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ABSTRACT

Fertility and the Real Exchange Rate*

We use a quinquennial data set covering 87 countries between 1975 and 2005 to investigate the relationship between fertility and the real effective exchange rate. Theoretically a country experiencing a decline in its fertility rate can be expected to have higher savings, lower investment, a current account surplus, and accordingly a real depreciation. We test and confirm this hypothesis, controlling for a host of potential determinants such as PPP deviations and the Balassa-Samuelson effect. We find a statistically significant and robust link between fertility and the exchange rate. Our point-estimate is that a decline in the fertility rate of one child per woman is associated with a depreciation of approximately .15% in the real effective exchange rate.

JEL Classification: F32 and J13
Keywords: cross-country, data, demographic, effective, empirical, multilateral and panel

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Submitted 10 May 2007

1. Introduction

Massive demographic changes are in play across the world. UN data show that average world life expectancy rose from 59.9 years in 1975-1980 to 65.4 just twenty-five years later, an increase of 9.2%. Even more dramatically, the fertility rate (average births per woman) fell from 3.92 to 2.65 during the same period of time, a larger change of some 48%. This paper is concerned with the consequences of such demographic issues for international finance. In particular, we ask "What effect does the fertility rate have on the real exchange rate?"

The theoretical linkage between the fertility rate and the real exchange rate seems intuitive. Suppose the fertility rate declines for some exogenous reason (e.g., an improvement in female education or a decrease in the cost of contraception). Life-Cycle theory argues that child-rearing is associated with increased consumption and thus reduced savings (children tend to consume more than they produce); a drop in fertility can be expected to raise savings. Investment may also drop if there is a decline in the future equilibrium capital stock resulting from a smaller populace. If savings rise and investment falls, the current account improves and a real depreciation of the exchange rate is part of the equilibrium response to an exogenous decline in fertility. This much seems straightforward (though there is dispute; more on this below). However, as with so many theories concerning exchange rates, the real question is whether these theoretical implications are borne out empirically or remain blurred by a myriad of other factors. This paper is an exploratory data analysis of the linkage between fertility and the exchange rate.

We gather a broad long panel of data that covers 87 countries between 1975 and 2005, a sample of great demographic change and cross-country heterogeneity. Unlike many theoretical predictions for exchange rates, ours seems borne out by the data. We find a strong link between the real effective exchange rate and the fertility rate. We use fixed-effects panel methods, and account for a number of other reasons why exchange rates adjust, including deviations from Purchasing Power Parity, the Balassa-Samuelson effect, the effects of trade liberalization, government spending, net foreign assets, and so on. Yet we still find that a 1-point decline in the fertility rate is associated with a real effective depreciation of around .15%. This result seems sensible and plausible; it is also quite robust.

In section 2 we provide a brief survey of the literature; this section can be skipped without loss. Our data set and methodology are presented in section 3, while section 4 contains our key results. Section 5 presents extensive sensitivity analysis, followed by a brief examination of other demographic phenomena. The paper closes with a brief conclusion.

2. Quick Survey of the Literature

Most research that studies the macroeconomic effects of demographic change is concerned with these phenomena at a national level. Nevertheless, a strand of the literature has developed recently that is concerned with the implications of demographic changes for the global economy. Two recent references of value are FRB Kansas City (2004) and Reserve Bank of Australia (2006); see also the excellent survey by Bosworth et al (2004).

It seems unnecessary to develop another theoretical model to link fertility to savings, investment, and the real exchange rate, since these interactions have been well explored in the literature. For instance, Higgins and Williamson (1996) use an overlapping generations model of growth to explore the impact of demographics on savings and the macroeconomy. Their model is entirely conventional except that instead of two it has three periods of life (youth, prime-age, and the elderly); a sketch of their model is provided in an appendix. They show that higher steady-state fertility lowers savings, raises investment and thus leads to current account deficits.

They also show that the same predictions describe the early stages of a typical "demographic transition" where fertility rises before falling (as seems to characterize the postwar period for many countries).

While an appreciation seems the likely result of increased fertility, the theoretical impact of increased fertility on the real exchange rate is not undisputed. Bryant et al (2004) develop a two-country model which implies that the real exchange rate may be expected to depreciate in the long run following a fertility increase. Cantor and Driskill (2000) show that the sign of the effect depends on whether or not the country is a net debtor; see also the references in Bosworth et al (2004).[1] In this paper, we add to the debate by using empirical rather than theoretical techniques.

Indeed, most of the existing work in this area is theoretical, often using simulation techniques for quantification. For instance, Bryant et al (2004) use simulation methods to study the effects of fertility declines in the international economy, and find that in their model a decline in fertility leads to a real appreciation; see also Bosworth et al (2004) and Bryant (2004, 2006). Boersh-Supan et al (2001 use simulations to study pension reform and international capital flows that stem from population aging in a multi-country overlapping generations model. Feroli (2003) also uses theory and simulation techniques and finds that demographic differences can explain much of the size and timing of some key current account imbalances.

The extant empirical work that estimates the effects of demographic changes tends to be concerned with macroeconomic quantities (such as the current account, or savings and investment rates) rather than international prices. For instance, Higgins (1998) uses a cross-country panel and finds strong evidence that demographic changes are correlated with current accounts. In particular, his evidence is consistent with standard theory that predicts declines in savings with the proportions of both the young and the elderly in the population; see also Domeij and Flodén (2004), Helliwell (2004), and Herbertsson and Zoega (1999). Kim and Lee (2005) find that increases in the proportion of dependents in the population lead to decreased savings and deteriorated current accounts using G-7 data and time-series techniques.

The closest antecedents to our paper are by Andersson and Österholm. In their (2005) paper, Andersson and Österholm use Swedish data and time-series techniques and find that using the distribution of the Swedish population across cohorts helps both determine and forecast the real exchange rate. Their analysis is bivariate and includes no real exchange rate determinants other than the age distribution. In Andersson and Österholm (2006), they extend the analysis to 25 OECD countries and use a panel of annual data. Again, they find some consistency between life-cycle theory and the effects of the age distribution on the real exchange rate. However, the age distribution does not help in forecasting the exchange rate, and some of their results are sensitive. They consider only one economic control, namely the real interest rate differential, which can be linked theoretically to the change in the real exchange rate. By way of contrast, our approach is medium-run in design, and is more focused on the effects of fertility on the exchange rate level. We cover a broader panel of countries, and allow for a large number of alternative real exchange rate determinants; we also conduct extensive sensitivity analysis.

