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ABSTRACT

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The experimental literature has documented that there is overbidding in second-price auctions, regardless of bidders' valuations. In contrast, in first-price auctions there tends to be overbidding for large valuations, but underbidding for small valuations. We show that the experimental evidence can be explained by a simple extension of the standard auction model, where bidders anticipate positive or negative emotions caused by the mere fact of winning or losing. Even if the "emotional" (dis-)utility is very small, the seller's optimal reserve price r* may be significantly different from the standard model. Moreover, r* is decreasing in the number of bidders.

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1 Introduction

In traditional auction theory, it is assumed that a bidder cares only about whether or not he gets the object to be auctioned off and the payment he has to make. If a bidder wins, his utility is given by his intrinsic valuation of the object minus his payment. If a bidder does not win, his utility is zero, just as if he had not participated in the auction.

While this may be a good approximation in many circumstances, it cannot be denied that bidders in auctions are just human beings. As such, they experience emotions.[1] A bidder may well enjoy the thrill and excitement of being a winner. He may thus derive some utility from the mere fact of winning, over and above his value from receiving the object. Similarly, a bidder might be annoyed and disappointed when he loses, because in contrast to someone who did not participate, he had a chance to win. He participated with the aim of winning, so being a loser might well cause anger.

The aims of this paper are threefold. First, we want to study the robustness of the standard symmetric auctions model with risk-neutral players and private independent values. Specifically, we are interested in the implications for the resulting bidding behavior when bidders anticipate (small) positive emotions of winning and (small) negative emotions of losing.[2]

Second, we investigate whether the introduction of such anticipated emotions into the standard auction model may shed light on various experimental findings on the bidding behavior in auctions with independent private values. Specifically, in laboratory experiments there tends to be overbidding in second-price auctions, regardless of the bidders' valuations (see, e.g., Kagel et al., 1987; Kagel and Levin, 1993; Kirchkamp and Reiss, 2004; Cooper and Fang, 2007). In contrast, in first-price auctions there tends to be overbidding when bidders have large valuations, but underbidding when bidders have small valuations. In particular, Cox et al. (1988) document overbidding in first-price auctions, but also emphasize that, when bids are approximated by linear bidding functions, intercepts tend to be negative. Cox et al. (1988) point out that some "subjects bid a much smaller proportion (sometimes zero) of their value" when their realized valuation was sufficiently small.[3]

[1] Recently, it has also been emphasized in neuroeconomic research that emotions influence economic decisions. See, e.g., Sanfey et al. (2003) and Cohen (2005).

[2] The fact that economic decisions may be influenced by *anticipated* emotions has been pointed out in experimental psychology. See, e.g., Mellers et al. (1999) and Camille et al. (2004).

[3] See Cox et al. (1988, p. 77–78). For a review of these findings, see Kagel (1995). Cox et al. (1985, 1988) call the observation that some bidders with very small valuations bid zero the "throw away" bid phenomenon. They

More recently, to investigate this issue further, Ivanova-Stenzel and Sonsino (2004) and Kirchkamp and Reiss (2004, 2006) facilitate underbidding at low valuations by considering experimental settings where the lowest possible valuation of buyers is strictly positive.[4] They find that the average amount of overbidding in first-price auctions is increasing in bidders' valuations (see also Goeree et al., 2002), and that, whenever valuations are sufficiently small, there is indeed substantial underbidding (i.e., bids are strictly positive, but below their risk-neutral equilibrium value).[5] Moreover, Kirchkamp and Reiss (2006) document that subjects indeed expect underbidding at low valuations by other players.

As is well known, if bidders anticipate a joy of winning, bids will be larger than in the standard model, both in first-price and second-price auctions. However, perhaps more surprisingly, we show that if bidders anticipate to be annoyed when they lose, the implications depend on the auction format. In a second-price auction, bidders who still participate bid more when they anticipate negative emotions, because they are more eager to avoid losing. In particular, in a second-price auction all participating bidders overbid by the same amount. Bidders with very low valuations will not participate at all. In a first-price auction, bidders with relatively small valuations might again not participate, but those who do, when anticipating that losing is painful, bid less than in the standard model. In contrast, bidders with high valuations bid more. The reason that in a first-price auction there may be underbidding is that anticipated anger reduces the number of participating bidders. In a first-price auction, the winner has to pay his own bid, so that bids are smaller when there are fewer bidders (and hence the probability of winning is larger). Provided that a bidder's valuation is sufficiently small, this effect dominates the willingness to bid more in order to avoid negative emotions. Hence, our simple extension of the standard symmetric auctions model is able to rationalize the above discussed observations that have emerged in the experimental literature on bidding behavior in first-price and second-price auctions.

Third, we derive novel predictions regarding the optimal reserve price, i.e., the reserve price that maximizes the seller's expected revenue. It turns out that the seller's optimal reserve price

argue that these bidders did not "seriously" participate in the auction. Note that actual non-participation was not a feasible option in these laboratory experiments.

[4] In most of the experimental literature, the lower bound of valuations is set equal to zero.

[5] For graphical illustrations, cf. Figure 6 and Table 2 in Kirchkamp and Reiss (2004) and Figure 8 in Cox et al. (1988). In Ivanova-Stenzel and Sonsino (2004), 7.4 percent of bids are even below the lowest possible valuation. For second-price auctions, Kirchkamp and Reiss (2004) replicate the earlier finding of relatively constant overbidding across all valuations.

goes up when winners enjoy positive emotions, while it goes down when losers experience negative emotions. Perhaps more surprisingly, we can show that the optimal reserve price is decreasing in the number of potential buyers. This result is in stark contrast to the standard model, where the optimal reserve price is independent of the number of potential buyers (see the seminal work of Myerson, 1981, and Riley and Samuelson, 1981). Intuitively, a reserve price increases the seller's revenue when it is binding, but it also reduces participation. When there are more buyers, a given reserve price will be binding with a smaller probability. It thus becomes relatively more important to compensate the fact that the anticipation of negative emotions already reduces participation. Hence, the optimal reserve price will go down. Finally, we also demonstrate that if there are many bidders, the optimal reserve price can be significantly smaller than the one in the standard model, even when the extent of the emotions seems to be almost negligible compared to the expected value of the object.

