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ABSTRACT

Unlocking Value: Equity Carve outs as Strategic Real Options*

Equity carve outs, the partial listing of a corporate subsidiary, appear to be transitory arrangements, usually dissolved within a few years by either a complete sale or a buy back. Why do firms perform expensive listings just to reverse them thereafter? We interpret carve outs as strategic options to attract information from the market over the relative value of a productive unit as an independent entity and thus to improve the decision process on whether to sell out or to retain control. The separate listing is costly, as it reduces coordination of production, but generates valuable information from the market over the optimal allocation of ownership. We compute the optimal timing for the final sale or buy back decisions, the value of the strategic options embedded in the carve out and the optimal shares retained. The model explains the temporary nature of carve outs, and suggests an explanation for many empirical findings. In particular, it explains why carve outs are more common in sectors with high uncertainty and in more informative markets.

JEL Classification: g13, g32 and g34
Keywords: buy back, equity carve out, real options, spin off and vertical integration

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1 Introduction

In an equity carve out (ECO), a parent company sells a portion of a subsidiary's common stock through an initial public offering, creating a listed unit. ECOs constitute a significant fraction of IPOs in the US. In the 1990s, over 10% of the IPOs were ECOs; by 1993, five of the six largest IPOs in the US capital market history were ECOs. ECOs are common in industries with high value uncertainty, high sales growth and considerable investments in R&D and marketing (Allen and McConnell, 1998). Carved-out firms appear to have higher growth potential as indicated by their high price/earnings (Schipper and Smith, 1986), market-to-book ratios and R&D expenses relative to the parent (Powers, 2003). The selling firm is usually a large company for which the subsidiary represents a small fraction of its activities.

Intriguingly, carve out arrangements appear to be temporary: most ECOs cease to exist as independent listings within a few years, as they are either sold off or reacquired by the parent. Schipper and Smith (1986) found that 44 out of 73 ECOs are reacquired by the parent, completely divested, spun-off, or liquidated within a few years. Klein et al. (1991) found that 56% of all ECOs are reacquired, while 38% are followed by a complete sell-off. Hand and Skantz (1999b) found instead that 56% of ECOs are sold off or spun off while 17,4% are re-acquired. This dual exit route suggests that at the time of the carve out decision the future optimal allocation of control over the unit is uncertain.[1]

In a survey of managers involved in ECOs by Schipper and Smith (1986), the ECO decision is justified as due either to a shift in corporate focus, greater autonomy for the subsidiary, or to increase the stock market's awareness of its activities. A public listing also produces information for the parent company on the prospects of the subsidiary as a separate unit (Nanda (1991) and Slovin et al. (1995)) and allows market-based managerial incentives (Holmström and Tirole, 1993). This informational gain is to be weighed against the benefits of integration. Coordination between two jointly controlled units may be more efficient than between two independent firms (Hart and Moore (2000) and Hart (1995)). On the other hand

[1] Similar results are reported by Miles et al. (1999) and Boone (2002). A generic study by McKinsey indicates that after five years, just 8% of carved out firms remains public under the control of the parent (Annema et al., 2002).

integration can also reduce efficiency because of managerial rent-seeking behavior induced by the parent's capital allocation procedure (Scharfstein and Stein, 2000).[2]

A carve out transforms a firm as an organizational entity from a subsidiary to a listed firm. While both these aspects have been studied in many empirical papers, their joint effect remains unexplored theoretically. An exception is Nanda (1991), who argues that a parent firm performs a carve out to finance a new investment when it perceives its own shares to be undervalued.

We provide a theoretical model on why a carve out can be value enhancing, and yet be a temporary solution. A parent can choose whether a subsidiary remains fully integrated, partially sold (carved out) or completely separated. The value of an independent unit evolves stochastically over time, and its value can only be observed with noise by the parent. However, equity market participants have the ability to gather information on the value of the unit as an independent entity.[3] Thus, a parent firm may float a fraction of its subsidiary's shares in order to learn more from the market about the value of being fully integrated relative to being spun off. This improves the decision over the ultimate control of the unit.

Floating a stake in a subsidiary, however, is costly in our model because partial integration leads to weaker central managerial incentives and therefore reduces the firm's overall profits. The carve out can be interpreted as a costly "learning option", which allows the parent to acquire information from the financial market at the price of a decrease of efficiency at the organizational level. The listing of a unit while retaining control grants the parent company, next to a "put option to sell", also a "call option to reacquire" the subsidiary. These options are exercised optimally on the basis of the market-generated information received on

[2]Lack of focus may create conflicting business interests; John and Ofek (1995) and Comment and Jarrell (1995) study how corporate focus enhances firm value, while Allen and McConnell (1998), Mulherin and Boone (2000) and Vijh (2002) investigate the corporate focus as a reason to carve out. Moreover, a separate listing allows direct incentives, may reduce internal power conflicts, and encourage initiative (Rajan and Zingales, 2001).

[3]A number of papers explore the link between informed trading in financial markets and firm's real decisions (e.g Subrahmanyam and Titman (1999), Dow and Rahi (2003), Goldstein and Guembel (2006) and Habib et al. (1997)). Luo (2005), Chen et al. (forthcoming) offer evidence that management use stock market information to take investment decisions. Gilson et al. (2001) and Fu (2002) find that carve outs result in higher analyst coverage and lower information asymmetry.

the synergies between the subsidiary and the parent. The parent firm can regain control if profits from integration turn out in the future to outweigh the gain from being independent.[4] Alternatively, the parent may simply sell out the subsidiary, accepting an irreversible loss of control. If changes in technology, regulation or demand cause coordination of activities to become profitable again, the parent firm may not be able any longer to reacquire control over the subsidiary after a complete sale. The firm may have fallen under the control of competitors, might have been restructured in an incompatible way, or its management may not be willing to lose its independence.

The key element in our model is the distinction between uncertainty of the synergies and the noise contaminating this information. Uncertainty enhances the value of a real option and therefore delays the decision. Noise has the opposite effect, diminishing the optimal waiting time because it reduces the option's value (see Childs et al. (2001) for an application to the real estate sector). Carving out allows the parent firm to acquire information from the financial market and reduce the noise with which it observes the synergies. Hence carving out increases the option values to sell out and to reacquire the subsidiary.

Our theoretical model explains the life cycle of equity carve outs as a set of strategic decisions taken in continuous time. The optimal timing to perform the carve out as well as the subsequent timing to exercise the sell out or buy back options depends critically on uncertainty over synergies and the capability of the market to generate more precise information. An independent listing which improves information available to the parent helps assessing the value of either strategy (sell out or buy back) and hence enhances the quality of the decision process.

As the informativeness of market increases, the firm will resort earlier to a carve out, because it learns more for a given loss in production efficiency during the carve out phase. At the same time, the parent will wait longer before selling out or buying back, because the information made available by the market is less noisy and therefore the value to wait and take a more informed decision is higher.

[4]CBS Corporation, a major TV network facing declining profitability, carved out its Infinity Broadcasting division, active in radio stations, in 1999 arguing that the market did not recognize its prospects. Yet two years later CBS reacquired the stake sold, citing loss of synergies across divisions (Wall Street Journal, 2000).

The model is clearly consistent with the temporary nature of the ECOs. Critically, our model can also explain specific empirical findings in the ECO literature. In particular, we can explain the timing of the final decisions between a sell out and reacquisition, and how it is correlated with the percentage of the subsidiary's shares retained by the parent. Our model predicts that ECOs are more likely to be sold off eventually when the initial stake sold in the ECO is smaller. Moreover, the model can explain why a sell out should occur more often than a buy back, and why, as the elapsed time from the ECO increases, the likelihood of a reacquisition increases. All these predictions are consistent with the existing empirical evidence (Klein et al. (1991) and Boone (2002)).

