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**ASSET PRICING WITH LIMITED
RISK SHARING AND
HETEROGENEOUS AGENTS**

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ABSTRACT

Asset Pricing with Limited Risk Sharing and Heterogeneous Agents*

We solve a model with incomplete markets and heterogeneous agents that generates a large equity premium, while simultaneously matching stock market participation and individual asset holdings. The high risk premium is driven by incomplete risk sharing among stockholders, which results from the combination of aggregate uncertainty, borrowing constraints and a (realistically) calibrated life-cycle earnings profile subject to idiosyncratic shocks. We show that it is challenging to simultaneously match asset pricing moments and individual portfolio decisions, while limited participation has a negligible impact on the risk premium, contrary to the results of models where it is imposed exogenously.

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1 Introduction

We present an asset pricing model that closely matches aggregate asset pricing moments (mean and standard deviation of stock returns and T-bill returns), while simultaneously matching individual allocations (stock market participation rate and asset holdings). The key ingredients of the model are household heterogeneity and market incompleteness.

Households are heterogeneous along several different dimensions. First, they receive different uninsurable labor income shocks. Second we have a life-cycle model and therefore young agents, mid-life households and retirees all behave differently. Third, we introduce a fixed cost of stock market participation, and thus agents who have paid the cost have access to a larger investment opportunity set. Fourth, households have Epstein-Zin preferences (Epstein and Zin (1989)) and we consider heterogeneity in both risk aversion and elasticity of intertemporal substitution.

Market incompleteness results from both aggregate and (uninsurable) idiosyncratic shocks, combined with borrowing constraints. Idiosyncratic uncertainty is driven by a (realistically) calibrated life-cycle stochastic earnings profile that agents cannot fully insure, and against which they cannot borrow. The sources of aggregate uncertainty in the model include aggregate productivity shocks and shocks to the capital depreciation rate. Our baseline model yields a high risk premium (3.83%) and a low riskless rate (2.58%) with coefficients of relative risk aversion no larger than 5. The volatility of consumption growth, for both stockholders and non-stockholders, is also consistent with the data. Since we do not consider adjustment costs of capital, stochastic depreciation is crucial for delivering the right level of stock return volatility in the model. Without those shocks our economy would still have a high market price of risk, but a negligible equity premium. We find that the main determinant of the price of risk in the model are the borrowing constraints. Idiosyncratic labor income shocks play a smaller role, as expected from Lucas (1994) and Heaton and Lucas (2000). However, we argue that it is still important to include the labor income shocks, as otherwise wealth accumulation would be counterfactually low. We show that removing these shocks from the model would produce a highly biased calibration of the preference parameters.

Previous literature (e.g. Saito (1995), Basak and Cuoco (1998), or Guvenen (2005)) has

argued that (exogenous) limited stock market participation can help explain the equity risk premium puzzle.[1] In our model limited participation is derived endogenously and, consistent with the data, the non-participants are significantly less wealthy than households that own stocks.[2] Therefore, excluding those households from the equity market has a *negligible* impact on the risk premium. Only by assuming extremely high entry costs, or by imposing the participation constraint exogenously, would it be possible to exclude rich households and thus produce a significant increase in the risk premium.[3] Nevertheless, we argue that taking limited participation into account is important when calibrating asset pricing models. In the data, the consumption growth of non-stockholders has much lower volatility and it is much less correlated with stock returns than stockholders' consumption growth (Mankiw and Zeldes (1991) and Malloy, Moskowitz and Vissing-Jørgensen (2006)). In a model with limited stock market participation it is possible to calibrate these differences, and thus use the moments of stockholders' consumption growth to explain equilibrium asset returns. Models without limited participation must match aggregate consumption moments, and thus can only rationalize a lower equity premium. In summary, having a model that captures the heterogeneity between the consumption growth of rich and poor households (such as ours) is important for matching the equity premium. Whether the poor households are stockholders or non-stockholders (as they currently are in the data) does not matter, exactly because those households do not have much wealth.

We rationalize the modest participation rates observed in the data by introducing a small stock market entry cost. The incentive to pay this entry fee naturally increases with the household's financial wealth. The most important determinants of wealth accumulation in our life-cycle model are the retirement replacement ratio, the degree of uncertainty in income risk, the age-earnings profile, and the preference parameters. In the baseline model we only

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[1] The models in Heaton and Lucas (2000) and Polkovnichenko (2004) also have exogenous participation constraints, but obtain a smaller impact on the equity premium.

[2] According to the latest numbers from the Survey of Consumer Finances, the participation rate is 88.84% among households with wealth above the median, and only 15.21% for those with wealth below the median. The median wealth for stockholders is $154,600$, while the median wealth for non-stockholders is $7,300$.

[3] In Cao, Wang and Zhang (2006) limited participation, which can arise endogenously due to heterogeneity in uncertainty aversion, can actually *decrease* the equity premium.

consider heterogeneity in risk aversion (RA) and elasticity of intertemporal substitution (EIS). Households with very low RA and low EIS care less about hedging background risk and about saving for retirement, respectively. Therefore they smooth earnings shocks with a small buffer stock of assets, do not accumulate significant wealth and have a limited incentive to pay the fixed cost. On the other hand, investors with high RA and high EIS participate in the stock market from early on.

Vissing-Jørgensen (2002a), Brav, Constantinides and Geczy (2002) and Malloy, Moskowitz and Vissing-Jørgensen (2006) estimate preference parameters for both non-stockholders and stockholders using the Euler equations on their consumption and savings decisions. These Euler equations do not identify the risk aversion coefficient of non-stockholders because those households do not have a portfolio decision. The resulting estimates are therefore for elasticities of intertemporal substitution (EIS).[4] Consistent with our model, the estimated EIS is indeed lower for nonstockholders. With respect to the unobserved heterogeneity in risk aversion, we emphasize that this does not matter as long as they *endogenously* choose not to participate in the stock market. The only important feature for our results is that non-participants are significantly less wealthy than stockholders, as they are in the data. The specific mechanism for achieving this within the model is not crucial. More precisely, we show that the same exact *quantitative* conclusions are obtained in versions of the model where nonstockholders have the same risk aversion or even higher risk aversion than stockholders.

Asset-pricing models usually assume that the riskless asset exists in zero net supply.[5] We instead explicitly include government bonds that exist in positive net supply. In equilibrium, the zero-net-supply assumption implies that the representative consumer must invest all of her wealth in the risky asset. Furthermore, in models with limited stock market participation, this implies that a significant fraction of the population must hold levered positions in equities. Those predictions are in clear contrast with the empirical evidence on household-level asset holdings (see, for instance, Poterba and Samwick (2001), Ameriks and Zeldes (2001), or Guiso, Haliassos and Japelli (2002)).

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[4]Naturally, in a model with CRRA preferences these estimates also identify the risk aversion coefficient since this is just the inverse of the EIS, but with Epstein-Zin preferences that is no longer the case.

[5]Alternatively the riskless rate is exogenously specified, and the supply of bonds is perfectly elastic.

We quantify an important trade-off between consistency of the model with asset pricing moments, and its consistency with household portfolios. If the riskless asset exists in positive supply, as opposed to zero net supply, households require a higher riskless rate to induce them to hold bonds. In addition, their consumption volatility falls as a smaller fraction of their wealth is now invested in the risky security. These two effects lead to a lower equity premium. In fact, as we decrease the supply of the riskless asset towards zero, our model can more easily match the historical equity premium and a low riskless rate. Naturally, the household portfolio allocations in such a version of the model are highly inconsistent with the data.

Our paper is part of a large literature investigating the implications of heterogeneous agent models with incomplete markets and/or limited stock market participation for asset pricing.[6] We also build on the literature on asset allocation with undiversifiable labor income risk.[7] The closest papers to ours are Saito (1995), Basak and Cuoco (1998), Heaton and Lucas (2000), Constantinides, Donaldson and Mehra (2002), Guvenen (2005) and Storesletten, Telmer and Yaron (2006). Storesletten et al. (2006) and Constantinides, Donaldson and Mehra (2002) consider life-cycle models, but do not have participation costs, preference heterogeneity or retirement income (a social security system). We use the first two features to match the stock market participation rate, and the third to obtain a more accurate measure of households' earnings uncertainty. In particular, Storesletten et al. (2006) obtain a very high market price of risk in their economy, but they cannot generate a high risk premium without a counterfactually high volatility of consumption. This is even more problematic in their set-up since they do not have limited participation, and therefore must match aggregate

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[6]For instance, Aiyagari and Gertler (1991), Telmer (1993), Lucas (1994), He and Modest (1995), Heaton and Lucas (1996), Basak and Cuoco (1998), Luttmer (1999), Heaton and Lucas (2000), Abel (2001), Constantinides, Donaldson and Mehra (2002), Calvet, Gonzalez-Eiras, and Sodini (2004), Guvenen (2005), and Storesletten, Telmer and Yaron (2006). See also the empirical work by Brav, Constantinides and Geczy (2002) and Vissing-Jorgensen (2002b).

[7]See, for instance, Heaton and Lucas (1997), Bertaut and Haliassos (1997), Viceira (2001), Haliassos and Michaelides (2003), Gomes and Michaelides (2005), Cocco, Gomes and Maenhout (2005), Cocco (2005), Yao and Zhang (2005), Davis, Kubler and Willen (2006), Polkovnichenko (2006), and Benzoni, Collin-Dufresne and Goldstein (2006).

consumption volatility.[8] Heaton and Lucas (2000) solve an overlapping generations exchange economy with incomplete risk sharing but households only live for two periods, and they model limited participation exogenously. Guvenen (2005), Basak and Cuoco (1998) and Saito (1995) impose limited stock market participation exogenously and have perfect risk sharing among stockholders. These differences are crucial since our risk premium is driven by the imperfect risk sharing across shareholders, and is almost unaffected by the *endogenous* limited participation. In addition, we have a life-cycle model with a calibrated earnings process and retirement income, thus closely matching the level of earnings uncertainty in the data. Finally, unlike all these papers, we explicitly take into account government debt, which has important asset pricing implications.

