DISCUSSION PAPER SERIES

No. 6224

# WHOLESALE MARKETS IN TELECOMMUNICATIONS

Marc Bourreau, Johan Hombert, Jérôme Pouyet and Nicolas Schutz

*INDUSTRIAL ORGANIZATION*

Centre for Economic Policy Research

www.cepr.org

Available online at: www.cepr.org/pubs/dps/DP6224.asp

ISSN 0265-8003

# WHOLESALE MARKETS IN TELECOMMUNICATIONS

**Marc Bourreau,** ENST and CREST-LEI
**Johan Hombert,** CREST-ENSAE and PSE
**Jérôme Pouyet,** Ecole Polytechnique and CEPR
**Nicolas Schutz,** PSE

Discussion Paper No. 6224
April 2007

Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

CEPR Discussion Paper No. 6224

April 2007

# ABSTRACT

## Wholesale Markets in Telecommunications*

In telecommunications some operators have deployed their own networks whereas some others have not. The latter firms must purchase wholesale products from the former to be able to compete on the final market. We show that, even when network operators compete in prices and offer perfectly homogenous products on the wholesale market, that market may not be competitive. Based on our theoretical analysis, we derive some policy implications for the broadband and the mobile telephony markets.

JEL Classification: L13 and L51
Keywords: telecommunications, upstream and downstream markets and vertical integration

Marc Bourreau
Département Sciences Économiques et Sociales
46, rue Barrault
75634 Paris Cedex 13
FRANCE
Email: marc.bourreau@enst.fr

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=152077

Johan Hombert
ENSAE
Bureau E11 - Timbre J120
3, Avenue Pierre Larousse
92245 Malakoff cedex
FRANCE
Email: johan.hombert@ensae.fr

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=164230

Jérôme Pouyet
Department of Economics
Ecole Polytechnique
91128 Palaiseau Cedex
FRANCE
Email: jerome.pouyet@polytechnique.edu

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=156915

Nicolas Schutz
Paris-Jourdan Sciences Economiques
48 Boulevard Jourdan
75014 Paris
FRANCE
Email: nicolas.schutz@polytechnique.org

For further Discussion Papers by this author see:
www.cepr.org/pubs/new-dps/dplist.asp?authorid=166140

* We gratefully acknowledge France T´el´ecom and CEPREMAP for the intellectual and financial support. We also thank Philippe F´evrier, Bernard Caillaud, Bruno Jullien, Laurent Linnemer, Patrick Rey, Bernard Salani´e, Jean Tirole and seminar participants for helpful comments on an earlier draft.

Submitted 27 February 2007

# 1 Introduction

An important step towards competition in many network industries has been to allow third-parties to access the infrastructure owned by the historical incumbent. Indeed, given that high fixed costs have already been sunk, duplication of the existing networks was deemed economically inefficient, so that forcing incumbent operators to lease the access to their infrastructure was the only regulatory tool to foster competition on final markets.

In telecommunications, technological innovations have made it possible for so-called 'facility-based' firms to roll out proprietary networks and to rely mainly on their own infrastructures to provide services to end-customers. Contrary to facility-based firms, 'service-based' firms do not invest in facilities but lease access to the networks of facility-based firms to offer services on retail markets.[1]

A priori, the presence of facility-based firms does not only increase directly the competitive pressure on the final market, but also indirectly, by providing service-based firms with additional possibilities to get access to final customers. The advent of potential competition on the market for access, called wholesale markets, was viewed as a way to boost competition in this industry. If service-based operators could get cheaper access to end-users, so the argument goes, the competitive pressure on final markets should increase, with a direct benefit passed through to end-users in the form of price cuts. Therefore, among practitioners, one question comes recurrently: When facility-based competition is in place, will the wholesale market deliver its promises? There is so far no clear consensus on the answer: Some telecoms regulators question that a wholesale market can be competitive and thus regulate the wholesale market on such grounds; others consider that in an unregulated environment facility-based firms would lease the access to their infrastructures at a competitive price, allowing service-based operators to compete on a level-playing field with facility-based firms.[2,3]

The main objective of this paper is to provide an economic analysis of the functioning of such wholesale markets in telecommunications. Using a stylized model, we show that these wholesale markets may be non competitive even when all the usual ingredients of

[1]Note, however, that facility-based firms might lease some network elements to the historical incumbents, and that service-based firms might have to install some telecommunications equipments. But, in a nutshell, a facility-based firm builds more than it leases, and a service-based firm leases more than it builds. Section 2 provides some industry background on the broadband and mobile telephony markets to illustrate this phenomenon.

[2]E.g., the UK telecoms regulatory authority, Ofcom, in its review of the wholesale broadband market (Ofcom, 2004) argues that: "Under competitive market conditions, both cable and BT would have an incentive to offer a wholesale product. [...] In a competitive market, cable's and BT's upstream (network) and downstream (retail) divisions would each earn a normal return".

[3]From an academic perspective, Hazlett (2005) argues that unregulated infrastructure-based competition should lead to the emergence of a wholesale market, whereas Woroch (2004) assumes that facility-based firms do not voluntarily give access to their networks.

Bertrand competition are in place, and that they cannot be studied in isolation of the related retail markets.

In our model, two facility-based, or integrated, firms compete in prices with differentiated products on a downstream market with a service-based, or pure downstream, firm. The goods sold to end-users are derived from an intermediate input that the integrated firms only can produce in-house, while the pure downstream firm must buy it from either of the integrated firms. Integrated firms, first, compete on the upstream market to provide the input to the pure downstream firm, and, second, compete on the downstream market with the pure downstream firm. The upstream market exhibits the usual ingredients of tough competition: integrated firms compete in prices, produce a perfectly homogeneous upstream good and incur the same constant marginal cost. Throughout most of the paper, we do not specify the downstream demand and cost functions and make no particular assumptions on the nature of the strategic interaction on the downstream market.

If one considers the upstream market in isolation of the downstream one, one could be tempted to believe that integrated firms would have incentives to cut prices to gain wholesale revenues until the marginal cost is attained, as the usual logic underlying Bertrand competition would predict. While attractive, we shall show that this logic is largely flawed.

The reason lies in the accommodation effect, according to which the integrated firm which supplies the upstream market with a strictly positive price-cost margin adopts a softer behavior on the downstream market than its rival integrated competitor. Indeed, realizing that final customers lost on the downstream market may be recovered indirectly via the upstream market, the upstream supplier is willing to preserve its upstream profit through its downstream pricing. We show then that, because this accommodating behavior of the upstream supplier favors its integrated rival on the downstream market, the upstream supplier earns less downstream profit (i.e., profit made on the downstream market only, not accounting for upstream revenues) than the integrated firm which does not supply the upstream market.

A crucial consequence is that, even once the upstream profits are accounted for, the integrated firm which supplies the upstream market might earn less total profit than the integrated firm which does not supply the upstream market. In particular, the outcome which would prevail if the upstream market were exogenously monopolized, i.e., if only one of the integrated firms could supply the intermediate good to the pure downstream firm, may still be an equilibrium when both integrated firms compete on the upstream market. Indeed, when one integrated firm offers the upstream price that maximizes its profit, conditionally on the other integrated firm exiting the upstream market, the other integrated firm benefits from the accommodation effect and may actually prefer to exit the upstream market. When this is the case, there exists an equilibrium in which the upstream market outcome coincides exactly with the exogenous monopoly outcome: The monopoly outcome persists even under the threat of potential competition on the upstream market.

The next step of the analysis consists in determining whether other equilibria may exist or not. Under reasonable conditions, we show that the competitive outcome, in which integrated firms set prices on the upstream market equal to the upstream unit cost, is also an equilibrium. However, in our general framework, nothing precludes the emergence of other equilibria in which upstream offers are identical but supra- or super-competitive (i.e., above or below the upstream unit cost).

In order to delineate more precisely the conditions under which non-competitive equilibria exist, we propose various illustrations. The first example highlights the role of product differentiation at the downstream level on the competitiveness of the upstream market. When final products are strongly differentiated, downstream demands are almost independent and, hence, the accommodation effect is weak. As a result, undercutting on the upstream market is always profitable and the upstream market ends up being competitive. Conversely, when downstream products are strong substitutes, the accommodation effect is strong and the monopoly outcome is an equilibrium. This comparative statics result exhibits a tension between downstream competitiveness and upstream competitiveness.

The second example illustrates the role of the strategic interaction on the downstream market on the competitiveness of the upstream market.[4] As downstream prices become more strategic complements, the accommodation effect weakens. Hence, the less strategic complements downstream prices are, the more likely is the emergence of non-competitive equilibria on the upstream market.

Analyzing the mobile telephony and the broadband markets, we give some empirical supports to these conclusions and deduce from our theoretical results several policy implications. First, we look at the hotly-debated issue of the vertical separation of the incumbent to foster competition on the upstream market. We show that the presence of a pure upstream firm is likely to generate a competitive upstream market only when downstream prices are not too strong strategic substitutes. Otherwise, there exists an equilibrium in which integrated firms are not active on the upstream market, which is served at a non-competitive price by the upstream firm only.

Then, we use our analysis to study various regulatory attempts that have been made to develop competition on wholesale markets in telecommunications. For instance, some regulators have removed the price cap on the incumbent's upstream offer in order to make entry on the upstream market more attractive to integrated firms. We argue that such a policy, which does not account for the interactions with the downstream markets, is likely to yield opposite effects to those expected. By contrast, a price cap on

[4] In the first illustration, downstream prices are always strategic complements.

at least one of the integrated firm's upstream offer may generate a competitive upstream market.

Overall, the results point towards the following warning: Any competition policy or regulatory recommendations regarding wholesale markets in telecommunications should be based on the specifics of both the upstream and the downstream markets. Analyzing the upstream market on a stand-alone basis is likely to yield wrong conclusions.

Our paper is related to different strands of the literature. The literature on network competition in telecommunications has, for a substantial part, developed around Laffont, Rey, and Tirole (1998a, 1998b). They consider competition on a downstream market only between facility-based firms. They focus on the impact of two-way interconnection charges on downstream prices. We abstract away from such a consideration, but focus instead on wholesale market competition, i.e., competition between network operators to attract service-based firms.

Similarly, the literature on one-way access pricing deals with situations in which a service-based firm must gain access to the network of a facility-based firm; see, among others, Laffont and Tirole (2001), Armstrong (2002), De Bijl and Peitz (2002), Foros (2004) and Bourreau and Dogan (2006). These papers are, by definition, silent on the issue of the competitiveness of wholesale markets, which is central to our analysis.

Brito and Pereira (2006) have independently developed a framework to study competition between several facility-based firms on a wholesale market for access. With specific demand and cost functions, their main point is that facility-based firms face a prisoner's dilemma when they decide whether or not to supply a service-based firm on the wholesale market: provided that the upstream market is supplied, the only subgame-perfect equilibrium in their framework is the competitive outcome. Our paper challenges that view and shows that this result has limitations: Whether the wholesale market exhibits competitive or non competitive features is rooted in the accommodation effect.

Finally, our paper is closely related to the literature on vertical foreclosure. The so-called traditional foreclosure theory argues that integrated firms have incentives to raise their non-integrated rivals' costs through their pricing of the intermediate input. After having been challenged by the Chicago School, this theory has been given firmer theoretical grounds by several authors; see, among others, Salinger (1988), Ordover, Saloner and Salop (1990) and Hart and Tirole (1990). Our accommodation effect, which states that the upstream pricing affects the downstream pricing of integrated firms, is reminiscent of Chen (2001).

The paper is organized as follows. Section 2 proposes an industry background on the broadband and mobile telephony industries, which motivates our modeling and analysis. Section 3 describes the model. Section 4 presents our main results, namely the possibility of non-competitive equilibria on the upstream market, and illustrates them.

Section 5 discusses several extensions and robustness checks of our basic framework. Section 6 builds on the theoretical analysis to derive some policy implications, in terms of regulation and competition policy, for the telecommunications industry. Section 7 concludes and presents alleys of future research.

# 2 Industry Background

In the telecommunications industry, most markets have a two-tier structure, with a wholesale level and a retail level. Initially, the incumbent operators (ILECs) were the sole vertically integrated (facility-based) firms, combining a wholesale unit and a retail unit. However, in the last few years, facility-based competitors have developed. One consequence has been the emergence of wholesale markets, where facility-based firms compete to provide wholesale services to service-based firms. Service-based firms use the wholesale services to design their retail offers and compete with facility-based firms in the retail market.

As we shall show below, this trend has been observed, in particular, in the broadband market, and more markedly, in the mobile telephony market. It might constitute a key feature of telecommunications markets in the future, and the model that we will present in Section 3 fits with this stylized fact. In what follows, we provide a brief description of these emerging wholesale markets for the broadband and mobile industries.

## 2.1 The broadband market

**Description of the market.** Broadband services enable consumers to access the Internet at high-speed rates. In most industrialized countries, broadband is developing fast. For instance, there were 37.9 million broadband lines in the United States (US) and 33.4 million lines in the European Union (EU) in 2004, compared to 28.2 million lines and 19.4 million lines, respectively, one year earlier.[5]

Different technologies can deliver broadband, but two of them dominate local broadband markets worldwide: the cable modem platform (which is the most widely used in the US)[6] and the copper-based digital subscriber line (DSL) platform (which is the dominant technology in the EU).[7]

The broadband market has a three-tiered structure, with three types of products: local access (which is an input for the wholesale broadband service), the wholesale

[5]See FCC (2005a) and EC (2006a).

[6]According to FCC (2005a), in 2004, cable providers served 56.4% and ADSL 36.5% of the broadband lines in the US.

[7]According to EC (2006a), in 2004, DSL platforms served 80.4% of the broadband lines in the EU, compared to 16.8% of lines provided by cable and 2.8% by other access technologies.

broadband service (also known as 'bitstream access'), and finally the retail broadband Internet access service.

Therefore, there are potentially two wholesale markets for broadband: the market for local access, and the wholesale broadband market.

In many countries, facility-based competition already exists between cable networks and the local loop of the ILEC, and new fiber optic networks are developing. Therefore, a wholesale market for local access might emerge; on such a market, entrants would trade-off between the local loop unbundling (LLU) offer of the incumbent[8] and an equivalent offer of a cable or fiber operator. In Section 6, we use this interpretation of the wholesale market to discuss vertical separation of vertically-integrated firms.

But for the moment, we choose to focus on the wholesale broadband market. The ILECs, the cable operators, and LLU operators compete at the retail level; they can also compete to provide wholesale broadband services to pure downstream firms.

**The wholesale broadband market.** In some European countries, a wholesale broadband market has emerged. For instance, in France, in 2005, two LLU operators, Cegetel and Neuf Telecom owned between 30 and 50 percent of the national wholesale broadband market, while the ILEC (France Telecom) served the rest.[9] In the UK too, LLU operators provide wholesale broadband services in dense areas,[10] and compete with the incumbent, BT, in the wholesale broadband market.

Though competition in the wholesale market is not always observed, in many countries, broadband wholesale offers are available. In particular, in most European countries, ILECs provide wholesale broadband services to competitors; this is not the case in the US. Besides, there are a few examples of wholesale broadband offers over cable in Europe,[11] the US,[12] Canada and Israel,[13], as well as examples of wholesale offers from LLU CLECs. Therefore, it might be argued, there is potential competition in the wholesale market.

Once a wholesale broadband market has emerged, a key regulatory question is whether existing regulation (if any) can be removed. Otherwise stated, can we ex-

[8] Local loop unbundling is a regulatory measure, which obliges the ILEC to give access to its copper lines that connect the customer premises to the local exchanges. It enables entrants to install DSL equipments to offer broadband services to end customers.

[9] See DGCCRF, Decision C2005-44 related to the merger between Neuf Telecom and Cegetel. The merger was accepted; one condition though required the merged company to continue to provide its wholesale services.

[10] See Ofcom, "Review of the wholesale broadband access markets 2006/07," 21 November 2006.

