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FINANCIAL CONSTRAINTS AND THE COSTS AND BENEFITS OF VERTICAL INTEGRATION

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ABSTRACT

Financial Constraints and the Costs and Benefits of Vertical Integration*

Does vertical integration reduce or increase transaction costs with external investors? This paper analyzes an incomplete contracts model of vertical integration in which a seller and a buyer with no cash need to finance investments for production. The firm is modeled as a "nexus of contracts" across the intermediate input supply and the financing transaction. The costs and benefits of vertical integration depend on the relative importance of a positive "contractual centralization" effect against a negative "de-monitoring" effect: the firm centrally organizes the nexus of contracts reducing the extent of contractual externalities while the market disciplines decisions driven by private benefits. Larger projects, more specific assets, and low investors protection are determinants of vertical integration.

JEL Classification: D23, G32, K12, L22 and O10
Keywords: contractual externalities, investors protection, limited liability, theory of the firm and vertical integration

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1 Introduction

There is substantial evidence that firms are constrained in their investment decisions in developing and developed countries as well.[1] Poor access to finance is often offered as a key explanation for the shape of the size distribution of firms in developing countries, which often displays very high shares of employment in enterprises with less than 5 employees, while most of the rest work for very large firms, leaving little room for employment in medium sized firms (the so called "missing middle"). This is in stark contrast with the richest countries where a significant share of workers is employed by large and medium-size enterprises. Finally, there is anecdotal evidence that supply chains are organized differently across countries with different levels of financial development. For instance, Rajan and Zingales (1995) document that corporate balance sheets in developed countries have large items that reflect interactions among firms (as suppliers and customers). Consistently with this evidence, Khanna and Palepu (1997, 2000) suggest that in response to widespread market failures firms in less developed countries tend to be more vertically integrated, echoing the findings reported by Acemoglu et al. (2005) on cross-country differences in industrial composition.

Motivated by this evidence, this paper focuses on the consequences of financial constraints on vertical integration, asking whether vertical integration reduces or increases transaction costs with external investors. The focus on vertical integration is motivated by practical as well as theoretical concerns. At a practical level, it is important to understand how financial constraints influence the organization of the supply chain (for example forward integration into distribution channels or backward integration for the procurement of specialized inputs). At a theoretical level, the analysis of vertical integration has been at the center stage of modern theories of the firm (see e.g. Williamson (1971, 1975 and 1985) and Hart (1995)).

I build a model in which a seller produces a good that can be used by a buyer, or sold on a spot market. The buyer and the seller have no cash, need to finance the investments for production, and cannot foresee in advance whether the input is most efficiently traded on the spot market or with

[1] A survey of firms around the world finds that access to finance is systematically reported as one of the key constraints on creation, expansion and performance of firms (Batra et al. (2001)). The World Business Enterprises Survey at the World Bank available online shows that in countries as diverse as Pakistan, Egypt or Ecuador more than 80% of surveyed firms report that collateral requirements of banks and financial institutions are problematic (the average value of the collateral required as a percentage of the loan value is as high as 311% in Zambia, 226% in Morocco and 203% in Nicaragua). Banerjee and Munshi (2004), Banerjee and Duflo (2004), Banerjee, Duflo and Munshi (2004) provide convincing evidence from India.

each other (contracts are incomplete). I make two key assumptions. First, I assume that ownership of physical assets gives control over contracting rights to those assets, i.e. it determines who has the right to sign trade contracts (residual control rights) and compare two alternative configurations. Under *non-integration* both sides can refuse to trade with the other party while under (buyer) *vertical integration* the buyer can decide the trade configuration by fiat, i.e. she can impose to the two divisions of the integrated firms to trade with each other or on the spot market. Second, I assume that, since (part of the) returns cannot be verified, financial streams get transferred with ownership. This is line with the idea that control rights entail the right to sign contracts with third parties, and contracts can be used to generate private benefits for the party in control.[2]

I compare vertical integration against non integration in terms of their pledgeable income, i.e. the highest expected returns that can be credibly transferred to external investors. The relative costs and benefits of vertical integration in terms of pledgeable income depend on the intensities of a positive "*contractual centralization*" effect against a negative "*de-monitoring*" effect. On the one hand, vertical integration centrally organizes the nexus of contracts, greatly reducing the extent of contractual externalities. In other words, external investors face fewer holdups and profits are collectively pledged. I label this first positive effect of vertical integration *contractual centralization* effect. This positive effect, comes at the cost of suppressing valuable sources of information associated with market transactions. Since the seller might refuse to trade with the buyer when such trade produces low joint profits, the bargaining process between two independent firms becomes an indirect source of information from the point of view of external investors. I label this second negative effect of vertical integration "*de-monitoring*" effect.[3]

[2] These two assumptions are well in line with the work of legal scholars and business historians. The firm in the model is a "nexus of contracts" across the intermediate input supply and the financing transaction (see e.g. Cheung (1983)) characterized by centralized allocation of control rights *and* joint liability of the financed investments. Hausmann and Kraakman (2001) argue that "To serve effectively as a nexus of contracts, a firm must generally have two attributes. The first is well defined decision-making authority. More particularly, there must be one or more persons who have ultimate authority to commit the firm to contracts" ... "The second attribute a firm must have, ... is the ability to bond its contracts credibly .... Bonding commonly requires that there exist a pool of assets that the firm's managers can offer as satisfaction for the firm obligations". Similarly, in an analysis of organizational forms in the United States in the nineteenth century, Lamoreaux (1998) argued that "no clear economic boundary distinguished ordinary contracts from those considered by the law to be firm (...) business people could choose from a range of contractual forms that offered varying degrees of firmness". Firmness in turn, is defined along two main dimensions: liability and firm's autonomy, the latter referring to the extent to which the firm had legal existence, - the possibility of writing binding contracts - beyond that of its members. Similar issues are discussed in the debate on "piercing the corporate veil" (see e.g. Posner (1976) and Landers (1976)).