The remainder of the paper is organized as follows. In section 2, we discuss the related literature, and in section 3, the model is introduced. In section 4, bidding behavior in second-price auctions and first-price auctions is analyzed. In section 5, the implications of positive and negative emotions for the seller's optimal reserve price are studied. Concluding remarks follow in section 6.

2 Related literature

Our paper is related to three strands of the literature. First, at a general level, the paper is part of the research program of behavioral economics that tries to enrich economic theory by taking into consideration psychological insights. Papers that are thus related in spirit include recent work on the implications of fairness, ethics, and anticipated emotions in mechanism design theory.[6]

Second, and more specifically, our paper contributes to a growing theoretical literature that aims at explaining the bidding behavior observed in the laboratory experiments discussed above. One candidate explanation that has been put forward is risk aversion, and various authors have shown that risk aversion may indeed lead to overbidding in first-price auctions (see, e.g., Riley and Samuelson, 1981; Cox et al., 1983, 1988). However, risk aversion cannot account for overbidding in

[6] For surveys, see, e.g., Rabin (2002), Tirole (2002), Camerer and Loewenstein (2003), and Fehr and Schmidt (2003). As far as emotions and economic theory are concerned, see Elster (1998). There is also an emerging theoretical literature on the competing role of cognition and *currently experienced* emotions in decision-making (see, e.g., Fudenberg and Levine, 2006). See also Loewenstein et al. (2001), who make a distinction between anticipatory emotions (cf. Caplin and Leahy, 2001) and anticipated emotions that are expected to be experienced in the future.

second-price auctions (where bidding one's valuation remains a weakly dominant strategy),[7] and predicts overbidding for all valuations in first-price auctions.

Other proposed explanations are based on bounded rationality of participants. For example, Kagel et al. (1987) argue that overbidding in second-price auctions might be driven by the illusion that overbidding increases the probability of winning with little cost (because only the second highest bid has to be paid).[8] Moreover, limited negative feedback from overbidding in this auction format might play a role.[9] Kirchkamp and Reiss (2004) consider the case that experimental subjects follow simple rules-of-thumb. Namely, they assume that participants simply bid their valuation minus some constant by which they shade their bids. In first-price auctions, this can indeed lead to underbidding at low valuations and overbidding at high valuations, but it also leads to equilibrium behavior in many of the second-price auction experiments studied in the literature. More recently, Crawford and Iriberri (2007) study a non-equilibrium model of level-k strategic thinking. This model leads to equilibrium behavior in second-price auctions and in first-price auctions with a uniform distribution of valuations (which has frequently been used in experiments). However, given a non-uniform distribution, the model can explain observed behavior in first-price auctions.[10]

The present paper belongs to a further strand of explanations that considers the role of emotions in auctions. We build on Cox et al. (1988, 1992) who introduce a (constant) joy of winning into standard auction models. Note that, in a first-price auction, a pure joy of winning model would predict (constant) overbidding independent of the valuation, which would be inconsistent with the experimental evidence discussed above. With respect to negative emotions, Filiz and Ozbay (2007) study the implications of anticipated loser regret, which may lead to overbidding in first-price auctions, but has no effect in second-price auctions (see also the related earlier work by Engelbrecht-Wiggans, 1989). Nevertheless, even in a second-price auction, losers may well feel disappointed and annoyed, i.e., there may be negative emotions different from regret. Morgan et al. (2003) introduce a spite motive, i.e., the utility of bidders is negatively affected by the surplus of the winning rival.

[7] Evidence casting doubt on the (exclusive) role of risk aversion is also provided by Cox, Smith, and Walker (1985), who employ the binary lottery procedure to induce risk neutral behavior.

[8] In the context of first-price auction experiments, Harrison (1989) has raised the methodological issue that deviating from equilibrium strategies might come at small costs. See also Harrison (1992) and the related discussion in the December 1992 issue of the *American Economic Review*.

[9] However, see Kagel and Levin (1993), Harstad (2000), and Cooper and Fang (2007) for evidence that overbidding does not seem to vanish with experience.

[10] See also Kirchkamp and Reiss (2006), who in an experimental study investigate the role of incorrect expectations with respect to the behavior of other players.

Such a spite motive predicts overbidding in second-price auctions (which is decreasing in valuations) as well as overbidding in first-price auctions (which is increasing in valuations), but never results in underbidding. To summarize, while other explanations may very well play a role, our simple model of positive and negative emotions offers a consistent explanation that can rationalize the observed bidding behavior both in second-price auctions and first-price auctions.[11]

Third, as discussed above, our paper is related to the literature on optimal reserve prices in auctions. To the best of our knowledge, the present paper is the first to investigate how positive emotions from winning and negative emotions from losing affect optimal reserve prices, and in contrast to the standard model, we show that the optimal reserve price may be decreasing in the number of potential buyers.[12]

3 The model

A monopolistic seller possesses a single, indivisible object. There are $n \geq 2$ potential buyers. The seller conducts a (second-price or first-price) sealed-bid auction with a reserve price $r \geq 0$. Thus, a buyer participating in the auction must at least bid r. If bidder i wins the object and pays the price t_i according to the rules of the auction, then his utility is given by $v_i - t_i + \varepsilon$, where $v_i \in [\underline{v}, \overline{v}]$ denotes his intrinsic valuation of the object to be auctioned off ($\overline{v} > \underline{v} \geq 0$). If buyer i does not win the auction, his utility is given by $-\gamma$. The case $\varepsilon = \gamma = 0$ is the standard case analyzed in the literature on auctions. A strictly positive ε captures the joy that a bidder feels when he wins the auction. Similarly, a strictly positive γ captures the anger that a bidder feels when he loses. As will become clear below, even if ε and γ are relatively small, the presence of anticipated emotions may affect equilibrium behavior substantially.

Apart from the (possibly very small) modification with regard to $\varepsilon > 0$ and $\gamma > 0$, our model is identical to the standard model of the independent private values environment with symmetric bidders that completely ignores positive and negative emotions.[13] Buyer i's type v_i is the realiza-

[11] For recent interesting experiments on the role of emotions in auctions, see e.g., Andreoni et al. (2007), Cooper and Fang (2007), and Filiz and Ozbay (2007).