The paper is structured as follows. Section 2 outlines the model and calibration while Section 3 discusses the baseline results. Section 4 studies the determinants of the equity premium, while Section 5 offers additional comparative statics. Section 6 concludes.

2   The Model

2.1   Outline

The model is solved at an annual frequency. Households have a finite horizon divided in two main phases: working life and retirement. During working life they receive a wage income subject to uninsurable shocks, and against which they cannot borrow. At retirement they receive a pension, financed by taxes on current workers' wages. Households can invest in two alternative assets: a claim to the risky capital stock (equity) and a riskless government bond. Before investing in equity for the first time they must pay a fixed participation cost.

Firms are perfectly competitive, and combine capital and labor, using a constant returns to scale technology, to produce a non-durable consumption good. The government taxes wages to finance the social security scheme (pension income), while taxes on both capital

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[8]We should be clear that the Storesletten et al. (2006) paper actually preceeds ours by several years. In fact our paper builds on their framework and, as a result, it is only natural that we have a more general model.

gains and bequests are used to finance government expenditures and the interest payments on public debt.

2.2 Production technology

2.2.1 Production function

The technology in the economy is characterized by a standard Cobb-Douglas production function, with total output at time t given by

$$Y_t = Z_t K_t^\alpha L_t^{1-\alpha} \tag{1}$$

where K is the total capital stock in the economy, L is the total labor supply and Z is a stochastic productivity shock which follows the process

$$Z_t = G_t U_t \tag{2}$$

$$G_t = (1 + g)^t \tag{3}$$

Secular growth in the economy is determined by the constant g (>0), while the productivity shocks U_t follow a two-state Markov chain capturing the average business cycle duration.

Firms make decisions after observing aggregate shocks. Therefore, they solve a sequence of static maximization problems with no uncertainty, and factor prices (wages, W_t, and return on capital, R_t^K) are given by the firm's first-order conditions

$$W_t = (1 - \alpha) Z_t (K_t / L_t)^\alpha \tag{4}$$

$$R_t^K = \alpha Z_t (L_t / K_t)^{1-\alpha} - \delta_t \tag{5}$$

where δ_t is the depreciation rate.

2.2.2 Stochastic depreciation

Standard frictionless production economies cannot generate sufficient return volatility, since agents can adjust their investment plans to smooth consumption over time (see Jermann (1998) or Boldrin, Christiano and Fisher (2001)). This usually motivates adjustment costs

for capital, which create fluctuations in the price of capital and increase return volatility. Since we have incomplete markets, different stockholders have different stochastic discount factors. They will therefore disagree on the solution to the optimal intertemporal decision problem of the firm (see Grossman and Hart (1979)). This is not a concern here because there is no intertemporal dimension to the firm's problem, but introducing adjustment costs would change that.[9] Recent incomplete markets production economy models have captured the adjustment cost effect by assuming a stochastic depreciation rate for capital (Storesletten et al. (2006), Gottardi and Kubler (2004) and Krueger and Kubler (2006)). We follow the same route and assume that the depreciation rate is given by

$$\delta_t = \delta + s * \eta_t \tag{6}$$

where η_t is an i.i.d. standard normal and s is a scalar. Therefore, δ_t is a general measure of economic depreciation, combining physical depreciation, adjustment costs, capital utilization and investment-specific productivity shocks.[10] We will assume that η_t is uncorrelated with the productivity shock U_t, but our results remain unchanged if we allow for correlation.

2.3 The government sector and social security

A social security system is important to provide the model with a realistic labor income process. If we were to ignore social security transfers we would significantly increase household income risk and wealth accumulation. The government sector is also crucial by issuing government bonds that exist in positive net supply, so as to match the average portfolio allocations in the data.

2.3.1 The social security system

The social security budget is balanced in all periods so we can discuss it separately. Given a replacement ratio of working life earnings (λ), the (proportional) social security tax rate on

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[9] Guvenen (2005) introduces adjustment costs in a model with restricted stock market participation. However, in his model the participation constraint is exogenous and there is perfect risk sharing among stockholders. Therefore, there is a unique stochastic discount factor for pricing capital.

[10] Greenwood, Hercowitz and Huffman (1988) use the same approach to model fluctuations in capital utilization.

labor income (τ_s) is determined endogenously. This tax rate ensures that the social security taxes are equal to total retirement benefits, taking into account the demographic weights and survival probabilities.

2.3.2 The government sector

The government's budget constraint (excluding social security) is

$$C_t^G + R_t^B B_t = B_{t+1} - B_t + T_t \tag{7}$$

where C^G is government consumption, B is public debt, R^B is the interest rate on government bonds, and T denotes the tax revenues. Tax proceeds arise from proportional taxation on capital (tax rate τ_K) and a 100% tax rate on bequests (E). The steady-state level of government debt (as a fraction of GDP) is calibrated to the data. Government expenditures do not enter the agents' utility functions are determined as the residual from (7), given the (exogenous) level of debt, the (exogenous) tax rate on capital, and the (endogenous) interest rate on bonds.

2.4 Households and financial markets

2.4.1 Preferences

Time is discrete. We follow the convention in life-cycle models and let adult age (a) correspond to effective age minus 19. Each period corresponds to one year and agents live for a maximum of 81 (A) periods (age 100). The probability that a consumer is alive at age ($a + 1$), conditional on being alive at age a, is denoted by p_a ($p_0 = 1$). At each point in time there is a stationary age distribution of households in the economy with no population growth. These households have Epstein-Zin preferences (Epstein and Zin (1989) and Weil (1990)) defined over consumption of a single non-durable good (C_a),

$$V_a = \left\{ (1 - \beta) C_a^{1-1/\psi} + \beta \left(E_a (p_a V_{a+1}^{1-\rho}) \right)^{\frac{1-1/\psi}{1-\rho}} \right\}^{\frac{1}{1-1/\psi}} \tag{8}$$

where ρ is the coefficient of relative risk aversion, ψ is the elasticity of intertemporal substitution and β is the discount factor.

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2.4.2 Labor endowment

Before retirement all households supply labor inelastically. The stochastic process for individual labor income (H_{at}^i) is given by[11]

$$H_{at}^i = W_t L_a^i \tag{9}$$

where L_a^i is the household's labor endowment (labor supply scaled by productivity) and W_t is the aggregate wage per unit of productivity. The household's labor endowment is specified to match the standard stochastic earnings profile in life-cycle models of savings and portfolio choice. More precisely, labor income productivity combines both permanent (P_a^i) and transitory (ε^i) shocks with a deterministic age-specific profile:

$$L_a^i = P_a^i \varepsilon^i \tag{10}$$

$$P_a^i = \exp(f(a)) P_{a-1}^i \xi^i \tag{11}$$

where $f(a)$ is a deterministic function of age, capturing the typical hump-shape profile in life-cycle earnings. We assume that $\ln \varepsilon^i$, and $\ln \xi^i$ are each independent and identically distributed with mean $\{-.5*\sigma_\varepsilon^2, -.5*\sigma_\xi^2\}$, and variances σ_ε^2 and σ_ξ^2, respectively. Retirement is exogenous and deterministic. All households retire at age 65 ($a^R = 46$) and retirement earnings are given by: $\lambda P_{a^R}^i W_t$, where λ is the replacement ratio.

2.4.3 Financial markets

There are two financial assets: a one-period riskless asset (government bonds), and a risky investment opportunity (capital stock). The riskless asset return is $R_t^B = \frac{1}{P_{t-1}^B} - 1$, where P^B denotes the government-bond price. The return on the risky asset is denoted by R_t^K. Before investing in stocks for the first time the investor must pay a fixed lump sum cost equal to $FP_a^i W_t$. This represents both the explicit transaction cost from opening a brokerage account and the (opportunity) cost of acquiring information about the stock market. The fixed cost (F) is scaled by the current value of the permanent component of labor income (P_a^i) and by the aggregate wage (W_t). This significantly simplifies the model's solution and is consistent with the opportunity cost interpretation.

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[11] We now include a superscript i to identify household specific variables.

2.4.4 Wealth accumulation

Total liquid wealth (cash-on-hand) can be consumed or invested in the two assets. At each age (a), agents enter the period with wealth invested in the bond market, B_a^i, and (potentially) in stocks, K_a^i, and receive $L_a^i W_t$ as labor income. Let the dummy variable I_P^i denote the time in which the participation cost is paid. Cash-on-hand at time t is given by

$$X_{at}^i = K_{at}^i(1 + (1 - \tau_K)R_t^K) + B_{at}^i(1 + (1 - \tau_K)R_t^B) + L_{at}^i(1 - \tau_s)W_t - I_P^i FP_{at}^i W_t \quad (12)$$

before retirement $(a < a^R)$, and by

$$X_{at}^i = K_{at}^i(1 + (1 - \tau_K)R_t^K) + B_{at}^i(1 + (1 - \tau_K)R_t^B) + \lambda P_{at^R}^i W_t - I_P^i FP_{at^K}^i W_t \quad (13)$$

during retirement $(a \geqslant a^R)$.

Households cannot borrow against their future labor income, and cannot short any asset. More precisely,

$$B_{at}^i \geq 0 \quad (14)$$

$$K_{at}^i \geq 0 \quad (15)$$

2.5 The individual optimization problem

2.5.1 Household expectations

Households are price takers and maximize utility given their expectations about future asset returns and aggregate wages. Under rational expectations, the latter are given by equations (4) and (5): future returns and wages are determined by future capital and labor, and by the realizations of aggregate shocks. Labor supply is exogenous as are the distributions of the aggregate shocks. The capital stock, however, is endogenous. Forming rational expectations of future returns and wages is, therefore, essentially equivalent to forecasting the future capital stock. Even if returns depend on the previous realizations of many state variables, that can only matter through one channel: the current value of the capital stock.

Therefore in the household's optimization problem we need to include state variables that will allow the agent to forecast K_t, for example the infinite-dimensional wealth distribution.

