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## ABSTRACT

### Choosing the Legal Retirement Age in Presence of Unemployment\*

The aim of this paper is to better understand the impact of unemployment on the design of Pay-As-You-Go pension systems, in the context of population aging. We consider a model in which people differ according to age and face in every period a given probability of becoming unemployed. We first determine the optimal pension system, which consists in a payroll tax rate, a pension benefit level and a retirement age and study its comparative statics with respect to a change of the unemployment rate and the length of life. We then characterize the issue-by-issue voting equilibrium and compare it to the optimal pension scheme. It is shown that the median voter in general chooses a retirement age lower than the optimal one as well as a higher payroll tax rate.

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# 1 Introduction

It is now a well known and documented fact that the demographic evolution threatens the viability of existing Pay-As-You-Go (PAYG) pension systems. Both the political debate and the scientific literature aim at finding sustainable reforms to solve the financial unbalance of public pensions. Three policies are usually pointed out: raising contributions, cutting benefits and raising the retirement age. Among those, a rise in the retirement age has received a special attention: a wide literature (see for instance Crawford and Lilien (1991) or Cremer, Lozachmeur, and Pestieau (2004)) studies the incentives to early retirement as an explanation for the empirical trend in many countries to retire earlier and earlier (documented in Gruber and Wise (1998)). A general consequence of the literature (Cremer and Pestieau (2003)) is to advocate policies that encourage continued activity as a way to improve financial problems of pension systems.

This literature rests on the assumption that labour markets are competitive and can completely absorb increases in the labour stock. However, if we depart from the assumption of perfectly competitive labour markets and consider unemployment, the issue of the retirement age cannot set aside labour market features. A rise in the legal retirement age can be (partially) ineffective if older workers become unemployed (not only contributions will not increase, but more unemployment benefits will have to be paid).

In this paper, we study the desirability of pension reforms, in particular concerning the legal retirement age,<sup>1</sup> when there is unemployment. In order to address this problem, we build up a continuous overlapping-generations (OLG) model in which agents inelastically supply labour in each period but express preferences over how many periods they want to work, given that labour gets costlier for older people. In each period, they face the risk of being unemployed. Redistribution occurs towards the retirees and the unemployed, and benefits are financed through a payroll tax on labour.

In this setting, our objective is twofold: we first clarify how the individually optimal policy concerning the pension system and unemployment subsidies, depends on the demography and the labour market characteristics. We then characterize the political choice over this policy and compare it to the optimal policy.

Our results are the following.

We first consider a setting in which unemployment benefits are optimally chosen and

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<sup>1</sup>In our simple setting, the legal retirement age, the effective retirement age and the age at which retirement benefits can be claimed are the same.

the decision over the retirement age has no impact on the unemployment rate. We show that, when confronted to a rise in the length of life or in the unemployment rate, individuals at the beginning of the life cycle want to increase the payroll tax rate, decrease pension benefits and increase the retirement age.

We then turn to exogenous unemployment benefits. The comparative statics with respect to the length of life are unaffected in this modified framework. However, conclusions change as far as an increase in unemployment is concerned. When unemployment benefits are low, it is now possible that individuals respond to a rise in unemployment by anticipating retirement.

The second part of the paper is devoted to the analysis of the political equilibrium. We consider issue-by-issue voting: individuals vote on the payroll tax rate and on the level of pension benefits independently. We show that a corner equilibrium with a maximum tax rate (equal to 1) and the age of retirement being equal to the age of the median voter always exists. Interior equilibria may also exist, depending on the parameters of the model. In these equilibria, the retirement age is larger and the payroll tax rate is lower than at the corner equilibrium. When comparing the political equilibrium with the optimal pension scheme, we find that the median voter has some incentives to retire earlier and to choose a higher tax rate than what optimality commands.

A few other papers (Demmel and Keuschnigg (2000), Corneo and Marquardt (2000), Keuschnigg and Keuschnigg (2004)) merge the labour market and pension reform issues but they aim at proving that wide public pension systems are harmful for unemployment. They focus on the consequences of high social security contributions on labour market equilibrium rather than on the impact of existing unemployment for pension reform: in a union wage-setting model, Corneo and Marquardt (2000) show that contributions to the retirement system have a negative effect on growth and that a shift from a PAYG to a funded system entails a Pareto improvement. Keuschnigg and Keuschnigg (2004) calibrate an OLG model with search unemployment to Austrian data and study the impact of a rise in the retirement on unemployment: they show that this policy is welfare improving, that it boosts growth and lowers unemployment.

Our paper is also related to the literature on voting on pensions. Casamatta, Cremer, and Pestieau (2005) study the equilibrium of a simultaneous vote on the contribution rate and on the tax on the retirement benefit, in a setting when the retirement decision is endogenous. Their focus is different from ours in that the retirement decision is individual

(there is no global determination of a mandatory retirement age); they are more interested in understanding the role of the implicit tax on continued activity for individuals with different productivity. Lacomba and Lagos (2007) focus on the political determination of the retirement age in a model with savings. They study the role of the redistribution degree of the pension system for the political outcome and show that more redistribution induces people to retire later. There are many differences with our model: they do not consider unemployment, they cannot say anything about the effect of an increase in the length of life and they assume that pensions can only be reformed by changing the retirement age. We show that changes in the retirement age are linked to reforms on the other dimensions, the contribution rate and the level of pensions.

The paper is organized as follows. Section 2 presents the main features of the model. Section 3 studies the life cycle utility maximization problem, investigates the role of unemployment and performs a sensitivity analysis on the length of life. Section 4 focuses on the determination of the political equilibrium and its comparison with the optimal policy.

## 2 The model

### 2.1 Production

We set up a continuous time model. At each time  $t$ , an homogeneous good is produced in the economy with a single input technology: labour  $L$  is used in the production function  $f_t(L_t)$ , which has non-increasing returns to scale. We assume that the labour market does not clear, so that there is unemployment and denote  $\pi_t$  the unemployment rate. We take the market imperfection as given.

The cost of labour is  $w_t$  and assumed not to depend on  $\pi$ , which is consistent with the minimum wage model of unemployment.

### 2.2 Consumption

A new generation is born at each point in time  $t$ , with a certain and finite time horizon  $T$ . Life can be divided into two broad periods: until  $R$ , the individual is active. After  $R$ , she retires.

### 2.2.1 The young

At  $t$ , each individual born between  $t - R$  and  $t$  inelastically supplies one unit of labour to the firm. With probability  $\pi_t$ , exogenous to the individual, she is employed and receives the wage  $w_t(1 - \tau_t)$  where  $\tau$  is the proportional tax on labour income. With probability  $1 - \pi_t$  she does not find a job and receives an unemployment benefit  $b_t^U$  (not taxed). Note that history does not play any role in the model: at  $t + 1$  individuals face the same probability  $\pi_{t+1}$  to be employed, irrespective of their employment status at time  $t$ .

