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ABSTRACT

Task Scheduling and Moral Hazard*

We study a two-period moral hazard problem with risk-neutral and wealth-constrained agents and three identical tasks. We show that the allocation of tasks over time is important if there is a capacity constraint on the number of tasks that can be performed in one period. We characterize the optimal schedule of tasks over time and the optimal assignment of tasks to agents conditional on the outcomes of previous tasks. In particular, we show that delaying tasks is optimal if and only if the effect of an agent's effort on the probability of success is relatively low.

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Keywords: hidden actions, job design, limited liability and task assignment

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1 Introduction

Almost every business project can be viewed as a multitask activity which is extended over time and which is subject to some technological or human capacity constraints. This paper studies the optimal allocation of tasks over time and the assignment of tasks to agents in a two-period moral hazard problem with risk-neutral and wealth-constrained agents. In our model, there are three identical tasks and there is a capacity constraint according to which one agent can perform at most two tasks in one period. We demonstrate that the allocation of tasks over time matters, even though this would not be the case in the absence of moral hazard. In particular, we characterize conditions under which delaying tasks is optimal due to incentive considerations.

Our model can be used to address several questions about the optimal organization of projects. For example, consider an entrepreneur who is in the business of painting houses. She employs workers on a case by case basis to fulfill orders she receives. It takes one worker half a day to paint an average-sized building. The quality of the outcome, which will be judged by the client, depends on the amount of the worker's effort invested into preparing surfaces, mixing of ingredients, and painting. Furthermore, there is always some uncertainty about whether the client will be satisfied with the job. Now imagine that there is an order for three buildings that have to be painted within two days, and the client inspects the work done at the end of a day. What is the optimal course of action for the entrepreneur? *Scheduling.* Should she schedule all three buildings to be painted in one day by employing several agents? Or is it better to employ one agent at a time and spread the work over two days? If the latter is the case, is it better to paint two houses on the first day and one house on the second day or vice versa? *Assignment of tasks.* Should the worker continue to paint the remaining buildings if the client is unhappy with his work or should he be replaced with another worker? *Wages.* How should the worker's remuneration depend on the satisfaction of the client?

The optimal contract in our model is determined by three factors. The first factor is well-known from the multitask agency literature.[1] The principal would like to assign as many projects as possible to one agent, because this reduces the rents that have to be paid in order

[1] See e.g. Che and Yoo (2001), Laux (2001), and Laffont and Martimort (2002, ch. 5). Related observations have also been made by Baron and Besanko (1992), Dana (1993), Hirao (1994), Gilbert and Riordan (1995), and Jackson and Sonnenschein (2007).

to induce high effort. The second factor is that rents can also be saved if the agent is retained whenever the first period was a success and rewarded only if both periods were successful. In the second period the effort costs endured previously are sunk and therefore the same rent can be used twice to motivate the agent in both periods.[2] The novel and decisive third factor is that the probability that the agent is replaced and hence additional rents have to be paid to a new agent in the second period depends on the number of tasks scheduled in the first period.

The interaction of these three factors results in a trade-off between paying lower expected rents to an agent employed in the first period and a higher probability of paying additional rents to a new agent in the second period. This trade-off is non-trivial because the *total* rent in the first period, i.e. the agent's wage minus his effort costs, which is required for two tasks is *smaller* than the rent which is required for one task. In other words, allocating one more first-period task to an agent does not require paying any additional rent for this task and in fact reduces the rent paid for the first task. Therefore, if two tasks are assigned to an agent in the first period, the expected rents of this agent can be made relatively low. At the same time, however, the probability that the agent does not succeed on all tasks is relatively high. If in this case the agent is replaced, additional rents must be paid to a new agent. If the original agent is retained, his first-period incentives are dulled, so he must be motivated by a larger rent.

There are several difficulties in the analysis that are absent in standard models, which assume a given number of tasks per period. First, the sets of the binding incentive compatibility and limited liability constraints differ depending on how many tasks are assigned to an agent and how they are distributed over time. Second, it is tedious to identify the optimal replacement of agents conditional on the first-period outcomes. We demonstrate that the analysis can be performed in terms of the agents' information rents, which significantly simplifies the exposition and the proofs.