[11] For instance, in the UK, one of the two cable operators has signed a wholesale broadband agreement with a third-party ISP (see Ofcom, 2004, §3.15, p. 69).

[12] Following its merger with AOL, Time Warner signed wholesale agreements with Earthlink and other few ISPs (see Hausman and Sidak, 2005, for other examples). 'Open access' of cable networks gave rise to a heated debate around 2000.

[13] See also ERG (2005).

pect the wholesale market to be competitive, or is there still a need for a regulatory intervention? We will address this question with our setting in Section 4.

## 2.2 The mobile market

**Description of industry.** The mobile telephony market is one of the most dynamic telecommunications market. In 2004, there were 184.7 million mobile users in the US and 383 million users in the EU. In the EU, the average penetration rate in the 25 Member States in 2005 was above 90%.[14]

However, due to very high fixed costs (e.g., the costs of the spectrum license and the costs of a mobile network), entry in the mobile markets has been limited and most national markets currently host only a few mobile network operators (MNOs). For instance, in most European countries, there are typically 3 to 4 MNOs offering 2G services.[15] But since 1999 a new class of competitors has emerged, known as 'mobile virtual network operators' (MVNOs).[16] MVNOs are service-based competitors; that is, they do not have a spectrum license nor a mobile network,[17] and, hence, they have to purchase a wholesale mobile service from MNOs.

MVNOs position themselves either as 'low cost' service providers (e.g., this is the case of Easy Mobile in the UK) or target specific niche market segments (e.g., teenagers for Virgin Mobile in the UK or NRJ Mobile in France). These different types of strategy raise questions about whether differentiation between MNOs and MVNOs can affect the competitiveness of the wholesale market. We will address this question with an illustration, at the end of Section 4.

**The wholesale mobile market.** In most national markets, a wholesale mobile market has emerged, with multiple upstream providers, and many MVNOs operate. According to Hazlett (2005), in 2005, there were around 20 MVNOs in the US.[18] In the European Union, there were a total of 214 MVNOs in 14 Member States out of 25 (EC, 2006b). In most countries, MNOs have given access to their networks voluntarily, but in some countries (e.g., Norway), they were forced to.

The wholesale mobile market is currently left unregulated in most countries. Typically, the prices for the wholesale product are set on a retail minus basis. Therefore, though several MNOs usually offer wholesale mobile services, competition has not driven

[14] See FCC (2005b) and EC (2006a).

[15] In 2005, there were 79 2G mobile operators in the EU (EC, 2006a).

[16] In 1999, the first MVNO, Virgin Mobile, entered in the UK market.

[17] Some MVNOs are merely resellers, hence do not have any infrastructure. Others (e.g., "full" MVNOs) own some mobile network elements, which gives them higher possibilities of differentiation.

[18] The two main MVNOs were TracFone which served over 3.5 million customers and Virgin Mobile USA, which served over 3 million customers.

wholesale prices to costs.

This might explain why, in some countries (e.g., France and Ireland), regulators have been willing to control the conditions of access for MVNOs, but no regulation has been implemented yet.[19] The relevant question to determine whether regulation is called for is the central focus of this paper: Can we expect competition between vertically-integrated firms (MNOs) to drive wholesale prices for service-based firms (MVNOs) to costs?

# 3 Model

Based on the factual analysis developed in the previous section, we now present our theoretical model.

**Firms.** There are two vertically integrated firms, denoted by 1 and 2, and one pure downstream firm, denoted by $d$. Integrated firms are composed of an upstream and a downstream unit, which produce the intermediate input and the final good, respectively. The pure downstream competitor is composed, initially, of a downstream unit only. In order to be active on the final market, it must either purchase the intermediate input from one of the integrated firms on the upstream market or it must build its own network at a fixed cost $I > 0$.

We assume that the upstream good is produced with a constant returns-to-scale technology, and that the unit cost $c_u$ is the same for both integrated firms. The downstream product is derived from the intermediate input on a one-to-one basis at a cost $c_k(.)$ for firm $k \in \{1, 2, d\}$. We assume that integrated firms have the same downstream cost function: $c_1(.) = c_2(.)$.

**Markets.** All firms compete in prices on the downstream market and provide imperfect substitutes to final customers. Let $p_k$ be the downstream price set by firm $k \in \{1, 2, d\}$ and $p \equiv (p_1, p_2, p_d)$ the vector of final prices. Firm $k$'s demand is denoted by $D_k(p)$; it depends negatively on its price and positively on its competitors' prices: $\partial D_k/\partial p_k < 0$ and $\partial D_k/\partial p_{k'} > 0$ for $k \neq k' \in \{1, 2, d\}$. Symmetry of the integrated firms is assumed again: $D_1(p_1, p_2, p_d) = D_2(p_2, p_1, p_d)$ and $D_d(p_1, p_2, p_d) = D_d(p_2, p_1, p_d)$ for all $p$.

On the upstream market, integrated firms compete in prices and offer perfectly homogeneous products. We denote by $a_i$ the upstream price set by integrated firm $i \in \{1, 2\}$.[20] The structure of the model is summarized in Figure 1.

[19]The Irish decision was eventually annulled while the French regulation project was rejected by the

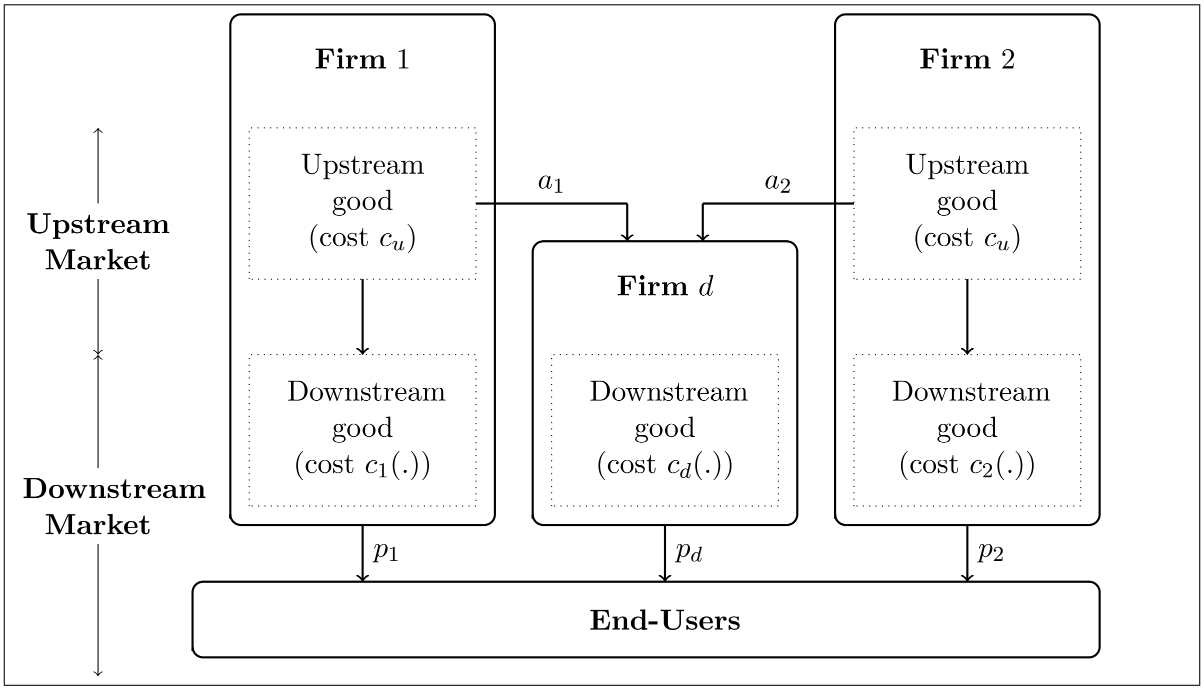


**Figure 1:** Structure of the model (when firm $d$ remains a pure downstream firm).

**Timing.** The sequence of decision-making is as follows:

*Stage 1 – Upstream competition*: The vertically integrated firms set simultaneously and non-cooperatively their prices on the upstream market.

*Stage 2 – Build or buy decision*: The pure downstream firm either elects one upstream provider or decides to build its own network.

*Stage 3 – Downstream competition*: All firms choose simultaneously and non-cooperatively their prices on the downstream market.

The outcome on the upstream market has two distinct impacts. First, it affects the pure downstream firm's choice between building its own network and becoming a facility-based competitor, and relying on an upstream provider to enter the downstream market. Second, if firm $d$ remains service-based, the level of the upstream price and the identity of the upstream supplier impact incentives on the downstream market. In Section 5, we consider that firm $d$ makes its build or buy decision before the upstream and downstream competition stages, and we show that our findings are not affected.

European Commission.

[20]Throughout the paper, subscripts $i$ and $j$ refer to integrated firms only, whereas subscript $k$ refers either to an integrated firm or to the pure downstream firm.

Throughout our analysis, we focus on pure strategies subgame-perfect equilibria and reason by backward induction.

**Profits.** Let $i \in \{1,2\}$. If firm $i$ supplies the upstream market, its profit is given by:[21]

$$\tilde{\pi}_i^{(i)}(p, a_i) = (p_i - c_u)D_i(p) - c_i(D_i(p)) + (a_i - c_u)D_d(p).$$

When integrated firm $j \neq i \in \{1,2\}$ supplies the upstream market or when firm $d$ produces the intermediate input in-house, firm $i$ earns:

$$\tilde{\pi}_i^{(j)}(p, a_j) = \tilde{\pi}_i^{(d)}(p) = (p_i - c_u)D_i(p) - c_i(D_i(p)).$$

When the pure downstream firm $d$ purchases the upstream good from integrated firm $i$, its profit writes as:

$$\tilde{\pi}_d^{(i)}(p, a_i) = (p_d - a_i)D_d(p) - c_d(D_d(p)),$$

whereas if it builds its own network, it earns:

$$\tilde{\pi}_d^{(d)}(p) = (p_d - c_u)D_d(p) - c_d(D_d(p)) - I.$$

In words, at cost $I > 0$, firm $d$ can build its network and bypass the upstream market. It then competes on a level-playing field with the integrated firms.

Notice that when the upstream price is equal to the upstream unit cost, i.e., $a_i = c_u$, there is no upstream profit and all firms compete on a level-playing field. Such an outcome depicts a perfectly competitive upstream market. Moreover, in that case, firm $d$ would never invest, a socially optimal decision.

# 4 Main Results

In this section, we develop our main argument: competition on the upstream market (stage 1) does not always yield the perfect competition outcome, even though integrated firms offer homogenous products and compete in prices on that market.

## 4.1 Preliminaries

**Downstream market competition (stage 3).** Two cases have to be considered, depending on whether or not firm $d$ relies on the upstream market.

[21] Throughout the paper, superscript in parenthesis indicates the identity of the upstream supplier.

*No Investment.* We consider first that firm $d$ has chosen firm $i \in \{1, 2\}$ as its upstream supplier. Let $j \neq i \in \{1, 2\}$.

In order to streamline the analysis and to focus on the relevant cases, we shall make the following standard assumptions. The best-response in downstream price of firm $k \in \{1, 2, d\}$, defined by $BR_k^{(i)}(p_{k'}, p_{k''}, a_i) = \arg\max_{p_k} \tilde{\pi}_k^{(i)}(p, a_i)$, is unique, bounded above and characterized by the corresponding first-order condition.[22] Moreover, we assume that for all $k' \neq k \in \{1, 2, d\}$, $|\partial BR_k^{(i)} / \partial p_{k'}| < 1$.[23]

We denote by $p_k^{(i)}(a_i)$, the equilibrium price of firm $k \in \{1, 2, d\}$; $p^{(i)}(a_i)$ denotes the vector of these downstream prices. At the equilibrium of this subgame, firms' profits are given by: $\pi_k^{(i)}(a_i) \equiv \tilde{\pi}_k^{(i)}(p^{(i)}(a_i), a_i)$. Notice that $\pi_i^{(i)}(c_u) = \pi_j^{(i)}(c_u)$ and $p_i^{(i)}(c_u) = p_j^{(i)}(c_u)$.

*Investment.* Consider that firm $d$ builds its own network. Since, first, the pure downstream firm now produces the intermediate input at its unit cost, and, second, there are no upstream profits, the equilibrium downstream prices are given by: $p_k^{(d)} = p_k^{(i)}(c_u)$ for firm $k \in \{1, 2, d\}$. Therefore, the profits of the vertically integrated firms are: $\pi_i^{(d)} = \pi_i^{(i)}(c_u) = \pi_j^{(i)}(c_u) = \pi_j^{(d)}$. Since it has to bear the investment cost to build its network, firm $d$'s profit is given by: $\pi_d^{(d)} - I$, with $\pi_d^{(d)} = \pi_d^{(i)}(c_u)$.

We assume for the moment, and discuss later on, that the investment is not worthless, or:

**Assumption 1.** $\pi_d^{(d)} - I > 0$.

**Build or buy decision (stage 2).** The pure downstream firm $d$ must choose between buying the upstream product from either of the integrated firms and building its own infrastructure. Hence, it chooses the former provided that:

$$\max_{i \in \{1,2\}} \pi_d^{(i)}(a_i) \geq \pi_d^{(d)} - I.$$

In words, firm $d$ does not exercise its outside option (i.e., it does not build its own network) provided that its profit when electing the most advantageous upstream provider is larger than its profit if it competes on a level-playing field with the integrated firms but bears the investment cost.[24]

[22] Uniqueness is made for expositional purposes only and could be relaxed without altering the essence of our results.

[23] This corresponds to the usual stability condition for a duopoly but this is less stringent than the stability requirement in a oligopoly with $n > 2$ firms, which can be stated as $\sum_{k' \neq k} \left| \partial BR_k^{(i)} / \partial p_{k'} \right| < 1$. See Vives (1990).

[24] We assume that when it is indifferent between investing and not investing, the pure downstream firm does not invest.

**Monopoly benchmark.** For later use, it is useful to study the situation in which the upstream market is exogenously monopolized by firm $i \in \{1,2\}$.

If firm $i$ sets an upstream price which triggers investment from firm $d$'s side, then it earns $\pi_i^{(d)}$. Moreover, Assumption 1 implies that firm $i$ is not able to fully foreclose firm $d$. Hence, the upstream supplier can always secure itself a (weakly) larger profit by ensuring that firm $d$ does not bypass the upstream market. Define the constrained upstream monopoly price as follows:

$$\hat{a} \in \arg\max_{a_i} \pi_i^{(i)}(a_i) \text{ s.t. } \pi_d^{(i)}(a_i) \geq \pi_d^{(d)} - I.$$

To focus on the interesting cases, we assume that $\hat{a}$ is uniquely defined[25] and such that:

**Assumption 2.** $\hat{a} > c_u$.

This assumption means that monopoly market power on the upstream market allows the upstream supplier to impose a strictly positive markup on the price of the upstream good; however, the implicit threat exerted by firm $d$'s outside option may limit the upstream supplier's market power. Hence, partial foreclosure of the pure downstream firm arises with an exogenous upstream monopoly.[26]

Under these assumptions, $\pi_i^{(i)}(\hat{a}) > \pi_i^{(d)}$: Therefore, in a subgame-perfect equilibrium, the pure downstream firm does not invest.

## 4.2 Persistence of the monopoly outcome

We now study the first stage of our game in which integrated firms compete on the upstream market and establish the central result of the paper. We show that the usual mechanism of Bertrand competition is flawed on the upstream market and that non-competitive equilibria may exist.

Assume that integrated firm $i$ has made an acceptable upstream offer to firm $d$, $a_i > c_u$, and let us see whether integrated firm $j$ is willing to corner the upstream market, as it is the case with standard (single-market) Bertrand competition. The integrated firms' downstream prices are characterized by the set of first-order conditions:

$$\frac{\partial \tilde{\pi}_i^{(i)}}{\partial p_i}(p, a_i) = D_i + (p_i - c_i'(D_i) - c_u)\frac{\partial D_i}{\partial p_i} + (a_i - c_u)\frac{\partial D_d}{\partial p_i} = 0, \quad (1)$$

$$\frac{\partial \tilde{\pi}_j^{(i)}}{\partial p_j}(p, a_i) = D_j + (p_j - c_j'(D_j) - c_u)\frac{\partial D_j}{\partial p_j} = 0. \quad (2)$$

[25]This implies no loss of generality.

