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STRATEGIC MERGER WAVES: A THEORY OF MUSICAL CHAIRS

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ABSTRACT

Strategic Merger Waves: A Theory of Musical Chairs*

This paper proposes an explanation of merger waves based on the interaction between competitive pressure and irreversibility of mergers in an uncertain environment. A set of acquirers compete over time for scarce targets. At each point in time, an acquirer can either postpone a takeover attempt, or raid immediately. By postponing the takeover attempt, an acquirer may gain from more favourable future market conditions, but runs the risk of being preempted by rivals. First, a complete information model is considered, and it is shown that the above tradeoff leads to a continuum of subgame perfect equilibria in monotone strategies that are strictly Pareto ranked. All these equilibria share the feature that all acquirers rush simultaneously in merger waves. The model is then extended to a dynamic global game by introducing slightly noisy private information about merger profitability. This game is shown to have a unique Markov perfect Bayesian equilibrium in monotone strategies, and the timing of the merger wave can thus be predicted. Last, the comparative dynamics predictions of the model are related to stylized facts.

JEL Classification: C73, D92, G34 and L13

Keywords: dynamic global games, merger waves, preemption and real options games

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STRATEGIC MERGER WAVES: A THEORY OF MUSICAL CHAIRS

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February 2007

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1. INTRODUCTION

Mergers and acquisitions (M&A) come in waves, both economy-wide and industry-wide. In particular, the existence of economy-wide waves is a well documented and robust stylized fact. Moreover, at the disaggregate level, a number of studies show that aggregate waves tend to cluster in particular industries and that economy-wide waves stem directly from waves at the industry level.¹

Since billions of dollars of assets change hands during these waves, the economic importance of mergers and acquisitions is hard to ignore. Accordingly, a vast empirical literature

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¹Golbe and White (1988) and Weston, Chung and Hoag (1990) show the existence of economy-wide waves and survey a number of other studies yielding similar results. For industry versus aggregate waves, see e.g. Nelson (1959), Gort (1969), Mitchell and Mulherin (1996), Andrade and Stafford (2004), Andrade, Mitchell and Stafford (2001).

has sought to uncover the forces leading to mergers. The evidence suggests that macroeconomic variables play an important role in determining the timing of mergers. Specifically, merger activity is found to be highly procyclical, slightly leading the business cycle. Other research has documented a positive relation between merger activity and factors such as economy-wide dispersion in Tobin's q (Jovanovic and Rousseau, 2002) and industrial production (Gort, 1969 and Mitchell and Mulherin, 1996). On the other hand, the business and popular press often stress that managers take into account other managers' actions when deciding on if and when to merge.

The aim of this paper is to build a theory that can explain why mergers happen in waves, incorporating both dependence on exogenous factors (such as aggregate activity) and strategic interdependence between firms' decisions.

Merger wave theories can be categorized according to whether or not they incorporate strategic elements. I will refer to them as *strategic* and *non-strategic* theories, respectively. Strategic theories of merger waves explicitly account for the mechanism through which one merger is related to the other. For example, the industrial organization literature has focused (almost exclusively) on strategic interaction through the product market.²

Non-strategic theories of merger waves emphasize the effects of exogenous factors such as deregulation, globalization or the introduction of new technologies. In this context, merger waves are characterized by the fact that it is not the merger activity of other firms *per se* that induce firms to merge, but rather an exogenous shift in the economic environment that simultaneously makes all mergers attractive.³ Contributions along these include Jovanovic and Rousseau (2002), Alvarez and Stenbacka (2005), Thijssen (2005), Smit et al. (2005), Mason and Weeds (2005), Rhodes-Kropf and Viswanathan (2004) and Toxvaerd (2005).

In practice, both strategic and non-strategic elements seem to play an important role in creating merger waves. Blair and Schary (1993) discuss these issues at length in the context of the 1980s merger wave and conclude that "...[the evidence] suggests a formal model of [merger] activity as a function of a set of macroeconomic and industry-specific conditions [...]. [Mergers are] triggered when those conditions reach some threshold point". Similarly, Mitchell and Mulherin (1996) study the influence of macroeconomic variables as well as industry specific shocks on the timing of M&A activity and conclude that "our results suggest that a fruitful research design would consider the joint effect of macroeconomic and industry-level factors in modeling the behavior of takeovers over time." Last, Cabral (2000) states that "real-world examples of [merger waves] in particular industries suggest that both exogenous and endogenous effects are present".

This calls for new theory that encompasses both features. In the present work, I propose a dynamic model of merger activity in which waves occur as an equilibrium phenomenon. An underlying economic fundamental influencing merger profitability is modeled as an exogenous stochastic process, but merger waves occur as a result of strategic interaction. A strategic merger wave in the current setting will be interpreted as a situation in which the exogenous economic conditions that would prompt a firm to seek a merger vary discontinuously with the merger activity of other firms. That is, given a set of exogenous economic conditions, only a strategic response to other firms' mergers will lead a firm to seek a merger itself.

The model builds on three simple ingredients. First, I pose that there is relative *scarcity* of potential desirable targets. This is a plausible assumption, given that there are often multiple suitors for specific targets. As a practical matter, there is usually no problem in distinguishing between potential targets and acquirers, where the identities of the acquirer

²See e.g. Fauli-Oller (2000) and Fridolfsson and Stennek (2005).

³Gort (1969) and Mitchell and Mulherin (1996) report evidence that M&A activity is significantly correlated with technological shocks and generally with disturbance to the economy or a specific industry.

and the target are determined by some notion of efficiency such as Tobin's q or some notion of size, e.g. capacity, market share or market capitalization. For example, in the world airline industry, there is a natural distinction between European and North American airlines. There is also a sense among the latter that potential European targets for takeovers or strategic alliances are scarce. Note that the interpretation of scarcity of targets need not be literal. An alternative interpretation is that the targets own or control scarce resources or assets. Such assets could be access to restricted (geographical) markets, existing customer bases, patents, business practices or as in the airlines example, landing slots in key European hubs.

The second ingredient driving my model is the recognition that mergers, viewed as investments, are partly irreversible and that they are carried out under conditions of considerable uncertainty. This leads to a *value of delay*, i.e. there is an *options value* in waiting to acquire a target, at least over a non-trivial period of time. Viewing an acquisition as an irreversible investment (or at least partially irreversible in the short run) is plausible; the merger decision can then reasonably be viewed as the problem of optimally exercising a real option. Delaying a merger may allow firms to look for the best fit; or it may be that the returns from the merger are realized in the future (when new markets are created), whereas implementation costs are borne immediately after the merger. Also, technological progress or convergence of hitherto separate industries may make it optimal not to merge straight away. Last, waiting may be valuable in resolving uncertainty.

The third ingredient of the present model is that competition for targets is imperfect. In general, imperfections in the price mechanism can arise because of private information, target management idiosyncrasies or agency problems. The existence of white knights and the fact that target management sometimes accepts offers that are not the most attractive, suggest that this is a plausible modeling assumption.

With these three assumptions in place, I first consider a complete information model where a measure of raiders compete over time for a smaller measure of targets. I show that there exists a continuum of subgame perfect equilibria. In all equilibria, *all potential acquiring firms raid the target firms simultaneously*, a feature that may be interpreted as a merger wave. The intuition for this type of equilibrium is simple. While waiting is optimal when all other firms wait (because of the value of delay), fear of being stranded without a firm to merge with can lead firms to attempt a preemptive takeover. This in turn vindicates the belief that there will be a merger wave, thus leading all firms to raid.

Although all equilibria share the same qualitative features, multiplicity is problematic since it is impossible to predict the timing of the mergers. To resolve the multiplicity, I extend the model by introducing incomplete information. This is achieved by letting acquirers receive slightly imperfect private information about the realizations of the randomly evolving economic fundamental variable. In this setting, it is shown that there exists a *unique perfect Markovian Bayesian equilibrium in monotone strategies*. The timing of the merger wave can thus be predicted and comparative analysis performed.

The analysis and results of this model explore the tradeoff between the value of delay and competitive considerations. Such a tradeoff between preemption (strategic/competitive considerations) and exogenous economic factors (non-strategic considerations) is often viewed as crucial for the decision of if and when to seek a merger by practitioners and industry participants alike. For example, the musical chairs metaphor is routinely used in the business press to describe the environment and conditions leading to merger waves. A commentator described an expected merger wave in the international industry for legal services by stating that "One of the images accountants like to use when describing the strategic thinking of law firms is that of an enormous and slightly lascivious game of musical chairs. The music is almost over and all the big Australian law firms are circling the room, trailing their coats in

the direction of a handful of global law firms and the Big Five professional services firms. If the Australians are lucky, the music might last just long enough for them to attract a merger partner [...]. But if they delay, all the international merger candidates will be snapped up by the lucky few [...].”

The remainder of the paper is organized as follows. The basic setup is described in Section 2, which also contains a discussion of the micro foundations of the model. In Section 3, the complete information version of the model is analyzed and merger wave equilibria are characterized. Section 4 extends the model to a dynamic global game by introducing incomplete information and shows that when information is very precise, the timing of the merger wave can be uniquely determined. Comparative analysis is performed in Section 5. Section 6 offers concluding remarks and discussion.

2. THE MODEL

Time is discrete and indexed by the non-negative integers $t = 0, 1, 2, \dots$. There is a continuum of *targets* and a continuum of *acquirers* with unit demand for a target. All acquirers are risk neutral and discount the future with the common factor $0 < \delta < 1$. In every period, each acquirer faces the choice between *raiding* and *waiting*. Let $a_t^i \in A_t^i = \{0, 1\}$ be player i 's action at time t , with $a_t^i = 0$ denoting *waiting* and $a_t^i = 1$ denoting *raiding*. An acquirer who waits, remains inactive until the next period.

Let x_t and y_t be the measures of remaining targets and acquirers respectively at time t and $z_t \in [0, y_t]$ be the measure of acquirers who choose to raid (*raiders*). Once an acquirer decides to raid, he participates in an *allocation game* with expected payoff $R(z_t, x_t, \theta_t)$ and remains inactive in all future periods. The stochastic variable $\theta_t \in \mathbb{R}$ represents some economic fundamental that influences merger profitability.