Intriguingly, our model also predicts that carve outs should be more frequent, and the stake sold smaller, when financial markets are more informative. This can explain why the frequency of carve outs in Europe is much lower than in the US. Elsas and Loffler (2001) show that there are much fewer carve outs in Germany and that retained stakes are significantly lower in Germany than in the US (57% on average against the 69% of Allen and McConnell (1998)). The intuition is that more informative financial markets allow to gather more information with a smaller floated stake, and thus a smaller decrease in profitability during the ECO period. Hence, more informative financial markets enhance firm value by helping assessing opportunities at a lower cost.

An additional implication, to our knowledge not yet explored empirically, is that carve outs should be more prevalent in industries where synergies are more uncertain, such as sectors subject to technological change. The intuition is that the informational role of markets is particularly valuable in this case. Furthermore, in such cases we predict a longer average duration before the final decision of selling out or buying back.

In the next section we present the basic model. Section 3 discusses the comparative statics and the empirical implications. A final section summarizes the results and concludes. The proofs are in the Appendix.

2 The Model

We first model the productive activities of two firms, distinguishing whether they are fully, partially or not at all integrated. We next describe how financial markets contribute to information aggregation as a function of the stake floated. Subsequently we solve for the dynamic optimal choice for the parent firm as to whether and when to carve out, as well as the optimal timing of a prospective sell-out or buy-back. Finally we compare the carve out decision with the alternative to sell out.

2.1 Production

We set up a simple model to represent how the productive side of two firms is affected for different levels of integration, following Hart (1995).

There are two firms, the parent, p, and the subsidiary, s, which combine their output to obtain the final product. The quantity of the final product is given by the sum of the outputs of the two firms. When the firms are fully or partially integrated, the parent firm retains control over the subsidiary. While (partially) integrated, production requires coordination by central management, where the effort spent in each firm is denoted by e_i with $i \in \{p,s\}$. In other words, costly managerial attention is allocated between the parent and the subsidiary output. Its cost is increasing and convex in each e_i, specifically $c(e_p,e_s) = e_p^2 + e_s^2$. The parent's management chooses how much effort to exert to increase the degree of coordination and hence each firm's output, q_p and q_s. Effort translates linearly into output: $q_i = e_i$. For notational convenience we normalize the price of output to 2, so that total profits are simply $2(q_p+q_s)$. The profits from selling the output are equally divided between the two firms.[5] The parent receives (q_p+q_s) as direct revenue flow and $\alpha(q_p+q_s)$ as revenue flow from owning α shares of the subsidiary. Hence, when the parent owns a fraction α of the subsidiary, it has a total revenue flow $(1+\alpha)(q_s+q_p)$.

When the two firms are independent, the parent has no control on the other firm's decisions. Each firm has its own management and no effort is exerted on coordination. In such a case, the profit flow for each firm is given by y.[6]

[5] This assumption is made for analytical simplicity, but it is not crucial for the results.

[6] The assumption that after the separation each firm earns the same profits is not essential.

The profit flow for an independent unit, y, varies over time due to changes in technology, regulation or demand. More precisely, we assume it follows an Arithmetic Brownian Motion without drift: $dy = \sigma_y dz_y$ where σ_y is the instantaneous standard deviation per unit of time and dz_y is the increment of a standard Wiener process. The difference between the profit flow of the two firms as either integrated or separate units, $2(q_p + q_s) - q_p^2 - q_s^2 - 2y$, is a measure of net synergies in a conglomerate, relative to the value generated by two separate entities. Accordingly, it may be positive or negative. The decision of the parent to be integrated or not is based on the synergies' values.[7]

When (partially) integrated management chooses effort so as to maximize the profit flow of the parent company.[8] Hence, the problem of the management in a (fully or partially) integrated firm is the following:

$$\max_{q_p, q_s} (1+\alpha)(q_s + q_p) - q_p^2 - q_s^2 \tag{1}$$

Note that $\alpha = 1$ indicates the case where the parent fully owns the subsidiary, when there is no distortion in incentives.

Define by $\pi_p(\alpha)$ the profit flow to the parent which owns a fraction α of the subsidiary and by $\pi_I(\alpha)$ the profit flow to the outside investors who own the remaining $(1-\alpha)$ shares. The first order conditions derive the optimized profit flow as a function of α:

$$\pi_p(\alpha) = \frac{(1+\alpha)^2}{2}, \qquad \pi_I(\alpha) = 1 - \alpha^2 \tag{2}$$

Intuitively, when the parent firm owns only a fraction of the subsidiary, it exerts less effort and the profit flow per share is lower than under full ownership. The disincentive effect inherent in partial ownership constitutes the cost of carving out the subsidiary.

2.2 Information Gathering by Investors

The parent does not observe y directly, but receives a noisy signal, Z_1, which reflects the actual process of y plus some noise ε which follows a Wiener process, z_ε, with a standard

[7]The assumption that only y is stochastic is done to obtain stochastic synergies and keep the model's setting simple.

[8]We assume that there is no conflict of interest between shareholders and management.

deviation σ_ε. More precisely:

$$dZ_1 = \sigma_y dz_y + \sigma_\varepsilon dz_\varepsilon \tag{3}$$

We define with $I(t)$ the information at time t and $m_1(t)$ represents the expected value of y conditional on the available information: $m_1(t) = E[y(t)|I(t)]$. The expected value of y is a weighted average of Z at time 0 and at time t and how it varies over time applying the results of Liptser and Shiryayev (1978):[9]

$$m_1(t) = \frac{\sigma_\varepsilon^2}{\sigma_y^2+\sigma_\varepsilon^2} Z_1(0) + \frac{\sigma_y^2}{\sigma_y^2+\sigma_\varepsilon^2} Z_1(t) \tag{4}$$

$$dm_1(t) = \frac{\sigma_y^2}{\sigma_y^2+\sigma_\varepsilon^2} dZ_1 \tag{5}$$

Once the subsidiary has been carved out, outside investors produce information which increases the precision of the signal. Following Holmström and Tirole (1993), we assume that the higher the stake sold, the higher is the precision of market-generated information.

As soon as the parent carves out the subsidiary, this noisy signal (3) ceases to exist, and a more informative signal, defined as Z_α, is observed by all. We assumed path continuity, so that the new process starts where the previous process ends. This appears the most natural assumption, since in our setting there is no new information released upon the carve out. The degree of the precision of the signal depends on the fraction floated. In particular:

$$dZ_\alpha = \sigma_y dz_y + \alpha^n \sigma_\varepsilon dz_\varepsilon \tag{6}$$

Hence, floating $1-\alpha$ results in a reduction of the noise component of the signal from 1 to α^n. The parameter $n \geq 1$ reflects the degree of market informativeness, and may correspond to measures of liquidity or of analyst following. If n increases, the signal noise from a floatation drops more sharply.

The expected value of y is conditional to the available information, defined as $m_\alpha(t)$, which is given by the weighted average of the inferred value when the process starts and the value of the signal at time 0. As the starting point of Z_α is the same as the ending point of

[9] See also Childs et al. (2001) for an application in real estate.