Krusell and Smith (1998) and den Haan (1997) suggest that, for this class of incomplete-markets economies, it is possible to approximate this infinite-dimensional state space with a small set of moments. As discussed in the appendix, our model can accurately forecast the capital stock using its lagged mean (last-period's aggregate capital stock, K_t) and the realizations of the two aggregate shocks (productivity, U_t, and stochastic depreciation, η_t):

$$K_{t+1} = \Gamma_K(K_t, U_t, \eta_t) \tag{16}$$

Since government bonds are only riskless over one period, households must forecast future bond prices (P_{t+1}^B). The forecasting rule for P_{t+1}^B is

$$P_{t+1}^B = \Gamma_P(P_t^B, K_t, U_t, \eta_t) \tag{17}$$

and details are given in Appendix 1. This introduces four additional state variables in the individual's maximization problem (P_t^B, K_t, U_t, and η_t).

2.5.2 The dynamic programming problem

We can now write the individual's recursive optimization problem. We normalize all individual variables by the current level of the permanent component of household's labor income ($P_a^i G_t^{\frac{1}{1-\alpha}}$) and all aggregate variables (wage and capital) by scaled aggregate productivity growth ($G_t^{\frac{1}{1-\alpha}}$). This induces stationarity in the model and reduces the dimensionality of the state vector by one variable. Normalized variables are denoted by lower case letters.[12] The value function is denoted by $V_a(x_{at}^i, F_a^i; k_t, U_t, \eta_t, P_t^B)$, where a is age, x_{at}^i is individual normalized cash on hand, F_a^i denotes the stock market participation status (a zero-one variable indicating whether the fixed cost has been paid or not), and the other four inputs are the aggregate variables from the forecasting equations ((16) and (17)). The individual optimization problem now becomes:

$$V_a(x_{at}^i, F_a^i; k_t, U_t, \eta_t, P_t^B) = \underset{\{k_{a+1,t+1}^i, b_{a+1,t+1}^i\}_{a=1}^A}{Max} \{(1-\beta)(c_{at}^i)^{1-1/\psi} \tag{18}$$

$$+ \beta(E_t[(\frac{P_{a+1}^i}{P_a^i}(1+g)^{\frac{1}{1-\alpha}})^{1-\rho} p_a V_{a+1}^{1-\rho}(x_{a+1,t+1}^i, F_{a+1}^i; k_{t+1}, U_{t+1}, \eta_{t+1}, P_{t+1}^B)])^{\frac{1-1/\psi}{1-\rho}}\}^{\frac{1}{1-1/\psi}}$$

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[12]Specifically, household-specific variables are normalized as $x_{at}^i \equiv \frac{X_a^i}{P_a^i G_t^{\frac{1}{1-\alpha}}}$, $c_{at}^i \equiv \frac{C_a^i}{P_a^i G_t^{\frac{1}{1-\alpha}}}$, $b_{a+1,t+1}^i \equiv \frac{B_{a+1,t+1}^i}{P_a^i G_t^{\frac{1}{1-\alpha}}}$, $k_{a+1,t+1}^i \equiv \frac{K_{a+1,t+1}^i}{P_a^i G_t^{\frac{1}{1-\alpha}}}$ while aggregate variables are normalized as $k_t \equiv \frac{K_t}{G_t^{\frac{1}{1-\alpha}}}$, and $w_t \equiv \frac{W_t}{G_t^{\frac{1}{1-\alpha}}}$.

subject to the constraints,

$$k^i_{a+1,t+1} \geq 0 \quad , \quad b^i_{a+1,t+1} \geq 0 \tag{19}$$

$$c^i_{a,t} + b^i_{a+1,t+1} + k^i_{a+1,t+1} = x^i_{a,t} \tag{20}$$

and with the laws of motion,

$$x^i_{a+1,t+1} = \begin{cases} \dfrac{\left[k^i_{a+1,t+1}(1+(1-\tau_K)R^K_{t+1})+b^i_{a+1,t+1}(1+(1-\tau_K)R^B_{t+1})\right]}{[(P^i_{a+1}/P^i_a)(1+g)^{\frac{1}{1-\alpha}}]} + \varepsilon^i(1-\tau_s)w_{t+1} - I^i_P F w_{t+1} & a < a^R \\[2em] \dfrac{\left[k^i_{a+1,t+1}(1+(1-\tau_K)R^K_{t+1})+b^i_{a+1,t+1}(1+(1-\tau_K)R^B_{t+1})\right]}{[(P^i_{a+1}/P^i_a)(1+g)^{\frac{1}{1-\alpha}}]} + \lambda w_{t+1} - I^i_P F w_{t+1} & a > a^R \end{cases} \tag{21}$$

$$R^K_{t+1} = R(k_{t+1}, U_{t+1}) \quad , \quad w_{t+1} = W(k_{t+1}, U_{t+1}) \tag{22}$$

$$k_{t+1} = \Gamma_K(k_t, U_t, \eta_t) \quad , \quad P^B_{t+1} = \Gamma_P(k_t, U_t, \eta_t, P^B_t) \tag{23}$$

2.6 Equilibrium

The equilibrium consists of endogenously determined prices (bond prices, wages, and equity returns), a set of cohort specific value functions, and policy functions, $\{V_a, b_a, k_a\}^A_{a=1}$, and rational expectations about the evolution of the endogenously determined variables, such that:

1. Firms maximize profits by equating marginal products of capital and labor to their respective marginal costs: equations (4) and (5).

2. Individuals choose their consumption and asset allocation by solving (18).

3. Markets clear and aggregate quantities result from individual decisions. Specifically,

$$k_t = \int_i \int_a P^i_{a-1} k^i_{at} \, da \, di \quad , \quad b_t = \int_i \int_a P^i_{a-1} b^i_{at} \, da \, di \tag{24}$$

The aggregation equation for labor supply is redundant since there is no labor-leisure choice. Once these two equations are satisfied, Walras' law implies that total expenditure (government consumption, investment and household consumption) must equal total output:

$$\frac{C^G_t}{G^{\frac{1}{1-\alpha}}_t} + (1+g)^{\frac{1}{1-\alpha}} k_{t+1} - (1-\delta_t)k_t + \int_i \int_a P^i_a c^i_{at} \, da \, di = U_t k^\alpha_t L^{1-\alpha}_t \tag{25}$$

4. The social security system is balanced at all times,

$$\int_i \int_{a \in I_W} \tau_s L^i_a w_t \, da \, di = \int_i \int_{a \in I_R} [\lambda \exp(f(a^R)) w_t P^i_{a^R}] \, da \, di \tag{26}$$

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where the left hand side is integrated over all workers ($a \in I_W$), while the right hand side over retirees ($a \in I_R$). This equation determines τ_s for a given value of λ.

5. The government budget (equation (7)) is balanced every period to sustain a given ratio of government debt to GDP (b_t/y_t).

6. Market prices expectations are verified in equilibrium.

Analytical solutions to this problem do not exist and we therefore use a numerical solution method (details are given in appendix 1).

2.7 Calibration

2.7.1 Aggregate variables

Decisions are made at an annual frequency. The productivity shock follows a first-order Markov process with two values. The probability of remaining in the current state (π) is 2/3, yielding an average business cycle duration of six years, and the standard deviation (σ_u) is 1%. Given all other parameter values, we set this volatility to match the standard deviation of aggregate output in the data. Capital's share of output (α) is set to 36%, and the average annual depreciation rate (δ) is 10%. The parameter s that determines the return volatility is set at 16%. We discuss the calibration of s (and σ_u) in more detail below. The aggregate supply of bonds is set at 38% of GDP, based on the average value of U.S. Treasury securities held by the U.S. public taken from the Congressional Budget Office. The capital tax rate is 20%, while bequests are taxed at 100%.[13]

2.7.2 Household variables

Agents begin working life at age 20, retire at age 65, and can live up to 100 years. We use the mortality tables of the National Center for Health Statistics to parameterize the conditional survival probabilities. We then use these survival probabilities to compute the stationary age distribution in the model. The deterministic labor income profile reflects the hump shape of earnings over the life-cycle. The corresponding parameter values, just like the retirement

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[13]Total bequests are a very small fraction of total government revenues (both in the model and in the data), and this assumption is only made for simplicity.

transfers ($\lambda = (1 - \tau_s)0.68$), are taken from Cocco, Gomes and Maenhout (2005). From equation (26) this generates an endogenous social security tax (τ_s) of approximately 10%. The volatilities of the idiosyncratic shocks are taken from Carroll (1992): 10 percent per year for σ_ε, half of the estimated value to take into account for measurement error, and 8 percent per year for σ_ξ.

We consider two groups of agents with the same population size and different preference parameters. In the baseline version of the model, type-A have low risk aversion ($\rho^A = 1.1$) and low elasticity of intertemporal substitution ($\psi^A = 0.1$), while type-B have higher risk aversion ($\rho^B = 5$) and slightly higher EIS ($\psi^B = 0.4$). The discount factor (β) is equal to 0.99 for both groups, corresponding to an average discount rate of 2.8%, once adjusted for the survival probabilities. We consider two cases for the fixed cost of participation: one where the cost is zero and another corresponding to 6.0% of the household's expected annual income. For the average household with an annual labor income of $\$35,000$, this corresponds to $\$2,100$. Paiella (2001) and Vissing-Jørgensen (2002b) obtain implied participation costs of $\$75$ to $\$200$ per-period, from Euler equation estimations. Our one-time cost is comparable in present-value terms. Alan (2006) estimates a CRRA life-cycle model with one-time fixed costs and obtains similar values. Later on we show that we can also match the data with lower values of F (e.g. 2.5%).