Our main contribution is to show how incentive considerations caused by moral hazard can determine the distribution of tasks over time. The optimal assignment of tasks in principal-

[2] This observation is related to the idea of deferred compensation (see Lazear, 1981) and the efficiency wage literature (see Shapiro and Stiglitz, 1984). See also the recent contributions of Schmitz (2005) and Tamada and Tsai (2007).

agent problems with moral hazard is by now well-understood and has been studied by several authors such as Holmström and Milgrom (1991), Itoh (1992, 1994), and Hemmer (1995).[3] Nevertheless, to the best of our knowledge, this is the first paper that simultaneously considers assignment and scheduling of tasks over time. Most of the existing literature on the assignment of tasks under moral hazard considers risk-averse agents and restricts attention to a subset of feasible contracts. In contrast, following some of the more recent contributions such as Che and Yoo (2001) and Laux (2001), we study a model with risk-neutral and wealth-constrained agents.[4] An advantage of this model is that it is more tractable and can be solved without imposing restrictions on the set of feasible contracts.

Finally, O'Donoghue and Rabin (1999) study the optimal allocation of tasks over time by an individual whose intertemporal preferences are described by a hyperbolic-discounting function. They demonstrate that an individual might delay performing a task because of her time-inconsistency. In contrast, in our model the principal might prefer to postpone performing two tasks until the second period in order to reduce her agency costs.

The remainder of the paper is organized as follows. In the next section, the model is introduced. In section 3, the incentive compatibility constraints are characterized. Our main results are presented in section 4. Concluding remarks follow in section 5. Some technical details have been relegated to the appendix.

2 The model

Consider a principal who wants to have three tasks accomplished within two periods of time. The principal cannot accomplish the tasks by herself. There is a set of (three or more) identical agents, each of whom could work on up to two tasks in a given period.[5] At most one agent can work on any given task, and each task can be performed only once. All parties

[3] Task assignment has also been studied in models with adverse selection, see e.g. Dana (1993), Gilbert and Riordan (1995), and Severinov (2005)

[4] This version of the moral hazard problem has also been used by, e.g., Crémer (1995), Baliga and Sjöström (1998), and Tirole (1999, 2001). See also the important early work of Innes (1990) and see Laffont and Martimort (2002, ch. 4) for an excellent textbook exposition.

[5] It will turn out that in the optimal contract, the principal will not make use of more than two agents. We will discuss the case in which there is only one agent available in Remark 2.

are risk-neutral. The agents have no wealth and their reservation utilities are given by zero.

At the beginning of the first period, the principal can offer non-renegotiable contracts to the agents. We impose no ad hoc restrictions on the class of feasible contracts; i.e., there is complete contracting in the sense of Tirole (1999).

An agent performing task $\tau \in \{1,2,3\}$ can exert either low or high effort, $e_\tau \in \{0,1\}$, which is unobservable. An agent's disutility from exerting effort e_τ on task τ is given by $e_\tau c$, where $c > 0$. The verifiable outcome of task τ is either good ($x_\tau = 1$) or bad ($x_\tau = 0$). The probability of a good outcome is p_1 if the agent exerts high effort ($e_\tau = 1$) and p_0 otherwise, where $0 < p_0 < p_1 \leq 1$. Let $X = \{0,1\}^3$ denote the set of possible outcomes of all three tasks and let x denote its generic element.

Next, let T_I denote the set of tasks performed in the first period and let T_I^a denote its subset assigned to agent a.[6] Each T_I induces a set of outcomes $X_I = \{0,1\}^{|T_I|}$ that can be observed at the end of the first period. For example, if two tasks are performed in the first period, then $X_I = \{0,1\}^2$. Let x_I be a generic element of this set.

Similarly, let T_{II} be the set of tasks performed in the second period and let $x_{II} \in X_{II} = \{0,1\}^{|T_{II}|}$ denote the second-period outcome. The assignment of tasks to agents in the second period can depend on the outcome of the first period. Hence, let $T_{II}^a(x_I)$ denote the set of tasks assigned to agent a conditional on the first-period outcome x_I.

At the end of the second period, the contractually specified payments are made. The wage payment to agent a is given by $w^a(x) \geq 0$. A contract (or a mechanism) consists of a collection of assignments and wage functions, $m = [T_I^a, T_{II}^a(\cdot), w^a(\cdot)]$, for all agents a to whom tasks are assigned.

Our goal is to characterize how the principal can induce high effort on all three tasks at minimal wage costs. It will turn out that the principal does not schedule more than two tasks in one period. Then, without loss of generality, let task $\tau = 1$ be allocated to the first period and task $\tau = 2$ to the second period. Besides finding the optimal wage schemes, the principal

[6] We restrict attention to deterministic assignments. This is without loss of generality, because the set of optimal stochastic assignments always contains a deterministic one: First, it is clear that random assignment of tasks in the first period cannot improve the payoff of the principal because the assignment takes place before the agents make any choices. Second, in Remark 1 we demonstrate that the principal's payoff cannot be improved by a stochastic assignment of tasks in the second period conditional on the outcome in the first period.

thus has to make the following organizational decisions.