[12] Rosenkranz and Schmitz (2007) have analyzed how reserve prices in auctions can play the role of reference points in the sense of Kahneman and Tversky (1979), so that a larger reserve price can have a positive impact on the bidders' willingness-to-pay. They show that then the optimal reserve price may be increasing in the number of bidders.

[13] See Riley and Samuelson (1981). This model has been called the "benchmark model" of auction theory in McAfee and McMillan's (1987) survey article. See also Matthews (1995), Krishna (2002), and Menezes and Monteiro (2005) for more recent expositions. Since a winning bidder has a negative impact on all other bidders, our model is also related to the more general literature on auctions with externalities (see, e.g., Jehiel et al., 1999).

tion of a random variable $\check{v}_i$. Each $\check{v}_i$ is independently and identically distributed on $[\underline{v}, \overline{v}]$. The cumulative distribution function F is strictly increasing and the density function is denoted by f. We make the usual monotone hazard rate assumption, according to which $[1 - F(v)]/f(v)$ is decreasing in v.[14] Only buyer i knows his realized value v_i, while the other components of the model are common knowledge. The seller sets the reserve price r that maximizes her expected revenue, and each buyer is interested in maximizing his expected utility. Throughout, we will focus our attention on symmetric equilibria.

4 Bidding behavior

We now study participation and bidding behavior in both second-price auctions and first-price auctions when anticipated emotions play a role.

Second-price auction. If at least two bidders participate, in a second-price auction the buyer submitting the highest bid wins the object, but he has to pay only the second-highest bid.[15] If only one bidder participates, he wins and has to pay the reserve price r. It is well known that in the standard model (where $\varepsilon = \gamma = 0$), each buyer i with $v_i \geq r$ will participate in the auction and bid his type v_i. In the present framework with $\varepsilon > 0$, $\gamma > 0$, this result can be generalized as follows.

Proposition 1 *In a second-price auction, only buyers of types $v \geq \tilde{v}$ will participate, where $[\tilde{v} - r + \varepsilon + \gamma]F(\tilde{v})^{n-1} = \gamma$. Their symmetric equilibrium bidding strategies are given by $b^S(v) = v + \varepsilon + \gamma$.*

Proof. Consider a buyer i and assume that all other buyers follow the strategy given in the proposition. If buyer i participates and bids $b^S(v_i)$, he wins and pays t_i if $b^S(v_i) > t_i$, where t_i is the maximum of the other bids if there are any, and $t_i = r$ otherwise. Consider a downward deviation to some $\tilde{b} < b^S(v_i)$. If $t_i < \tilde{b} < b^S(v_i)$, he still wins and pays t_i, so his utility is not changed. If $\tilde{b} < b^S(v_i) < t_i$, his utility $-\gamma$ also remains unchanged. If $\tilde{b} < t_i < b^S(v_i)$, he now loses and his utility is $-\gamma$, while he could have attained the utility $v_i - t_i + \varepsilon$ by bidding $b^S(v_i)$. The latter utility level would have been larger, because $v_i - t_i + \varepsilon > v_i - b^S(v_i) + \varepsilon = -\gamma$. A similar argument shows that an upward deviation $\tilde{b} > b^S(v_i)$ cannot be profitable for the buyer. Finally, a

[14] As a consequence, we are in Myerson's (1981) "regular case," i.e., the "virtual valuation" $v - [1 - F(v)]/f(v)$ is increasing in v.

[15] If there is a tie, so that there is more than one bidder with the highest bid, let the object go to each of them with equal probability. A similar assumption can be made in the first-price auction. However, the tie-breaking rule is not important, because the probability of a tie will be zero.

buyer with the smallest valuation for which it is not yet unprofitable to participate in the auction will win (and pay r) only if all other buyers have smaller valuations. Hence, a buyer will participate whenever $v \geq \tilde{v}$, where $\tilde{v} \in (\underline{v}, \overline{v})$ is implicitly given by $[\tilde{v} - r + \varepsilon]F(\tilde{v})^{n-1} - \gamma[1 - F(\tilde{v})^{n-1}] = 0$.[16] ■

If there were no negative emotions of losing ($\gamma = 0$), then every buyer with $v \geq r - \varepsilon$ would participate and bid $b^S(v) = v + \varepsilon$, which is more than the usual second-price auction bid v, because a bidder now anticipates that he will feel happy when he wins. Yet, if a buyer also anticipates that it is painful to lose ($\gamma > 0$), his valuation must be larger than $r - \varepsilon$ in order to make participation individually rational for him.[17] Specifically, the larger is n, the larger will be the threshold valuation $\tilde{v}$ below which a buyer will not participate,[18] because the probability of losing increases when there are more bidders. Notice also that $\tilde{v}$ is increasing in r and that $\tilde{v} > r$ if and only if r is relatively small, i.e., $r < \bar{r}$, where $F(\bar{r})^{n-1} = \gamma/(\varepsilon + \gamma)$ and.$\bar{r} \in (\underline{v}, \overline{v})$.[19]

Corollary 1 *In a second-price auction with a given reserve price $r \geq 0$, the following results hold. (i) Increasing ε increases participation, while increasing γ reduces participation. (ii) The bids are increasing in ε and γ. (iii) The amount of overbidding of a participating bidder, $b^S(v) - v = \varepsilon + \gamma$, is independent of v.*

Proof. Part (i) follows from the implicit definition of $\tilde{v}$. Specifically, the threshold valuation $\tilde{v}$ below which buyers do not participate satisfies

$$\frac{d\tilde{v}}{d\varepsilon} = \frac{-F(\tilde{v})^n}{F(\tilde{v})^n + \gamma(n-1)f(\tilde{v})} < 0$$

and

$$\frac{d\tilde{v}}{d\gamma} = \frac{1 - F(\tilde{v})^{n-1}}{F(\tilde{v})^{n-1} + \gamma(n-1)f(\tilde{v})/F(\tilde{v})} > 0.$$

Parts (ii) and (iii) of the corollary are obvious. ■

[16] Note that $\tilde{v}$ is uniquely defined. When v moves from $\underline{v}$ to $\overline{v}$, then $\gamma[1 - F(v)^{n-1}]$ decreases from γ to 0. If $r \in [0, \varepsilon]$, then $[v - r + \varepsilon]F(v)^{n-1}$ increases from 0 to $\overline{v} - r + \varepsilon$, hence an intermediate value argument can be applied. Similarly, if $r \in (\varepsilon, \overline{v} + \varepsilon]$, then $[v - r + \varepsilon]F(v)^{n-1}$ increases from 0 to $\overline{v} - r + \varepsilon$ when v moves from $max\{\underline{v}, r - \varepsilon\}$ to $\overline{v}$. The seller will never set the reserve prices r larger than $\overline{v} + \varepsilon$, because then no one would participate (even though a buyer's bid may well be larger than $\overline{v} + \varepsilon$).

