DISCUSSION PAPER SERIES

No. 6278

THE DISTRIBUTIONAL CONSEQUENCES OF DIVERSITY-ENHANCING UNIVERSITY ADMISSIONS RULES

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PUBLIC POLICY

Centre for Economic Policy Research

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Available online at: **www.cepr.org/pubs/dps/DP6278.asp**

ISSN 0265-8003

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Discussion Paper No. 6278
May 2007

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CEPR Discussion Paper No. 6278

May 2007

ABSTRACT

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This paper examines public attitudes towards university admissions rules by focusing on the imposition of the costs of racial diversity across majority citizens. High-income majority citizens, who tend to have better academic qualifications, favour more diversity under affirmative action, which imposes its costs on marginal majority candidates. Lower-income majority citizens prefer less diversity under affirmative action and would rather achieve diversity by de-emphasizing academic qualifications. Increasing income inequality among majority citizens tends to reduce the median citizen's support for affirmative action. Our results explain why affirmative action has become increasing uppopular among white voters, and why white voters who oppose affirmative action may support top-x-percent rules like those recently introduced in Texas, California and Florida.

JEL Classification: D72, H75 and I23
Keywords: affirmative action, college admissions and university admissions

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* We thank seminar participants at Berkeley, NYU, Oxford, Penn, the Public Economic Theory Conference in Beijing, and University of Hong Kong for their comments, as well as Nicholas Hill and Daniel Singer for research assistance. This paper derives from a previous one entitled “Admission Impossible? Self Interest and Affirmative Action.”

Submitted 04 April 2007

The Distributional Consequences of Diversity-Enhancing University Admissions Rules

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Abstract

This paper examines public attitudes towards university admissions rules by focusing on the imposition of the costs of racial diversity across majority citizens. High-income majority citizens, who tend to have better academic qualifications, favor more diversity under affirmative action, which imposes its costs on marginal majority candidates. Lower-income majority citizens prefer less diversity under affirmative action and would rather achieve diversity by de-emphasizing academic qualifications. Increasing income inequality among majority citizens tends to reduce the median citizen's support for affirmative action. Our results explain why affirmative action has become increasing uppopular among white voters, and why white voters who oppose affirmative action may support top-x-percent rules like those recently introduced in Texas, California and Florida.

1 Introduction

Ethnic and racial diversity at American colleges and universities has recently become a major issue of political debate. Because applicants from minority groups tend to have lower high-school grades and standardized-test scores than their majority counterparts, universities cannot produce diverse student bodies simply by filling their ranks with the applicants boasting the highest grades and test scores. American universities have traditionally chosen to achieve diversity mainly through affirmative-action policies, which set a lower admissions standard for underrepresented minorities.[1] But these policies are unpopular among white voters. Starting with Proposition 209 in California and the *Hopwood v. Texas* ruling of the Fifth Circuit Court of Appeals in 1996, a recent series of court rulings, state legislations, and ballot referenda has banned or curtailed race-conscious

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[1] In this paper, the term "affirmative action" refers exclusively to race-conscious admissions standards and excludes other programs promoting diversity such as minority outreach.

admissions in many states.[2]

Universities reacted to bans by discounting traditional measures of academic achievement in admissions. Shortly after affirmative action was banned at the University of California, its Berkeley campus broadened its measure of academic achievement to incorporate several factors other than grades and standardized-test scores.[3] In 1997, the Texas state legislature passed a law requiring public universities (e.g., UT Austin or Texas A&M) to admit any Texas resident graduating in the top ten percent of her high-school class, where rank is solely determined by high-school grades.[4] Because high schools vary in quality and exhibit substantial racial segregation, admitting students based on high-school class rank rather than standardized-test scores raises minority enrolment. Indeed, after falling substantially immediately after the bans, minority representation at elite public universities in Texas rebounded significantly under the new rules (Chan and Eyster, 2003).[5] Following the success of the Texas law, the University of California adopted a similar plan that admits the top four percent of each California high-school class to its system, and Florida recently replaced affirmative action with a rule granting admission to the top twenty percent of each high-school class in the state.

A small literature has examined the causes and consequences of these policy reforms. Chan and Eyster (2003) show that universities that care about diversity may respond to a ban on race-based admissions by using a noisy admissions test. They point out that a noisy test is an inefficient way to create diversity since, unlike affirmative action, a noisy test does not select the most academically

[2] In 1998, Washington state voters passed Proposition 200, which is identical to California's 209. In the 2001 case *Johnson v. University of Georgia*, a Federal Circuit court struck down affirmative action at the University of Georgia. But in the 2000 case *Smith v. University of Washington*, the Ninth Circuit Court of Appeals upheld affirmative action in law school admissions.

In June 2003, the Supreme Court ruled on the legality of affirmative-action programs in undergraduate and law school admissions at the University of Michigan (*Gratz v. Bollinger* and *Grutter v. Bollinger*, respectively). The court ruled that "student body diversity is a compelling state interest that can justify using race in admissions ... The Law School engages in a highly individualized, holistic review of each applicant's file ... There is no policy, either *de jure* or *de facto* of automatic acceptance of rejection based on any 'soft' variable" (Summary, *Grutter v. Bollinger*). By contrast, in the context of undergraduate admissions, "the court finds that the University's current policy, which automatically distributes 20 points, or one-fifth of the points needed to guarantee admission, to every single 'underrepresented minority' applicant solely because of race, is not narrowly tailored to achieve educational diversity" (Summary, *Gratz v. Bollinger*). Thus, affirmative action—as long as it is not explicitly defined—is legal. The ruling neither invalidates California's Proposition 209 nor reverses Texas's changes in admission policy.

[3] As a result, average SAT I scores at Berkeley fell from 1309 in 1997, the last year of affirmative action, to 1290 in 2001 (even as the number of applicants per matriculant rose from 7.6 to 9.4, and their average score rose from 1239 to 1251).

[4] The pre-ban admissions policy in Texas considered high-school grades, standardized-test scores, and a number of non-academic factors. Under the new system, these factors are used only to evaluate students outside the top-ten percent of their high-school classes.

[5] Long (2004) examines the effectiveness of the Texas's top-ten-percent rule in achieving diversity; he concludes that too few many minority applicants belong to the top ten percent of their high-school classes for a top-ten-percent rule to replace traditional affirmative action.

qualified candidates from either the majority or minority group. Epple, Romano, and Sieg (2007) also explore how universities that care about diversity may react to a ban on affirmative action. Using a computational model of admissions, they show that universities may use income and test score as a proxy for race in setting tuition policies. Despite these attempts to bolster minority enrolment, it remains substantially lower under a ban than under affirmative action. Fryer, Loury, and Youret (forthcoming) show in a related model that universities may react to a ban on by changing admissions rule to depend on factors correlated with race.