- Scheduling. Should task $\tau = 3$ be scheduled to the first period or to the second period (see Figure 1)?
- Task assignment. Who should be in charge of the tasks? For example, should one agent be responsible for all tasks, or should different agents work on different tasks? Should the assignment of tasks to agents in the second period depend on the outcome of the first period?

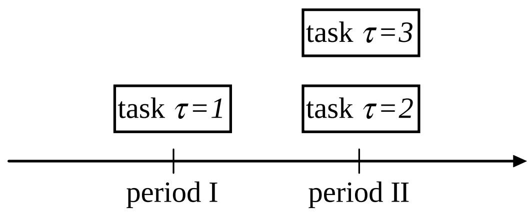

Figure 1a. One task in the first period.

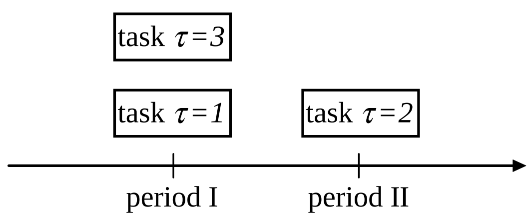

Figure 1b. Two tasks in the first period.

Note that the technology has been deliberately kept as simple as possible, so that in a first-best world (i.e., in the absence of moral hazard), the principal would be completely indifferent with regard to these organizational decisions. The principal would have to pay $3c$ in order to implement high effort, regardless of the scheduling and regardless of the assignment of agents to tasks. Hence, if we find a strict preference of the principal for a particular scheduling and a particular task assignment, then this result must be due to incentive considerations only.

3 Incentive compatibility

Let e_I denote the effort levels associated with the tasks that are scheduled in the first period. Hence, $e_I = e_1$ if $T_I = \{1\}$, $e_I = (e_1, e_3)$ if $T_I = \{1, 3\}$, and $e_I = (e_1, e_2, e_3)$ if $T_I = \{1, 2, 3\}$. Similarly, let e_{II} be the second-period effort levels.

Agent a's second-period expected payoff given the first-period outcome x_I and the second-period effort e_{II} is

$$U_{II}^a(x_I, e_{II}) = \sum_{x_{II} \in X_{II}} \Pr\{x_{II}|e_{II}\} w^a(x) - \sum_{\tau \in T_{II}^a(x_I)} e_\tau c.$$

The ex ante expected payoff of agent a is given by

$$U_I^a(e_I, e_{II}) = \sum_{x_I \in X_I} \Pr\{x_I|e_I\} U_{II}^a(x_I, e_{II}) - \sum_{\tau \in T_I^a} e_\tau c.$$

Notice that the first-period effort costs are sunk in the second period. Therefore, these costs are included in the ex ante payoff but not in the second-period payoff.

We are looking for a contract that always implements high effort, $e_1^* = e_2^* = e_3^* = 1$, at minimal costs for the principal. Let $e_{II} = (e_{II}^a, e_{II}^{-a})$, where e_{II}^a denotes the effort levels associated with the second-period tasks that are assigned to agent a. Furthermore, let e_{II}^*, e_{II}^{a*}, and e_{II}^{-a*} denote high effort levels. Define similarly the effort levels associated with the first-period tasks, $e_I = (e_I^a, e_I^{-a})$, e_I^*, e_I^{a*}, and e_I^{-a*}.

In an equilibrium in which all agents exert high effort, an agent a who is in charge of second-period tasks (i.e., $|T_{II}^a(x_I)| \neq 0$) finds it optimal to exert high effort if

$$U_{II}^a(x_I, e_{II}^{a*}, e_{II}^{-a*}) \geq U_{II}^a(x_I, e_{II}^a, e_{II}^{-a*}) \quad \text{for all } e_{II}^a. \tag{1}$$

Similarly, an agent a who is in charge of first-period tasks (i.e., $|T_I^a| \neq 0$) finds it optimal to exert high effort if

$$V_I^a := U_I^a(e_I^{a*}, e_I^{-a*}, e_{II}^*) \geq U_I^a(e_I^a, e_I^{-a*}, e_{II}^*) \quad \text{for all } e_I^a. \tag{2}$$

We call a contract incentive compatible if conditions (1) and (2) are satisfied. The expected payment from the principal to all agents in equilibrium is equal to

$$\sum_{a \in A} \sum_{x \in X} \Pr\{x|e_I^*, e_{II}^*\} w^a(x) = \sum_{a \in A} V_I^a + 3c.$$

A contract is optimal if it minimizes the principal's agency costs given by the sum of the agents' ex ante expected rents, $\sum_{a \in A} V_I^a$, among all incentive compatible contracts.