[17] Notice that anticipated negative emotions are thus somewhat related to entry fees or participation costs (however, the emotional costs are incurred by losers only and they are not payments that accrue to the seller). On auctions with costly participation, see, e.g., Samuelson (1985) and Menezes and Monteiro (2000).

[18] Formally, this follows immediately from the implicit definition of $\tilde{v}$ in Proposition 1.

[19] Hence, even if the reserve price is set equal to zero, in the presence of emotions, bidders with sufficiently low valuations will not participate.

Consider now a buyer of type $v \geq \tilde{v}$. The expected payment of the buyer to the seller reads

$$T^S(v) = rF(\tilde{v})^{n-1} + \int_{\tilde{v}}^{v} (w + \varepsilon + \gamma) dF(w)^{n-1},$$

because the buyer will win only if he has the highest value. He then must pay r if all other buyers have types smaller than $\tilde{v}$ (which happens with probability $F(\tilde{v})^{n-1}$), and he must pay $b^S(w) = w + \varepsilon + \gamma$ if $w \in (\tilde{v}, v)$ is the highest value of the other $n-1$ buyers.

First-price auction. If the seller conducts a first-price auction, the bidder with the highest bid wins and has to pay what he has bid. In the standard framework (where $\varepsilon = \gamma = 0$), there is a symmetric equilibrium in which each bidder bids less than his true type. In the present model, this result can be generalized, so that a bidder who participates in a first-price auction bids $b^F(v)$, which is less than $b^S(v)$. Specifically, the following result can be obtained.

Proposition 2 *In a first-price auction, only buyers of types $v \geq \tilde{v}$ will participate. Their symmetric equilibrium bidding strategies are given by*

$$b^F(v) = \left(v + \varepsilon - \frac{1 - F(v)^{n-1}}{F(v)^{n-1}} \gamma - \int_{\tilde{v}}^{v} \frac{F(w)^{n-1}}{F(v)^{n-1}} dw \right).$$

Proof. Consider buyer i and suppose that all other buyers follow the strategy given in the proposition. Since $b^F(v)$ is increasing, it is never profitable for buyer i to bid more than $b^F(\overline{v})$, because otherwise he would win for sure and could increase his payoff by slightly reducing his bid. Therefore, buyer i considers to bid $b \in [r, b^F(\overline{v})]$. There exists a value $z \in [\tilde{v}, \overline{v}]$ such that $b^F(z) = b$. Hence, buyer i's expected payoff from bidding b can be written as follows:

$$\begin{aligned} & \left(v_i - b^F(z) + \varepsilon\right) F(z)^{n-1} - \gamma[1 - F(z)^{n-1}] \\ = \quad & (v_i - z)\, F(z)^{n-1} + \int_{\tilde{v}}^{z} F(w)^{n-1} dw. \end{aligned}$$

If buyer i bids $b^F(v_i)$, his expected payoff thus is $\int_{\tilde{v}}^{v_i} F(w)^{n-1} dw$. Since

$$\begin{aligned} & (v_i - z)\, F(z)^{n-1} + \int_{\tilde{v}}^{z} F(w)^{n-1} dw - \int_{\tilde{v}}^{v_i} F(w)^{n-1} dw \\ = \quad & \int_{v_i}^{z} \left[F(w)^{n-1} - F(z)^{n-1}\right] dw \leq 0, \end{aligned}$$

it cannot be profitable for buyer i to deviate from $b^F(v_i)$. Finally, in analogy to the proof of Proposition 1, it is individually rational for buyer i to participate whenever $v \geq \tilde{v}$. ■

Just as in the second-price auction, anticipated joy of winning increases the bids. Yet, while anticipated anger increases a buyer's bid in the second-price auction (provided the buyer still participates), it may decrease his bid in the first-price auction. When a buyer wants to avoid the pain of losing, in a second-price auction he either does not participate or he bids more aggressively (recall that the bid determines the winner, but not his payment). In contrast, in a first-price auction, a winning bidder must pay his own bid. When the buyers anticipate anger, fewer bidders will participate. Thus, on the one hand, a participating bidder will win with a larger probability, so he may reduce his bid. On the other hand, anticipated anger increases a bidder's desire to win, so he might be willing to bid more. The first effect is stronger than the second effect if the bidder's valuation is sufficiently small. These findings can be summarized as follows.

Corollary 2 *In a first-price auction with a given reserve price $r \geq 0$, the following results hold. (i) Increasing ε increases participation, while increasing γ reduces participation. (ii) The bids are increasing in ε. They are increasing in γ if v is sufficiently large and decreasing in γ if v is sufficiently small. (iii) The amount of overbidding of a participating bidder, $b^F(v) - \left(v - \int_r^v \frac{F(w)^{n-1}}{F(v)^{n-1}} dw\right)$, is increasing in v. Moreover, there is always overbidding for large valuations, and there is underbidding for small valuations whenever $r < \bar{r} \in (\underline{v}, \overline{v})$.*[20]