The expected payoff $R(z_t, x_t, \theta_t)$ from participating in the allocation game is called the *raiding value* and should be thought of as the expected value of obtaining, through some bidding process, an infinite flow of future profits. The current *waiting value* depends on the discounted expected future raiding and waiting values and can be expressed recursively as follows:

$$W(z_t, x_t, \theta_t) = \delta E_t \max [R(z_{t+1}, x_{t+1}, \theta_{t+1}), W(z_{t+1}, x_{t+1}, \theta_{t+1})]$$

Note that since an acquirer always has the option of waiting indefinitely, the waiting value in any period is non-negative. Finally, the *net waiting value* $\Delta(z_t, x_t, \theta_t)$ is defined as the difference between the waiting and raiding values, i.e. $\Delta(z_t, x_t, \theta_t) \equiv W(z_t, x_t, \theta_t) - R(z_t, x_t, \theta_t)$ and is simply the options value of deferring a takeover attempt. When the net waiting value is negative (positive), then waiting (raiding) is the dominant strategy. When it is zero, acquirers are indifferent between waiting and raiding.

Next, make the following assumptions:

- A1** The measure of initial acquirers is larger than the measure of initial targets, i.e. $y_0 > x_0$.
- A2** The raiding value $R(z_t, x_t, \theta_t)$ is bounded, continuous in all arguments, strictly increasing in θ_t , weakly decreasing in z_t and weakly increasing in x_t with $R(z_t, 0, \theta_t) = 0$.
- A3** The process $\{\theta_t\}_{t=0}^\infty$ is first-order Markov such that $\theta_t | \theta_{t-1} \sim G$, with density function g and for $\theta_{t-1} > \theta'_{t-1}$, $G(\theta_t | \theta'_{t-1}) > G(\theta_t | \theta_{t-1})$.
- A4** The raiding value and the stochastic process $\{\theta\}_{t=0}^\infty$ are such that for all z_{t+1}, x_{t+1}

$$R(z_t, x_t, \theta_t) - \delta E[R(z_{t+1}, x_{t+1}, \theta_{t+1}) | \theta_t]$$

is strictly increasing in θ_t .

- A5** A merger is irreversible, i.e. if $a_t^i = 1$ then $A_s^i = \emptyset$ for all $s > t$.

Assumption A1 captures the notion that targets are scarce. Assumption A2 ensures that both the raiding value and the waiting value are bounded and well behaved and furthermore that expected payoffs from raiding are decreasing in the intensity of competition (measured either as an increase in the measure of competitors z_t or as the absolute scarcity of remaining targets x_t). Last, the value of raiding is assumed to be increasing in the economic fundamental, such that higher realizations increase the benefits of merging, controlling for the level of competition. Assumption A3 states that there is persistence in the evolution of the economic fundamental, such that a higher realization of θ_t today shifts the distribution of future realizations in the sense of first-order stochastic dominance. Assumption A4 is a joint condition on the raiding value and the stochastic process which ensures that the raiding value increases at a higher rate than the waiting value. Last, assumption A5 simply states that a merger is irreversible.

Note that assumptions A2-A5, i.e. irreversibility and persistent shocks to merger profitability yield a value of delay, a standard insight of the real options literature. The requirement that the raiding value be strictly increasing in the economic fundamental imposes restrictions on the allocation mechanism. Specifically, it amounts to the assumption of market imperfections such that, controlling for the level of competition, the expected benefit from merging is non-negative for the raider.

It is implicitly assumed that no raider gets rationed in the allocation game as long as the measure of acquirers who chose to raid is smaller than the measure of targets. Thus, if an acquirer decides to raid, he will either merge with a target and leave the game, or be rationed, in which case the game is over. Thus, it follows that the laws of motion for the endogenous state variables are given by

$$\begin{aligned} x_t &\equiv \max \left\{ x_0 - \sum_{r=0}^{t-1} z_r, 0 \right\} \\ y_t &\equiv \max \left\{ y_0 - \sum_{r=0}^{t-1} z_r, y_0 - x_0 \right\} \end{aligned}$$

This couple of identities captures the main strategic element in the interaction between acquiring firms. The higher the measure of raiders in any given period, the scarcer targets become in future periods, thereby eroding the options value of waiting.

3. THE COMPLETE INFORMATION GAME

Before studying the details of the complete information game, some key concepts and properties of the model will be introduced. First, consider when acquirers will choose to raid. It turns out that there are levels of the economic fundamental such that the raiding behavior of acquirers is disconnected from strategic considerations. In particular, for low enough levels of the fundamental, raiders cannot break even, while for high enough levels of the fundamental, the options value of waiting is so low that waiting is suboptimal even when ignoring competitive pressure. In between these two extremes though, strategic considerations play a crucial role. In order to characterize the model, the extreme realizations just outlined will be properly defined.

Assume throughout that $x_t > 0$ and denote by $z^t \equiv \{z_s\}_{s=t}^\infty$ a sequence of current and future measures of raiders. Two such sequences z^t and \hat{z}^t are ordered as $z^t \geq \hat{z}^t$ if and only if $z_s^t \geq \hat{z}_s^t$ for all $s \geq t$. Define the following:

Definition 1. (*Merger Triggers*)

Marshallian Trigger: $\underline{\theta} \equiv \inf\{\theta_t : R(z_t, x_t, \theta_t) \geq 0\}$

Strategic Trigger. $\tilde{\theta}(z^t) \equiv \inf\{\theta_t : \Delta(z_t, x_t, \theta_t) \leq 0\}$

First-Best Trigger: $\bar{\theta}(z^t) \equiv \inf\{\theta_t : \Delta(z_t, x_t, \theta_t) \leq 0 \text{ for } z_s = 0, s \geq t\}$

That is, the Marshallian trigger $\underline{\theta}$ is the lowest value of θ_t at which the raiding value is non-negative. Next, the strategic trigger $\tilde{\theta}(z^t)$ is the lowest value of θ_t such that raiding in the current period dominates waiting, given a sequence of current and future measures of raiders. Last, the first-best trigger $\bar{\theta}(z^t)$ is the lowest value of θ_t such that, even in the absence of competitive pressure, delaying a takeover one period further is not optimal. The first-best trigger $\bar{\theta}(z^t)$ is simply the strategic trigger $\tilde{\theta}(z^t)$ evaluated at z^t with $z_s = 0, s \geq t$. Note that $\tilde{\theta}(z^t) \in [\underline{\theta}, \bar{\theta}(z^t)]$ and that $\tilde{\theta}(z^t) \rightarrow \bar{\theta}(z^t)$ as $x_t \rightarrow y_t$.

Next, I present two results which ensure that the above merger triggers exist and are unique.

Lemma 2. (State Monotonicity). For any sequence z^t , there exists a unique strategic trigger $\tilde{\theta}(z^t) \geq \underline{\theta}$ such that the net waiting value is (a) positive for $\theta_t < \tilde{\theta}(z^t)$, (b) equal to zero for $\theta_t = \tilde{\theta}(z^t)$ and (c) negative for $\theta_t > \tilde{\theta}(z^t)$. Furthermore, $\tilde{\theta}(z^t)$ is weakly increasing in x_t and weakly decreasing in z_t and z^t .

Lemma 3. (Strategic Complementarities). For any sequence z^{t+1} and $\theta_t \in [\underline{\theta}, \tilde{\theta}(z^{t+1})]$ there exists a unique $z_t^* \in [x_t, y_t]$ such that the net waiting value is (a) positive for $z_t < z_t^*$, (b) equal to zero for $z_t = z_t^*$ and (c) negative for $z_t > z_t^*$. Furthermore, z_t^* is weakly increasing in x_t and weakly decreasing in θ_t .

Proof: See Appendix A ■

These results have a straightforward interpretation. Lemma 2 shows that, given any level of future and present competition, the expected value for an acquirer contemplating whether to raid or wait increases in the economic fundamental. In fact, both the value of raiding and waiting increase. But as the economic fundamental increases, the options value of delay is eroded and ultimately the raiding value overtakes the value of waiting. Lemma 3 states that, given a level of the economic fundamental, an increase in the measure of raiders reduces the measure of future targets, thereby increasing the opportunity cost of postponing a merger.

The three triggers define four regions where θ_t can take values, namely $(-\infty, \underline{\theta})$, $[\underline{\theta}, \tilde{\theta}(z^t))$, $[\tilde{\theta}(z^t), \bar{\theta}(z^t))$ and $[\bar{\theta}(z^t), \infty)$. I discuss these in turn. First, for the outer regions $(-\infty, \underline{\theta})$ and $[\bar{\theta}(z^t), \infty)$, it is always optimal to wait or raid respectively and strategic considerations play no role. For the lower region $(-\infty, \underline{\theta})$, raiders cannot break even and thus waiting is a dominant strategy. The existence of the upper region $[\bar{\theta}(z^t), \infty)$ stems from the interaction between irreversibility and uncertainty. In particular, note that in the current analysis, the value of raiding is interpreted as a flow, which is a function not only of the current realization, but also of the future evolution of the economic fundamental. In other words, the value of being merged remains subject to random fluctuations in the economic environment. Since an acquisition is irreversible, the acquirer must be confident that the value of being merged is not likely to disappear or become negative. In order to make it worthwhile to risk being stuck with a loss making merger, the acquirer will demand a “premium” above the level of the economic fundamental which makes him break even. In other words, were the takeover decision fully reversible, acquirers would raid at the point at which they break even, i.e. when the fundamental is at or above $\underline{\theta}$ and simply undo the merger should future conditions render it unprofitable. With irreversibility however, the options value of delaying implies

that a takeover has an opportunity cost, since it effectively kills the option. Since, by the assumptions imposed on payoffs and the stochastic process, the raiding value increases in the economic fundamental at a higher rate than the waiting value, there exists a trigger such that raiding becomes a dominant action above this trigger level. The region $[\underline{\theta}, \bar{\theta}(z^t))$ is the standard real options hysteresis band (where competition is ignored), i.e. a range for the economic fundamental in which the acquirers are characterized by inertia. Last, once competition is taken into account, the hysteresis band shrinks to $[\underline{\theta}, \tilde{\theta}(z^t))$, since in the region $[\tilde{\theta}(z^t), \bar{\theta}(z^t))$ it is always optimal to raid. Within this new (reduced) hysteresis band, only a strategic response to competitive pressure will prompt an acquirer to raid. This reduction of the hysteresis band is a result of the following reasoning. Since competition increases future target scarcity, it decreases the value of delay, thereby effectively narrowing the region of inertia. As competition becomes less important, $\tilde{\theta}(z^t) \rightarrow \bar{\theta}(z^t)$.

