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ABSTRACT

Non Linear Taylor Rules and Asymmetric Preferences in Central Banking - Evidence from the UK and the US*

Objectives and Methodology: This paper explores theoretically and empirically the view that, due to asymmetric central bank preferences, Taylor rules are often non-linear and that the nature of those asymmetries changes over different policy regimes. Our theoretical model uses a standard New-Keynesian framework to establish equivalence relations between the shape of non-linearities in Taylor rules and asymmetries in monetary policy objectives. We then estimate and test these relations for the UK and the US over various subperiods by means of smooth transition regressions.

Results: There is often evidence in favor of non-linear rules in both countries, and their character changes substantially over subperiods. The pre inflation targeting period in the UK is characterized by a concave rule supporting recession avoidance preferences, while the inflation targeting period is characterized by a convex rule supporting inflation avoidance preferences on the part of monetary policymakers. The inflationary Vietnam War era in the US displays a convex rule supporting inflation avoidance while the stable Greenspan period is characterized by a concave rule supporting recession avoidance on the part of the Fed. Our findings from both countries support the view that reaction functions and the symmetry properties of the underlying loss functions change in line with the main macro problem of the day.

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1 Introduction

The positive theory of monetary policy in developed economies has reached a broad consensus in recent years. Monetary policy is conceptualized as follows: policy authorities minimize a linear combination of the quadratic deviations of inflation and of output from their respective targets (the inflation and the output gaps in what follows), and the main policy instrument is the short term rate of interest[1]. Much of the literature of the last decade has implemented this perspective by estimating "Taylor rules" or reaction functions in which the short term rate is a linear function of the, currently expected, future values of the inflation and of the output gaps.[2] It is well known that, as a theoretical matter, such linear reaction functions are obtained when the expected value of a loss function that is quadratic in the inflation and output gaps is minimized subject to a linear economic structure.

More recently some of the literature has considered the possibility that, as a positive matter, the loss function of monetary policymakers may not be quadratic and consequently the Taylor rules derived from such functions are not necessarily linear. There is both anecdotal as well as systematic evidence to support this view. On the anecdotal side (Blinder (1998, pp. 19, 20) states that "*In most situations the central bank will take far more political heat when it tightens preemptively to avoid higher inflation than when it eases preemptively to avoid higher unemployment*". This suggests that during normal times the central bank (CB) in the US may be more averse to negative than to positive output gaps.[3] We refer to CB loss functions that

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[1]This obviously includes strict inflation targeting as the particular case in which the weight on the output gap is equal to zero.

[2]A partial list of references includes Clarida *et al. (*1998, 2000) Muscatelli *et al.* (2002), Muscatelli and Trecroci (2002).

[3]In fact, notwithstanding its analytical convenience, a quadratic loss function that penalizes equally sized positive and negative output gaps to the same extent does not appear to be realistic. Obviously, policymakers are averse to negative output gaps but it is less obvious that they are, given inflation, equally averse to positive output gaps. They may even, given inflation, be indifferent between different magnitudes of positive output gaps. Cukierman (2000, 2002) shows that in the last case, and in the presence of uncertainty and rational expectations there is an inflation bias even if the output target of the CB is equal to the potential level.

display this type of asymmetry as **recession avoidance preferences** (RAP). In the presence of uncertainty about future shocks such an asymmetry leads the CB to take more precautions against negative than against positive output gaps.

Using a natural rate framework and time series data for the US, Ruge-Murcia (2003) tests this hypothesis empirically and finds that it provides a better explanation for the behavior of US inflation than the Kydland-Prescott (1977), Barro-Gordon (1983) inflation bias model. Using a natural rate framework and cross sectional data for the OECD countries Cukierman and Gerlach (2003) find evidence supporting a half quadratic specification of output gap losses.[4]

On the other hand, during periods of inflation stabilization in which monetary policy-makers are trying to build credibility up, they may be more averse to positive than to negative inflation gaps of equal size. We refer to objective functions that display this type of asymmetry as **inflation avoidance preferences** (IAP). In the presence of uncertainty about future shocks this leads policymakers to react more vigorously to positive than to negative inflation gaps. Although there is no current systematic evidence to support such a view, this might have been the case in the UK after the introduction of inflation targets in 1992 and during the gradual Israeli disinflation in the second half of the nineties.[5]

Asymmetric objectives normally lead to non linear reaction functions. This paper utilizes time series data from the UK and the US to test whether there is evidence to support the view that the interest rate reaction functions in those two countries exhibit non linearities, thereby pointing to asymmetric policy preferences.

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[4]In this specification losses from negative output gaps are quadratic and, given inflation, there are no losses from positive output gaps.

[5]During this period the inflation target in Israel was missed much more frequently from below than from above, lending credence to the view that the Bank of Israel took more precautions against upward than against downward deviations from the target. Nobay and Peel (1998) and Ruge-Murcia (2000) show that in such cases a deflationary bias may arise. Mishkin and Posen (1997) express a similar view for inflation targeters. A somewhat different view appears in Sussman (2007) who interprets the Israeli experience to imply that the Bank of Israel pursued an implicit inflation target that was lower than the one set by government.

To understand the meaning of such non linearities for the loss functions of UK and US monetary policymakers we start, in section 2, by providing a general formulation of preferences in which recession avoidance and inflation avoidance coexist. We then analyze the character of non linearity in the Taylor rules that emerge. This is done for a New-Keynesian economic structure of the type surveyed in Clarida, Gali and Gertler (1999). The main theoretical implication is that, when recession avoidance preferences dominate, the reaction function is concave in both the output and inflation gaps, and when inflation avoidance preferences dominate, the reaction function is convex in both gaps. This result suggests that we may expect to find different patterns of non linearities between normal periods and periods of inflation stabilization.

Section 3 provides an empirical test of these hypotheses by allowing the response of the interest rate to the expected values of both gaps to change gradually with the magnitude of each expected gap. This specification, due to Bacon and Watts (1971) and Seber and Wild (1989), utilizes hyperbolic tangents to formulate the responses of the interest rate to the expected values of the inflation and output gaps. When the response increases with the expected gap the reaction function is convex in the expected gap, and when the response decreases in the expected gap, the reaction function is concave in it. An important advantage of this specification is that the concavity-convexity properties of the reaction function depend on only two parameters that can be estimated by means of the same GMM methods used to estimate linear Taylor rules.

When applied to appropriate subperiods, in the UK and the US, this methodology yields several main results.[6] First, in both the UK and the US non-linear Taylor rules are more prevalent than linear rules. Second, the type of non-linearity changes between subperiods. In particular, the non-linear Taylor rule for the UK in the 1979-1990 period is concave while the one for the 1992-2005 period is convex. This supports the view that prior to the introduction of inflation targeting in 1992, RAP dominated whereas, after the introduction of inflation targets,

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[6] Our sample period is 1979-3 to 2005-4 for the UK and 1960-1 to 2005-4 for the US.

when stabilization of inflation became a prime objective of policy, IAP became dominant.

Third, except under Volcker's chairmanship the Fed's reactions functions are all non linear. Fourth, the character of those non linearities varies substantially across different US Fed chairs. In particular, while the McChesney Martin inflationary period is characterized by convexity, the Taylor rule is concave under Greenspan. This lends support to the view that, when inflation and inflationary expectations were on the rise during the Vietnam war, inflation aversion dominated policy preferences. On the other hand, after inflationary expectations had been anchored by Volcker, recession aversion became dominant under Greenspan. Finally, there is weaker, but still significant, evidence of dominant recession avoidance preferences during the Burns/Miller era.