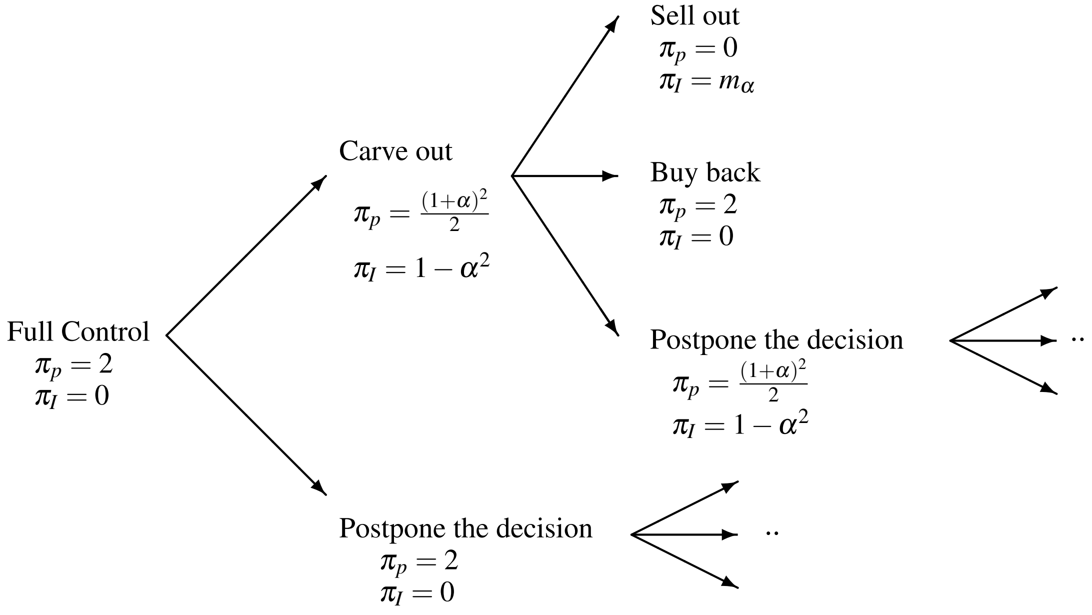

Figure 1: The Decision Tree

the Z_1 we have:

$$m_\alpha(t) = \frac{\alpha^{2n}\sigma_\varepsilon^2}{\sigma_y^2+\alpha^{2n}\sigma_\varepsilon^2}\left(\frac{\sigma_\varepsilon^2}{\sigma_y^2+\sigma_\varepsilon^2}Z_1(0)+\frac{\sigma_y^2}{\sigma_y^2+\sigma_\varepsilon^2}Z_\alpha(0)\right)+\frac{\sigma_y^2}{\sigma_y^2+\alpha^{2n}\sigma_\varepsilon^2}Z_\alpha(t) \quad (7)$$

$$dm_\alpha(t) = \frac{\sigma_y^2}{\sigma_y^2+\alpha^{2n}\sigma_\varepsilon^2}dZ_1 \quad (8)$$

2.3 Dynamic Ownership Choices

At date 0 the parent fully owns the subsidiary, $\alpha = 1$. At each point in time, the parent can carve out the subsidiary or retain full control. The decision is made contingent on the expected value of y, m_1, i.e. on the (relative) profitability of being independent.

After the carve out, the parent has at each instant 3 options: remain in the carve out status, buy back and be again in the full control ($\alpha = 1$) or sell out the remaining shares and become independent. Figure 1 illustrates the parent firm's decision tree.

The decision to carve out can be interpreted as an infinitely lived American compound option (Dixit and Pindyck, 1994). Childs et al. (2001) compute a simple infinitely lived American option when the underlying process is observed with noise. We apply their results to compute the compound option to carve out.

The value of shares on the market equals the present value of expected future cash flows for the investors. In a sell out, the price is set at the expected present value of the subsidiary's future profits as an independent firm. In a buy back, the price reflects the present value of the cash flows investors would have if staying in the carve out state. The price at the carve out reflects the present value of profits when the subsidiary is partially owned plus the present value of the expected cash flow in case of final sale or repurchase. This is computed taking into account that the parent times optimally the sell out or buy back decision.

2.4 The optimal thresholds

Let $V_F(m_1)$, be the value of the parent when integrated and m_i^*, with $i = \{1, \alpha\}$ the optimal threshold of the expected values of the profitability of being independent conditional on the information available. The subscript F, C, B and S indicate the variable respectively in the status of full control, carve out, buy back and sell out. Define:

$$\begin{aligned}
\underline{\beta}_\alpha &= \frac{(1+\alpha)\left((1+\alpha)^3 - (1-\alpha)\sqrt{\alpha^4+2\alpha^3-2\alpha^2+2\alpha+1}\right)}{2(1-\alpha)(\alpha^3+\alpha)} \\
\overline{\beta}_\alpha &= \frac{(1+\alpha)\left((1+\alpha)^3 + (1-\alpha)\sqrt{\alpha^4+2\alpha^3-2\alpha^2+2\alpha+1}\right)}{2(1-\alpha)(\alpha^3+\alpha)} \\
\beta_1 &= \frac{\sqrt{2r\left(\sigma_y^2+\sigma_\varepsilon^2\right)}}{\sigma_y}, \qquad \beta_\alpha = \frac{\sqrt{2r\left(\sigma_y^2+\alpha^{2n}\sigma_\varepsilon^2\right)}}{\sigma_y}
\end{aligned} \tag{9}$$

Theorem 1 *When*

$$\underline{\beta}_\alpha \leq \beta_\alpha \leq \overline{\beta}_\alpha \tag{10}$$

there exists an optimal threshold level m_C^* above which a profit maximizing firm performs a carve out. It subsequently buys back all shares once m_α falls below a threshold level m_B^*, and sells out once it passes a threshold level m_S^*.

The exact values of m_C^*, m_B^* and m_S^* as functions of the parameters are given in the proof in the appendix. As shown there, the initial value of the firm prior to the carve out takes the form

$$V_F(m) = A_1 \exp^{\beta_1 m_1} + \frac{2}{r} \tag{11}$$

where A_1 is a function (see equation (34) in the appendix for the exact values).

β_1 and β_α (see equation (9)) are the two parameters related to the market signal precision through which the noise and uncertainty affect the decision to carve out. They decrease in the ratio of the volatility of the unit value to the noise of the associated signal. The value of a profit maximizing firm is thus given by the value of an integrated unit, $2/r$, plus the compounded option value of performing the carve out, $A_1 \exp^{\beta_1 m_1}$.

Lemma 1 *The optimal thresholds have the following boundaries:* $m_S^* \geq \frac{1+\alpha}{2}$, $m_B^* \leq \frac{1+\alpha^2}{(1+\alpha)}$ *and* $m_C^* \geq m_B^*$.

Intuitively, as long as the conditional expected value of a complete sale is low, the parent prefers to keep complete control of the subsidiary, as ECO is costly and an independent unit is unlikely to be more valuable. As m_1 increases, the value of an independent unit increases and the parent is willing to incur the reduction in profits associated with a carve out in order to access to better information (see Fig. 2).[10] At the optimal threshold level to carveout, m_C^* the expected profits from being independent are greater than the foregone profits of being integrated.

After the carve out, information becomes more precise, which allows a more informed decision on eventually selling out or buying back the subsidiary. Thus the decision to sell out reflects the following trade off. If the parent sells out, it earns the present value of the profit flow m_α. When m_α is sufficiently high, the parent will sell out the subsidiary. As long as the unit remains in the carve out state, the parent has a lower profit flow, but keeps alive the option to buy it back in case the benefits of integration increase enough. The parent never sells out when m_α is so low that the conditional expected value of an independent unit is lower than the expected profits in the carve out state.

Finally, when deciding on a buy back, the parent compares the gain in profitability from an integrated unit against losing the option to sell out. When m_α becomes very low, it is unlikely that a sell out will be profitable, and the parent prefers to buy back the unit. In

[10] This is consistent with Schipper and Smith (1986) who show that ECOs are performed with the aim to increase corporate focus and autonomy in the longer term (i.e., complete sell out) and in order to pursue this end the parent firm needs to increase market awareness (i.e., generate information through a market listing and trading).