3 Baseline Results

3.1 Individual allocations

3.1.1 Consumption and savings

Figure 1 plots the life-cycle profiles of consumption, wealth and labor income for the average household. From age 65 onwards labor income refers to retirement income. Early in life, households accumulate a small buffer-stock of wealth. Later on they start saving for retirement. The combination of idiosyncratic shocks and preference heterogeneity induces significant cross-sectional heterogeneity in wealth accumulation. This is highlighted in Table 1 where we report the different quartiles of the wealth-to-income ratio distribution from the

baseline model. For comparison, we include the same numbers from the 2001 SCF. The model's values are consistent with those in the data. In particular the model matches the right tail of the wealth distribution relatively well.[14]

Early in life households face liquidity constraints and therefore they save only for precautionary reasons. As a result, at this life-cycle stage, savings are mostly determined by prudence/risk aversion (ρ). The less risk-averse households accumulate less wealth since they are less concerned about background risk. At mid-life, the savings behavior is determined mostly by the preference for low-frequency consumption smoothing, i.e. by the elasticity of intertemporal substitution (EIS, ψ). As a result, at this life-cycle stage households with lower EIS save less. Combining these two results, we conclude that type-A households (with low RA and low EIS) accumulate significantly less wealth over the life-cycle. On the contrary, the type-B households (more risk-averse and with higher EIS) constitute the majority of the wealthy population.

3.1.2 Participation rates

Decreasing risk aversion increases the optimal share invested in stocks but, (as shown in the previous subsection) also decreases wealth accumulation at every life-cycle stage. The impact of a change in risk aversion on the participation decision will therefore depend on which of these two effects dominates. Consistent with life-cycle asset allocation models (e.g. Gomes and Michaelides (2005)) we find that the wealth effect dominates. The stock market participation rate is much lower for the less-risk averse (type-A) households: 7.4% versus 98.8% (see Table 2). Overall, the participation rate in the model is very close to the one in the data: 53.1% versus 51.9%, respectively.

This form of heterogeneity is consistent with the existing empirical evidence. Vissing-Jørgensen (2002a), Brav, Constantinides and Geczy (2002) and Malloy, Moskowitz and Vissing-Jørgensen (2006), for example, estimate the elasticity of intertemporal substitution for both non-stockholders and stockholders using the Euler equations on their consumption and savings decisions. Consistent with our model, the estimated EIS is indeed lower for non-

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[14]We note however that, for higher percentiles (e.g. 90th or 95th), the values in the data are almost twice as high as those implied by the model.

stockholders. For instance, Vissing-Jørgensen (2002a) obtains estimates of the EIS greater than 0.3 for risky asset holders, while for the remaining households these estimates are small and insignificantly different from zero.

By definition, it is impossible to estimate empirically the risk aversion coefficient of non-stockholders from Euler equations, because those households do not have a portfolio decision. However, it is important to mention that our results *do not depend* on non-stockholders having lower risk-aversion and/or lower EIS. As we discuss below (in section 4.1.3), the preference parameters of the stock market non-participants are irrelevant for determining equilibrium asset pricing moments. What is crucial is that non-participants are less wealthy than the stock market participants, which is the equilibrium outcome once we endogenize the participation decision.

3.1.3 Asset shares

In the model the average equity share for stockholders is 79.1%, which might seem counter-factually high since the average equity share in household portfolios is 54.8% (from the 2001 SCF). However, the raw SCF numbers are only directly comparable with the predictions of portfolio allocation models (e.g. Gomes and Michaelides (2005)), they *are not* directly comparable with the predictions of general equilibrium models. In a general equilibrium model, unless we specifically model corporate debt, we are not modeling net household holdings of corporate bonds (direct and indirect, through equities and bank accounts). Therefore, when compared with direct data on household portfolios (e.g. SCF data) the equity share from the general equilibrium model must be scaled down by the average debt-to-equity ratio of the corporate sector (28% from Rajan and Zingales (1995)). If we do this, then the implied equity share in our model is 56.9% which is very close the 2001 SCF numbers (54.8%).[15]

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[15]This is a standard problem in the literature, and one that actually works against these models. Since firms in the model are financed by unlevered equity, while in the data the average corporate debt-to-equity ratio is 28%, then the relevant comparison should be with a (lower) unlevered historical risk premium.

3.2 Asset prices and consumption volatility

The model is calibrated to match the equity return volatility using the standard deviation of the stochastic depreciation (s). We use the EIS of the type-B agents (ψ^B), and the discount factor (β, which is identical for both types) to match three key moments: the level and volatility of the riskless rate, and the consumption growth volatility (discussed below). We choose the preference parameters of the type-A agents (EIS and RA) to match the participation rate, which has almost no impact on asset pricing moments, as discussed in more detail in the next sections. This leaves one free parameter to match the risk premium: the type-B agents' risk aversion.[16]

3.2.1 Asset returns

Table 3 reports the main asset pricing moments implied by the model, along with their empirical U.S. counterparts taken from Campbell (1999). The mean risk-free rate and the volatility of equity returns are closely matched.[17] The standard deviation of the risk-free rate is 3.26% compared with 5.33% in the data.[18] This result is particularly good since, in an economy with a non-trivial production sector, obtaining a low standard deviation of the riskless asset is quite challenging. Boldrin et al. (2001) report standard deviations between 17.4% and 25.4%, while Guvenen (2005) and Jermann (1998) obtain 5.7% and 11.5%, respectively. With a risk aversion coefficient of 5 for the type-B agents, the implied equity premium is 3.83%, corresponding to approximately 60% of the risk premium in the data. The serial correlation of equity returns is essentially zero as in the data, while the risk-free rate in the model is more persistent than its empirical counterpart.

Our model generates some time variation in excess returns but not much. Excess returns

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[16]Since we have a model with incomplete markets this calibration is more complicated. For example, risk aversion also affects the risk-free rate, and the EIS also affects the risk premium. We discuss these relationships in more detail below.

[17]We could easily increase the volatility of equity returns to match the data exactly, by increasing s, and naturally the corresponding equity premium would be even higher. However, this would also lead to a higher volatility of both consumption growth and the riskless rate.

[18]We undershoot the risk-free rate volatility since we do not have inflation in the model.

are not predictable by dividend yields but they are marginally predictable by the *cay* variable (Lettau and Ludvigson (2001)). This predictability however is much weaker than in Lettau and Ludvigson (2001). Even though we obtain very similar coefficients for the cointegrating vector, the R^2 of the predictive regression is at least 10 times lower, and the coefficient is not statistically significant at horizons of less than 3 years. If we repeat this analysis using only stockholders consumption the results are only marginally stronger, while if we use non-stockholder's consumption they disappear completely.

3.2.2 Consumption volatility

In a limited participation equilibrium, equity is priced by the stochastic discount factors of stockholders. Therefore, it is important to check that the consumption risk implied by the model is consistent with the data. Table 4 presents the standard deviation of the average growth rate of stockholders and non-stockholders consumption. We compare these with the estimates in Malloy, Moskowitz and Vissing-Jørgensen (2006) based on data from the Consumer Expenditure Survey (CES). Both in the model and in the data the volatility of consumption growth for stockholders is much higher than that for non-stockholders. The consumption volatilities in the model are higher than in the data but it is important to remember the difference in time periods. The standard deviation of aggregate consumption growth is 3.3% in the 1890-1997 sample (Campbell (1999)) that we use to calibrate the moments in the paper, compared with only 1.7% in the 1982-2004 sample from Malloy, Moskowitz and Vissing-Jørgensen (2006). Therefore, it is natural to expect that the volatility of both stockholders' and non-stockholders' consumption growth over the longer sample period will have been higher than the one estimated from the recent CES sample.[19]

These results highlight the importance of taking limited participation into account when

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[19]In our model the convariance between stockholders' consumption growth and the risk premium is 0.0074. In the Campbell (1999) sample the covariance between total consumption growth and stock returns is 0.0037. If we scale this by the ratio of stockholders' consumption volatility to total consumption volatility from Malloy, Moskowitz and Vissing-Jørgensen (2006) ($2.1 = 0.036/0.017$), we obtain 0.0078 which is extremely close to the number implied by our model. This highlights again the importance of considering limited participation in the model.

calibrating asset pricing models. Models with limited stock market participation, such as ours, can replicate the differences between consumption growth of stockholders and non-stockholders, and can thus use the moments of the former to explain equilibrium asset returns. Models without limited participation must match the aggregate consumption moments and can only rationalize a lower equity premium.

3.2.3 Correlation between labor income shocks and stock returns

In a frictionless one-sector production economy the marginal productivity of capital and labor are both driven by the same productivity shock. The correlation between aggregate wages and the return on capital is therefore very high. This might suggest that our equity premium is driven by a counterfactually high correlation between household earnings shocks and stock returns. This is not the case, however. Consistent with the empirical evidence (e.g., Davis and Willen (2001)), the endogenous correlation between household-level labor income shocks and stock returns is close to zero (1.2%). Two features of the model explain this. First, stochastic depreciation shocks (η_t) reduce the correlation between stock returns (R_t^K) and aggregate wages (w_t), just as adjustment costs of capital would. Second, for a given correlation between w_t and R_t^K, the idiosyncratic productivity shocks imply a much lower correlation between stock returns (R_t^K) and household-level income (H_t^i).

4 Decomposing the Equity Premium

In this section we explore the explanatory contribution to the equity premium of the four main features of the model: endogenous limited participation, preference heterogeneity, incomplete risk sharing among stockholders and positive bond supply. Appendix 2 discusses additional comparative statics.

4.1 Limited participation

4.1.1 Model without limited participation

We consider a version of the model with the stock market entry cost (F) set to zero, keeping all other parameter values unchanged. The results are shown in Table 5 and are almost identical to those obtained for the baseline model. Most importantly, there is a very modest reduction in the equity premium: 3 basis points. This might seem surprising given the results obtained in Saito (1995), Basak and Cuoco (1998), or Guvenen (2005). These papers argue that limited stock market participation can contribute in explaining the equity risk premium puzzle. In those models, however, stock market participation is exogenous, while we derive it endogenously.[20] As a result, in our model the non-participants are significantly less wealthy than the rest of the population. This is consistent with the data. According to the latest numbers from the Survey of Consumer Finances, the participation rate is 88.84% among households with wealth above the median, and only 15.21% for those with wealth below the median. The median wealth for stockholders is $154,600$, while the median wealth for non-stockholders is $7,300$. In our model, the ratio of median stockholders' wealth to median non-stockholders' wealth is approximately 30. Therefore, excluding these households from the equity market has a *negligible* impact on the risk premium.