For the complete information game, it is assumed that both past and current realizations of the economic fundamental θ_t are common knowledge. Let $h_t = (\theta_0, \dots, \theta_{t-1}; z_0, \dots, z_{t-1})$ denote history at time t and H_t the set of all possible histories at time t .

In this setting, a monotone Markovian strategy is defined as follows:

Definition 4. A monotone Markovian strategy for the complete information game is a mapping $a : \mathbb{R}^{2t} \times \mathbb{R} \rightarrow \mathbb{R}$ such that $(h_t, \theta_t) \mapsto a(h_t, \theta_t) \equiv k_t$.

Thus, for a history h_t and current fundamental θ_t , a strategy picks a real number k_t with the interpretation that an acquirer raids whenever $\theta_t \geq k_t$ and waits whenever $\theta_t < k_t$. Given a strategy k_t , the chosen action a_t will thus be

$$a_{k_t}(\theta_t) = \begin{cases} 1 & \text{for } \theta_t \geq k_t \\ 0 & \text{for } \theta_t < k_t \end{cases}$$

where 1 stands for *raid*, 0 stands for *wait* and $a_{k_t}(\theta_t)$ denotes an indicator function. With this definition in place, the following result can be stated:

Proposition 5. (Merger Waves) For any sequence z^{t+1} and history $h_t \in H_t$, any cutoff $k_t \in [\underline{\theta}, \tilde{\theta}(z^{t+1})]$ constitutes an equilibrium strategy. Furthermore, there exist no equilibria in asymmetric strategies.

Proof: The first part of the proposition follows immediately from the definitions and Lemma 3. To see the second part, fix a sequence of future cutoff strategies $\{k_s\}_{s=t+1}^\infty$ (and thus z^{t+1}) and consider period t . Let a measure μ^i , $i = a, b$ use strategies with cutoff k_t^i with $k_t^a < k_t^b$ and $\mu^a + \mu^b = y_t$. Recall that a cutoff k_t is only an equilibrium strategy if it is optimal for *any* realization of the economic fundamental θ_t . For $\theta_t \in [\underline{\theta}, k_t^a]$ all acquirers wait and the asymmetric strategies can coexist in equilibrium. Similarly, for $\theta_t \in [k_t^b, \bar{\theta}(z^{t+1})]$ all acquirers raid, which is also an equilibrium. Now consider the case where $\theta_t \in [k_t^a, k_t^b]$. Given the considered strategies, a realization in this range prompts a measure μ^a to raid and a measure μ^b to wait. If $\mu^a \geq z_t^*$, the cutoff k_t^b cannot be an equilibrium strategy. Similarly, if $\mu^b < z_t^*$, the cutoff k_t^a cannot be an equilibrium strategy. Thus equilibria in asymmetric strategies do not exist ■

The proposition states that there is a continuum of equilibrium cutoffs in each period. These are strictly Pareto ranked, with higher cutoffs dominating lower ones. It follows that even the Pareto efficient subgame perfect equilibrium is strictly inefficient, as it induces firms to merge at a suboptimally low level of the economic fundamental. Note that all these equilibria are also perfect equilibria. An immediate consequence of Proposition 5 is the following:

Corollary 6. (*Indeterminacy*) A merger wave may be triggered in any period t where $\theta_t \in [\underline{\theta}(x_t, 0), \bar{\theta}(z^{t+1})]$. Furthermore, a merger wave may be triggered no earlier than $\underline{t} \equiv \inf\{t : \theta_t \geq \underline{\theta}\}$ and no later than $\bar{t} \equiv \inf\{t : \theta_t \geq \bar{\theta}(z^t)\}$.

As the corollary suggests, there is no clear way in which to determine the equilibrium outcome. At this stage, the only thing that can be said is that \underline{t} is the earliest time and \bar{t} the very latest time at which a rush can occur, where \bar{t} is just the first point in time where the fundamental is above the first-best trigger and \underline{t} is the first point in time where the fundamental is above the Marshallian trigger. Actually, this claim is weaker than necessary, for there is indeed no equilibrium with acquirers rushing at time \bar{t} . Postponing a takeover this long is inconsistent with the pressure caused by any reasonable threat of preemption. An immediate result is thus that in all equilibria, a merger wave will happen in finite time with probability one. In other words, there is no equilibrium in which all acquirers postpone their takeovers indefinitely.

3.1. Aggregate Waves and Industry Waves. Before moving to the incomplete information game, the nature and interpretation of the model deserves some further discussion. As mentioned in the Introduction, merger waves at the aggregate level are a result of waves at the industry level. As Mitchell and Mulherin (1996) show, waves at the industry level tend to be much more intense than those at the economy level, who tend to be more drawn out. Arguably, the more prolonged economy-wide wave patterns are due to imperfect synchronization of the constituent industry merger waves. However, this should not lead to the conclusion that the underlying causes of merger activity is wholly disconnected across industries. In fact, it is entirely plausible that industries' responses to aggregate shocks and changes at the economy level are determined by industry-specific factors. In terms of the present model, this leaves two possible choices of interpretation. The first is to view it directly as a model of the aggregate economy. The second option is to view the model as one of mergers waves within a particular industry or sector of the economy. The model assumes that targets are somewhat interchangeable, suggesting the second interpretation as an industry model. If adopted, this view suggests a straightforward extension of the basic model into one of industry-wide merger waves. One can construct an economy with industries each characterized by the features of the present model. The underlying economic fundamental is determined at the aggregate level, but each industry is characterized by different degrees of target scarcity and possibly different shapes of reduced form payoff functions, reflecting basic differences such as the intensity and mode of product market competition, cost structures and such. If the industries are not too differentiated along these dimensions, aggregate merger activity will not be randomly distributed over time, but roughly follow those at the industry level. Furthermore, because of the inter-industry differentiation, aggregate waves will be more drawn out than those at the industry level, which in turn accords well with the industry-economy merger pattern emerging from the data.

3.2. Beyond the Reduced Form. The model presented here is based on a reduced form specification of acquirers' payoffs from raiding. In particular, it leaves unspecified two distinct features, namely the takeover process and the source of the gains from merger. While the main focus of this work is the phenomenon of merger waves, it is in order to place the present model and analysis in the context of the available evidence and literature. As I will argue below, many of the stylized facts on mergers and acquisitions previously identified in the literature fit well with the basic modeling assumption made here.

The Gains from Merger. Although mergers have been the object of research for more than half a century, there still seems to be little consensus on exactly what prompts

firms to seek mergers. Nevertheless, disparate strands of research can be brought together to create a consistent and plausible view of the causes of mergers. This view has economic shocks and disturbances to the economy and industries at its root and emphasizes the role played by efficiency and capital reallocation in determining the timing of takeovers as well as the identities of acquirers and targets.

One of the most well-established stylized facts about merger activity is that mergers are positively related to economic shocks (see e.g. Gort, 1969, Mitchell and Mulherin, 1996, Andrade et al., 2001 and Andrade and Stafford, 2004). Economic shocks, be they to technology, demand, relative input prices or optimal production scale, may render existing market organization suboptimal, thus prompting firms to seek to reorganize themselves with a view to reallocate assets and achieve efficiency. This motive for mergers underlies recent work that proposes growth models in which inefficient firms sell their assets to efficient firms through mergers and thereby put resources to their best use. This view of mergers has implications in terms of Tobin's q .⁴ Firms with inefficient use of assets should tend to have low levels of q , while efficient firms should tend to have relatively high levels of q . In fact, this is also reflected in the data and the respective levels of q are one of the best predictors of the identities of target and acquirer and also for the gains brought about by merger. Specifically, it has been shown that (i) acquiring firms usually have higher q than do targets and that high q firms are more likely to launch takeover attempts than low q firms and (ii) target firms have lower q than an industry control sample and the average industry level of q .⁵ Taken together, these findings are suggestive of a "high q buys low q " scenario. Fortunately, the match of high q acquirers and low q targets is found to create the largest surplus.⁶

Viewing merger activity as a means for low q firms to transfer assets to high q firms also suggests that a key determinant in creating merger activity is inter-firm dispersion of Tobin's q . Jovanovic and Rousseau (2002) show that it is indeed the case that increases in the dispersion of Tobin's q are positively related to merger activity, a finding which lends strong support to the q theory of mergers. In fact, they find that dispersion of Tobin's q leads mergers, an observation that ties in well with the fact that merger activity leads the overall business cycle.

As a last observation, consider the relation between mergers, the business cycle, demand and industrial production. Merger activity is found to be positively related to increases in industrial production. Maksimovic and Phillips (2001) find that inefficient firms are more likely to sell in times of economic expansion while efficient firms are more likely to buy in the same circumstances.⁷

Summing up, the evidence and existing literature suggest that economic disturbances create profitable reorganization opportunities by distorting the efficiency of firms' current use of assets, effectively driving a wedge between efficient firms and inefficient firms who can be characterized by their levels of Tobin's q . In terms of the present model, the potential acquirers may be regarded as the high q firms and the targets as the low q firms.

In light of the above stylized facts, the economic fundamental θ in the present model is best interpreted as aggregate shocks to the economy, such as technology or demand shocks.

⁴See Jovanovic and Rousseau (2002, 2004).

⁵Andrade et al. (2001) and Andrade and Stafford (2004).

⁶Lang, Stulz and Walkling (1989) find that in tender offers, bidder, target and total surplus are larger when high q acquirers take over low q targets (and vice versa). Servaes (1991) extends the analysis to a sample covering both tender offers and mergers and confirms this result. Last, Maksimovic and Phillips (2001) provide compelling evidence that efficient firms are usually the acquiring firms and inefficient ones the targets.

⁷For an explicit model with this type of reallocation which fits in the current framework after straightforward modification, see Maksimovic and Phillips (2002).

The important feature of these shocks is that they create opportunities for reallocation of assets across firms from inefficient ones to efficient ones.

The Takeover Process. Under Delaware law, target management must act “as an auctioneer charged with getting the best price for the stock-holders at a sale of the company”.⁸ In consequence, the theoretical literature on the microstructure of takeovers is largely based on auctions theory. This raises the question of the appropriate modeling of the auction environment. While some contributions adopt private values settings, the evidence seems to suggest that takeover battles are predominantly common values environments. This conclusion also sits well with the finding that broad industry or economy-wide factors influence the value and timing of mergers (see e.g. Cramton and Schwartz, 1991 for a discussion).