### 2.2.2 The old

We assume that the retirement age  $R$  is imposed and common to everybody. Therefore, at  $t$ , each individual born between  $t - T$  and  $t - R$  is a retiree. She receives a retirement benefit  $b_t^R$  (not taxed). The pension system is Pay-As-You-Go (PAYG), financed with the payroll tax on labour at time  $t$ .

## 2.3 Population dynamics

We denote  $C_t$  the number of individuals born at time  $t$ :

$$C_t = C_0 e^{nt},$$

where  $n$  is the rate of population growth.

In the following, we consider a stationary population, i.e. we set  $n = 0$ . If we denote  $N_t$  the total population,  $N_t^R$  the number of retirees, and  $N_t^A$  the number of young people living at time  $t$ , we therefore have that  $N_t = C_0 T$ ,  $N_t^A = C_0 R$  and  $N_t^R = C_0(T - R)$ .

In other words, with a stationary population, at each  $t$  the population is the same so that all subscripts  $t$  can be canceled.

## 2.4 On the unemployment rate

As noted at the beginning of this section, we do not model the origin of  $\pi$ . We therefore consider the most general framework in which unemployment depends not only on the technology but also on the labour supply and its characteristics. In other words, we take into account the possibility that the unemployment rate depends on  $R$  (which, in our model, determines the stock of labour).

There is no clear empirical evidence on the link between the rate of unemployment and the retirement age. Keuschnigg and Keuschnigg (2004) find a negative general equilibrium

effect on unemployment of an increase in  $R$ . However, Diamond (2005), using data from Gruber and Wise (1998), finds no relationship between the unemployment rate for males and the log of the tax force.

Taking this absence of clear evidence into account, our model accommodates all cases.

Given these features of the model, in what follows we are interested in two different issues:

1. What does the presence of unemployment change for the choice of the individually optimal policies? In other words, how do preferences about the fiscal instruments,  $R$ ,  $\tau$  and  $b^R$  respond to changes in unemployment?
2. How do preferences about the fiscal instruments,  $R$ ,  $\tau$  and  $b^R$ , change when there is a shift in the length of life, both with and without unemployment?

### 3 The First-best: Maximization of life-cycle utility

#### 3.1 Preferences

The utility of an individual of age  $a \in [0, T]$  is given by

$$\begin{aligned}
 \text{if } a < R \quad U &= \int_a^R ((1 - \pi(R))[u(c) - v(s)] + \pi(R)u(b^U)) ds + \int_R^T u(b^R) ds \\
 &= (1 - \pi(R))(R - a)u(c) + \pi(R)(R - a)u(b^U) - (1 - \pi(R)) \int_a^R v(s) ds + (T - R)u(b^R)
 \end{aligned} \tag{1}$$

$$\text{if } a \geq R \quad U = (T - a)u(b^R)$$

where  $c$  is consumption, which is equal to the net wage  $w(1 - \tau)$ .<sup>2</sup> We assume that  $u(\cdot)$  is increasing and concave and that  $v(\cdot)$ , which denotes the marginal disutility of labour, is increasing with age.

The life-cycle utility is obtained when we set  $a = 0$ .

#### 3.2 The resource constraint

The payroll tax on labour is used to finance both the unemployment and the retirement benefits from the same budget. Given the stationarity assumptions about the population,

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<sup>2</sup>People do not have the possibility to save or borrow in our model. The only possibility to “transfer” wealth across time is through the pension system. This is of course an important limitation of our approach. With a credit market, the preferences of a given individual on the pension system would depend on accumulated savings and thus on the previous retirement age. This would introduce complicated dynamics. See Lacomba and Lagos (2007).



the government budget constraint (GBC) does not depend on  $t$ . It is given by:

$$\tau w(1 - \pi(R))R - \pi(R)Rb^U - (T - R)b^R = 0. \quad (2)$$

### 3.3 Optimal policies

We are now interested in the optimal values of the fiscal instruments for an individual at the beginning of his life, i.e. we want to maximize his life-cycle utility.

The individual solves

$$\begin{aligned} \max_{\tau, b^R, b^U, R} \quad & (1 - \pi(R))Ru(c) + \pi(R)Ru(b^U) - (1 - \pi(R)) \int_0^R v(a)da + (T - R)u(b^R) \\ \text{s.t.} \quad & (2). \end{aligned}$$

Denoting  $\mathcal{L}$  the Lagrangian function and  $\lambda$  the Lagrange multiplier, the first-order conditions (FOC) are given by:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \tau} &= -u'(c) + \lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial b^R} &= u'(b^R) - \lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial b^U} &= u'(b^U) - \lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial R} &= -\pi'(R)Ru(c) + (1 - \pi(R))u(c) + \pi'(R)Ru(b^U) + \pi u(b^U) + \pi'(R) \int_0^R v(a)da \\ &\quad - (1 - \pi(R))v(R) - u(b^R) + \lambda(\tau w(1 - \pi(R)) - \tau w\pi'(R)R - \pi'(R)Rb^U - \pi b^U + b^R) = 0 \\ \frac{\partial \mathcal{L}}{\partial \lambda} &= \tau w(1 - \pi(R))R - \pi(R)Rb^U - (T - R)b^R = 0 \end{aligned}$$

which can be simplified as:

$$\begin{aligned} u'(c) &= u'(b^R) \\ u'(c) &= u'(b^U) \\ \underbrace{\pi'(R) \int_0^R v(s)ds}_{\boxed{1}} - \underbrace{(1 - \pi(R))v(R)}_{\boxed{2}} + \underbrace{u'(b^R)w}_{\boxed{3}} - \underbrace{u'(b^R)w\pi(R)}_{\boxed{4}} - \underbrace{u'(b^R)wR\pi'(R)}_{\boxed{4}} &= 0 \quad (3) \\ \tau w(1 - \pi(R))R - \pi(R)Rb^U - (T - R)b^R &= 0. \end{aligned}$$

The first and second conditions do not depend on  $\pi(R)$ , which means that the results are the same when there is no unemployment, when unemployment is constant and when it depends on  $R$ : we obtain that there is full insurance against unemployment and that there is perfect consumption smoothing over the life cycle.