Proof. Part (i) is identical to Corollary 1. As far as part (ii) is concerned, from Proposition 2 and Corollary 1 it follows that

$$\frac{db^F(v)}{d\varepsilon} = 1 + \frac{F(\tilde{v})^{n-1}}{F(v)^{n-1}} \frac{d\tilde{v}}{d\varepsilon} = 1 - \frac{F(\tilde{v})^{n-1}}{F(v)^{n-1}} \frac{F(\tilde{v})^n}{F(\tilde{v})^n + \gamma(n-1)f(\tilde{v})},$$

which is positive for $v \geq \tilde{v}$. Similarly, it follows that

$$\begin{aligned} \frac{db^F(v)}{d\gamma} &= -\frac{1 - F(v)^{n-1}}{F(v)^{n-1}} + \frac{F(\tilde{v})^{n-1}}{F(v)^{n-1}} \frac{d\tilde{v}}{d\gamma} \\ &= -\frac{1 - F(v)^{n-1}}{F(v)^{n-1}} + \frac{1}{F(v)^{n-1}} \frac{1 - F(\tilde{v})^{n-1}}{1 + \gamma(n-1)F(\tilde{v})^{-n} f(\tilde{v})}, \end{aligned}$$

so that

$$\frac{db^F(v)}{d\gamma} < 0 \Leftrightarrow \frac{1 - F(\tilde{v})^{n-1}}{1 + \gamma(n-1)F(\tilde{v})^{-n} f(\tilde{v})} < 1 - F(v)^{n-1}.$$

Hence, when γ grows, a bidder will reduce his bid if v is sufficiently close to $\tilde{v}$, while he will increase his bid if v is sufficiently close to $\overline{v}$.

[20] Note that this condition is satisfied in the experimental papers discussed in the introduction.

With regard to part (iii), recall that $\tilde{v} = r$ in the standard model where $\varepsilon = \gamma = 0$, so that the amount of overbidding is

$$\begin{aligned}\Delta(v, \varepsilon, \gamma, r) &= b^F(v) - \left(v - \int_r^v \frac{F(w)^{n-1}}{F(v)^{n-1}} dw\right) \\ &= \varepsilon - \frac{1 - F(v)^{n-1}}{F(v)^{n-1}}\gamma + \int_r^{\tilde{v}} \frac{F(w)^{n-1}}{F(v)^{n-1}} dw.\end{aligned}$$

To see that the amount of overbidding is increasing, note that

$$\frac{d}{dv}\Delta(v, \varepsilon, \gamma, r) = (n-1)f(v)F(v)^{-n}\left(\gamma - \int_r^{\tilde{v}} F(w)^{n-1} dw\right),$$

which is positive, because (using the fact that F is an increasing function)

$$\begin{aligned}\gamma - \int_r^{\tilde{v}} F(w)^{n-1} dw &= [\tilde{v} - r + \varepsilon + \gamma]F(\tilde{v})^{n-1} - \int_r^{\tilde{v}} F(w)^{n-1} dw \\ &> [\tilde{v} - r + \varepsilon + \gamma]F(\tilde{v})^{n-1} - (\tilde{v} - r)F(\tilde{v})^{n-1} > 0.\end{aligned}$$

Consider the case $r < \bar{r}$ and hence $r < \tilde{v}$, so that the smallest valuation of a bidder who participates both in the standard model (where $\varepsilon = \gamma = 0$) and in our model is $v = \tilde{v}$. There is underbidding at $v = \tilde{v}$, because (again using the definition of $\tilde{v}$)[21]

$$\begin{aligned}\Delta(\tilde{v}, \varepsilon, \gamma, r) &= \frac{1}{F(\tilde{v})^{n-1}}\left[(\varepsilon + \gamma)F(\tilde{v})^{n-1} - \gamma + \int_r^{\tilde{v}} F(w)^{n-1} dw\right] \\ &< \frac{1}{F(\tilde{v})^{n-1}}\left[(\varepsilon + \gamma)F(\tilde{v})^{n-1} - \gamma + (\tilde{v} - r)F(\tilde{v})^{n-1}\right] = 0.\end{aligned}$$

Consider now the case $r \geq \tilde{v}$, so that the smallest valuation of a bidder who participates in the standard model as well as in our model is $v = r$. There is overbidding at $v = r$, because

$$\begin{aligned}\Delta(r, \varepsilon, \gamma, r) &= \frac{1}{F(r)^{n-1}}\left[(\varepsilon + \gamma)F(r)^{n-1} - \gamma - \int_{\tilde{v}}^r F(w)^{n-1} dw\right] \\ &> \frac{1}{F(r)^{n-1}}\left[(\varepsilon + \gamma)F(r)^{n-1} - (r - \tilde{v})F(r)^{n-1} - \gamma\right] \\ &= \frac{1}{F(r)^{n-1}}(\tilde{v} - r + \varepsilon + \gamma)\left[F(r)^{n-1} - F(\tilde{v})^{n-1}\right] \\ &= \frac{1}{F(r)^{n-1}}\frac{\gamma}{F(\tilde{v})^{n-1}}\left[F(r)^{n-1} - F(\tilde{v})^{n-1}\right] > 0.\end{aligned}$$

Finally, at $v = \overline{v}$, there is overbidding, because

$$\Delta(\overline{v}, \varepsilon, \gamma, r) = \varepsilon + \int_r^{\tilde{v}} F(w)^{n-1} dw,$$

[21] Notice that in this case $b^F(\tilde{v})$ can be even smaller than $\underline{v}$. Specifically, $b^F(\tilde{v}) < \underline{v}$ holds whenever $r \leq \underline{v}$.

which is obviously positive if $r < \tilde{v}$. Otherwise, if $r \geq \tilde{v}$, then there is overbidding at $v = r$ already.

■

As an illustration, consider Figure 1, where $F(v) = v$ for $v \in [0, 1]$, $r = 0$, and $n = 3$. The dotted line is the usual equilibrium bidding strategy when $\varepsilon = \gamma = 0$, while the solid curve is the equilibrium bidding strategy $b^F(v)$ when $\varepsilon = \gamma = 1/20$. Note that in the latter case, bidders with a valuation smaller than $\tilde{v} \approx 0.338$ do not participate, bidders with a valuation between $\tilde{v}$ and $\hat{v} \approx 0.609$ bid less than in the standard model, and bidders with a valuation larger than $\hat{v}$ bid more.