[3] In his first classical paper on vertical integration and market imperfections, Williamson (1971) argue that " (...) unable to monitor the performance of *large, complex* organizations in any but the crudest way (...) investors

While there exists a large literature on the working of internal capital markets which is closely related to this work (see Stein (2003) for an excellent survey), the two effects underlined in the model are conceptually distinct from previous contributions in the literature and are (relatively more) specific to a setting in which the two projects are linked by a "common destiny" (as it is the case in specialized supply chains). The positive "*contractual centralization*" effect of integration is different from advantages of centralized finance related to diversification (see e.g. Diamond (1984) and Inderst and Mueller (2003)), or to better allocation of capital within hierarchies (see e.g. Williamson (1975), Stein (1997) and Matsusaka and Nanda (2002)). On the other hand, the negative "*de-monitoring*" effect of integration is different from arguments identifying the costs of centralized finance in terms of higher degrees of rent seeking (see e.g. Rajan, Servaes and Zingales (2000), Scharfstein and Stein (2000)), influence activities (see e.g. Meyer, Milgrom and Roberts (1992)) or soft budget constraints (see e.g. Dewatripont and Maskin (1995)).[4]

Factors affecting the net balance of the trade-off between the two effects are expected to be determinants of vertical integration. The model predicts that a higher degree of specificity linking the two projects favours vertical integration. Assets that are likely to be used together should also be managed and financed together. The main reason is that when the two units are likely to trade together the indirect monitoring role of market transactions is reduced, and the benefits of non-integration do not compensate for the costs. The model also predicts that larger projects (in terms of investment, profits and quasi-rents) are more likely to be financed under the umbrella of a single firm and that vertical integration becomes relatively more likely in environments characterized by low investor protection and low verifiability of cash flows.

Finally, since the analysis of the determinants of vertical integration has been a central theme in the modern literature on the theory of the firm, it is worth relating this work to that literature. In contrast to the property rights theory of the firm (Grossman and Hart (1986), Hart and Moore (1990) and Hart (1995)), I de-emphasize the importance of the assignment of control rights in shaping incentives to undertake *ex-ante* non contractible investments.[5] The model instead focuses

demand larger returns as finance requirements become progressively greater, *ceteris paribus*". The model in this paper suggests that vertically integrated firms are more complex and inherently more difficult to monitor because they bring the bargaining process within firm's boundaries.

[4] While the model does not provide an analysis of large and publicly held corporations, the "de-monitoring" effect echoes the idea that separate firms with separate stock prices have more stock market monitoring. However, in the present context the monitoring is indirectly done by the specific input market, not by the stock market.

[5] The property right approach has been applied to the analysis of the financial structure of firms (see Hart and

on the consequences that firm's boundaries have on the *ex-post governance* of the firm, and in particular on the transaction costs with external investors.[6] Since the model presents the firms as a nexus of contracts, the input transaction (i.e. the make-or-buy decision), is not anymore the exclusive focus of the analysis. The organization of the transaction between a buyer and a seller depends on the nexus of contracts linking these parties to other economic actors (in this case, external investors). In its emphasis on the notion that control over assets gives control over contracting rights on the use of those assets, the model in this paper is mostly related to Holmstrom (1999) (see also Holmstrom and Tirole (1987, 1991)).

The paper is organized as follows. Section 2 presents the model and the main assumptions. Section 3 derives the pledgeable income under vertical integration and under non-integration. Section 4 compares the pledgeable income under the two organizational forms, and presents the main results on the determinants of vertical integration. Section 5 discusses in more detail the relationship between various forms of contractual externalities and vertical integration, as well as joint liability contracts among independent firms. Section 6 offers some concluding remarks.

2 The Model

Set up: Technology and Private Benefits

Consider two managers, a buyer and a seller, $j \in \{d, u\}$ respectively in charge of two different projects: a downstream (d) unit and an upstream (u) unit. The upstream unit produces a good that can be used by the downstream unit, or sold to an external market. The two managers are aware of the possibility that certain features of the input may make the input best suited to be traded on the spot market or with each other, but they cannot foresee in advance the nature of these features, and hence cannot write an ex-ante contract which is contingent on the nature of ex-post trade.

Trade between the two units (henceforth "internal trade") generates joint profits V_i while trade

Moore (1994), Aghion and Bolton (1992) and Bolton and Sharfstein (1992)). These contributions treat the firm as a single entrepreneur and hence can not ask what determines firm's boundaries. The property right approach has also been extended to consider cash constrained agents (Aghion and Tirole (1994, 1997), Legros and Newman (2004), Acemoglu et al. (2006)). The general theme emerging from this literature is that control rights may be exchanged for cash.