3. Methodology

We estimate the following equation as our default:

$$\log(reer)_{it} = \beta fert_{it} + \gamma_1 PPP_{it} + \gamma_2 y/yus_{it} + \gamma_3 open_{it} + \gamma_4 TL_{it} + \gamma_5 G/Y_{it} + \gamma_6 growth_{it} + \gamma_7 \log(pop)_{it} + \gamma_8 \log(y)_{it} + \Sigma_t \varphi_t + \Sigma_i \theta_i + e_{it} \qquad (1)$$

where

- $\log(\text{reer})_{it}$ is the natural logarithm of the average real effective CPI exchange rate for country i over quinquennial time period t (taken from IFS),
- fert is the fertility rate measured in children per woman (UN),
- PPP is the deviation from purchasing power parity relative to the United States (PWT),
- y/yus is the ratio of current per capita GDP relative to the United States (PWT),
- open is the ratio of exports plus imports to GDP (PWT),
- TL is a measure of trade liberalization (Wacziarg-Welch),
- G/Y is the government share of GDP (PWT),
- growth is the growth rate of GDP (PWT),
- pop is the population (PWT)
- y is the level of real GDP per capita measured in international $ (PWT),
- $\{\gamma\}$ is a set of nuisance coefficients,
- $\{\varphi\}$ is a set of time-specific effects,
- $\{\theta\}$ is a set of country-specific effects, and
- e is a well-behaved disturbance term.

The coefficient of interest to us is β, the effect of fertility on the log real effective exchange rate. We estimate this reduced-form equation with least squares, including comprehensive sets of both time- and country-specific fixed effects. Thus, ours is a "within" estimator that accounts for time-invariant national phenomena (e.g., political institutions or geographic features). We are also allowing for phenomena that are common to a period of time (e.g., oil prices and exchange rate misalignments) when we estimate our key coefficient. Thus β can be interpreted as the exchange rate effect on a country when its fertility rate falls, holding the global fertility rate constant.[2]

Our "default" equation includes all the usual suspects for medium- and long-run real exchange rate determination; see, e.g., Drine and Rault (2001). The most important is the deviation from purchasing power parity (PPP). There is still dispute as to whether there is a strong tendency for real exchange rates to revert towards relative PPP, especially in the short- to medium-run; Taylor and Taylor (2004) provide a recent survey. It is certainly safest to include the deviation from PPP as a control rather than omit it and risk biasing our results while saving only a single degree of freedom.[3] We measure the PPP deviation using the Penn World Table 6.2 price level of GDP (mnemonic "P"), which is estimated purchasing power over GDP divided by the actual exchange rate (times 100). Both are expressed relative to the United States (the latter's price level is equal to 100), so this is a *bilateral* measure of PPP. We use it with our *multilateral* exchange rate index since a) the US is always one of the richest countries in the sample, and b) we include period-specific fixed effects to take account of common shocks, such as general fluctuations in the value of the American dollar. Since lower values represent a lower cost of living (compared to the US), we expect γ_1 to be positive.

Our second set of controls is intended to account for the much-discussed "Balassa-Samuelson" effect. This links productivity growth in tradables to wage growth which then spills out to wages and prices in the non-tradables sector. More generally, the Balassa-Samuelson effect links development to real exchange rate appreciation. A number of variables have been used as proxies for the effect; Froot and Rogoff (1994) provide a survey. Towards this end, we include: 1) the ratio of national to American real GDP per capita (measured in international \$); 2) the log of real GDP per capita; and 3) the growth rate. Including real GDP per capita may also be important since it is strongly negatively correlated with the fertility rate both across countries and across time.

Above and beyond controls for PPP and the Balassa-Samuelson effect, we include four other controls. These are: 1) openness (trade as a proportion of GDP); 2) a binary measure of trade liberalization; 3) the log of the country's population; and 4) the ratio of GDP spent directly by the government. Open countries with more liberal trade may have lower prices and exchange rates. Smaller countries may find it easier to pursue mercantilist exchange rate policy. Government spending tends to be disproportionately non-traded as noted by Froot and Rogoff (1994), so that the equilibrium real exchange rate may be a function of government size. There is no overwhelming evidence that any of these variables play a consistently strong role in real exchange rate determination in practice. Still, all are potentially relevant, and each has been suggested in the literature as being of possible import.

While we pursue a considerable amount of sensitivity analysis below, it is not all of equal interest. We often add two extra controls that are of particular relevance, and so create an "augmented" model. The variables are: 1) the ratio of net foreign assets to GDP emphasized by Cantor and Driskill (2000) and Lane and Milesi-Ferretti (2004, 2006), and 2) the current account imbalance, again measured as a percentage of GDP. Net foreign assets is a variable of considerable appeal, since net debtors need in principle to have depreciated exchange rates to generate the trade surpluses required to service debt payments. These variables are of theoretical interest, but their inclusion reduces the sample size considerably.[4]

The Data Set

Almost all of our series are drawn from standard data sets; the sources are set out in an appendix. The two biggest constraints we face are: a) the limited amount of comprehensive data on demographic phenomena, and its overlap with b) the real effective exchange rate.

The Population Division of the United Nations provides series for a large number of countries on life expectancy and the fertility rate. The changes in fertility rates during our sample period are dramatic. For instance, the Spanish fertility rate fell from 2.6 (children per woman) in 1975-1980 to only 1.3 in 2000-2005, while China's fertility fell from 3.3 to 1.7 during the same period of time. Fertility rates are also uneven; during the 2000-2005 period, both the Russian and Japanese fertility rates were 1.3, while that of Pakistan was 4.3 and the Nigerian fertility rate was 5.9. The UN also provides the size of a country's population both in total and divided into 21 five-year age cohorts (ages 0-4, 5-9, etc.). All this is provided for the entire populace, and each sex separately. The series are provided at five-year intervals and are averaged over quinquennial periods (so that they span e.g., July 1975 through June 1980). The series begin in 1950 and are forecast through 2050.

Figure 1 contains quinquennial box and whiskers plots for the fertility rate data we use. The box covers the range of data between the 25th and 75th percentiles; the median is marked inside the box with a bar (it is slightly lower than the global average); the whiskers extend out to the rest of the distribution.[5] Two distinct features of the data are apparent. First, fertility rates are trending lower over time. Second, there is enormous cross-country variation in fertility. Our methodology takes both phenomena into account.

The International Monetary Fund provides data on the real effective exchange rate (REER) for over ninety countries through its *International Financial Statistics* database, beginning in 1975. While six different variants are available, we rely on the only one available for a large number of observations; it is based on relative CPIs (IFS mnemonic "rec").[6] This variable is set such that 2000=100 for all countries; higher values represent more real appreciation. The series have been checked and corrected for errors.

Real exchange rate series are available from 1975 through 2005 for 91 countries; however the UN does not provide demographic data for four of these (Antigua & Barbuda, Dominica, Grenada, and St. Kitts & Nevis). Accordingly, our sample consists of six quinquennial observations for 87 countries between 1975 and 2005 for a maximum of 522 observations (though some are missing).[7] This is a span of data that would be considered appropriate for medium- or perhaps long-run analysis in international finance. However it is approximately the length of a single generation, and thus can not reasonably capture long-run or steady state equilibrium for demographic phenomena (such as those examined by Bryant, 2004, 2006). This is especially true since our data come from a period of time commonly considered to be part of a demographic transition towards lower fertility rates. During this sample, 76 of the 87 countries experienced declines in fertility. Most of these falls were large; 55 were declines of at least 25%. No country experienced a large increase in fertility.