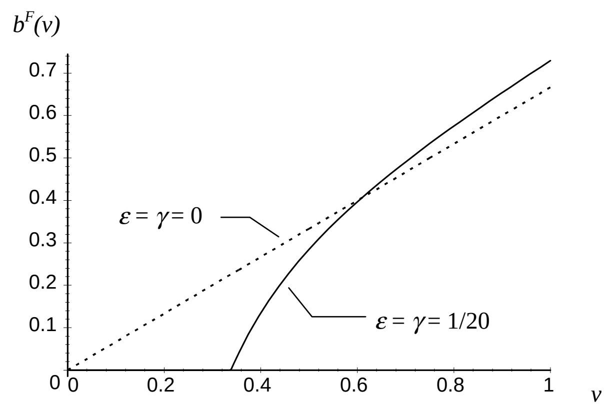

Figure 1. Bidding strategy in the first-price auction.

To summarize, our model can explain experimental findings according to which there is always overbidding in second-price auctions, while in first-price auctions there is overbidding for large valuations and underbidding for small valuations.

To conclude the analysis of the first-price auction, consider again a buyer of type $v \geq \tilde{v}$. He pays $b^F(v)$ if all other buyers have types smaller than v, so his expected payment to the seller is

$$T^F(v) = b^F(v)F(v)^{n-1} = (v + \varepsilon + \gamma)\, F(v)^{n-1} - \gamma - \int_{\tilde{v}}^{v} F(w)^{n-1} dw.$$

Using the definition of $\tilde{v}$, integration by parts immediately shows that $T^F(v) = T^S(v)$, which is in accordance with the well-known revenue equivalence principle.[22]

[22] We do not want to stress revenue-equivalence too much. It might well be possible that the extent of the emotions depends on the rules of the auction (i.e., ε and γ could be different in the two auctions formats), so that first-price and second-price auctions were no longer revenue-equivalent.

5 The optimal reserve price

We can now analyze how a revenue-maximizing seller should set the reserve price when she takes the bidders' emotions into account. The seller does not know the buyers' types. Hence, her expected revenue Π is given by n times the expected value of the payment that a buyer makes to the seller (which is $T^F(v) = T^S(v)$ if $v \geq \tilde{v}$, and 0 otherwise). With integration by parts, it follows that

$$\begin{aligned}
\Pi &= n \int_{\tilde{v}}^{\overline{v}} \left((v + \varepsilon + \gamma) F(v)^{n-1} - \gamma - \int_{\tilde{v}}^{v} F(w)^{n-1} dw \right) dF(v) \\
&= n \int_{\tilde{v}}^{\overline{v}} \left[\left((v + \varepsilon + \gamma) F(v)^{n-1} - \gamma \right) f(v) - F(v)^{n-1} + F(v)^{n-1} F(v) \right] dv \\
&= n \int_{\tilde{v}}^{\overline{v}} \left(v - \frac{1 - F(v)}{f(v)} + \varepsilon + \gamma \right) F(v)^{n-1} dF(v) - n\gamma [1 - F(\tilde{v})].
\end{aligned}$$

Note that if $\varepsilon = \gamma = 0$ (i.e., in the standard model), then Π is obviously maximized by $\tilde{v} = r = r_0$, where

$$r_0 - \frac{1 - F(r_0)}{f(r_0)} = 0,$$

so that the integrand is positive whenever $v \geq \tilde{v}$. The following proposition characterizes the optimal reserve price if $\varepsilon > 0$ and $\gamma > 0$, i.e., if there are positive emotions of winning and negative emotions of losing.[23]

Proposition 3 *The optimal reserve price r^* is given by*

$$r^* = \frac{1 - F(\tilde{v}^*)}{f(\tilde{v}^*)},$$

where $\tilde{v}^*$ is uniquely characterized by

$$\tilde{v}^* - \frac{1 - F(\tilde{v}^*)}{f(\tilde{v}^*)} = \frac{1 - F(\tilde{v}^*)^{n-1}}{F(\tilde{v}^*)^{n-1}} \gamma - \varepsilon.$$

Proof. The first derivative of the seller's expected profit with respect to the threshold valuation $\tilde{v}$ is given by

$$\frac{d\Pi}{d\tilde{v}} = -n \left(\tilde{v} - \frac{1 - F(\tilde{v})}{f(\tilde{v})} + \varepsilon + \gamma \right) F(\tilde{v})^{n-1} f(\tilde{v}) + n\gamma f(\tilde{v}).$$

[23] Notice that, using the envelope theorem, it is straightforward to check that the seller's expected profit when she sets the optimal reserve price is increasing in ε and decreasing in γ, as one would have expected. Hence, if, for example, ε is larger and γ is smaller in first-price than in second price auctions, then the seller will prefer to conduct a first-price auction.

The first-order condition of the seller's profit-maximization problem can thus be written as $\lambda(\tilde{v}^*) = \mu(\tilde{v}^*, \varepsilon, \gamma, n)$, where

$$\begin{aligned}\lambda(\tilde{v}) &= \tilde{v} - \frac{1 - F(\tilde{v})}{f(\tilde{v})}, \\ \mu(\tilde{v}, \varepsilon, \gamma, n) &= \frac{1 - F(\tilde{v})^{n-1}}{F(\tilde{v})^{n-1}}\gamma - \varepsilon.\end{aligned}$$

Note that given the monotone hazard rate assumption, there always exists a unique $\tilde{v}^* \in (\underline{v}, \overline{v})$ that satisfies the first-order condition. When $\tilde{v}$ moves from $\underline{v}$ to $\overline{v}$, then $\lambda(\tilde{v})$ strictly increases from $\underline{v} - 1/f(0)$ to $\overline{v}$, while $\mu(\tilde{v}, \varepsilon, \gamma, n)$ strictly decreases from $+\infty$ to $-\varepsilon$. Due to continuity, an intermediate value argument can thus be applied. The optimal reserve price can then be derived from $[\tilde{v}^* - r^* + \varepsilon + \gamma]F(\tilde{v}^*)^{n-1} = \gamma$, which can be rewritten as $\tilde{v}^* - r^* = \mu(\tilde{v}^*, \varepsilon, \gamma, n)$, so that $r^* = \tilde{v}^* - \lambda(\tilde{v}^*)$ must hold.[24] ■

We can now analyze what implications anticipated joy and anger have for the optimal reserve price. As might have been expected, the joy of winning increases the optimal reserve price. Even though the pain of losing increases the (remaining) bids in a second-price auction (and the bids of buyers with high valuations in a first-price auction), the optimal reserve price goes down when anger is anticipated, because negative emotions already reduce the buyers' willingness to participate in the auction.