[6] Ex-post governance costs and benefits of integration were central in earlier transaction costs theories of integration (see e.g. Williamson (1971, 1975)), and are the focus of some recent contributions in the literature. For an excellent discussion of these issues and further references, see Gibbons (2004).

on the input spot market generates joint profits V_m. Whether internal trade or trade on the input spot market is more profitable depends on the state of nature. Let $V \in \{\underline{V}, \overline{V}\}$ denote the joint profits of the two units and $\Delta V = \overline{V} - \underline{V} > 0$ be the difference between the two levels of joint profits. I assume that with probability π internal trade generates joint profits $V_i = \overline{V}$ while trading on the spot market generates joint profits $V_m = \underline{V}$. With the complementary probability $(1 - \pi)$ joint profits from internal trade are $V_i = \underline{V}$ while joint profits from trade on the spot market are equal to $V_m = \overline{V}$. I assume that whenever the two units trade on the market, the downstream unit realizes profits equal to βV_m and the upstream unit realizes profits equal to $(1 - \beta)V_m$. The parameter π captures the specificity of the relationship. When π is close to one, the input is very specific to the needs of the buyer, and internal trade almost always generates higher surplus than trade on the spot market. The parameter β is instead determined by industry conditions. High β is more likely if, when the buyer purchases the input on the spot market, she realizes relatively high profits, perhaps because many firms compete in the upstream industry driving down input's price on the spot market. When β is relatively small instead, the seller can easily sell the input on the spot market at a fairly high price, perhaps because competition among several potential buyers drives input's price up.

I assume that with probability σ the buyer derives private benefits b from the trade configuration that yields low joint profits $\underline{V}$. These private benefits cannot be transferred, and are unverifiable. The level and realization of private benefits could be affected by factor as diverse as technology, belonging to social networks, or legal environment. For instance, in designing a product, the buyer may find it ex-post profitable (e.g. a reduction in non monetary costs) to tailor the good on the specification of an existing good readily available on the market even when the seller produces an input that would generate higher profits if used in the production process. Alternatively, the buyer may find out that her capabilities have a better fit, e.g. in terms of acquiring certain skills, with the specifications of the input produced by the seller, even if trading on the input market would generate higher joint profits. Another example, would be the possibility of trading with relatives or other members of a network that generates benefits that cannot be transferred to third parties, instead of trading with the partner (possibly on an anonymous market) that would guarantee higher joint profits.

In sum, four states of nature can be realized, depending on whether internal trade or trade on the spot market yields high joint profits, and on whether the buyer derives private benefits from the trade configuration yielding low joint profits. The realization of the state of nature is perfectly observed by the buyer, while the seller only observes whether internal trade generates high of low joint profits if she has control rights over some assets. Finally, I assume that third parties, such as investors and courts, do not observe the state of nature. I assume for simplicity that private benefits are completely unspecific to the relationship, i.e. that the realizations of private benefits and joint profits from the two trading configurations are independent. I also assume that the seller never derives private benefits from the trade configuration that generates low joint profits. It is possible to relax these two assumptions without gaining much further intuition.

Ownership

Ownership determines who has the right to decide on whether trade takes place between the two units or on the spot market. I focus for simplicity on two different configurations. Under *non-integration* both sides can veto internal trade. In the absence of a previous enforceable contract, two independent firms trade with each other if and only if the two managers agree. Under (buyer) *vertical integration* instead the manager of the downstream unit can decide the trade configuration by fiat, i.e. she can impose internal trade between the two divisions of the integrated firms or impose to both units to trade on the spot market.[7]

As a result of this assumption, one key difference between vertical integration and non integration is that in an integrated firm there is no bargaining (one party unilaterally decides the trade configuration) while under non integration when the two firms trade with each other they need to bargain to determine the price P for the transaction. For the bargaining process, I assume that the buyer has the right to make a take-it-or-leave-it offer with probability α, while with the complementary probability the seller has the right to make a take-it-or-leave-it offer.[8]

The second difference between vertical integration and non integration is related to financial streams. I assume that the financial streams get transferred with ownership. While it is true that

[7]Since the upstream manager never derives private benefits from the trade configuration that generates low joint profits, giving him ownership would be best from the point of view of external investors. To make the problem simple and interesting, I assume that this arrangement is not feasible. It is straightforward to relax these assumptions (see Macchiavello (2005)).

[8]Another difference between integration and non integration is that the seller observes which trade configuration yields high joint profits only if the two firms are non integrated. This assumption, while not essential for the analysis, is reasonable if one assumes that in practice the state of the world is at least in part learned through the exercise of contracting rights (e.g. negotiating input prices, hiring consultants, etc...).

parts of the financial streams and decisions rights can be transferred through contracts, in reality there is typically a connection between the right to decide and the financial responsibility for the outcome of the decision. This assumption is consistent with the idea that separating the return streams of the productive assets from the decision rights is not entirely feasible, because (part of) the returns cannot be verified. For example, control rights typically entails the right to sign contracts with third parties, and these contracts can be used to generate private benefits for the party in control. To formalize these ideas, I assume that whenever a manager has control, she can hide profits making them unverifiable and keep a fraction ϕ of those profits for herself. I interpret the parameter ϕ as a proxy for the degree of external investor protection in the economy.[9] Under vertical integration, the downstream manager has control over the entire profits V, while under non integration the buyer and the seller have control over the profits of their respective firm ($\beta\ V_m$ and $(1-\beta)V_m$ respectively when they trade on the input market, and $V_i - P$ and P when they trade with each other). Subject to the limits imposed by the possibility of "stealing", profits are contractible, and payments to external investors can be made contingent on their realization.