All the models mentioned above assume universities can choose their own admissions policies (in compliance with the law on affirmative action). But while American universities enjoy considerable freedom, they, particularly public universities, are not immune from political pressure.[6] In this paper, we examine how different admissions policies affect different majority citizens. We show that different diversity-enhancing policies have different distributional consequences for majority citizens, and, as a result, engender different levels of political support.[7]

Section 2 introduces a simple model of university admissions. Its basic premises are that academic ability correlates positively with both income and socioeconomic status and that test score provides an informative signal of academic ability. Section 3 explores majority citizens' support for diversity under affirmative action given a fixed admissions test. Since majority citizens from high socioeconomic groups tend to have higher academic ability and test scores than those from lower socioeconomic groups, they are less likely to have marginal test scores conditional on being accepted. Relative to the benefits that they receive from diversity—more valuable education when admitted—higher-ability majority citizens pay a lower share of its costs—the likelihood of being displaced. Thus, even with the same inherent taste for diversity, majority citizens from higher socioeconomic classes demand more diversity (i.e., support more affirmative action) than those from lower socioeconomic classes.

When admissions is highly competitive, income inequality tends to reduce the median voter's support for diversity. Specifically, when admissions is sufficiently competitive, fixing the median voter's income (i.e., ability), transferring income from poorer majority candidates to wealthier ones causes the median voter to demand less diversity because it worsens her competitive position relative to the admitted class. In recent years, income inequality has widened as the number of

[6] Recall that it was the Texas state legislature that passed the Texas ten-percent rule.

[7] Whereas this paper investigates how income inequality among majority citizens affects demand for diversity and the means of achieving it, Austen-Smith and Wallerstein (2006) explore how racial diversity affects majority citizens' demand for reducing income inequality. They find that affirmative action acts as a substitute for income redistribution.

applicants to elite colleges and universities has grown. Our result suggests that these factors may partially explain affirmative action's increasing unpopularity among white voters.

Section 4 considers majority citizens' support for diversity when affirmative action is banned. In this case diversity can be created through new programs such as those mentioned above. Relative to affirmative action, programs de-emphasizing test score shift more of the cost of racial diversity—displaced majority candidates—onto majority candidates from higher socioeconomic classes; these programs not only displace marginal majority citizens but also infra-marginal majority citizens, who more likely belong to higher socioeconomic groups. We identify conditions under which voters from lower socioeconomic classes demand more diversity when affirmative action is banned. Thus, while programs like the Texas ten-percent rule may create diversity less efficiently than affirmative action in terms of overall student quality, they may increase the support for diversity among lower socioeconomic classes.

2 The Model

An economy is composed of citizens from two ethnic groups: a majority group, W, and a minority group, N (e.g., white and non-white). The sole good in the economy, college education, is supplied by a college with a fixed capacity of $C < W + N$ students. Education costs nothing to produce, and the college charges no tuition. Instead, it rations education according to some admissions rule.

The majority group is divided into three socioeconomic types, $I \equiv \{l, m, h\}$, which we refer to as low, middle, and high types, respectively. The size of the majority group is W, and the size of type i is W_i. Each citizen of type i is endowed with the same income λ_i. Throughout, we assume that $0 < \lambda_l \leq \lambda_m \leq \lambda_h$. The minority group has size N, and all minority citizens have the same income $\lambda_N \in (0, \lambda_l)$.[8] We assume that $\lambda_l < W/2$ and $\lambda_l + \lambda_m > (W + N)/2$. To capture the strong empirical correlation between academic achievement and parental income, we make the extreme simplifying assumption that income equals ability and denote both by λ.[9] The assumption allows us to focus on the distributional consequences of different admissions rules and abstract away from the efficiency consideration emphasized by Chan and Eyster (2003). We discuss how the model can be extended to the case where income and ability are imperfectly correlated in Section 4. Because the economy consists of a single good, income operates solely through its effect on ability.

[8] Our qualitative results would not change if $\lambda_N > \lambda_l$, so long as λ_N is less than the mean majority income. We use the stronger assumption only to simplify analysis.

[9] For evidence on the correlation among SAT, demographics, and univeristy performance, see Rothstein (2005).

2.1 Citizens

Every citizen applies to the college and matriculates if admitted. We capture the idea that people value racial diversity with the assumption that citizens who attend college benefit from a diverse student body.[10] Let n denote the fraction of the student body composed of minority citizens and $v(n)$ denote the value of education when diversity is n.[11] We assume that v is everywhere twice differentiable, with $v(0) = 0$, $v'(n) > 0$ and $v''(n) < 0$ for all n. Let $\pi(n, \lambda)$ denote the probability that a citizen with ability λ is admitted when diversity is n. The expected utility of citizen λ is $\pi(n, \lambda)v(n)$; her payoff is zero when un-admitted. The assumption that citizens value diversity only when admitted implies that every citizen, regardless of her probability of admission, is willing to lower her probability of admission by $\frac{nv'(n)}{v(n)}$ percent in return for a one-percent increase in diversity. Because this does not depend on λ, preferences are "income neutral". We assume income neutrality not because we believe that there is no relationship between inherent taste for diversity and ability or income (or race), but instead to focus on how inequality among majority citizens creates differences in preferences for diversity.

The assumption that preferences are income neutral implies the existence of a unique diversity level that is Pareto efficient among majority citizens. Let n denote the share of seats going to minority citizens, and w_i the share of majority-citizen seats going to income group i. Within each group, citizens are admitted with equal probability. Under an allocation (n, w_l, w_m, w_h) a majority citizen of type i is admitted with probability $w_i(1-n)C/W_i$ and receives the expected utility $v(n)w_i(1-n)C/W_i$. Let n^* denote the level of diversity that maximizes $(1-n)v(n)$; since it maximizes each majority type's utility, it maximizes their total utility. Starting from any allocation (n, w_l, w_m, w_h) where $n \neq n^*$, every majority socioeconomic group can be made better off by moving to n^* in the following manner: each group i foregoes (or gains) share w_i of the change in majority seats necessary to move the minority group's share from n to n^*. Hence, n^* is the unique Pareto-efficient-among-majority-citizens level of diversity when the university can implement any allocation unconstrained by test score; it provides a useful benchmark to measure the majority citizens' demand for diversity under the various admissions policies discussed below.

Minority citizens' preferred level of diversity maximizes $nv(n)$. They wish to increase diversity

[10] Many college graduates believe that an ethnically diverse student body contributed to the value of their educations. For example, Bowen and Bok (1998) report a survey of recent alumni from elite colleges and universities in which fifty-five percent of whites consider the "ability to work effectively and get along well with people of different races/culture" to be a "very important" life skill; sixty-three percent of whites believe that their undergraduate experience was of considerable value in developing this skill.