[26]Were Assumption 2 not satisfied, there would be no gain to introduce a competitive pressure on the upstream market.

The comparison between (1) and (2) shows that the upstream supplier has more incentives to raise its downstream price than its integrated rival. Indeed, it internalizes the fact that, when it increases its downstream price, some of the customers it loses will purchase from the pure downstream firm, thereby increasing its upstream revenues. This mechanism implies that, in equilibrium, the upstream supplier charges a higher downstream price than its integrated rival.

**Lemma 1.** *If firm $i$ supplies the upstream market at price $a_i > c_u$, then it charges a larger downstream price than its integrated rival $j$:*

$$p_i^{(i)}(a_i) > p_j^{(i)}(a_i).$$

*Proof.* See Appendix A.1. □

The literature on vertical foreclosure has long emphasized that an integrated firm may have incentives to preserve its downstream profit through its upstream offer.[27] Lemma 1 highlights that the reverse mechanism also exists: The integrated firm which supplies the upstream market has incentives and is able to preserve its upstream profit through its downstream pricing. Realizing that final customers lost on the downstream market may be recovered via the upstream market, the upstream supplier is less aggressive on the downstream market in order not to jeopardize its upstream profit. We shall refer to that mechanism as the 'accommodation effect'.[28]

The accommodation effect implies that the upstream supplier is a soft competitor on the downstream market. This tends to favor its integrated rival. Indeed, benefiting from the accommodation effect, the integrated firm which is inactive on the upstream market earns more downstream profits than the upstream supplier.

**Lemma 2.** *If the upstream market is supplied by integrated firm $i$ at an upstream price $a_i > c_u$, then firm $j$ earns strictly larger downstream profits than firm $i$:*

$$\left[p_i^{(i)}(a_i) - c_u\right] D_i(p^{(i)}(a_i)) - c_i\left(D_i(p^{(i)}(a_i))\right) < \left[p_j^{(i)}(a_i) - c_u\right] D_j(p^{(i)}(a_i)) - c_j\left(D_j(p^{(i)}(a_i))\right).$$

*Proof.* See Appendix A.2. □

A key consequence of that result is that we cannot tell unambiguously which of the integrated firms earns more total profits. On the one hand, the upstream supplier extracts revenues from the upstream market. On the other hand, its integrated rival benefits from larger downstream profits thanks to the accommodation effect. It may

[27] See Ordover, Saloner and Salop (1990), Hart and Tirole (1990), Salinger (1988), Chen (2001), and Rey and Tirole (2005) for an extensive survey.

[28] This result is reminiscent to Chen (2001), who shows that an integrated firm sets a higher downstream price when it supplies the upstream market than when a pure upstream competitor does.

well be the case that the integrated firm which does not supply the upstream market earns larger total profits, if the additional downstream profits outweigh the foregone upstream revenues. Hence, the usual logic of Bertrand competition may not work anymore: An integrated firm may not always want to undercut its integrated rival on the upstream market, even though the upstream price is above the marginal cost. This potentially opens the door to non-competitive equilibria on the upstream market, in which the upstream product would be priced above its marginal cost. In particular, the monopoly outcome can be an equilibrium, as stated by the following result.

**Proposition 1.** *If* $\pi_j^{(i)}(\hat{a}) \geq \pi_i^{(i)}(\hat{a})$*, then there exists a monopoly-like equilibrium, i.e., a subgame-perfect equilibrium in which the upstream market is supplied by the integrated firm* $i$ *only at price* $\hat{a}$*.*

*Proof.* Immediate. □

Because losers on the upstream market become winners on the downstream market, the usual competitive forces may collapse. This does not hinge on any commitment device for the integrated firms to exit the upstream market, nor to any kind of overt or tacit collusion. Note that, by symmetry, two monopoly-like equilibria exist if the condition stated in Proposition 1 is satisfied.

As Proposition 1 highlights, the existence of monopoly-like equilibria depends on a comparison between profit levels. In Subsection 4.4, we provide two illustrations which allow to understand more deeply that condition.

We can already hint at a key ingredient to the persistence of the monopoly outcome: the degree of differentiation of the pure downstream firm. Indeed, suppose that the entrant is on a niche market, in the sense that its demand does not depend on the prices set by the rival downstream firms and vice-versa.[29] In that situation, the wholesale profit of the upstream supplier is fully disconnected from its retail behavior: The upstream market can no longer be used to soften competition. Hence, with a pure downstream firm on a niche market, the upstream market is always competitive at equilibrium. This intuition will be refined in our first illustration in Subsection 4.4.

At this stage, our result deserves a slight digression on a related literature, namely the vertical foreclosure theory. That theory points out that competition on the upstream market may be flawed since buyers and sellers interact on the downstream market, so that integrated firms have incentives to raise the pure downstream firms' costs.

The Chicago School has forcefully criticized the validity of this argument on the following ground: Even if integrated firms have incentives to raise their rivals' costs, these incentives are not strong enough to offset the competitive forces on the upstream market. Differently put, as long as the upstream price would remain above the marginal

[29] Formally, this requires that: $\partial D_d/\partial p_i = \partial D_i/\partial p_d = 0$ for $i \in \{1, 2\}$.

cost, an integrated firm could always undercut its integrated rival by a small amount, thereby stealing the upstream profit without modifying the downstream outcome.

Our results highlight that an important point is missing in the Chicago School critique: The accommodation effect implies that undercutting on the upstream market does have an important adverse impact on the downstream outcome. This may lead to partial foreclosure.

### 4.3 Other equilibria

Given the effects that we have unveiled previously, one could legitimately ask whether other equilibria may exist. However, in full generality, their characterization turns out to be cumbersome. Therefore, we shall introduce two economically meaningful assumptions.

**Assumption 3.** *The profit of the pure downstream firm is decreasing in the upstream price:* $(\pi_d^{(i)})'(.) < 0$, $i \in \{1, 2\}$.

This assumption allows us to focus on the cases in which, when it decides not to invest, the pure downstream firm chooses the cheaper upstream providers.[30] Would firm $d$ prefer to choose the most expensive upstream provider, we would have another, somewhat trivial, reason for the existence of non-competitive equilibria on the upstream market. Assumption 3 allows to rule out these cases.

Under Assumption 3, the highest upstream price that can be charged by the upstream supplier firm $i$ is denoted by $\overline{a}$ and is characterized by:

$$\pi_d^{(i)}(\overline{a}) = \pi_d^{(d)} - I.$$

Since $\pi_d^{(i)}(c_u) = \pi_d^{(d)}$ and $I > 0$, we have $\overline{a} > c_u$. Moreover, $\overline{a}$ increases with $I$: As the investment cost increases, the outside option of building its own network becomes less attractive to firm $d$, and a larger upstream price can be charged by its upstream provider.

The next assumption, together with the uniqueness of the constrained monopoly upstream price, allows to rule out irregular cases in which $\pi_i^{(i)}(.)$ would have multiple local maxima.

[30] An increase in firm $d$'s cost has typically two impacts on its profit. First, the price-cost margin is directly reduced, leading unambiguously to a lower profit. Second, the best-response (in downstream price) of firm $d$ shifts upward, which affects the equilibrium of the final market. These first two effects are standard in any IO models with price competition and product differentiation. In our context, there is also a third effect since the best-response of the upstream supplier also shifts upward (the accommodation effect). The overall impact on firm $d$'s profit is a priori ambiguous and depends typically on the strategic interaction on the downstream market. In line with most IO models, Assumption 3 implies that the direct effect outweighs the strategic ones.

**Assumption 4.** *The profit of the upstream supplier is concave in the upstream price:* $(\pi_i^{(i)})''(.) \leq 0$.

Then, we obtain the following proposition.

**Proposition 2.** *Under Assumptions 1-4, there exists a matching-like equilibrium at price* $a_*$*, i.e., a subgame-perfect equilibrium in which the outcome on the upstream market is such that both integrated firms offer the same upstream price* $a_*$*, iff* $\pi_i^{(i)}(a_*) = \pi_j^{(i)}(a_*)$ *and* $a_* \leq \hat{a}$.

*Proof.* See Appendix A.3. □

A particular matching-like equilibrium corresponds to the case $a_* = c_u$, in which both integrated firms offer an upstream price equal to their upstream marginal cost. As the intuition suggests, the competitive outcome on the upstream market is an equilibrium.

**Corollary 1.** *Under Assumptions 1-4, there exists an equilibrium with a competitive upstream market:* $a_1 = a_2 = c_u$.

*Proof.* Immediate. □

However, nothing precludes a priori the existence of other matching-like equilibria featuring either a supra-competitive upstream market (i.e., $a_* > c_u$) or a super-competitive upstream market (i.e., $a_* < c_u$). The existence of these equilibria also hinges on the accommodation effect. For $a_* > c_u$, the integrated firm which does not supply the upstream market benefits from the accommodation effect and may not want to undercut. For $a_* < c_u$, the accommodation effect is reversed. The upstream supplier offers an aggressive downstream price to reduce the upstream demand, which hurts its integrated rival. Even though the upstream supplier makes losses on the upstream market, it does not want to exit that market since it would then suffer from an adverse accommodation effect.

We conclude that paragraph with the following result:

**Proposition 3.** *Under Assumptions 1-4:*

- *A subgame-perfect equilibrium is either monopoly-like or matching-like.*
- *From the viewpoint of the integrated firms, any monopoly-like equilibrium Pareto-dominates any matching-like equilibrium.*

*Proof.* See Appendix A.4. □

Propositions 1, 2 and 3 provide a characterization of all the possible equilibria of our game. Moreover, the monopoly-like equilibria, when they exist, Pareto-dominate all other equilibria from the integrated firms' standpoint. Therefore there is a strong presumption that, if they exist, the monopoly-like outcomes emerge in equilibrium.

Finally, notice that the multiplicity of equilibria is reminiscent of the analysis of auctions with externalities studied in Jehiel, Moldovanu and Stachetti (1996) and in Jehiel and Moldovanu (1996). Indeed, the upstream price competition subgame can be viewed as an auction run by the service-based firm in which bidders are the facility-based firms. The payoffs earned by the bidders depend both on the identity of the winner and on the price set by that winner; hence, it may be interesting not to participate in order to influence the continuation subgame of downstream price competition.

## 4.4 Illustrations

We now show that our assumptions are satisfied for standard demand and cost specifications. The first illustration enables one to analyze the impact of differentiation at the downstream level on the competitiveness of the upstream market. The second one highlights the role of the strategic interaction at the downstream level.

**The dilemma between upstream and downstream competitiveness.** In this first illustration, the demand that addresses to firm $k$, $k \in \{1, 2, d\}$, is given by: $D_k(p) = 1 - p_k - \gamma(p_k - \bar{p})$, where $\bar{p}$ is the average of all downstream prices and $\gamma \geq 0$ is a substitutability index. All costs are set to zero: $c_u = 0$ and $c_k(.) = 0$. With that specification, profit functions satisfy all the assumptions we made so far. Figure 2 offers a graphical representation of the profit functions $\pi_i^{(i)}(.)$, $\pi_j^{(i)}(.)$ and $\pi_d^{(i)}(.)$.

Then, we obtain the following proposition.

**Proposition 4.** *Consider the linear demands and zero costs case. There exists $\gamma' > 0$ and $I(\gamma)$ such that:*
*If $\gamma \geq \gamma'$ and $I \geq I(\gamma)$, then there exist four subgame-perfect equilibria:*

- *a* Bertrand-like *outcome in which both integrated firms offer the upstream good at its marginal cost, i.e., $a_1 = a_2 = c_u = 0$;*
- *a* matching-like *outcome in which both integrated firms offer the upstream good at the same upstream price strictly above the upstream marginal cost, i.e., $a_1 = a_2 = a_* > c_u$ such that $\pi_i^{(i)}(a_*) = \pi_j^{(i)}(a_*)$;*
- *two* monopoly-like *outcomes in which an integrated firm offers the upstream good at the monopoly upstream price $\hat{a}$ and the other integrated firm makes no offer.*

*Otherwise, i.e., when $\gamma < \gamma'$ or $I < I(\gamma)$, there exists only one subgame-perfect equilibrium in which the Bertrand-like outcome emerges on the upstream market.*

*Proof.* See Appendix A.5. □

As illustrated in Figure 2 and discussed in Subsection 4.2, when $a_i > c_u$, two opposite effects are at work. On the one hand, the upstream supplier derives profit from the upstream market. On the other hand, its integrated rival benefits from the accommodation effect on the downstream market. When the upstream price is not too large, the upstream profit effect dominates and $\pi_i^{(i)}(a_i) > \pi_j^{(i)}(a_i)$. When the upstream price is large enough, upstream revenues shrink, the accommodation effect is strengthened and $\pi_i^{(i)}(a_i) < \pi_j^{(i)}(a_i)$.

The threat of the pure downstream firm exercising its outside option forces the integrated firms to set not too high upstream prices. In particular, if the investment cost $I$ is lower than $I(\gamma)$, then $\bar{a}$, the highest price that can be charged by the upstream supplier, cannot exceed $a_*$. For these rather low upstream prices, both integrated firms prefer to be the upstream supplier of the pure downstream firm. As a result, only the Bertrand outcome emerges when $I < I(\gamma)$.

Suppose now that $I$ is sufficiently large so that the pure downstream firm's outside

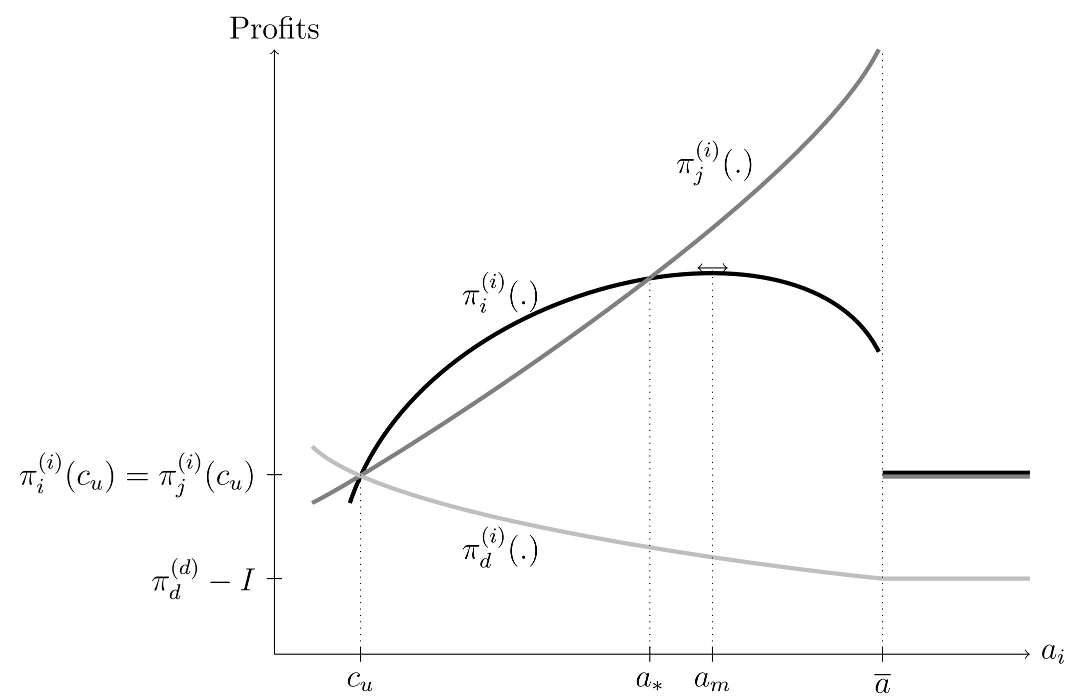


**Figure 2:** Stage 1 profits in the linear case with zero costs (under the assumptions $\gamma \geq \gamma'$ and $I \geq I(\gamma)$).

option is not a binding threat to the upstream supplier. Suppose that the upstream supplier sets the upstream monopoly price. When the substitutability between final products is strong, the integrated firm which supplies the upstream market is reluctant to set too low a downstream price since this would strongly contract its upstream profit. An implication is that the integrated firm which does not supply the upstream market benefits from a substantial accommodation effect, and, as a result, has no incentives to become the upstream supplier: There exists a monopoly-like equilibrium when downstream products are sufficiently substitutes (i.e., $\gamma \geq \gamma'$).