Initial Contract and Timing of Events

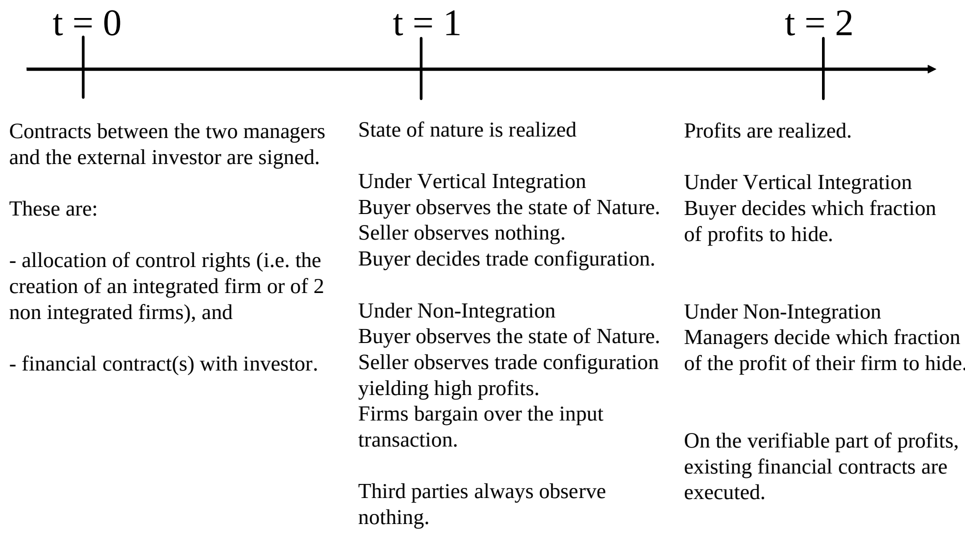
I assume that the two managers are essential for production, but have no cash. The downstream units has a fixed set up cost k_d while the upstream unit has a fixed set up cost k_u. The two managers hence need to borrow in order to start the (two) firm(s). I assume that the two managers have an outside option equal to zero. There is a unique risk neutral investor that contracts with both managers and that has all the ex-ante bargaining power.[10] Contracts are thus designed to maximize the pledgeable income of the two projects.

Given the simplicity of the payoff structure, I focus on simple debt-like contracts. In particular I assume that the investor holds a debt-like claim B over the profits of a firm. When the firm is integrated there is a unique B. When the two firms are not integrated, the investor holds claims B_d and B_u on the profits of the downstream and upstream firm respectively. When the two firms are non integrated, I assume for now that the repayment of each firm is not made contingent on

[9] External investors' protection from expropriation is the single most important factor explaining large cross-countries differences in the access of firms to external finance (see e.g. La Porta et al. (1997)). An alternative interpretation would link ϕ to the extent to which firm's investments are "tangible".

[10] I discuss the implications of allowing for multiple investors in section 5. Note however that the two units might also try to maximize the cash that they can raise from external investors in order to use this money to transfer rents according to the initial distribution of bargaining power.

Figure 1: Timing of Events



the repayment of the other firm, i.e. I rule out joint liability contracts.[11]

To summarize, the timing of events is as follows. At date 0 all the feasible contracts are signed. These are the financial contracts signed with the external investor, and the creation of an integrated firm or of two separate firms (the allocation of control rights). At date 1, the state of nature is realized and observed by the buyer. If the seller has control rights over its unit, she observes which trade configuration yields high joint profits, otherwise she observes nothing. Third parties always observe nothing. If the firm is integrated, the buyer decides whether the two divisions should trade the input with each other or on the spot market. Under non integration instead the two separate firms bargain over the price, according to the process specified above. At date 2 profits are realized. If the firm is integrated the buyer decides whether to hide profits or not. If the

[11] I discuss the implications of relaxing this assumption in Section 5. I also assume that the investor can not holds claims which are contingent on the identity of the trading partners. When the firm is integrated it seems natural to assume that the investors can not easily distinguish whether the input was traded between the two divisions of the firm, or on the spot market. When the two firms are non integrated instead, the investor could hold different claims on the profits of the two firms depending on whether the two firms trade with each other or not. The two managers may find profitable to sign contracts with third parties that would undo these provisions.

firm is not integrated, each manager decides whether to hide the profits of her firm. Finally, on the verifiable part of profits, existing financial contracts are executed. The timing of the game is illustrated in Figure 1.

3 Organizational Form and Pledgeable Income

In order to focus on the main results, and to avoid a long taxonomy of cases that do not add further intuition, I introduce the following assumptions on the parameters of the model.

Assumptions

A1: $\max\{2-\frac{1}{\beta},0\}\Delta V \leq \phi\Delta V < b < \beta\Delta V$,

A2: $\beta\overline{V} < \Delta V$,

A3: $\pi \leq \frac{\beta\overline{V}}{\Delta V+\beta\underline{V}}$,

A4: $k_u + k_d = K$ and $\underline{V} < \frac{K}{1-\phi} < \overline{V}$

Assumption A1 implies that private benefits that the buyer obtains from the low joint profits configuration are larger than the private benefits she can possibly create by hiding the difference in the joint profits ($b > \phi\Delta V$), and assures that the agency conflict with the buyer has potentially some bite. On the other hand, private benefits are sufficiently small so that if the low joint profits trade configuration is implemented, not only transferrable profits are lower, but also aggregate surplus is reduced ($b < \beta\Delta V$). Assumption A1 also implies that transaction costs associated with hiding profits ϕ are bounded ($\phi \in [2-\frac{1}{\beta},\beta]$) and simplifies the analysis.

Assumption A2 introduces an upper bound on the share of profits that the buyer can realize by trading on the spot market and implies that the downstream firm cannot convince the upstream firm to undertake internal trade whenever it generates low joint profits $(1-\beta)\overline{V} > \underline{V}$.[12] Assumption A3 puts an upper bound on the ex-ante likelihood that internal trade yields high joint profits. Finally A4 implies that the two projects have a joint positive net present value ($K < (1-\phi)\overline{V}$) and yet the financing problem is not trivial ($(1-\phi)\underline{V} \leq K$). Assumptions A2, A3 and A4 simplify the exposition and are not crucial for the main results.

[12] The fact that the two managers do not have cash is important, as it prevents the buyer from bribing the seller to accept trade. Related to this point, I consider cases in which the investor does not lend more than K.