The first-order condition with respect to  $R$ , conversely, depends on the assumption about  $\pi(R)$ :

- When there is no unemployment, the terms  $\boxed{1}$ ,  $\boxed{2}$ ,  $\boxed{3}$  and  $\boxed{4}$  disappear. The right hand side of equation (3) is equal to  $-v(R) + u'(b^R)w$ . The optimal value of  $R$  strikes a balance between the disutility of labour at the age of retirement (working one more year entails an increase in the disutility of labour corresponding to that additional year) and the marginal gain in terms of additional resources due to the fact that individuals work one more year;
- When unemployment is fixed, i.e. when it does not depend on the (mandatory) retirement age and thus on the labour stock in the population, the terms  $\boxed{1}$  and  $\boxed{4}$  vanish, but  $\boxed{2}$  and  $\boxed{3}$  remain. After simplification, we get that equation (3) is the same as in the competitive case. This implies that for life cycle maximization, the optimal value of retirement age depends on unemployment only through the GBC.
- When unemployment varies with  $R$ , we also have to consider the remaining terms, on top of the effects described up to now. Let us assume that  $\pi'(R) > 0$ .  $\boxed{1}$  measures the lower loss in terms of labour disutility from an increase in  $R$  due to the fact that increasing  $R$  makes it more likely to become unemployed and therefore not to incur that disutility loss. Conversely,  $\boxed{4}$  points to the fact that an increase in  $R$  is costly due to the increase in unemployment that induces more spending for the additional unemployed and fewer resources for the elderly.

As noted above, we do not know the relationship between unemployment and retirement age; however, this relationship is not the main focus of our paper. Therefore, in order to simplify computations, *we assume that, unless indicated, the unemployment rate does not depend on  $R$*  from now on in the paper.<sup>3</sup> After substituting the condition on  $b^U$  (which states that there is perfect insurance: consumption while working equals

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<sup>3</sup>The assumptions that the both the wage level and the unemployment rate are independent of the retirement age may seem incompatible at first glance. A raise in the retirement age increases the supply of labour and thus, given that labour demand is unchanged, should reflect in a higher unemployment rate. We can however show that these two assumptions are satisfied simultaneously in a slightly enriched model with two types of labour (high skilled and low skilled) and a CES production function. The labour market for the skilled being perfectly competitive, an increase in labour supply leads to an increase in the quantity of skilled labour employed at equilibrium. This in turn leads to an increase in the demand for low skilled workers that, under the CES specification, just offsets the initial increase in labour supply.

unemployment benefits), the first-order conditions become in this case

$$c = b^R \tag{4a}$$

$$wu'(b^R) = v(R) \tag{4b}$$

$$b^R = \frac{R}{T-R}w(\tau - \pi). \tag{4c}$$

The first condition means that individuals at the beginning of the life cycle want to equalize consumption during the working period with retirement consumption. The second one states that individuals choose the retirement age so as to equalize the marginal benefit of working one more year with the marginal disutility of labour. It is useful to look at this condition in greater details. In this purpose, we rewrite the derivative of the Lagrangian with respect to  $R$ , in the case where  $\pi'(R) = 0$ :

$$\frac{\partial \mathcal{L}}{\partial R} = (1 - \pi)u(c) + \pi u(b^U) - u(b^R) - (1 - \pi)v(R) + \lambda(\tau w(1 - \pi) - \pi b^U + b^R).$$

Increasing the retirement age for one year leads the individual to work for one more year with probability  $(1 - \pi)$  and thus to enjoy consumption  $c$  during this year (term  $(1 - \pi)u(c)$ ) and to be unemployed with probability  $\pi$ , earning the unemployment compensation  $b^U$  ( $\pi u(b^U)$ ). On the other hand, the individual renounces to one year on pension benefit ( $-u(b^R)$ ). Under perfect insurance and perfect consumption smoothing ( $b^U = c = b^R$ ), these three terms cancel out. Next, increasing the retirement age for one additional year allows to relax the government budget constraint: the number of contributors to the pension system increases and the number of beneficiaries decreases.<sup>4</sup> This is reflected in the last term of the above equation. Using the optimality conditions ( $b^U = c = b^R$ ) and replacing the value of the Lagrange multiplier, this term becomes  $w(1 - \pi)u'(b^R)$ . This explains first-order condition (4b).

Finally, the third condition is simply the government budget constraint, taking into account the perfect insurance condition ( $b^U = c$ ).

### 3.3.1 Sensitivity of the retirement age to an increase in the life length

We investigate in the next proposition how preferences at the beginning of the life-cycle are affected by a change in the life length.

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<sup>4</sup>Although the number of unemployed people and thus the amount of unemployment compensation payments also increases, the net effect on the pension level is positive.

**Proposition 1.** *When the length of life increases, individuals at the beginning of the life-cycle adjust their preferred policy: they are willing to pay more taxes and receive a lower pension benefit. Moreover the optimal retirement age is increased.*

*Proof.* See appendix A. ■

The intuition for this result is the following. Condition (4a) tells us that  $b^R$  and  $\tau$  necessarily move in opposite direction following a change in a parameter of the model. Assume that, consequently to an increase in the length of life  $T$ ,  $b^R$  increases and  $\tau$  decreases. Inspecting (4c) leads us to the conclusion that  $R$  should increase (otherwise  $b^R$  could not increase). This contradicts equation (4b): if  $b^R$  and  $R$  increase, this condition cannot be satisfied anymore, as the LHS and the RHS move in opposite directions. Therefore,  $b^R$  must decrease and  $\tau$  must increase following an increase in the length of life. Understanding this, it is clear that  $R$  should be increased so as to satisfy (4b).

Later on in the paper, we will focus on the particular case in which the disutility of working one more year does not depend on age ( $v(R) = v$ ). In this case, condition (4b) implies that the level of pension benefits is unaffected by a change in an exogenous variable, the reason being that the RHS is fixed. Then (4a) implies that  $c$  (and therefore  $\tau$ ) is also unaffected. Following an increase in the length of life, the only way to satisfy the GBC (4c) is thus to increase  $R$ . In the particular case where  $v(R)$  is constant, individuals at the beginning of the life cycle react by adjusting the age of retirement only, keeping the payroll tax rate and the level of pension benefits unchanged.

Before moving to the next section, we shall return for a moment to the more general case where  $\pi'(R) > 0$ . Considering for convenience a linear relationship between the unemployment rate and the retirement age ( $\pi''(R) = 0$ ) and a constant disutility of labour, the expression  $dR/dT$  in appendix 1 becomes:

$$\frac{dR}{dT} = \frac{1}{|M|} \left[ w^2 u''(b^R) \frac{1-\tau}{1-\pi} (1 - \pi - R\pi'(R)) \right].$$

When the unemployment rate depends on the retirement age, it is not true anymore that individuals always choose to delay retirement in the face of an increased longevity. They may want to retire earlier if and only if the elasticity of the unemployment rate with respect to the retirement age is sufficiently large:

$$\frac{\pi'(R)}{\pi/R} > \frac{1}{\pi} - 1.$$

The probability of unemployment is endogenous. If we assume that  $\pi(R) = \alpha R$  with  $0 \leq \alpha \leq 1/T$ , the above condition becomes

$$\alpha > \frac{1}{2R}.$$

One should be cautious in interpreting this formula as  $R$  endogenously depends on  $\alpha$ . Further investigation is needed to determine the precise conditions in which individuals may choose to advance retirement in the face of an increased longevity.