[11] We use the term "diversity" to mean minority enrolment. The utility of a majority citizen not attending college is normalized to zero.

both to raise their own chances of admission and for its positive effect on the value of education.

2.2 Admissions Rules

For the remainder of the paper, we assume that admission is based upon citizens' scores on some admissions test. An admissions rule consists of an admissions test as well as cut-off scores for the majority and the minority groups, denoted by t_W and t_N, respectively. The rule operates by admitting any citizen who scores above her group's cut-off. The set of feasible admissions tests is denoted by the extended ray $[\underline{x}, \infty]$ for $\underline{x} > 0$. A feasible admissions rule is characterized by an admissions test $x \in [\underline{x}, \infty]$ and a pair of cutoffs (t_W, t_N) that satisfy the capacity constraint that the number of admitted citizens equals C. On test x, citizen i with ability λ scores $t = \lambda + x\varepsilon_i$, where ε_i is random term with zero mean and finite variance. One test x is more accurate than another x' if $x < x'$; thus $\underline{x}$ provides an upper bound on the accuracy of feasible tests. Let ϕ and Φ denote the density and cumulative distribution functions of ε, respectively. We assume ϕ is everywhere strictly positive on $\mathbb{R}$, twice differentiable, and satisfies the following assumption.

Assumption 1 *ϕ is strictly log-concave.*

Formally, Assumption 1 is equivalent to the strict monotone likelihood ratio property (SMLRP) that for any $y > z$, $\phi\left(\frac{t-y}{x}\right)/\phi\left(\frac{t-z}{x}\right)$ strictly increases in t: the higher the test score, the greater the likelihood that it comes from a higher-ability citizen. This in turn implies that on any test x the expected ability of citizens scoring t strictly increases in t, a natural property of any meaningful test. For instance, if a test contains multiple questions, and higher-ability citizens answer each question correctly with higher probability, then test score satisfies Assumption 1. Many common distributions, including the normal and the logistic, have strictly log-concave densities.[12]

The parameter x may be interpreted as an exogenous characteristic of a test or as noise deliberately added to existing measures of qualification. University admissions policies also take into account such non-academic factors as "character" or "leadership," in which case x may be viewed as the weight given to these factors relative to academic achievement. Finally, whereas admissions in our model depends upon a single test, in reality universities use several measures of qualification. Part of an admission office's expertise comes in combining such measures to best predict ability. Combining several tests in a suboptimal way is analogous to adding noise to a single test in our model.

[12] For more on log concavity, see Bagnoli and Bergstrom (2005).

When $t_N = t_W$, citizens from the two groups are treated identically. When $t_N < t_W$, there is affirmative action. While in reality it may be illegal to set $t_W < t_N$, here for simplicity we ignore this constraint. Under the admissions rule (x, t_N, t_W), the fraction of the class belonging to the majority group is

$$1 - n = \frac{\sum_{i \in I} \left(1 - \Phi\left(\frac{t_W - \lambda_i}{x}\right)\right) W_i}{C} \tag{1}$$

and the fraction belonging to the minority group is

$$n = \frac{\left(1 - \Phi\left(\frac{t_N - \lambda_N}{x}\right)\right) N}{C}. \tag{2}$$

It is clear from Equations 1 and 2 that for any x, any $n \in (0, 1)$ is achievable by choosing the right t_W and t_N. Henceforth, we represent an affirmative-action admissions rule by the duple $(x, n) \in [\underline{x}, \infty] \times (0, 1)$. Equations 1 and 2 show how x and n determine the cutoffs $t_W(x, n)$ and $t_N(x, n)$.

Under a test-score-based admissions system, diversity can be promoted in two ways. First, admissions can include affirmative action—setting $t_W > t_N$. (In our setting, this is equivalent to raising n while holding x constant.) Second, the test can be made less accurate by setting $x > \underline{x}$. Since minority citizens have lower academic ability, "adding noise" to the test increases minority representation. When the test is pure noise ($x = \infty$), minority citizens are represented in proportion to their share of the population: $n = \frac{N}{N+W}$. While both methods can achieve any $n \leq \frac{N}{N+W}$ (so long as n is no smaller than it would be without affirmative action on test $\underline{x}$), they have different implications for the way seats are divided among majority citizens from the different socioeconomic groups. As a result, although different types of majority citizens are endowed with identical innate preferences over diversity, they may have different *induced* preferences or "demand" over diversity under a test-score-based admissions system.[13]

3 Demand for Diversity under Affirmative Action

We first analyze majority citizens' demand for affirmative action under a fixed admissions test. Under admissions rule (x, n) a majority citizen of type i receives the expected utility

$$U_i(x, n) = \left(1 - \Phi\left(\frac{t_W(x, n) - \lambda_i}{x}\right)\right) \lambda_i v_W(n). \tag{3}$$

[13] Naturally, in a richer model where minority citizens also belong to different socioeconomic groups, these different admissions rules also would affect different minority citizens in different ways.

Write $\phi_i(x,n)$ for $\phi\left(\frac{t_W-\lambda_i}{x}\right)$ and $\Phi_i(x,n)$ for $\Phi\left(\frac{t_W-\lambda_i}{x}\right)$. The marginal utility of n (taking into account its effect on the probability of admission) for a majority citizen of type i is

$$\frac{\partial U_i}{\partial n} = \frac{-\phi_i(x,n)\,C}{\sum_{j\in I}\phi_j(x,n)\,W_j}\lambda_i v_W(n) + (1-\Phi_i(x,n))\,\lambda_i v_W'(n). \tag{4}$$

Increasing diversity raises the citizen's value of education when admitted. But it also necessitates raising the cut-off t_W, which displaces majority citizens scoring at t_W. Hence, a citizen's preference for diversity depends both upon $(1-\Phi_i(x,n))$, the probability that she scores above t_W and enjoys the benefits of diversity, as well as upon $\phi_i(x,n)$, the probability that she scores at the cut-off t_W. Lemma 1 comes directly from the log-concavity of ϕ.

Lemma 1 *Under Assumption 1, for each x, n, and $i>j$, $\frac{\phi_i(x,n)}{1-\Phi_i(x,n)} < \frac{\phi_j(x,n)}{1-\Phi_j(x,n)}$ (Theorem 2, Bagnoli and Bergstrom 1989).*

It follows from (4) and Lemma 1 that $U_j' \geq 0$ implies $U_i' \geq 0$ for $i>j$. That is, if a lower type wants to increase diversity starting from some n, then a higher type also wants to increase diversity. Let $n_i^*(x)$ denote type i's preferred level of diversity under admission test x.