Before I determine the pledgeable income under the two organizational forms, I consider the trading configuration implemented if the two managers have deep pockets.

Lemma 1 *Integration generates total surplus* $\overline{V} - K$. *Non integration generates total surplus* $\overline{V} - \pi\sigma(1-\alpha)(\Delta V - b) - K$ *if* $\sigma\Delta V < b$ *and* $\overline{V} - K$ *otherwise.*

Under integration, the buyer is the residual claimant of the profits of the firm, and A1 ensures that $\overline{V} > \underline{V} + b$. The unique residual claimant of firm's profits would always implement the high profits action, which also maximizes social surplus. Under non integration instead the two firms have to bargain over the price P. As a benchmark, it is useful to consider the case $\alpha = 1$. Under this scenario, the buyer always has the right to make a take-it-or-leave-it offer and the ex-post bargaining process is efficient. When this is the case, in the absence of non contractible investments, the organizational form does not matter for efficiency. When $\alpha < 1$ instead the ex-post bargaining process occurs under asymmetric information with probability $(1-\alpha)$, and it is thus, in general, not efficient. In particular, if $\sigma\Delta V > b$ the seller bargains too aggressively to extract the highest possible price at the expense of total efficiency. The simple model thus captures the original intuition of the transaction cost literature (see e.g. Williamson (1971), (1975)) that vertical integration by replacing the bargaining process with fiat reduces the inefficiencies caused by contract incompleteness.[13] In this context, α is a measure of the degree of inefficiency in the ex-post bargaining process, as well as a measure of the bargaining power of the buyer.

In the next subsections I analyze the situation when the buyer and the seller do not have deep pockets and need to finance the initial investments k_d and k_u. I first derive the highest joint pledgeable income under vertical integration and under non-integration. In the next section I compare the two organizational forms in terms of pledgeable income.

3.1 Vertical Integration

Under vertical integration, the buyer decides which trade configuration should be implemented and has control over the joint profits generated by the two units. Let's consider the investor's problem.

[13] If the upstream managers also derives non observable private benefits from the trade configuration that yields low joint profits, integration would always dominate. On the other hand, the assumption that bargaining occurs under assymetric information with probability $(1-\alpha)$ is not essential for our results (see the first corollary in subsection 3.2). Schmitz (2006) and Matouschek (2003) are examples of models of vertical integration with inefficient ex-post bargaining.

The investor chooses debt level B in order to maximize the pledgeable income of the integrated firm. Since control rights over the trade configuration are centralized, the two trade configurations are perfectly symmetric and there are only two relevant states of the world from the point of view of the external investor: with probability σ the buyer derives private benefits b from the action yielding low cash flows, and with complementary probability $(1-\sigma)$ she does not.

Let $\mathbf{I}(s \mid B) \in \{0,1\}$ be an indicator function taking values equal to one if, given debt B, in state s the buyer repays debt. Let π_s be the probability that state s is realized. The investor solves

$$\max_{B} \mathbf{\Sigma}_s \pi_s B \cdot \mathbf{I}(s \mid B)$$

As in standard models with hidden action, the investor trade-off a higher debt level B with a higher probability that the debt is repaid. The financial contract uses debt B to provide incentives on two dimensions. First, the investor wants to encourage the buyer to select the appropriate course of action in terms of cash flows. Second, she wants to make sure that the buyer does not hide cash flows. Denoting by P_{int} the solution to the investor's problem, i.e. the highest pledgeable income of a vertically integrated firm, I prove in the Appendix the following proposition.

Proposition 1 *Under Assumption A1,* $P_{int} = \max\{(1-\phi)(1-\sigma)\overline{V}, \overline{V} - b - \phi\underline{V}\}$.

The investor can choose a relatively low level of debt $B_{int} = \overline{V} - b - \phi\underline{V}$ that leaves to the manager rents equal to the private benefits $b + \phi\underline{V}$ that can be acquired exercising the control rights over the financial streams and the trading decisions, or she can instead set an higher level of debt $B_{int} = (1-\phi)\overline{V}$, knowing that whenever private benefits b from the low profits action are realized the debt will not be repaid.

As expected, the pledgeable income of an integrated firm is decreasing in the level of private benefits b (weakly) and in the likelihood of the agency conflict (higher σ). Moreover, the pledgeable income of an integrated firm is increasing in the degree of investor protection (decreases in ϕ) and in the joint profits generated by the two firms (higher $\overline{V}$). However, it is decreasing in $\underline{V}$, a direct measure of the importance of taking the right trading decision, but also an inverse measure of the cost of opportunism for the manager.

The pledgeable income does not depend on π, β and α. P_{int} does not depend on π since the buyer is the full residual claimant of the firm's profits, and depending on the level of debt B_{int}, will chose the appropriate trade configuration with either probability 1 or $(1-\sigma)$. This is a direct consequence of the fact that under integration, the two trading configurations become symmetric from the point of view of the borrower. The same logic also explain why P_{int} does not depend on β. The buyer, being the residual claimant, does not care about the distribution of profits between the two divisions of the firm. Finally, P_{int} does not depend on α as vertical integration eliminates all bargaining process between the two units.

It is also clear that $P_{int} < (1-\phi)\overline{V} < \overline{V}$, and hence there exist profitable investment opportunities that cannot be financed because of the agency conflict. Note also that if, contrary to Assumption A2, I assumed $b < \phi\Delta V$, I would have that $P_{int} = (1-\phi)\overline{V}$ and the only agency conflict that would have some bite is the possibility of hiding profits.