### 3.3.2 Comparison of the optimal retirement age with and without unemployment

In this section, we seek to understand better how the optimal retirement age varies with the unemployment rate, in order to be able to compare the competitive case with the (fixed) unemployment framework. For that purpose, using the fact that  $b^R - b^U = 0$ , we differentiate the system of first-order conditions (4a)-(4c) with respect to  $\pi$  and use Cramer rule to get the following

**Proposition 2.** *When the unemployment rate increases, agents at the beginning of the life-cycle want to postpone retirement, increase the payroll tax rate and decrease pension benefits.*

*Proof.* See appendix B. ■

Overall, this proposition is very close in spirit to the previous one. From the system (4a)-(4c) we can see that unemployment does not influence preferences; it has only an impact on the budget constraint: a higher unemployment rate implies that there are fewer contributors and more unemployed (and thus more money spent for unemployment benefits). In order to compensate the decrease in the government resources, agents want to postpone retirement. Moreover, for the same reasons as in the previous section, agents want to decrease the pension benefit and to increase the payroll tax rate.

To summarize, we have shown that the retirement age and the payroll tax rate desired by an individual at the beginning of his life increase whereas the level of retirement benefits decrease when there is more unemployment or when the length of life increases.

## 3.4 Exogenous unemployment benefits

We solve in this section the life-cycle maximization problem when unemployment benefits are given. The reason for doing so is twofold. First, as we saw in the previous sections, the

optimum with endogenous benefits involves perfect insurance. This result is due to the fact that individuals do not choose their labour supply within time periods. If it was the case, perfect insurance would not occur as nobody would have an incentive to continue working. This result is thus totally dependent on the simplified nature of our model and would disappear in a more elaborated model. Second, we consider in the next section dealing with the political equilibrium exogenous benefits. Considering exogenous benefits in the life-cycle maximization problem then allows to make appropriate comparisons between the two settings.

The life-cycle program writes

$$\begin{aligned} \max_{\tau, b^R, R} \quad & (1 - \pi)Ru(c) + \pi Ru(b^U) - (1 - \pi) \int_0^R v(a)da + (T - R)u(b^R) \\ \text{s.t.} \quad & \\ & \tau w(1 - \pi)R - \pi Rb^U - (T - R)b^R = 0. \end{aligned}$$

Denoting  $\mathcal{L}$  the Lagrangian function and  $\lambda$  the Lagrange multiplier, the first-order conditions (FOC) are given by:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \tau} &= -u'(c) + \lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial b^R} &= u'(b^R) - \lambda = 0 \\ \frac{\partial \mathcal{L}}{\partial R} &= (1 - \pi)u(c) + \pi u(b^U) - (1 - \pi)v(R) - u(b^R) + \lambda(\tau w(1 - \pi) - \pi b^U + b^R) = 0 \\ \frac{\partial \mathcal{L}}{\partial \lambda} &= \tau w(1 - \pi)R - \pi Rb^U - (T - R)b^R = 0 \end{aligned}$$

which can be simplified as:

$$c = b^R \tag{5a}$$

$$\pi(u(b^U) - u(b^R)) + \frac{T}{R}b^R u'(b^R) = (1 - \pi)v(R) \tag{5b}$$

$$b^R = \frac{R}{T - R}(\tau w(1 - \pi) - \pi b^U). \tag{5c}$$

The first condition is the same as in the first-best program: individuals at the beginning of the life-cycle want to smooth consumption between the working and the retirement periods. The second condition, that corresponds to the first-order condition on the retirement age, is modified by the fact that there is no perfect insurance anymore. When retirement is postponed for one year, the individual gets consumption  $c$  with probability  $(1 - \pi)$  and unemployment benefits  $b^U$  with probability  $\pi$ . He also loses pension benefits

for that year. The total effect on utility is thus  $(1 - \pi)u(c) + \pi u(b^U) - u(b^R)$ . Under consumption smoothing ( $c = b^R$ ), this becomes  $\pi(u(b^U) - u(b^R))$ , which corresponds to the first term in (5b). This term is new compared to (4b), as in the case with perfect insurance  $b^U = b^R$ . The second term in (5b) comes from the fact that postponing retirement allows to increase pension benefits. It corresponds to the term  $w(1 - \pi)u'(b^R)$  in (4b) and is identical to it when  $b^U = b^R$ .<sup>5</sup> Finally the term on the RHS of (5b) simply means that working one more year entails a disutility  $v(R)$ . Observe that when  $b^U$  is such that  $b^U = b^R$ , the solutions to the problems with endogenous and exogenous unemployment benefits coincide.

The first-order condition on  $R$  is more complex in the case of exogenous unemployment insurance. In particular, it does now depend on  $\pi$ , which was not the case with endogenous insurance. This has significant implications for the comparative statics of the pension policy, as stated in the next proposition, proven in appendix C.

**Proposition 3.** *Comparative statics on the optimal policy with exogenous unemployment benefits.*

(i) *When the length of life increases, agents at the beginning of the life-cycle want to postpone retirement, increase the payroll tax rate and decrease pension benefits.*

(ii) *When the unemployment rate increases, the change in the retirement age, the payroll tax rate and pension benefits is ambiguous. In particular, the retirement age may increase or decrease following an increase in the unemployment probability.*

The first part of the proposition says that the comparative statics of the optimal pension policy with respect to a change in the life length are qualitatively unchanged when unemployment benefits are exogenous: the payroll tax rate and the retirement age increase whereas pension benefits decrease. The reason for this result is similar to the case with endogenous unemployment benefits.

The comparative statics with respect to  $\pi$  are however different. The expression derived in appendix C show that the effect of  $\pi$  on the pension policy cannot be unambiguously signed anymore. To illustrate this point, we reproduce below equation (17c) in the appendix:

$$\frac{dR}{d\pi} = \frac{w}{|M|} \left[ -(u(b^R) - u(b^U) - v(R))(T - \pi R) - (\tau w + b^U)Ru'(b^R)\left(\frac{T}{R}(1 - R_r(b^R)) - \pi\right) \right].$$

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<sup>5</sup>The factor  $(1 - \pi)$  does not appear in (4b) as it is present on both sides of the equation and so has been cancelled out.