Only by assuming extremely high entry costs or by imposing the participation constraint exogenously, would it be possible to exclude rich households, thus producing a significant increase in the risk premium. More precisely, in Saito (1995), for parameter values that generate an equity premium in excess of 3%, the shadow value of the fixed cost that would keep households out of the stock market ranges from 22.5% to 54.1% of total wealth.[21] This conclusion might seem at odds with our discussion in section 3.2 stressing the importance of limited participation for calibrating the moments of consumption growth. The two results are however perfectly consistent. The discussion in section 3.2 shows that having a model that

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[20]Heaton and Lucas (2000) and Polkovnichenko (2004) also have exogenous participation constraints, but they do not exclude a significant fraction of wealthy households. Therefore, their results are closer to ours.

[21]Since this cost is computed as a function of total wealth, the implied cost is even larger for the median-wealth household, when expressed in terms of current income (as we do in our model).

captures the heterogeneity between the consumption growth of rich and poor households, such as ours, is important for matching the equity premium. Whether these poor households are stockholders or non-stockholders (as they currently are in the data) does not matter, exactly because they do not have much wealth.

4.1.2 Lower participation cost

We also consider a lower fixed cost value (F). As expected from our previous discussion, we can again match the participation rate if we re-calibrate the preference parameters of the type-A agents. Moreover, this has almost no impact on the other aggregate variables. More precisely, we can decrease the fixed cost from 6% to 2.5% (for example), and obtain a participation rate of 47.3% by reducing ρ^A and ψ^A (the risk aversion and EIS of 'non-stockholders') to 1.05 and 0.05, respectively. All other variables remain essentially unchanged, and even the consumption growth of non-stockholders is only marginally affected.[22] Matching the participation rate with a lower entry cost makes our previous conclusion stronger: the stock market non-participants are now even less wealthy, and therefore excluding them from the stock market has a smaller impact on the equity premium. Only if we consider that our previous fixed cost value ($F = 6.0\%$) is already too low, could we question the conclusion that the participation constraint has almost no impact on asset prices.

4.1.3 Equal or higher risk-aversion for non-participants

As previously mentioned, the only important input for the limited participation results is that non-participants are significantly less wealthy than stockholders, as they are in the data. More precisely, for a given risk aversion of stock-market participants, it does not matter if the risk aversion of non-participants is higher or lower, as long as these households *endogenously* choose not to participate in the stock market. In a life-cycle model household savings are determined by the retirement replacement ratio, earnings risk, the age-income profile, and the preference parameters. In the baseline version of the model we only considered heterogeneity in risk aversion and elasticity of intertemporal substitution. In this subsection we explore

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[22]The full set of results is available upon request.

heterogeneity along other dimensions.

In the first case we assume that all households have the same risk aversion coefficient ($\rho^A = \rho^B = 3$), but non-stockholders have higher discount rate ($\beta^A = 0.7$ and $\beta^B = 0.99$), lower EIS ($\psi^A = 0.05$ and $\psi^B = 0.6$) and higher replacement ratio ($\lambda^A = 0.8$ and $\lambda^B = 0.6$). This calibration is chosen to match the empirical wealth accumulation profiles of 'non-stockholders', as in the baseline model. The results are shown in table 6, fourth column. The impact of limited participation on asset prices is again negligible, and the intuition is the same as before: since the participation decision is endogenous all wealthy households choose to invest in stocks. In the fifth column of table 6 we change the previous calibration slightly, by assuming that non-stockholders actually have *higher* risk aversion ($\rho^A = 4$), and obtain the same conclusion.[23] These results show that the equilibrium quantities are essentially the same with and without limited participation (even the moments of the riskless rate), regardless of the preference parameters of 'non-stockholders'. Naturally, if we change the parameters of the 'stockholders' this is no longer true. For example, if we compare any of these models with our baseline calibration (column three) we find that they have a lower equity premium (3.12% versus 3.83%) as a result of the lower risk aversion coefficient for stockholders.

4.2 Preference heterogeneity

We now consider an economy where all agents have the same preference parameters. We consider parameter values similar to those of the type-B agents (in the baseline model), because we want to generate a high equity premium. We ignore the participation cost since it has almost no impact on these investors' behavior. All other parameter values remain the same as in the baseline case. The results are shown in column four of Table 7. In column three we report the results for the two-types model without the fixed cost, for comparison.[24]

If we were to take the preference parameters of the type-B agents ($\rho = 5$, $\psi = 0.4$, $\beta = 0.99$) the single-agent economy would have much higher (average) wealth accumulation

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[23]For this calibration we need to assume that non-stockholders have a lower discount rate than in the previous case ($\beta^A = 0.6$), to "compensate" for the higher risk aversion.

[24]These are essentially the same as the baseline results, as shown above (see table 3).

and thus much lower average returns on both assets. Therefore we re-calibrate the model to try to match the same aggregate moments as in the baseline case: the level and volatility of the riskless rate, the risk premium, and the volatility of consumption growth. Under the new calibration the risk aversion coefficient and the discount factor remain unchanged, but the elasticity of intertemporal substitution is set equal to 0.15. The results are shown in column four of Table 7. The economy without preference heterogeneity yields a slightly higher risk premium with a lower riskless rate. The volatility of consumption growth decreases, as a result of the lower EIS (with the same risk aversion coefficient), but in this model that should be matched against the volatility of aggregate consumption growth (not stockholder's consumption growth). Although the two models deliver very similar implications for the aggregate variables, as previously shown, if we want to match stock market participation rates, and/or the cross-sectional wealth distribution, then we must consider our baseline model. The wealth distribution, in the economy without preference heterogeneity, does not replicate the bottom half of its empirical counterpart, and therefore this model would require an extremely large fixed cost to deliver significant non-participation.

4.3   Incomplete risk sharing among stockholders

We have shown that the equity premium in our model is not driven by limited participation. This contrasts with the results in Saito (1995), Basak and Cuoco (1998), or Guvenen (2005). However, in those models there is perfect risk sharing among stockholders, while in ours that is not the case. We have incomplete risk sharing due both aggregate and idiosyncratic shocks, and the liquidity constraints.

4.3.1   Results without stochastic depreciation

If we set both aggregate shocks equal to zero, stocks and bonds become perfect substitutes. Therefore we only set s equal to zero, and keep all other parameter values unchanged. The results are shown in Table 8. The stock returns volatility is now close to zero, and the volatility of consumption growth also falls substantially: it is now 0.22% for stockholders

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and 0.06% for non-stockholders.[25] Even though equity returns are almost riskless, the consumption growth volatility of stockholders and non-stockholders is not the same. These groups still have different ratios of financial wealth to labor income. Despite the extremely low volatility of returns and consumption growth, we still obtain a 0.83% equity premium. The presence of borrowing constraints and undiversifiable idiosyncratic risk generates a high 'equilibrium market price of risk' thus delivering a non-trivial equity premium even for a smaller value of aggregate uncertainty. Given this market price of risk, the level of aggregate uncertainty essentially scales the risk premium up or down.[26] It is also interesting to report that, the correlation between labor income shocks and stock returns is now much higher: 54% versus 1% in the baseline case. This was expected since returns on capital are now driven exclusively by the aggregate productivity shock, just like wages. However, it is interesting to note that this correlation is still significantly below 1, and this is due to the presence of the idiosyncratic labor income shocks.

4.3.2   Calibration of aggregate uncertainty

In standard endowment economies, aggregate consumption must equal aggregate dividends. Therefore, consumption growth volatility is essentially an exogenous parameter that can be calibrated directly. In models with a non-linear production technology this is no longer the case. Based on our previous discussion we have set the parameters s and σ_u to match aggregate risk in the economy. For example, in the economy with very low aggregate uncertainty (Table 8) the volatility of (stockholders') consumption growth is only 0.22% as opposed to 3.6% in the data. The distinction between s and σ_u is important. With σ_u alone (and $s = 0$) we could match either the volatility of consumption (and output) or the stock return volatility, but not both simultaneously. Matching consumption volatility would produce a very low return volatility, while matching the standard deviation of stock returns would imply a high standard deviation of consumption growth. We therefore choose s and σ_u so that we can

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[25]We still have non-stockholders even though stocks are almost riskless. Some households never accumulate enough wealth to justify paying the fixed cost, given the modest equity premium of 0.83%.

[26]This is the same mechanism that would obtain in the presence of explicit adjustment costs (e.g. Jermann (1998), Boldrin et al. (2001) or Guvenen (2005)) .

match both.

4.3.3   The role of idiosyncratic risk and borrowing constraints

The previous results (Table 8) show that, in the absence of the main aggregate shock (stochastic depreciation), the price of risk is still high but the quantity of risk in the economy is negligible, and thus the implied equity premium is very small. We still have a high price of risk due to the presence of household-specific risk: borrowing constraints and idiosyncratic income shocks. In this section we report results for the opposite experiment: keep aggregate risk unchanged, but decrease the level of household risk. The results are shown in table 9, where we consider the impact of both borrowing constraints and idiosyncratic income shocks. Column 4 of Table 9 shows that the idiosyncratic shocks play a modest role, as expected from Lucas (1994) and Heaton and Lucas (2000): the equity premium only falls by 0.34% when we remove this source of uncertainty in the model, i.e. when set $\sigma_\varepsilon = \sigma_\xi = 0$. However, we argue that it is still important to include labor income shocks in the model, as otherwise the wealth accumulation would be counterfactually low. Consequently if we calibrate such a model to match the empirical evidence, it would bias the calibration of the preference parameters. This is quite visible in the implied aggregate moments, as for the same preference parameters both the return on equity and riskless rate must increase significantly to clear markets. In particular the riskless rate is now close to 5%.

The study of the *quantitative* impact of borrowing constraints in our set-up is not a straightforward exercise. There are several important issues to take into account in a model where borrowing is allowed. For instance, do we allow households to borrow for financing additional investment in equity, or only to finance consumption? In our analysis, we consider only the second case as there is very limited evidence of households with levered equity portfolios in the data.[27] Second, in discrete-time models with log-normally distributed income, if we want to avoid default then the maximum borrowing limit is actually zero.[28] Therefore,

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[27] Although we acknowledge that, if we were to allow for this, the model would conterfactually generate such behavior for a non-trivial fraction of households.