4 The main results

Let us analyze first what happens if the principal does not make use of any dynamic incentive considerations.

Proposition 1 *Consider an agent who is hired for one period only and who is induced to work hard.*

(i) If the agent works on one task, his expected rent is

$$V_1^* = \frac{p_0 c}{p_1 - p_0}.$$

(ii) If the agent works on two tasks, his expected rent is

$$V_2^* = \frac{2p_0^2 c}{p_1^2 - p_0^2} < V_1^*.$$

Proof. (i) Consider an agent who works on one task only. Let w_0 and w_1 be the agent's wage after failure and after success respectively. Then, the incentive compatibility of high effort can be written as

$$V_1 = p_1 w_1 + (1 - p_1)w_0 - c \geq p_0 w_1 + (1 - p_0)w_0,$$

or, equivalently,

$$w_1 \geq \frac{c}{p_1 - p_0} + w_0. \tag{3}$$

It follows that V_1 is minimized by setting $w_0 = 0$ and $w_1 = \frac{c}{p_1 - p_0}$, which gives $V_1 = V_1^*$.

(ii) Consider now an agent who works on two tasks. Let w_{00}, w_{10}, w_{01}, and w_{11} be the agent's wage after failure on both tasks, after success on one of the tasks, and after success on both tasks, respectively. The incentive compatibility constraint of high effort with respect to the deviation to exerting low effort on both tasks is given by

$$V_2 = p_1^2 w_{11} + p_1(1-p_1)(w_{10}+w_{01}) + (1-p_1)^2 w_{00} - 2c \geq p_0^2 w_{11} + p_0(1-p_0)(w_{10}+w_{01}) + (1-p_0)^2 w_{00},$$

or, equivalently,

$$w_{11}(p_1^2 - p_0^2) \geq 2c + (w_{10} + w_{01})(p_1 - p_0)(p_1 + p_0 - 1) + w_{00}((1 - p_0)^2 - (1 - p_1)^2), \tag{4}$$

If $p_1 + p_0 \geq 1$, all the terms on the right hand side of the above expression are non-negative and V_2 is minimized by

$$w_{00} = w_{10} = w_{01} = 0, \qquad w_{11} = \frac{2c}{p_1^2 - p_0^2}, \tag{5}$$

which gives $V_2 = V_2^*$.

If, however, $p_1 + p_0 < 1$, higher values of w_{10} and w_{01} may potentially allow to save on w_{11}. In this case, the linearity of V_2 and the incentive constraint in wages imply that V_2 is minimized either by (5), in which case its value is V_2^*, or by

$$w_{00} = w_{11} = 0, \qquad w_{10} + w_{01} = \frac{2c}{(p_1 - p_0)(1 - (p_1 + p_0))},$$

in which case, however, $V_2 = \frac{2p_0(1-p_0)c}{(p_1-p_0)(1-(p_1+p_0))} > V_2^*$. Finally, it is straightforward to check that if the wages are given by (5), then the incentive compatibility constraints of high effort with respect to the deviation of exerting low effort on one task only (rather than on both tasks) are also satisfied. ■

Under our assumptions, if all three tasks are scheduled in one period, then it is optimal for the principal to hire two agents and the agency costs are $V_2^* + V_1^*$. Indeed, because one agent can perform at most two tasks in one period, the principal has to hire at least two agents. If two agents are hired, so that one of them works on two tasks and the other agent works on one task, the agency costs are $V_2^* + V_1^*$. If three agents are hired, each of whom works on one task, then the total agency costs are $3V_1^* > V_2^* + V_1^*$. By assigning two tasks to one agent, the principal saves on the rent that has to be paid to the agents.

The same agency costs, $V_2^* + V_1^*$, will be incurred by the principal if one of the agents is assigned to perform one task in one period and another agent is assigned to perform two tasks in the other period. As long as the agent who works in the first period is never retained, nothing could be gained if his wage were conditioned on the outcome of the second period. In particular, it would make no difference whether two tasks are scheduled in the first period and one task in the second period or vice versa.

However, the principal can decrease the agency costs if she makes use of dynamic incentive considerations and sometime employes the same agent in both periods. Specifically, consider an agent who works on one task in the first period. In order to implement high effort in the first period, the agent must be rewarded in case of a first-period success. Now suppose that the payment of this reward is postponed to the end of the second period and the agent gets the reward only if (i) there is a first-period success and (ii) the agent also is successful on both remaining tasks in the second period. It turns out that in this way the rent V_1^* that is necessary to implement high effort in the first period is large enough to also induce the agent to exert high efforts in the second period.