3.2 Non-Integration

Under non-integration the two units are two independent firms managed by two separate managers. Let's first consider the investor's problem. The investor chooses debts levels B_d and B_u in order to maximize the joint pledgeable income of the two non integrated firms. The problem has to be solved backward. For a given vector of debt levels B_d and B_u, I first analyze the bargaining game between the two managers depending on whether

- internal trade or trade on the spot market produces high joint profits,
- the buyer derives private benefits or not from the low cash flow action, and
- the buyer or the seller has the right to make a take-it-or-leave-it offer for the price of the input.

In any state s (there are eight, i.e. $2 \times 2 \times 2$, states), the bargaining process determines the profits of the two firms, and because of the possibility of hiding profits, B_u and B_d may or may not be repaid. Let $\mathbf{I}_d(s \mid B_d, B_u) \in \{0,1\}$ and $\mathbf{I}_u(s \mid B_d, B_u) \in \{0,1\}$ be two indicator functions taking values equal to one if, given B_u and B_d, in state s the outcome of the bargaining process is such that B_d and B_u are respectively repaid. Let π_s be the probability that state s is realized.

The investor solves

$$\max_{B_d, B_u} \mathbf{\Sigma}_s \pi_s \left(\mathbf{\Sigma}_{j \in \{d,u\}} B_j \cdot \mathbf{I}_j(s \mid B_d, B_u) \right)$$

As for the case of vertical integration, the investor will have to trade-off a higher debt levels B_d and B_u, with a higher probability that the debt is repaid. The main difference with respect to the case if vertical integration, is that the profits of the two firms, and hence the maximum debt levels that firms can credibly commit to repay, are determined by the bargaining process between the two managers.

In order to focus on the main intuition, I present the results for the two polar cases $\alpha = 1$ and $\alpha = 0$. Denote with $P_{ni}(\alpha)$ the highest joint pledgeable income of two non integrated firms as a function of α: in the Appendix I prove the following

Proposition 2 *Under Assumptions A1-A3,*

$$P_{ni}(1) = (1 - \phi) \left[\beta \overline{V} + (1 - \beta) \max\{\underline{V}, \overline{V}(1 - \pi)\} \right]$$

and

$$P_{ni}(0) = (1 - \phi) \left[\beta \max\{\underline{V}, \overline{V}(1 - \pi)\} + \max\{(\overline{V} - \beta \underline{V})\pi(1 - \sigma), (1 - \beta)\overline{V}\} \right]$$

As for the case of vertical integration, the highest joint pledgeable income of the two non integrated firms is increasing in the level of investor protection (lower ϕ) and in the level of joint profits $\overline{V}$. However, certain important differences can be noted with respect to the case of vertical integration.

Consider first the case $\alpha = 1$. In this case the buyer always makes a take-it-or-leave-it offer, and the bargaining process is efficient. To gain some intuition on how the bargaining process influences the choice of debt levels B_d and B_u, consider, as an illustration, the case of the seller. The investor can choose a debt level $B_u = (1 - \phi)(1 - \beta)\underline{V}$ which will always be repaid by the seller, since any offer made by the buyer has to be greater than $(1 - \beta)\underline{V}$, which is the minimum outside option for the seller in the bargaining process. Alternatively, the investor can set an higher debt $B_u = (1 - \phi)(1 - \beta)\overline{V}$, but whenever trade should take place internally, the buyer offers such a low price that the seller accepts, but does not find it profitable to repay the debt B_u (this happens with probability π).

As a consequence of this logic, in contrast to the case of vertical integration, when $\alpha = 1$, P_{ni} is (weakly) increasing in $\underline{V}$. While under vertical integration the profits from the low joint profits action are rents that have to be given to the borrower to induce her to repay the debt ($\phi\underline{V}$), under non integration a fraction of these profits becomes the outside option of the party that does not have bargaining power, and can guarantee a minimum level of debt repayment.

P_{ni} depends on the degree of input specificity π and on the industry structure, as summarized by β. Consider again the case $\alpha = 1$. P_{ni} is (weakly) decreasing in π. The decentralized allocation of control rights over whether the two firms should trade internally or on the spot market, creates a basic trade-off under non integration. On the one hand, the buyer does not have enough money to "bribe" the seller to trade internally when trade on the spot market produces higher joint profits but the buyer has private benefits from trading internally. On the other hand, when internal trade generates high joint profits but the buyer has private benefits from trading on the spot market, she has the right to impose trade on the spot market. The higher π, the less valuable is the option of having the seller block a low joint profit trade configuration.

When the buyer has high bargaining power she can extract higher profits from the two firms if β is high. This increases the pledgeable income of the buyer, reducing the pledgeable income of the seller. The seller's pledgeable income however is reduced proportionally to the seller outside option, and not to the joint profits. In other words, the higher bargaining power of the buyer, helps the investor in pumping out money from the two firms.

The discussion so far focussed on the case in which the buyer has all the bargaining power. What happens if the seller has all the bargaining power? Before I discuss the result in Proposition 3 for the case $\alpha = 0$, I present a direct consequence of the discussion above.

Corollary

Assume that the seller observes the realization of b. Then

$$P_{ni}(0) = (1-\phi)\left[(1-\beta)\,\overline{V} + \beta\max\{\underline{V}, \overline{V}(1-\pi)\}\right]$$

If the seller has all the power in the bargaining game, and observes the realization of the private benefits b, the bargaining game is completely symmetric to the case $\alpha = 1$, with the exception that the role played by industry structure β is inverted. We than see that, under efficient bargaining,

$P_{ni}(1)$ is increasing in β, while $P_{ni}(0)$ is decreasing in β, and the pledgeable income is higher when the buyer has full bargaining power if $P_{ni}(1) > P_{ni}(0)$, i.e. if $\beta > \frac{1}{2}$. From the point of view of the investor, α and β are hence complements, in the sense that higher bargaining power of the buyer is good news in industries in which most of the value is kept or produced downstream. The corollary also shows that with respect to predictions regarding π, $\phi, \overline{V}$ and $\underline{V}$ it is immaterial whether we assume that the seller observes the realization of buyer's private benefits b or not.