The countries with both demographic and REER observations are tabulated in Appendix Table A1. The top-left graph in Figure 2 contains a scatter of the REER plotted against the fertility rate; immediately to the right is the analogue in natural logarithms. The wide dispersion of the observations masks the positive unconditional correlation between the real exchange and fertility rates. Accordingly, we scatter the log of the real exchange rate against the level of fertility (the log-level specification that we use in much of our empirical work) in the lower-right of the figure, but only after trimming the 5% outliers from both tails of both variables. Standard tests reject the hypothesis of a unit-root in the (log-) real exchange rate for this panel.[8]

We use conventional sources for our other series. The Penn World Table 6.2 provides series on: PPP-deviations, the ratio of national to American real GDP per capita, openness, growth, population, the level of real GDP per capita, and the ratio of government spending to output. The binary measure of trade liberalization is provided by Wacziarg and Welch (2003); Lane and Milesi-Ferretti (2006) provide net foreign assets. Further details are provided in a data appendix, and some simple bivariate scatter-plots of the log REER against important control regressors are provided in Figure 3. Descriptive statistics are provided in Appendix Table A2. Simple correlation coefficients for both the default and augmented models are presented in Appendix Table A3.

4. The Baseline

Our benchmark results are presented at the left-hand side of Table 1. Half of the control variables are statistically significant. Reassuringly, the deviation from PPP is correctly (positively) signed and highly significant with a t-ratio in excess of 12. More open and more liberalized economies tend to have appreciated currencies, while growth is associated with depreciation. The latter effect is opposite to what one would expect if the Balassa-Samuelson term dominated the sample. However, neither of the income terms is particularly important; nor is size or the government spending effect. It is difficult to explain variation in real exchange rates across countries. The equation explains only a small fraction of the variation in the (log-) real effective exchange rate. The within-country R^2 is a respectable .68, but the between-countries R^2 is a dismal .01; the overall R^2 is .only .1.

The most important result is tabulated in the top row, which presents β, the effect of the fertility rate on the (log-) REER, *ceteris paribus*. Consistent with standard life-cycle theory, it is positive; decreases of fertility from say 3 to 2 children per woman leads to a real depreciation of .15%. This is of plausible economic size, being neither trivial nor incredibly large. It is of great statistical significance, having a t-ratio of 5.9 and thus being different from zero at all confidence levels.

The column immediately to the right augments the default specification by adding net foreign assets and the current account imbalance, both measured as percentages of GDP. Neither of these plausible variables turns out to have much relevance, though their inclusion reduces the sample size. However, this has little effect on either the economic or statistical significance of the fertility rate. The same is true in the four additional variants of the basic setup that are tabulated to the right of the table. In all cases, the effect of fertility on the real exchange rate is positive, and both economically and statistically significant.

5. Sensitivity Analysis

Table 2 contains the results of a large number of robustness checks intend to check whether estimates of β are sensitive to our exact econometric methodology. We perturb the model in over twenty ways, and present estimates for the fertility rate coefficient in both the default model of Table 1 as well as the model augmented by the inclusion of net foreign assets and the current account (nuisance coefficients are not reported to conserve space). Readers in a hurry can skip this section; our key finding is robust.

We begin by adding other variables that are potential determinants of the real exchange rate. While the nominal exchange rate regime is often linked to the *volatility* of the *nominal* exchange rate in the short run, there is little evidence that it is tied to the *level* of the nominal exchange rate over the medium run, let along the medium-run level of the *real* exchange rate.

Nevertheless, we have successively added dummy variables representing *de jure* fixed exchange rate regimes (from Ghosh, Gulde and Wolf, 2002), and *de facto* fixed exchange rate regimes (we use two standard measures, those of Reinhart and Rogoff, 2004 and Levy-Yeyati and Sturzenegger).[9] Next, we have separately added two standard measures of political institutions, namely the Polity 2 measure of democracy, and also a measure of constraints on the executive drawn from the same data set. Finally, we added a lagged dependent variable.[10] However, none of these additions much affects either the size or significance of our key coefficient. The same is true of dropping either the time- or country-specific fixed effects.

We have also changed the estimation strategy in a number of ways. First, we model the country-effects as random instead of fixed; this results in little substantive change, as does weighting by either log-population or log-GDP. Our default equation models the natural logarithm of the exchange rate as a function of the level of the fertility rate. We check this semi-log model against both log-log and level-level alternatives, and find that they all result in economically and statistically significant positive coefficients. We also checked whether the exact data frequency is critical by moving from quinquennial to decadal averages. Again, we find essentially similar results.

Finally we have cut the sample up in a number of different ways. First, we dropped all observations with residuals that lie at least two standard deviations from zero. To check the sensitivity of the results over time, we successively drop early and late observations, and also present cross-sectional results for three separate years.[11] We then used the World Bank's Country Classifications to group our countries by region, allowing us to drop successively developing countries from: 1) Latin America and the Caribbean; 2) Europe and Central Asia; 3) the Middle East and North Africa, 4) Sub-Saharan Africa; 5) East Asia; and 6) South Asia. We separately also drop all high-income countries. However, none of these checks shake our confidence in the basic result. The effect of the fertility rate on the log of the real effective exchange rate is always estimated to be positive, with a semi-elasticity of around .15. Almost all the estimates of β reported in Table 2 are precisely estimated and significantly different from zero; it is never significantly negative.

Non-Linearities in the Relationship

We can see little evidence in the data of any particular non-linear relationship between the real exchange rate and fertility. The lower-left graph of figure 2 is a scatter-plot of the log real effective exchange rate against the fertility rate, after each variable has had the effects of all the other controls removed.[12] There are few indications of any economically sensible relationship more complicated than that of linearity, and indeed simple checks for asymmetric effects of fertility increases/decreases deliver nothing. Nevertheless, in Figure 4 we plot the fitted values of a non-parametric relationship between the (log of the real effective) exchange and fertility rates, once all other effects have been purged through linear regressions.[13] Most of the fitted values fit inside the +/- 2 standard error confidence interval around the linear relationship, which is also provided. We conclude that there is little evidence of any important non-linearity in the relationship between fertility and the real exchange rate.[14]

Simultaneity/Measurement Error

We think that the fertility rate is plausibly exogenous with respect to the real exchange rate. Nevertheless, it is easy to find good instrumental variables for fertility, since this has been closely linked in the demographic literature to female education. For measures of the latter, we take advantage of the updated Barro-Lee data set, which provides series on education at 5-year intervals from 1960. We use combinations of three instrumental variables for the fertility rate: 1) the percentage of 15+ females without schooling; 2) the percentage of 15+ females who attained secondary school; and 3) the average years of school for 15+ females.[15] Table 3 presents our IV results for our default model, as well as two of its variants. We tabulate estimates of β when we use all of three of the IVs (IV set 1), and then drop each of the three IVs one by one (thus IV set 2/3/4 drops the first/second/third IV.).