Consider a isoelastic utility function,  $u(x) = x^{1-\varepsilon}/(1-\varepsilon)$ , with  $\varepsilon > 1$ . As  $u(b^U) \rightarrow -\infty$  when  $b^U \rightarrow 0$ , it is clear that  $dR/d\pi$  is negative for  $b^U$  sufficiently small and positive when  $b^U$  and  $b^R$  are close. This contrasts with the endogenous insurance case in which the retirement age always increases with the unemployment rate. The reason for this difference can be found by inspecting (5b). The marginal benefit of postponing retirement is composed of two terms,  $\pi(u(b^U) - u(b^R))$  and  $(T/R)b^R u'(b^R)$ . The first of these two terms expresses the fact that with probability  $\pi$ , the individual is unemployed in the marginal working year and thus earns  $b^U$  instead of  $b^R$ . When the unemployment compensation is small and the disutility of living one year without resources is large (which is true as long as  $\varepsilon \geq 1$ ), the marginal cost of postponing retirement is very important and cannot be offset by the marginal gain in terms of increased pension benefits,  $(T/R)b^R u'(b^R)$ .

## 4 Issue-by-issue voting equilibrium

We determine in this section the political equilibrium when individuals vote on one issue at a time.<sup>6</sup> The two issues determined by the vote are the payroll tax rate and the retirement age. We take the unemployment benefit as given, the pension benefit level being residually determined by the government budget constraint.

### 4.1 Voting on $\tau$ for a given $R$

#### 4.1.1 Individually optimal tax rates

We have to distinguish two cases. If, at the time of the vote, the individual is a retiree, he will vote for the tax rate that maximizes his pension level, namely  $\tau = 1$ . Therefore we have:

$$\text{If } a \geq R, \tau^* = 1.$$

We now consider the case of a worker. The government budget constraint (2) can be rewritten:

$$b^R(\tau, R) = \frac{\tau w(1-\pi)R - \pi R b^U}{(T-R)}. \quad (7)$$

Substituting this expression into the life-cycle utility of an aged  $a$  worker (see (1)), the program of this latter is:

$$\max_{\tau \in [(\pi b^U)/w(1-\pi), 1]} \int_a^R ((1-\pi)[u(c) - v(s)] + \pi u(b^U)) ds + \int_R^T u(b^R(\tau, R)) da.$$

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<sup>6</sup>This equilibrium concept has been proposed by Shepsle (1979).



The lower bound on  $\tau$ ,  $\tau_{\min} \equiv (\pi b^U)/(w(1 - \pi))$ , comes from the fact that, even if the pension level is set to 0, some taxes have to be collected so as to finance unemployment benefits. After some manipulations, we obtain the following first-order condition:

$$\frac{u'(c)}{u'(b^R)} = \frac{R}{R - a}. \quad (8)$$

This condition tells us that  $\forall a > 0$ ,  $c < b^R$  ( $c = b^R$  when  $a = 0$ ). At the beginning of the life-cycle, people want to smooth consumption perfectly. As they get older, the return from the PAYG system increases and the substitution effect pushes toward consuming more during retirement than during the working period. Note that this result would not hold with a perfect credit market in which agents would be able to borrow as much as they want. In such a case, they would choose the pension system that is the most generous possible (which corresponds to setting the payroll tax rate equal to 1) and would use borrowing to smooth consumption over time.

#### 4.1.2 Majority voting tax rate

By differentiating the utility function (1), we obtain that the slope of an indifference curve in the  $(\tau, b^R)$  plane is:

$$\frac{db^R}{d\tau} = \frac{w(R - a)u'(c)}{(T - R)u'(b^R)}.$$

The slope of indifference curves decreases with age. This implies both that *the optimal payroll tax rates increase with age* and that *the single-crossing condition holds*. This latter property in turn implies that a Condorcet winner exists. The majority voting solution is the preferred tax of the individual with median age:  $a = T/2$ . Using (7) and (8), the majority voting tax rate,  $\tau^{mv}$ , is implicitly defined by the following condition:

$$\left(R - \frac{T}{2}\right)u'(w(1 - \tau^{mv})) - Ru'((R/(T - R))(\tau^{mv}w(1 - \pi) - \pi b^U)) = 0. \quad (9)$$

## 4.2 Voting on $R$ for a given $\tau$

### 4.2.1 Majority voting retirement age

In general, the optimal choice of the retirement age by any individual depends on the status quo retirement age through its effect on savings. Agents adjust their optimal savings depending on the retirement age. If this latter is low for example, the number of contributors is small relatively to the beneficiaries and the pension level is accordingly low. In such a situation, individuals may want to make important savings in order to finance

their old age consumption. When, on the other hand, pensions are generous, individuals do not have to save a lot. Therefore the level of savings depend on the current retirement age and in turn they affect the newly chosen retirement age: the optimal retirement choice of a given individual at a given date depends on his accumulated wealth. We abstract from these difficulties in this model by considering that individuals do not have the possibility to save.

The choice of the retirement age also depends on whether it is assumed that the current retirees go back to the labour market or not when the newly voted retirement age is superior to their age. If the current retirees do not return to the labour market, the status of the individual, whether he is active or not, makes a difference. Two individuals of the same age but who do not have the same status (one is a worker and the other a retiree) do not share the same preferences. The retiree favours a retirement age as large as possible as it does imply a lower dependency ratio and thus a larger pension. The worker trades-off the gain in pension and wage benefit with the disutility of working an additional year and the loss of pension benefit for that year. We shall assume in what follows that *the current retirees return to the labour market if the new retirement age is larger than their age*. Under this assumption, we do not have to make the distinction between active workers and retirees. All that matter for the determination of the optimal retirement decision is the age of the individual.

We first argue that an individual has no interest in choosing a retirement age strictly lower than his own age. In such a situation, he would benefit from increasing the retirement age, as it would increase the number of contributors to the pension system and decrease the number of beneficiaries.<sup>7</sup> An individual aged  $a$  thus solves

$$\max_{R \in [a, T]} V(R) = \int_a^R ((1 - \pi)[u(c) - v(s)] + \pi u(b^U)) ds + (T - R)u(b^R(\tau, R)).$$

The first-order conditions for an interior solution is

$$\frac{\partial V}{\partial R} = (1 - \pi)u(c) + \pi u(b^U) - u(b^R) - (1 - \pi)v(R) + \frac{T}{R}b^R u'(b^R) = 0. \quad (10)$$

This condition does not depend on age  $a$ . Consequently, when they choose an interior solution, individuals all choose the same value for the retirement age. In other words, there is no conflict over this decision. The majority voting retirement age is thus the preferred retirement age of the individual with median age,  $a = T/2$ .<sup>8</sup>

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<sup>7</sup>Increasing the retirement age also increases the number of unemployed and thus total unemployment payments. One can show however that the gain in alleged pension payments outweigh this cost.

<sup>8</sup>To be precise, if the median age individual wants a retirement age larger than his own age, all

### 4.3 Equilibrium

An interior equilibrium is characterized by equations (9) and (10) that we rewrite below:

$$(R^e - \frac{T}{2})u'(c) - R^e u'(b^R) = 0 \quad (11)$$

$$(1 - \pi)u(c) + \pi u(b^U) - u(b^R) - (1 - \pi)v(R^e) + \frac{T}{R^e}b^R u'(b^R) = 0, \quad (12)$$

where

$$\begin{aligned} c &= w(1 - \tau^e) \\ b^R &= \frac{R^e}{T - R^e}(\tau^e w(1 - \pi) - \pi b^U). \end{aligned}$$

#### 4.3.1 Existence and characterization

We argue in the following proposition that a stable corner equilibrium always exists whereas the existence of a stable interior equilibrium is not guaranteed.

