\, v_{t+1}^m, \tag{6.1}$$

where

$$\theta_m \;=\; 2\,(\alpha-1)^{1/2}\,\left(\frac{1+\alpha}{1+\alpha^2}\right)^{1/2}\left(\frac{\sigma_u^2}{\sigma_m^2}\right)^{1/2}, \qquad \theta_{m+1} \;=(\alpha-1)^{1/2}\,\left(\frac{1+\alpha}{1+\alpha^2}\right)^{1/2}\left(\frac{\sigma_u^2}{\sigma_m^2}\right)^{1/2}.$$

Proof. See Appendix.

Even in this specification Corollary 1 applies, in that stationary, linear RE equilibria can exist only if $\alpha > 1$. For $\alpha > 1$, fixed points for a mapping analogous to that reported in equation (2.3) identify stationary, linear RE equilibria for the FX market. As in previous specifications there can be either one, or two (left panel) or no stationary, linear RE equilibrium.

One can derive results analogous to those reported in Proposition 2.

Proposition 8 *With private information on monetary fundamental innovations, when stationary, linear RE equilibria exist, the conditional precision of the generic FX dealer, d, of next day monetary fundamental innovation, v_{t+1}^m, is equal to*

$$\pi_{m,1} \;\equiv\; Var\left(\tilde{v}_{t+1}^m \mid \Omega_t^d\right)^{-1} \;=\; \frac{2\alpha^2}{1+\alpha^2}\,\pi_m,$$

while the unconditional expected profits of the informed trader are

$$E\,[\,\tilde{\Pi}_t\,] \;=\; 2\,(1+\alpha)\,\left[\frac{\alpha^2\,(1+\alpha)}{(\alpha-1)(1+\alpha^2)}\right]^{1/2}\left[1 \;-\; \frac{4+(\alpha-1)^2}{4\alpha^2\,(1+\alpha^2)}\right]^{1/2}\left(\sigma_m^2\,\sigma_u^2\right)^{1/2}.$$

Proof. See Appendix.

With respect to the results outlined in Proposition 2, the major difference is that it is no longer the case that the conditional precision of the generic FX dealer of next day monetary fundamental innovation more than doubles as consequence of the information he extracts from the order flow he observes. Indeed, on the contrary we see that $\pi_m < \pi_{m,1} < 2\pi_m$ for $\alpha > 1$. In other words, if an informed trader possesses private information on an impending monetary shock, she finds it optimal to consume *less* than half of her informational advantage.

To shed more light on the differences with the properties of the benchmark specification we now turn to a brief comparative statics exercise.

6.2 Comparative Statics

The following Corollary outlines the interdependence which now exists between the private value of information and the efficiency of the FX market.

Corollary 9 *With private information on monetary fundamental innovations, when stationary, linear RE equilibria exist, the conditional precision of the generic FX dealer, $\pi_{m,1}$, of next day monetary fundamental innovation, v^m_{t+1}, is an increasing function of the semi-elasticity of money demand, α.*

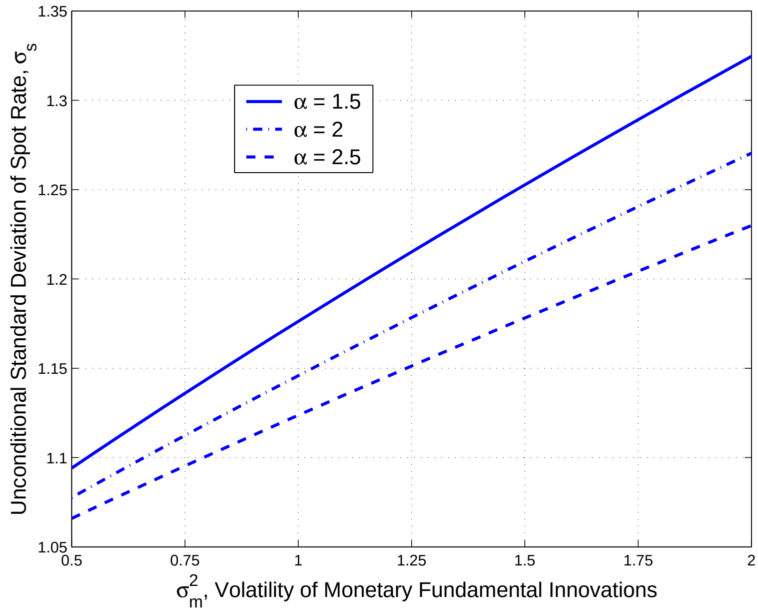
The unconditional expected profits of the informed trader, $E[\tilde{\Pi}_t]$, are an increasing function of the volume of noise trading in the FX market, σ^2_u, and of the volatility of the monetary fundamental innovation, σ^2_m.