Therefore, when the downstream market has a pro-competitive structure, the competitive forces on the upstream market vanish. Conversely, when the downstream market has poorly competitive features, the upstream market is highly competitive. In other words, there exists a tension between competitiveness on the downstream market and competitiveness on the upstream market. Intuitively, the same downstream interactions which strengthen the competitive pressure on the downstream market, are those which soften the competitive pressure on the upstream market.

**The role of the strategic interaction.** In the previous illustration, downstream prices were always strategic complements.[31] Our second illustration considers another specification which allows to study in greater depth the role of the strategic interaction. Downstream competition is described by the circular extension of the Hotelling model (see Salop, 1979). A unit mass of consumers is uniformly distributed around a circle of unit length. Firms 1, 2 and $d$ are located symmetrically on that circle. A consumer located at distance $x_k$ from firm $k \in \{1, 2, d\}$ and who purchases the final product from it, obtains the net utility $u_g - p_k - tx_k$, where $u_g$ is the gross utility associated to the final product and $t > 0$ is a transportation cost. Throughout that illustration, we assume that the $u_g$ is large enough so that the downstream market is covered in every equilibrium configurations. As is well-known, the downstream demands are equal to:[32] $D_k(p) = 1/3 - 1/(2t)(2p_k - p_{k'} - p_{k''})$ where $\{k, k', k''\} = \{1, 2, d\}$. Downstream costs are quadratic: $c_k(D_k) = c_d D_k + cD_k^2$, with $c_d > 0$ large enough (to ensure positivity of costs and prices at equilibrium), and, $c > -5t/6$ (to ensure the various second-order conditions). The sign of $c$ indicates whether the average cost is increasing or decreasing, i.e., whether there are decreasing or increasing returns-to-scale.

With this specification, we obtain a neat characterization of the strategic interaction between downstream prices: If $c \leq -t/2$ then prices are strategic substitutes, otherwise ($c \geq -t/2$), they are strategic complements. Roughly speaking, with sufficiently increasing returns-to-scale, when a firm cuts its downstream price, a rival firm experi-

[31] We remind that prices are strategic complements if $\partial^2 \tilde{\pi}_k^{(i)} / \partial p_k \partial p_{k'} \geq 0$, $k \neq k'$, and strategic substitutes otherwise; see Bulow, Geanakoplos and Klemperer (1985) and Fudenberg and Tirole (1984).

[32] See Tirole (1988).

ences a reduction of its demand, and, therefore, an increase of its marginal cost, which ultimately leads this rival to increase its own price.

In this context, the investment cost plays a qualitatively similar role as in the previous illustration: In particular, if $I$ is low enough, then non-competitive outcomes never emerge in equilibrium. Therefore, we assume from now on that $I$ is large enough and we obtain:

**Proposition 5.** *Consider the Hotelling demands and quadratic costs case. If $c \leq t$, then there exist monopoly-like equilibria.*[33] *Otherwise, the Bertrand-like outcome is the unique subgame-perfect equilibrium.*

*Proof.* See Appendix A.6. □

This proposition gives us insights on the link between the strategic interaction and the emergence of non-competitive equilibria. We know from Lemma 1 that the upstream supplier charges a larger downstream price than its integrated rival. We referred to that result as the accommodation effect. Parameter $c$, which relates to the strength of the strategic interaction, might also be viewed as an inverse proxy for the difference between $p_i^{(i)}$ and $p_j^{(i)}$, i.e., for the intensity of the accommodation effect.

To see why, it is convenient to make the following thought experiment. Assume that the upstream supplier $i$ does not take into account its upstream profit when setting its downstream price. Assume also that the pure downstream firm purchases the intermediate input at price $a_i \geq c_u$. All the first-order conditions on the downstream market remain the same, except firm $i$'s one, which becomes:

$$D_i + (p_i - c_i'(D_i) - c_u)\frac{\partial D_i}{\partial p_i} = 0. \tag{3}$$

In this hypothetical situation, integrated firms $i$ and $j$ set the same price $\tilde{p}_i = \tilde{p}_j$, while firm $d$ charges $\tilde{p}_d$.

Suppose now that firm $i$ takes into account its upstream profit when maximizing its profit, and consider the impact on equilibrium downstream prices. Firm $i$'s first-order condition changes from (3) to (1): Its best-response shifts upwards. Therefore, the upstream supplier increases its downstream price. The subsequent behavior of firm $j$ depends on the nature and the strength of the strategic interaction. If prices are strategic substitutes ($c < -t/2$), then firm $j$ lowers its downstream price, which enlarges the gap between equilibrium downstream prices, making the accommodation effect more important. If prices are weakly strategic complements, firm $j$ best-responds by increasing its price moderately, implying a weaker accommodation effect. If prices are strongly

[33]Matching-like equilibria which differ from the Bertrand-like equilibrium usually exist too. These equilibria may feature either a supra-competitive or a super-competitive upstream market. See the Appendix for the complete analysis.

strategic complements, firm $j$'s price hike is so important that it almost matches the initial price increase of firm $i$: The accommodation effect almost disappears.[34]

Therefore, as downstream prices become more strategic complements, the accommodation effect weakens and the incentives to undercut on the upstream market are reinforced. This illustrates another crucial relation between upstream and downstream markets: The less strategic complements downstream prices are, the more likely is the emergence of non-competitive equilibria on the upstream market.

# 5 Extensions and Discussions

We now discuss some extensions of our basic setting. This allows us both to check for the robustness of our results and to support several policy implications developed in the next section.

**Pure upstream competitor.** Throughout all the paper, we assumed that the upstream market could only be supplied by integrated firms. In this extension, we assume that a pure upstream competitor, firm $u$, is able to produce the intermediate input at constant marginal cost $c_u$. Denote by $\pi_d^{(u)}(a_u)$ the profit earned by firm $d$ when the upstream market is supplied by firm $u$ at price $a_u$. We suppose that $\pi_d^{(u)}(.)$ is strictly decreasing: Together with Assumption 3, this implies that, whatever the upstream supplier, the profit of the pure downstream firm decreases in the upstream price.

It is natural to wonder whether the mere presence of firm $u$ is sufficient to break the interactions between the upstream and the downstream markets, and to prevent non-competitive outcomes from emerging. The following proposition states that this is indeed the case provided that downstream prices are strategic complements.

**Proposition 6.** *Under Assumptions 1-4, if $\pi_d^{(u)}(.)$ is strictly decreasing and downstream prices are strategic complements, then there is no subgame-perfect equilibrium in which the upstream market is supplied at a price strictly above the marginal cost.*

*Proof.* See Appendix A.7. □

Let us briefly give the intuition underlying that proposition. Assume that firm $u$ supplies the upstream market at price $a_u > c_u$. We claim that both firm $i$ and firm $d$ are made strictly better off if firm $i$ matches firm $u$, and firm $d$ elects firm $i$ as its

[34] Several things are worth pointing out here. First, we assumed stability of the downstream equilibrium: Therefore it makes sense to use such a heuristic reasoning. Second, the initial price increase of firm $i$, followed by the reaction of firm $j$, has a lot of subsequent effects: Firm $d$ would also want to change its price, which would imply further price changes. We chose to abstract from these second-order considerations, which do not affect qualitatively the intuition.

upstream supplier. Indeed, if firm $d$ accepts to purchase the input from firm $i$ at price $a_u$, this creates an accommodation effect: Firm $i$ raises its downstream price, and, by strategic complementarity, firms $j$ and $d$ react by increasing their prices as well. By a revealed preference argument, these price increases benefit firms $i$ and $d$. Besides, when firm $i$ matches firm $u$, it also earns upstream profits, which provides additional incentives to match.

Conversely, if the upstream market is supplied by integrated firm $i$ at a price $a_i > c_u$, then firm $u$ always wants to undercut, for its sole source of profit comes from the upstream market. Moreover, firm $u$ is always able to attract firm $d$, provided that it offers sufficiently low an upstream price.[35]

Having said that, it becomes clear that the upstream market cannot be supplied at a supra-competitive price in equilibrium. One may then wonder which outcome will arise on the upstream market. As already said, in our basic setting, Proposition 2 implies that the competitive outcome is always a subgame-perfect equilibrium. Obviously, adding a pure upstream competitor does not affect this result. However, nothing precludes a priori the existence of super-competitive equilibria.

One may wonder whether Proposition 6 remains valid when the strategic complementarity assumption is relaxed. The following proposition shows that this assumption is indeed crucial (again, to streamline the analysis, we consider that the investment cost $I$ is large enough):

**Proposition 7.** *Consider the Hotelling demands and quadratic costs case, and, a pure upstream firm u. Denote $\hat{a}_u$ the upstream monopoly price of firm u.*

- *If $c > -53t/69$, then the competitive outcome is the only subgame-perfect equilibrium.*
- *If $c < -53t/69$, then there exists a monopoly-like equilibrium in which firm u sets $\hat{a}_u$ and both integrated firms exit the upstream market.*[36]

*Proof.* See Appendix A.8. □

When an integrated firm considers undercutting a pure upstream firm on the upstream market, it has to take two effects into account. The upstream profit effect, i.e., the fact that the undercutting firm steals all upstream profits, is always positive. There is also a strategic effect. When firm $i$ supplies the upstream market, it becomes more accommodating on the downstream market. If prices are strategic complements, both

[35]Indeed, $\pi_d^{(u)}(.)$ and $\pi_d^{(i)}(.)$ are decreasing and $\pi_d^{(u)}(c_u) = \pi_d^{(i)}(c_u)$.

[36]For expositional purposes, we do not state all subgame-perfect equilibria when $c < -53t/69$; see the Appendix for the complete analysis.

firms $j$ and $d$ raises their prices, making everybody better off. However, under strategic substitutability, firms $j$ and $d$ best-respond by decreasing their prices: a negative strategic effect. Proposition 7 tells us that when the degree of strategic substitutability is high enough, the negative strategic effect outweighs the upstream profit effect, removing all incentives to undercut the pure upstream firm $u$ and allowing the emergence of non-competitive equilibria.

To conclude this extension, two remarks are in order. First, Propositions 6 and 7 extend to the case of a pure upstream competitor the intuition obtained in Proposition 5: The integrated firms' incentives to undercut on the upstream market become stronger when downstream prices become more strategic complement. Second, with at least two pure upstream firms, the Bertrand-like outcome always emerges at equilibrium.

**Build or buy decision and total foreclosure.** One may wonder whether our results hinge too strongly on the timing we chose. Assume that the pure downstream firm makes its investment decision before the upstream competition stage. In the continuation subgame in which the downstream firm becomes facility-based, nothing changes: The three firms compete on a level playing field and earn operating profits $\pi_i^{(d)}$ for integrated firm $i \in \{1, 2\}$ and $\pi_d^{(d)} - I$ for firm $d$. Consider now the subgame in which firm $d$ remains service-based.[37] We have to take another element into account in our analysis: once the upstream competition stage is reached, firm $d$ no longer has an outside option. If no upstream offer is made, firm $d$ cannot be active on the downstream market. In other words, the total foreclosure of the pure downstream competitor may be an issue. Let us deal with it formally.

Denote by $A = \{a \in \mathbb{R} | D_d(p(a)) > 0\}$ the set of upstream prices such that $d$ serves a positive quantity at the downstream equilibrium. If integrated firms make no upstream offer, they both earn the same profits: $\pi_1(\emptyset) = \pi_2(\emptyset) \equiv \pi(\emptyset)$. This allows us to define the unconstrained upstream monopoly price:

$$\hat{a} = \arg \max_{a \in A \bigcup \{\emptyset\}} \pi_i^{(i)}(a).$$

If the solution is interior, namely, if $\hat{a}$ is such that firm $d$ serves a strictly positive quantity in equilibrium, Propositions 1, 2 and 3 remain valid. Conversely, assume that $\hat{a} = \emptyset$. Then total foreclosure is obviously an upstream equilibrium: Provided that firm $i$ does not make an upstream offer, firm $j$ does not want to make an upstream offer either, by definition of $\hat{a}$. In other words, monopoly-like equilibria still exist, and, once again, upstream competition may not affect the outcome with respect to the monopolized benchmark. Propositions 2 and 3 are also obtained immediately. Finishing

[37] The analysis that follows also applies to the initial timing when Assumption 1 is not satisfied, i.e., when the investment cost $I$ is larger than the operating profits $\pi^*$.

the resolution is straightforward: Firm $d$ builds a network if and only if it anticipates that the equilibrium upstream price $a$ in the no-investment subgame will be strongly non-competitive, namely if: $\pi_d^{(i)}(a) \leq \pi_d^{(d)} - I$.

The main message of our article is that we should not expect too much from upstream market competition. We have just seen that the same point can be made when total foreclosure is possible. If a monopolized upstream market leads to total foreclosure, introducing upstream competition will not help. Similarly, if a monopolized upstream market leads to partial foreclosure, namely if the cost of firm $d$ gets raised in equilibrium, upstream competition will be a poor medicine as well. In a nutshell, upstream competition does not modify the degree of vertical foreclosure.

**The role of upstream pricing.** The accommodation effect stems from the fact that the upstream tariff incorporates a variable part. By contrast, suppose that the integrated firms are unable to implement a unit price, presumably because they cannot meter the usage of the intermediate input, or because some government regulation forbids it. In that case, they are constrained to set upstream fixed fees only, which obviously prevents them from affecting their upstream profit through their downstream behavior.

Conversely, the mere presence of the variable part in the upstream tariff is sufficient to generate the accommodation effect. In particular, the possibility for non-competitive equilibria carries over to the case of two-part tariffs $\{(a_i, T_i), (a_j, T_j)\}$ on the upstream market. The equilibrium upstream variable part maximizes the joint profit of the upstream supplier and the pure downstream firm, i.e., $a_i^{tp} = \arg\max_{a_i} \pi_i^{(i)}(a_i) + \pi_d^{(i)}(a_i)$.[38] The fixed fee $T_i$ must be such that, first, firm $j$ does not want to undercut, and, second, firm $d$ is willing to accept that tariff, or: $T_i \leq \min\{\pi_j^{(i)}(a_i^{tp}) - \pi_i^{(i)}(a_i^{tp}); \pi_d^{(i)}(a_i^{tp}) - (\pi_d^{(d)} - I)\}$.[39]

Having said that, the characterization of the subgame-perfect equilibria of the game with two-part tariffs becomes straightforward.

If $\pi_i^{(i)}(a_i^{tp}) + \pi_d^{(i)}(a_i^{tp}) \leq \pi_j^{(i)}(a_i^{tp}) + \pi_d^{(d)} - I$,[40] then the only two equilibria are such that one of the integrated firm charges the variable part $a_i^{tp}$ and a fixed fee equal to $\pi_d^{(i)}(a_i^{tp}) - (\pi_d^{(d)} - I)$, which extracts all the rent from the downstream firm, while the other integrated firm makes no upstream offer. Under two-part tariff competition, this is a monopoly-like equilibrium.