The IV estimates are all positive and significantly different from zero at conventional levels. They tend to be about twice as large as the estimates from Tables 1-2, though they are less precisely estimated. These instrumental variables are not weak; the smallest F-test for excluding the IVs from the first-stage regression is significantly different from zero at the .004 level. Further, since we use three instrumental variables for one coefficient, we can test the exclusion restrictions (using IV sets 2-4) by adding the dropped instrumental variables to the second-stage equation one by one. When we do this, none of the IVs enter the exchange rate equation significantly. Succinctly, our instrumental variables appear to satisfy both standard requirements, and act to reinforce our finding of a positive linkage between the fertility and real exchange rates.

6. Other Demographic Measures

While the focus of this paper is on the relationship between fertility and the real exchange rate, we also briefly examine other demographic measures. In Table 4, we use the same default model as in Tables 1-2, and merely substitute different demographic features in place of the fertility rate. We find results that are, on the whole, consistent with those of fertility, but substantially weaker. Our hypothesis is that an exogenous increase in fertility can be expected to lower savings and perhaps raise investment, thereby causing a current account deficit and an equilibrium real appreciation. Consistent with that, an increase in the birth rate or decreases in infant or child mortality are also associated with appreciations. However, only the infant mortality results are significantly different from zero.

The results from Table 4 show that increases in the ratio of either the young (below 20) or elderly (above 65) as a proportion of the populace, or a decrease in the ratio of the active labor force (20-65) are also associated with real appreciations. Again, while all these findings are consistent with theory, none are significantly different from the hypothesis of no effect at all. Still, such ratios are relatively coarse measures of the age distribution; finer detail is available from the UN. We take advantage of the latter by modeling the effects on the real exchange rate of the entire age distribution, the latter modeled by 21 age cohorts. We use the default empirical model of Tables 1-2, but replace the fertility rate with the entire age distribution of the population. In particular, we are interested in:

$$\log(reer)_{it} = \beta^1 prop^1{}_{it} + \beta^2 prop^2{}_{it} + \ldots + \beta^{21} prop^{21}{}_{it} + \gamma_1 PPP_{it} + \ldots + \gamma_8 \log(y)_{it} + \Sigma_t \varphi_t + \Sigma_i \theta_i + e_{it} \qquad (1')$$

To avoid multicollinearity, we use an unconstrained quadratic polynomial to parameterize the cohort distribution; our technique follows Higgins (1998).[16] That is we constrain $\beta^j = \delta_0 + \delta_1 j + \delta_2 j^2$, so that we actually estimate:

$$\log(reer)_{it} = \delta_0 \Sigma_j prop^j{}_{it} + \delta_1 \Sigma_j (j * prop^j{}_{it}) + \delta_2 \Sigma_j (j^2 * prop^j{}_{it})$$

$$+ \gamma_1 PPP_{it} + \ldots + \gamma_8 \log(y)_{it} + \Sigma_t \varphi_t + \Sigma_i \theta_i + e_{it} \qquad (1'')$$

The results are reported in Table 5. As the age distribution shifts towards the young (as in a more fertile country), the real exchange rate appreciates. The effect dies off as older cohorts become larger, and actually turns negative during the prime earning (and thus saving) years, and eventually starts to turn around at the approximate age of retirement. All this is consistent with life-cycle theory and the results of Andersson and Österholm (2006), though the level of statistical confidence in these results is low.

We expect the real exchange rate to appreciate with an increase in fertility, since we expect savings to fall, and investment (perhaps) to rise concomitantly. Is there evidence that either savings or investment actually respond to fertility rate changes as expected? We provide a tidbit of information in Table 6. From the Penn World Table, we gather information on the proportion of output spent on investment, as well as the analogue for national savings (defined as national income minus both private consumption and government spending). We do not develop models for either the savings or investment rates, and simply regress them separately on fixed time- and country-specific fixed effects, as well as the fertility rate. That is, we include no conditioning variables at all. Since we take no account of the source or nature of historical and anticipated shocks, these should be viewed as indicative findings at most. Still, the coefficients are consistent with the basic life-cycle framework; fertility rate increases are associated with a fall in savings and an increase in the investment rate. These results are consistent with Herbertsson and Zoega (1999), Kim and Lee (2005), and most of the literature (discussed in Section 2 above). Curiously, when we use the World Bank's measure of the current account (derived from balance of payments data) the results are weaker.[17]

Almost all the results of this section are consistent with our fertility finding, and collectively they add somewhat to our level of confidence. However, many are statistically weak and we do not wish to overstate their strength. This remains an area for future research.

7. Conclusion

This is a dry, straightforward paper, by design. It does not attempt to make any contribution in terms of theory, methodology, or data. Any interest in this paper lies in its estimates.

Real exchange rates are difficult to model empirically; while theories abound, there is only weak empirical evidence supporting quite plausible theories like purchasing power parity and the Balassa-Samuelson effect. In this paper, we provide a little good news for researchers in the area. Consistent with standard life-cycle theory, we find that a country experiencing a decline in its fertility rate experiences a real depreciation, holding other things constant. This effect seems plausible and robust. We think of this a good place to pass the torch to others.

Table 1: Benchmark Results

	Default	Augment	Variant 1	Variant 2	Variant 3	Variant 4
Fertility Rate	.15** (.03)	.15** (.02)	.16** (.02)	.13** (.02)	.17** (.02)	.11** (.02)
PPP-deviation	.0089** (.0007)	.0096** (.0007)	.0093** (.0007)	.0091** (.0007)	.0081** (.0006)	.0100** (.0008)
Real Income p/c as % US Income p/c	-.002 (.002)	.004 (.002)	-.001 (.002)	-.002 (.002)		
Openness %GDP	.0014** (.0005)	.0007 (.0005)	.0011* (.0005)	.0007 (.0004)		
Trade Liberalization Measure	.17** (.04)	.12** (.03)	.15** (.04)			
Government Spending % GDP	.002 (.003)	.008** (.003)	.005 (.003)			
Growth real GDP per capita	-.010** (.004)	-.005 (.004)				
Log(population)	-.13 (.15)	.00 (.13)				
Log(real GDP per capita, $)	.06 (.07)	-.25** (.09)		.06 (.06)		
Net Foreign Assets % GDP		-.0001 (.0003)			.0001 (.0003)	
Current Account %GDP		-.001 (.002)				-.002 (.002)
Observations	332	282	336	380	311	340
R^2	.10	.26	.19	.20	.25	.10

OLS coefficients; those significantly different from zero at 5% (1%) marked by one (two) asterisk(s).
Standard Errors in parentheses.
Regressand is natural logarithm of real effective exchange rate based on CPI.
Fixed period and country effects included but not reported.
Quinquennial (five-year averages) for 87 countries, spanning 1975 through 2004.