[28] Even if we allow households to roll-over debt the same conclusion is valid as long as they are not allowed to die in debt. In a model with survival probabilities, such as ours, this is immediate since households can

either we explicitly model default and introduce a default penalty in the model or we assume an exogenous borrowing limit that does not exceed the lower bound on the income process, given by the discretization of the log-normal process.[29] We find show that such a borrowing limit equal to 50% of average next period earnings is already enough to decrease the equity premium by 0.66%, even though stock return volatility remains unchanged (columns 5 and 6 of Table 9). Naturally, if we were to increase the borrowing limit further, the equity premium would be even lower.

4.4   Level of government debt

Asset pricing models usually assume that the riskless asset exists in zero net supply. This implies that the representative consumer must invest all of her wealth in the risky asset. In models with limited stock market participation, such as ours, this also implies that a significant fraction of the population must hold levered positions in equities. These predictions clearly contrast with the empirical evidence on household-level asset holdings (see, for instance, Poterba and Samwick (2001), Ameriks and Zeldes (2001), or Guiso, Haliassos and Japelli (2002)). We explicitly allow for government bonds in positive supply, and calibrate the model to match the historical ratio of government debt to GDP. However, that makes it harder to match the unconditional asset pricing moments. To show this, we decrease the ratio of government debt to GDP from 38% to 15% and compare the results with those obtained in the baseline economy.[30]

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die every period. In a model with a determinisitc terminal date households have a maximum borrowing limit of zero in the previous period, which by backward induction then leads to a maximum borrowing limit of zero every period.

[29]Naturally the strictly positive lower bound on income only arises because of the discretization of the log-normal process. In a model with a zero borrowing limit, as in our baseline case, this does not affect the results. However, when allowing for borrowing constraints this determines the maximum exogenous borrowing limit compatible with a zero-default assumption. Cocco, Gomes and Maenhout (2005) discuss these issues in detail in the context of a partial equilibrium model with endogenous borrowing constraints.

[30]We do not consider a case with zero government debt because we still want to match the stock market participation rates. Since agents that do not participate in the stock market must hold positive amounts of bonds, the other agents would then be forced to hold negative amounts. Therefore, this would require

We consider an economy with the same parameter values as in the baseline case. The results are shown in column four of Table 10. Naturally, as we decrease the bond supply, the bond price increases and the risk-free rate falls. Households are required to invest a larger fraction of their wealth in stocks. This leads to a higher equity premium and to an increase in the consumption growth volatility. More precisely, for the same parameter values as in the baseline case, the risk-free rate drops to 0.89% and the equity premium increases to 4.62%, while the standard deviation of consumption growth for stockholders is now 5.56%. Naturally, if we were to re-calibrate the model we would be able to increase its fit even more. Therefore, as we converge to the standard assumption (zero bond net supply), the equity premium in the model approaches the historical value.

4.5 Long-run consumption risks

Recently, Bansal and Yaron (2004) have proposed "long-run consumption risks" as a mechanism for generating a large equity premium. Unfortunately, in our model this effect is not present.[31] In this section we briefly explain why that is the case, and why it would be challenging to introduce such effects in our economy. There are two important components in the "long-run risks" mechanism proposed in the literature. First, the existence of a "slow-moving" component in consumption growth, which is correlated with dividend growth (i.e. long-run risk must exist). Second, agents must have a preference for early resolution of uncertainty (i.e. long-run risk must earn a positive risk premium).

The second ingredient can be obtained in a model with Epstein-Zin preferences when the EIS is larger than the inverse of the coefficient of relative risk aversion. This is indeed the case in our baseline economy, but the difference is very small ($EIS = 0.4$ and $RRA = 5$). In contrast, Bansal and Yaron (2004) consider an EIS of 1.5 and risk aversion of 10. The first feature can be assumed directly in pure exchange economy models, since the dividend process is exogenous. However, in a production economy, both the dividend and the consumption processes are endogenous. The exogenous process in now the one for the productivity shocks. Therefore, if we want to generate long-run risks in a production economy we must be able

relaxing the short-selling constraints on bonds, and the two models would no longer be directly comparable.

[31]More precisely, $cov(Ln(C_{t+s}/C_t), R_t^K - R_t^B)$ is essentially constant as a function of s.

to map those productivity shocks into a consumption process with a "slow-moving" growth rate. Kaltenbrunner and Lochstoer (2006) explore this possibility in the context of a infinite-horizon representative agent production economy model. They show that a high EIS, by increasing inter-temporal consumption smoothing, induces persistent time-variation in the expected aggregate consumption growth following a productivity shock. However, they also show that only an EIS larger than 1 will deliver quantitatively significant effects. Therefore, not only does our model fail to generate long-run risks, even if it did it would assign them a negligible risk premium.

To change that conclusion we would need to consider a very high EIS. However, in the context of a life-cycle model, this would generate strongly counterfactual wealth profiles and consumption volatility. Table 11 reports the results obtained from increasing the EIS (of type-B agents). We find that as we increase the EIS, the volatility of (stockholders') consumption growth rises quite fast: for $\psi^B = 0.75$ this volatility is already 7.45%. In addition, wealth accumulation is also much higher leading to a significant reduction in both the return on equity and the riskless rate. In fact, with $\psi^B = 0.75$, the riskless rate is already very close to zero (0.14%), and the equity premium is still only marginally affected. In order to obtain large effects in the equity premium we would need an EIS larger than 1, but in our model this would imply a volatility of consumption growth above 10% and a negative riskless rate.[32] Kaltenbrunner and Lochstoer (2006) also show that, in their set-up, productivity shocks must be extremely persistent to induce changes in long-run expected consumption growth "of the right sign". However, this intuition does not carry over that easily to our model, for two reasons. First, in our model we actually have four different shocks. Two household-specific productivity shocks (permanent and transitory), aggregate productivity shocks and stochastic depreciation shocks. All of these determine both household-level future expected consumption growth and the risk premium. Second, we have an overlapping generations model with borrowing constraints, a retirement period and "exponentially-decreasing" survival probabilities. Therefore, household-level expected future consumption growth has a hump-shape response pattern, regardless of the nature of the underlying shocks. Naturally,

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[32]Naturally the negative riskless rate only arises because households are not allowed to simply keep the money "under their beds" and earn a zero rate of return.

more persistent shocks will generate a consumption response with larger half-life but, unlike in an infinite horizon-model, there are no "pure" transitory shocks, and no "pure" permanent shocks either. In fact, we have increased the persistence of the productivity shocks (to both 0.9 and 0.975), and the quantitative results were not significantly affected. Likewise, we have also increased the persistence of the stochastic depreciation shocks. Here we find that the equity premium increases significantly but this comes at a the cost of higher volatility for both stockholders' consumption growth and the riskless rate.[33]

5   Conclusion

We present an asset pricing model with incomplete markets and heterogeneous agents that reproduces key aggregate moments, while being consistent with observed individual asset allocation decisions. More precisely, the model matches the level and volatility of the risk-free rate, the stock market participation rate and the micro-evidence on asset allocation decisions. Further, the model explains approximately two thirds of the historical equity risk premium. Market incompleteness results from both aggregate and (uninsurable) idiosyncratic shocks. Idiosyncratic uncertainty is driven by a (realistically) calibrated life-cycle income profile that agents cannot fully insure, and against which they cannot borrow. This is a very important feature of the model. Replicating the level of risk that households actually face is not possible in an infinite horizon model, without a retirement period, for example.

We then decompose each component's contribution to the risk premium. The risk premium in our model is generated by incomplete risk sharing among stockholders. Aggregate risk, mostly stochastic depreciation of capital, matches the quantity of risk in the economy, namely stock return volatility, and stockholders' consumption growth volatility. Idiosyncratic risk and (mostly) borrowing constraints generate a high market price risk. The combination delivers a risk premium of 3.8% with a risk aversion coefficient of 5.

We show that, accounting for the positive supply of government bonds drives down the equity premium, making it harder to match the historical numbers. If we relax this assumption, the model is able to match the historical equity premium with a realistic consumption

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[33]This set of results is available upon request.

growth volatility. We find that limited stock market participation has a *negligible* impact, contrary to previous results in the literature where the participation decision is specified exogenously. However, we argue that taking limited participation into account is important for the calibration of asset pricing models, since stockholders' consumption growth is much more volatile and more correlated with stock returns. Finally we show that our model is not able to endogenously generate "long-run risk effects" without yielding counterfactually high consumption growth volatility and wealth accumulation.

One should be careful about interpreting our limited participation results. Namely, these results do not imply that all changes in the participation rate will have (close to) zero impact on asset prices. In particular, a social security reform that changes households' savings incentives (or creates forced savings), will simultaneously increase the participation rate and the wealth accumulation of (the previous) non-stockholders. Equilibrium returns could then be significantly affected. Our results do, however, imply that reductions in the transaction costs associated with stock market participation (e.g. reduction in brokerage fees, expansion of online discount brokers, etc.) will have almost no impact on the steady-state distribution of asset returns.

Appendix 1: Solving the model

1) Solution method outline

The solution method builds on den Haan (1997), Krusell and Smith (1998) and Storeslet-ten et al. (2006). We start by presenting the outer loop of the code and discuss the details afterwards.

The numerical sequence works as follows:

i) Specify a set of forecasting equations (Γ_K and Γ_P).

ii) Solve the household's decision problem, taking prices as given, and using the forecast-ing equations to form expectations (details in 2).

iii) Given the policy functions, simulate the model (5500 periods) while computing the market clearing variables at each period (details in 3).

iv) Use the simulated time-series to update the forecasting equations (details in 4).

v) Repeat ii), iii) and iv) with the new forecasting equations until convergence. We have two convergence criteria:

- Stable coefficients in the forecasting equations.

- Forecasting equations with regression R^2 above 99%.