This Corollary proposes a significative difference for the properties of the equilibrium from those outlined in Corollary 2 for the benchmark specification,[14] in particular with respect to those pertaining to the conditional precision of the impending fundamental innovation ($\pi_{q,1}$ in the benchmark specification and $\pi_{m,1}$ in the new one). Thus, differently from the benchmark specification, the larger the semi-elasticity of money demand, the more aggressive the informed trader and the more precise the information she reveals through trading.[15] We see, in fact, that the dependence of the order flow coefficient, λ_o, on α differs from that which prevails in the benchmark specification. Thus, to a very larger α corresponds a smaller value of $|\lambda_o|$, ie. a

[14]The intuition for dependence of $E[\tilde{\Pi}_t]$ on σ^2_m and σ^2_u is straightforward. It is analogous to that which applies to σ^2_q and σ^2_u in Corollary 2.

[15]For $\alpha \downarrow 1$, $\pi_{m,1} \downarrow \pi_m$, while for $\alpha \uparrow \infty$ $\pi_{m,1}$ converges to $2\,\pi_m$.

Figure 13. Comparative statics: Exchange rate volatility



Notes: We plot the unconditional standard deviation of the spot rate, σ_s, against the semi-elasticity of money demand, α, for different values of the volatility of monetary shocks, σ_m^2. Parameter values are $\gamma = 0.5$, $\sigma_q^2 = 1$, $\sigma_u^2 = 0.000256$.

smaller value for the slope of the price function against which the hedge fund trades. She will then trade more aggressively placing a more informative market order.

With respect to the liquidity properties of the FX market we now have the following Corollary of Proposition 7, which instead proposes analogous results to those outlined in Corollary 3 for the benchmark specification.

Corollary 10 *With private information on monetary fundamental innovations, when stationary, linear RE equilibria exist, the absolute value of the order flow coefficient,* $|\lambda_o|$, *is an increasing function of the volatility of the monetary fundamental shock,* σ_m^2, *and a decreasing function of the volume of noise trading in the FX market,* σ_u^2.

Finally, consider the volatility of the exchange rate. In Figure 13 we reproduce the dependence of the unconditional standard deviation of the spot rate innovation on the volatility of the monetary fundamental innovation, σ_m^2, for three values of the semi-elasticity of money demand, α. Figure 13 shows that, not surprisingly, the larger the variance of the monetary fundamental shock, σ_m^2, the more volatile the exchange rate turns out to be. As seen in Figure 4 a similar result applies to the benchmark specification.

Concluding Remarks

We have formulated a microstructure model of exchange rate determination, where private information reaches the FX market via FX dealers' customers which observe signals on impending fundamental shifts and where order flow influences the equilibrium value of a foreign currency via both portfolio-balance and information effects.