More generally, results from both countries support the view that the dominant type of asymmetry in monetary policy (recession avoidance versus inflation avoidance) often changes with the economic environment.

2 The relation between recession and inflation avoidance preferences and the character of non linearities in Taylor rules

Non linear reaction functions may arise because the loss functions of central bankers are not quadratic, or because the economic structure is non linear, or because of both. In this paper we focus on the first class of reasons for non linearities in Taylor rules, by maintaining the assumption that the economic structure is linear.[7]

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[7]Dolado, Pedrero and Ruge Murcia (2004) find that there is no evidence against a linear aggregate supply schedule for the US. A discussion of our findings within a broader perspective that entertains the possibility of non linear Phillips curves appears at the end of the concluding section.

To motivate the analysis of this section, consider the case in which the loss function of the CB displays RAP so that the bank is more averse to negative than to positive output gaps of equal size. In the face of uncertainty this type of asymmetry should induce the CB to try to lean more heavily against negative than against positive output gaps, implying that the interest rate response to an expected change in the gap should be weaker at positive than at negative output gaps. Provided the aversion to output gaps weakens gradually as the algebraic value of the gap increases, intuition suggests that, in the presence of RAP, the reaction function should be concave with respect to the output gap.

Similarly, the loss function could also display IAP in the sense that positive inflation gaps are more costly than negative inflation gaps of the same size. IAP induce the CB to lean more heavily against positive, than against negative inflation gaps. Provided the aversion to the inflation gap weakens gradually as the algebraic value of the gap decreases, intuition suggests that, in the presence of inflation avoidance, the CB will react more strongly to a change in inflation when this gap is algebraically higher. Hence the reaction function should be convex with respect to the inflation gap.

This section illustrates and rigorously confirms this broad intuition for a New Keynesian economic structure of the type proposed by Clarida, Gali and Gertler (1999). This is done by deriving the interest rate reaction function of a CB whose loss function is characterized by **both** recession and inflation avoidance preferences. The analysis includes the cases in which there is only recession avoidance or only inflation avoidance, as particular cases. This formulation has two advantages. First it establishes the concavity/convexity properties of the reaction function with respect to **both** gaps rather than only with respect to the gap that is subject to an avoidance motive. Second, it allows both avoidance motives to operate **simultaneously**. This provides a convenient framework for the interpretation of estimated non linear reaction functions in section 3. We turn next to the details of the analysis.

2.1 General specification of inflation and recession avoidance preferences

The objective of the monetary authority is to minimize

$$E_0 \sum_{t=0}^{\infty} \delta^t L_t \tag{1}$$

where δ is the discount factor and L_t is given by equation (2).

$$L_t = A f(x_t) + h(\pi_t - \pi^*). \tag{2}$$

Here A is a positive coefficient, π^* is the inflation target and the functions $f(x_t)$ and $h(\pi_t - \pi^*)$ possess the following properties.

$$
\begin{aligned}
f'(x_t) &< 0 \; for \; x_t < 0, \; f'(x_t) \geq 0 \; for \; x_t \geq 0, \; f(0) = f'(0) = 0, \\
f''(x_t) &> 0, \; f'''(x_t) \leq 0, \\
h'(\pi_t - \pi^*) &\leq 0 \; for \; \pi_t - \pi^* \leq 0, \; h'(\pi_t - \pi^*) > 0 \; for \; \pi_t - \pi^* > 0, \\
h(0) &= h'(0) = 0, \; h''(\pi_t - \pi^*) > 0, \; h'''(\pi_t - \pi^*) \geq 0
\end{aligned} \tag{3}
$$

where the tags attached to the functions $f(x_t)$ and $h(\pi_t - \pi^*)$ designate partial derivatives whose order is given by the number of tags. The specification in equation (2) states that losses from both the output gap and the inflation gap attain their minimal levels when those two gaps are zero and that losses are larger, at least weakly, the larger the absolute value of the gaps. The first order partial derivatives of both $f(x_t)$ and $h(\pi_t - \pi^*)$ are zero when the gaps are zero. As with the quadratic, the second partial derivatives are assumed to be positive, but unlike the quadratic $f(x_t)$ and $h(\pi_t - \pi^*)$ need not be symmetric around zero.

Potential asymmetries with respect to final objectives are introduced by means of assumptions on third partial derivatives. Stronger aversion to negative than to positive output gaps (a RAP) is characterized by a negative third derivative of the function, $f(x_t)$. A negative value of $f'''(x_t)$ means that the marginal loss of being away from potential output shrinks with the algebraic value of the output gap. In particular, it implies that the marginal loss at a given negative output gap is larger than the marginal loss at a positive output gap with the same absolute value.[8] When $f'''(x_t) = 0$ output gap losses are symmetric and there is no RAP. This is the standard quadratic case.

Analogously, stronger aversion to positive than to negative inflation gaps (an IAP) is characterized by a positive third derivative of the function $h(\pi_t - \pi^*)$. A positive value of $h'''(\pi_t - \pi^*)$ means that the marginal loss of deviating from the inflation target increases with the algebraic value of the inflation gap. In particular, it implies that, for a given absolute value of the inflation gap, the marginal cost of a positive deviation from the target is larger than the marginal cost of a negative deviation. This kind of asymmetry may arise during periods of inflation stabilization in which the buildup of credibility is a primary consideration. In the particular case $h'''(\pi_t - \pi^*) = 0$ inflation gap losses are symmetric and there is no IAP.[9]

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[8]There is a formal analogy between this formulation of recession aversion and the formulation of utility of a consumer possessing a precautionary demand for savings. Kimball (1990) shows that a necessary and sufficient condition for the existence of the latter is that the marginal utility of consumption or income be a convex function of income (a positive third partial derivative). Similarly, the condition $-f'''(x_t) > 0$, that characterizes recession avoidance means that the marginal utility, to policymakers, of an increase in output is a convex function of output. However, the analogy is only partial since the precautionary demand for savings affects the **stock** of savings while recession aversion affects the **interest rate** chosen by the CB.

[9]Geraats (2006) utilizes a similar modeling of RAP (prudence in her terminology) to provide an explanation for the relation between inflation and its variability within a Barro-Gordon framework.

2.2 Economic Structure

The behavior of the economy is characterized by means of a stylized New-Keynesian, sticky prices framework due to Clarida *et al.* (1999) and is briefly summarized in what follows.[10] In this framework inflation depends on the currently expected future inflation and on the output gap, and the output gap depends on the real rate of interest and on its own expected future value. The CB affects the economy through its choice of the nominal rate of interest which, given inflationary expectations, affects the real rate. Formally

$$x_t = -\varphi(i_t - E_t\pi_{t+1}) + E_t x_{t+1} + g_t \tag{4}$$

$$\pi_t = \lambda x_t + bE_t\pi_{t+1} + u_t \tag{5}$$

where x_t and π_t are the output gap and inflation, $E_t x_{t+1}$ and $E_t\pi_{t+1}$ are the expected values of those variables conditioned on the information available in period t, i_t is the nominal rate of interest, g_t is a demand shock, u_t is a cost shock and φ, λ and b are positive parameters. The innovations, g_t and u_t, are mutually and temporally independent white noise processes whose realizations are not forecastable on the basis of information available to the CB prior to period t.[11]

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[10] For the deeper motivation and assumptions underlying this well-known structure the reader is referred to the original article.