2) Solving the household's decision problem

2.1) Discretization of the state space

Age (a) is a discrete state variable taking 81 possible values. We discretize the cash-on-hand dimension (x_t^i) using 51 points, with denser grids closer to zero to take into account the higher curvature of the value function in this region. With respect to the other two continuous state variables, we use 16 points to discretize k_t, and 25 points to discretize P_t^B. The remaining state variables, depreciation shock (η_t), aggregate productivity shock (U_t) and participation status (F_t^i) are all discrete, each taking only two possible values. Therefore no approximation is required.[34]

The grid range for the continuous state variables is verified ex-post by comparing with the values obtained in the simulations, and with the results obtained when this range is in-creased. The number of grid points for x_t is particularly important for producing an accurate

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[34]In the baseline version of the model this leads to a state space of dimension $13, 219, 200$.

participation decision, since the policy functions for consumption and asset allocation would not require this many points. A smaller number of grid points for k_t and for P_t^B would not affect the policy functions directly. It would, however, affect the R-square of the forecasting equations and the convergence of their respective coefficients.

2.2) Maximization

We solve the maximization problem for each agent type using backward induction. For every age a prior to A, and for each point in the state space, we optimize using grid search. We need to compute the value associated with each set of controls (consumption, decision to pay the fixed cost, and share of wealth invested in stocks). From the Bellman equation,

$$
\begin{aligned}
&V_a(x_{at}^i, F_a^i; k_t, U_t, \eta_t, P_t^B) \\
&= \underset{\{k_{a+1,t+1}^i, b_{a+1,t+1}^i\}_{a=1}^A}{Max} \{(1-\beta)(c_{at}^i)^{1-1/\psi} \quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad\quad (\text{A1}) \\
&\quad + \beta(E_t[(\frac{P_{a+1}^i}{P_a^i}(1+g)^{\frac{1}{1-\alpha}})^{1-\rho} p_a V_{a+1}^{1-\rho}(x_{a+1,t+1}^i, F_{a+1}^i; k_{t+1}, U_{t+1}, \eta_{t+1}, P_{t+1}^B)])^{\frac{1-1/\psi}{1-\rho}} \}^{\frac{1}{1-1/\psi}}
\end{aligned}
$$

these values are given as a weighted sum of current utility $((c_{at}^i)^{1-1/\psi})$ and the expected continuation value $(E_a V_{a+1}(.))$, which we can compute once we have obtained V_{a+1}. In the last period, regardless of whether the fixed cost has already been paid, the policy functions are trivial and the value function corresponds to the indirect utility function. This gives us the terminal condition for our backward induction procedure. Once we have computed the value of all the alternatives we pick the maximum, thus obtaining the policy rules for the current period. Substituting these decision rules in the Bellman equation we obtain this period's value function $(V_a(.))$, which is then used to solve the previous period's maximization problem. This process is iterated until $a = 1$.

We use the forecasting equations $(\Gamma_K$ and $\Gamma_P)$ to form expectations of the aggregate variables, and we perform all numerical integrations using Gaussian quadrature to approximate the distributions of the innovations to the labor income process $(\varepsilon^i$ and $\xi^i)$ and the aggregate shocks $(\eta_t$ and $U_t)$. For points which do not lie on state space grid, we evaluate the value function using a cubic spline interpolation along the cash-on-hand dimension, and a bi-linear interpolation along the other two continuous state variables $(k_t$ and $P_t^B)$. Bi-linear interpolation works well along these two dimensions because households are price takers, and

therefore these state variables are not affected by the control variables.

3) Simulating the model and clearing markets

3.1) Simulation

We use the policy functions for the two agent types (A and B) to simulate the behavior of 2000 agents of each type in each of the 81 cohorts (total of $324,000$ households) over 5500 periods. The realizations of the aggregate random variables (stochastic depreciation η_t and aggregate productivity U_t) are drawn from their original two-point distributions, while the idiosyncratic productivity shocks (ε^i and ξ^i) are drawn from the corresponding log-normal distributions. All other random variables are endogenous to the model. The realizations of the exogenous random variables are held constant within the outer loop, i.e. across iterations, so as not to affect the convergence criteria.

3.2) Market clearing

For every time period we simulate the households' behavior for every possible bond price (i.e. every point in the grid for P_t^B). We then aggregate the individual bond demands and use a linear interpolation to determine the market clearing bond price. All household equilibrium allocations (consumption and asset holdings) are then obtained from a linear interpolation with the same coefficients, while the aggregate variables (capital and output) are computed by aggregating these market clearing allocations. This then determines the state variables for simulating the next period's decisions.

4) Updating the forecasting equations:

Using the simulated time-series (after discarding the first 500 observations) we estimate the following OLS regressions, for every pair of productivity shock (U_t) and depreciation shock (η_t) realizations,

$$\ln(k_{t+1}) = q_{10} + q_{11}\ln(k_t) \tag{A2}$$

and

$$\ln(P_{t+1}^B) = q_{20} + q_{21}\ln(k_t) + q_{22}\ln(P_t^B) \tag{A3}$$

This gives us 8 equations and 8 sets of coefficients that forecast capital (k_{t+1}) and the bond price (P_{t+1}^B). We iterate the code until we have converged on the coefficients and on

the R-squared of these regressions. For the first set of equations (A2) we obtain R-squared values around 99.99%. For the second set of equations (A3), the R-squared values are in the 95% range when we only use $\ln(k_t)$ as a regressor and increase to above 99.5% when we add $\ln(P_t^B)$.

We should mention that, the real process for k_t is *not* assumed in the model and then verified in equilibrium. In our equilibrium, the forecast of k_t is an AR(1) process with state-specific intercepts and slopes (i.e. interacted with the other state variables), but this is not imposed as the real process for k_t when solving the model. Instead this process is left as a "free" equilibrium outcome that can potentially depend on all the state variables in the economy (at all possible lags). It is only in the simulations, that we verify that the household's conjecture was indeed "correct" (extremely accurate). All that is required is to find a good forecast for this process. As long as we can forecast it very well, any function will be fine.[35]

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[35]We have considered alternative forecasting equations for k_t. Namely we've added $(\ln(P_{t+1}^B)$, $(\ln(k_t))^2$, $(\ln(P_{t+1}^B))^2$, and $(\ln(k_t))(\ln(P_{t+1}^B))$ as additional regressors. These specifications also delivered an R^2 of (at least) 99.9%, and exactly because of that, they give the same forecast and hence the same equilibrium (up to 2 decimal places).

Appendix 2: Additional Comparative Statics

Preference parameters

Table $A1$ reports changes in key moments of the model from perturbing the main preference parameters. There is no change in the standard deviation of equity returns (therefore not reported) because stock market volatility is determined almost exclusively by stochastic depreciation (s).

Decreasing ψ^B (the type$-B$ investors' EIS) or decreasing β (the discount factor) produces similar results. In both cases there is a reduction in wealth accumulation (K/Y falls) implying an increase in the average returns of both assets. The supply of the riskless asset is constant while the capital stock is endogenous. Market clearing therefore implies that the risk-free rate increases by more than the return on capital. Hence, in equilibrium, the equity premium falls. The stockholders' consumption growth volatility decreases slightly since there is a reduction in the ratio of financial wealth to labor income. Naturally, changing the discount factor also significantly impacts the volatility of non-stockholders consumption growth, since this parameter has the same value for both types.

As we decrease the type-B investors' risk aversion (ρ^B), we have two effects on returns. First, total wealth accumulation falls leading to an increase in the average returns on both assets. Second, these investors are now more willing to hold equity, which simultaneously increases the risk-free rate and decreases the expected return on capital. These two effects lead to a clear increase in the risk-free rate. In addition, we should observe a larger decrease in the equity premium than when decreasing ψ^B or β. The impact on the capital stock and the expected equity return, however, is ambiguous. For these parameter values, the capital output ratio decreases slightly thus increasing the equity return. The standard deviation of stockholders' consumption growth falls, again due to a reduction in the ratio of financial wealth to labor income.[36]

Finally, the participation rate is almost unaffected in all cases. Stock market participation

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[36]The standard deviation of non-stockholders consumption growth is also marginally affected for two reasons. First, there is the indirect impact on the type-A investors (most of whom are non-stockholders) through the change on the level and volatility of the risk-free rate. Second, some of the type-B investors are also non-stockholders.

is essentially determined by the fixed cost and the preference parameters of the type-A investors.

Other parameters

In Table $A2$ we present comparative statics on the technology parameters. An increase in σ_U is reflected in higher volatility of output (not reported) and consumption. As aggregate uncertainty increases so does precautionary savings. Consequently, the capital-output ratio increases and asset returns fall. Since bond supply is constant the riskless rate must adjust more than the return on capital, leading to a modest increase in the equity premium. A higher π (persistence of U) makes the transitory shocks harder to smooth. Therefore agents save less and the returns on both assets increase. This effect is very small, though, and it depends on the level of π being considered.

A lower depreciation rate (δ) makes equity relatively more attractive, causing the capital stock to increase substantially. Mechanically, a 2% reduction in δ would lead to an equal increase in the average return on capital. However, since the capital stock is also higher, the equilibrium average equity return only increases by 0.77%. The risk-free rate also rises, reflecting the change in the relative demand for the two assets.

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Table 1: Moments from the wealth-to-income ratio distribution.

The table reports values from the baseline model and the data (2001 Survey of Consumer Finances). Income is defined as the sum of wages and salaries, unemployment or worker's compensation and pensions, annuities, or other disability or retirement programs. Wealth is defined as liquid assets plus home equity. Liquid wealth is made up of all types of transaction accounts, certificates of deposit, total directly-held mutual funds, stocks, bonds, total quasi-liquid financial assets, savings bonds, the cash value of whole life insurance, other managed assets (trusts, annuities and managed investment accounts) and other financial assets. Home equity is defined as the value of the home less the amount still owed on the first and 2nd/3rd mortgages and the amount owed on home equity lines of credit.

	Model	Data
25th percentile	0.63	0.33
Median	2.81	1.75
75th percentile	5.47	5.25

Table 2: Stock market participation rates $(\overline{P})$.