Proposition 4 *The optimal reserve price r^* is increasing in ε and decreasing in γ.*

Proof. From the implicit definition of the optimal threshold valuation $\tilde{v}^*$ in Proposition 3, it follows that

$$\frac{d\tilde{v}^*}{d\varepsilon} = -\frac{1}{\lambda_{\tilde{v}}(\tilde{v}^*) - \mu_{\tilde{v}}(\tilde{v}^*)} < 0,$$

where subscripts denote partial derivatives. The denominator is positive, because the monotone hazard rate condition implies that $\lambda_{\tilde{v}}(\tilde{v}^*)$ is positive and $\mu_{\tilde{v}}(\tilde{v}^*) = -\gamma(n-1)F(\tilde{v}^*)^{-n}f(\tilde{v}^*) < 0$. Since $[1 - F(\tilde{v}^*)]/f(\tilde{v}^*)$ is decreasing, this implies that $dr^*/d\varepsilon > 0$.

Similarly, it follows that

$$\frac{d\tilde{v}^*}{d\gamma} = \frac{1 - F(\tilde{v}^*)^{n-1}}{[\lambda_{\tilde{v}}(\tilde{v}^*) - \mu_{\tilde{v}}(\tilde{v}^*)]F(\tilde{v}^*)^{n-1}} > 0$$

[24] For completeness, notice that the proposition also holds if $\varepsilon = 0$ and $\gamma > 0$. If $\gamma = 0$ and $\varepsilon \geq 0$, then two cases have to be distinguished: (i) If $\varepsilon < 1/f(0) - \underline{v}$, then the optimal reserve price is as characterized in Proposition 3. (ii) If $\varepsilon \geq 1/f(0) - \underline{v}$, then the optimal reserve price is ε.

and thus $dr^*/d\varepsilon < 0$. ■

Let us now turn to the impact of the number n of potential buyers on the optimal reserve price. Interestingly, it turns out that the optimal reserve price r^* is decreasing in the number of buyers. This result is in stark contrast to the standard result, according to which the optimal reserve price r_0 is independent of the number of buyers. Intuitively, when buyers anticipate negative emotions, they are less inclined to participate. In order to increase participation, it makes sense to reduce the reserve price. This effect is the more important, the larger is the number n of potential buyers.

Proposition 5 *The optimal reserve price r^* is decreasing in n.*

Proof. In the proof, we treat n as if it were a continuous variable. From the implicit definition of $\tilde{v}^*$ in Proposition 3 it follows that

$$\frac{d\tilde{v}^*}{dn} = \frac{\mu_n(\tilde{v}^*)}{\lambda_{\tilde{v}}(\tilde{v}^*) - \mu_{\tilde{v}}(\tilde{v}^*)} > 0,$$

where the denominator is positive (see the proof of Proposition 4) and $\mu_n(\tilde{v}^*) = -\gamma F(\tilde{v}^*)^{1-n} \ln F(\tilde{v}^*) > 0$. As a consequence, it follows that $dr^*/dn < 0$ must hold. ■

Remark 1 *Even if ε and γ are very small, the optimal reserve price r^* can be significantly smaller than the reserve price r_0 that is optimal in the absence of emotions, provided the number of potential buyers is sufficiently large.*

For example, consider the uniform distribution on the unit interval, so that $r_0 = 0.5$. Assume that $\varepsilon = \gamma = 0.01$. If we measured payoffs in 100 dollar units, this would mean that while the expected valuation is 50 dollars, the monetary equivalent of a bidder's joy of winning or his anger when he loses is only 1 dollar. If there are only two potential buyers, the effects of joy and anger just cancel out, so that the optimal reserve price is still $r^* = r_0 = 0.5$. However, if $n = 25$, then the optimal reserve price is significantly reduced to $r^* \approx 0.162$.

6 Concluding remarks

It seems to be fair to say that most human beings are excited when they win and they are sad when they lose. We simultaneously include such positive and negative emotions in the standard auction model. We certainly do not claim that the simple way in which we have tried to capture joy and anger in our model is the ultimate way to introduce emotions into auction theory. Clearly (but also at the cost of additional complexity), there might be private information about emotions, emotions might dependent on various variables such as the winning bid, the second-highest bid, the number of bidders, etc. However, we think it is interesting to see that the present simple extension of the standard auction model can simultaneously rationalize patterns of overbidding and underbidding in first-price auctions and second-price auctions observed in the earlier experimental literature. We also study optimal reserve prices and show that, in contrast to the standard result, in the presence of emotions the optimal reserve price might be decreasing in the number of bidders. As we are not aware of any conclusive experimental evidence regarding how the optimal reserve price varies with the number of bidders, this question might be an interesting topic for future experimental research.

7 References

Andreoni, J., Che, Y.-K., and Kim, J. (2007). Asymmetric information about rivals' types in standard auctions: an experiment. Games and Economic Behavior, vol. 59, pp. 240–259.

Camerer, C.F. and Loewenstein, G. (2003). Behavioral economics: past, present, future. In C.F. Camerer, G. Loewenstein and M. Rabin, eds., Advances in Behavioral Economics, Princeton University Press.

Camille, N., Coricelli, G., Sallet, J., Pradat-Diehl, P., Duhamel, J.-R., and Sirigu, A. (2004). The involvement of the orbitofrontal cortex in the experience of regret. Science, vol. 304, pp. 1167–1170.

Caplin, A. and Leahy, J. (2001). Psychological expected utility theory and anticipatory feelings. Quarterly Journal of Economics, vol. 116, pp. 55–79.

Cohen, J.D. (2005). The vulcanization of the human brain: a neural perspective on interactions between cognition and emotion. Journal of Economic Perspectives, vol. 19, pp. 3–24.