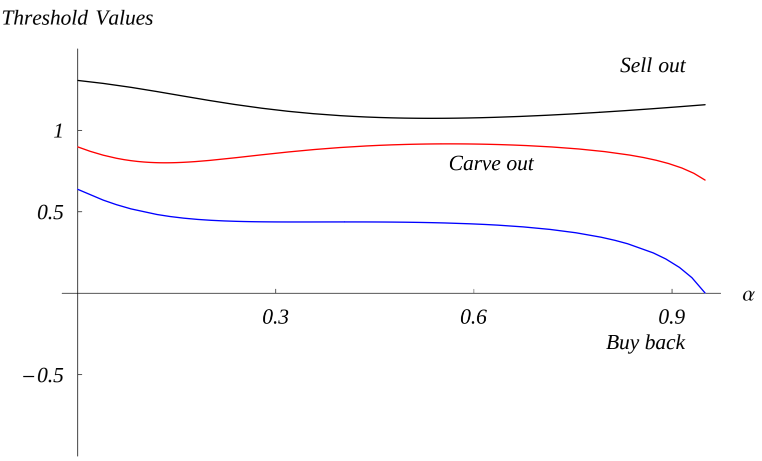

Figure 2: Threshold values in terms of retained stake ($r = 0.01$, $\sigma_y = 0.2$, $\sigma_\varepsilon = 0.3$, $n = 1$.)

particular, the option to buy back is never exercised when m_α is greater than 1, which is the minimum profit flow in the carve out status.

The increase in precision associated with an ECO increases the value to postpone the decision to irreversibly sell out or to buy back. The buy back threshold is always smaller than the threshold to carve out, as the signal becomes more reliable.

When a large stake is sold and the noise is low, the decrease in profitability is substantial. The parent would wait for high conditional expected values of the profits of being independent, m_1, to carve out (Fig. 2). At the same time because of the low noise the gain from the increase in information precision is limited and the parent would therefore prefer to sell out immediately. Hence when condition (10) is not satisfied, the threshold level to carve out, m_C^* is greater than the sell out one, m_S^*.

In the extreme case, when there is little noise, selling a large stake is too expensive as the gain in information acquisition does not compensate the loss in profits. The parent then never wants to carve out (formally m_C^* does not exist). Hence with little noise few shares have to be floated (see condition 10). Note however that $\overline{\beta}_\alpha$ goes to $+\infty$ both as $\alpha \to 0$ and as $\alpha \to 1$. This implies there always exists a α for which the above condition is satisfied, that is there always exists a carve out threshold. The same is true when noise is too high. The parent would not gain much from carving out in terms of information acquisition.

2.5 Carve out or Spin off?

So far we have analyzed the case where the parent firm has only the opportunity to carve out. The parent firm could also simply spin off selling all shares in the subsidiary. We now consider the decision of the parent to optimally carve out or spin off the subsidiary.

A spin off decision can only be taken on the basis of m_1 thus without any information from the market. Define with m^*_{so} the optimal threshold to spin off.

Lemma 2 *Assume a carve out is not possible. Then, the parent spins off the subsidiary when:*

$$m^*_{so} = \frac{1+\beta_1}{\beta_1} \tag{12}$$

As we saw in the previous section, the threshold level to carve out, m^*_C, can be greater than the threshold level to sell out, m^*_S, in which case the sell out occurs immediately after the carve out. In reality this does not occur as in general there is a lock up period. We then assume that in this case the parent does not carve out, but spins off the subsidiary without taking it public. In particular, m^*_C is less than m^*_S when $m^*_C \in [\hat{m}_a, \hat{m}_b]$ where:[11]

$$\hat{m}_a = \frac{4\beta_1 + 3\beta_\alpha^2\left(1-\alpha^2\right) - \sqrt{16\beta_1^2 - 8\left(1-4\alpha+3\alpha^2\right)\beta_\alpha^2\beta_1 - \left(1-\alpha^2\right)\beta_\alpha^2\left(16-\left(1-\alpha^2\right)\beta_\alpha^2\right)}}{4(1-\alpha)\beta_\alpha} \tag{13}$$

$$\hat{m}_b = \frac{4\beta_1 + 3\beta_\alpha^2\left(1-\alpha^2\right) + \sqrt{16\beta_1^2 - 8\left(1-4\alpha+3\alpha^2\right)\beta_\alpha^2\beta_1 - \left(1-\alpha^2\right)\beta_\alpha^2\left(16-\left(1-\alpha^2\right)\beta_\alpha^2\right)}}{4(1-\alpha)\beta_\alpha} \tag{14}$$

We next compare the optimal threshold level to carve out and spin off.

Proposition 2 *When $m^*_C \in [\hat{m}_a, \hat{m}_b]$ and $m^*_C < m^*_{so}$, the parent prefers to carve out at m^*_C, otherwise it spins off the subsidiary at $m_1 = m^*_{so}$.*

When the noise of the signal is low, the parent does not gain much from carving out in terms of information acquisition, while it loses part of the synergies. Hence when the information on the profitability of being independent is sufficiently precise, the carve out is too expensive and the parent prefers to spin off the subsidiary directly.

[11] As the starting point of m_α is the ending point of m_1, $\hat{m}_a$ and $\hat{m}_b$ are values both of m_1 and m_α.

3 Comparative Statics and Empirical Implications

This section explores the effect of information precision, uncertainty and market informativeness. It then assesses the effect of shares retained on the optimal timing of exercise of the strategic options associated with the carve out.

As the threshold values cannot be analyzed explicitly, we present numerical simulations and illustrate them as graphs. We carry out comparative statics analytically whenever possible.

3.1 Information precision and uncertainty

We analyze now the role of uncertainty and noise on the decision to carve out.

Proposition 3 *The optimal sell out threshold level, m_S^* is monotonically increasing in σ_y and decreasing in σ_ε. The optimal buy back threshold level, m_B^* is monotonically decreasing in σ_y and increasing in σ_ε.*

In line with traditional option theory, higher uncertainty increases option value and hence the value of waiting to exercise the options to sell out or to buy back. The optimal exercise levels to sell out and buy back therefore move to more extreme values (Figure 3). Higher uncertainty implies a higher probability that the value of the synergies might drop drastically so it is better to wait for higher (lower) synergy value before giving up the flexibility offered by waiting. Hence, the sell out threshold increases, while the buy back threshold falls.

By contrast, the less precise is the signal, the smaller is the advantage of waiting as noted by Childs et al. (2001). Higher noise implies less information and reliability in the signal. Waiting for a high (low) signal to sell out (buy back) to be more confident on the profitability of the decision is not worthwhile when the signal is not very reliable. Hence, the threshold level to sell out is lower and the threshold level to buy back is higher (Figure 4).

The effect of uncertainty and noise on the carve out threshold level is complicated, as the option to carve out is a compound option. Changes in noise and uncertainty therefore affect both the payoff when the option is exercised and the option value itself. Higher uncertainty on one side induces the parent to wait longer and see how synergies evolve before irreversibly carving out (i.e. uncertainty increases the option value to wait). At the same

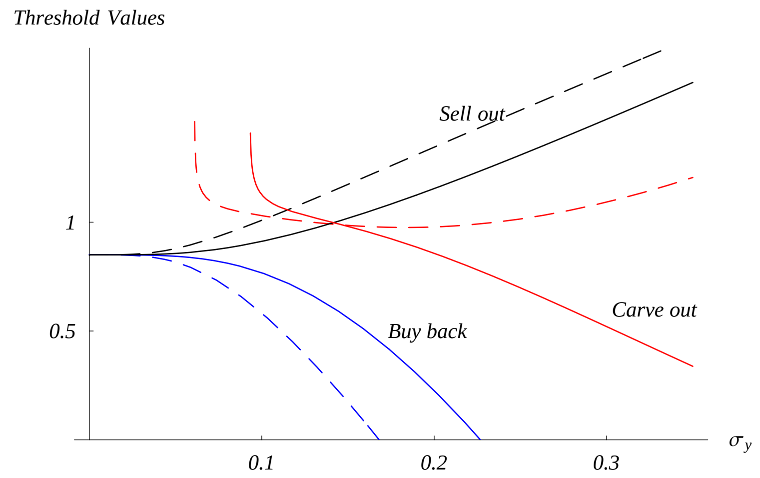

Figure 3: Threshold values as a function of uncertainty, for two values of noise ($r = 0.01$, $\alpha = 0.6$, $n = 1$. Dashed line: $\sigma_\varepsilon = 0.1$; Normal line: $\sigma_\varepsilon = 0.3$).