If $\pi_i^{(i)}(a_i^{tp}) + \pi_d^{(i)}(a_i^{tp}) \geq \pi_j^{(i)}(a_i^{tp}) + \pi_d^{(d)} - I$, then the only equilibrium features both integrated firms charging the variable part $a_i^{tp}$ and a fixed fee equal to $\pi_j^{(i)}(a_i^{tp}) - \pi_i^{(i)}(a_i^{tp})$,

[38] See Bonanno and Vickers (1988).

[39] Notice that the equilibrium fixed part $T_i$ may be negative.

[40] This inequality holds, for instance, with Hotelling-Salop demand specifications and constant marginal costs, provided that the investment cost is low enough.

which makes them indifferent between supplying the upstream demand or not. This is a matching-like equilibrium.

**Proposition 8.** *Under two-part tariff competition on the upstream market, the equilibrium is either monopoly-like or matching-like.*

*Proof.* Immediate. □

Once again, upstream competition does not modify the outcome with respect to the monopoly benchmark. If the upstream equilibrium is the monopoly-like, both the fixed and the variable part of the tariffs remain the same; obviously, downstream prices are not affected either. If the equilibrium is a matching one, competition modifies the fixed part only, without affecting any downstream price. In other words, the only impact of competition is to redistribute some profits from the integrated firms to the pure downstream firm.

**Quantity competition.** The accommodation effect exists if the upstream supplier can enhance its upstream profits by behaving softly on the downstream market. As discussed previously, this requires that it actually interacts with the pure downstream firm. One may wonder whether the accommodation effect hinges on the assumption of price competition on the downstream market. Indeed, if the downstream strategic variables are quantities and all firms play simultaneously, then the upstream supplier can no longer impact its upstream profit through its downstream behavior. However, if for instance integrated firms are Stackelberg leaders on the downstream market, then the upstream supplier's quantity choice modifies its upstream profit, and the accommodation effect is still at work. To summarize, the question is not whether firms compete in prices or in quantities, but whether the strategic choice of a firm can affect its rivals' quantities.[41]

**Discriminatory downstream pricing.** The inability of firms to discriminate among downstream customers is also an important ingredient for the accommodation effect. The Hotelling-Salop example with constant linear costs is convenient to grasp the intuition. Assume that each firm is able to set a different price for each of the two Hotelling-Salop segments on which it is active. Then the upstream supplier is induced to set a high price on the segment it disputes with the pure downstream competitor, and a low price on the segment it disputes with its integrated rival. In other words, the upstream supplier no longer seems accommodating to its upstream rival, which removes the disincentives of the latter to undercut on the upstream market. This restores the competitiveness of the upstream market.

[41] With a linear demand function and quantity competition, if integrated firms are Stackelberg leaders on the downstream market, then there always exist a monopoly-like equilibrium.

**Single upstream supplier assumption.** Throughout all the paper, we assumed that the pure downstream firm was constrained to choose one upstream supplier only. Consider now that, when upstream offers are identical, the pure downstream firm splits its demand equally between integrated firms. We can still think about the upstream market in terms of accommodation effect and upstream profit effect. To see this, suppose that integrated firms $i$ and $j$ share the upstream market. Firm $i$ obviously earns upstream profits. Besides, it also benefits from an accommodation effect: Firm $j$ has incentives to be less aggressive on the downstream market, since it also supplies the upstream market.

It becomes clear that non-competitive equilibria can still exist. Assume that firm $i$ supplies the upstream market at some price $a_i$. If firm $j$ chooses to exit the upstream market, by setting a price higher than $a_i$, it benefits fully from the accommodation effect at the cost of forgiving upstream profits. If it undercuts its rival, it gets all the upstream profits, but it loses the accommodation effect. If it matches its rival, i.e., if it also sets $a_i$, it benefits partly from both effects. As before, there is no reason why firm $j$ would always want to undercut, since there is no reason why the upstream profit effect would always dominate the accommodation effect. For instance, solving the model with Hotelling-Salop demand specifications and constant marginal costs, we get the following subgame-perfect equilibria: the two monopoly-like outcomes, the competitive outcome, and a continuum of non-competitive equilibria in which both integrated firms set the same price and share the upstream demand.

# 6 Policy Implications

In this section, we first apply our results to discuss the competitiveness of wholesale markets in the broadband and mobile industries. Then, we analyze which regulatory tools might be used to stimulate competition in the wholesale market.

## 6.1 Competitiveness of wholesale markets

The main message of our paper is that we should not expect too much from competition in the wholesale market. Facility-based competition has clear benefits; as facility-based firms do not rely on ILECs (or to some limited extent) to provide services to end consumers, the regulatory burden can be lifted to some extent. However, it seems unlikely that facility-based competition will also lead to a competitive wholesale market, and hence, stimulate the development of service-based competition.

One good example is the broadband market. Facility-based competition has developed, with the development of cable modem networks and LLU operators. Therefore, potential competition in the wholesale broadband market exists, and a wholesale market has even emerged in some countries (e.g., France or the UK). However, there is little evidence that these markets are competitive. This is consistent with our analysis, which has highlighted that competitive outcomes are unlikely.

We have also suggested that the nature of competition in the retail market, and in particular, the degree of differentiation between firms in that market, might affect the outcome in the wholesale market. More precisely, with our first illustration, we showed that if differentiation in the retail market was sufficiently high, the wholesale market was competitive, and that otherwise, non-competitive outcomes were also possible.

This result provides interesting insights for the mobile industry. As we outlined in Section 2, in the mobile market, some MVNOs target niche markets (such as, specific ethnic groups). Since these niche MVNOs and MNOs are strongly differentiated, our model predicts that the wholesale market is likely to be competitive. Therefore, one should not consider that entry of niche MVNOs will have little impact because they do not compete head-to-head with MNOs on the retail market. A more general assessment is called for, as their impact on the wholesale market might be substantial.

In a similar spirit, our model suggests that strongly differentiated MVNOs are more likely to enter the final markets, as they are more likely to benefit from attractive wholesale offers by MNOs. Hence, in the mobile industry we might expect a high degree of product diversity and soft price competition on the downstream market.

Finally, we showed that possibilities of bypass (when the pure downstream firm builds its own facility) had an impact on the competitiveness of the wholesale market. More precisely, the lower the cost of building an alternative infrastructure for the pure downstream firm, the more competitive the wholesale market is likely to be.

This result has interesting policy implications. For the broadband market, it implies that favorable conditions for local loop unbundling (e.g., a low rate for the unbundled lines) might stimulate the development of the wholesale broadband market. In the mobile industry, it means that favorable terms for spectrum licences (e.g., terms for ungranted mobile licences, or for Wimax licences) might increase MNOs' incentives to set low wholesale prices for MVNOs.

Once again, this reasoning outlines that the wholesale markets should not be considered in isolation; one should also take into account the interplay with the possibilities of bypass and competition in the retail markets.

## 6.2 Regulatory tools

We have shown that, if left unregulated, a wholesale market is unlikely to be competitive. We consider below two types of regulatory intervention.

**Price cap.** The regulator could decide to regulate the wholesale offer of one vertically integrated firm (e.g., the incumbent firm) while leaving the offer of the other vertically integrated firm (if any) unregulated. The broadband market provides an example of this form of asymmetric regulation on the wholesale market. In some European countries, the broadband wholesale offers of ILECs are regulated, while those of CLECs are left unregulated.

If the regulator knows the upstream cost, $c_u$, then the optimal regulation is straightforward; it involves setting the regulated upstream price, $a_r$, at $c_u$.

However, the regulator can have imperfect information about the upstream cost of the regulated firm. In such a case, the regulator can improve upon the unregulated outcome by setting a sufficiently low price cap, $a_r$. For instance, consider our first illustration in Section 4.4. If the regulator sets $a_r < a_*$, the Bertrand-like outcome becomes the unique equilibrium. Indeed, with this price cap constraint, both integrated firms prefer to be the upstream supplier of the pure downstream firm.

To summarize, a sufficiently low price cap is an appropriate remedy to enhance the competitiveness of the wholesale market.

**Structural separation.** It is sometimes advocated that vertical separation of the incumbent operator, into two independent upstream and downstream divisions, can also promote competition in the telecommunication industry. In that scenario, the upstream division of the ILEC sets the price of its wholesale offer, without taking into account the impact on the downstream division, and reciprocally.

In particular, vertical separation has been considered to stimulate competition the broadband market. Two types of separation could be implemented. First, the local access unit of the ILEC could be separated from its downstream units (it corresponds to the first interpretation of the wholesale market in Section 2). Second, the Internet service provider unit of the ILEC could be separated from the upstream unit.[42]

In Section 5, we have analyzed how the competitiveness of the wholesale market is affected, when a pure upstream competitor is introduced. We have shown that if downstream prices are strategic complements (or not too strategic substitutes), the introduction of a pure upstream firm leads to a competitive wholesale market. However, if the degree of strategic substitutability is high enough, non-competitive equilibria can also emerge.

Therefore, whether vertical separation (and hence, the introduction of a pure upstream provider) can lead to a competitive wholesale market is not clear. If prices are strategic complements, the intuition that separation has a pro-competitive effect is

[42]This type of situation has been observed in some countries. For instance, in France, some years ago, the ILEC's Internet service provider, Wanadoo, was a subsidiary of its parent company, France Télécom.

correct. But this is not true if prices are strategies substitutes, which can occur with increasing returns to scale.

There are two other types of situations in which a pure upstream unit can operate. First, municipalities can decide to invest in broadband networks and to offer wholesale broadband services to stimulate competition in the wholesale market. Again, our setting suggests that this objective is not necessarily attained.

Second, some private companies can decide to enter as pure upstreamproviders. For instance, in the broadband market, firms like Covad or Northpoint in the US, or Mangoosta in France, adopted this strategy. In the mobile market, so-called mobile virtual enablers (MVNEs) are also pure upstream firms. Our model suggests that this type of strategy is viable, only under certain restrictive conditions.

# 7 Conclusion

Our analysis has focused on the links between vertically-related markets, when the upstream good is a complementary input to the downstream product, and when the competitors on the upstream market are also rivals on the downstream one. Such a theoretical framework depicts well wholesale markets in telecommunications.

One of the main insights conveyed in the paper is that these upstream markets might not be competitive. Put differently, the monopoly outcome, which obtains when the upstream market is exogenously monopolized, might persist even when competition in that market is possible. The main reason lies in the accommodation effect, according to which an integrated firm supplying the upstream market tends to be a soft competitor on the downstream market. This implies in turn that the rival integrated firm might not be willing to compete at all on the upstream market. Various examples have been proposed to illustrate this mechanism and some of its determinants: product differentiation and strategic interaction at the downstream level. Roughly speaking, product differentiation and strategic interaction affect the strength of the accommodation effect, and, therefore, the potential competitiveness of the upstream market.

The theoretical analysis has been the basis of several policy implications and robustness checks. Let us reemphasize that, from a competition policy perspective, our analysis suggests that analyzing upstream markets in isolation of the related downstream markets is logically flawed: Taken on a stand-alone basis, our upstream market has all the competitive features of a perfectly competitive market; once the downstream market is added to the analysis, the picture is much more mixed. Overall, the potential competitiveness of upstream markets hinges as much on the economic fundamentals pertaining to that market as on the fundamentals of the related downstream markets.

Various extensions would certainly be worth studying.

From a more industry-specific perspective, and as regards the broadband market, it would be worth investigating the role of the local loop in our setting. Indeed, most facility-based firms rely to some extent on the local loop owned by the incumbent to offer broadband services to final customers. It would be interesting to understand how the competition on the wholesale market is affected by the regulation of the access to the local loop. In the same vein, as regards the mobile telephony market, interconnection flows between MNOs, and their regulation, may impact the competition between MNOs to attract MVNOs.

From a more theoretical viewpoint, the role of the market structures remains to be studied. A first set of questions relates to the impact of the number of vertically integrated firms and of pure downstream firms on the competitiveness of the wholesale market. A second set of questions relates to the entry process in this industry: Are vertically integrated firms more likely to enter than pure downstream ones?

All these questions are left for future research.

# A Appendix

## A.1 Proof of Lemma 1

To begin with, we prove the following fixed point lemma:

**Lemma 0.** *Let $f : [0, \infty) \to [0, \infty)$ be a $\mathcal{C}^1$ function. Assume that $f$ is bounded and $|f'(.)| < 1$.*
*Then, $f$ admits a unique fixed point $x^*$. Besides, $f(x) > x$ if, and only if, $x < x^*$; and $f(x) < x$ if, and only if, $x > x^*$.*

*Proof.* Define $g(x) \equiv f(x) - x$. Computing its first derivative, we see that $g(.)$ is strictly decreasing, since $f'(.) < 1$. $g(0) = f(0) \geq 0$, and $\lim_{x\to\infty} g(x) = -\infty$, since $f(.)$ is bounded. By continuity, there exists a unique $x^* \geq 0$ such that $g(x) = 0$. Since $g(.)$ is strictly decreasing, $g(x) > 0$ if, and only if, $x < x^*$. And $g(x) < 0$ if, and only if, $x > x^*$. This concludes the proof. □

Assume that integrated firm $i \in \{1, 2\}$ is the upstream supplier at price $a_i > c_u$, and let us show that $p_i^{(i)}(a_i) > p_j^{(i)}(a_i)$. By definition of the unique downstream equilibrium,

$$p_i^{(i)}(a_i) \quad = \quad BR_i^{(i)}\left(BR_j^{(i)}\left(p_i^{(i)}(a_i), p_d^{(i)}(a_i), a_i\right), p_d^{(i)}(a_i), a_i\right).$$

We see from first-order condition (2) that firm $j$'s best response function does not depend on $a_j$: $BR_j^{(i)}(.,., a_i) = BR_j^{(i)}(.,., c_u)$. By contrast, (1) implies that firm $i$'s best-response is increasing in the upstream price: $BR_i^{(i)}(.,., a_i) > BR_i^{(i)}(.,., c_u)$. Hence:

$$p_i^{(i)}(a_i) > BR_i^{(i)}\left(BR_j^{(i)}\left(p_i^{(i)}(a_i), p_d^{(i)}(a_i), c_u\right), p_d^{(i)}(a_i), c_u\right). \tag{4}$$

By assumption, $|\partial BR_i^{(i)}/\partial p_j| < 1$ and $BR_i^{(i)}$ is bounded. We can apply Lemma 0 to function $BR_i^{(i)}(., p_d^{(i)}(a_i), c_u))$: There exists a unique $p^*$ such that $BR_i^{(i)}(p^*, p_d^{(i)}(a_i), c_u)) = p^*$. Besides, since $BR_i^{(i)}(.,., c_u) = BR_j^{(i)}(.,., c_u)$, we also have that $BR_j^{(i)}(p^*, p_d^{(i)}(a_i), c_u)) = p^*$. $p^*$ is the downstream price set by integrated firms in the hypothetical situation in which firm $i$ would supply the upstream market at $c_u$ and firm $d$'s price would be exogenously fixed at $p_d^{(i)}(a_i)$.

Define the following function:

$$f : x \mapsto BR_i^{(i)}\left(BR_j^{(i)}\left(x, p_d^{(i)}(a_i), c_u\right), p_d^{(i)}(a_i), c_u\right).$$

Notice that $f(p^*) = p^*$, by definition of $p^*$. Obviously, $f$ is bounded and $|f'(.)| < 1$. Thus, it admits a unique fixed point: $p^*$. Besides, $f(x) < x \Leftrightarrow x > p^*$. We deduce from (4) that $p_i^{(i)}(a_i) > p^*$.