[11] More generally we could have specified each shock as being composed of a component that is known in period t plus a component that is unknown in that period. However, since the known component does not play any role in the subsequent analysis, we normalize it to zero for simplicity.

2.3 The policy process and the policy rule

An important aspect of monetary policymaking is that the interest rate has to be chosen before the realization of economic shocks is known with certainty by policymakers. This is captured here by the fact that the innovations g_t and u_t are unknown at the time policymakers pick the nominal interest rate i_t. The policy rule can be found by minimizing the expected value in equation (1) subject to the behavior of the economy as given by equations (4) and (5). Inserting those equations into equation (2), substituting the resulting expression into equation (1) and taking the expected value conditional on period's 0 information, the problem of the central bank is to choose the current interest rate, i_0, and the sequence of future interest rates i_t, $t > 0$ so as to minimize the following expression

$$
E_0 \sum_{t=0}^{\infty} \delta^t \left\{ \begin{array}{c} Af\left[-\varphi(i_t - E_t\pi_{t+1}) + E_t x_{t+1} + g_t\right] + \\ h\left[\lambda(-\varphi(i_t - E_t\pi_{t+1}) + E_t x_{t+1} + g_t) + bE_t\pi_{t+1} + u_t - \pi^*\right] \end{array} \right\}. \tag{6}
$$

Under discretion policymakers take expectations of future variables as given and reoptimize each period. Since there are no lags this problem can be decomposed into a series of separate one period problems. The typical first order condition for an internal maximum is therefore

$$
AE_t f'[.] + \lambda E_t h'[.] = 0, \quad t = 0, 1, 2, .. \ . \tag{7}
$$

where the expectation E_t is taken over the distributions of the (unknown in period t) innovations g_t and u_t. Period's t condition determines the interest rate chosen in that period as a function of the inflation and output gaps expected for period $t + 1$. Note that, since policy is discretionary and neither the economic structure nor the objective function contain lagged terms, the current choice of interest rate does not affect current expectations of future periods' inflation and the

output gap.[12]

2.4   Comparative statics

The first order condition in equation (7) implicitly determines the optimal choice of interest rate by the CB as a function of expected inflation and of the expected output gap. Since, except for the general restrictions implied by recession and inflation avoidance preferences, the functional forms of $f(.)$ and $h(.)$ are not further restricted it is generally not possible to solve explicitly for the reaction function of the CB as in the quadratic case. However it is still possible to derive some features of the reaction function, and their relations to the two types of asymmetric preferences, by performing comparative static experiments.

Totally differentiating the first order condition for $t = 0$ in (7) with respect to $E_0\pi_1$ and rearranging

$$\frac{di_0}{dE_0\pi_1} = \frac{1}{\varphi}\frac{\varphi A E_0 f_0''[.] + \lambda(\varphi\lambda + b)E_0 h_0''[.]}{A E_0 f_0''[.] + \lambda^2 E_0 h_0''[.]} \equiv \frac{\varphi A E_0 f_0''[.] + \lambda(\varphi\lambda + b)E_0 h_0''[.]}{\varphi D} \qquad (10)$$

where

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[12]This can be seen by taking expected values, as of t, of the economic structure in equations (4) and (5) which yields

$$E_t x_{t+1} = -\varphi(E_t i_{t+1} - E_t \pi_{t+2}) + E_t x_{t+2} + E_t g_{t+1} \qquad (8)$$

$$E_t \pi_{t+1} = \lambda E_t x_{t+1} + b E_t \pi_{t+2} + E_t u_{t+1} \qquad (9)$$

$$f_0''[.] \equiv f''[-\varphi(i_0 - E_0\pi_1) + E_0x_1 + g_0],$$

$$h_0''[.] \equiv h''[\lambda(-\varphi(i_0 - E_0\pi_1) + E_0x_1 + g_0) + bE_0\pi_1 + u_0 - \pi^*]. \tag{11}$$

Since, as stated in equation (3), all the second partial derivatives are positive the expression in equation (11) is positive, implying that policymakers react to an increase in expected inflation by raising the nominal interest rate. Furthermore, since the numerator is larger than the denominator, the nominal rate increases by more than the increase in inflationary expectations implying that policymakers raise the ex ante real rate in response to an increase in inflationary expectations.

In a similar manner, totally differentiating the first order condition for $t = 0$ in (7) with respect to E_0x_1 and rearranging[13]

$$\frac{di_0}{dE_0x_1} = \frac{1}{\varphi} \frac{A(1 + \varphi\lambda)E_0 f_0''[.] + \lambda^2(1 + \varphi\lambda + b)E_0 h_0''[.]}{D}. \tag{12}$$

The specification of asymmetric objectives in equation (3) implies that the response of the interest rate to an increase in the expected output gap, as given by equation (12) is positive.

2.5 The relationship between different types of asymmetric preferences and the character of non linearities in Taylor rules

The main implications of the presence of asymmetric preferences for the shape of non linearities in Taylor rules is summarized in the following proposition. The proof of the proposition is in the appendix.

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[13]Note that this expression takes into consideration that a unit increase in E_0x_1 induces, via equation (9), an increase of size λ in $E_0\pi_1$.

Proposition 1: *Given a New-Keynesian economic structure of the type described in equations (4) and (5) the following hold*

(i) In the presence of recession avoidance preferences (RAP), but no inflation avoidance preferences (IAP), the reaction function is concave in both the inflation and output gaps.

(ii) In the presence of IAP, but no RAP, the reaction function is convex in both the inflation and output gaps.

(iii) In the presence of **both** *IAP and RAP the reaction function may be either linear or non linear. The first case occurs when the effects of the two types of asymmetries on the curvature of the reaction function are of similar magnitude. When the second case occurs, the reaction function is concave in both gaps if the RAP dominates the IAP and convex in both of them when the reverse holds.*

(iv) The reaction function of a strict inflation targeter is linear, independently of wether IAP is present or not.

The proposition confirms the intuition discussed at the beginning of this section. It also shows that the concavity/convexity properties of each type of asymmetric objective carry over to the "cross gap" in the reaction function. For example, when the RAP dominates the IAP, the Taylor rule is concave not only in the expected output gap, but in the expected inflation gap **as well.**[14]

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[14]Part (iv) of the proposition is a consequence of the fact that the second derivatives of the interest rate with respect to both expected inflation and the expected output gaps turn out to be equal to products of the parameter A and of other terms (see equations (21) and (22) of the Appendix). Since, for strict inflation targeters $A = 0$, this implies that the second derivatives vanish so that the Taylor rule is linear.

However, when the CB also has a preference for interest rate smoothing this is no longer the case implying that the reaction function of a strict inflation targeter is not necessarily linear. This can be demonstrated by repeating the analysis in the first part of the appendix for

$$L_t = h(\pi_t - \pi^*) + \frac{c}{2}(i_t - i_{t-1})$$

where c is a positive constant that captures a preference for interest rate smoothing.

3   Estimation and testing for non linearities in Taylor rules

This section proposes and implements a procedure for the estimation of non linear reaction functions that: first, allows non linearity tests by nesting the linear model as a restricted case of a more general non-linear one; second, is sufficiently flexible to let the data determine the nature of non linearity (eg: concavity versus convexity) when such non linearity is present. In conjunction with proposition 1, the results of this estimation makes it possible to make inferences about the dominant type of asymmetry, if any, and to examine its temporal stability.