Indeed, the model presented here should be seen as such a common values environment, as the underlying economic fundamental determining merger profitability affects all potential acquirers in the same way. In practice, it is quite difficult to derive comparative statics results for common value auctions under general distributional assumptions. Three main assumptions on expected bidder returns are needed for the present analysis, namely that they decrease as more acquirers raid, that they decrease as targets become more scarce and that they increase as the value of mergers increases. The first two of these seem quite uncontroversial in common values auctions, while the third will require that further restrictions are imposed on the bidders’ valuation functions. To obtain sharp comparative statics results, some authors have resorted to numerical analysis of specific distribution functions.

The formal literature on takeover auctions is large and diverse, but share some key features. Of the more important in the current context are the following: (i) the number of bidders is usually exogenously fixed, (ii) they are reduced form in the sense that they abstract from the sources of the gains from takeovers, (iii) targets are passive in that they accept all offers above some exogenously determined level and (iv) targets are assumed scarce (i.e. there is one target and multiple bidders) and acquirers have unit demand. Some of these features are shared with the present model and thus deserve further discussion.

Competition and the Division of Surplus. An important feature of this model is that rents for bidders are not entirely dissipated by competition. There are a number of reasons why competition may not drive bidder returns to zero. First, asymmetric information may yield bids that are lower than the actual value of targets, as is standard in common values auctions. Second and keeping in mind a situation with dispersed ownership, there are several ways by which a raider may be able to expropriate some of the rents from a takeover and avoid the celebrated free rider problem. The most important reasons are toeholds and dilution, respectively. A bidding firm is said to have a toehold when it has acquired a fraction of the target’s shares prior to the takeover auction. Such toeholds may diminish the competitiveness of the auction by discouraging other bidders from aggressive bidding. Another way that the bidder may benefit from the takeover is by dilution of target shares. After the merger is consummated, bidder and target shares are swapped into shares of the new entity. If the bidder is allowed to determine the conversion ratio of target shares to new shares, he may effectively change the terms of trade in his own favour.

3.3. An Example. In order to make the results less abstract, a very stylized but explicit model is now presented that fits in the general framework presented so far. Consider a setting where two separate industries believe that at some uncertain point in the future, there will be demand of products whose production requires the participation of both industries. As an example, one may think of providers of media and content (e.g. AOL and Time-Warner).

⁸See Cramton and Schwartz (1991) and Cramton (1998) for more details on the auctions view of takeovers.

Let T denote the point in time where this new market opens and let V denote the profits accruing to a supplier in this market. Suppose that upon merging, the parties incur a fixed implementation cost $c > 0$. Discounted profits from a merger are thus given by $\delta^T V - c$. For $T \leq 0$ an immediate merger is optimal, while for T sufficiently large, a merger yields negative profits. The surplus created through the merger is thus strictly increasing and bounded in $\theta \equiv -T$. Last, assume that there is uncertainty about the date at which the market will open.

Turning to the allocation game, assume that a target facing a single bidder engages in some bargaining over the terms of the takeover. A target facing multiple bidders picks a single bidder with probability α and engages in bilateral bargaining.

With probability $1 - \alpha$, the target conducts an auction. In the auction, assume that target i receives N_i bids. The value for the target of the offer from bidder j is given by

$$U_{ij} = b_j + v_{ij}$$

where the idiosyncratic component is random and identically and independently distributed over i, j and satisfies standard assumptions of the probit model and b_j is bidder j 's bid. Underlying these preferences lie non-modeled factors such as the tastes of target management, differences in corporate culture etc. Bidder j seeks to maximize

$$P_{ij}(b_j) [\pi(\theta) - b_j]$$

where

$$P_{ij}(b_j) = \frac{\exp[b_j/\mu]}{\sum_{k=1}^{N_i} \exp[b_k/\mu]}$$

is the probability that bidder j wins the target and $\pi(\theta) = \delta^T V - c$. In symmetric equilibrium all bids are equal, leading to

$$b^* = \pi(\theta) - \frac{\mu N_i}{1 - N_i}$$

This in turn yields equilibrium payoffs given by

$$\frac{\mu}{N_i - 1}$$

The payoff from the auction is independent of the exact value of the target, an instance of the Bertrand trap. Denote by $\pi(z_t, x_t, \theta_t)$ the expected share of the surplus obtained by a raider in the bargaining game. I explicitly let this share depend on z_t and x_t as these may influence the relative bargaining powers. The expected payoff to a raider is $\pi(z_t, x_t, \theta_t)$ for $z_t \leq x_t$. For $z_t > x_t$, expected payoffs are given by

$$\frac{x_t}{z_t} \left[\alpha \pi(z_t, x_t, \theta_t) + (1 - \alpha) \left(\frac{\mu}{N_i - 1} \right) \right]$$

Summing up, the raiding value is given by

$$R(z_t, x_t, \theta_t) = I_{[0, x_t]}(z_t) \pi(z_t, x_t, \theta_t) + I_{[x_t, y_t]}(z_t) \frac{x_t}{z_t} \left[\alpha \pi(z_t, x_t, \theta_t) + (1 - \alpha) \left(\frac{\mu}{N_i - 1} \right) \right]$$

where $I_{[a, b]}(z_t)$ is the indicator function. This game is simple yet realistic and it is straightforward to show that it satisfies the assumptions made about $R(z_t, x_t, \theta_t)$.

4. THE INCOMPLETE INFORMATION GAME

In practice, assumptions of complete information and common knowledge seem hard to justify and one should in general expect at least some degree of informational differentiation. For this reason, I now enrich the model by assuming incomplete information. This assumption will have radical implications for the equilibrium set. Namely, it will be shown that once incomplete information is introduced, there exists a unique Markovian perfect Bayesian equilibrium in monotone strategies. The analysis exploits results developed by Morris and Shin (2003) for static games.

The model is extended to a dynamic global game by assuming that the realization of the fundamental variable θ_t is no longer common knowledge at time t . Recall that $G(\theta_t | \theta_{t-1})$ denotes the distribution of θ_t , conditional on past information and denote by g the corresponding probability density function. The next step is to specify the information technology, which is characterized by the following assumptions:

A6 At time t , acquirer i receives signal $s_{it} = \theta_t + \sigma \varepsilon_{it}$ with $\varepsilon_{it} \sim F$ (density f) identically and independently distributed over time and across acquirers.

A7 For $a > b$, $f(a - \theta)/f(b - \theta)$ is increasing in θ .

In Assumption A6, the scalar $\sigma > 0$ measures the precision of the signal. This assumption means that in each period, acquirers receive information on the current realization of the economic fundamental. Assumption A7 states that the distribution of noise F satisfies the monotone likelihood ratio property, i.e. an increase in any signal shifts the distribution of other signals in the sense of first-order stochastic dominance. This means that an acquirer who receives a high signal assigns a large probability to other acquirers receiving high signals too. This fact is important, as signals are not only used to estimate θ_t , but also to make inferences about other acquirers' signals. The monotone likelihood ratio property is implied by the assumption that the variables θ_t and $\{s_{it}\}_{i \in Y_t}$ are affiliated.

In this setting, define the following:

Definition 7. A monotone Markovian strategy for the incomplete information game is a mapping $a : \mathbb{R}^{2t} \times \mathbb{R} \rightarrow \mathbb{R}$ such that $(h_t, s_t) \mapsto a(h_t, s_t) \equiv k_t$.

Given a strategy k_t , the chosen action a_t will thus be given by

$$a_{k_t}(s_t) = \begin{cases} 1 & \text{for } s_t \geq k_t \\ 0 & \text{for } s_t < k_t \end{cases}$$

where again 1 denotes *raid* and 0 denotes *wait* and $a_{k_t}(s_t)$ denotes an indicator function. Thus, the choice variable is the cutoff level k_t . Given these strategies, for a given cutoff k_t , the measure of raiders is determined by the distribution of signals. But given θ_t , the distribution of a signal s is given by $F\left(\frac{s-\theta_t}{\sigma}\right)$. One can use this to express the measure of raiders as

$$z_t = y_t \left[1 - F\left(\frac{k_t - \theta_t}{\sigma}\right) \right]$$

Since the game is dynamic, the optimal action at any given point in time will depend on competitors' play in subsequent periods. Thus, players must forecast other players' actions, which in turn implies that players must forecast other players' forecasts. Finally, it is important to realize that history only serves to the extent that it yields a prior belief $G(\theta_t | \theta_{t-1})$ on θ_t . Past actions have no useful informational content and only feed through to the current decisions through their influence on the state variables x_t and y_t .

The following result shows that under the maintained assumptions, when information becomes very precise, there exists a unique Markovian perfect Bayesian equilibrium in monotone strategies.

Proposition 8. (*Uniqueness in Incomplete Information Game*) As $\sigma \rightarrow 0$, for each history $h_t \in H_t$ there exists a unique cutoff signal s_t^* such that the net waiting value is (a) positive for $s_t < s_t^*$, (b) equal to zero for $s_t = s_t^*$ and (c) negative for $s_t > s_t^*$.

Before giving the detailed proof of this proposition, I will first sketch the intuition of the basic steps. The idea of the proof is to first break up the dynamic game into a sequence of properly constructed static games. These static games simply correspond to the stages of the dynamic game, with the “waiting value” in the static game corresponding to the equilibrium continuation value in the dynamic game. Each of these static games are amenable to the techniques developed by Morris and Shin (2003). In order to follow this approach, the equilibrium continuation values must be well defined as a function of current data. That this is indeed the case is shown by actually constructing the equilibrium continuation value by induction and showing that they are well defined. One may then focus on a sequence of static games. The key to the proof of uniqueness at each stage is to note that as noise becomes negligible, an acquirer cannot, based on his signal, make inferences about the relative rank of his signal among the signals of the population of acquirers. This implies that the proportion of rivals choosing different actions is a uniformly distributed random variable. But given a uniform distribution of actions and the single crossing property of the acquirer’s net waiting value in θ_t , there is a unique signal at which indifference obtains. Once the existence of a unique indifferent acquirer has been established, the last step is to verify that for signals below the cutoff, waiting is optimal, while for signals above the cutoff, raiding is optimal. In the complete information game, it was proved that there are no equilibria in asymmetric strategies. This result carries over to the incomplete information game and thus one can restrict attention to symmetric cutoffs k_t .