We can now apply the mean value inequality to function $BR_j^{(i)}(., p_d^{(i)}(a_i), c_u)$ between $p^*$ and $p_i^{(i)}(a_i)$:

$$\left| BR_j^{(i)}\left(p_i^{(i)}(a_i), p_d^{(i)}(a_i), c_u\right) - BR_j^{(i)}\left(p^*, p_d^{(i)}(a_i), c_u\right)\right| \\ \leq \sup_{p_i \in \left[p^*, p_i^{(i)}(a_i)\right]} \left| \frac{\partial BR_j^{(i)}}{\partial p_i}\left(p_i, p_d^{(i)}(a_i), c_u\right)\right| \left| p_i^{(i)}(a_i) - p^* \right|.$$

Since the upper bound is taken over a compact set, it is strictly lower than 1. Besides, since firm $j$'s best response does not depend on the upstream price, and by definition of $p_j^{(i)}$, $BR_j^{(i)}(p_i^{(i)}(a_i), p_d^{(i)}(a_i), c_u) = p_j^{(i)}(a_i)$. By definition of $p^*$, $BR_j^{(i)}(p^*, p_d^{(i)}(a_i), c_u) = p^*$. Using the fact that $p^* < p_i^{(i)}(a_i)$, the mean value inequality can then be rewritten as:

$$\left| p_j^{(i)}(a_i) - p^* \right| < p_i^{(i)}(a_i) - p^*.$$

In particular, $p_j^{(i)}(a_i) - p^* < p_i^{(i)}(a_i) - p^*$, hence, $p_j^{(i)}(a_i) < p_i^{(i)}(a_i)$.

## A.2 Proof of Lemma 2

Let integrated firm $i \in \{1, 2\}$ be the upstream supplier at price $a_i > c_u$. Its downstream profit is given by:

$$(p_i^{(i)}(a_i) - c_u)D_i(p_i^{(i)}(a_i), p_j^{(i)}(a_i), p_d^{(i)}(a_i)) - c_i\left(D_i(p_i^{(i)}(a_i), p_j^{(i)}(a_i), p_d^{(i)}(a_i))\right), \quad (5)$$

with $p_i^{(i)}(a_i) > p_j^{(i)}(a_i)$ by Lemma 1. Define $\hat{p} > p_i^{(i)}(a_i)$ such that:

$$D_i(\hat{p}, p_i^{(i)}(a_i), p_d^{(i)}(a_i)) = D_i(p_i^{(i)}(a_i), p_j^{(i)}(a_i), p_d^{(i)}(a_i)).$$

Downstream profit (5) is smaller than:

$$(\hat{p} - c_u)D_i(\hat{p}, p_i^{(i)}(a_i), p_d^{(i)}(a_i)) - c_i\left(D_i(\hat{p}, p_i^{(i)}(a_i), p_d^{(i)}(a_i))\right).$$

By symmetry between integrated firms, this can be rewritten as:

$$(\hat{p} - c_u)D_j(p_i^{(i)}(a_i), \hat{p}, p_d^{(i)}(a_i)) - c_j\left(D_j(p_i^{(i)}(a_i), \hat{p}, p_d^{(i)}(a_i))\right).$$

By revealed preferences, this profit is smaller than the downstream profit of firm $j$:

$$(p_j^{(i)}(a_i) - c_u)D_j(p_i^{(i)}(a_i), p_j^{(i)}(a_i), p_d^{(i)}(a_i)) - c_j\left(D_j(p_i^{(i)}(a_i), p_j^{(i)}(a_i), p_d^{(i)}(a_i))\right),$$

which concludes the proof.

## A.3 Proof of Proposition 2

Suppose first that both integrated firms offer the same upstream price $a_* \leq \hat{a}$, such that $\pi_i^{(i)}(a_*) = \pi_j^{(i)}(a_*)$. The pure downstream firm chooses indifferently one of them as its upstream supplier, and both integrated firms earn the same profit.

Consider an upward deviation of integrated firm 1, be it the upstream supplier or not. Now, by Assumption 3, firm $d$ strictly prefers buying the upstream good from firm 2 at price $a_*$, and firm 1's profit is unchanged. Consider now a downward deviation: $a_i < a_*$. Firm $d$ strictly prefers to buy from firm 1, which then earns $\pi_i^{(i)}(a_i)$. By Assumption 4, since $a_* \leq \hat{a}$, this profit is smaller than $\pi_i^{(i)}(a_*) = \pi_j^{(i)}(a_*)$. That situation is therefore an equilibrium.

Conversely, consider that both integrated firms offer the same upstream price $a_*$, and assume that one of the assumptions of Proposition 2 is not satisfied. Suppose first that $a_* > \hat{a}$. Then the upstream supplier has a strictly profitable deviation: propose $\hat{a}$.

If $\pi_i^{(i)}(a_*) < \pi_j^{(i)}(a_*)$, then the upstream supplier would rather set an upstream price above $a_*$ to earn $\pi_j^{(i)}(a_*)$.

If $\pi_i^{(i)}(a_*) > \pi_j^{(i)}(a_*)$, then the integrated firm which does not supply the upstream market would rather set an upstream price slightly smaller than $a_*$ to earn a profit almost equal to $\pi_i^{(i)}(a_*)$.

## A.4 Proof of Proposition 3

Consider by contradiction an equilibrium configuration in which $a_i < a_j$ and $a_i \neq \hat{a}$. By Assumption 3, the upstream supplier is firm $i$.

If $a_j > \hat{a}$, it is a strictly profitable deviation for firm $i$ to offer $\hat{a}$. If $a_j \leq \hat{a}$, firm 1 would rather charge any upstream price in $(a_i, a_j)$, since $\pi_i^{(i)}(.)$ is increasing in this interval by Assumption 4.

Let us now show that a monopoly-like equilibrium Pareto-dominates any matching equilibrium with upstream price $a_*$, from the viewpoint of integrated firms. We have $\pi_j^{(i)}(\hat{a}) \geq \pi_i^{(i)}(\hat{a})$ by Proposition 1. Consider a matching-like equilibrium at upstream price $a_*$. By definition of $\hat{a}$, $\pi_i^{(i)}(\hat{a}) \geq \pi_i^{(i)}(a_*) = \pi_j^{(i)}(a_*)$. This concludes the proof.

## A.5 Proof of Proposition 4

The proof proceeds in several steps. We first compute the downstream equilibrium, and check that all the assumptions we need are satisfied. We show that the build or buy decision imposes an upper bound on the upstream prices offered by integrated firms.

We can then compute the monopoly benchmark and make some comparisons between integrated firms' profits. This allows us to apply Propositions 1, 2 and 3, and get all existing subgame-perfect equilibria.

**Downstream equilibrium.** Integrated firm $i$ supplies the upstream market at price $a_i$, and we denote by $j$ its integrated rival. For all downstream and upstream prices, we have:

$$\frac{\partial^2 \tilde{\pi}_k^{(i)}}{\partial p_k^2} = -2(1 + \frac{2}{3}\gamma) < 0, \quad \forall k \in \{1, 2, d\}.$$

This ensures that the best-response functions are uniquely defined. They are equal to:[43]

$$\begin{aligned} BR_i^{(i)}(p_j, p_d, a_i) &= \frac{3 + a_i\gamma + \gamma(p_j + p_d)}{6 + 4\gamma}, \\ BR_j^{(i)}(p_1, p_d, a_i) &= \frac{3 + \gamma(p_i + p_d)}{6 + 4\gamma}, \\ BR_d^{(i)}(p_i, p_j, a_i) &= \frac{3 + 3a_i + 2a_i\gamma + \gamma(p_i + p_j)}{6 + 4\gamma}. \end{aligned}$$

The stability condition is satisfied, since, for all $k \neq k'$, we have:

$$\left| \frac{\partial BR_k^{(i)}}{\partial p_{k'}} \right| = \frac{\gamma}{6 + 4\gamma} < 1.$$

There is a unique downstream equilibrium, which can be calculated by solving the set of first-order conditions. We get:

$$\begin{aligned} p_i^{(i)}(a_i) &= \frac{18 + 15\gamma + 9a_i\gamma + 5a_i\gamma^2}{36 + 42\gamma + 10\gamma^2}, \\ p_j^{(i)}(a_i) &= \frac{18 + 15\gamma + 3a_i\gamma + 3a_i\gamma^2}{36 + 42\gamma + 10\gamma^2}, \\ p_d^{(i)}(a_i) &= \frac{18 + 18a_i + 15\gamma + 21a_i\gamma + 7a_i\gamma^2}{36 + 42\gamma + 10\gamma^2}. \end{aligned}$$

[43] At first sight, these best-response functions seem to be unbounded, which would violate one of our assumptions. If downstream demands are defined more carefully, as $D_k = \max\{1 - p_k - \gamma(p_k - \bar{p}), 0\}$, then it can be shown that downstream prices have to lie below a certain threshold for the three firms to be active. We abstract from these considerations in the following, since we are only interested in configurations in which all firms supply a positive quantity.

They are well-defined if the equilibrium quantity served by downstream firm $d$ is positive, which is equivalent to:

$$a_i \leq a_{\max}(\gamma) \quad \equiv \quad \frac{6+5\gamma}{6+7\gamma+\gamma^2} > 0.$$

**Build or buy decision.** Assumption 3 is satisfied, since:

$$\pi_d^{(i)}(a_i) = \frac{3(1+\gamma)^2(6+\gamma)^2(3+2\gamma)}{4(3+\gamma)^2(6+5\gamma)^2}[a_i - a_{\max}(\gamma)]^2,$$

thus $\pi_d^{(i)}(.)$ is decreasing for $a_i \leq a_{\max}(\gamma)$. Therefore, there exists $\overline{a}(\gamma, I)$, such that the pure downstream firm prefers buying the upstream good at price $a_i$ rather than building a network if, and only if:

$$a_i \leq \overline{a}(\gamma, I).$$

$\overline{a}(\gamma, I)$ increases from 0 to $a_{\max}(\gamma)$ when $I$ increases from 0 to $\pi_d^{(d)}$.

**Monopoly benchmark.** Assumption 4 is ensured by:

$$\frac{d^2\pi_i^{(i)}}{da_i^2} = -\frac{648+1944\gamma+2205\gamma^2+1158\gamma^3+269\gamma^4+20\gamma^5}{2(18+21\gamma+5\gamma^2)^2} < 0,$$

implying that $\pi_i^{(i)}(.)$ is concave. Firm $i$'s (unconstrained) maximum is reached for:

$$a_i = a_m(\gamma) \quad \equiv \quad \frac{324+594\gamma+360\gamma^2+75\gamma^3}{648+1296\gamma+909\gamma^2+249\gamma^3+20\gamma^4} \in (0, a_{\max}(\gamma)).$$

The (constrained) monopoly upstream price is:

$$\hat{a}(\gamma, I) \quad \equiv \quad \min\{a_m(\gamma), \overline{a}(\gamma, I)\} \in (0, a_m(\gamma)].$$

**Comparison of integrated firms' profits.** $\pi_i^{(i)}(.)$ and $\pi_j^{(i)}(.)$ are parabolas, they cross each other twice, in $a_i = c_u$ and in:

$$a_i = a_*(\gamma) \quad \equiv \quad \frac{9(12+16\gamma+5\gamma^2)}{108+180\gamma+93\gamma^2+13\gamma^3}.$$

$\pi_i^{(i)}(.)$ is concave, and $\pi_j^{(i)}(.)$ is convex since:

$$\frac{d^2\pi_j^{(i)}}{da_i^2} = \frac{3(3+2\gamma)\gamma^2(1+\gamma)^2}{2(3+\gamma)^2(6+5\gamma)^2} > 0.$$

Hence, we have:

$$\pi_i^{(i)}(a_i) \geq \pi_j^{(i)}(a_i) \quad \Leftrightarrow \quad a_i \in [0, a_*(\gamma)]. \tag{6}$$

Let us now check whether or not $a_m(\gamma) \in [0, a_*(\gamma)]$:

$$a_m(\gamma) - a_*(\gamma) = \frac{3(3+\gamma)(6+5\gamma)(-648-1296\gamma-864\gamma^2-183\gamma^3+5\gamma^4)}{(108+180\gamma+93\gamma^2+13\gamma^3)(648+1296\gamma+909\gamma^2+249\gamma^3+20\gamma^4)}.$$

Analyzing the above function, we obtain that there exists $\overline{\gamma} > 0$, such that:

$$a_m(\gamma) \geq a_*(\gamma) \quad \Leftrightarrow \quad \gamma \geq \overline{\gamma}.$$

Define $I(\gamma) \geq 0$ such that $\overline{a}(\gamma, I(\gamma)) = a_*(\gamma)$. We have:

$$a_*(\gamma) \leq \overline{a}(\gamma, I) \quad \Leftrightarrow \quad I \geq I(\gamma).$$

**Upstream equilibrium.** Since $\pi_i^{(i)}(0) = \pi_j^{(i)}(0)$ and $0 \leq \hat{a}(\gamma, I)$, Proposition 2 implies that the Bertrand outcome is always an equilibrium.

If $\gamma < \overline{\gamma}$, then $0 < \hat{a}(\gamma, I) \leq a_m(\gamma) < a_*(\gamma)$. (6) implies that $\pi_i^{(i)}(\hat{a}(\gamma, I)) > \pi_j^{(i)}(\hat{a}(\gamma, I))$, and by Proposition 1 there is no monopoly-like equilibrium. Moreover $a_*(\gamma) > \hat{a}(\gamma, I)$ implies by Proposition 2 that there is no other matching-like equilibrium than the Bertrand equilibrium.

If $\gamma \geq \overline{\gamma}$, then $a_m(\gamma) \geq a_*(\gamma)$. The necessary and sufficient condition for the existence of monopoly-like equilibria is, by (6), that $\overline{a}(\gamma, I) \geq a_*(\gamma)$. This is also a necessary and sufficient condition for the matching-like equilibrium with upstream price $a_*$. That condition is equivalent to $I \geq I(\gamma)$, which concludes the proof.

## A.6 Proof of Proposition 5

To begin with, let us prove the following proposition, which gives a complete characterization of subgame-perfect equilibria in the Hotelling case with quadratic costs, and thus contains Proposition 5:

**Proposition 9.** *Consider the Hotelling case with quadratic costs. Define the following threshold:* $I(c,t) = \frac{(c+t)(4c+3t)(18c+17t)}{3(15c+13t)^2}$. *Then:*

- *The competitive outcome is always a subgame-perfect equilibrium.*
- *The two monopoly-like outcomes are subgame-perfect equilibria if, and only if,* $c \leq -3t/4$, *or* $c \leq t$ *and* $I \geq I(c,t)$.
- *There exists a supra-competitive matching-like equilibrium if, and only if,* $-3t/4 < c \leq t$ *and* $I \geq I(c,t)$.
- *There exists a super-competitive matching-like equilibrium if, and only if,* $-22t/27 \leq c < -3t/4$.

*These are the only subgame-perfect equilibria.*

*Proof.* As before, the proof proceeds in several steps. We first compute the downstream equilibrium, and check that all the assumptions we need are satisfied. We show that the build or buy decision imposes an upper bound on the upstream prices offered by integrated firms. We can then compute the monopoly benchmark. Proposition 2 gives us all matching-like equilibria. Proposition 1 provides necessary and sufficient conditions for monopoly-like equilibria. We conclude with Proposition 3, which states that we have found all subgame-perfect equilibria.

**Downstream equilibrium.** In the following, we assume that the upstream market is supplied by firm $i$ at price $a_i$, and denote by $j$ the firm which does not supply the upstream market.

Straightforward computations show that, for all $k \neq k' \in \{1,2\}$, $\partial^2 \tilde{\pi}_k^{(i)}/\partial p_k \partial p'_k = (2c+t)/(2t^2)$. Therefore, prices are strategic complements if $c \geq -t/2$, and strategic substitutes otherwise.

The three profit functions are concave: For all $k$, $\partial^2 \tilde{\pi}_k^{(i)}/\partial p_k^2 = -2(c+t)/t^2$, which is strictly negative, since $c > -5t/6$. Since all three first-order conditions have a unique solution, we deduce the best-response functions on the downstream market:[44]

$$BR_i^{(i)}(p_j, p_d, a_i) = \frac{3a_i t + 4ct + 6c_d t + 3c_u t + 2t^2 + (p_j + p_d)(6c+3t)}{12(c+t)},$$
$$BR_j^{(i)}(p_i, p_d, a_i) = \frac{4ct + 6c_d t + 6c_u t + 2t^2 + (p_i + p_d)(6c+3t)}{12(c+t)},$$
$$BR_d^{(i)}(p_i, p_j, a_i) = \frac{6a_i t + 4ct + 6c_d t + 2t^2 + (p_i + p_j)(6c+3t)}{12(c+t)}.$$

[44]Once again, these best-responses seem to be unbounded. If we take into account the consumers' reservation utility, then the best-response functions are indeed bounded. We abstract from these considerations, since we focus on situations in which the downstream market is covered.