Our analysis has provided several results. Firstly, our model generates empirical implications which are consistent with some recent empirical research on FX market microstructure. Thus, we find that order flow possesses a persistent impact on exchange rate returns, that cumulative order flow and the exchange rate are cointegrated and that the spot rate is a weak predictor of future fundamental values. Secondly, we have clarified the impact that private information has on the characteristics of FX markets. In particular, we have uncovered the close link which exists between the trading activity of privately informed traders, the information that order flow signals and exchange rate dynamics. We have also seen how such link influences the efficiency and liquidity of the FX markets.

Through extensive comparative statics exercises and the analysis of several formulations of the model, we have outlined how the impact of private information on exchange rate dynamics and the characteristics of the FX market crucially hinge on the quality and nature of private information, and on its dispersion among sophisticated traders. Thus, we have seen that when an informed trader has access to private signals of poorer quality, the FX market turns out to be less efficient, but more liquid, while the informed trader gains smaller profits. We have also seen that when private information is dispersed among more informed traders the FX market becomes more fragile, as the risk of a market crash increases, but that when an equilibrium exists, this exhibits larger efficiency and liquidity.

Finally, we have seen how crucial the nature of private information is for its effect on the FX market. In particular, we have considered two alternative specifications: in the benchmark specification the informed trader has access to private information on innovations which possess a *permanent* impact on the *real* component of the fundamental value of the exchange rate; in an alternative formulation, such innovations have a *transitory* impact on the *monetary* component of the fundamental value. Interestingly, under the assumptions of the second specification we see that order flow presents a smaller information content, so that the FX market turns out to aggregate information less efficiently.

An important assumption we have made is that private information is short-lived, so that the informed trader is forced to exploit immediately and in full her informational advantage. However, more insight on the impact of private information on FX markets would be derived from a formulation in which informed traders have access to signals on fundamental innovations which enter into the public domain in the distant future. With long-lived private information, an informed trader can decide how optimally consume her informational advantage over time. This modifies and enriches the transmission link between private information, order flow and exchange rate dynamics. Analyzing the impact of long-lived private information on the FX market is however a very challenging endeavour that we leave to future research.

Appendix

Proof of Proposition 1. First consider that all FX dealers possess symmetric information. This means that all FX dealers possess the same information sets, which in day t include current and past observations of order flow $(o_t, o_{t-1}, \ldots)$ and of the fundamental components $(v_t^m, v_t^q, v_{t-1}^m, v_{t-1}^q, \ldots)$. Thus, in equation (1.9) we can apply the law of iterated expectations in that $\bar{E}_t^k[\tilde{v}_{t+k}] = E_t^d[\tilde{v}_{t+k}]$ and $\bar{E}_t^k[\tilde{z}_{t+k}] = E_t^d[\tilde{z}_{t+k}]$. Hence, consider that in a RE equilibrium all FX dealers will formulate the same conjecture on the trading strategy of the informed trader. In particular, assume that according to such a conjecture in any day t the informed trader submits a market order, I_t,[16] according to the formulation given in equation (2.2) where θ_m and θ_q are two generic constants.

Under these assumptions for $k > 0$, $E_t^d[\tilde{v}_{t+k}^m] = 0$, $E_t^d[\tilde{v}_{t+k}^q] = v_t^q + E_t^d[\tilde{\epsilon}_{t+1}^q]$ and $E_t^d[\tilde{z}_{t+k}] = z_t$, so that equation (1.9) reduces to

$$s_t \quad = \quad \frac{1}{1+\alpha}\, v_t^m \;+\; v_t^q \;+\; \frac{\alpha}{1+\alpha}\, E_t^d\,[\tilde{\epsilon}_{t+1}^q] \;-\; \frac{\alpha}{\nu}\, z_t. \tag{A.1}$$

[16]Where if I_t is positive (negative) the informed trader actually sells (purchases) the foreign currency.