The second row reports data from the 2001 Survey of Consumer Finances; the third row reports the unconditional results from baseline model; the fourth and fifth rows report the average participation rates for each type of agent. Type-A agents have risk aversion equal to 1.1, and elasticity of intermporal substitution equal to 0.1, and type-B agents have risk aversion equal to 5 and elasticity of intertemporal substitution equal to 0.4.

	$\overline{P}$
Data	51.9%
Model (average)	53.1%
Model (Type-A)	7.4%
Model (Type-B)	98.8%

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Table 3: Asset pricing moments from the data (Campbell (1999)) and from the baseline model.

Variable	Moment	Model	Data
Risk-free Rate	Mean	2.58%	1.58%
	Std. Dev.	3.26%	5.33%
	AR(1)	0.89	0.52
Equity Return	Mean	6.41%	8.31%
	Std. Dev.	16.31%	19.81%
	AR(1)	-0.05	-0.06
Risk Premium	Mean	3.83%	6.74%

Table 4: Consumption volatility.

The second row reports the standard deviation of average stockholders' consumption growth from the baseline model and from the data (Consumer Expenditure Survey, numbers taken from Malloy, Moskowitz and Vissing-Jørgensen (2006) and annualized). The third row reports the same statistics for nonstockholders' consumption growth.

Variable	Moment	Model	Data
Cons.Growth (S)	Std. Dev.	5.02%	3.6%
Cons.Growth (NS)	Std. Dev.	2.65%	1.4%

Table 5: Asset pricing moments from the data (Campbell (1999)) and from the model with different values of the fixed entry cost (F).

Variable	Moment	$F = 0.06$	$F = 0.00$	Data
Riskless Rate	Mean	2.58%	2.57%	1.58%
	Std. Dev.	3.26%	3.25%	5.33%
Equity Return	Mean	6.41%	6.37%	8.31%
	Std. Dev.	16.31%	16.31%	19.81%
Risk Premium	Mean	3.83%	3.80%	6.74%

Table 6: Asset pricing moments from alternative versions of the model with different sources of household heterogeneity.

In both models the type-B agents ('stockholders') have $\rho^B = 3$, $\psi^B = 0.6$, $\beta^B = 0.99$ and $\lambda^B = 0.6$. In Model 1 the type-A agents ('non-stockholders') have $\rho^A = 3$, $\psi^A = 0.05$, $\beta^A = 0.7$ and $\lambda^A = 0.8$. In Model 2 the type-A agents ('non-stockholders') have $\rho^A = 4$, $\psi^A = 0.05$, $\beta^A = 0.6$ and $\lambda^A = 0.8$. The last row ('ΔRisk Premium $(F = 0)$') reports the change in risk premium resulting from the corresponding version of each model with $F = 0$.

Variable	Moment	Baseline	Model 1	Model 2
Riskless Rate	Mean	2.58%	2.54%	2.55%
	Std. Dev.	3.26%	3.00%	3.00%
Equity Return	Mean	6.41%	5.66%	5.67%
	Std. Dev.	16.31%	16.26%	16.26%
Risk Premium	Mean	3.83%	3.12%	3.12%
ΔRisk Premium $(F = 0)$	Mean	0.03%	0.03%	0.03%

Table 7: The role of preference heterogeneity.

The results from the model are reported in columns three and four. Column three ("Two types") shows the results for the baseline calibration (with $F = 0$). Column four ("One type"), refers to the model where all agents have the same preference parameters ($\rho = 5$, $\psi = 0.15$ and $\beta = 0.99$). The asset pricing data is taken from Campbell (1999) while the consumption data is from Malloy, Moskowitz and Vissing-Jørgensen (2006).

Variable	Moment	Two types	One type	Data
Riskless Rate	Mean	2.57%	0.69%	1.58%
	Std. Dev.	3.25%	3.29%	5.33%
Equity Return	Mean	6.37%	4.65%	8.31%
	Std. Dev.	16.31%	17.13%	19.81%
Risk Premium	Mean	3.80%	3.97%	6.74%
Cons.Growth (B)	Std. Dev.	5.02%	3.06%	3.6%

Table 8: Impact of aggregate uncertainty.

Results with the baseline value of s (16%), and with $s = 0$. The asset pricing data is taken from Campbell (1999) while the consumption data is from Malloy, Moskowitz and Vissing-Jørgensen (2006).

Variable	Moment	Baseline	$s = 0$
Riskless Rate	Mean	2.58%	4.49%
	Std. Dev.	3.26%	0.22%
Equity Return	Mean	6.41%	5.33%
	Std. Dev.	16.31%	0.37%
Risk Premium	Mean	3.83%	0.83%
Cons.Growth (S)	Std. Dev.	5.02%	0.22%
Cons. Growth (NS)	Std. Dev.	2.65%	0.06%

Table 9: Impact of idionsyncratic risk and borrowing constraints.

Coulmn three reports our baseline results. In columns four through six we consider different deviations from this baseline case. In column four the volatility of the two idiosyncratic shocks (σ_ε and σ_ξ) is set equal to zero. In columns five and six we allow for borrowing subject to different limits, expressed as a fraction of the household's expected next-year's income ($\overline{B}$). The asset pricing data is taken from Campbell (1999) while the consumption data is from Malloy, Moskowitz and Vissing-Jørgensen (2006).

Variable	Moment	Baseline	$\sigma_\varepsilon=\sigma_\xi=0$	$\overline{B}=25\%$	$\overline{B}=50\%$
Riskless Rate	Mean	2.58%	4.81%	2.98%	4.14%
	Std. Dev.	3.26%	3.25%	3.24%	3.47%
Equity Return	Mean	6.41%	8.29%	6.60%	7.10%
	Std. Dev.	16.31%	16.42%	16.32%	16.37%
Risk Premium	Mean	3.83%	3.49%	3.62%	3.17%
Cons. Growth (S)	Std. Dev.	5.02%	4.57%	4.88%	4.72%
Cons. Growth (NS)	Std. Dev.	2.65%	2.64%	2.63%	2.34%

Table 10: Model with different levels of government debt.

The results from the model are reported in columns three and four. Column three ("$B/Y = 38\%$") is our baseline model. Column four ("$B/Y = 15\%$") reports the results for the same parameter values as the baseline calibration except that the debt to GDP ratio (B/Y) is now equal to 15%. The asset pricing data is taken from Campbell (1999) while the consumption data is from Malloy, Moskowitz and Vissing-Jørgensen (2006).

Variable	Moment	B/Y=38%	B/Y=15%	Data
Riskless Rate	Mean	2.58%	0.89%	1.58%
	Std. Dev.	3.26%	3.61%	5.33%
Equity Return	Mean	6.41%	5.51%	8.31%
	Std. Dev.	16.31%	16.30%	19.81%
Risk Premium	Mean	3.83%	4.62%	6.74%
Cons.Growth (S)	Std. Dev.	5.02%	5.56%	3.6%
Cons. Growth (NS)	Std. Dev.	2.65%	2.72%	1.5%

Table 11: Changing the EIS

Coulmn three reports our baseline results. In columns four through six we consider different values of ψ^B (elasticity of intertemporal substitution of the type-B agents). The asset pricing data is taken from Campbell (1999) while the consumption data is from Malloy, Moskowitz and Vissing-Jørgensen (2006).

Variable	Moment	Baseline	$\psi^B = 0.5$	$\psi^B = 0.6$	$\psi^B = 0.75$
Riskless Rate	Mean	2.58%	1.60%	0.89%	0.14%
	Std. Dev.	3.26%	2.89%	2.77%	2.59%
Equity Return	Mean	6.41%	5.52%	4.86%	4.17%
	Std. Dev.	16.31%	16.29%	16.26%	16.23%
Risk Premium	Mean	3.83%	3.92%	3.97%	4.03%
Cons. Growth (S)	Std. Dev.	5.02%	5.65%	6.37%	7.45%
Cons. Growth (NS)	Std. Dev.	2.65%	2.63%	2.62%	2.61%

Table A1: Perturbations in the preference parameters.

This table reports changes relative to the values obtained in the baseline model ($\psi^B = 0.4$, $\beta = 0.99$ and $\rho^B = 5$). All other parameter values are those used in the baseline model. All reported values are arithmetic differences relative to the baseline results, except for the capital output ratio which is a percentage difference.

Moment	$\psi^B = 0.5$	$\psi^B = 0.3$	$\beta = 0.98$	$\rho^B = 4$
Mean Riskless Rate	−0.99%	2.17%	1.28%	1.06%
Std. Dev. Riskless Rate	−0.14%	0.34%	−0.01%	−0.09%
Mean Equity Return	−0.89%	1.89%	1.16%	0.71%
Mean Risk Premium	0.10%	−0.28%	−0.13%	−0.35%
Std. Dev. Cons. Growth (S)	0.86%	−1.09%	−0.23%	−0.14%
Std. Dev. Cons. Growth (NS)	0.00%	−0.02%	−0.02%	−0.02%
Capital-Output Ratio	5.6%	−10.3%	−6.83%	−4.30%
Stock Market Participation Rate	−0.40%	1.35%	−0.10%	−0.08%

Table A2: Perturbations in the technology parameters.

This table reports changes relative to the values obtained in the baseline model ($\sigma_u = 0.01$, $\pi = 2/3$ and $\delta = 0.1$). All other parameter values are those used in the baseline model. All reported values are arithmetic differences relative to the baseline results, except for the capital output ratio which is a percentage difference.

Moment	$\sigma_u = 0.02$	$\pi = 0.8$	$\delta = 0.08$
Mean Riskless Rate	−0.01%	0.08%	0.28%
Std. Dev. Riskless Rate	0.01%	−0.06%	0.25%
Mean Equity Return	−0.00%	−0.02%	0.77%
Mean Risk Premium	0.01%	−0.1%	0.49%
Std. Dev. Cons. Growth (S)	0.02%	−0.02%	0.52%
Std. Dev. Cons. Growth (NS)	0.39%	−0.04%	−0.03%
Capital-Output Ratio	0.03%	−0.33%	8.44%
Stock Market Participation Rate	−0.1%	−0.01%	−0.12%

Figure 1 - Life-Cycle Wealth Accumulation, Labor Income and Consumption