When $\alpha = 0$ and the seller *does not* observes the realization of b the intuitions presented above still hold, but a new effect has to be taken into account. In this case the seller makes a take-it-or-leave-it offer, and the bargaining process may be inefficient. The seller may end up bargaining too aggressively, in order to extract an high price from the buyer. With probability $\pi\sigma$ however the buyer derives private benefits from trading on the spot market and rejects the offer of the seller, which is then left with low profits and does not repay the debts. This further effect explains why $P_{ni}(0)$ is potentially non monotonic in π and β and why it is increasing in $(1-\sigma)$. It is interesting to note the following

Corollary

If $\alpha = 0, \pi \to \frac{\beta\overline{V}}{\Delta V + \beta\underline{V}}$ *and* $(1-\sigma) \to 1$, *the pledgeable income under inefficient bargaining is higher than the pledgeable income under efficient bargaining if* $\beta < \frac{1}{2}$.

The Corollary makes clear that the inefficiency of the bargaining process between the buyer and the seller is not necessarily bad news from the point of view of the investor. If the likelihood of internal trade yielding high joint profits (high π), and the chances that aggressive bargain still lead to debt repayment (low σ) are high, the aggressive behavior of the seller in the bargaining game allows the investor to set a higher B_u and to "pump" money out of the two firms more easily. This effect will be stronger the lower β. When this is the case, the aggressive bargaining induced by asymmetric information reinforces the complementarity between low β and $\alpha = 0$.

So far I have compared the pledgeable income under non integration with respect to different configurations of the bargaining process (buyer vs. seller bargaining power, and efficient vs. inefficient bargaining). In the next section I compare the highest pledgeable income under integration and under non integration, and derives the main results of the paper.

4 Determinants of Vertical Integration

In this section I compare the pledgeable income under two alternative organizational forms: vertical integration and non integration. The primary objective of the analysis is to shed some light on the determinants of vertical integration. The focus on the pledgeable income is relevant since, in environments in which investors have ex-ante bargaining power, the organizational form that generates higher pledgeable income is more likely to be chosen to organize the two projects.

As in the previous section, I focus on the two polar cases $\alpha \in \{0, 1\}$. The next proposition provides necessary and sufficient conditions under which vertical integration has higher pledgeable income than non integration.

Proposition 3 *Consider* $\alpha \in \{0, 1\}$. *Under Assumption A1-A3* $P_{ni}(\alpha) < P_i$ *if and only if* $\pi > \pi^*(\alpha, \beta, \phi, \sigma, V, b) = \frac{1}{\alpha(1-\beta)+(1-\alpha)\beta} \min\{\frac{b-\phi\Delta V}{(1-\phi)\overline{V}}, \sigma, \frac{1}{(1-\alpha)} \frac{\beta(1-\beta)\overline{V}}{(1-\sigma)(\overline{V}-\beta\underline{V})}\}$.

Depending on parameters, the joint pledgeable income of the two projects might be higher under vertical integration or under non-integration. The net balance of the costs and benefits of vertical integration depends on the relative intensities of two opposing effects. On the one hand vertical integration ensures that the investor faces fewer holdups and that profits can be collectively pledged as collateral ensuring more financing. In contrast, since financial streams get transferred with ownership and returns cannot be completely verified, non-integration multiplies the sources of opportunism and makes the financing process harder. I label this first positive effect of vertical integration *contractual centralization* effect.

On the other hand, the bargaining process associated with market transactions can be an indirect source of (soft) information from the point of view of investors. In particular, the seller might refuse to trade with the buyer when such trade produces low joint profits, even if the buyer would privately benefit from such trade. In contrast, vertical integration, by centralizing control rights suppresses the bargaining process and allow the manager of the integrated firm to implement the course of action yielding higher private benefits but lower profits relatively more often. I label this second negative effect of vertical integration "*de-monitoring*" effect.[14] In other words, vertical

[14] On an empirical ground, Garcia-Appendini (2006) provides some evidence that banks base their credit-granting decisions on information about the trade credit relationships between the firm and its input suppliers. While information on trade credit can sometime be obtained from credit information brokers such as Dun & Bradstreet (hard

integration reduces opportunism related to the possibility of hiding profits at the cost of increasing opportunism related to the possibility of implementing a course of action that yield lower profits.

Factors affecting the net balance of this trade-off in favor of the profit-pooling effect are potential determinants of vertical integration. In particular, I now consider how changes in various parameters affect the likelihood of vertical integration. To apply the model to actual data, I allow for the presence of other factors affecting the integration decision. To do this, I assume that the degree of specificity of the assets is given by $\pi = \overline{\pi} + \varepsilon$, where ε is a random variable distributed according to a continuously differentiable cumulative function $F(\cdot)$ with associated density $f(\cdot)$ strictly positive on the domain and zero mean. The realization of ε is unobserved to the econometrician. To fix ideas, it could be geographic location affect the transportation costs incurred to ship and procure the input on the external market. Under these assumptions, vertical integration will be observed whenever $\pi > \pi^*(\overline{\pi}, \alpha, \beta, \phi, \sigma, V, b)$, i.e. with probability

$$L(\overline{\pi}, \alpha, \beta, \phi, \sigma, V, b) = F(\overline{\pi} - \pi^*(\alpha, \beta, \phi, \sigma, V, b)) \tag{1}$$

The next proposition identifies the determinants of vertical integration.