3.1   A linear benchmark

In the absence of asymmetries the CB loss function is quadratic in both the inflation and the output gaps. As is well known, this leads to linear reaction functions. In the presence of asymmetries the interest rate reaction function may be non linear. As a benchmark, we start from linear reaction functions of the type estimated by Clarida *et al.* (1998, 2000). In this type of reaction function the desired interest rate, i_t^*, is given by

$$i_t^* = \alpha + \beta(E[\pi_{t,k}|\Omega_t] - \pi^*) + \gamma E[x_{t,q}|\Omega_t] \qquad (13)$$

where π^* is the inflation target, Ω_t is the information set of the CB in period t, and $E[\pi_{t,k}|\Omega_t]$ (and $E[x_{t,q}|\Omega_t]$) are the rate of inflation (and the output gap) expected to materialize in period $t + k$ (and $t + q$), given this information set. In practice, it is commonly assumed that central banks adjust the policy rate gradually so that the actual rate converges to the desired rate

through a partial adjustment mechanism (Clarida *et al.*, 1998, 2000)[15]:

$$i_t = \rho i_{t-1} + (1 - \rho)i_t^*. \tag{14}$$

Combining equations (13) and (14) and adding an exogenous interest rate control error, v_t, yields

$$i_t = (1 - \rho)\{\alpha + \beta(E[\pi_{t,k}|\Omega_t] - \pi^*) + \gamma E[x_{t,q}|\Omega_t]\} + \rho i_{t-1} + v_t \tag{15}$$

Adding and subtracting $\beta E[\pi_{t,k}|\Omega_t]$ and $\gamma E[x_{t,q}|\Omega_t]$ to equation (15) and rearranging

$$i_t = (1 - \rho)\{\widetilde{\alpha} + \beta\pi_{t,k} + \gamma x_{t,q}\} + \rho i_{t-1} + \varepsilon_t \tag{16}$$

where $\widetilde{\alpha} = \alpha - \beta\pi^*$ and the error term, ε_t, is a linear combination of the forecast errors of inflation and of the output gap, and of the interest rate control error.[16]

We start by estimating the linear benchmark in equation (16).[17] Estimation is done by means of Hansen's (1982) Generalized Method of Moments (GMM), using as instruments four lags of the policy targets (output and inflation) and four lags of the policy instrument (the interest rate) and of the real price of oil. Clarida *et al. (op. cit.)* experiment with various leads for the output gap and expected inflation, but in fact $q = k = 1$ seems to fit the data reasonably well, and the estimates of the parameters are not too sensitive to changes in q and k over the range of 1 to 4 quarters.

Information about the data used appears in part 4 of the appendix.[18] The data used

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[15]For a justification of interest-smoothing behaviour, see Cukierman (1990, 1992). Svensson (2000) and Muscatelli *et. al.* (2002) develop models that include the costs of interest rate adjustment in the optimization exercise.

[16]Its explicit form is $\varepsilon_t = -[\beta(\pi_{t+k} - E[\pi_{t,k}|\Omega_t]) + \gamma(x_{t+q} - E[x_{t,q}|\Omega_t])] + v_t$. Here π_{t+k} and x_{t+q} are the actual values of inflation and of the output gap in periods $t + k$ and $t + q$ respectively.

[17]We do not show these estimates here for reason of space.

[18]We use official estimates of potential output from OECD and CBO sources to construct series for the output

is quarterly and the sample period is 1979-3 to 2005-4 for the UK and 1960-1 to 2005-4 for the US. The beginning of the sample in the US is dictated by data availability. In the case of the UK, the full sample is available from about 1976, three years earlier than the start of our sample. We chose not to utilize those additional three years of data and to start the UK sample in 1979 because in that year a Conservative government headed by Margaret Thatcher replaced the previous Labour government and implemented a change in monetary regime, with a switch towards an explicitly monetarist policy.

3.2   Parametrization of non linearities

To parametrize the non linearities we focus on smooth-transition models (STR). Such models allow the **marginal** reaction of the short term interest rate to expected output and inflation gaps to change smoothly over the range of the reaction function. STR naturally fit with the smooth specification of recession and inflation avoidance preferences in the previous section. This specification is based on the notion that it is unlikely that interest rate responses in the reaction function remain constant over a wide range and then change discontinuously at particular values of the inflation or output gaps.

An STR model can be built following the modeling strategy proposed by Granger and Terasvirta (1993). This method allows the coefficients of variables that might enter non linearly to depend on a transition variable z_t that governs the smooth transition. For example one may parametrize the marginal response of the interest rate to future inflation in equation (16) as

$$\beta = \beta_1 + \beta_2 F(z_t) \tag{17}$$

gap. In contrast, Muscatelli *et al.* (2002) use a Kalman Filter based estimation to allow for gradual learning by the authorities of changes in the processes generating inflation and the output gap. Using official data on the output gap makes our results more directly comparable to most of the existing empirical literature on interest rate reaction functions.

where $F(z_t)$ is an appropriate non linear and continuous function of z_t. A similar procedure can be applied to the coefficient, γ, of the future output gap in equation (16). Granger and Terasvirta (1993) propose either the logistic or exponential functions to model the smooth transition. However, we obtained a better fit by using an hyperbolic tangent ($tanh$) function to capture the gradual transition of the marginal response.[19] Following are the details of this parametrization. Let $y_z(z_t - \theta_z) \equiv \psi_z(z_t - \theta_z)$ be a linear function of z_t where ψ_z and θ_z are parameters to be determined. The hyperbolic tangent of $y(z_t)$ is given by

$$F\left[y_z(z_t - \theta_z)\right] = \tanh\left[y_z(z_t - \theta_z)\right] \equiv \frac{e^{y_z(.)} - e^{-y_z(.)}}{e^{y_z(.)} + e^{-y_z(.)}}. \tag{18}$$

The hyperbolic tangent is a monotonically increasing function of $y_z(.)$. As $y_z(.)$ varies between minus and plus infinity, $tanh$ varies between -1 and +1. When $y(.) = 0$, $tanh$ is zero and has an inflection point at zero. For negative values of $y_z(.)$, $tanh$ is convex and for positive values of $y_z(.)$, it is concave in $y_z(.)$. The additional sub-parametrization of $y_z(.)$ in terms of z_t governs the location and the spread of the hyperbolic tangent in terms of the transition variable z_t. In particular, θ_z determines the location of the point of inflection of $tanh$ and ψ_z determines how quickly it moves up with the transition variable z_t.

The analysis in section 2, implies that the inflation gap is an appropriate transition variable for β and that the output gap is an appropriate transition variable for γ. This leads to the following extension of equation (16)

$$
\begin{aligned}
i_t &= (1-\rho)\{\widehat{\alpha} + \beta_1 \pi_{t,1} + \gamma_1 x_{t,1} + \beta_2 \pi_{t,1} \tanh\left[\psi_\pi(\pi_{t,1} - \pi^*)\right] + \gamma_2 x_{t,1} \tanh\left[\psi_x(x_{t,1})\right]\} \\
&\quad + \rho i_{t-1} + \varepsilon_t
\end{aligned} \tag{19}
$$

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[19]Such a parametrization has been proposed by Bacon and Watts (1971), and Seber and Wild (1989).