A quick inspection at these best-response functions shows that, for all $k, k'$, $|\partial BR_k^{(i)}/\partial p_{k'}| = |(2c+t)/(4(c+t))|$, which is strictly smaller than 1, since $c > -5t/6$.

There is indeed a unique downstream equilibrium, which can be calculated by solving the set of first-order conditions. We get:

$$p_i^{(i)}(a_i) = \frac{1}{6}(3a_i + 4c + 6c_d + 3c_u + 2t),$$
$$p_j^{(i)}(a_i) = \frac{1}{30}\left(9a_i + 20c + 30c_d + 21c_u + 10t + \frac{36c(a_i - c_u)}{6c+5t}\right),$$
$$p_d^{(i)}(a_i) = \frac{1}{30}\left(21a_i + 20c + 30c_d + 9c_u + 10t - \frac{36c(a_i - c_u)}{6c+5t}\right).$$

Substituting these prices into the profit functions, we get:

$$\pi_i^{(i)}(a_i) = \frac{1}{18}\left(9(a_i - c_u) + 2(c+t) - \frac{27(a_i - c_u)^2}{6c+5t}\right),$$
$$\pi_j^{(i)}(a_i) = \frac{(c+t)(9(a_i - c_u) + 12c + 10t)^2}{36(6c+5t)^2},$$
$$\pi_d^{(i)}(a_i) = \frac{(c+t)(-9(a_i - c_u) + 12c + 10t)^2}{36(6c+5t)^2}.$$

Demands at the downstream equilibrium are given by: $D_i^{(i)}(a_i) = 1/3$, $D_j^{(i)}(a_i) = 1/3 + 3(a_i - c_u)/(2(6c+5t))$ and $D_d^{(i)}(a_i) = 1/3 - 3(a_i - c_u)/(2(6c+5t))$. Before going further, we need to find upper and lower bounds for $a_i$, i.e., we have to define the strategy space. To do so, we look for the set of values of $a_i$, such that each firm serves a positive quantity and earns a positive profit at the downstream equilibrium. After some algebra, we find that firm $i$ is active if, and only if, $a_i \in [c + c_u + 5t/6 - \sqrt{(6c+5t)(26c+23t)}/(6\sqrt{3}),\ c + c_u + 5t/6 + \sqrt{(6c+5t)(26c+23t)}(6\sqrt{3})]$; firm $j$ is active if, and only if $a_i \geq c_u - 4c/3 - 10t/9$; and firm $d$ is active if, and only if $a_i \leq c_u + 4c/3 + 10t/9$. These thresholds can be ranked, and used to define the strategy space as $[a_{min}, a_{max}]$,[45] with $a_{max} = c_u + 4c/3 + 10t/9$ and:

$$a_{min} = \begin{cases} c + c_u + \frac{5}{6}t - \frac{\sqrt{(6c+5t)(26c+23t)}}{6\sqrt{3}} & \text{if } c \geq -\frac{22}{27}t, \\ c_u - \frac{4}{3}c - \frac{10}{9}t & \text{otherwise.} \end{cases}$$

$\pi_d^{(i)}(.)$ is quadratic and convex in $a_i$. It reaches its minimum for $a_i = c_u + 4c/3 + 10t/9$. Therefore, $\pi_d^{(i)}(.)$ is decreasing for $a_i \leq a_{max}$. Hence, Assumption 3 is satisfied. One also sees immediately that $\pi_i^{(i)}(.)$ is concave, hence Assumption 4 is satisfied.

[45] Notice that these upper and lower bounds are actually functions of $c$ and $t$. We choose to omit the arguments for the sake of clarity.

**Build or buy decision** When the pure downstream firm chooses to build a network, the operating profits earned by the three firms are given by: $\pi_i^{(d)} = \pi_d^{(d)} = (c+t)/9$. This allows us to rephrase Assumption 1: $(c+t)/9 > I$.

We can now calculate $\bar{a}$, the threshold above which firm $d$ decides to build a network. By definition, $\bar{a}$ is the solution of equation $\pi_d^{(i)}(a_i) = \pi_d^{(d)} - I$. This equation has only one solution inside the strategy space:

$$\bar{a} = c_u + \frac{4c}{3} + \frac{10t}{9} - \frac{2}{9}(6c+5t)\sqrt{1 - \frac{9I}{c+t}}.$$

The assumptions we made on $c$ and $I$ ensure that $\bar{a} > c_u$.

**Monopoly benchmark** Let us compute the unconstrained upstream monopoly price, $a_m = \arg\max_{a_i} \pi_i^{(i)}(a_i)$. We get $a_m = c + c_u + 5t/6$, which is obviously larger than $c_u$ and lower than $a_{max}$.

Since $\pi_i^{(i)}(.)$ is quadratic and concave, we easily deduce the value of the constrained monopoly upstream price from the above discussion: $\hat{a} = \min\{\bar{a}, a_m\}$. Again, we have $c_u < \hat{a} < \bar{a}$.

**Matching-like equilibria.** We can now apply Proposition 2, which states that if $a_i$ satisfies $\pi_i^{(i)}(a_i) = \pi_j^{(i)}(a_i)$ and $a_i \leq \hat{a}$, then there is a matching-like equilibrium at price $a_i$. First of all, let us solve equation $\pi_i^{(i)}(a_i) = \pi_j^{(i)}(a_i)$ in $a_i$. We find two roots: $c_u$ and $a_* = c_u + 2(4c+3t)(6c+5t)/(45c+39t)$.

As we said before, $c_u < \hat{a}$; besides, $\pi_i^{(i)}(c_u) = \pi_j^{(i)}(c_u)$. Therefore, the competitive outcome is a subgame-perfect equilibrium.

Let us see whether a matching-like outcome can arise at price $a_*$. Notice first, that if $c < -22t/27$, $a_*$ is lower than $a_{min}$, i.e., $a_*$ is outside the strategy space and cannot be played by both firms in equilibrium.

Assume now $c \geq -22t/27$. If $c < -3t/4$, $a_*$ is below $c_u$. Since $\hat{a} > c_u$, $a_* < \hat{a}$. By Proposition 2, if $-22t/27 \leq c < -3t/4$, there exists a super-competitive matching-like equilibrium at price $a_*$.

If $c = -3t/4$, then $a_* = c_u$, and we already know that the competitive outcome is an equilibrium.

If $c > -3t/4$, we need to go through messier calculations. Notice that $a_* \leq \hat{a}$ if, and only if $a_* \leq a_m$ and $a_* \leq \bar{a}$, since $\hat{a} = \min\{a_m, \bar{a}\}$. After some algebra, we get that $a_* \leq a_m$ if and only if $c \leq t$. We also get that $a_* \leq \bar{a}$ if and only if:

$$I \geq \frac{(c+t)(4c+3t)(18c+17t)}{3(15c+13t)^2} \equiv I(c,t).$$

One shows easily that $0 \leq I(c,t) < \pi_d^{(d)}$ for $c > -3t/4$.

This gives us a complete characterization of matching-like equilibria.

**Monopoly-like equilibria.** Now, we use Proposition 1 to investigate whether monopoly-like outcomes are subgame-perfect equilibria.

Define $\Phi(a_i) = \pi_j^{(i)}(a_i) - \pi_i^{(i)}(a_i)$. $\Phi(.)$ is quadratic and convex. One also knows that it has two roots, $c_u$ and $a_*$. Therefore, $\Phi(a) \geq 0$ if, and only if $a \geq \max\{a_*, c_u\}$, or $a \leq \min\{a_*, c_u\}$. We know that $\hat{a}$ is larger than $c_u$. As a result, $\Phi(\hat{a}) \geq 0$ if, and only if, $\hat{a} \geq \max\{a_*, c_u\}$.

If $c \leq -3t/4$, we have seen that $a_* \leq c_u$. We deduce that $\Phi(\hat{a}) = \pi_j^{(i)}(\hat{a}) - \pi_i^{(i)}(\hat{a}) \geq 0$ if, and only if, $\hat{a} \geq c_u$, which always holds. Hence, using Proposition 1, if $c \leq -3t/4$, there are always two monopoly-like equilibria.

If $c > -3t/4$, then $a_* > c_u$. Therefore, $\pi_j^{(i)}(\hat{a}) - \pi_i^{(i)}(\hat{a}) \geq 0$ if, and only if, $\hat{a} \geq a_*$. We have already analyzed this inequality: It holds if, and only if, $I \geq I(c, t)$ and $c \leq t$.

To conclude the proof, all we need to do now is use Proposition 3, which tells us that we have found all subgame-perfect equilibria.

□

## A.7 Proof of Proposition 6

The proof is made easier by noticing that our game exhibits some supermodular features. More precisely, assume that firms 1 and $u$ propose the same upstream price $a_1 = a_u = a$, while firm 2 makes no upstream offer. The game $(\mathbb{R}; (p_k, p_{-k}, i) \mapsto \tilde{\pi}_k^{(i)}(p_k, p_{-k}, a), i = u, 1; k = 1, 2, d)$ is strictly supermodular (with the order relation $u < 1$).[46] Indeed, for all $k$, $\tilde{\pi}_k^{(i)}(p_k, p_{-k}, a)$ has increasing differences in $(p_k, i)$, and $\tilde{\pi}_1^{(i)}(p_1, p_{-1}, a)$ has strictly increasing differences in $(p_k, i)$. Besides, the downstream equilibrium is, by assumption, unique. Supermodularity theory (see Vives (1990), p.35) tells us that the equilibrium of that game is strictly increasing in $i$, i.e., $p_k^{(u)}(a) < p_k^{(1)}(a)$ for $k = 1, 2, d$.

Let us now show by contradiction that the upstream market can not be supplied at an upstream price strictly larger than $c_u$.

Suppose that integrated firm 1 supplies the upstream market at $a_1 > c_u$. Pure upstream firm $u$ earns zero profit. However it is able to corner the upstream market and earn a positive profit by offering $a_u > c_u$. Indeed $\pi_d^{(u)}(c_u) = \pi_d^{(1)}(c_u) > \pi_d^{(1)}(a_1)$, thus the pure upstream firm can undercut with an upstream price close enough to $c_u$.[47]

[46] The new notations are similar to the previous ones: $\tilde{\pi}_k^{(u)}(., ., a)$ denotes the out-of-equilibrium profit of firm $k$ when the upstream market is supplied by firm $u$ at price $a$, while $\pi_k^{(u)}(a)$ (resp. $p_k^{(u)}(a)$) denotes its profits (resp. downstream price) at downstream equilibrium.

[47] It can not undercut with $a_1 - \varepsilon$ since, as we shall see below, firm $d$ would still prefer to buy from firm 1 in that case.

Suppose now that the upstream market is supplied by pure upstream firm $u$ at $a_u = a > c_u$. Let us show that, if it offers $a_1 = a_u = a$, integrated firm 1 corners the upstream market and enhances its profit. First, firm $d$ strictly prefers to purchase from firm 1. Indeed, when it purchases from firm $u$, it earns:

$$\pi_d^{(u)}(a) = (p_d^{(u)}(a) - a)D_d(p_1^{(u)}(a), p_2^{(u)}(a), p_d^{(u)}(a)) - c_d\left(D_d(p_1^{(u)}(a), p_2^{(u)}(a), p_d^{(u)}(a))\right).$$

Since $p_k^{(u)}(a) < p_k^{(1)}(a)$ for $k = 1, 2$, there exists $\hat{p} > p_d^{(u)}(a)$ such that:

$$D_d(p_1^{(1)}(a), p_2^{(1)}(a), \hat{p}) = D_d(p_1^{(u)}(a), p_2^{(u)}(a), p_d^{(u)}(a)).$$

Then,

$$\pi_d^{(u)}(a) < (\hat{p} - a)D_d(p_1^{(1)}(a), p_2^{(1)}(a), \hat{p}) - c_d\left(D_d(p_1^{(1)}(a), p_2^{(1)}(a), \hat{p})\right),$$

and, by revealed preference,

$$\pi_d^{(u)}(a) < (p_d^{(1)}(a) - a)D_d(p_1^{(1)}(a), p_2^{(1)}(a), p_d^{(1)}(a)) - c_d\left(D_d(p_1^{(1)}(a), p_2^{(1)}(a), p_d^{(1)}(a))\right),$$

which is equal to $\pi_d^{(1)}(a)$. Therefore, firm $d$ prefers to purchase from firm 1. It remains to show that firm 1's profit is larger when it supplies the upstream market. When $u$ supplies the upstream market firm 1 earns:

$$\pi_1^{(u)}(a) = D_1(p_1^{(u)}(a), p_2^{(u)}(a), p_d^{(u)}(a)) - c_1\left(D_1(p_1^{(u)}(a), p_2^{(u)}(a), p_d^{(u)}(a))\right).$$

Using that $p_k^{(u)}(a) < p_k^{(1)}(a)$ for $k = 2, d$, there exists $\tilde{p} > p_1^{(u)}(a)$ such that:

$$D_1(\tilde{p}, p_2^{(1)}(a), p_d^{(1)}(a)) = D_1(p_1^{(u)}(a), p_2^{(u)}(a), p_d^{(u)}(a)).$$

Then,

$$\pi_1^{(u)}(a) < (\tilde{p} - c_u)D_1(\tilde{p}, p_2^{(1)}(a), p_d^{(1)}(a)) - c_1\left(D_1(\tilde{p}, p_2^{(1)}(a), p_d^{(1)}(a))\right),$$

which is smaller than:

$$(\tilde{p} - c_u)D_1(\tilde{p}, p_2^{(1)}(a), p_d^{(1)}(a)) - c_1\left(D_1(\tilde{p}, p_2^{(1)}(a), p_d^{(1)}(a))\right) + (a - c_u)D_d(\tilde{p}, p_2^{(1)}(a), p_d^{(1)}(a))$$

since $a > c_u$. Finally we obtain by revealed preference that:

$$\pi_1^{(u)}(a) < (p_1^{(1)}(a) - c_u)D_1(p_1^{(1)}(a), p_2^{(1)}(a), p_d^{(1)}(a)) - c_1\left(D_1(p_1^{(1)}(a), p_2^{(1)}(a), p_d^{(1)}(a))\right) + (a - c_u)D_d(p_1^{(1)}(a), p_2^{(1)}(a), p_d^{(1)}(a)),$$

which is equal to $\pi_1^{(1)}(a)$. This concludes the proof.

## A.8 Proof of Proposition 7

As in section A.6, we prove a more general result, which gives a full characterization of subgame-perfect equilibria in the Hotelling case with quadratic costs and a pure upstream firm. Obviously, that result contains Proposition 7.