Proposition 1 *Given Assumption 1, for each test $x \in [\underline{x},\infty)$, if majority type j prefers diversity level n' to $n<n'$ given test x, then so too do all majority types higher than j. Thus, $n_h^*(x) \geq n_m^*(x) \geq n_l^*(x)$, and $n_m^*(x)$ is the Condorcet winner among all n given x.*[14]

Since admitted higher types tend to score higher than admitted lower types, they are less likely to score at the cut-off than admitted lower types. Consequently, admitted higher types are less likely to be displaced by affirmative action. Since higher types have the same preferences over diversity as lower types—but give up a smaller share of their seats—majority citizens' demand for n satisfies a single-crossing property: for any test x, whenever low types wish to admit more minority citizens, so too do high types. The single-crossing condition implies that low and middle types prefer $n_m^*(x)$ to any $n>n_m^*(x)$, and middle and high types prefer $n_m^*(x)$ to any $n<n_m^*(x)$. Finally, minority citizens always prefer a higher n. Hence, $n_m^*(x)$ is the Condorcet winner among all n.

In general, a more informative test always causes low types to demand less diversity. But its effect on high and middle types is ambiguous. Figure 1 shows the three types of majority citizens' demand for diversity as a function of x.[15] The baseline corresponds to n^*, the preferred level of

[14] Alternative x from the set X is a Condorcet winner if a majority of citizens prefers x to any $x' \in X$.

[15] Figure 1 uses the following parameter values: for each $i \in I$, $W_i = 1$, $C = 0.5$, $\varepsilon \sim N(0,1)$, $\lambda_h = 0.8$, $\lambda_m = 0.7$, $\lambda_l = 0$ and $v(n) = n^{0.2}$.

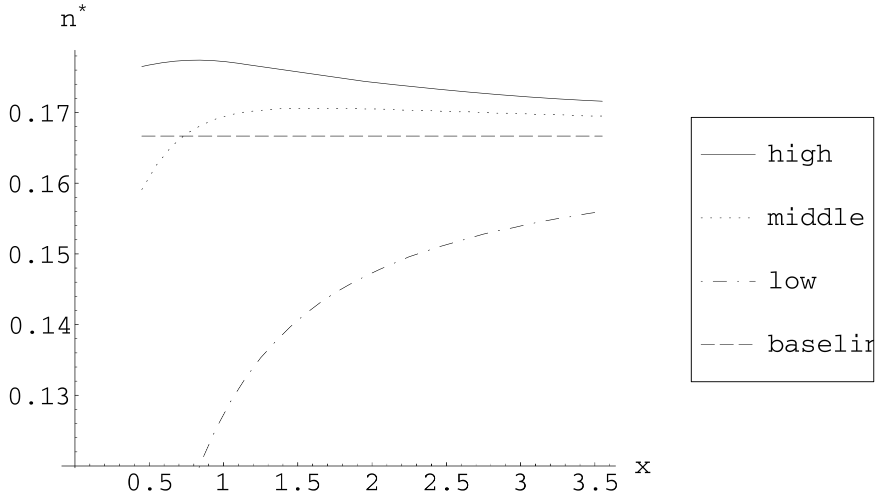

Figure 1: Majority Citizens' Preferred Diversity Levels under Affirmative Action under Different Admissions Tests

diversity when all majority citizens have equal income. When $x = \infty$, test scores are uncorrelated with academic ability. In this case, the three majority types are represented in proportion to their shares of all majority citizens, and hence each type is in the same position at the cutoff and demands n^*. Starting from the right of the figure (large x), low types prefer less diversity as x grows smaller, whereas high and middle types first demand more diversity before demanding less.

For small x—a precise admissions test—high types comprise nearly all admitted majority citizens, which means that they reap all the benefits of diversity but also bear all of its costs. As we have seen, this implies that they demand n close to n^*. As x increases, more low and middle types are admitted, and high types benefit less from diversity but also pay a smaller part of the costs. For small x, an increase in x raises the share of low and middle types scoring at the cutoff more than the share scoring above the cutoff. In this range, because increases in x reduce high types' share of the costs of diversity faster than their share of the benefits, increasing x increases their demand for diversity.

Proposition 1 shows how majority citizens' demand for diversity relates to the income distribution. In our model, a majority citizen prefers more diversity than n^* if and only if her income group bears a smaller share of the cost of diversity (in terms of lower probability of admission) than her group share of seats occupied by majority citizens. When income is evenly distributed, all majority citizens prefer $n = n^*$. When income is unevenly distributed ($\lambda_l < \lambda_m < \lambda_h$), high types bear the smallest share of the cost of diversity while low types bear the largest. As a result,

$n_h^*(x) > n^* > n_l^*(x)$. In general, middle types may bear a larger or smaller share than average, depending on the selectivity of admissions and whether λ_m is closer to λ_h than λ_l. However, because affirmative action is used almost exclusively in *highly selective* colleges, income inequality likely diminishes middle types' support for diversity.

Proposition 2 *Suppose ϕ is single-peaked and C is sufficiently small that for each x and n*

$$\phi_h(x,n) > \phi_m(x,n) > \phi_l(x,n).^{16} \tag{5}$$

Then for any admissions test x any redistribution of income to higher types from low or middle types causes $n_m^*(x)$ to decrease.

It is not surprising that transferring income from middle to high types causes middle types to demand less diversity. The main point of Proposition 2, however, is that even a redistribution from low to high types, which does not change the income of middle types, lowers $n_m^*(x)$.

To understand the intuition behind Proposition 2, it is instructive to rewrite the first-order condition for middle types, $\partial U_m(x, n_m^*)/\partial n = 0$, as

$$\frac{h_m(x,n_m^*)}{\sum_{j\in I} h_j(x,n_m^*)\, w_j(x,n_m^*)} = \frac{v_W'(n_m^*)(1-n_m^*)}{v_W(n_m^*)} \tag{6}$$

where $h_i(x,n) \equiv \frac{\phi_i(x,n)}{1-\Phi_i(x,n)}$ and $w_i(x,n) \equiv \frac{(1-\Phi_i(x,n))W_i}{\sum_{j\in I}(1-\Phi_j(x,n))W_j}$, group i's share of admitted majority citizens. The right-hand side of (6) is decreasing in n_m^*. It follows that a redistribution of income form low to high types lowers n_m^* if it raises the hazard rate of the middles types relative to the average hazard rate of the admitted class. Since more high than low types score at the cut-off, redistributing income from low to high types raises the total number of majority citizens scoring above the cut-off. As a result, the cut-off must be raised to meet the capacity constraint, raising the hazard rate of middle types. Furthermore, such a redistribution increases the share of the admitted class made up of high types, whose hazard rate is lower than middle and low types', lowering the average hazard rate of the admitted majority citizens.

Intuitively, even though middle types' competitive position, as measured by their hazard rate, may not deteriorate relative to the entire majority population, it does relative to the subset against whom they compete. This raises the share of the cost of diversity borne by the middle types, causing them to prefer less diversity.[17]

[16] C need not be extremely small. For example, for a symmetric ϕ it suffices that $C < W_h/2$.