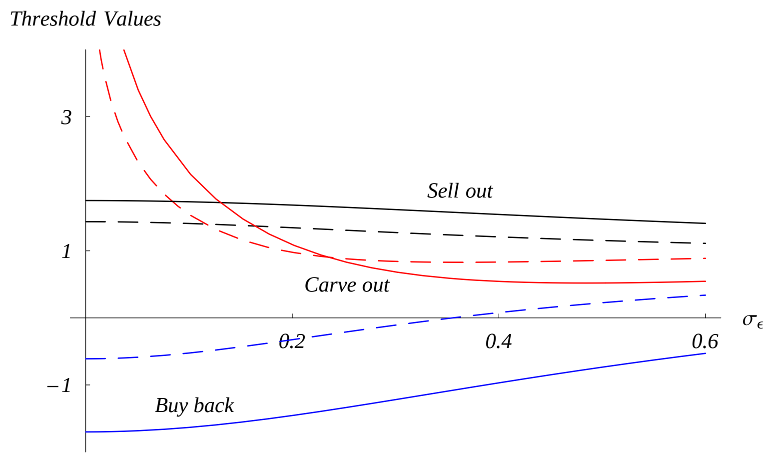

Figure 4: Threshold values as a function of noise, for two values of uncertainty ($r = 0.01$, $\alpha = 0.6$, $n = 1$. Dashed line: $\sigma_y = 0.2$; Normal line: $\sigma_y = 0.3$).

time, however, higher uncertainty makes carving out more valuable. This is because a carve out gives access to the options to buy back and sell out, which in turn are more valuable as uncertainty increases. The overall effect is described in Figure 3. The most significant case is when noise is high. In general, the effect on the payoff prevails and the parent tends to carve out earlier (lower m_1). Only for low levels of noise, the carve out threshold is increasing in uncertainty. In this case the effect on the payoff value due to the associated options is smaller than its effect on the option to carve out.

Noise affects the carve out threshold in a complex way. Higher noise acts in the opposite way to uncertainty. Hence, it induces the parent to carve out earlier as the signal on which the decision is based, is less reliable. At the same time, it induces the parent to carve out later as the gain from carving out is not that high because of the lower value of the option to buy back and sell out. Furthermore, carving out many shares, noise is reduced and the value of taking such a decision increases.

The overall effect is described in Figure 4. In general when the signal is very noisy, the carve out becomes attractive and it is performed earlier. Only if the uncertainty is low and not many shares are floated, the information gain from a carve out is limited and hence the parent tends to carve out at higher conditional expected values.

Thus carve outs should be more common when synergies are uncertain and observed noisily. Many ECOs will arise in sectors where uncertainty on the gains from integration are highly uncertain due to technological innovation or demand changes, but where often the information acquisition at the firm level is difficult or extremely expensive. In this case the information aggregation in the financial market offers a cheaper tool to reduce noise and take a more informed decision.

3.2 Informativeness of the Financial Market

An interesting issue is to which extent financial market characteristics can affect carve out decisions. In our specification the precision of market information increases with n. A higher n can be interpreted as higher financial analyst following or higher market liquidity which increases traders' information production incentives. This parameter may vary among

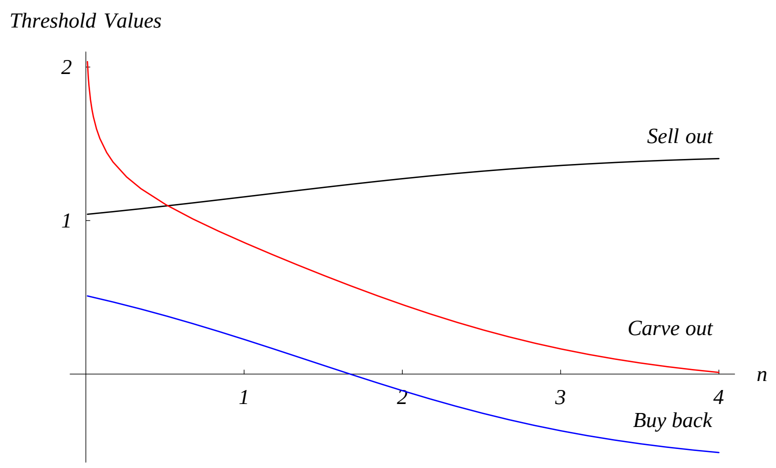

Figure 5: Threshold value in terms of market informativeness ($r = 0.01$, $\sigma_y = 0.2$, $\alpha = 0.6$, $\sigma_\varepsilon = 0.3$).

financial markets but also among firms in the same market.[12]

Proposition 4 *As market informativeness, n, increases, the threshold to sell out increases and the thresholds to buy back and carve out decrease.*

The parent firm needs to carve out fewer shares and hence loses less in terms of production efficiency. Thus when the financial market conveys more information, the parent firm is more willing to carve out because it learns more for a given loss in production efficiency during the carve out phase. At the same time, the parent will wait longer before selling out or buying back, because the information made available by the market is less noisy and therefore there is value to waiting to learn more from the market (see Figure 5).

In markets where information aggregation is more precise, we expect to see more carve outs, and the subsidiary should persist longer in this state. This might explain why ECOs are more common in the US than in other countries like continental Europe where financial markets are less liquid. For example in Germany only 50 carve outs have been carried out between 1984 and 2000 against the 450 carried out in the US the '90s (Elsas and Loffler, 2001).

[12]Allen and Gale (2000) suggest that informational properties of financial markets differ, depending on whether they are part of a 'market' based financial system (UK, US) or a 'bank' based system (Germany, Japan).

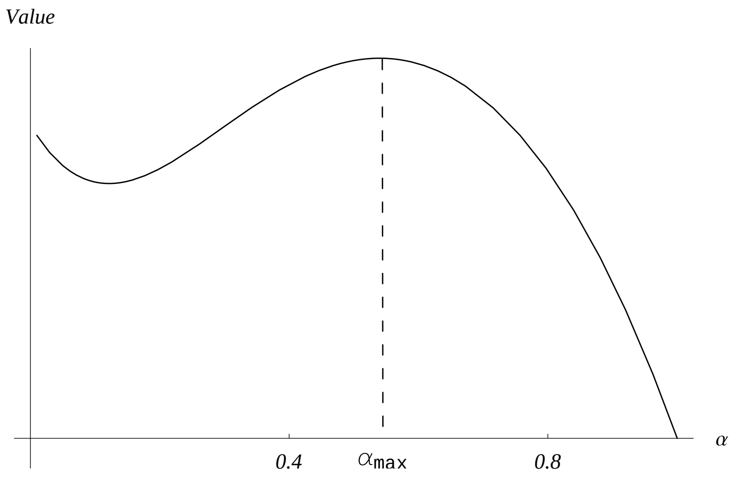

Figure 6: The option value to carve out as function of the shares retained at the optimal threshold ($r = 0.05$, $\sigma_y = 0.2$, $\sigma_\varepsilon = 0.7$, $n = 1$).