We make an economically meaningful assumption: Firm $u$ never makes an upstream offer below the upstream marginal cost. If such a strategy were allowed, we would get, for some values of $c$, subgame-perfect equilibria in which the pure upstream firm sets an upstream price $a_u < c_u$, and an integrated firm undercuts by setting $a_i < c_u$. The existence of the other subgame-perfect equilibria is, of course, not affected by that assumption. We now state our general result:

**Proposition 10.** *Consider the Hotelling case with quadratic costs and a pure upstream firm. Define the following thresholds:* $I_u(c,t) = \frac{8(c+t)(5c+4t)(21c+19t)}{3(51c+43t)^2}$ *and* $\underline{a}_u(c,t) = c_u + \frac{2(5c+4t)(6c+5t)}{51c+43t}$. *Denote by* $\hat{a}_u$ *the monopoly upstream price of firm* $u$. *Then:*

- *The competitive outcome is a subgame-perfect equilibrium.*
- *There exists an equilibrium in which the upstream market is supplied by an integrated firm at a price lower than* $c_u$ *if and only if* $-22t/27 \leq c < -3t/4$.
- *If* $-5t/6 < c \leq -53t/69$, *there exists a monopoly-like equilibrium, in which firm* $u$ *supplies the upstream market at price* $\hat{a}_u$, *while both integrated firms make no upstream offers.*
- *If* $-5t/6 < c \leq -4t/5$ *(respectively, if* $-4t/5 < c \leq -53t/69$ *and* $I \geq I_u(c,t)$*), there exists a continuum of equilibria, in which the pure upstream firm supplies the upstream market at* $a_u \in (c_u, \hat{a}_u)$ *(respectively,* $a_u \in [\underline{a}_u(c,t), \hat{a}_u)$*), while an integrated firm sets a slightly higher price.*

*These are the only subgame-perfect equilibria.*

*Proof.* The sketch of the proof is similar to the previous ones. To begin with, we compute the downstream equilibrium. We rule out some outcomes, which can obviously not be subgame-perfect equilibria. We then apply Proposition 9 to characterize matching-like equilibria. Proposition 6 allows us to focus on the case $c \in (-5t/6, -t/2]$, which proves more cumbersome.

**Downstream equilibrium.** We already know that all our assumptions are satisfied when the upstream market is supplied by an integrated firm (see Appendix A.6). In the following, we assume that firm $u$ supplies the upstream market at price $a_u$.

For all $k \neq k'$ in $\{1, 2, d\}$, $\partial^2 \tilde{\pi}_k^{(u)}/\partial p_k \partial p'_k = (2c+t)/(2t^2)$. Therefore, as before, downstream prices are strategic complements if, and only if, $c \geq -t/2$.

The second partial derivative of a firm's profit with respect to its own price is equal to $-2(c+t)/t^2$, which is negative since $c > -5t/6$. Solving for the equation $\partial \tilde{\pi}_k^{(u)}/\partial p_k = 0$, we get a unique solution, which yields the following best-response functions:[48]

$$BR_i^{(u)}(p_j, p_d, a_u) = \frac{4ct + 6c_d t + 6c_u t + 2t^2 + (p_j + p_d)(6c + 3t)}{12(c+t)},$$
$$BR_d^{(u)}(p_1, p_2, a_u) = \frac{6a_u t + 4ct + 6c_d t + 2t^2 + (p_1 + p_2)(6c + 3t)}{12(c+t)}.$$

The (positive) slopes of these best-response functions are equal to $|(2c+t)/(4(c+t))|$. As before, they are smaller than 1 if, and only if, $c > -5t/6$.

Downstream equilibrium prices are given by:

$$p_1^{(u)}(a_u) = p_2^{(u)}(a_u) = \frac{1}{15}\left(3a_u + 10c + 15c_d + 12c_u + 5t + \frac{12c(a_u - c_u)}{6c + 5t}\right),$$
$$p_d^{(u)}(a_u) = \frac{1}{15}\left(9a_u + 10c + 15c_d + 6c_u + 5t - \frac{24c(a_u - c_u)}{6c + 5t}\right).$$

Substituting these expressions into the profit functions, we get:

$$\pi_1^{(u)}(a_u) = \pi_2^{(u)}(a_u) = \frac{(c+t)(6c + 3(a_u - c_u) + 5t)^2}{9(6c + 5t)^2},$$
$$\pi_d^{(u)}(a_u) = \frac{(c+t)(6c - 6(a_u - c_u) + 5t)^2}{9(6c + 5t)^2},$$
$$\pi_u^{(u)}(a_u) = \frac{(a_u - c_u)(6c - 6(a_u - c_u) + 5t)}{3(6c + 5t)}.$$

Demands at downstream equilibrium are given by: $D_1^{(u)}(a_u) = D_2^{(u)}(a_u) = 1/3 + (a_u - c_u)(6c + 5t)$ and $D_d^{(u)}(a_u) = 1/3 - 2(a_u - c_u)/(6c + 5t)$.

[48] If the consumers' reservation utility is taken into account, these best-response functions are bounded. See footnote 44.

We can look for the strategy space for firm $u$, i.e., the set of upstream prices such that firm $d$ demands a positive quantity on the upstream market, and firm $u$ makes a positive profit. Straightforward calculations show that $a_u$ has to lie in $[c_u, c_u + c + 5t/6]$.

We finally check that the derivative of $\pi_d^{(u)}(.)$, equal to $(4(6a_u - 6(c + c_u) - 5t)(c + t))/(3(6c + 5t)^2)$, is negative inside the strategy space.

**Simple computation-free results.** In the following, we apply Propositions 6 and 9, and rule out some trivial outcomes.

First of all, we can state that the competitive outcome is always an equilibrium. Indeed, by Proposition 6, we know that if the upstream market is supplied at the upstream marginal cost, then, integrated firms do not want to undercut. Besides, it is obvious that firm $u$ does not want to undercut either, since it would then make negative profits.

A similar point can be made regarding super-competitive matching-like equilibria. If $c \in [-22t/27, -3t/4)$, there exists a subgame-perfect equilibrium in which both integrated firms set the same upstream price below $c_u$, and firm $u$ exits.

It is obvious that the upstream market cannot be supplied above $c_u$ by an integrated firm. Indeed, the pure upstream firm would undercut, which is possible, since both $\pi_d^{(u)}(.)$ and $\pi_d^{(i)}(.)$ are decreasing, and $\pi_d^{(u)}(c_u) = \pi_d^{(i)}(c_u)$.

If $c \geq -t/2$, downstream prices are strategic complements. Using Proposition 6, we deduce that there are no subgame-perfect equilibria in which the upstream market is supplied above the upstream marginal cost.

Now that these simple points have been made, all we need to do is investigate whether supra-competitive equilibria can exist when $-5t/6 < c < -t/2$.

**Supra-competitive equilibria under strategic substitutability.** Let us define $a_{um} = \arg\max_{a_u}\{\pi_u^{(u)}(a_u)\}$, the unconstrained monopoly upstream price of firm $u$. $\pi_u^{(u)}(.)$ being quadratic and concave, $a_{um}$ is well-defined and characterized by the first-order condition, which yields: $a_{um} = c_u + c/2 + 5t/12$. Since $\pi_d^{(u)}(.)$ is decreasing, we can also define an upstream price threshold $\bar{a}_u$, such that firm $d$ builds a network if, and only if $a > \bar{a}_u$. Solving the build or buy equation, we get:

$$\bar{a}_u = c_u + \left(c + \frac{5t}{6}\right)\left(1 - \sqrt{1 - \frac{9I}{c+t}}\right).$$

This allows us to express simply the constrained monopoly upstream price: $\hat{a}_u = \min\{\bar{a}_u, a_{um}\}$.

In the following, we assume that the upstream market is supplied by firm $u$ at price $a_u > c_u$, and investigate whether some firm wants to deviate.

Clearly, if $a_u$ were above $\hat{a}_u$, firm $u$ would rather deviate downwards and set $\hat{a}_u$. Hence, we assume in the following that $a_u \leq \hat{a}_u$.

We now look for the largest price firm $i$ can charge if it is willing to replace firm $u$ on the upstream market. Formally, we solve the equation $\pi_d^{(u)}(a_u) = \pi_d^{(i)}(a_i)$ in $a_i$. This equation has only one solution inside the strategy space: $a_i(a_u) = (4a_u - cu)/3$. One can check easily that $a_i(a_u) \leq a_m$. Therefore, if firm $i$ wants to supply the upstream market, its profit-maximizing strategy is to set $a_i(a_u)$.

Let us now see whether firm $i$ actually wants to replace firm $u$ on the upstream market. Formally, this involves comparing $\pi_i^{(i)}(a_i(a_u))$ and $\pi_i^{(u)}(a_u)$. After some algebra, we get:

$$\pi_i^{(i)}(a_i(a_u)) - \pi_i^{(u)}(a_u) = -\frac{(a_u - c_u)(-2(5c+4t)(6c+5t) + (a_u - c_u)(51c+43t))}{3(6c+5t)^2}.$$

The above expression is quadratic and concave in $a_u$. Therefore, it is negative outside its roots. These roots are $c_u$ and:

$$\underline{a}_u(c,t) = c_u + \frac{2(5c+4t)(6c+5t)}{51c+43t}.$$

Hence, $\pi_i^{(i)}(a_i(a_u)) \leq \pi_i^{(u)}(a_u)$ if, and only if $a_u \geq \max\{c_u, \underline{a}_u(c,t)\}$.

Simple calculations show that $\underline{a}_u(c,t) \leq c_u$ if, and only if, $c \in (-5t/6, -4t/5]$. In this case, integrated firms never want to replace the pure upstream firm on the upstream market. In particular, if firm $u$ sets its upstream monopoly price, integrated firms let it serve the upstream market: This is a monopoly-like equilibrium. Less interestingly, we also get a continuum of equilibria, in which some integrated firm sets a price $a_i > c_u$, and firm $u$ best-responds by setting $a_u = \min\{\hat{a}_u, a_u(a_i)\}$, with $a_u(a_i) = (c_u + 3a_i)/4$.

If $c > -4t/5$, then $\underline{a}_u(c,t)$ is above $c_u$. We then need to check whether or not $\underline{a}_u(c,t) > \bar{a}_u$. If so, there cannot exist any equilibria in which firm $u$ supplies the upstream market at a supra-competitive price.

First, if $c > -53t/69$, then $\underline{a}_u(c,t)$ is larger than $a_{um}$. In particular, $\underline{a}_u(c,t) > \hat{a}_u$. As we said before, if firm $u$ sets a price above $\hat{a}_u$, it wants to deviate and charge $\hat{a}_u$. But if it sets an upstream price below $\hat{a}_u$, integrated firms want to undercut. Therefore, when $c > -53t/69$, supra-competitive equilibria do not exist.

Second, if $-4t/5 < c \leq -53t/69$, then $\underline{a}_u(c,t) \leq \bar{a}_u$ if, and only if $I \leq I_u(c,t)$, with:

$$I_u(c,t) \equiv \frac{8(c+t)(5c+4t)(21c+19t)}{3(51c+43t)^2}.$$

We can now conclude: If $-4t/5 < c \leq -53t/69$ and $I \leq I_u(c,t)$, there exists a continuum of non-competitive equilibria, in which firm $u$ sets a price $a_u \in [\underline{a}_u(c,t), \hat{a}_u]$ and an integrated firm charges $a_i(a_u)$. In particular, there is a subgame-perfect equilibrium in which firm $u$ supplies the upstream market at its monopoly price $\hat{a}_u$. □

# References


Armstrong, M., 2002, "The Theory of Access Pricing and Interconnection", in *Handbook of telecommunications economics*, eds. M. Cave, S. Majumdar and I. Vogelsang, North-Holland, Amsterdam.

Bonanno, G. and J. Vickers, 1988, "Vertical Separation", *The Journal of Industrial Economics*, vol. 36(3), pp. 257-265.

Bourreau, M. and P. Dogan, 2006, "Build or Buy Strategies in Local Markets", *American Economic Review*, Papers and Proceedings, vol. 96(2), pp. 72-76.

Brito, D. and P. Pereira, 2006, "Access to Bottleneck Inputs under Oligopoly: A Prisoners' Dilemma?", Portuguese Competition Authority Working Paper 16.

Bulow, J., J. Geanakoplos, and P. Klemperer, 1985, "Multimarket Oligopoly: Strategic Substitutes and Complements", *The Journal of Political Economy*, vol. 93(3), pp. 488-511.

Chen, Y., 2001, "On Vertical Mergers and Their Competitive Effects", *The RAND Journal of Economics*, vol. 32(4), pp. 667-685.

De Bijl, P. and M. Peitz, 2002, *Regulation and entry into telecommunications markets*, Cambridge University Press.

European Commission, 2006a, "European Electronic Communications Regulation and Markets 2005 (11th Report)", Commission of the European Communities, COM(2006)68.

European Commission, 2006b, "Annex to the European Electronic Communications Regulation and Markets 2005 (11th report)", Commission Staff Working Document, Volume I.

European Regulators Group, 2005, "Bitstream Access. ERG Common Position - Adopted on 2nd April 2004 and amended on 25th May 2005", ERG (03) 33rev2.

Federal Communications Commission, 2005a, "High Speed Services for Internet Access, Status as of December 31, 2004".

Federal Communications Commission, 2005b, "Annual Report and Analysis of Competitive Market Conditions With Respect to Commercial Mobile Services", FCC 05-173.

Foros, O., 2004, "Strategic Investments with Spillovers, Vertical Integration and Foreclosure in the Broadband Access Market", *International Journal of Industrial Organization*, vol. 22(1), pp. 1-24.

Fudenberg, D. and J. Tirole, 1984, "The Fat-Cat Effect, the Puppy-Dog Ploy, and the Lean and Hungry Look", *The American Economic Review*, vol. 74(2), Papers and Proceedings of the Ninety-Sixth Annual Meeting of the American Economic Association, pp. 361-366.

Hart, O. and J. Tirole, 1990, "Vertical Integration and Market Foreclosure", *Brookings Papers on Economic Activity. Microeconomics*, vol. 1990, pp. 205-286.

Hazlett, T., 2005, "Rivalrous Telecommunications Networks With and Without Mandatory Sharing", Working Paper 05-07, AEI-Brookings Joint Center for Regulatory Studies.

Hausman, J. and G. Sidak, 2005, "Did Mandatory Unbundling Achieve its Purpose? Empirical Evidence from Five Countries", *Journal of Competition Law and Economics*, vol. 1(1), pp. 173-245.

Jehiel, P. and B. Moldovanu, 1996, "Strategic Nonparticipation", *The RAND Journal of Economics*, 27(1), pp. 84-98.

Jehiel, P., B. Moldovanu and E. Stacchetti, 1996, "How (Not) to Sell Nuclear Weapons", *The American Economic Review*, vol. 86(4), pp. 814-829.

Laffont, J.-J., Rey, P. and J. Tirole, 1998a, "Network Competition: I. Overview and Nondiscriminatory Pricing", *The RAND Journal of Economics*, vol. 29(1), pp. 1-37.

Laffont, J.-J., Rey, P. and J. Tirole, 1998b, "Network Competition: II. Price Discrimination", *The RAND Journal of Economics*, vol. 29(1), pp. 38-56

Laffont, J.-J., and J. Tirole, 2001, *Competition in telecommunications*, The MIT Press, Cambridge, Massachusetts.

Ordover, J., G. Saloner and S. Salop, 1990, "Equilibrium Vertical Foreclosure", *The American Economic Review*, vol. 80(1), pp. 127-142.

Ofcom, 2004, "Review of the Wholesale Broadband Access Markets, Final Explanatory Statement and Notification", 13 May 2004.

Ofcom, 2006, "Review of the Wholesale Broadband Access Markets", 2006/07.

Rey, P. and J. Tirole, 2005, "A Primer on Foreclosure", *Handbook of industrial organization*, M. Armstrong and R. Porter Eds, vol. III, North Holland.

Salinger, M., 1988,, "Vertical Mergers and Market Foreclosure", *The Quarterly Journal of Economics*, vol. 103-2), pp. 345-356.

Salop, S., 1979, "Monopolistic Competition with Outside Goods", *Bell Journal of Economics*, vol. 10(1), pp. 141-156.

Tirole, J., 1988, *The theory of industrial organization*, The MIT Press, Cambridge, Massachusetts.

Vives, X., 1990, *Oligopoly pricing: Old ideas and new tools*, The MIT Press, Cambridge, Massachusetts.

Woroch, G., 2004, "Open Access Rules and Equilibrium Broadband Deployment", in *Frontiers of Broadband, Electronic and Mobile Commerce*, Gary Madden and Russel Cooper eds., Physica-Verlag.