Table 2: Sensitivity Analysis

	Default	Augmented
Add *de jure* exchange rate peg dummy	.17** (.05)	.05 (.04)
Add RR *de facto* exchange rate peg dummy	.19** (.03)	.17** (.03)
Add LYS *de facto* exchange rate peg dummy	.17** (.04)	.22** (.03)
Add Polity 2	.16** (.03)	.15** (.02)
Add Executive Constraints	.12** (.03)	.10** (.02)
Add lagged dependent variable	.10** (.03)	.08** (.03)
Without time effects	.12** (.03)	.13** (.03)
Without country-specific fixed effects	.09** (.01)	.10** (.01)
Country-Specific Random effects	.12** (.02)	.12** (.02)
Weighted by log population	.15** (.03)	.16** (.02)
Weighted by log GDP	.15** (.02)	.16** (.02)
Level (not log) of REER as regressand	36** (5)	39** (5)
Log (not level) of fertility as key regressor	.37** (.08)	.32** (.07)
Decadal data	.13** (.05)	.08* (.04)

Table 2: Sensitivity Analysis, continued

	Default	Augmented
Without >|2σ| outliers	.15** (.02)	.16** (.02)
1980-85 cross-section	.10* (.04)	.23** (.06)
1990-95 cross-section	.09** (.03)	.10** (.03)
2000-04 cross-section	-.03 (.04)	-.02 (.04)
Drop 1975-1984	.11** (.03)	.09** (.03)
Drop 1991-2004	.16** (.03)	.15** (.03)
Drop Latin America, Caribbean	.15** (.03)	.16** (.02)
Drop European developing countries, Central Asia	.14** (.03)	.16** (.02)
Drop Middle East, North Africa	.07* (.03)	.10** (.03)
Drop Sub-Saharan Africa	.18** (.03)	.15** (.02)
Drop East Asia	.15** (.03)	.15** (.02)
Drop South Asia	.14** (.03)	.15** (.02)
Drop high income countries	.19** (.04)	.22** (.04)

Coefficient of fertility rate; standard errors in parentheses. Coefficients significantly different from zero at 5% (1%) marked by one (two) asterisk(s).
Regressand is natural logarithm of real effective exchange rate based on CPI.
OLS estimation. Regressors included in default specification but not recorded are: 1) log population; 2) log real GDP per capita; 3) openness (% GDP); 4) government spending (% GDP); 5) PPP-deviation; 6) real income per capita as % of US income per capita; 7) growth in real GDP per capita; 8) Wacziarg-Welch measure of trade liberalization; 9) time effects; 10) country-specific fixed effects. Augmented specification adds: 1) net foreign assets (% GDP); and 2) current account (% GDP).
Quinquennial (five-year averages) for 87 countries, spanning 1975 through 2004, except where noted.

Table 3: Instrumental Variable Results

	IV set 1	IV set 2	IV set 3	IV set 4
Default	.25** (.07)	.32** (.09)	.20* (.10)	.29** (.08)
Augmented	.20** (.05)	.28** (.07)	.17* (.08)	.26** (.06)
Variant 1	.27** (.05)	.30** (.06)	.23* (.10)	.29** (.06)

Coefficient of fertility rate; standard errors in parentheses. Coefficients significantly different from zero at 5% (1%) marked by one (two) asterisk(s).
Regressand is natural logarithm of real effective exchange rate based on CPI.
Regressors included in default specification but not recorded are: 1) log population; 2) log real GDP per capita; 3) openness (% GDP); 4) government spending (% GDP); 5) PPP-deviation; 6) real income per capita as % of US income per capita; 7) growth in real GDP per capita; 8) Wacziarg-Welch measure of trade liberalization; 9) time effects; 10) country-specific fixed effects. Augmented specification adds: 1) net foreign assets (% GDP); and 2) current account (% GDP). Variant 1 subtracts: 1) log population; 2) log real GDP per capita; and 3) growth in real GDP per capita.
Instrumental Variables estimation; IVs taken from Barro-Lee. Set 1 includes: 1) percentage of 15+ females without schooling; 2) percentage of 15+ females attained secondary school; and 3) average years of school for 15+ females. Sets 2-4 drop each (one) of the three IVs in turn.
Quinquennial (five-year averages) for 87 countries, spanning 1975 through 1999.

Table 4: Different Measures of Demographics and the Real Exchange Rate

	Default	Augmented
Birth Rate	.009 (.006)	-.001 (.005)
Infant Mortality Rate	-.020** (.007)	-.014** (.005)
Child Mortality Rate	-.007 (.006)	-.002 (.006)
Ratio of Young (<20) to Population	.005 (.006)	.005 (.006)
Ratio of Elderly (>65) to Population	.014 (.014)	.011 (.011)
Ratio of Active (20-65) to Population	-.010 (.007)	-.009 (.006)
Life Expectancy	-.009 (.005)	-.009* (.005)

Coefficient of demographic measure; standard errors in parentheses. Coefficients significantly different from zero at 5% (1%) marked by one (two) asterisk(s).
Regressand is natural logarithm of real effective exchange rate based on CPI.
OLS estimation. Regressors included in default specification but not recorded are: 1) log population; 2) log real GDP per capita; 3) openness (% GDP); 4) government spending (% GDP); 5) PPP-deviation; 6) real income per capita as % of US income per capita; 7) growth in real GDP per capita; 8) Wacziarg-Welch measure of trade liberalization; 9) time effects; 10) country-specific fixed effects. Augmented specification adds: 1) net foreign assets (% GDP); and 2) current account (% GDP).
Quinquennial (five-year averages) for 87 countries, spanning 1975 through 2004.

Table 5: Adding Different Age Cohorts to the Real Exchange Rate Equation

	Cohort	Coefficient	Std. Error
. *	% 0-4	.021	.013
. *	% 5-9	.018	.012
. *	% 10-14	.015	.011
. *	% 15-19	.012	.011
. *	% 20-24	.009	.010
. *	% 25-29	.007	.010
. *	% 30-34	.005	.010
. *	% 35-39	.003	.010
.*	% 40-44	.002	.011
*	% 45-49	.001	.011
*.	% 50-54	-.000	.011
*.	% 55-59	-.001	.011
* .	% 60-64	-.002	.012
* .	% 65-69	-.002	.012
* .	% 70-74	-.002	.012
* .	% 75-79	-.002	.012
*.	% 80-84	-.001	.012
*.	% 85-89	-.001	.012
*	% 90-94	.000	.013
.*	% 95-99	.002	.013
. *	% 100+	.003	.013
	Sum	.09	.22

Coefficient of percentage age cohorts (both sexes), constrained to lie on quadratic polynomial.
Regressand is natural logarithm of real effective exchange rate based on CPI.
OLS estimation. Regressors included but not recorded are: 1) log population; 2) log real GDP per capita; 3) openness (% GDP); 4) government spending (% GDP); 5) PPP-deviation; 6) real income per capita as % of US income per capita; 7) growth in real GDP per capita; 8) Wacziarg-Welch measure of trade liberalization; 9) time effects; 10) country-specific fixed effects.
Quinquennial (five-year averages) for 87 countries, spanning 1975 through 2004.

Table 6: Bivariate Effect of Fertility on Savings, Investment, and the Current Account

Regressand	Fertility Effect	Observations
Savings Rate, % GDP (PWT)	-1.67* (.75)	452
Investment Rate, % GDP (PWT)	2.41** (.43)	452
Current Account, % GDP (WDI)	-.21 (.67)	393

Coefficient of fertility rate; standard errors in parentheses. Coefficients significantly different from zero at 5% (1%) marked by one (two) asterisk(s).
Regressand is rate based on CPI.
OLS estimation. Time and country-specific fixed effects included but not recorded.
Quinquennial (five-year averages) for 87 countries, spanning 1975 through 2004.