3.3 Shares retained

We here solve numerically for the optimal stake sold in a carve out.[13] When noise is relatively high compared to uncertainty, there exists an optimal amount of shares retained in the interval $\alpha \in (0,1)$. Intuitively, the parent faces the trade off of selling many shares to reduce noise but incurring a higher loss in profitability. Hence, if the market signal is relatively precise, the parent performs a carve out in order to sell out the unit soon after. If the signal is very noisy, the parent will sell all shares. When the noise is high but not too high the fraction of shares retained in this case can be quite high (see Figure 6). This is consistent with the empirical studies which find that in general the parent retains more than 50% of the subsidiary (Schipper and Smith (1986), Klein et al. (1991) and Hand and Skantz (1999b)).

Thus parent firms in less informative markets should float a higher fraction of shares to obtain the same level of information precision. No systematic studies across financial markets on carve out have been carried out so far. However, Elsas and Loffler (2001) show evidence that German carve outs, on average sell significantly more shares than US firms.

The fraction of shares retained at the carve out stage affects the timing of the subsequent decisions. We now characterize the optimal exercise to sell out and buy back for a given stake retained.

[13]Empirically, both tax and control considerations should be taken into account

Proposition 5 *The optimal sell out threshold, m_S^* is increasing in α if*

$$\beta_\alpha > \frac{2(1+\alpha)}{(1-\alpha)^3} \tag{15}$$

Intuitively, the fraction of shares retained, α, has two effects on the threshold to sell out. On the one hand, the higher is the retained control of the parent, the smaller is the integration loss. Hence the parent is less willing to sell out quickly, ad will do so only when the value of an independent unit is high enough. On the other hand, the higher the stake retained, the less informative is the market signal. Under a less precise signal, the parent learns less by waiting and hence tends to sell out earlier.

The overall effect is the sum of these two effects, and the relationship may be non-monotonic in α (see Figure 2). In general, when market information is very noisy (either because few shares have been sold or because the signal is intrinsically imprecise), that is when condition (15) is satisfied, the parent tends to sell out earlier. When instead the signal is more precise, i.e. condition (15) is not satisfied, the parent can rely more on the signal and hence will wait longer.

The stake retained affects the buy back decision in the following way.

Proposition 6 *The market signal level m_B^* at which the parent buys back the unit, is monotonically decreasing in α if:*

$$\beta_\alpha < \frac{(1+\alpha)\sqrt{1+6\alpha+\alpha^2}}{\sqrt{2}(1-\alpha)^2} \tag{16}$$

Again the threshold is affected in two different ways. Retaining more shares makes waiting less costly because the organizational inefficiency is small. Therefore, the difference in payoff between carve out and buy back states is small and the parent can wait before taking the final decision. At the same time, when the signal is more noisy, the parent relies less on the informational gain and hence it is less willing to wait. Again the total effect is the sum of these two effects. In general the effect due to the loss of efficiency prevails. When the parent sells more shares it tends to wait less. (Figure 2). Only if the signal noise is extremely high carving out many shares improves the signal precision and hence the firm tends to wait longer. [14]

[14]The buyback threshold however is not a mirror image of the sell out case. When the parent retains almost

The effect of shares retained on the threshold to carve out is more complicated as it affects the option values of buying back and selling out and in addition affects the cost of a carve out (Figure 2). When uncertainty is high relative to noise, the parent tends to wait longer before carving out. The gain in noise reduction is limited and hence it is more profitable for the parent to wait before incurring the loss in efficiency. This effect is stronger the higher the stake sold.

When instead noise is high relative to uncertainty, the effect is the opposite: the carve out threshold m_C^* increases in the shares retained. When retaining many shares, the parent does not gain much in terms of information acquisition and the options to buy back and sell out are not very valuable as the noise remains high and hence the parent waits longer (i.e. higher m_C^*). The opposite applies when many shares are sold. The noise reduction is very high and so the parent is more eager to carve out.

Our model offers predictions on the timing of the sell out and the buy back. The expected time to sell out (buy back) since the carve out, is proportional to the distance between the threshold values, i.e. the threshold value to sell out and the threshold value to carve out (the threshold value to carve out and the one to buy back). Figure 7 shows that difference between these distances. This difference is positive indicating that the expected time elapsed between the carve out and buy back is longer than the time between the carve out and the sell out. It also follows that the sell outs will occur more often than buy backs. This is consistent with the findings of the empirical studies which focus on the buy back and sell out as events after a carve out (Klein et al. (1991), Hand and Skantz (1999a) and Boone (2002)). Figure 7 shows also that the higher is the noise the larger is the difference between threshold values. Hence, in sectors where noise is higher we should expect sell outs to be more frequent than buy backs.

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all the shares it tends to wait infinitely long because by exercising the option to buy back it would not gain much in terms of profits in the specialized contract while it would forego the option to sell out.

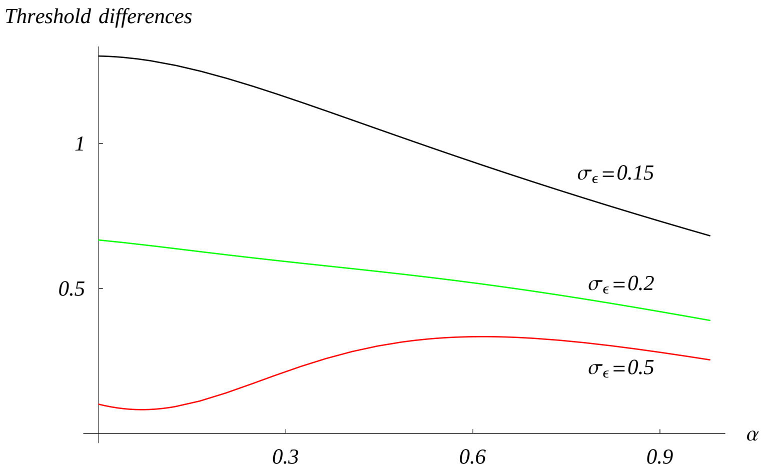

Figure 7: Difference between the distance of the carve out threshold value to the buy back one and the distance of the sell out threshold value to sell out to the carve out one ($r = 0.05$, $\sigma_y = 0.2$, $\sigma_\varepsilon = 0.3$, $n = 1$). A higher value indicates that the expected time to buy back is longer than the one to sell out.

4 Concluding Remarks

Equity carve outs are often presented as a popular strategy for large companies to refocus businesses without relinquishing strategic control over the carved-out units. The empirical evidence suggests that ECOs are temporary strategies which are either reversed or lead to a complete sale. We argue that these strategies depend on market information learnt during the ECO stage. A separate listing allows the use of the market generated information on the relative value of the unit as an independent firm, and thus allows an optimal decision over whether the unit should remain integrated. This may well account for the popularity of ECOs in high growth and highly uncertain sectors such as those more affected by technological change. Here the information gathering function of the financial market is particularly useful. An equity carve out, however, also implies some opportunity costs, which ultimately drive the decision on the allocation of ownership over the unit.

This paper models these trade-offs in a framework with compound real options. We endogenize both the cost and benefit of a listing. When the parent carves out the subsidiary, profitability is reduced because less coordinating effort is exerted by the management. At the same time, it gains more precise information on the value of the subsidiary as an independent entity, and thus reduces the chance for errors in the decision over control allocation. A carve out is thus a temporary configuration, aimed at improving the quality of the decision process on whether to remain integrated or to sell out the subsidiary.

Our approach thus describes the potential benefits of equity carve outs as the acquisition of strategic information to improve decision making. By engaging the market to generate information on the relative value of conglomeration costs or positive synergies. Unlike other real option models, our approach distinguishes between the favorable effect of volatility over synergies on strategic value, against the unfavorable effect of noise, which reduces the precision of information. We show that the signal to noise ratio drives the optimal timing of exercise of embedded strategic options.