Similarly, consider an agent who works on two tasks in the first period. Now suppose that the agent gets a reward only if (i) he succeeds on both first-period tasks and (ii) the agent also is successful on the remaining task in the second period. The rent V_2^* that is necessary to implement high efforts in the first period is large enough to also induce the agent to exert high effort in the second period.

Proposition 2 *(i) Consider an agent who works on one task in the first period and who is retained if and only if the first-period task was a success. The expected rent that must be given to the agent in order to implement high efforts on all three tasks is V_1^*.*

(ii) Consider an agent who works on two tasks in the first period and who is retained if and only if both first-period tasks were successful. The expected rent that must be given to the agent in order to implement high efforts on all three tasks is V_2^*.

Proof. (i) Consider an agent who works on one task in the first period. Let w_1 and w_0 be the agent's expected continuation payoffs (ignoring the effort cost sunk in the first period) conditional on success and failure in the first period, respectively. Then, the incentive compatibility constraint for high effort in the first period is given by (3) and the argument in the proof of Proposition 1 implies that the agent's rent cannot be lower than V_1^*. Now set the agent's wage in the second period equal to $\frac{1+2(p_1-p_0)}{(p_1-p_0)p_1^2}c$ if all tasks are successful and zero otherwise. It is straightforward to check that the incentive constraints for high effort are satisfied in both periods and the rent is V_1^*.

(ii) The argument for the agent who works on two tasks in the first period is analogous. In this case, the agent's wage is $\frac{2+p_1^2-p_0^2}{(p_1^2-p_0^2)p_1}c$ if all tasks are successful and zero otherwise. ■

Let us now calculate the principal's agency costs if the agent is retained in the second period after success in the first period.

- **Schedule "1-2."** Suppose that the principal schedules one task in the first period. If the first period is a success, the agent is retained and the principal must pay the rent V_1^* only. If the first period is a failure, the agent responsible for the first period gets nothing and a different agent is hired for the two remaining tasks in the second period. Hence, the total agency costs are

$$V_1^* + V_2^* - p_1 V_2^*,$$

where the last term is the retention benefit (i.e., the rent saved by the principal if the first-period agent is not replaced because the first-period task was a success).

- **Schedule "2-1a."** Suppose that the principal schedules two tasks in the first period. Assume that the agent responsible in the first period is retained if and only if both first-period tasks are successful. In this case, the principal must pay him the rent V_2^* only. If at least one first-period task was a failure, the responsible agent gets nothing and a different agent is hired for the remaining task in the second period. Then the total agency costs are

$$V_2^* + V_1^* - p_1^2 V_1^*,$$

where again the last term is the retention benefit (i.e., the rent saved by the principal if the first-period agent is not replaced because he succeeded on both first-period tasks).

It is obvious that there is a trade-off between the schedules "1-2" and "2-1a." If only one task is scheduled in the first period, then the magnitude of the retention benefit is smaller ($V_2^* < V_1^*$), but it is experienced with a larger probability ($p_1 > p_1^2$) than in the case in which two tasks are scheduled in the first period. Clearly, the principal prefers schedule "1-2" whenever $p_1 V_2^* > p_1^2 V_1^*$, which can be rewritten as follows.

Proposition 3 *Schedule "1-2" is more profitable for the principal than schedule "2-1a" whenever* $p_1 < \frac{1}{2}\left[\sqrt{p_0^2 + 8p_0} - p_0\right]$.

In other words, if p_1 is sufficiently large (i.e., close to 1), then two tasks should be scheduled in the first period: As $p_1 \to 1$, the probability that a retention benefit is experienced is almost equal in both cases, so that only the magnitude matters. Hence, schedule "2-1a" is more profitable, because $V_2^* < V_1^*$. In contrast, if p_1 is small (i.e., close to p_0), then one task should be scheduled in the first period: As $p_1 \to p_0$, we have $V_1^*/V_2^* = (p_0 + p_1)/2p_0 \to 1$. The magnitude of the retention benefit is almost the same for both schedules, so that the probability effect dominates and schedule "1-2" is more profitable (because $p_1 > p_1^2$).

The following schedule is an alternative to schedule "2-1a" that also makes use of dynamic incentives and assigns two tasks to one agent in the first period:

- **Schedule "2-1b."** Suppose that the principal schedules two tasks in the first period. Assume that the agent responsible in the first period is retained if and only if at least one of the first-period tasks was successful. If both first-period task were failures, then the responsible agent gets nothing and a different agent is hired for the remaining task in the second period.