The proof of Proposition 8 uses the following four-step procedure. First, the infinite horizon game is truncated to obtain a finite horizon game, denoted by $\Gamma(T) \equiv \{\Gamma_t\}_{t=0}^T$. Second, due to the recursive structure of $\Gamma(T)$, it is possible, for all t , to associate Γ_t with a simplified (*associated*) static game Γ_t^* , for which uniqueness can be shown by using the techniques developed for static games by Frankel, Morris and Pauzner (2003) and Morris and Shin (2003). This association is achieved by showing that in each period t , the function constituting the waiting value in Γ_t is well defined as a function of current information and actions. By solving the game backwards, the players are faced with an essentially static problem in each period. The third step is to show that the truncated (*underlying*) game $\Gamma(T)$ (where payoffs depend on the realization of the state and where history is informative about the distribution of this period’s realization) converges uniformly to the sequence of associated games $\Gamma^*(T) \equiv \{\Gamma_t^*\}_{t=0}^T$, as noise becomes negligible. This implies that the underlying truncated game $\Gamma(T)$ has a unique perfect Bayesian equilibrium in Markovian cutoff strategies. The last step is to show that equilibria of the truncated game converge as the horizon recedes so that equilibria of $\Gamma(T)$, as $T \rightarrow \infty$, coincide with the equilibria of the infinite horizon game $\Gamma(\infty)$. Using the terminology of Morris and Shin (2003), the problem to be solved in each period satisfies *action single crossing* (established in Lemma 3), *state monotonicity* (established in Lemma 2), *limit dominance* (follows from the existence of $\underline{\theta}$ and $\bar{\theta}(z^t)$) and a *monotone likelihood ratio property* of the signal distribution (Assumption A7). Under these and some continuity conditions, Morris and Shin (2003) show that there exists a unique Bayesian equilibrium in monotone strategies.

Before continuing with the analysis, I will state a straightforward lemma that determines the posterior distribution and density of θ given a signal s , that will be useful in the sequel.

Lemma 9. (*Posteriors*) The posterior density $f_{\theta|s}$ and distribution $F_{\theta|s}$ of θ given some

signal s are given by

$$f_{\theta|s}(\vartheta|s) = \frac{g(\vartheta)f\left(\frac{s-\vartheta}{\sigma}\right)}{\int_{-\infty}^{\infty} g(\theta)f\left(\frac{s-\theta}{\sigma}\right)d\theta} \quad (1)$$

$$F_{\theta|s}(\vartheta|s) = \frac{\int_{-\infty}^{\vartheta} g(\theta)f\left(\frac{s-\theta}{\sigma}\right)d\theta}{\int_{-\infty}^{\infty} g(\theta)f\left(\frac{s-\theta}{\sigma}\right)d\theta} = \frac{\int_{\frac{s-\vartheta}{\sigma}}^{\infty} g(s-\sigma u)f(u)du}{\int_{-\infty}^{\infty} g(s-\sigma u)f(u)du} \quad (2)$$

Proof: Omitted. See Toxvaerd (2004) ■

To formally state the proposition of the existence of a unique perfect Bayesian equilibrium in monotone strategies, assume for now that the waiting value is well defined as a function of current information and strategies. That this is indeed the case will be verified shortly. Consider a simplified associated game $\Gamma_t^*(T)$, where it is assumed that the received signal is a sufficient statistic of the state and θ is drawn from a uniform distribution on the real line. Throughout, an asterisk will denote quantities pertaining to the associated game. Although the prior of θ is an improper distribution (has infinite probability mass), it is possible to apply Lemma 9 by normalizing the prior density to one, i.e. $g(\theta) = 1$. The density of the posterior is then given by $f_{\theta|s}^*(\theta|s) = \sigma^{-1}f\left(\frac{s-\theta}{\sigma}\right)$ and the distribution by $F_{\theta|s}^*(\theta|s) = 1 - F\left(\frac{s-\theta}{\sigma}\right)$.

Let $\Delta_\sigma^*(s, k)$ denote the expected payoff gain to “waiting” with respect to the posterior after having received signal s and believing that all other players use strategies with cutoffs k .⁹ This is given by

$$\begin{aligned} \Delta_\sigma^*(s, k) &\equiv E_{\theta|s}^* \left[\Delta \left(y \left[1 - F \left(\frac{k-\theta}{\sigma} \right) \right], x, s \right) \right] \\ &= \int_{-\infty}^{\infty} \Delta \left(y \left[1 - F \left(\frac{k-\theta}{\sigma} \right) \right], x, s \right) \sigma^{-1} f \left(\frac{s-\theta}{\sigma} \right) d\theta \end{aligned} \quad (3)$$

For comparison, consider the underlying game at time t and denote by $\Delta_\sigma(s_t, k_t)$ the expected payoff gain to waiting when signal s_t has been observed and all other acquirers use cutoffs k_t . By Lemma 9, this is given by

$$\begin{aligned} \Delta_\sigma(s_t, k_t) &\equiv E_{\theta|s} \left[\Delta \left(y_t \left[1 - F \left(\frac{k_t-\theta_t}{\sigma} \right) \right], x_t, \theta_t \right) \right] \\ &= \frac{\int_{-\infty}^{\infty} \Delta \left(y_t \left(1 - F \left(\frac{k_t-\theta}{\sigma} \right), x_t, \theta \right) \right) g(\theta) f \left(\frac{s_t-\theta}{\sigma} \right) d\theta}{\int_{-\infty}^{\infty} g(\theta) f \left(\frac{s_t-\theta}{\sigma} \right) d\theta} \end{aligned} \quad (4)$$

The differences between (3) and (4) are twofold. First, in (3), the signal s replaces the economic fundamental θ . Second, the posterior distributions over the economic fundamental are generated by different prior beliefs. All other properties are shared.

Assume for now that the functions $\Delta_\sigma(s_t, k_t)$ and $\Delta_\sigma^*(s_t, k_t)$ are well defined and that all players receiving identical signals would have identical beliefs about the exact shapes of the functions. With these definitions in place, the following lemmata needed for the proof of the uniqueness result can be stated:

Lemma 10. (*Uniqueness in Associated Game*) For any history $h_t \in H_t$, there exists a unique cutoff signal s_t^* in the associated static game Γ_t^* such that: $\Delta_\sigma^*(s_t^*, s_t^*) = 0$, $\Delta_\sigma^*(s_t, s_t^*) > 0$ for $s_t < s_t^*$ and $\Delta_\sigma^*(s_t, s_t^*) < 0$ for $s_t > s_t^*$.

⁹Since the associated game is essentially static, time subscripts are omitted for ease of notation.

Proof: To prove Lemma 10, two separate results need to be established. First, it is shown that there is a unique signal such that indifference obtains exactly when receiving signal $s_t = s_t^*$. Second, it is shown that for lower signals waiting is optimal, while for higher signals raiding is optimal.

First rewrite $\Delta_\sigma^*(s, k)$, by changing variables using $z = y [1 - F(\frac{s-\theta}{\sigma})]$:

$$\Delta_\sigma^*(s, k) = \int_0^y \Delta(z, x, k) y^{-1} dz$$

For $k = s$,

$$\Delta_\sigma^*(s, s) = \int_0^y \Delta(z, x, s) y^{-1} dz$$

In other words, the function $\Delta_\sigma^*(s, k)$ has been rewritten such that it is an integral over a uniform distribution of z over $[0, y]$. But generically, there is a unique s^* that solves

$$\int_0^y \Delta(z, x, s^*) y^{-1} dz = 0$$

Thus that there is exactly one cutoff signal s^* at which an agent is exactly indifferent between raiding and waiting. It now has to be verified that there exists an equilibrium where the agent raids whenever $s > s^*$ and waits whenever $s < s^*$. In order to do this, recall that the game displays *action single crossing* (follows from Lemma 3), *state monotonicity* (follows from Lemma 2) and that the noise distribution has the monotone likelihood ratio property (Assumption A7).

The expected payoff gain to waiting, given signal s , when all other players use cutoffs k is given by

$$\begin{aligned} \Delta_\sigma^*(s, k) &\equiv \int_{-\infty}^{\infty} \Delta\left(y\left(1 - F\left(\frac{k-\theta}{\sigma}\right)\right), x, s\right) \sigma^{-1} f\left(\frac{s-\theta}{\sigma}\right) d\theta \\ &= \int_{-\infty}^{\infty} \Delta(y[1 - F(-m)], x, s) f\left(\frac{s-k}{\sigma} - m\right) dm \end{aligned}$$

by changing variables so that $m = \sigma^{-1}(\theta - k)$. Now rewrite the above expression as

$$\begin{aligned} \Delta_\sigma^*(s, k) &= \tilde{\Delta}(s, k, s') \\ &\equiv \int_{-\infty}^{\infty} \gamma(m, s') \varphi(s, m) dm \end{aligned}$$

where

$$\begin{aligned} \gamma(m, s') &= \Delta(1 - F(-m), x, s') \\ \varphi(s, m) &= f\left(\frac{s-k}{\sigma} - m\right) \end{aligned}$$

Because of the monotone likelihood ratio property, $\tilde{\Delta}(\cdot, k, s')$ preserves the single crossing property of $\Delta(z, x, \theta)$ by a result by Athey (2002). That is, there exists $s^*(k, s')$ such that

$$\begin{aligned} \tilde{\Delta}(s, k, s') &< 0 \quad \text{if } s > s^*(k, s') \\ \tilde{\Delta}(s, k, s') &> 0 \quad \text{if } s < s^*(k, s') \end{aligned}$$

By state monotonicity, $\tilde{\Delta}(s, k, s')$ is strictly decreasing in s' . Now let $s > s'$ and suppose that

$$\tilde{\Delta}(s, k, s) = 0$$

It follows that

$$\tilde{\Delta}(s', k, s') > \tilde{\Delta}(s', k, s) > \tilde{\Delta}(s, k, s) = 0$$

where the first inequality comes from state monotonicity and the second comes from the action single crossing property. A symmetric argument holds for $s < s'$. This implies that there exists a best response function $\beta : \mathbb{R} \rightarrow \mathbb{R}$ such that

$$\begin{aligned}\Delta_\sigma^*(s, k) &< 0 & \text{if } s > \beta(k) \\ \Delta_\sigma^*(s, k) &= 0 & \text{if } s = \beta(k) \\ \Delta_\sigma^*(s, k) &> 0 & \text{if } s < \beta(k)\end{aligned}$$

But there exists a unique s^* that solves

$$\Delta_\sigma^*(s^*, s^*) = \int_0^y \Delta(z, x, s^*) y^{-1} dz = 0$$

Therefore $\beta(k) = k$. It has thus been shown that with a uniform prior, there exists a unique equilibrium in cutoff strategies such that

$$a_k(s) = \begin{cases} 1 & \text{if } s > s^* \\ 0 & \text{if } s < s^* \end{cases}$$

This proves Lemma 10 ■

Lemma 11. (*Limit Uniqueness in Underlying Game*) For any history $h_t \in H_t$, as $\sigma \rightarrow 0$, $\Delta_\sigma(s_t, s_t - \sigma\xi) \rightarrow \Delta_\sigma^*(s_t, s_t - \sigma\xi)$ uniformly.