Cooper, D.J. and Fang, H. (2007). Understanding overbidding in second price auctions: an experimental study. Economic Journal, forthcoming.

Cox, J.C., Smith, V.L., and Walker J.M. (1983). Tests of a heterogenous bidders theory of first price auctions. Economics Letters, vol. 12, pp. 207–212.

Cox, J.C., Smith, V.L., and Walker J.M. (1985). Experimental development of sealed-bid auction theory; calibrating controls for risk aversion. American Economic Review, vol. 75, pp. 160–165.

Cox, J.C., Smith, V.L., and Walker J.M. (1988). Theory and individual behavior of first-price auctions. Journal of Risk and Uncertainty, vol. 1, pp. 61–99.

Cox, J.C., Smith, V.L., and Walker J.M. (1992). Theory and misbehavior of first-price auctions: comment. American Economic Review, vol. 82, pp. 1392–1412.

Crawford, V.P. and Iriberri, N. (2007). Level-k auctions: can a non-equilibrium model of strategic thinking explain the winner's curse and overbidding in private-value auctions? Econometrica, forthcoming.

Elster, J. (1998). Emotions and economic theory. Journal of Economic Literature, vol. 36, pp. 47–74.

Engelbrecht-Wiggans, R. (1989). The effects of regret on optimal bidding in auctions. Management Science, vol. 35, pp. 685–692.

Fehr, E. and Schmidt, K.M. (2003). Theories of fairness and reciprocity - evidence and economic applications. In M. Dewatripont, L. Hansen and S. Turnovsky, eds., Advances in Economics and Econometrics, Eighth World Congress of the Econometric Society, Vol. 1, Cambridge: Cambridge University Press, pp. 208–257.

Filiz, E. and Ozbay, E.Y. (2007). Auctions with anticipated regret: theory and experiment. American Economic Review, forthcoming.

Fudenberg, D. and Levine, D.K. (2006). A dual self model of impulse control. American Economic Review, vol. 96, pp. 1449–1476.

Goeree, J.K., Holt, C.A., and Palfrey, T.R. (2002). Quantal response equilibrium and overbidding in private-value auctions. Journal of Economic Theory, vol. 104, pp. 247–272.

Harstad, R.M. (2000). Dominant strategy adoption and bidders' experience with pricing rules. Experimental Economics, vol. 3, pp. 261–280.

Harrison, G.W. (1989). Theory and misbehavior of first-price auctions. American Economic Review, vol. 79, pp. 749–762.

Harrison, G.W. (1992). Theory and misbehavior of first-price auctions: reply. American Economic Review, vol. 82, pp. 1426–1443.

Ivanova-Stenzel, R. and Sonsino, D. (2004). Comparative study of one-bid versus two-bid auctions. Journal of Economic Behavior and Organization, vol. 54, pp. 561–583.

Jehiel, P., Moldovanu, B., and Stacchetti, E. (1999). Multidimensional mechanism design for auctions with externalities. Journal of Economic Theory, vol. 85, pp. 258–293.

Kagel, J.H. (1995). Auctions: a survey of experimental research. In Kagel, J.H. and A. Roth, eds., Handbook of Experimental Economics, Princeton: Princeton University Press, pp. 501–586.

Kagel, J.H., Harstad, R.M., and Levin, D. (1987). Information impact and allocation rules in auctions with affiliated private values: a laboratory study. Econometrica, vol. 55, pp. 1275–1304.

Kagel, J.H. and Levin, D. (1993). Independent private value auctions: bidder behavior in first-, second-, and third-price auctions with varying numbers of bidders. Economic Journal, vol. 103, pp. 868–879.

Kahneman, D. and Tversky, A. (1979). Prospect theory: an analysis of decision under risk. Econometrica, vol. 47, pp. 263–291.

Kirchkamp, O. and Reiss, P. (2004). The overbidding-myth and the underbidding-bias in first-price auctions. Discussion Paper 04–32, SFB 504, Mannheim.

Kirchkamp, O. and Reiss, P. (2006). Expectations in first-price auctions. Discussion Paper, University of St. Andrews.

Krishna, V. (2002). Auction Theory, San Diego: Academic Press.

Loewenstein, G.F., Weber, E.U., Hsee, C.K., and Welch, N. (2001). Risk as feelings. Psychological Bulletin, vol. 127, pp. 267–286.

Matthews, S.A. (1995). A technical primer on auction theory I: independent private values. Discussion Paper No. 1096, Northwestern University.

McAfee, R.P. and McMillan, J. (1987). Auctions and bidding. Journal of Economic Literature, vol. 25, pp. 699–738.

Mellers, B.A., Schwartz, A., and Ritov, I. (1999). Emotion-based choice. Journal of Experimental Psychology: General, vol. 128, pp. 332–345.

Menezes, F.M. and Monteiro, P.K. (2000). Auctions with endogenous participation. Review of Economic Design, vol. 5, pp. 71–89.

Menezes, F.M. and Monteiro, P.K. (2005). An Introduction to Auction Theory, Oxford: Oxford University Press.

Morgan, J., Steiglitz, K., and Reis, G. (2003). The spite motive and equilibrium behavior in auctions. Contributions to Economic Analysis & Policy, vol. 2, article 5.

Myerson, R.B. (1981). Optimal auction design. Mathematics of Operations Research, vol. 6, pp. 58–73.

Rabin, M. (2002). A perspective on psychology and economics. European Economic Review, vol. 46, pp. 657–685.

Riley, J.G. and Samuelson, W.F. (1981). Optimal auctions. American Economic Review, vol. 71, pp. 381–392.

Rosenkranz, S. and Schmitz, P.W. (2007). Reserve prices in auctions as reference points. Economic Journal, vol. 117, 637–653.

Samuelson, W.F. (1985). Competitive bidding with entry costs. Economics Letters, vol. 17, pp. 53–57.

Sanfey, A.G., Rilling, J.K., Aronson, J.A., Nystrom, L.E., and Cohen, J.D. (2003). The neural basis of economic decision-making in the ultimatum game. Science, vol. 300, pp. 1755–1758.

Tirole, J. (2002). Rational irrationality: some economics of self-management. European Economic Review, vol. 46, pp. 633–655.