In schedule "2-1b," the agent can earn a positive rent even if one of the first-period tasks is a failure. Hence, the agency costs associated with the agent responsible in the first period are larger than V_2^* (the agency costs associated with the first-period agent in schedule "2-1a").

Proposition 4 *Consider an agent who works on two tasks in the first period and who is retained if and only if at least one first-period task was a success. The expected rent that must be given to the agent in order to implement high efforts on all three tasks is* $V_2^* + 2p_0p_1V_1^*/(p_0 + p_1)$.

Proof. Let w_{00}, w_{10}, w_{01}, and w_{11} be the agent's continuation payoffs in the second period (ignoring the effort cost sunk in the first period) after failure on both tasks, after success on one of the tasks, and after success on both tasks, respectively. Because high effort has to be incentive compatible in the second period after success on at least one of the tasks in the first period, we necessarily have $w_{11}, w_{10}, w_{01} \geq V_1^*$. The first period incentive compatibility constraint of high effort with respect to the deviation to exerting low effort on both tasks is given by (4). The agency cost is then minimized by setting

$$w_{11} = \frac{2c}{p_1^2 - p_0^2} + \frac{2V_1^*(p_1 + p_0 - 1)}{p_1 + p_0}, \qquad w_{10} = w_{01} = V_1^*, \qquad w_{00} = 0,$$

which gives the result. ■

Hence, in the case of schedule "2-1b" the total agency costs are

$$V_2^* + \frac{2p_0p_1}{p_0 + p_1}V_1^* + V_1^* - [2p_1(1 - p_1) + p_1^2]V_1^*,$$

where the last term is again the principal's retention benefit (i.e., the rent she saves when at least one first-period task was successful, which happens with probability $2p_1(1 - p_1) + p_1^2$). In other words, in comparison to schedule "2-1a," the principal now gives a larger rent to the agent responsible in the first period in order to gain the retention benefit more often.

Note that the agency costs can be rewritten as

$$V_2^* + 2p_1^2\frac{p_1 + p_0 - 1}{p_0 + p_1}V_1^* + V_1^* - p_1^2V_1^*.$$

Hence, a comparison with the agency costs $V_2^* + V_1^* - p_1^2 V_1^*$ in the case of schedule "2-1a" immediately leads to the following result.

Proposition 5 *Schedule "2-1b" is more profitable for the principal than schedule "2-1a" whenever* $p_1 + p_0 < 1$.

However, even when schedule "2-1b" is better than "2-1a," it may still happen that it is more profitable to schedule only one task in the first period. This is the case if

$$p_1 V_2^* > \left(p_1^2 - 2p_1^2 \frac{p_1 + p_0 - 1}{p_0 + p_1}\right) V_1^*.$$

Note that when $p_1 \to p_0$, then the coefficient of V_1 converges to $p_0 - p_0^2$, which is smaller than p_0. Hence, if p_1 is close enough to p_0, then it is again better to schedule only one task in the first period. Specifically, the preceding condition can be rewritten as follows.

Proposition 6 *Schedule "1-2" is more profitable for the principal than schedule "2-1b" whenever* $p_1 < 1 - \frac{1}{2}\left[p_0 + \sqrt{p_0^2 - 12p_0 + 4}\right]$.

Propositions 3, 5, and 6 completely characterize the principal's optimal scheduling and task assignment. Indeed, it is straightforward to show that she cannot do better with any of the remaining conceivable arrangements.

Remark 1 *(i) The principal cannot gain anything if she uses a (potentially stochastic) retention policy different from schedules "1-2", "2-1a", and "2-1b". (ii) Moreover, the principal could not gain anything if she employed two agents in the period in which two tasks are scheduled.*

Proof. (i) We will consider the setting in which the principal assigns two tasks to one agent in the first period. The argument for the other case is analogous. Let w_{00}, w_{10}, w_{01}, and w_{11} be the agent's continuation payoff in the second period (ignoring the effort cost sunk in the first period) after failure on both tasks, after success on one of the tasks, and after success on both tasks, respectively. In addition, denote by q_{00}, q_{10}, q_{01}, and q_{11} the probability of retaining the agent after failure on both tasks, after success on one of the tasks, and after success on both tasks, respectively. Because high effort has to be incentive compatible in the second period if the agent is retained, we necessarily have

$$w_{00} \geq q_{00} V_1^*, \quad w_{10} \geq q_{10} V_1^*, \quad w_{01} \geq q_{01} V_1^*, \quad w_{11} \geq q_{11} V_1^*.$$