Proof: It was shown in Lemma 10 that in the associated game with uniform prior and private values, there is a unique equilibrium sequence of cutoffs. What remains to be shown is that the game with general prior and private values comes arbitrarily close to the associated private values uniform priors game, as noise vanishes.

Recall that $\Delta_\sigma(s, k)$ is

$$\begin{aligned}\Delta_\sigma(s, k) &= E_{\theta|s} \left[\Delta \left(y \left[1 - F \left(\frac{k - \theta}{\sigma} \right) \right], x_t, \theta_t \right) \right] \\ &= \int_{-\infty}^{\infty} \Delta \left(y \left[1 - F \left(\frac{k - \theta}{\sigma} \right) \right], x_t, \theta \right) dF_{\theta|s}(\theta|s)\end{aligned}\tag{5}$$

where $F_{\theta|s}(\theta|s)$ is the posterior distribution. To do a change of variables using $z = y [1 - F(\frac{s-\theta}{\sigma})]$, first note that

$$\theta = k - \sigma F^{-1} \left(\frac{y-z}{y} \right)$$

Next, let $\Psi_\sigma(z; s, k)$ be the posterior distribution evaluated at this θ . From Lemma 9 it follows that

$$\Psi_\sigma(z; s, k) = F_{\theta|s} \left(k - \sigma F^{-1} \left(\frac{y-z}{y} \right) | s \right) = \frac{\int_{\frac{s-k}{\sigma} + F^{-1}(\frac{y-z}{y})}^{\infty} g(s - \sigma u) f(u) du}{\int_{-\infty}^{\infty} g(s - \sigma u) f(u) du}$$

Therefore (5) becomes,

$$\Delta_\sigma(s, k) = \int_0^y \Delta\left(z, x_t, k - \sigma F^{-1}\left(\frac{y-z}{y}\right)\right) d\Psi_\sigma(z; s, k)$$

Recall from the proof of Lemma 10 that for the associated game

$$\Delta_\sigma^*(s, k) = \int_0^y \Delta(z, x, k) y^{-1} dz = \int_0^y \Delta(z, x, s) d\Psi_\sigma^*(z; s, k)$$

where $\Psi_\sigma^*(z; s, k) = F_{\theta|s}^*\left(k - \sigma F^{-1}\left(\frac{y-z}{y}\right) | s\right) = 1 - F\left(\frac{s-k}{\sigma} + F^{-1}\left(\frac{y-z}{y}\right)\right)$. Thus $\Psi_\sigma^*(z; s, s) = \frac{y-z}{y}$, which is the distribution function of the uniform distribution on $[0, y]$.

Returning to $\Psi_\sigma(z; s, k)$, note that for some small $|\xi|$

$$\Psi_\sigma(z; s, s - \sigma\xi) = \frac{\int_{\xi+F^{-1}\left(\frac{y-z}{y}\right)}^{\infty} g(s - \sigma u) f(u) du}{\int_{-\infty}^{\infty} g(s - \sigma u) f(u) du} \xrightarrow{\sigma \rightarrow 0} 1 - F\left(\xi + F^{-1}\left(\frac{y-z}{y}\right)\right) = \Psi_\sigma^*(z; s, s - \sigma\xi)$$

Therefore, $\Delta_\sigma(s, s - \sigma\xi) \rightarrow \Delta_\sigma^*(s, s - \sigma\xi)$ continuously as $\sigma \rightarrow 0$.

What remains to be shown is that $\Delta_\sigma(s, s - \sigma\xi) \rightarrow \Delta_\sigma^*(s, s - \sigma\xi)$ uniformly as $\sigma \rightarrow 0$. In other words, one must ensure that the equivalence of the two games is not a result of a discontinuity at $\sigma = 0$. Instead of showing uniform convergence directly, I will proceed by showing convergence with respect to the uniform convergence norm. Convergence in this norm implies uniform convergence. First, note that there exist extreme signals \underline{s} and \bar{s} such that for all k : $\Delta_\sigma(s, k) > 0$ for $s < \underline{s}$ and $\Delta_\sigma(s, k) < 0$ for $s > \bar{s}$. This follows from the existence of dominance regions $[-\infty, \underline{\theta}]$ and $[\bar{\theta}, \infty]$, where there is a unique optimal action. One can thus pick any pair \underline{s} and \bar{s} such that $\underline{s} < \underline{\theta}$ and $\bar{s} > \bar{\theta}$, and restrict attention to the compact interval $S \equiv [\underline{s}, \bar{s}]$. Since S is compact and the second argument of the Δ_σ function is continuous with respect to s (i.e. the function $s - \sigma\xi$), the set $K \equiv [\underline{s} - \sigma\xi, \bar{s} - \sigma\xi]$ is also compact. Hence $\Delta_\sigma(s, k)$ takes values in a compact set. Next, define the sup-norm (or *uniform convergence norm*)

$$\|\Delta\| \equiv \sup_{s, k} \{|\Delta(s, k)|\}$$

It has to be shown that $\Delta_\sigma(s, k)$ is continuous in the uniform convergence topology. I start by showing continuity of $\Delta_\sigma(s, k)$ with respect to the Euclidean metric. Fix s', k' . Since the function is continuous in both arguments, it follows that

$$\begin{aligned} \forall \varepsilon_1 > 0, \exists \delta_1 : |s - s'| < \delta_1 \Rightarrow |\Delta_\sigma(s, k) - \Delta_\sigma(s', k)| < \varepsilon_1 \forall k \\ \forall \varepsilon_2 > 0, \exists \delta_2 : |k - k'| < \delta_2 \Rightarrow |\Delta_\sigma(s, k) - \Delta_\sigma(s, k')| < \varepsilon_2 \forall s \end{aligned}$$

This in turn implies that

$$\sqrt{(s - s')^2 + (k - k')^2} < \bar{\delta} \equiv \sqrt{\delta_1^2 + \delta_2^2}$$

But then by the triangle inequality it follows that

$$\begin{aligned} |\Delta_\sigma(s, k) - \Delta_\sigma(s', k')| &= |\Delta_\sigma(s, k) - \Delta_\sigma(s', k) + \Delta_\sigma(s', k) - \Delta_\sigma(s', k')| \\ &\leq |\Delta_\sigma(s, k) - \Delta_\sigma(s', k)| + |\Delta_\sigma(s', k) - \Delta_\sigma(s', k')| \\ &\leq \varepsilon_1 + \varepsilon_2 \equiv \varepsilon \end{aligned}$$

and continuity with respect to the Euclidean metric follows. Denoting by $\mathbf{C}(S \times K)$ the space of continuous functions on $S \times K$, it follows that $\Delta_\sigma(s, k) \in \mathbf{C}(S \times K)$. But showing uniform convergence is equivalent to showing that as $\sigma \rightarrow 0$,

$$\|\Delta_\sigma - \Delta_\sigma^*\| = \sup_{s,k} \{|\Delta_\sigma(s, k) - \Delta_\sigma^*(s, k)|\} \rightarrow 0$$

with respect to the sup-norm. By substituting for the relevant functions and taking limits, the result follows ■

Lemma 10 states that any static associated game has a unique Bayesian equilibrium. Lemma 11 shows that when private information is very precise, any finite horizon version of the underlying game becomes arbitrarily close to a sequence of simplified static associated games. In other words, as $\sigma \rightarrow 0$, the period t expected relative payoff function in the underlying game, converges uniformly to the relative payoff function of some simplified static game which has a unique Bayesian equilibrium in monotone strategies. Having established these results, the sought proof follows:

Proof of Proposition 8: It is first established that $\Delta_\sigma(s_t, k_t)$ and $\Delta_\sigma^*(s_t, k_t)$ are well defined. Consider the truncated game, where play is exogenously terminated after some period T . At time T , optimality dictates that remaining acquirers raid for all signals that convince them of receiving a non-negative payoff. Note that this is irrespective of what other players do (i.e. independent of z_T). Now consider the (possibly trivial) decision at time $T - 1$. Because equilibrium actions are well defined (and unique) at time T , the expected waiting value at time $T - 1$ is well defined. But then, so is the expected net waiting value $\Delta_\sigma(s_{T-1}, k_{T-1})$. The problem to be solved at time $T - 1$ is essentially a static game as the one considered in Lemma 10 and thus there exists a unique equilibrium with cutoff s_{T-1}^* . Having established uniqueness at time $T - 1$, assume that at time $\tau < T - 1$ there exists a unique sequence $\{s_t^*\}_{t=\tau+1}^T$ of equilibrium cutoffs. With this inductive assumption, the next step is to show that at time $\tau - 1$ there exists a unique sequence of equilibrium cutoffs $\{s_t^*\}_{t=\tau}^\infty$. To see this, recall that the expected payoff gain from waiting is given by $\Delta_\sigma(s_\tau, k_\tau)$. This function shares all the properties of the function $\Delta_\sigma(s_{T-1}, k_{T-1})$ and thus there exists a unique equilibrium in monotone strategies with cutoff s_τ^* . Having shown uniqueness for arbitrary finite horizon version of the model, the infinite horizon game is considered. First note that the optimal strategy at any point in time optimally trades off the value of waiting with the value of raiding, i.e. the function $\Delta_\sigma(s_t, k_t)$. Clearly, $\Delta_\sigma(s_t, k_t)$ converges to a unique limit as $T \rightarrow \infty$, since both the value of raiding and that of waiting are bounded monotone functions of T . But then $s_t^*(T) \rightarrow s_t^*(\infty)$ as $T \rightarrow \infty$, where $s_t^*(T)$ is the equilibrium cutoff in period t in the game truncated after period T and $s_t^*(\infty)$ is the equilibrium cutoff at time t in the infinite horizon game ■

An immediate result of proposition 8 is the following:

Corollary 12. (*Unique Timing of Merger Wave*) The merger wave will be triggered at time $t^* \equiv \inf\{t : \theta_t \geq \theta_t^*\}$.