[17] While our model assumes a linear relationship between income and academic ability, Proposition 2 holds more generally, so long as the relationship is not too concave.

Let $\overline{\lambda}$ denote the mean income of the society. When income is evenly distributed among majority citizens ($\lambda_l = \lambda_m = \lambda_h = \overline{\lambda}$), all majority citizens prefer $n = n^*$. It follows from Proposition 2 that $n_m^*(x) < n^*$ whenever $\lambda_l \leq \lambda_m \leq \overline{\lambda} < \lambda_h$. This pattern is described in the following corollary.

Corollary 1 *Suppose ϕ is single-peaked and C is sufficiently small that (5) is satisfied. Then the median majority voter prefers strictly less diversity than n^* when the majority group's median income is equal to its mean income or less.*

Propositions 1 and 2 explain how the competitive environment shapes majority citizens' attitudes toward diversity. In particular, unequal access to higher education, caused by income inequality and competitive admissions, likely lowers the low and middle types' demand for diversity. The results suggest that programs equalizing educational resources or lessening the impact of inequality, such as school-finance reforms and class-based affirmative action, may promote diversity not only by aiding disadvantaged minority students but also by softening majority voters' opposition to affirmative action.

4 Demand for Diversity under a Ban

When affirmative action is banned, the university can only promote diversity by adding noise to its admissions test. For each diversity level $n \leq \frac{N}{W+N}$, define $\widetilde{x}(n)$ such that

$$t_N(\widetilde{x}(n), n) = t_W(\widetilde{x}(n), n). \tag{7}$$

The test $\widetilde{x}(n)$ implements diversity n under the equal-treatment constraint. Since minority citizens have lower test scores than majority citizens, $\widetilde{x}(n)$ increases in n. The marginal utility of n for a type i majority citizen under a ban is

$$\frac{dU_i}{dn} = \frac{\partial U_i}{\partial n} + \frac{\partial U_i}{\partial x}\frac{d\widetilde{x}}{dn}.$$

The second term on the right-hand side captures the effect of a less informative test (holding n constant). Let

$$E_W[\lambda | t_W, x] \equiv \frac{\sum_{i \in I} \phi_i(x, n) W_i \lambda_i}{\sum_{i \in I} \phi_i(x, n) W_i}$$

be the expected ability of a majority candidate scoring at the cut-off t_W. Proposition 3 characterizes the sign of $\frac{\partial U_i}{\partial x}$.

Proposition 3 *For each x and n, $\frac{\partial U_i}{\partial x} \geq 0$ if and only if $E_W[\lambda | t_W, x] \geq \lambda_i$, and $\frac{\partial U_i}{\partial x} = 0$ if and only if $E_W[\lambda | t_W, x] = \lambda_i$. It follows that, for each n, $j > i$, and $x' > x$, if $U_j(x', n) \geq U_j(x, n)$ then $U_i(x', n) > U_i(x, n)$.*

A majority citizen benefits from a more random test if and only if her income is below the average income of majority citizens scoring at the cut-off. Hence, lower types prefer a more random test than higher types. Fixing n, the marginal value of x to a majority citizen of type i is

$$\frac{\partial U_i}{\partial x} = -\frac{\phi_i(x,n)}{x}\left(\frac{\partial t_W(x,n)}{\partial x} - \frac{t_W(x,n)-\lambda_i}{x}\right)\lambda_i v(n).$$

A more random test $x+dx$ affects both the citizen's test score and the cut-off for majority citizens. Since t_W is continuous in x, a small increase in noise only matters to those citizens scoring near the original cut-off $t_W(x,n)$. A citizen with ability λ_i scores t_W under (x,n) only when $\varepsilon = \frac{t_W-\lambda_i}{x}$. Under the new test $x+dx$ her score changes by $\frac{t_W-\lambda_i}{x}dx$. To satisfy the capacity constraint, the cut-off must adjust by an amount equal to the average change in test score among citizens scoring t_W. Thus,

$$\frac{\partial t_W}{\partial x} = \frac{t_W - E_W[\lambda|t_W,x]}{x}.$$

Hence, type i is better off under $(x+dx,n)$ than under (x,n) when $\frac{t_W-\lambda_i}{x} > \frac{t_W-E[\lambda|t_W,x]}{x}$. Cancelling common terms yields Proposition 3.

Since a noisier admissions test places a bigger share of the burden of diversity on higher types than does affirmative action, lower and middle types may support more diversity under a ban. This explains why Florida voters may support replacing affirmative action by a race-neutral admissions rule that admits the same number of minority students. Let n_i^b denote type i's preferred level of diversity under a ban. The next proposition provides sufficient conditions for n_i^b, $i \in \{l,m\}$, to exceed $n_i(x)$, the type's preferred diversity level under affirmative action.

Proposition 4 *Suppose ϕ is single peaked and C is sufficiently small that for each $n \leq \frac{N}{W+N}$*

$$\phi_h(\widetilde{x}(n),n) > \phi_m(\widetilde{x}(n),n) > \phi_l(\widetilde{x}(n),n) > \phi_N(\widetilde{x}(n),n). \tag{8}$$

Then for $i \in \{l,m\}$, $n_i^b \geq n_i(x)$ if (i) λ_i is less than the mean income of all citizens or (ii) λ_i is less than the mean income of all majority citizens, and $n_i(\widetilde{x}(n_i(x))) \geq n_i(x)$.

The single-peakedness of ϕ and condition (8) imply that the average income of citizens scoring at the cut-off is always higher than that of all citizens. When type i citizens have income below the population mean, increasing n by raising x also raises their admissions probability.[18] Hence, they always support more diversity under a ban. When type i citizens have income below the majority-citizen mean, fixing n they always prefer more x. If, in addition, they demand more diversity under

[18] The logic of Proposition 3 applies to the whole population as well. In this case, admitting one more minority citizen displaces more than one high-type majority citizen.

test $\widetilde{x}(n_i(x))$ (the test that creates $n_i(x)$ without affirmative action) than under x, then under a ban any n less than $n_i(x)$ will be strictly dominated by $n_i(\widetilde{x}(n_i(x)))$.