The first period incentive compatibility constraint of high effort with respect to the deviation to exerting low effort on both tasks is given by (4). The agency cost in the first period is then minimized by setting

$$w_{11} = \frac{2c}{p_1^2 - p_0^2} + V_1^* \left((q_{10} + q_{01}) \frac{p_1 + p_0 - 1}{p_1 + p_0} + q_{00} \frac{2 - (p_1 + p_0)}{p_1 + p_0} \right),$$

$$w_{10} = q_{10} V_1^*, \quad w_{01} = q_{01} V_1^*, \quad w_{00} = q_{00} V_1^*.$$

The total agency cost is given by the agency cost in the first period plus the additional agency cost in the second period if the agent is not retained after the first period,

$$\left\{ p_1^2 (1 - q_{11}) + p_1(1 - p_1)(2 - q_{10} - q_{01}) + (1 - p_1)^2 (1 - q_{00}) \right\} V_1^*.$$

Now observe that the total agency cost is linear in the probability of retaining the agent, implying that there exists an optimal deterministic retaining policy, $q_{00}, q_{10}, q_{01}, q_{11} \in \{0, 1\}$. Specifically, either schedule "2-1a" ($q_{00} = q_{10} = q_{01} = 0, q_{11} = 1$) or schedule "2-1b" ($q_{00} = 0, q_{10} = q_{01} = q_{11} = 1$) are optimal.

(ii) The principal does not want to employ two agents in one period because she would have to pay higher agency costs to two agents working on one tasks each than to one agent working on two tasks. To demonstrate this formally one derives minimal agency costs for the case when two agents are employed in one period and shows that they are larger than the costs derived for the schedules "1-2", "2-1a", and "2-1b". This can be done, in a straightforward manner, by using arguments similar to the one in the first part of the proof. ■

To summarize, as is illustrated in Figure 2, there are three regions. In region "1-2", only one task is scheduled in the first period. The agent in charge of the first period is replaced in the second period whenever the first-period outcome was bad. In regions "2-1a" and "2-1b", two tasks are scheduled in the first period. In region "2-1a", the agent is replaced if at least one first-period outcome was bad. In region "2-1b", the agent is replaced only if both first-period outcomes were bad.

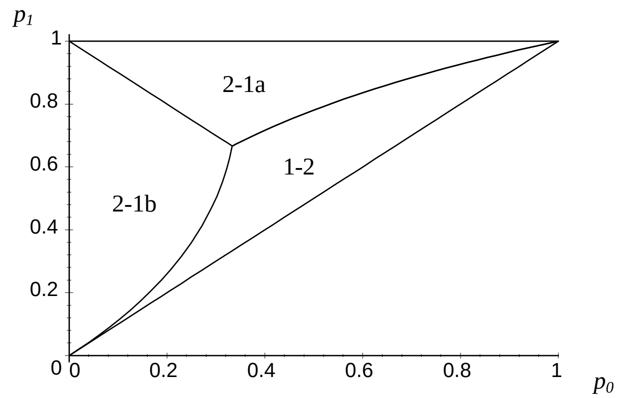

Figure 2. Optimal scheduling.

Finally, we conclude with the following observation. Note that the principal will optimally replace the agent who is in charge of the first period with positive probability. Therefore, one might ask what happens if the principal cannot make use of this possibility (because, say, there is only one qualified agent available or replacing the agent is impossible for legal reasons).

Remark 2 *If there was only one agent, the principal would be indifferent between scheduling two tasks in the first or in the second period.*

Proof. Suppose that the principal schedules two tasks in the first period and one task in the second period. Let w_{00}, w_{10}, w_{01}, and w_{11} be the agent's continuation payoffs in the second period (ignoring the effort cost sunk in the first period) after failure on both tasks, after success on one of the tasks, and after success on both tasks, respectively. Since high effort has to be incentive compatible in the second period, we must have $w_{00}, w_{10}, w_{01}, w_{11} \geq V_1^*$. The first period incentive compatibility constraint of high effort with respect to the deviation to exerting low effort on both tasks is given by (4). The agency cost in the first period is minimized by setting

$$w_{11} = \frac{2c}{p_1^2 - p_0^2} + V_1^*, \qquad w_{10} = w_{01} = w_{00} = V_1^*,$$

giving the total agency cost of $V_1^* + V_2^*$. An analogous argument establishes that if two tasks are scheduled in the second period, the total agency cost is also equal to $V_1^* + V_2^*$. ■

If the principal had no possibility to replace the agent after a bad first-period outcome, her agency costs would always be given by $V_1^* + V_2^*$, regardless of the schedule. Note that these are the same agency costs as in the case in which the principal always replaced the agent. Using always the same agent in both periods does not allow the principal to save rents compared to a situation where the agent is always replaced, because even after a bad first period the agent must be motivated in the second period.