Recall that under complete information, there was a continuum of realizations of the economic fundamental which could constitute equilibrium cutoffs. The striking feature of Proposition 8 is that under incomplete but very precise information, there exists only one equilibrium cutoff θ^* (which is of course a function of the state variables).¹⁰ The cutoff θ^* will be referred to as the *risk-dominant trigger*. When an acquirer observes a signal equal to

¹⁰Of course $\theta^* = \underline{\theta}$ cannot be excluded, in which case the equilibrium is degenerate.

the risk-dominant trigger, he is exactly indifferent between raiding and waiting. For higher signals, waiting is too risky; for lower signals, waiting is expected to yield higher payoffs.

Continuing the discussion of the previous section, the fact that equilibrium is unique allows acquirers to incorporate the risk of preemption properly into their calculations, without having to resort to speculation about which equilibrium will be played. Thus, the risk-dominant trigger reflects the required return making an acquirer willing to raid, correctly adjusting this return for the riskiness of delaying a takeover attempt.

Before presenting the comparative analysis of the model, it is in order to place it in the context of a large literature on global games and games with strategic complementarities, some of which includes dynamic models. While this paper's main contribution is not to add to that literature per se, the model is not a trivial extension of it. The setup considered here is quite different from the existing global games literature in several respects. First, the tradeoff between waiting and raiding, which leads to strategic complementarities and state monotonicity, is not present in any of the static models, e.g. that of Morris and Shin (1998). The options value of delay, a particular feature of the present model, reflects the fact that mergers, like many other investments, are taken in an environment of continuing uncertainty. In the bulk of the dynamic global games literature, including Heidhues and Melissas (2006), Goldstein and Pauzner (2005), Angeletos et al. (2003) and Steiner (2005), randomness in the economic environment is only present at the start of the game. Other related work includes that of Abreu and Brunnermeier (2003) on booms and crashes in stock markets and that of Morris and Shin (1999) on currency attacks. In contrast to the present model, the latter model has no strategic intertemporal links while the former mainly focuses on learning.

5. COMPARATIVE ANALYSIS

Although the model presented here is reduced form, it still allows for an identification of factors influencing the timing of mergers. Figure 1 illustrates how the different triggers are ordered and a sample path of the economic fundamental. In this section I discuss the comparative dynamics of the model along three dimensions, which are presented in turn.

Measures of Targets and Acquirers. The first point of interest is the stocks of target firms x_t and acquirers y_t . Ceteris paribus, a smaller measure of targets increases scarcity, thereby eroding the options value of delaying a takeover. The effect of lower x_t is thus to shift both the strategic and the risk-dominant triggers downwards. In the extreme case where there are very few targets, one should expect an almost immediate rush, although this may not be identified empirically as a merger wave, since it involves very few takeovers. Furthermore, an increase in the measure of potential raiders y_t has the exact opposite effect as a decrease in x_t .

Evolution of Economic Fundamental. The evolution of the economic fundamental determining merger profitability has a direct implication for the expected timing of the wave. The higher the growth rate in the stochastic process, the less damaging it is for an acquirer who has already merged to be in the non-profitability region of the economic fundamental. Holding fixed the volatility of the process, a higher growth rate means that even if current conditions yield losses from being merged, the process will on expectation soon move into the profitable region. In turn, this narrows the region of inertia, leading to a decrease in the strategic triggers and shifts the risk-dominant trigger downwards. The model thus predicts that the higher the growth rate of the economic fundamental, the earlier the wave occurs.

Volatility of Economic Fundamental. Next, consider the effects of the volatility of the process $\{\theta_t\}_{t=0}^{\infty}$. Recall that the options value of delaying a takeover attempt stems from the possibility of benefiting from very high future realizations, while being effectively shielded

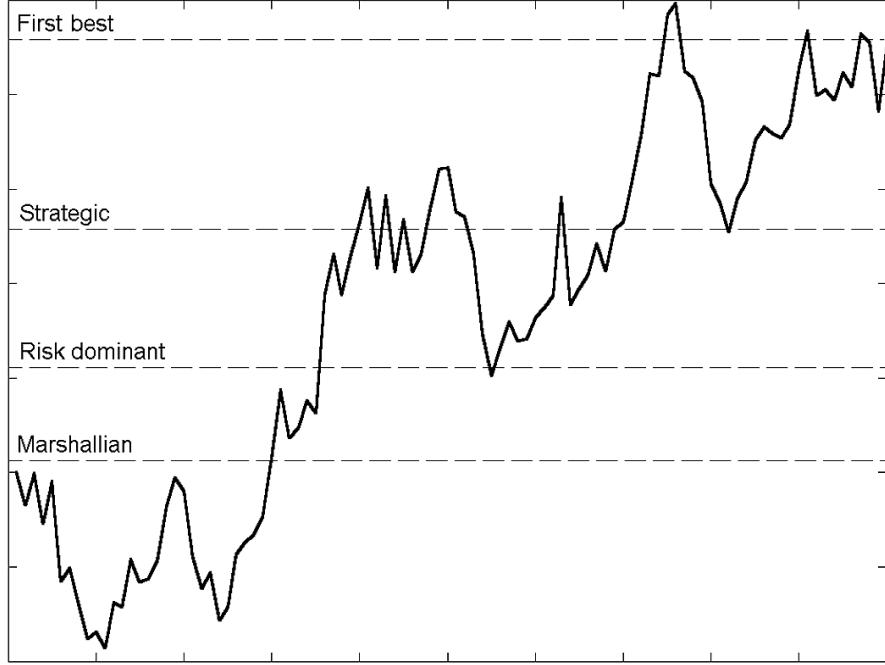


Figure 1: Triggers and sample path of θ_t .

from very low realizations. This suggests that the first-best trigger $\bar{\theta}(z^t)$ is increasing in the volatility of the process. While this is certainly the case in most real options models, it does not hold a priory in the current setting without further assumptions.

To obtain the result, assume that the raiding value $R(z_t, x_t, \theta_t)$ is linear in θ_t .¹¹ Then the expected waiting value is convex in θ_t and so is the expected net value of waiting. Next, consider a mean preserving spread of the distribution $G(\theta_t | \theta_{t-1})$. Such a spread increases the probability mass in the tails of the distribution. Since the options value of delaying a takeover attempt is convex, standard results show that the options value is increased by the spread in the distribution. Thus, an increase in volatility increases the opportunity cost of an immediate raid, thereby making it optimal to postpone it further. Similarly, with very low volatility there is no incentive to postpone a merger (supposing of course that $\theta_t \geq \underline{\theta}$ so that the raiders break even). Without imposing linearity of the raiding value, not much can be said since the net waiting value is not in general convex. This result suggests that there may be an empirically identifiable relation between economic volatility and the intensity in M&A activity. But given the strong assumption of linearity imposed on the model in order to obtain it, this possible relation should not be pressed too far.

While the effect of increased volatility just discussed is important, there is a second and somewhat subtler effect of increasing the level of uncertainty. Recall that knowledge of the

¹¹If $R(z_t, x_t, \theta_t)$ is linear, boundedness can be achieved by restricting the state space to some bounded set $\Theta \subset \mathbb{R}$ which includes both the Marshallian and the first-best triggers. While the analysis has been carried out under the assumption that the economic fundamental takes values on the real line, parallel techniques can be applied to this reduced (but very large) state space.

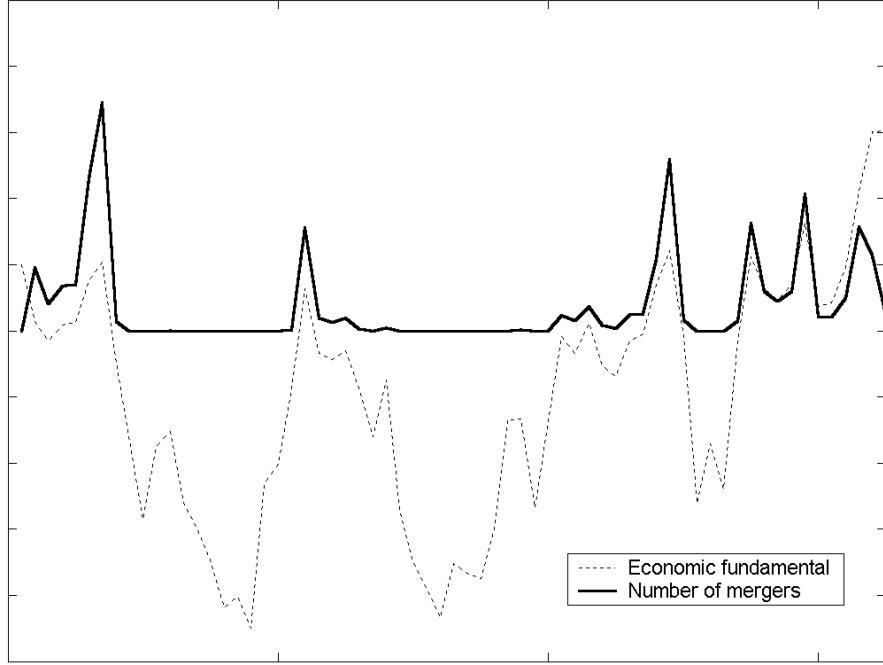


Figure 2: Simulated merger activity.

process $\{\theta_t\}_{t=0}^{\infty}$ has two uses, namely forecasting the future evolution and for generating a prior distribution $G(\theta_t | \theta_{t-1})$. The latter is influenced by the volatility of the stochastic process since it determines how informative the prior distribution is. Specifically, the more volatile the process $\{\theta_t\}_{t=0}^{\infty}$ is, the less precise is public information. This has the effect of weakening the requirement of signal precision needed for uniqueness. Morris and Shin (2003) show that with general Lipschitz continuous payoff functions, normal prior and normal noise, there exists a threshold of the relative informativeness of private and public information such that uniqueness obtains whenever the relative informativeness is lower than the threshold. Specifically, uniqueness obtains when new information is much more informative than history. Their results could be adapted and applied to the present model. Thus, in a very volatile environment, uniqueness of a perfect Bayesian equilibrium in monotone strategies should obtain even if there is significant noise in private information. In turn, increased noise in private information increases the probability of dispersed equilibria, in which not all acquirers raid simultaneously. If a small measure of acquirers raid in a given period, the unique merger trigger in the proceeding period decreases, in turn making it more likely that some acquirers will receive signals above the new (and now lower) trigger. In this way, the wave may gain momentum and ultimately result in a rush to merge.