We conclude that the choice of an equity carve out is just the first part of a sequential contingent strategy to either sell-off the subsidiary in a phased manner or re-acquire it if favorable information on its internal strategic value emerges. Accordingly, the ECO is a valuable tool in the financial markets are highly informative for firms facing large uncertainty over strategic synergies but lack information. Finally, we have produced novel empirical implications for the frequency of ECOs, buy backs and sell outs, across industry characteristics, measures of strategic uncertainty and financial markets transparency.

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A Appendix

A.1 Proof of Theorem 1

We first set up the Bellman equation for each possible situation: full control, carve out, buy back and sell out. Subsequently, we set the boundary conditions to uniquely identify the optimal threshold values. Let $V(m_i)$, with $i \in \{1, \alpha\}$ be the value of the parent when either (partially or fully) integrated, or when not owning the the subsidiary.

Full control. The Bellman equation when the parent owns fully the subsidiary is:

$$\frac{1}{2}\frac{\sigma_y^4}{\sigma_y^2+\sigma_\varepsilon^2}V''(m_1) - rV(m_1) + \pi_p(1) = 0 \tag{17}$$

The general solution of this differential equation is given by:

$$V_F(m_1) = A_1 \exp(\beta_1 m_1) + A_2 \exp(-\beta_1 m_1) + \frac{2}{r} \tag{18}$$

where A_1 and A_2 are the differential constant that have to be found and:

$$\beta_1 = \frac{\sqrt{2r\left(\sigma_y^2+\sigma_\varepsilon^2\right)}}{\sigma_y} \tag{19}$$

From the boundary condition $V_F(-\infty) = \frac{2}{r}$, we can set $A_2 = 0$.

Carve out. Similarly, the Bellman equation in the carve out status is given by:

$$\frac{1}{2}\frac{\sigma_y^2}{\sigma_y^2+\alpha^2\sigma_\varepsilon^2}V''(m_\alpha)-rV(m_\alpha)+\pi_p(\alpha)=0 \tag{20}$$

Its solution is:

$$V_C=B_1\exp(\beta_\alpha m_\alpha)+B_2\exp(-\beta_\alpha m_\alpha)+\frac{(1+\alpha)^2}{2r} \tag{21}$$

with

$$\beta_\alpha=\frac{\sqrt{2r\left(\sigma_y^2+\alpha^{2n}\sigma_\varepsilon^2\right)}}{\sigma_y} \tag{22}$$

where B_1 and B_2 are the differential constants.

Buy back and sell out. Here no options are embedded, so the traditional NPV can be applied:

$$V_B=\frac{2}{r},\qquad V_S=\frac{m_\alpha}{r} \tag{23}$$

Optimal timing. In order to solve for the optimal exercise thresholds, we impose the value matching and smooth pasting conditions.

When the parent holds full control, the optimal threshold to carve out has to satisfy the following conditions:

$$V_F(m_C^*)=V_C(m_C^*)+\frac{1-\alpha^2}{r}+(1-\alpha)G(m_C^*) \tag{24}$$

$$V_F'(m_C^*)=V_C'(m_C^*)+(1-\alpha)G'(m_C^*) \tag{25}$$

where $G(m)$ is the extra value of the subsidiary for the outside investors given the possibility that the subsidiary in the future can be bought back or sold out. Computing $G(m_\alpha)$ requires computing $E[\exp(-rT)]$ for the sell out and buy back where T is the time to such event. As in Dixit and Pindyck (1994), this means solving the following differential equation where the boundary conditions are determined by the absorbing barriers at $m=m_B^*$ and $m=m_S^*$:

$$1/2\frac{\sigma_y^2}{\sigma_y^2+\alpha^2\sigma_\varepsilon^2}G''(m_\alpha)+rG(m_\alpha)=0$$

This has the general solution:

$$G(m_\alpha)=g_1\exp(-\beta_\alpha m_\alpha)+g_2\exp(\beta_\alpha m_\alpha) \tag{26}$$

where g_1 and g_2 are constants to be determined. As m_α hits m_B^* the parent buys back the shares paying the market price, while when m_α hits m_s the parent sells out the remaining shares and the conditional expected value of the subsidiary is m_S^*/r. Hence the conditions are:

$$G(m_B^*) = \frac{\pi_I}{r} + G(m_B^*) - \frac{\pi_I}{r} - G(m_B^*) = 0$$
$$G(m_S^*) = \frac{m_S^*}{r} - \frac{\pi_I}{r} - G(m_s^*)$$

where $G(m)$ is given in equation (26). Solving this system of equations for g_1 and g_2 and substituting in (26), we obtain:

$$G(m_\alpha) = \frac{\exp(2\beta_\alpha m_\alpha) - \exp(2\beta_\alpha m_B^*)}{2\left(\exp\left(2\beta_\alpha m_S^*\right) - \exp(2\beta_\alpha m_B^*)\right)} \exp(\beta_\alpha (m_S^* - m_\alpha)) \frac{\left(m_S^* - \pi_I\right)}{r} \tag{27}$$

When the subsidiary has been carved-out, the optimal threshold to buy back has to satisfy the value matching and the smooth pasting conditions:

$$V_C(m_B^*) = V_B(m_B^*) - \frac{1-\alpha^2}{r} \tag{28}$$
$$V_C'(m_B^*) = V_B'(m_B^*) \tag{29}$$

In the case of a switch from carve out to sell out, at the optimal threshold the following conditions have to be satisfied:

$$V_C(m_S^*) = V_S(m_S^*) + \alpha\frac{m_S^*}{r} \tag{30}$$
$$V_C'(m_S^*) = V_S'(m_S^*) + \frac{\alpha}{r} \tag{31}$$

This system of six equations ((24),(25),(28),(29),(30),(31)) determines the three thresholds and the three remaining constants. The constants are:

$$B_1 = \frac{(1+\alpha)\, e^{\beta_\alpha m_S^*}}{\left(e^{2\beta_\alpha m_S^*} - e^{2\beta_\alpha m_B^*}\right) r\beta_\alpha}, \tag{32}$$

$$B_2 = e^{2\beta_\alpha m_B^*} B_1, \tag{33}$$

$$A_1 = -\frac{e^{m_S^*\beta_\alpha - m_C^*(\beta_1+\beta_\alpha)}\left(1+\alpha - m_S^*\beta_\alpha(1-\alpha)\right)\left(e^{2m_B^*\beta_\alpha} - e^{2m_C^*\beta_\alpha}\right)}{\left(e^{2m_S^*\beta_\alpha} - e^{2m_B^*\beta_\alpha}\right) r\beta_1} \tag{34}$$

There are two solutions for each threshold. However, to guarantee positive option values we need $m_S^* > m_B^*$. This gives the unique solution:

$$m_S^* = \frac{(1+\alpha)+\sqrt{\beta_\alpha^2(1-\alpha)^4+(1+\alpha)^2}}{2(1+\alpha)\beta_\alpha} \left(1+(1+\alpha)\beta_\alpha - \frac{\beta_\alpha^2(1-\alpha)^4(1-(1+\alpha)\beta_\alpha)}{\left(1(1+\alpha)+\sqrt{\beta_\alpha^2(1-\alpha)^4+(1+\alpha)^2}\right)^2}\right) \tag{35}$$

$$m_B^* = m_S^* + \frac{1}{2\beta_\alpha}\ln\left[\frac{2m_S^*-2-(1+\alpha)\beta_\alpha}{2m_S^*+2-(1+\alpha)\beta_\alpha}\right] \tag{36}$$

The equation that provides the solutions for m_C^* is of the type $F(z)=0$ where $F(z) = (k_1\exp[z]+\exp[-z])-k_2$, z is the unknown and k_1 and k_2 are constants with $k_2<0$. If $k_1<0$, there are two solutions and we select the one where $F(z)$ is increasing. At this point the payoff from carving out becomes greater than the option. If $k_1>0$ no solution exist. In our setting this means, a solution exists when $\underline{\beta}_\alpha \leq \beta_\alpha \leq \overline{\beta}_\alpha$