5 Concluding remarks

Throughout, we have deliberately considered a framework where in a first-best world the principal would be indifferent between having a task performed today or tomorrow, so that we could isolate incentive considerations from purely technological effects.

It is straightforward to extend our analysis in order to analyze cases where for technological reasons, the principal would prefer a particular schedule even in a first-best world. In such a modified model, incentive effects could still overcompensate technological considerations, so that we could easily explain "procrastination" (a task is performed in the second period, although the first period would be optimal in a first-best world) and "preproperation" (a task is performed in the first period, although the second period would be optimal in a first-best world). Note that we do not have to assume time-inconsistent preferences to generate these effects.[7]

[7] In contrast, in the literature on hyperbolic discounting (see e.g. O'Donoghue and Rabin, 1999), procrastination and preproperation are explained in single-person decision problems, assuming that preferences are time-inconsistent (with regard to present and future "selfs" of a person).

References

Baliga, S. and Sjöström, T. "Decentralization and Collusion." *Journal of Economic Theory,* Vol. 83 (1998), pp. 196–232.

Baron, D.P. and Besanko, D. "Information, Control, and Organizational Structure." *Journal of Economics and Management Strategy,* Vol. 1 (1992), pp. 237–275.

Che, Y.-K. and Yoo, S.-W. "Optimal Incentives for Teams." *American Economic Review,* Vol. 91 (2001), pp. 525–541.

Crémer, J. "Arm's Length Relationships." *Quarterly Journal of Economics,* Vol. 110 (1995), pp. 275–295.

Dana, J.D., Jr. "The Organization and Scope of Agents: Regulating Multiproduct Industries." *Journal of Economic Theory,* Vol. 59 (1993), pp. 288–310.

Gilbert, R.J. and Riordan, M.H. "Regulating Complementary Products: A Comparative Institutional Analysis." *RAND Journal of Economics,* Vol. 26 (1995), pp. 243–256.

Hemmer, T. "On the Interrelation between Production and Technology, Job Design, and Incentives." *Journal of Accounting and Economics,* Vol. 19 (1995), pp. 209–245.

Hirao, Y. "Task Assignment and Agency Structure." *Journal of Economics and Management Strategy,* Vol. 2 (1994), pp. 299–323.

Holmström, B. and Milgrom, P. "Multitask Principal-Agent Analyses: Incentive Contracts, Asset Ownership, and Job Design." *Journal of Law, Economics, and Organization,* Vol. 7 (1991), pp. 24–52.

Innes, R.D. "Limited Liability and Incentive Contracting with Ex-ante Action Choices." *Journal of Economic Theory,* Vol. 52 (1990), pp. 45–67.

Itoh, H. "Cooperation in Hierarchical Organizations: An Incentive Perspective." *Journal of Law, Economics, and Organization,* Vol. 8 (1992), pp. 321–345.

Itoh, H. "Job Design, Delegation and Cooperation: A Principal-Agent Analysis." *European Economic Review,* Vol. 38 (1994), pp. 691–700.

Jackson, M.O. and Sonnenschein, H.F. "Overcoming Incentive Constraints by Linking Decisions." *Econometrica* (2007), forthcoming.

Laffont, J.-J., and Martimort, D. *The Theory of Incentives: The Principal-Agent Model.* Princeton, N.J.: Princeton University Press, 2002.

Laux, C. "Limited-Liability and Incentive Contracting with Multiple Projects." *RAND Journal of Economics*, Vol. 32 (2001), 514–526.

O'Donoghue, T. and Rabin, M. "Doing It Now or Later." *American Economic Review*, Vol. 89 (1999), 103–124.

Schmitz, P.W. "Allocating Control in Agency Problems with Limited Liability and Sequential Hidden Actions." *RAND Journal of Economics,* Vol. 36 (2005), pp. 318–336.

Severinov, S. "The Value of Information and Optimal Organization." Discussion Paper, 2005.

Shapiro, C. and Stiglitz, J. "Equilibrium Unemployment as a Worker Discipline Device." *American Economic Review,* Vol. 74 (1984), pp. 433–444.

Tamada, Y. and Tsai, T.-S. "Optimal Organization in a Sequential Investment Problem with the Principal's Cancellation Option." *International Journal of Industrial Organization* (2007), forthcoming.

Tirole, J. "Incomplete Contracts: Where Do We Stand?" *Econometrica,* Vol. 67 (1999), pp. 741–781.

Tirole, J. "Corporate Governance." *Econometrica,* Vol. 69 (2001), pp. 1–35.