To calculate $E_t^d\,[\tilde{\epsilon}_{t+1}^q]$ we can apply the projection theorem for Normal distributions and find that

$$E_t^d\,[\tilde{\epsilon}_{t+1}^q] \quad = \quad -\,\frac{\pi_o}{\pi_q + \pi_o}\,\frac{1}{\theta_q}\left(o_t \;+\; \theta_m\,v_t^m\right), \tag{A.2}$$

where $\pi_o = \theta_q^2 \pi_u$, with $\pi_u = 1/\sigma_u^2$, and $\pi_q = 1/\sigma_q^2$. Therefore, inserting equation (A.2) into equation (A.1) we find that the exchange rate respects equation (2.1) where

$$\begin{aligned}
\lambda_m &= \frac{1}{1+\alpha} - \frac{\alpha}{1+\alpha}\,\frac{\pi_o}{\pi_q+\pi_o}\,\frac{1}{\theta_q}\,\theta_m = \frac{1}{1+\alpha} + \lambda_o\,\theta_m,\\
\lambda_o &= -\,\frac{\alpha}{1+\alpha}\,\frac{\pi_o}{\pi_q+\pi_o}\,\frac{1}{\theta_q},\\
\lambda_q &= 1,\\
\lambda_z &= -\,\frac{\alpha}{\nu}.
\end{aligned}$$

Suppose now that the informed trader conjecture that the spot rate respects equation (2.1). Thus, in day t, she will maximize the expected value of her end-of-day profits, $\tilde{\Pi}_t$, which respects the following definition

$$\tilde{\Pi}_t \quad = \quad -\,I_t\left(\tilde{s}_{t+1} \;-\; s_t \;+\; i_t^* \;-\; i_t\right) \;=\; -\,I_t\,\tilde{r}_t,$$

where r_t denotes the excess return on the foreign currency. The information set of the informed trader in day t, Ω_t^I, contains past observations of the spot rate, $(s_{t-1}, s_{t-2}, \ldots)$, current and past observations of the fundamental components, $(v_t^m, v_t^q, v_{t-1}^m, v_{t-1}^q, \ldots)$, and the observation of next day innovation to the real fundamental component, ϵ_{t+1}^q. Assuming that the spot rate respects equation (2.1), the informed trader can ex-post back-out the order flow variable o_t from the spot rate, s_t, as long as we assume that in the distant past the order flow variable was observed or announced. Then, we can claim that $E[\tilde{r}_t|\Omega_t^I] = \epsilon_{t+1}^q - \lambda_o I_t + (\frac{1}{\alpha} - \lambda_m)v_t^m$. The first order condition of her optimization problem yields

$$-\,I_t \;=\; -\,\frac{1}{2\lambda_o}\,(\mu_m\,v_t^m \;+\; \epsilon_{t+1}^q), \tag{A.3}$$

where $\mu_m = 1/\alpha - \lambda_m$, while the second order condition requires that $\lambda_o < 0$. Notice that equation (A.3) is consistent with the conjecture of the FX dealers in equation (2.1) as long as

$$\theta_m \;=\; -\,\frac{\mu_m}{2\lambda_o} \;=\; \mu_m\,\theta_q, \qquad \theta_q \;=\; -\,\frac{1}{2\lambda_o}.$$

Solving simultaneously for θ_q and λ_o we have that

$$\lambda_o = -\left(\frac{\alpha-1}{4(1+\alpha)}\right)^{1/2}\left(\frac{\sigma_q^2}{\sigma_u^2}\right)^{1/2},$$

$$\theta_q = \left(\frac{1+\alpha}{\alpha-1}\right)^{1/2}\left(\frac{\sigma_u^2}{\sigma_q^2}\right)^{1/2}.$$

For $\lambda_o < 0$ the second order condition of the optimization problem of the informed trader is met. Notice that employing the definition of λ_m we find that

$$\theta_m = \frac{2}{\alpha}\frac{1}{1+\alpha}\theta_q = \frac{2}{\alpha}\frac{1}{1+\alpha}\left(\frac{1+\alpha}{\alpha-1}\right)^{1/2}\left(\frac{\sigma_u^2}{\sigma_q^2}\right)^{1/2}.$$