**Proposition 4.** *Assume that the utility function is isoelastic:  $u(x) = x^{1-\varepsilon}/(1-\varepsilon)$ . The political equilibrium such that  $\tau^e = 1$  and  $R^e = T/2$  always exists. There might also exist interior equilibria, depending on the parameters of the model. In these equilibria,  $\tau^e < 1$  and  $R^e > T/2$ .*

*Proof.* See appendix D. ■

Issue-by-issue voting equilibria are represented on the figures below.

In order to get a better sense of the conditions ensuring the existence of interior equilibria, we solve the model with a logarithmic utility function,  $u(x) = \ln x$ , and a constant disutility of work,  $v(R) = v$ . Condition (12) becomes in this case:

$$(1 - \pi) \ln c + \pi \ln b^U - \ln b^R - (1 - \pi)v + \frac{T}{R^e} = 0. \quad (13)$$

From (11),

$$\frac{c}{b^R} = \frac{R^e - T/2}{R^e}.$$

Substituting into (13), we have:

$$\phi(R^e) \equiv \ln \frac{R^e - T/2}{R^e} + \pi \ln \frac{b^U}{c} - (1 - \pi)v + \frac{T}{R^e} = 0. \quad (14)$$

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individuals younger than him as well as part of the older ones have the same optimal retirement age. When the preferred retirement age of the median individual is equal to his own age, all younger (resp. older) individuals have a lower (resp. higher) optimal retirement age.

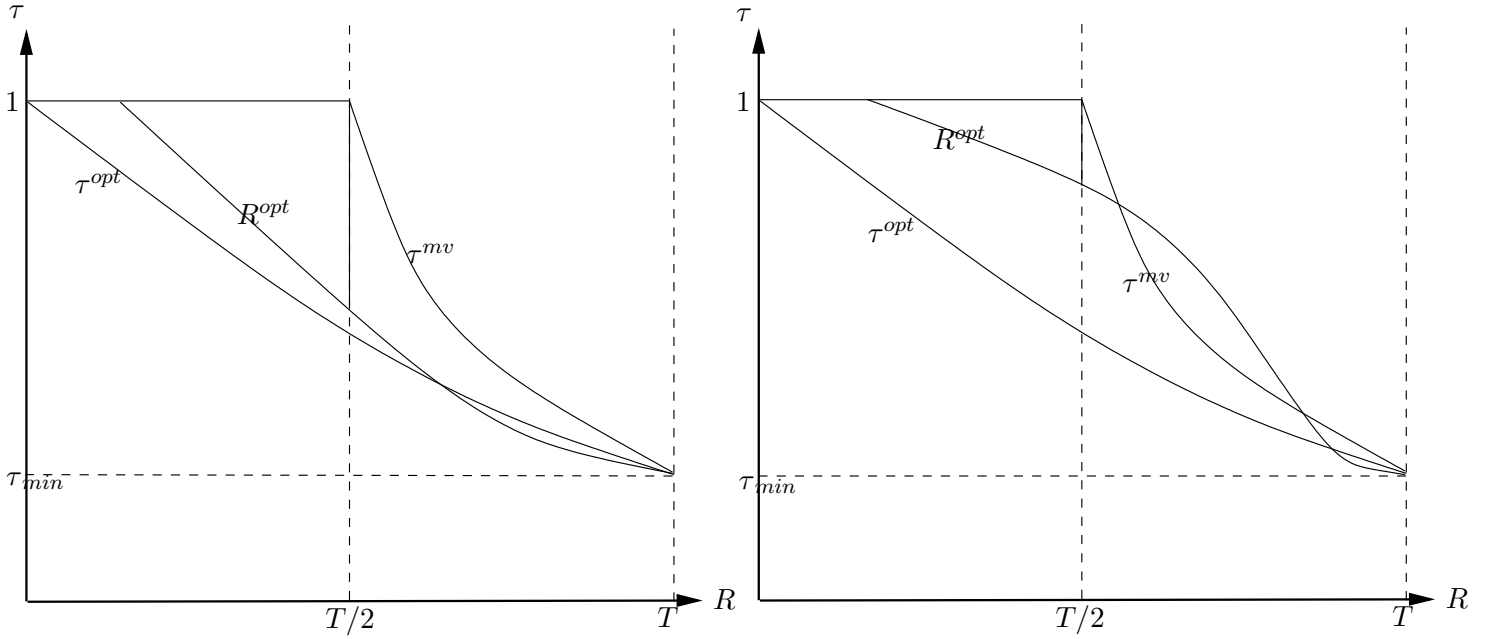


Figure 1: Issue-by-issue voting equilibria

After some manipulations, we obtain that

$$c = \frac{R - T/2}{(T/2)(1 + \pi) - \pi R} (w(1 - \pi) - \pi b^U).$$

Substituting into (14), this implies that  $\phi(\cdot)$  is increasing with  $b^U$ . When  $b^U \rightarrow 0$ , we readily deduce from (14) that  $\phi(R) \rightarrow -\infty$  and thus that there is no interior equilibrium. The intuition is straightforward. If the individual decides to postpone retirement for one year, he takes the risk of being unemployed with probability  $\pi$  during this year, being left with almost no resources when the unemployment compensation is low. Starting from any candidate equilibrium, he has thus an incentive to reduce the retirement age so as to avoid this risk.

Under perfect insurance ( $b^U = c$ ), the equilibrium condition writes:

$$\ln \frac{R^e - T/2}{R^e} - (1 - \pi)v + \frac{T}{R^e} = 0.$$

When  $R \rightarrow T/2$ ,  $\phi(R) \rightarrow -\infty$ . Moreover,

$$\phi(T) = \ln 1/2 - (1 - \pi)v + 2.$$

To have an interior equilibrium, it must be that  $\phi(T) > 0$  (it can be checked that  $\phi$  is increasing), that is:

$$\begin{aligned} \ln 1/2 - (1 - \pi)v + 1 &> 0 \\ \Leftrightarrow \pi &> 1 - \frac{\ln 1/2 + 1}{v}. \end{aligned} \tag{15}$$

Therefore, *an interior equilibrium with perfect insurance exists whenever the unemployment probability is sufficiently large*. The intuition for this result is that a large unemployment probability makes unlikely that the individual will work in the additional year if retirement is postponed. The expected labour disutility,  $(1 - \pi)v$ , is accordingly small. To decide whether to delay retirement or not, the individual compares the loss in consumption for that year  $u(c) - u(b^R) = \ln(c/b^R)$  (instead of being on retirement, he either works or is unemployed, enjoying the same level of consumption in these two states under perfect insurance) with the rise in pension benefits obtained when retirement is delayed. We have shown that, when the unemployment probability is large enough, there exist values of the retirement age such that the gain in pension benefits outweighs the sum of the consumption loss and the expected labour disutility.