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where the explicit form of tanh is given in equation (18) and where we allow, in principle, for the possibility that the parameters ψ and θ vary depending on whether the variable which determines the transition is the output gap or the inflation gap. More precisely, $\tanh\left[y_\pi(\pi_{t,1} - \theta_\pi)\right] = \tanh\left[\psi_\pi(\pi_{t,1} - \pi^*)\right]$ and θ_x is constrained to zero so that $\tanh\left[y_x(x_{t,1} - \theta_x)\right] = \tanh\left[\psi_x x_{t,1}\right]$. However, as explained later, the various models tried fit the data well with the restriction $\psi_x = \psi_\pi = \psi$ so that, in the estimated models, the two gaps enter the hyperbolic tangent function with the same linear coefficient, ψ.

To illustrate the consequences of this specification for the **total** response coefficients of inflation and of the output gap consider, for example, the implied behavior of β. As inflation varies around π^*, the value of the hyperbolic tangent function varies between -1 and 1. For concreteness, suppose that $\pi^* = 4\%$, and that $\psi_\pi = 1$. At low inflation rates (approximately 2% below the threshold) the tanh function is close to its lower asymptote of -1 so the long-run response of the nominal interest rate to expected inflation in this range is approximately $(\beta_1 - \beta_2)$. By contrast, at high inflation rates (approximately 2% above π^*, i.e. at 6% inflation) the *tanh* function is close to its upper asymptote of 1 so that the nominal interest rate response to expected inflation in this range is approximately $(\beta_1 + \beta_2)$. In between, β is a monotonic function of inflation and is bounded between $(\beta_1 - \beta_2)$ and $(\beta_1 + \beta_2)$. Depending on whether β_2 is positive or negative, the marginal response of the interest rate to inflation is (for the most part) an increasing or decreasing function of inflation implying that the coefficient β_2 determines whether the reaction function is convex or concave in expected inflation.[20] Analogous considerations apply to the response of the interest rate to the output gap.

Equation (19) is estimated by GMM for alternative values of the parameter ψ in the range between 0.1 and 1 with increments of 0.1. Since, in terms of goodness of fit, the best results are obtained, by and large, for $\psi_x = \psi_\pi \equiv \psi = 0.2$ all the results presented are for this

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[20]The analytical details underlying this statement are set out in the second part of the appendix.

common value of ψ. Four lags of all the explanatory variables are used as instrumental variables in the estimation. A convenient feature of the hyperbolic tangent smooth transition regressions (HTSTR) is that they make it possible to estimate the implicit inflation target, π^*, along with the other parameters even in the absence of an explicit inflation target.

3.3   Estimation procedures and results for the UK

Table 1 presents estimated HTSTR for the UK from the third quarter of 1979 till the fourth quarter of 2005 as well as for the subperiods 1979:3-1990:3 and 1992:3-2005:4.[21] The beginning of the sample is chosen to coincide with the beginning of the Conservative government under Margaret Thatcher. An explicit inflation target has been announced by the UK government since 1992. The breakdown of the sample into two subperiods is meant to capture potential differences in the reaction function between the first period, in which there was no explicit target, and the second one which was characterized by an explicitly announced inflation target. For the whole sample period, as well as for the first subperiod, the implicit inflation target is estimated along with the other parameters[22]. For the second subperiod, the officially announced inflation target (which is 2.5% in terms of the retail price index) was used.[23]

In all the regressions the linear coefficient, β_1, on expected inflation is positive and significant. The linear coefficients, γ_1, on the output gap are positive too but significant only in the subsamples. All regressions display the well known gradual adjustment of the interest rate to the desired level with a non adjustment coefficient that varies between 0.62 and 0.84. The

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[21] The period 1990:4-1992:2 has been excluded from the subsamples since, during this period, the UK belonged to the exchange rate mechanism of the EMS implying that UK monetary policy at that time was largely shadowing that of the German Bundesbank.

[22] We estimate π^* and ψ using a grid search procedure to provide the best fit for the GMM criterion function. As noted above, we found that $\psi = 0.2$ provided the best fit throughout and hence this is not tabulated.

[23] In December 2003, the UK government changed the Bank of England's inflation target from RPIX to the harmonised index of consumer prices (CPI), with a target of 2%. However, it is generally accepted that this new remit of a 2% target for CPI was, at the time, broadly consistent with a 2.5% target for RPIX.

coefficient β_2 that governs the smooth transition in the case of expected inflation is uniformly significant. The coefficient, γ_2 that governs the smooth transition in the case of the expected

Table 1:

Hyperbolic Tangent Smooth Transition Regressions (HTSTR): UK, 1979 - 2005

Period	Estimated Coefficients							Statistics
	$\widehat{\widehat{\alpha}}$	$\widehat{\beta}_1$	$\widehat{\beta}_2$	$\widehat{\gamma}_1$	$\widehat{\gamma}_2$	$\widehat{\rho}$	π^*	
1979 : 3 to 2005 : 4	$-6.06**$ (1.90)	$3.54**$ (0.51)	$-2.22**$ (0.39)	0.31 (0.38)	-0.46 (1.40)	$0.70**$ (0.08)	5.0	$\sigma = 0.83$ $J(15) = 12.63$
1979 : 3 to 1990 : 3	1.99 (3.88)	$1.71**$ (0.63)	$-0.76*$ (0.42)	$0.88*$ (0.54)	$-3.17*$ (1.82)	$0.62**$ (0.12)	7.5	$\sigma = 1.07$ $J(15) = 14.39$
1992 : 3 to 2005 : 4	4.29 (4.93)	$2.26*$ (1.23)	$4.45**$ (1.63)	$4.53**$ (1.52)	2.68 (2.88)	$0.84**$ (0.05)	2.5	$\sigma = 0.35$ $J(15) = 20.29$

Notes: Numbers in parenthesis indicate standard errors (using a consistent covariance matrix for heteroscedasticity and serial correlation); σ indicates the standard error of the estimate; J(n) is Hansen's test of the model's overidentifying restrictions, which is distributed as a $\chi^2(n+1)$ variate under the null hypothesis of valid overidentifying restrictions (n stands for the number of instruments minus the number of freely estimated parameters). Two stars designate a coefficient/statistic that is significant at the 5% level, and one star indicates significance at the 10% level. The interest rate, the output gap, inflation, the inflation target, π^*, and σ are all measured in percentages.

output gap is significant only in the first subperiod. Interestingly, the linear coefficients on expected inflation are all substantially higher than one. However, due to the non linearity of the Taylor rule, these coefficients do not suffice to establish whether the Taylor principle is satisfied

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or not. To do that we need to evaluate the total (non linear) response of the interest rate to inflation.[24] This response is given by

$$\frac{\partial i}{\partial \pi_{t,1}} = \beta_1 + \beta_2 \left[\tanh\left[\psi(\pi_{t,1} - \pi^*)\right] + \psi\pi_{t,1} \tanh'\left[\psi(\pi_{t,1} - \pi^*)\right] \right] \tag{20}$$

where $\tanh'[.]$ is the first derivative of tanh with respect to its argument. It is shown in the second part of the appendix that for most of the rates of inflation experienced in the UK over the sample period, this response is increasing (decreasing) with inflation if and only if β_2 is positive (negative). Consequently, the reaction function is convex or concave in expected inflation, depending on wether β_2 is positive or negative.[25] Table 1 shows that, for the whole sample period, $\beta_2 < 0$ implying dominant recession avoidance preferences (RAP).