The optimal carve out threshold level is:

$$m_C^* = -m_S^* + \ln\left[\left(\beta_1\beta_\alpha(1-\alpha)^2+\sqrt{\varphi(m_B^*, m_S^*)}\right)\xi(m_B^*, m_S^*)\right] \tag{37}$$

where

$$\varphi(m_B^*, m_S^*) = \frac{\exp(2m_S^*\beta_\alpha)-\exp(2m_B^*\beta_\alpha)}{4(\beta_1-\beta_\alpha)(1+\alpha-m_S^*(1-\alpha))}$$

$$\xi(m_B^*, m_S^*) = (1-\alpha)^4\beta_1^2\beta_\alpha^2 - 16\frac{(\beta_1^2-\beta_\alpha^2)\exp(2\beta_\alpha(m_S^*+m_B^*))}{(\exp(2m_S^*\beta_\alpha)-\exp(2m_B^*\beta_\alpha))^2}\left((1+\alpha)^2-m_S^*\beta_\alpha^2(1-\alpha)^2\right)$$

A.2 Proof of Proposition 1

To simplify the exposition of the proof of this proposition, we define $F_S(m_\alpha) = B_1\exp(\beta_\alpha m_\alpha)$, which is the option value to sell out, $F_B(m_\alpha) = B_2\exp(-\beta_\alpha m_\alpha)$, which is the option value to buy back and $F_C(m_1) = A_1\exp(\beta_1 m_1)$ the compounded option to carve out.

Proof that $m_S^* \geq \frac{1+\alpha}{2}$.

When the parent sells out:

$$F_S(m_S^*) + F_B(m_S^*) + \frac{(1+\alpha)^2}{2r} = (1+\alpha)\frac{m_S^*}{r} \tag{38}$$

As $F_S(m_\alpha) \geq 0$ and $F_B(m_\alpha) \geq 0$, it follows that

$$m_S^* \geq \frac{(1+\alpha)}{2}$$

Proof that $m_B^* \leq \frac{1+\alpha^2}{(1+\alpha)}$**.**

At the optimal threshold level to buy back, the payoff to buy back has to be higher than the payoff of selling out. More precisely the following condition has to be satisfied:

$$\frac{2}{r} - \frac{1-\alpha^2}{2r} \geq (1+\alpha)\frac{m_B^*}{r}$$

It follows that:

$$m_B^* \leq \frac{1+\alpha^2}{1+\alpha} \tag{39}$$

Proof that $m_C^* \geq m_B^*$**.**

Consider the smooth pasting conditions (24) and (28):

$$F_C(m_C^*) + \frac{2}{r} = F_B(m_C^*) + F_S(m_C^*) + \frac{(1+\alpha)^2}{2r} \tag{40}$$

$$\frac{2}{r} - \frac{1-\alpha^2}{r} = F_B(m_B^*) + F_S(m_B^*) + \frac{(1+\alpha)^2}{2r} \tag{41}$$

When the option to carve out is exercised, the gain from being in the carve out status has to be higher than being in the full control status, otherwise there is no gain from carving out, hence equation (40) is greater (or equal) than equation (41) In order to have this inequality satisfied, $m_C^* \geq m_B^*$.

A.3 Proof of Lemma 2

The proof is similar to a standard computation of an American option where the underlying is the expected payoff conditional on the noisy signal. The Bellman equation when the parent owns fully the subsidiary is given by equation (17). Define with $V_{F,so}$ the value of the parent when fully owning the subsidiary and having the option to sell out. The general solution of the differential equation is given by equation

$$V_{F,so}(m_1) = D_1 \exp(\beta_1 m_1) + D_2 \exp(-\beta_1 m_1) + \frac{2}{r} \tag{42}$$

where D_1 and D_2 are the differential constants that have to be found and β_1 is given in equation (9). $D_2 = 0$ as $V_{F,so}(-\infty) = \frac{2}{r}$. The value matching and the smooth pasting conditions are respectively:

$$V_{F,so}(m_{so}^*) = 2\frac{m_{so}^*}{r} \tag{43}$$

$$V_{F,so}'(m_{so}^*) = \frac{2}{r} \tag{44}$$

Solving the system yields equation (12).

A.4 Proof of Proposition 2

Comparing equations (35) and (37) we obtain that $m_C^* > m_S^*$ when $m_C^* \in [\hat{m}_a, \hat{m}_b]$ where $\hat{m}_a$ and $\hat{m}_b$ are given by equations (13) and (14). Otherwise the parents wants to sell out immediately as soon as the carve out is performed. At the same time when $m_C^* > m_{so}^*$ the parents prefers to do a spin off.

A.5 Proof of Proposition 5

Consider equation (35). The partial derivative with respect to α is:

$$\frac{\partial m_S^*}{\partial \alpha} = \frac{2(1+\alpha) - (1-\alpha)^3 \beta_\alpha}{2\sqrt{\beta_\alpha^2 (1-\alpha)^4 + 4(1+\alpha)^2}}$$

This is positive when $\beta_\alpha(\alpha) < \frac{2(1+\alpha)}{(1-\alpha)^3}$.

A.6 Proof of Proposition 6

Solving the system of equations (35) and (36), we obtain:

$$\begin{aligned} m_B^* = {} & \frac{2(1+\alpha) + \sqrt{\beta_\alpha^2(1-\alpha)^4 + 4(1+\alpha)^2}}{8(1+\alpha)\beta_\alpha} \\ & \left(2 + (1+\alpha)\beta_\alpha - \frac{\beta_\alpha^2\left((1+\alpha)\beta_\alpha - 2\right)(1-\alpha)^4}{\left(2(1+\alpha) + \sqrt{\beta_\alpha^2(1-\alpha)^4 + 4(1+\alpha)^2}\right)^2}\right) \\ & - \frac{1}{\beta_\alpha} \ln\left(\frac{2(1+\alpha) + \sqrt{\beta_\alpha^2(1-\alpha)^4 + 4(1+\alpha)^2}}{(1-\alpha)^2\beta_\alpha}\right) \end{aligned} \tag{45}$$

Furthermore:

$$\lim_{\alpha\to 0} s_B^* = \frac{\left(2+\sqrt{\beta_{\alpha=0}^2+4}\right)}{8\beta_{\alpha=0}} \left(2+\beta_{\alpha=0} - \frac{(\beta_{\alpha=0}-2)\,\beta_{\alpha=0}^2}{\left(2+\sqrt{\beta_{\alpha=0}^2+4}\right)^2} + \right) - \frac{1}{\beta_{\alpha=0}} \ln\left(\frac{2+\sqrt{\beta_{\alpha=0}^2+4}}{\beta_{\alpha=0}}\right) > 0 \tag{46}$$

$$\lim_{\alpha\to 1} s_B^* = -\infty \tag{47}$$

The second derivative is 0 when $\beta_\alpha = \frac{(1+\alpha)\sqrt{\alpha^2+6\alpha+1}}{\sqrt{2}(1-\alpha)^2}$. In this point the first derivative can be positive. Hence m_B^* can have a local maximum.

A.7 Proof of Proposition 3

The payoffs from selling out or buying back are independent from β_α. The payoff from being in the carve out status increases as β_α increases. Hence, the higher β_α, the smaller m_S^* and the higher m_B^*. Given equation (9), there is a negative (positive) relation between β_2 and σ_y (σ_ε).

A.8 Proof of Proposition 4

The payoff from selling out and buying back are not affected by β_α. An increase in n implies a reduction of β_α and hence an increase of the option value to sell out and buy back. This lowers the threshold level to sell out and increases the threshold level to buy back. A value increase of the options to buy back and to sell out means an increase of the payoff when carving out. n does not affect β_1, hence the higher is n the lower is the threshold to carve out.