Figure 1: Box and Whisker Plots of the Global Fertility Distribution

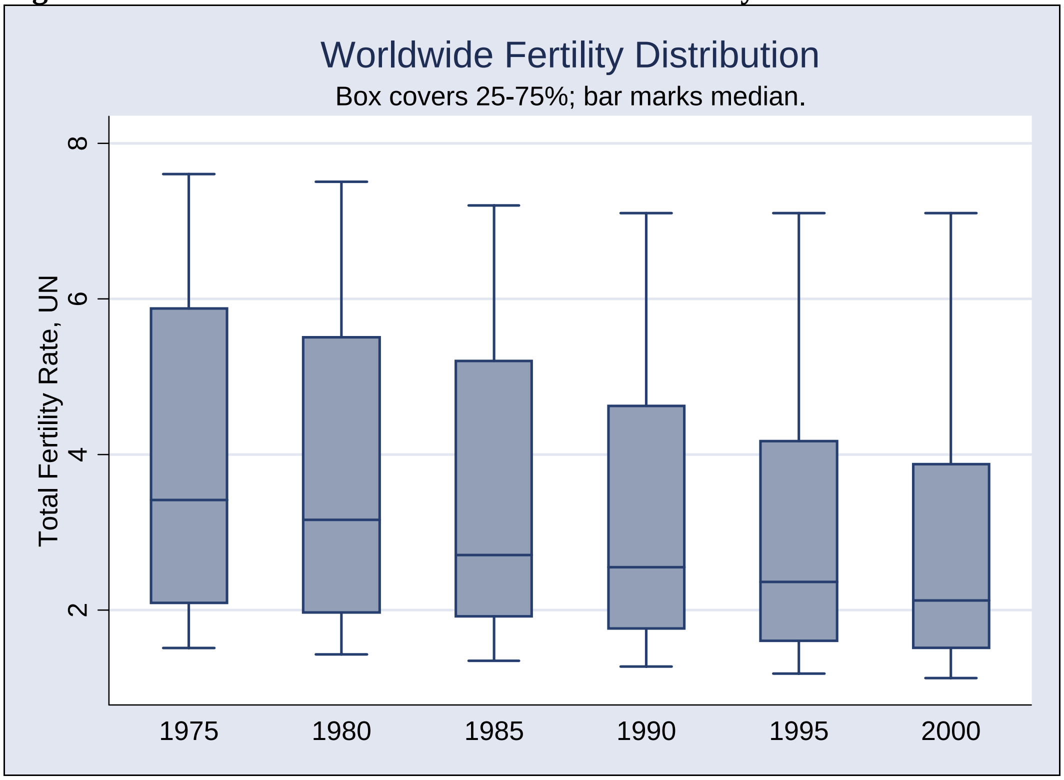


Figure 2: Bivariate Scatterplots of Key Variables

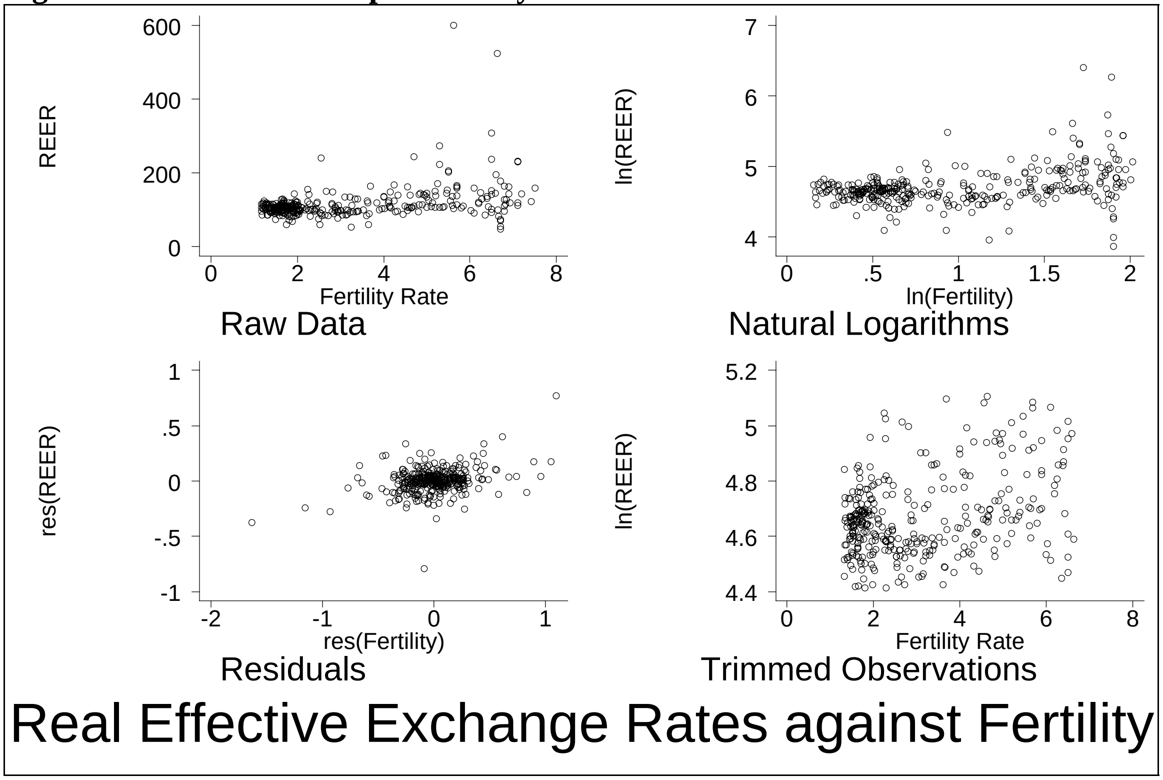


Figure 3: Bivariate Scatterplots of Control Variables

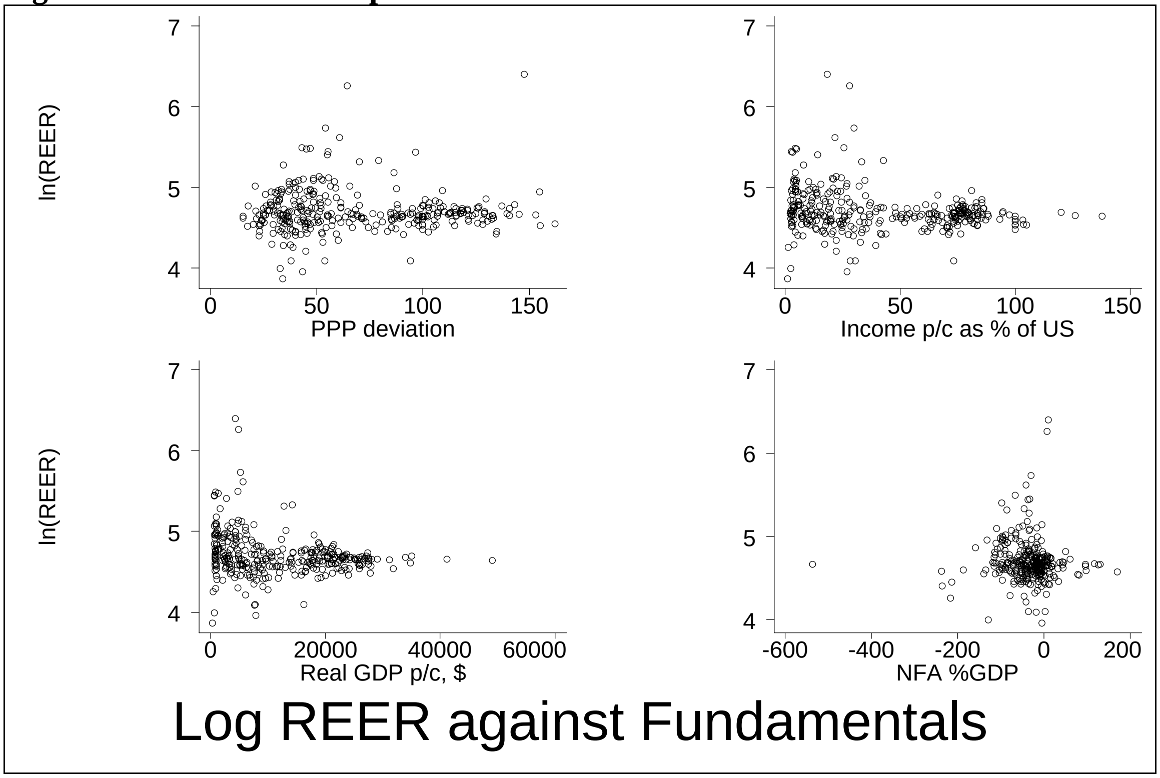
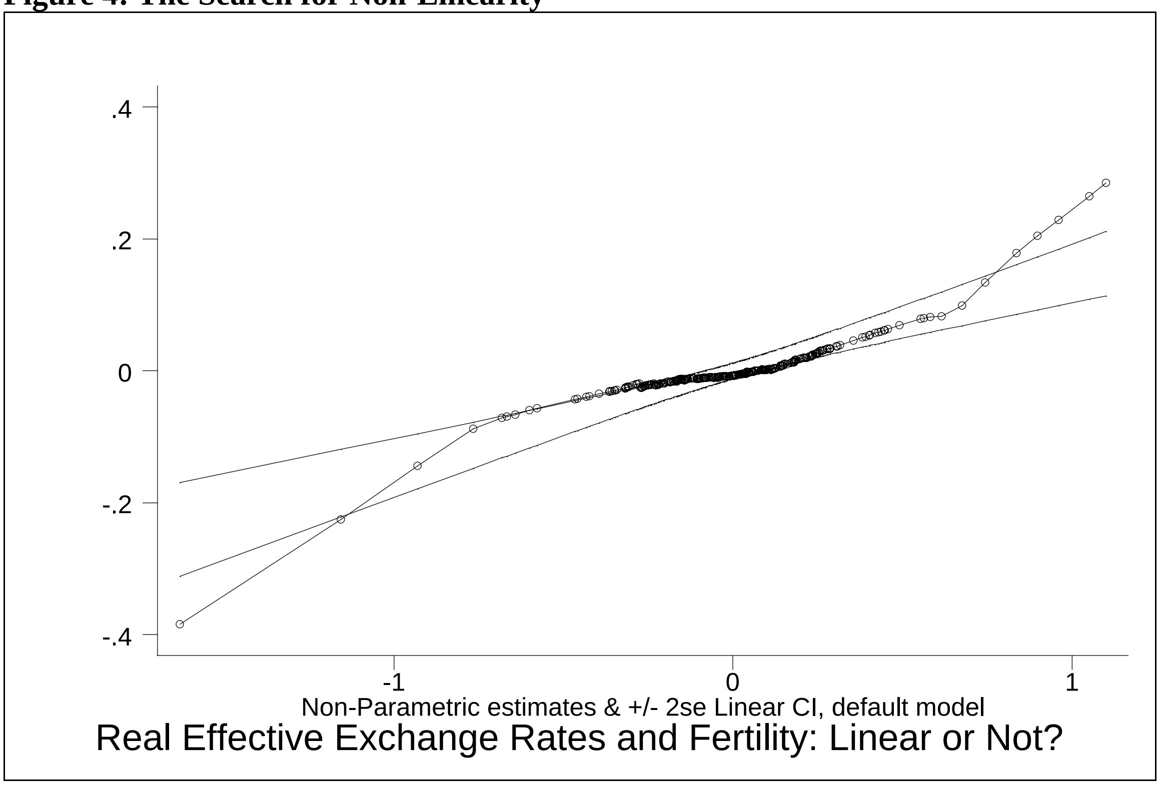


Figure 4: The Search for Non-Linearity



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Appendix Table A1: Country List

Algeria	Armenia	Australia
Austria	Bahamas	Bahrain
Belgium	Belize	Bolivia
Bulgaria	Burundi	Cameroon
Canada	Central African Rep.	Chile
China	Colombia	Congo/Zaire
Costa Rica	Croatia	Cyprus
Czech Republic	Côte d'Ivoire	Denmark
Dominican Republic	Ecuador	Equatorial Guinea
Fiji	Finland	France
Gabon	Gambia	Germany
Ghana	Greece	Guyana
Hungary	Iceland	Iran
Ireland	Israel	Italy
Japan	Lesotho	Luxembourg
Macedonia, FYR	Malawi	Malaysia
Malta	Moldova	Morocco
Netherlands	Netherlands Antilles	New Zealand