Under Condition 1 of Proposition 4, both low and middle types prefer complete randomization to any other admissions rule under a ban. This occurs in our model due to the perfect correlation in citizens' income and ability. Yet in reality, this correlation is far from perfect. If the returns to higher education are increasing in ability, then all citizens would prefer being admitted when their ability is high rather than low, and randomization, which admits all citizens with equal chance regardless of ability, would entail an efficiency loss (Chan and Eyster, 2003). As a result, when citizens know their income but not ability, even the lowest-income candidate may not prefer $x = \infty$. Nevertheless, conditional on any given level of diversity higher-income candidates still would prefer more informative tests than lower-income candidates. Our model extends to this more general case where income and ability are imperfectly correlated. Let a_i denote majority citizen i's academic ability. Suppose the relationship between income and academic ability is $a_i = \lambda_i + \xi_i$, where ξ_i is a noise term, but majority citizens observe only their incomes. The relationship between income and test scores then becomes: $t_i = \lambda_i + \varepsilon_i + \xi_i$, where ε_i is log concave. Proposition 1 will continue to hold if the random variable $\psi_i \equiv \varepsilon_i + \xi_i$ is strictly log-concave, a condition satisfied if ξ_i is log-concave (Karlin 1968, cited by Miravete 2002).

Throughout we assume that citizens care about diversity only when admitted. For many reasons, they also may care about it when un-admitted. In this case, majority citizens care not only about their likelihood of scoring at the cut-off conditional on admission but also about their likelihood of scoring at the cut-off unconditionally. In this case, the amount by which a majority citizen is willing to lower her admission probability to increase diversity, $\frac{d\pi}{dn}\frac{n}{\pi}$, decreases in π; hence, high types may not prefer more diversity than middle and low types. If concern for diversity when admitted is sufficiently large relative to concern when not admitted, then all of our results hold. Regardless, an overall concern for diversity will not change the different types' preferences over admissions tests. Hence, it remains true that tying n and x through banning affirmative action is likely to increase the support for diversity among majority citizens from lower socioeconomic groups.

Theoretically there may be other ways to tie x and n that are preferable to an outright ban. However, such rules may be impractical due to the difficulties in writing a contract specifying in advance how the admissions criterion would change with the racial composition of the student body. By contrast, to enforce a ban, one only need to make sure that candidates of all ethnicities are treated equally.

5 Conclusion

Because applicants from minority groups tend to have lower high-school grades and standardized-test scores than their majority counterparts, universities cannot produce diverse student bodies simply by admitting those applicants with the highest grades and test scores. There are two ways to remedy this imbalance: the first is through affirmative action; the second is by discounting academic achievement in admissions. American universities traditionally have relied on the first approach. But a recent series of court rulings, state legislations, and ballot referenda banning race-conscious admissions has led many universities to gravitate to the second.

Any means of creating diversity replaces more academically qualified majority candidates with less qualified minority ones. As Chan and Eyster (2003) point out, for each extra minority candidate admitted, affirmative action sacrifices the least quality because it replaces the least qualified majority candidate among those who would otherwise be admitted. This feature of affirmative action appeals to university officials keen on maintaining high overall student quality. Yet at the same time it renders affirmative action politically unpopular by placing a disproportionate burden on majority candidates with marginal qualifications, who more likely come from lower socioeconomic backgrounds. This paper constructs a simple model to illustrate how increasing income inequality may reduce the median voter's support for diversity and identifies conditions under which majority voters from lower socioeconomic groups many support more diversity when affirmative action is banned. Our results explain why affirmative action has become more unpopular among white voters, and why white voters who are against affirmative action may support top-x-percent rules like the ones recently introduced in Texas, California and Florida.

Finally, while de-emphasizing test scores may spread the burden of diversity more equally across different socioeconomic groups, it reduces the admissions process's ability to select the most qualified applicants. Combining race-based and class-based affirmative action may be a better way to foster political support for diversity than banning affirmative action.

6 Appendix

Proof of Lemma 1: The proof follows Bagnoli and Bergstrom (2005). For any $i > j$,

$$\begin{aligned}\frac{1-\Phi_i(x,n)}{1-\Phi_j(x,n)} &= \frac{\int_{t_W}^{\infty}\phi\left(\frac{t-\lambda_i}{x}\right)dt}{\int_{t_W}^{\infty}\phi\left(\frac{t-\lambda_j}{x}\right)dt} \\ &= \int_{t_W}^{\infty}\frac{\phi(\frac{t-\lambda_i}{x})}{\phi(\frac{t-\lambda_j}{x})}\frac{\phi\left(\frac{t-\lambda_j}{x}\right)}{\int_{t_W}^{\infty}\phi\left(\frac{t-\lambda_j}{x}\right)dt}dt \\ &> \frac{\phi_i(x,n)}{\phi_j(x,n)}.\end{aligned}$$

The right-hand side of the second equation expresses $\frac{1-\Phi_i(x,n)}{1-\Phi_j(x,n)}$ as a weighted average of $\frac{\phi\left(\frac{t-\lambda_i}{x}\right)}{\phi\left(\frac{t-\lambda_j}{x}\right)}$ for $t \in (t_W, \infty)$. The last inequality holds since Assumption 1 implies that $\frac{\phi\left(\frac{t-\lambda_i}{x}\right)}{\phi\left(\frac{t-\lambda_j}{x}\right)}$ strictly increases in t. Q.E.D.

Proof of Proposition 1: Suppose $U_i(x,n'') \geq U_i(x,n')$ for some $n'' > n'$. Then for all $j > i$,

$$\begin{aligned}\ln U_j\left(x,n''\right) - \ln U_j\left(x,n'\right) &= \int_{n'}^{n''}\left(\frac{\partial U_j(x,n)}{\partial n}\bigg/ U_j(x,n)\right)dn \\ &> \int_{n'}^{n''}\left(\frac{\partial U_i(x,n)}{\partial n}\bigg/ U_i(x,n)\right)dn \\ &\geq 0.\end{aligned}$$

The first inequality follows from Lemma 1. Hence, $U_j(x,n'') > U_j(x,n')$. The second part is an immediate consequence of the single-crossing property. Write $h_i(x,n)$ for $\phi_i(x,n)/(1-\Phi_i(x,n))$, type i's hazard rate. In our model type i's preferred level of diversity under test x, $n_i^*(x)$, satisfies the first-order condition

$$\frac{h_i(x,n_i^*(x))}{\sum_{j\in I}h_j(x,n_i^*(x))\,w_j(x,n_i^*(x))} = (1-n_i^*(x))\frac{v_W'(n_i^*(x))}{v_W(n_i^*(x))}, \tag{9}$$

where

$$w_j(x,n) \equiv \frac{(1-\Phi_j(x,n))\,W_j}{\sum_{k\in I}(1-\Phi_k(x,n))\,W_k}$$

is type i's share of admitted majority citizens.[19] Let n^* denote the level of diversity such that $(1-n^*)\frac{v_W'(n^*)}{v_W(n^*)} = 1$. We saw in Section 3 that

$$n_h^*(x) = n_m^*(x) = n_l^*(x) = n^*$$

[19] Equation 9 follows from substituting Equation 1 into Equation 4.

when $\lambda_h = \lambda_m = \lambda_l$. Since $(1-n)\frac{v'_W(n)}{v_W(n)}$ decreases in n, $n_i^*(x) \geq n^*$ for any x if and only if

$$\frac{h_i(x, n_i^*(x))}{\sum_{j \in I} h_j(x, n_i^*(x)) w_j(x, n_i^*(x))} < 1.$$

Thus a majority citizen prefers more diversity than n^* if and only if her types are to bear a smaller share of the cost (in terms of lower probability of admission) than other majority citizens. Since for each x and n, $h_l(x,n) > h_m(x,n) > h_h(x,n)$, for each x, $n_h^*(x) > n^* > n_l^*(x)$. Q.E.D.