Figure 2 illustrates simulated merger activity and a sample path of the economic fundamental with non-zero noise in private information. As is apparent from the graph, equilibria of this model generate distinct peaks in merger activity that resemble those observed in practice. For very low realizations of the fundamental, there is no merger activity. As the fundamental increases, merger activity picks up. Note that towards the end, merger activ-

ity becomes insensitive to increases in the economic fundamental. This is simply because by then, there are not many remaining targets. This is consistent with the observations of Weston et al. (1990), who note that “The fact that mergers peak before overall economic activity may reflect that there is at any one time a pool of firms suitable for acquisitions and, as they are acquired in a period of high merger activity, the pool is diminished and merger activity returns to a low level.”

6. DISCUSSION

As merger waves still constitute one of the most important unresolved puzzles of finance, this paper set forth a theory to explain the occurrence of merger waves. Merger waves were derived as an equilibrium phenomenon, in a simple timing game. The basic forces leading to merger waves in the current work is the interaction of, on one hand, an options value of delaying a takeover, caused by irreversibility and uncertainty and on the other hand, the risk of preemption by rivals, caused by a relative scarcity of desirable takeover targets. While the emphasis on this tradeoff clearly does not do justice to the multitude of factors influencing actual mergers, it does capture effects that are present in many mergers and merger waves. A simple extension of the model into one of incomplete information allowed the exact timing of the merger wave to be determined. Moreover, comparative dynamics analysis showed that numerous factors affect the timing of the merger wave, such as the volatility of the economic environment, the growth rate of the underlying economic forces influencing merger profitability, the competitiveness of the bidding game played by would-be acquirers and last, the effects of the sensitivity of the benefits from merger to changes in target scarcity and changes in the economic fundamental.

The presented model has several satisfactory features. First, it builds on simple and intuitive forces which are consistent with both empirical evidence, casual observation and received theory on many features of mergers. Furthermore, the identified tradeoff leading to waves is consistent with the thinking of practitioners, as evidenced from numerous public statements on specific merger waves. I believe this to be a desirable feature of a model built to understand actual behavior.

Importantly, the present model encompasses both dependence of the merger decision on macroeconomic variables and strategic considerations. This dual dependence has been repeatedly identified by empirical studies, but existing work has emphasized either purely strategic motives or purely non-strategic ones.

While the presented model is reduced form, the basic structure imposed on it does reflect empirically plausible forces. Furthermore, the predictions of the analysis are broadly consistent with the pattern of M&A activity which is in fact observed. First, mergers activity should pick up under beneficial economic conditions, consistent with the documented procyclicality of M&A activity. Second, because of competitive pressure, mergers happen earlier than suggested by pure profitability considerations. This suggests that merger activity should lead the business cycle. Again, this is consistent with the evidence. The model is also consistent with a particular pattern of single-bidder versus multiple-bidder takeover contests across the merger wave. In particular, it is consistent with multiple-bidder contests being concentrated at later stages of the merger wave, when competitive pressure becomes more important. This prediction does not seem to arise in existing work on merger waves. As such, this may serve to discriminate between theories in future empirical work.

APPENDICES

A. PROOF OF LEMMA 2 AND LEMMA 3

Consider the space of bounded functions Ω on $Y_t \times X_t \times \mathbb{R}$ and define the operator $M : \Omega[Y_t \times X_t \times \mathbb{R}] \rightarrow \Omega[Y_t \times X_t \times \mathbb{R}]$ by

$$MV(z_t, x_t, \theta_t) = \max\{R(z_t, x_t, \theta_t), \delta E[V(z_{t+1}, x_{t+1}, \theta_{t+1}) | \theta_t]\}$$

Fix a sequence of strategies, and by implication a sequence z^t . It will now be shown that for each t , M is a contraction mapping on the space $\Omega[Y_t \times X_t \times \mathbb{R}]$ with the sup-norm. With this norm, the space Ω is a Banach space. Let $V(z_t, x_t, \theta_t) > \hat{V}(z_t, x_t, \theta_t)$ for all (z_t, x_t, θ_t) . Then

$$\begin{aligned} MV(z_t, x_t, \theta_t) &= \max\{R(z_t, x_t, \theta_t), \delta E[V(z_{t+1}, x_{t+1}, \theta_{t+1}) | \theta_t]\} \\ &\geq \max\{R(z_t, x_t, \theta_t), \delta E[\hat{V}(z_{t+1}, x_{t+1}, \theta_{t+1}) | \theta_t]\} \\ &= M\hat{V}(z_t, x_t, \theta_t) \end{aligned}$$

Thus the mapping M satisfies monotonicity. Next, let $a > 0$. Thus

$$\begin{aligned} M[V(z_t, x_t, \theta_t) + a] &= \max\{R(z_t, x_t, \theta_t), \delta E[V(z_{t+1}, x_{t+1}, \theta_{t+1}) + a | \theta_t]\} \\ &= \max\{R(z_t, x_t, \theta_t), \delta E[V(z_{t+1}, x_{t+1}, \theta_{t+1}) | \theta_t] + \delta a\} \\ &\leq MV(z_t, x_t, \theta_t) + \delta a \end{aligned}$$

and the mapping M satisfies discounting. Therefore, by Blackwell's sufficiency conditions, M is a contraction mapping (with modulus δ) on $\Omega[Y_t \times X_t \times \mathbb{R}]$. Since by assumption $R(z_t, x_t, \theta_t)$ is bounded and continuous in all arguments, it follows by the contraction mapping theorem that there exists a unique fixed point $V(z_t, x_t, \theta_t)$ such that $MV(z_t, x_t, \theta_t) = V(z_t, x_t, \theta_t)$, and furthermore that this fixed point is bounded and continuous in (z_t, x_t, θ_t) (see e.g. Stokey and Lucas with Prescott, 1989 for details). Assume throughout that $x_t > 0$ and fix a sequence z^t .

Recall that by assumption $R(z_t, x_t, \theta_t)$ is strictly increasing in θ_t , weakly decreasing in z_t and weakly increasing in x_t . For $z_t < x_t$ it follows by the assumption of first-order stochastic dominance that the fixed point $V(z_t, x_t, \theta_t)$ is strictly increasing in θ_t . Also, for $\theta_t > \underline{\theta}$, $V(z_t, x_t, \theta_t)$ is weakly decreasing in z_t and weakly increasing in x_t .

Recall that $V(z_t, x_t, \theta_t) \geq 0$ for all θ_t . That is, an acquirer can always secure himself a payoff of zero by waiting indefinitely. On the other hand, $R(z_t, x_t, \theta_t) < 0$ for $\theta_t < \underline{\theta}$ which implies that in this range of the economic fundamental it is optimal to wait, i.e. $V(z_t, x_t, \theta_t) = \delta E[V(z_{t+1}, x_{t+1}, \theta_{t+1}) | \theta_t]$. Now let $\theta_t \geq \underline{\theta}$ and consider an increase in θ_t . Both the value of raiding and that of waiting will increase. A simple argument shows that for sufficiently high θ_t , the value of raiding overtakes that of waiting. Assume that for all θ_t

$$\delta E[V(z_{t+1}, x_{t+1}, \theta_{t+1}) | \theta_t] \geq R(z_t, x_t, \theta_t) > 0$$

Specifically, this implies

$$\sup_{\theta_t} V(z_t, x_t, \theta_t) = \sup_{\theta_t} \delta E[V(z_{t+1}, x_{t+1}, \theta_{t+1}) | \theta_t]$$

which contradicts $\delta \in]0, 1[$. Assumption A4, i.e. the assumption that

$$R(z_t, x_t, \theta_t) - \delta E[R(z_{t+1}, x_{t+1}, \theta_{t+1}) | \theta_t]$$

is strictly increasing in θ_t , ensures that there is a unique crossing since it implies that the value of raiding increases at a higher rate than the value of waiting.¹² In conclusion, for each sequence z^t there exists a unique finite $\tilde{\theta}(z^t) \in [\underline{\theta}, \infty[$ such that

$$R(z_t, x_t, \tilde{\theta}(z^t)) = \delta E[V(z_{t+1}, x_{t+1}, \theta_{t+1}) | \tilde{\theta}(z^t)]$$

Since $V(z_t, x_t, \theta_t)$ is weakly increasing in x_t , so is $\tilde{\theta}(z^t)$. Similarly, since $V(z_t, x_t, \theta_t)$ is weakly decreasing in z_t , $\tilde{\theta}(z^t)$ is weakly decreasing in z_t . This also holds for any future measure of raiders and thus $\tilde{\theta}(z^t)$ is also weakly decreasing in z^t . The first-best trigger $\bar{\theta}(z^t)$ is just the value of $\tilde{\theta}(z^t)$ for the sequence z^t with $z_s = 0$ for all $s \geq t$. Last, note that for all $\theta_t > \underline{\theta}$, and $z_t \geq x_t$, $R(z_t, x_t, \theta_t) > 0$ while $V(z_{t+1}, x_{t+1}, \theta_{t+1}) = 0$. Thus there exists a unique $z_t^* \in [x_t, y_t]$ such that

$$R(z_t^*, x_t, \theta_t) = \delta E[V(z_{t+1}, x_{t+1}, \theta_{t+1}) | \theta_t]$$

It follows from the discussion above that z_t^* is weakly decreasing in θ_t and weakly increasing in x_t ■

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¹²To see this, define the mapping $L\Lambda(z_t, x_t, \theta_t) = V(z_t, x_t, \theta_t) - R(z_t, x_t, \theta_t)$. Straightforward manipulation yields $L\Lambda(z_t, x_t, \theta_t) = \max \{0, -R(z_t, x_t, \theta_t) + \delta E[R(z_{t+1}, x_{t+1}, \theta_{t+1}) | \theta_t] + \delta E[\Lambda(z_{t+1}, x_{t+1}, \theta_{t+1}) | \theta_t]\}$. Under Assumption A3-A4, this defines a mapping L from $\Omega[Y_t \times X_t \times \mathbb{R}]$ into itself which is strictly increasing in θ_t . Furthermore, the mapping L is a contraction with a unique fixed point. Last, note that since $W(z_t, x_t, \theta_t) = \delta V(z_t, x_t, \theta_t)$, it follows that $\Lambda(z_t, x_t, \theta_t) = \max \{0, \Delta(z_t, x_t, \theta_t)\}$.

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