Nicaragua	Nigeria	Norway
Pakistan	Papua New Guinea	Paraguay
Philippines	Poland	Portugal
Romania	Russia	Samoa
Saudi Arabia	Sierra Leone	Singapore
Slovak Republic	Solomon Islands	South Africa
Spain	St. Lucia	St. Vincent & Grenadines
Sweden	Switzerland	Togo
Trinidad and Tobago	Tunisia	Uganda
Ukraine	United Kingdom	United States
Uruguay	Venezuela	Zambia

Appendix Table A2: Descriptive Statistics

	Obs.	Mean	Std. Dev.	Min	Max
Log(Real Effective Exchange Rate)	443	4.69	.27	3.9	6.4
Fertility Rate	522	3.36	1.86	1.1	7.6
PPP-deviation	472	66.59	34.31	15.3	195.2
Real Income p/c, % US Income	472	38.35	30.19	1.1	137.9
Openness, %GDP	472	80.24	53.93	8.9	406.7
Trade Liberalization Measure	462	.59	.48	0	1
Government Spending, %GDP	472	22.76	10.74	4.6	67.9
Growth real GDP per capita	462	1.67	3.68	-10.4	48.2
Log(population)	546	8.55	2.06	3.7	14.1
Log(real GDP per capita, $)	472	8.81	1.08	5.9	10.8
Net Foreign Assets, %GDP	387	-35.67	76.21	-980.	184.
Current Account, %GDP	409	-3.08	6.8	-30.4	21.4