Proof of Proposition 2: For each n, $i > j$, and $x < x'$

$$\begin{aligned} U_i(x', n) \geq U_i(x, n) \quad &\Leftrightarrow \quad \frac{t_W(x', n) - \lambda_i}{x'} \leq \frac{t_W(x, n) - \lambda_i}{x} \\ &\Leftrightarrow \quad t_W(x', n)\, x - t_W(x, n)\, x' \leq \lambda_i (x - x') \\ &\Rightarrow \quad t_W(x', n)\, x - t_W(x, n)\, x' < \lambda_j (x - x') \\ &\Leftrightarrow \quad U_j(x', n) > U_j(x, n). \end{aligned}$$

The proof of the second part is in the text. Q.E.D.

Proof of Proposition 3: Let $z^* \equiv \arg\max_{z \in \Re} \phi(z)$, $\lambda_h^* \equiv \sum_{i \in I} \lambda_i W_i / W_h$, and $\Lambda \equiv \left\{ (\lambda'_i)_{i \in I} : \sum_{i \in I} \lambda'_i W_i = \sum_{i \in I} \lambda_i W_i \right\}$. Since $t_W(x, n; C, (\lambda_i)_{i \in I})$ decreases in C, there exists $\overline{C}$ such that $t_W(\underline{x}, 0; \overline{C}, (\lambda_h^*, 0, 0)) - \lambda_h^* = \underline{x} z^*$. Since for any $(\lambda'_i)_{i \in I}$, $t_W\left(x, n; C, (\lambda'_i)_{i \in I}\right)$ increases in n and x, for all $C \in (0, \overline{C})$, $x \in [\underline{x}, \infty]$, $n \in [0,1]$, and $(\lambda'_i)_{i \in I} \in \Lambda$,

$$\phi\left(\frac{t_W - \lambda'_h}{x}\right) > \phi\left(\frac{t_W - \lambda'_m}{x}\right) > \phi\left(\frac{t_W - \lambda'_l}{x}\right).$$

The condition $C \leq \overline{C}$ serves two purposes. First, it guarantees that at $t_W(x,n)$, for each x and n, high types are better represented than middle types at t_W, who in turn are better represented than low types. Second, it guarantees that, for each i, type i's preferred level of diversity, n_i^*, uniquely satisfies

$$\frac{\partial U_i}{\partial n} = \frac{-h_i C}{\sum_{j \in I} \phi_j W_j} v(n) + v'(n) = 0.$$

It is straightforward to show that when $C \leq \overline{C}$, $\partial^2 U_i / \partial n^2 < 0$ when $\partial U_i / \partial n = 0$. The second property allows us to apply standard comparative-statics techniques to analyze the way n_i^* is affected by a redistribution of income.

A mean-preserving income redistribution from type j to high types that raises λ_h by 1 unit reduces λ_j by $\frac{W_h}{W_j}$. Hence, the effect of such a redistribution on type i's preferred diversity level is $\frac{\partial n_i^*}{\partial \lambda_h} - \frac{W_h}{W_j}\frac{\partial n_i^*}{\partial \lambda_j}$, where for $h_i \equiv \phi_i / (1 - \Phi_i)$. By the Implicit Function Theorem,

$$\frac{\partial n_i^*}{\partial \lambda_h} - \frac{W_h}{W_j}\frac{\partial n_i^*}{\partial \lambda_j} = -\left(\frac{\partial^2 U_i}{\partial n \partial \lambda_h} - \frac{W_h}{W_j}\frac{\partial^2 U_i}{\partial n \partial \lambda_j}\right) \Big/ \frac{\partial^2 U_i}{\partial n^2}.$$

Since $\frac{\partial^2 U_i}{\partial n^2} < 0$, $\frac{\partial n_i^*}{\partial \lambda_h} - \frac{\partial n_i^*}{\partial \lambda_j}\frac{W_h}{W_j}$ has the same sign as $\frac{\partial^2 U_i}{\partial n \partial \lambda_h} - \frac{W_h}{W_j}\frac{\partial^2 U_i}{\partial n \partial \lambda_j}$. Let $z_i\left(x, n; (\lambda_i)_{i \in I}\right) \equiv \left(t_W - \lambda_i\right)/x$. We can write

$$\frac{\partial^2 U_i}{\partial n \partial \lambda_h} - \frac{W_h}{W_j}\frac{\partial^2 U_i}{\partial n \partial \lambda_j} = \frac{-Cv(n)h_i}{\sum_{k \in I}\phi_k W_k}\left(\frac{dh_i}{dz_i}\frac{1}{h_i}\left(\frac{dz_i}{d\lambda_h} - \frac{W_h}{W_j}\frac{dz_i}{d\lambda_j}\right) - \frac{\sum_{k \in I}\frac{d\phi_k}{dz_k}\left(\frac{dz_k}{d\lambda_h} - \frac{W_h}{W_j}\frac{dz_k}{d\lambda_j}\right)W_k}{\sum_k \phi_k W_k}\right). \tag{A1}$$

From the capacity constraint (Equation 1), $\frac{dt_W}{d\lambda_i} = \phi_i W_i \big/ \sum_{k \in I}\phi_k W_k$. It follows that

$$\frac{dz_h}{d\lambda_h} - \frac{W_h}{W_j}\frac{dz_h}{d\lambda_j} = \frac{1}{x}\left(\frac{\left(\phi_h - \phi_j\right)W_h}{\sum_i \phi_i W_i} - 1\right) < 0.$$

For $i, j \in \{l, m\}$,

$$\frac{dz_i}{d\lambda_h} - \frac{W_h}{W_j}\frac{dz_i}{d\lambda_j} = \begin{cases} \frac{1}{x}\left(\frac{(\phi_h - \phi_j)W_h}{\sum_{k \in I}\phi_k W_k}\right) > 0 & \text{if } i \neq j \\ \frac{1}{x}\left(\frac{(\phi_h - \phi_j)W_h}{\sum_{k \in I}\phi_k W_k} + \frac{W_h}{W_j}\right) > 0 & \text{if } i = j. \end{cases}$$

Since $\frac{dz_i}{d\lambda_h} - \frac{W_h}{W_j}\frac{dz_i}{d\lambda_j} > 0$ for $i,\ j \in \{l, m\}$, the first term inside the parentheses in (A1) is positive for $i \in \{l, m\}$.