3.3.1 Subperiods results and their implications

When the entire sample period is broken down into two subperiods, the first without an explicit inflation target and the second with it, the type of non linearity changes dramatically. While in the first subperiod β_2 is still negative and significant implying that the reaction function is concave in inflation, it is positive and significant during the inflation targeting period, implying that the reaction function is convex in inflation during that period. A broadly similar picture emerges with respect to γ_2 which is negative and significant in the first period and positive during the inflation targeting period.

Those findings, in conjunction with the results of proposition 1, support the view that during the first period monetary policy was dominated by recession avoidance preferences (RAP), while during the second period it was dominated by inflation avoidance preferences (IAP). Inter-

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[24]Recall that, due to the features of the GMM estimation procedure, the coefficients of the actual future inflation rate are identical to the coefficient of expected inflation.

[25]Similarly, part 3 of the appendix shows that if γ_2 is positive (negative) the reaction function is convex (concave) with respect to the expected output gap.

estingly, monetary policy in the UK during the first subperiod was conducted by the Chancellor of the Exchequer. With the advent of inflation targeting at the beginning of the second period attainment of the target became a major objective of monetary policy.[26]

3.4   Estimation procedures and results for the US

The entire US sample spans the period between the first quarter of 1960 and the last quarter of 2005. It is well known that there were at least two important changes in policy regimes in the US. One when, at the beginning of his term, Volcker de-emphasized the interest rate instrument and focussed policy on an unborrowed reserve target, and another, when the Board of Governors of the Fed decided, in late 1982, to reinstate the interest rate instrument.[27] Excluding this three years period, Clarida, Gali and Gertler (2000) find that the coefficient of inflation in a linear reaction function was lower than one in the pre Volcker period and significantly higher than one from about 1982 onwards. They attribute a substantial part of the increased stability experienced in the US during the latter period to this change in regime. More generally, to allow for possible changes in monetary regimes across different chairmans of the Fed we decompose the entire sample period into four basic subperiods that correspond to the chairmanships of McChesney-Martin, Burns\Miller, Volcker and Greenspan.[28]

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[26]During the first five years of the second period the Chancellor was still formally in charge of monetary policy but consulted the Bank of England prior to taking decisions. From May 1997 onwards, with the granting of operational independence, the Bank of England is solely in charge of monetary policy.

[27]An enlightening institutional discussion of this period appears in Lindsey, Orphanides and Rasche (2005).

[28]The Burns and Miller eras are lumped together due to the brevity of Miller's chairmanship.

Table 2

Reaction Functions by Board Chairs: US, 1960:1 - 2005:4

Period	Estimated Coefficients							Statistics
	$\widehat{\widehat{\alpha}}$	$\widehat{\beta}_1$	$\widehat{\beta}_2$	$\widehat{\gamma}_1$	$\widehat{\gamma}_2$	$\widehat{\rho}$	π^*	
Martin 1960:1 1970:1	5.76 ∗∗ (1.52)	−1.80∗ (0.98)	3.21 ∗∗ (0.63)	0.34 ∗∗ (0.17)	0.091 ∗∗ (0.045)	0.76 ∗∗ (0.08)	1.3	$\sigma = 0.42$ $J(15) = 11.62$
Burns/ Miller 1970:2 1979:3	1.42 ∗∗ (0.72)	0.86 ∗∗ (0.09)	0.08 (0.72)	0.55 ∗∗ (0.19)	−0.90 ∗∗ (0.45)	0.42 ∗∗ (0.13)	3.2	$\sigma = 0.85$ $J(15) = 14.03$
Volcker 1982:4 1987:3	0.16 (1.31)	1.52 ∗∗ (0.29)	-	0.80 ∗∗ (0.40)	-	0.89 ∗∗ (0.03)	-	$\sigma = 0.84$ $J(17) = 11.56$
Greenspan 1987:4 2005:4	2.35 (1.08)	1.01 ∗∗ (0.36)	−0.92 (0.85)	0.88 ∗∗ (0.12)	−0.84 ∗∗ (0.14)	0.81 ∗∗ (0.03)	2.9	$\sigma = 0.38$ $J(15) = 13.38$

Notes: Numbers in parenthesis indicate standard errors (using a consistent covariance matrix for heteroscedasticity and serial correlation); σ indicates the standard error of the estimate; J(n) is Hansen's test of the model's overidentifying restrictions, which is distributed as a $\chi^2(n+1)$ variate under the null hypothesis of valid overidentifying restrictions (n stands for the number of instruments minus the number of freely estimated parameters). Two stars designate a coefficient/statistic that is significant at the 5% level, and one star indicates significance at the 10% level. The interest rate, the output gap, inflation, the inflation target, π^*, and σ are all measured in percentages.

HTSTR models are estimated for each subperiod. Except for the Volcker era there is some evidence in favor of non linearities in all periods.[29] Since to this day the US does not practice explicit inflation targeting, π^* is estimated, in the non linear cases, along with the other coefficients. As for the UK this is done by using a grid search that minimizes the criterion function over π^*. Table 2 summarizes the estimation results. The eleven quarters under Volcker, during which the operational target was unborrowed reserves were excluded from the Volcker sample period.

Table 2 suggests that the reaction function of the Fed varied in a non negligible manner across different chairs. Particularly striking is the difference between the McChesney-Martin and Greenspan reaction functions. Although there is evidence of non linearity under both chairs, the forms of non linearity differ. The significantly positive β_2 and γ_2 coefficients under McChesney-Martin are consistent with dominant IAP, whereas the negative signs of those coefficients under Greenspan are indicative of dominant RAP. Those findings are consistent with the view that McChesney-Martin, who operated during a period of high capacity utilization and rising inflation due to the Vietnam war, viewed inflation as the number one problem.[30] On the other hand Greenspan whose chairmanship started in a period in which inflationary expectations were already well anchored felt that he could devote more attention to recession avoidance.

Although there is no evidence of non linearity in inflation during the Burns/Miller era, γ_2 is negative and significant pointing towards non linearities in the output gap and recession avoidance preferences. Furthermore, in line with previous results, the coefficient on inflation

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[29]The unrestricted HTSTR estimation for the (relatively short) Volcker years displayed convergence problems and the estimates of $\beta_1, \beta_2, \gamma_1$ and γ_2 were all insignificant - - indicating overparametrisation. However, when the non linear parameters β_2 and γ_2 were restricted to zero, the GMM estimate of the reaction function converged and recovered the significant linear coefficients that appear in Volcker's row in table 2.

[30]Although the estimate of β_1 is negative during this period the **total** marginal response of the interest rate to inflation is positive and substantially larger than one for practically all inflation rates experienced under McChesney-Martin.

is smaller than one. Those finding are broadly consistent with Burns (1979, page 16) famous statement that the role of the Fed is to continue "probing the limits of its freedom to under-nourish...inflation". In view of the dominant inflation avoidance under McChesney-Martin the Burns/Miller period appears as an outlier since, in spite of rising inflation, it displays recession avoidance. One possible explanation is that Burns was subject to heavier political pressures than his predecessor due to the fact that, during his tenure, inflation was accompanied by a reduction in the rate of growth while, under McChesney-Martin inflation was expansionary. There is no evidence of non linearities under Volcker but the (constant) coefficient on inflation is significantly higher than one as in Clarida *et. al.* (2000).