Appendix Table A3: Simple Bivariate Correlations

Sample:	Default	Default	Augment	Augment
Correlation with:	Log REER	Fertility	Log REER	Fertility
Fertility Rate	.39		.48	
PPP-deviation	.00	-.53	.00	-.50
Real Income p/c, % US Income	-.19	-.73	-.20	-.70
Openness, %GDP	.05	-.17	.06	-.14
Trade Liberalization Measure	-.31	-.60	-.34	-.55
Government Spending, %GDP	.13	.08	.17	.12
Growth real GDP per capita	-.13	-.39	-.24	-.33
Log(population)	-.02	-.06	-.01	.00
Log(real GDP per capita, $)	-.22	-.85	-.27	-.83
Net Foreign Assets, %GDP			-.08	-.50
Current Account, %GDP			-.11	-.31
Observations	332	332	282	282

Approximate standard error for default (augmented) sample correlations = .05 (.06).

Appendix Table A4: Data Sources

World Population Prospects (2004 Revision): Population Division, Department of Economic and Social Affairs, United Nations Secretariat (available at http://esa.un.org/unpp)

- Fertility Rate
- Population by 5-year age groups
- Life Expectancy

Penn World Table (Mark 6.2) Center for International Comparisons, University of Pennsylvania (available at http://pwt.econ.upenn.edu/)

- Population
- Real GDP per capita (and growth of)
- Openness, % GDP
- Government Spending, % GDP
- Real Income per capita, % US Income
- PPP-deviation

International Financial Statistics (March 2007) International Monetary Fund (available at http://ifs.apdi.net)

- CPI-based Real Effective Exchange Rate

World Development Indicators (March 2007) World Bank (available at http://publications.worldbank.org/WDI/)

- Current Account, % GDP
- Government Budget Position, % GDP

World Bank Country Classifications (March 2007) World Bank (available at http://siteresources.worldbank.org/DATASTATISTICS/Resources/CLASS.XLS)

- Country Classifications

Reinhart-Rogoff Exchange Rate Regime Classifications (available at http://www.publicpolicy.umd.edu/faculty/reinhart/annual1.dta), gross classification.

Levy-Yeyati and Sturzenegger Exchange Rate Regime Classifications (available at http://200.32.4.58/~ely/Base_2005.zip), 3-regime classification.

Wacziarg-Welch Measure of Trade Liberalization (available at http://www.stanford.edu/~wacziarg/downloads/liberalization.xls)

Net Foreign Assets from Philip Lane and Gian-Maria Miles-Ferretti's "The External Wealth of Nations Mark II: Revised and Extended Estimates of Foreign Assets and Liabilities, 1970-2004" (available at http://www.imf.org/external/pubs/ft/wp/2006/data/wp0669.zip)

Appendix: Sketch of a Theoretical Model

In this brief appendix we provide the outlines of a bare-bones model, drawn from the literature, to illustrate the linkages between fertility and the real exchange rate.

We follow Higgins and Williamson (1996) in modeling agents as living for three periods in a world of overlapping generations. The middle-aged are the only productive agents, and must earn enough for their consumption, their children's consumption, and to save for their old age. The size of successive generations is linked since children are borne at an exogenous fertility rate. The middle-aged workers supply their labor inelastically in return for a wage, and spend this income consistent with a lifetime budget constraint. Preferences are additively separable, and while parents care about their children, they have no bequest motive.

Output is produced with a CRS production function which experiences exogenous technological progress; investment augments the capital stock. A small economy with open capital markets can borrow or lend at the exogenous foreign interest rate, which ties down the marginal product of capital and hence capital intensity and the capital stock. The interaction of savings and investment determines the current account, and the real exchange rate adjusts to ensure a balance between capital flows and the current account.

Given this machinery, Higgins and Williamson show that savings is then a function of capital intensity, the fertility rate, and the growth rate of productivity. They show that under plausible conditions, an increase in fertility lowers the savings rate. In all cases, higher fertility raises investment. However, even if higher fertility leads to increased savings, it always reduces the current account and thus appreciates the real exchange rate.

Endnotes

[1] Indeed, the savings rate need not fall in response to an increase in fertility. If the young save a positive amount, then the fact that they outnumber the old may actually lead to an increase in the savings rate; Obstfeld and Rogoff (1996, pp 151-2) provide an example.
[2] Bryant (2004) refers to this as an "asymmetric" effect. Note that if all national fertility rates fall by the same amount this effect would be picked up by the time effect and no exchange rate movement would be predicted.
[3] Especially since we find it to be quite significant!
[4] The current account may also be endogenous.
[5] There are no outliers in the sense of observations that lie more than 150% away from the edges of the interquartile range.
[6] The CPI-based multilateral real exchange rate is available for more than four times as many observations as its closest competitor (which is based on relative normalized unit labor costs).
[7] Other authors have also used five-year averages to smooth out short-run influences and reduce the autoregressive nature of the data; see e.g., Higgins (1996). We create our quinquennial observations by weighting annual data appropriately, wherever possible. For instance, in constructing the 1975-1980 observation, we put weights of .5 on both 1975 and 1980 observations and unity on those for the intervening years. For a number of variables we are missing the 2005 observation.
[8] The Levin, Lin and Chu test rejects the common unit root process at more than .0001, while the Im, Pesaran and Shin test for individual unit root processes also rejects at more than .0001. Parenthetically, the evidence for the fertility rate is more mixed, but the Levin, Lin and Chu test again rejects the common unit root process at more than .0001.
[9] The Levy-Yeyati and Sturzenegger data set is available at http://200.32.4.58/~ely/
[10] The lagged dependent variable is itself insignificant in both economic and statistical terms, but reduces our sample size considerably.
[11] Of the twelve cross-sectional estimates of β (6 periods, 2 models), seven are significantly positive, and five are close to zero, three positive and two negative. We have no particular interpretation for the 2000 results.
[12] It is thus a scatter-plot of residuals; the regression line has precisely the same slope as that reported in Table 1, by the Frisch-Waugh theorem.
[13] The non-parametric relationship is estimated with locally weighted regression, using a band width of .5.
[14] Cantor and Driskill (2000) show that the relationship between fertility and the exchange rate may depend on whether the country is a net debtor or not. Above and beyond adding net foreign assets as a control, we have tested for this non-linear relationship in a couple of ways. First, we split the sample by whether the country had positive NFA or not; in both cases β is estimated to be significantly positive for both the default and augmented models. Second, we added an interaction between NFA and fertility to the set of controls. This was typically positive and of marginal significance; β remains strongly positive and significant.
[15] The Barro-Lee data set is available at http://www.cid.harvard.edu/ciddata/ciddata.html. We do not use the 2000 education estimates.
[16] Adding a cubic terms results in little change, and the term is insignificant.
[17] Herbertsson and Zoega (1999), Helliwell (2004), and Higgins (1998) provide analysis and related results.