Since, by definition, $\sum_{k \in I}\phi_k\left(\frac{dz_k}{d\lambda_h} - \frac{W_h}{W_j}\frac{dz_k}{d\lambda_j}\right)W_k = 0$, and using $\frac{d\phi_k}{dz_k} = \frac{\phi_k'}{\phi_k}\phi_k$, the second term inside the parentheses in (A1) can be rewritten as

$$\frac{1}{x\sum_{k \in I}\phi_k W_k}\left(\left(\frac{\phi_h'}{\phi_h} - \frac{\phi_m'}{\phi_m}\right)W_h\phi_h\left(\frac{dz_h}{d\lambda_h} - \frac{W_h}{W_j}\frac{dz_h}{d\lambda_j}\right) - \left(\frac{\phi_m'}{\phi_m} - \frac{\phi_l'}{\phi_l}\right)W_l\phi_l\left(\frac{dz_l}{d\lambda_h} - \frac{W_h}{W_j}\frac{dz_l}{d\lambda_j}\right)\right) < 0.$$

The inequality holds because $\frac{\phi_h'}{\phi_h} > \frac{\phi_m'}{\phi_m} > \frac{\phi_l'}{\phi_l}$ and for $j \in \{l, m\}$, $\frac{dz_h}{d\lambda_h} - \frac{W_h}{W_j}\frac{dz_h}{d\lambda_j} < 0$ and $\frac{dz_l}{d\lambda_h} - \frac{W_h}{W_j}\frac{dz_l}{d\lambda_j} > 0$. It follows that for $i,\ j \in \{l, m\}$, $\frac{\partial n_i^*}{\partial \lambda_h} - \frac{W_h}{W_j}\frac{\partial n_i^*}{\partial \lambda_j} < 0$.

To complete the proof, consider any $(\lambda_i)_{i \in I}$ and $\left(\lambda_i'\right)_{i \in I}$ such that $\sum_{i \in I}\lambda_i W_i = \sum_{i \in I}\lambda_i' W_i$, and $\lambda_h > \lambda_h'$, $\lambda_m \leq \lambda_m'$ and $\lambda_l \leq \lambda_l'$. For $\alpha \in [0, 1]$, define $q_i = \lambda_i' + \left(\lambda_i - \lambda_i'\right)\alpha$. Note that for $i \in \{l, m\}$,

$$\begin{aligned}
& n_i^*((\lambda_i)_{i \in I}) - n_i^*(\left(\lambda_i'\right)_{i \in I}) \\
= \; & \int_0^1 \left(\frac{\partial n_i^*((q_i)_{i \in I})}{\partial q_h}\left(\lambda_h - \lambda_h'\right) + \frac{\partial n_i^*((q_i)_{i \in I})}{\partial q_m}\left(\lambda_m - \lambda_m'\right) + \frac{\partial n_i^*((q_i)_{i \in I})}{\partial q_l}\left(\lambda_l - \lambda_l'\right)\right)d\alpha \\
= \; & \int_0^1 \left(\frac{W_h}{W_m}\frac{\partial n_i^*((q_i)_{i \in I})}{\partial q_m} - \frac{\partial n_i^*((q_i)_{i \in I})}{\partial q_h}\right)\frac{W_m}{W_h}\left(\lambda_m - \lambda_m'\right)d\alpha \\
& + \int_0^1 \left(\frac{W_h}{W_l}\frac{\partial n_i^*((q_i)_{i \in I})}{\partial q_l} - \frac{\partial n_i^*((q_i)_{i \in I})}{\partial q_h}\right)\frac{W_l}{W_h}\left(\lambda_l - \lambda_l'\right)d\alpha \\
< \; & 0.
\end{aligned}$$

The second equality follows from $\sum_{i \in I}\lambda_i W_i = \sum_{i \in I}\lambda_i' W_i$. Q.E.D.

Proof of Proposition 4: (i) Condition (8) means that for all $n \leq \frac{N}{N+W}$

$$\frac{\sum_{k\in K}\phi_k\left(\widetilde{x}\left(n\right),n\right)W_k}{\sum_{k\in I}\phi_k\left(\widetilde{x}\left(n\right),n\right)W_k+\phi_N\left(\widetilde{x}\left(n\right),n\right)N} > \frac{\sum_{k\in K}W_k}{\sum_{k\in\{h,m,l\}}W_k+N}$$

for $K \in \{\{l,m,h\},\{m,h\},\{h\}\}$. Hence, for all $n < \frac{N}{N+W}$

$$\frac{\sum_{k\in I}\phi_k\left(\widetilde{x}\left(n\right),n\right)W_k\lambda_k+\phi_{Nk}\left(\widetilde{x}\left(n\right),n\right)N\lambda_N}{\sum_{k\in I}\phi_k\left(\widetilde{x}\left(n\right),n\right)W_k+\phi_{Nk}\left(\widetilde{x}\left(n\right),n\right)N} > E\left[\lambda\right].$$

That is, the mean ability of a citizen scoring at cutoff is always greater than the population mean. Following the logic of Proposition 3, we can show that $(1-\Phi_i\left(\widetilde{x}\left(n\right),n\right))$ increases in n when $\lambda_i < E\left[\lambda\right]$. Hence, type i always prefer a higher n (as v is also increasing in n).

(ii) Condition (8) means that for all $n \leq \frac{N}{N+W}$

$$\frac{\sum_{k\in I}\phi_k\left(\widetilde{x}\left(n\right),n\right)W_k\lambda_k}{\sum_{k\in I}\phi_k\left(\widetilde{x}\left(n\right),n\right)W_k} > E_W\left[\lambda\right].$$

That is, the mean ability of a majority citizen scoring at cutoff is always higher than the mean ability of a majority citizen. Suppose for type$i$, $\lambda_i < E_W\left(\lambda\right)$ and $n_i\left(\widetilde{x}\left(n_i\left(x\right)\right)\right) \geq n_i\left(x\right)$. Then, for any $n < n_i\left(x\right),$

$$U_i\left(\widetilde{x}\left(n\right),n\right) < U_i\left(\widetilde{x}\left(n_i\left(x\right)\right),n\right) \leq U_i\left(\widetilde{x}\left(n_i\left(x\right)\right),n_i\left(\widetilde{x}\left(n_i\left(x\right)\right)\right)\right).$$

The first inequality follows from $\lambda_i < E_W\left(\lambda\right)$ and $\widetilde{x}\left(n_i\left(x\right)\right) > \widetilde{x}\left(n\right)$; the second follows from the supposition that $n_i\left(\widetilde{x}\left(n_i\left(x\right)\right)\right)$ is the optimal n given $\widetilde{x}\left(n_i\left(x\right)\right)$. Q.E.D.

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