To sum up, in the log utility case, unemployment benefits should be high enough and the probability of unemployment large enough for an interior equilibrium to exist.

### 4.3.2 Comparison between the majority voting solution and the optimal one

In this last section, we aim at comparing the pension policy chosen optimally at the beginning of the life cycle and at the political equilibrium. We argue in the next proposition that the comparison depends critically on whether the optimal retirement age is lower or larger than the median age in the population.

**Proposition 5.** *The retirement age chosen at the political equilibrium is lower than the optimal retirement age if and only if this latter is larger than the median age,  $T/2$ .*

*Proof.* When  $R^{opt} < T/2$ , it is obviously lower than  $R^e$ , this latter being always greater than  $T/2$ . We now study the case  $R^{opt} > T/2$ . As emphasized in the proof of proposition 4, the curves  $R^{opt}(\tau)$  and  $R^{mv}(\tau)$  coincide when  $R \in (T/2, T)$ . On the other hand, both  $\tau^{opt}(R)$  and  $\tau^{mv}(R)$  converge to  $\tau_{\min}$  when  $R \rightarrow T$ . Moreover,  $\tau^{mv}(R)$  is everywhere larger than  $\tau^{opt}(R)$ . This implies that the intersection point of  $R^{opt}(\tau)$  and  $\tau^{opt}(R)$  (that corresponds to the optimal policy) is necessarily to the right of the intersection point of  $R^{mv}(\tau)$  and  $\tau^{mv}(R)$  (this point characterizing the political equilibrium). ■

This proposition is driven by the fact that the median voter, being closer to retirement than the newborn individual, chooses a larger tax rate. This in turn leads him to choose a lower retirement age.<sup>9</sup> This conclusion is not valid in the case where the optimal retirement

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<sup>9</sup>This comes from the fact that the curves  $R^{mv}(\tau)$  and  $R^{opt}(\tau)$  (that coincide when  $R \in (T/2, T)$ ) are decreasing. Recall that an increase in  $R$  induces a loss  $(1 - \pi)u(c) + \pi u(b^U) - u(b^R)$  and a gain

age is lower than  $T/2$  for the simple reason that the median voter never has an interest in choosing a retirement age lower than his own age, namely  $T/2$ .

## 5 Conclusion

This paper contributes to the literature on pension reform, necessary in a setting of population ageing, investigating the impact of unemployment on the reform. We start from the analysis of individually optimal preferences and then move to the discussion of the political decision making, based on the issue-by-issue equilibrium concept. Our results shed light on the interaction between unemployment insurance and the optimal design of the pension system. In particular, we have shown that the optimal adjustment of the pension system in a context of increasing unemployment depends dramatically on the size of the unemployment compensation. We have also emphasized the tendency of the political process to make people retire too early.

Two important assumptions are made that should be relaxed in future research. First, we consider that agents do not have the possibility to save or borrow. This of course makes our analysis simpler. However, the savings and retirement decisions are intimately related. Taking savings into account when analyzing retirement policy thus appears to be a desirable continuation of our work. Second, we have assumed exogenous labour supply within time periods. This allowed us to have a straightforward characterization of the optimal unemployment insurance scheme. A realistic approach of the labour market however makes desirable to endogenize labour supply.

Finally, further work, both empirical and theoretical, should be devoted to understanding the impact of the collective retirement decision on the unemployment rate and its implications for the design of pension systems.

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$(T/R)b^R u'(b^R)$ . Increasing  $\tau$  makes both this gain and this loss bigger. It can however be shown that the aggregate effect is negative. To compensate this drop in the marginal benefit, individuals choose to reduce  $R$ .

# Appendix

## A Proof of proposition 1

The differentiation with respect to  $T$  of the system of equations (4a)-(4c) (after substitution of the first condition into the third) can be written in matrix form:  $Mx = b$  where  $M$  is

$$\begin{pmatrix} w & 1 & 0 \\ 0 & u''(b^R)w & -v'(R) \\ T/(1-\pi) & 0 & 1 \end{pmatrix}$$

$x$  is the vector

$$\begin{bmatrix} d\tau/dT \\ db^R/T \\ dR/dT \end{bmatrix}$$

and  $b$  is the vector

$$\begin{bmatrix} 0 \\ 0 \\ (1-\tau)/(1-\pi) \end{bmatrix}.$$

We can compute

$$\det M = w^2 u''(b^R) - v'(R) \frac{T}{1-\pi} \quad (16)$$

and we see that  $\det M < 0$ . Applying Cramer's rule, we get

$$\begin{aligned} \frac{d\tau}{dT} &= -\frac{1}{|M|} \left[ v'(R) \frac{1-\tau}{1-\pi} \right] \geq 0 \\ \frac{db^R}{dT} &= \frac{1}{|M|} \left[ v'(R) w \frac{1-\tau}{1-\pi} \right] \leq 0 \\ \frac{dR}{dT} &= \frac{1}{|M|} \left[ w^2 u''(b^R) \frac{1-\tau}{1-\pi} \right] > 0. \end{aligned}$$

## B Proof of proposition 2

If we differentiate the first-order conditions (4a)-(4c) with respect to  $\pi$  (after substitution of the first condition into the third) we get:

$$\begin{aligned} -w \frac{d\tau}{d\pi} - \frac{db^R}{d\pi} &= 0 \\ v'(R) \frac{dR}{d\pi} + u''(b^R) \frac{db^R}{d\pi} w &= 0 \\ \frac{dR}{d\pi} + \frac{T}{1-\pi} \frac{d\tau}{d\pi} - \frac{T(1-\tau)}{1-\pi} &= 0 \end{aligned}$$

We can rewrite these equations in matrix form  $Mx = b$  where  $M$  is the matrix defined in appendix A,  $x$  is the vector

$$\begin{bmatrix} d\tau/d\pi \\ db^R/d\pi \\ dR/d\pi \end{bmatrix}$$

and  $b$  is the vector

$$\begin{bmatrix} 0 \\ 0 \\ (T(1 - \tau))/(1 - \pi)^2 \end{bmatrix}.$$

Applying Cramer's rule, we have

$$\begin{aligned} \frac{d\tau}{d\pi} &= \frac{1}{|M|} \left[ -v'(R) \frac{T(1 - \tau)}{(1 - \pi)^2} \right] \leq 0 \\ \frac{db^R}{d\pi} &= \frac{w}{|M|} \left[ v'(R) \frac{T(1 - \tau)}{(1 - \pi)^2} \right] \geq 0 \\ \frac{dR}{d\pi} &= \frac{1}{|M|} \left[ w^2 u''(b^R) \frac{T(1 - \tau)}{(1 - \pi)^2} \right] > 0. \end{aligned}$$