Using a Linex specification for asymmetric objectives and combining the Burns/Miller sample period together with that of McChesney-Martin, Surico (2007) finds evidence in favor of RAP over the combined pre-Volcker period. Due to pooling of the two sample periods, this result is not necessarily inconsistent with ours for the Burns/Miller era. But our results suggest that, due to the different character of non linearities under McChesney-Martin and Burns/Miller, lumping of those periods together is likely to be misleading. Surico also tests for non linearities in the Volcker-Greenspan era lumped into one period and finds no evidence in favor of non linearities. Although these findings are not inconsistent with ours for the Volcker period, they are inconsistent with the preference for recession avoidance we detect under Greenspan. Again, this difference may be due to the pooling of sub-samples.[31] One advantage of lumping is that the sample periods are longer. But this has to be weighted against the substantial differences in curvature of the reaction functions under different US chairs that we detect.

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[31] Besides differences in methodology (Linex versus HTSTR) and in definition of periods, Surico's measure of inflation is based on the personal consumption expenditure deflator while ours is based on the CPI.

4 Conclusion

Using hyperbolic tangent smooth transition regressions (HTSTR) this paper produces evidence in favor of non linearities in Taylor rules for the UK and the US. Our estimates of the parameters characterizing those non linearities support the following conclusions. During the pre inflation targeting period in the UK the Taylor rule is concave. Given a linear New Keynesian economy this finding is consistent with the existence of dominant recession avoidance preferences (RAP). During the inflation targeting period in the UK (from 1992 and on), the Taylor rule is convex supporting the existence of dominant inflation avoidance preferences (IAP). The UK findings suggest that the introduction of inflation targeting in the UK was accompanied by rather fundamental changes in monetary policy objectives, well beyond what the introduction of such targets is traditionally understood to involve.

Excluding the Volcker period as Fed chair, the evidence supports the existence of non linearities in US interest rate reaction functions under all chairs. Under McChesney-Martin the Taylor rule is convex, supporting a dominant IAP, while under Greenspan it is concave supporting a dominant RAP. Those results are consistent with the view that the dominant type of asymmetry changes with the most pressing economic problem of the recent past. The McChesney-Martin period was characterized by rising inflation and inflationary expectations due to the Vietnam war. By contrast, Greenspan became chairman after inflationary expectations have been firmly anchored by Volcker. Consequently, McChesney-Martin developed inflation avoidance whereas Greenspan could focus on recession avoidance. In a similar vein, the UK results are consistent with the view that, in addition to a substantial reduction in the target, the advent of inflation targeting, and latterly central bank independence, was associated with an increase in the importance of IAP relatively to RAP.

In principle non linear reaction functions of the type estimated in this paper may arise, even in the absence of asymmetric objectives, if the aggregate supply schedule is non linear.

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However, the fact that there are periodic changes in the curvature of those reaction functions (from concave to convex or vice versa) and that they tend to occur in parallel to known changes in policy regimes raises the odds that those non linearities mainly reflect asymmetric policy preferences. Thus, it seems more likely that the change in curvature of the policy rule between the pre inflation targeting period and the inflation targeting period in the UK is due to changes in policy preferences and policymaking institutions rather than to a change in the curvature of the Phillips curve. Similarly, the changes in the nature of non linearities between different chairs in the US are more likely to be due to changes in policy preferences than to changes in the curvature of the Phillips curve. One may argue along the lines of Lucas' critique that changes in the policy rule lead, at least after a while, to changes in the Phillips curve. Although this is an intriguing argument, it is consistent with the view that, to the extent that there was a change in economic structure it was caused by a change in policy preferences, rather than the other way around.[32]

5 Appendix

5.1 Proof of proposition 1

In general the effects of RAP and of IAP on the curvature of the reaction function with respect to the two gaps can be found by calculating the second derivatives of the interest rate with respect to the expected output gap and the expected rate of inflation.[33] Differentiating equations (12)

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[32]It is widely believed that when inflation and its variance go down the slope of the Phillips relation becomes flatter. Obviously, for large variations in inflation this may be formulated as a non linear relation. Note however that such a change is also consistent with Phillips curves with differing **linear** slopes across different policy regimes. In other words, for sufficiently cohesive policy regimes, linear aggregate supply schedules may suffice. Dolado, Pedrero and Ruge Murcia (2004) present evidence from the US that supports this view.

[33]Note that, since the inflation target, π^*, is invariant to expected inflation, the derivative with respect to expected inflation is identical to the derivative with respect to the expected inflation gap.

and (10) with respect to $E_0 x_1$ and $E_0 \pi_1$ respectively, we obtain after some algebra[34]

$$\frac{d^2 i_0}{d(E_0 x_1)^2} = \lambda^2 \frac{A\lambda b^2}{\varphi D^3} \left\{ A(E_0 f_0'')^2 E_0 h_0''' + \lambda(E_0 h_0'')^2 E_0 f_0''' \right\} \tag{21}$$

$$\frac{d^2 i_0}{d(E_0 \pi_1)^2} = \frac{A\lambda b^2}{\varphi D^3} \left\{ A(E_0 f_0'')^2 E_0 h_0''' + \lambda(E_0 h_0'')^2 E_0 f_0''' \right\} \tag{22}$$

where the functions' brackets have been deleted to simplify notation. Note that, except for $E_0 f_0'''$ all the terms in those two expressions are positive (see equations (3), (4) and (5)).

(i) In the presence of RAP but no IAP $E_0 f_0''' < 0$, $E_0 h_0''' = 0$, and those two equations reduce to

$$\frac{d^2 i_0}{d(E_0 x_1)^2} = \lambda^3 \frac{A\lambda b^2}{\varphi D^3} (E_0 h_0'')^2 E_0 f_0''' \tag{23}$$

$$\frac{d^2 i_0}{d(E_0 \pi_1)^2} = \frac{A\lambda^2 b^2}{\varphi D^3} (E_0 h_0'')^2 E_0 f_0''' \tag{24}$$

implying that the Taylor rule is concave in both gaps.

(ii) In the presence of IAP but no RAP $E_0 f_0''' = 0$, $E_0 h_0''' > 0$, and those two equations reduce to

$$\frac{d^2 i_0}{d(E_0 x_1)^2} = \lambda^2 \frac{A^2 \lambda b^2}{\varphi D^3} (E_0 f_0'')^2 E_0 h_0''' \tag{25}$$

$$\frac{d^2 i_0}{d(E_0 \pi_1)^2} = \frac{A^2 \lambda b^2}{\varphi D^3} (E_0 f_0'')^2 E_0 h_0''' \tag{26}$$

implying that the Taylor rule is convex in both gaps.

(iii) The coefficients of both $E_0 f_0'''$ and of $E_0 h_0'''$ in equations (21) and (22) are positive. Since $E_0 f_0''' < 0$ and $E_0 h_0''' > 0$ the sign of this expression is determined by the importance

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[34]Further details appear in Cukierman and Muscatelli (2003).

of the RAP, as measured by the absolute value of $E_0 f_0^{'''}$, relatively to the importance of the IAP as measured by $E_0 h_0^{'''}$. If the absolute values of $E_0 f_0^{'''}$ and of $E_0 h_0^{'''}$ are such that the first and the second terms in curly brackets on the right hand sides of (21) and (22) approximately offset each other, the reaction function is approximately linear in spite of the presence of both prudence motives.