Finally,

$$\lambda_m = \frac{\alpha-1}{\alpha(1+\alpha)}.$$

■

Proof of Proposition 2. The result is immediate. From the formulae of the conditional precision of the average FX dealer and the unconditional expected profits of the informed trader, by substituting the optimal trading strategy of the informed trader in equation (2.2) the expressions in the statement of the Proposition for the conditional precision and the unconditional expected profits are obtained. ■

Proof of Proposition 3. To show this Proposition holds it is sufficient to notice that $E[\tilde{\epsilon}_{t+1}^q|\Omega_t^I] = \phi_n\,\xi_t^q$. Then, one just needs to repeat the steps which lead to the proof of Proposition 1. ■

Proof of Proposition 4. The proof is analogous to that of Proposition 2. It is sufficient to insert in the formulae of the conditional precision of the average FX dealer and the unconditional expected profits of the informed trader the expression for the optimal trading strategy of the informed trader in equation (4.1). ■

Proof of Proposition 5. To show this Proposition holds it is sufficient to notice that if for $l' \neq l$, $I_t^{l'} = -\theta_m^{L+1}v_t^m - \theta_q^{L+1}\epsilon_{t+1}^q$, then $E[\tilde{r}_t \mid \Omega_t^{I^l}] = (1+\lambda_o L\theta_q^{L+1})\epsilon_{t+1}^q + (1/\alpha - \lambda_m + \lambda_o L\theta_m^{L+1})v_t^m - \lambda_0 I_t^l$. Hence, solving the optimization problem of informed trader l, one finds that her optimal order, I_t^l, is equal to $I_t^l = -\theta_m^l v_t^m - \theta_q^l\epsilon_{t+1}^q$, with $\theta_m^l = -(1/(2\lambda_o))(1/\alpha - \lambda_m + \lambda_o L\theta_m^{L+1})$ and

$\theta_q^l = -(1/(2\lambda_o))(1 + \lambda_o L \theta_q^{L+1})$. Imposing the symmetric restrictions $\theta_m^l = \theta_m^{L+1}$ and $\theta_q^l = \theta_q^{L+1}$ and repeating the remaining steps outlined in the proof of Proposition 1 concludes the proof. ■

Proof of Proposition 6. The proof is analogous to that of Proposition 2. It follows straightforwardly from the definitions of the conditional precision of the average FX dealer and the unconditional expected profits of the generic informed trader, and the equilibrium expression for the optimal trading strategy of the informed traders in equation (5.1). ■

Proof of Proposition 7. One can repeat the steps followed in the proof of Proposition 1. However, one should notice that in equation (A.1) $E[\tilde{\epsilon}_{t+1}^q|\Omega_t^I]$ is replaced by $E[\tilde{v}_{t+1}^m|\Omega_t^I]$, where this value is now

$$E_t^d\,[\tilde{v}_{t+1}^q] \;=\; -\,\frac{\pi_o}{\pi_m \,+\, \pi_o}\,\frac{1}{\theta_m^{+1}}\left(o_t \,+\, \theta_m\, v_t^m\right).$$

Similarly, now $E[\tilde{r}_t|\Omega_t^I] = \lambda_m\, v_{t+1}^m - \lambda_o I_t + (\frac{1}{\alpha} - \lambda_m)v_t^m$ so that in equation (A.3) is given by the following formula,

$$-\,I_t \;=\; -\,\frac{1}{2\lambda_o}\,(\mu_m\, v_t^m \;+\; \lambda_m\, v_{t+1}^m).$$

The result is then immediate. ■

Proof of Proposition 8. To prove this Proposition holds is sufficient to repeat the steps followed in the proof of Proposition 2 noticing that $\tilde{v}_{t+1}^m$ replaces $\tilde{\epsilon}_{t+1}^q$. ■

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