## C Proof of proposition 3

The differentiation of the system (5a)-(5c) with respect to  $T$  can be written in matrix form:  $Mx = b$  where  $M$  is

$$\begin{pmatrix} -w & -1 & 0 \\ 0 & T/R(u'(b^R) + b^R u''(b^R)) - \pi u'(b^R) & -(1 - \pi)v'(R) - (T/R^2)b^R u'(b^R) \\ w(1 - \pi)R & -(T - R) & (T/R)b^R \end{pmatrix}$$

$x$  is the vector

$$\begin{bmatrix} d\tau/dT \\ db^R/dT \\ dR/dT \end{bmatrix}$$

and  $b$  is the vector

$$\begin{bmatrix} 0 \\ -(1/R)b^R u'(b^R) \\ b^R \end{bmatrix}.$$

We then get

$$\begin{aligned} \frac{d\tau}{dT} &= \frac{1}{|M|} [b^R(1 - \pi)v'(R)] \geq 0 \\ \frac{db^R}{dT} &= \frac{1}{|M|} [-wb^R(1 - \pi)v'(R)] \leq 0 \\ \frac{dR}{dT} &= \frac{1}{|M|} \left[ -w \frac{T}{R} (b^R)^2 u''(b^R) \right] > 0, \end{aligned}$$



where

$$|M| = -w \frac{T^2}{R^2} (b^R)^2 u''(b^R) + w(1 - \pi)v'(R)(T - \pi R) > 0.$$

When  $v'(R) = 0$ , these expressions simplify to:

$$\begin{aligned} \frac{d\tau}{dT} &= 0 \\ \frac{db^R}{dT} &= 0 \\ \frac{dR}{dT} &= \frac{R}{T}. \end{aligned}$$

We next turn to the comparative statics of the optimal policy with respect to  $\pi$ , the unemployment probability. The vector  $b$  is modified to:

$$\begin{bmatrix} 0 \\ u(b^R) - u(b^U) - v(R) \\ (\tau w + b^U)R \end{bmatrix}.$$

This leads to:

$$\frac{d\tau}{d\pi} = \frac{1}{|M|} \left[ \frac{T}{R} b^R (u(b^R) - u(b^U) - v(R)) + (\tau w + b^U) R \left( \frac{T}{R^2} b^R u'(b^R) + (1 - \pi)v'(R) \right) \right] \quad (17a)$$

$$\frac{db^R}{d\pi} = \frac{-w}{|M|} \left[ \frac{T}{R} b^R (u(b^R) - u(b^U) - v(R)) + (\tau w + b^U) R \left( \frac{T}{R^2} b^R u'(b^R) + (1 - \pi)v'(R) \right) \right] \quad (17b)$$

$$\frac{dR}{d\pi} = \frac{w}{|M|} \left[ -(u(b^R) - u(b^U) - v(R))(T - \pi R) - (\tau w + b^U) R u'(b^R) \left( \frac{T}{R} (1 - R_r(b^R)) - (\pi) \right) \right]$$

## D Proof of proposition 4

Conditions (9) and (10) define two reactions curves  $\tau^{mv}(R)$  and  $R^{mv}(\tau)$  respectively in the  $(R, \tau)$  space. A political equilibrium is characterized by the intersection of these curves.

As stated before, the optimal tax rate of the median voter is 1 when  $R = T/2$ . Therefore  $\tau^{mv}(R)$  passes through the point  $(T/2, 1)$ . Moreover,  $\lim_{R \rightarrow T} \tau^{mv}(R) = \tau_{\min}$ : when  $R \rightarrow T$ , the only way to satisfy (9) is to have  $\tau \rightarrow \tau_{\min}$ ; otherwise,  $b^R \rightarrow +\infty$  and the first-order condition (9) cannot be satisfied.

We now study the reaction curve  $R^{mv}(\tau)$ , implicitly defined in (10). As a first step, we characterize this curve without imposing the constraint  $R^{mv} \geq T/2$  (recall that the median voter's preferred retirement age is larger than his own age). With a isoelastic utility function, the term  $-u(b^R) + (T/R)b^R u'(b^R)$  in (10) becomes  $u(b^R)((1 - \varepsilon)(T/R) - 1)$ . When  $R \rightarrow T$ ,  $b^R \rightarrow +\infty$ , unless  $\tau \rightarrow \tau_{\min}$ , which implies that  $u(b^R)((1 - \varepsilon)(T/R) - 1) \rightarrow \infty$ .

As all other terms in the first-order condition take finite values, the only way to satisfy this condition is to have  $\tau \rightarrow \tau_{\min}$ . Consequently,  $\lim_{\tau \rightarrow \tau_{\min}} R^{mv}(\tau) = T$ .

Next, we construct the curve  $\tau^{opt}(R)$ , that corresponds to the optimal tax rate chosen at the beginning of the life cycle, for a given  $R$ . This curve is implicitly defined in (5a). It is straightforward to show that this curve is decreasing and passes through the points  $(0,1)$  and  $(T, \tau_{\min})$ . The optimal pension policy at the beginning of the life cycle with exogenous unemployment benefits is obtained at the intersection point of  $\tau^{opt}(R)$  and  $R^{opt}(\tau)$ , this latter curve being implicitly defined in (5b). Note that  $R^{opt}(\tau)$  coincides with  $R^{mv}(\tau)$  when the median age individual chooses an interior retirement age. It can be shown that the optimization program at the beginning of the life cycle is convex. Therefore, the curve  $R^{opt}(\tau)$  crosses  $\tau^{opt}(R)$  once and only once (at the optimum). Moreover,  $R^{opt}(\tau)$  can be shown to be decreasing. These two properties imply that  $R^{opt}(\tau)$  crosses the vertical axis  $R = T/2$  at  $\tau = R^{opt^{-1}}(T/2)$ . For  $\tau \leq R^{opt^{-1}}(T/2)$ ,  $R^{mv}(\tau)$  coincides with  $R^{opt}(\tau)$  (and it is strictly larger than  $T/2$ ). For  $\tau \leq R^{opt^{-1}}(T/2)$ ,  $R^{mv}(\tau) = T/2$ . Reminding that  $\tau^{mv}(T/2) = 1$ , we have shown that  $\tau = 1$  and  $R = T/2$  is an equilibrium.

Other equilibria possibly exist. It depends whether the curves  $\tau^{mv}(R)$  and  $R^{mv}(\tau)$  cross in the region  $R \in (T/2, T)$ . Numerical examples show that both cases are possible: for some parameters values, interior equilibria do exist whereas they don't for other values.

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