If the absolute value of $E_0 f_0^{'''}$ is sufficiently large in comparison to $E_0 h_0^{'''}$ (the RAP dominates the IAP), the right hand sides of equations (21) and (22) are negative and the Taylor rule is concave in both gaps. If the converse holds the right hand sides of equations (21) and (22) are positive and the Taylor rule is convex in both gaps.

(iv) If the CB is a strict inflation targeter, $A = 0$, and the expressions in equations (21) and (22) are equal to zero, implying that the reaction function is linear. QED

5.2 An equivalence relation between the sign of β_2 and the curvature of the HTSTR with respect to expected inflation

This part of the appendix states and proves a condition on rates of inflation which establishes a clear cut equivalence between the sign of β_2 and the second derivative of the estimated hyperbolic tangent regressions with respect to expected inflation.

Proposition 2: For inflation gaps that are not too large in absolute value, the estimated hyperbolic tangent regressions are convex or concave in expected inflation depending on wether β_2 is positive or negative

Proof: Differentiating (20) with respect to $\pi_{t,1}$

$$\frac{\partial^2 i}{\partial \pi_{t,1}^2} = \beta_2 \psi \left[2 \tanh^{'} [\psi(\pi_{t,1} - \pi^*)] + \psi \pi_{t,1} \tanh^{''} [\psi(\pi_{t,1} - \pi^*)] \right] \tag{27}$$

where $\tanh^{'} [.]$ and $\tanh^{''} [.]$ are the first and second derivatives of the hyperbolic tangent with

respect to its argument. Since $\psi > 0$ the sign of $\frac{\partial^2 i}{\partial \pi_{t,1}^2}$ is the same as that of β_2 if and only if the expression in brackets on the right hand side of (27) is positive. Rearranging, using the explicit formula for the hyperbolic tangent in (18) to calculate $\tanh'[.]$ and $\tanh''[.]$ and the fact that it is convex (concave) when the inflation gap is negative (positive), the right hand side of (27) is positive if and only if

$$\pi_{t,1} > \frac{1}{\psi} \frac{1}{\tanh[\psi(\pi_{t,1} - \pi^*)]}, \quad \pi_{t,1} - \pi^* < 0 \tag{28}$$

$$\pi_{t,1} < \frac{1}{\psi} \frac{1}{\tanh[\psi(\pi_{t,1} - \pi^*)]}, \quad \pi_{t,1} - \pi^* > 0. \tag{29}$$

Since $\psi = 0.2$ in all the HTSTR those conditions reduce to

$$\pi_{t,1} > \frac{5}{\tanh[0.2(\pi_{t,1} - \pi^*)]}, \quad \pi_{t,1} - \pi^* < 0 \tag{30}$$

$$\pi_{t,1} < \frac{5}{\tanh[0.2(\pi_{t,1} - \pi^*)]}, \quad \pi_{t,1} - \pi^* > 0. \tag{31}$$

The right hand side of (30) is negative and that of (31) is positive. For rates of inflation (or deflation) sufficiently close to any positive inflation target, $\pi^* > 0$, the conditions in those equations are satisfied since in such a case the right hand side of (30) is a very large negative number and the right hand of (31) is a very large positive number. As the distance between $\pi_{t,1}$ and the target grows in either direction the margin by which those conditions are satisfied shrinks monotonically until the inequalities in equations (30) and (31) are reversed. But as long as the inflation gap is not too large in absolute value, the right hand side of (27) is positive and sign of $\frac{\partial^2 i}{\partial \pi_{t,1}^2}$ is the same as that of β_2, so that the HTSTR are convex or concave depending on wether β_2 is positive or negative. QED

The critical values at which the term in brackets on the right hand side of (27) becomes

negative are determined implicitly from

$$\pi_{cL} = \frac{5}{\tanh\left[0.2(\pi_{cL} - \pi^*)\right]}, \quad \pi_{cL} - \pi^* < 0$$

$$\pi_{cH} = \frac{5}{\tanh\left[0.2(\pi_{cH} - \pi^*)\right]}, \quad \pi_{cH} - \pi^* > 0. \tag{32}$$

So, as long as $\pi_{cL} < \pi_{t,1} < \pi_{cH}$ the result in proposition 2 applies and the response of the interest rate to expected inflation is convex or concave depending on wether β_2 is positive or negative. These critical values for the UK, for the 1979-92 and 1992-05 sub-samples respectively, are: $(-5.05 < \pi_{t,1} < 10.19$ and $-5.45 < \pi_{t,1} < 6.99)$. Except for the first two and and a half years of the first subperiod in the UK those conditions are always satisfied. In the case of the US, the critical values for the McChesney-Martin era are $(-5.65 < \pi_{t,1} < 6.46)$ and for the Greenspan era $(-5.38 < \pi_{t,1} < 7.19)$. These conditions are always satisfied.

5.3   An equivalence relation between the sign of γ_2 and the curvature of the HTSTR with respect to the expected output gap

Using arguments similar to those used in the proof of proposition 2 one can establish the following proposition[35]

Proposition 3: For output gaps that are smaller in absolute value than a critical gap, $\mid x_c \mid$, the estimated hyperbolic tangent regressions are convex or concave in the expected output depending on wether γ_2 is positive or negative. For $\psi = 0.2$, the critical value of the output gap is implicitly determined from

$$\mid x_c \mid \equiv \frac{5}{\tanh\left[\psi \mid x_c \mid\right]}.$$

For $\psi = 0.2$, $\mid x_c \mid = 5.998$. Practically all the values of the output gaps in our samples are

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[35]The argument are similar except for the fact that in the case of the output gap, the conditions can be formulated on the output gap itself rather than on the level of output. This simplifies the analysis.

below this critical value in absolute terms. For the UK this condition is always satisfied. For the US, except for five observations (the data point 1966-1 and the period 1982-1-to 1983-1), again this condition holds.

5.4 Data sources and definitions

UK:

Y – real output - Main Economic Indicators - OECD, Quarterly series: March 2006 Series [GBR.CMPGDP.VIXOBSA]

YBAR - OECD's Estimate of Potential GDP.

x - percentage deviations of Y from YBAR.

$i-$ Bank of England intervention rate - end quarter -

Source: Bank of England - compound series using BoE repo rate, and eligible bills discount rate

RPIX - UK Quarterly Index of Retail Prices - All items excluding mortgage interest payments

Source: Office for National Statistics (ONS)

π - annualized quarterly percentage change in the RPIX.

USA:

Y – real output - Main Economic Indicators - OECD, Quarterly series: March 2006 Series [USA.EXPGDP.LNBARSA]

YBAR - CBO's Estimates of Potential GDP (Congressional Budget Office).

x - percentage deviations of Y from YBAR.

CPI - Main Economic Indicators - OECD, Quarterly series: March 2006

Series USA CPI All items [USA.CPALTT01.IXOB] - seasonally adjusted

π - annualized quartertly percentage change in the CPI.

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i - FFR- United States (March 2006). - Source IFS - Quarterly series.

Series [60B..ZF FEDERAL FUNDS RATE]

Other variables used - Real oil price - computed from:

OIL - Quarterly series.

Series [76AAZZF PETROLEUM:UK BRENT] source IFS (Units: US Dollars per Barrel)

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