Proposition 4 *Under Assumption A1-A3 the likelihood of vertical integration is*

1. *increasing in assets specificity (higher $\overline{\pi}$),*
2. *(weakly) increasing in the level of profits (higher $\overline{V}$) and (weakly) decreasing in profits associated with the low profits trade configuration (higher $\underline{V}$),*
3. *(weakly) decreasing in the degree if investor's protection (lower ϕ),*
4. *(weakly) decreasing in the likelihood (higher σ) and size of buyer's private benefits (higher b),*

A first key determinant of vertical integration is $\overline{\pi}$, the likelihood that internal trade yields high joint profits, i.e. the degree of specificity linking the two projects. When $\overline{\pi}$ is high, vertical

information), more often banks can still check with the firm's suppliers (soft information). This second source of soft information, becomes unavailable if the firm is integrated, and the supplier is an employee of the firm, instead of the owner of a separate firm.

integration becomes more likely, i.e. assets that are likely to be used together should also be managed and financed together. The intuition for this result directly follows from the analysis of the pledgeable income under non integration. Non integration grants control rights over the trade configuration to multiple firms, inducing bargaining. This bargaining process generates useful information from the point of view of the investors, since the seller can prevent a low profit action to be taken when profits are higher if trade takes place on the spot market. The positive monitoring effect of non integration is stronger when $\overline{\pi}$ is low, that is when the optimal trade configuration from the point of view of the investors is likely to be trade on the input spot market.[15]

This result echoes the property rights results that complementary assets should be jointly owned. However here the source of complementarity does not derive from the incentives to undertake non contractible investments, but is instead determined by the likelihood that the two assets should trade together. As an example, we expect that firms that use specific inputs that are geographically localized and costly to transport should be jointly owned and financed, i.e. vertically integrated. This seems to be consistent with anecdotal as well as formal evidence (see the evidence discussed in Whinston (2003)).

A consequence of this result is also that in a cross-section of firms, vertically integrated firms are more likely to trade internally, instead that on the market. There are two distinct effects to be considered. First of all, Proposition 4 suggests that assets with high π will be owned and managed together. In other words, vertically integrated firms are inherently different from non integrated firms since they tend to use assets that are likely to trade together relatively more often. We can think of this first effect as a "*selection*" effect. The second effect, which might partially offsets the selection effect, derives from the fact that different allocation of control rights induce different likelihood that each trade configuration is implemented. Under vertical integration trade takes place internally with probability $\pi(1-\sigma)+(1-\pi)\sigma$ if $(1-\sigma) > \frac{\overline{V}-b-\phi\underline{V}}{(1-\phi)\overline{\overline{V}}}$, and with probability π otherwise. Under non integration instead, the two firms trade together with a probability equal to π when the bargaining process is efficient, and with even lower probability when the bargaining process is inefficient $(\alpha\pi + (1-\alpha)\pi(1-\sigma))$. Since the allocation of control rights directly influences the

[15] As noted above, in this environment seller integration would be optimal. The model can be extended to consider the case in which with some probability σ_u the seller also derives private benefits b_u from the action yielding low cash flows. When this is done, seller integration is not unambiguously optimal anymore. Moreover, it can be shown that an higher degree of correlation in private benefits realizations reduces pleadgeable income and favors vertical integration (see Macchiavello (2005)).

likelihood of internal trade, we can think of this effect as a "*governance*" effect. The governance effect (weakly) reinforces the selection effect in increasing the likelihood of internal trade under integration if $(1-\sigma) < \frac{\overline{V}-b-\phi \underline{V}}{(1-\phi)\overline{\overline{V}}}$, and reduces it otherwise. Mullainhathan and Scharfstein (2001) find evidence which is consistent with these predictions, although, by treating the integration decision as exogenous, they do not disentangle the two effects emphasized by the model.[16]

A second important determinant of vertical integration is the size of the two projects, as summarized by $\overline{V}$. Non integration multiplies the scope for opportunistic behavior by giving to multiple agents the possibility of hiding (part of the) profits. The higher the profits, the costlier this decentralization in the opportunities to steal profits, and hence the more likely is vertical integration to ensure higher returns to the investors. This result not suggests that: i) in a cross-section of firms, vertically integrated firms are larger, and ii) vertical integrated firms have larger plants. Both observations receive some empirical support (see e.g. Acemoglu et al. (2005) and Hortačsu and Syverson (2005)).

The positive relationship between $\overline{V}, \pi$ and vertical integration, also suggest that vertical integration positively depends on the *level* of quasi-rents on which the two firms bargain. A useful measure of quasi-rents in our environment is the ex-ante difference in expected profits between internal trade and trade on the input spot market, which is proportional to $R = (2\pi - 1)\,\Delta V$, and hence $\frac{\partial R}{\partial \overline{V} \partial \pi} > 0$. In an influential paper Whinston (2003) noted that the transaction cost and the property right theory of the firm are conceptually different, and that, while the former predicts a positive relationship between vertical integration and the level of the quasi-rents generated by specificity and incomplete contracts, the property rights theory of the firm does not have predictions on the relationship between the level of quasi-rents and vertical integration. Moreover, he noted that virtually all empirical work support the transaction cost intuition that higher quasi-rents are associated with either long term contracts or with vertical integration (while it is much harder to empirically identify margins). The prediction of the model are coherent with these pieces of evidence.

A third prediction of the model is that vertical integration becomes relatively more likely in

[16] These considerations abstract from the possibility that vertical integration might require more specific assets with lower resale (or collateral) value, making financing more difficult. Similarly, the allocation of ownership rights over assets could influence the care that managers put in mantaining those assets. Conceptually, we could think of the pleadgable income of integration and non integration as respectively given by $P_{int} + \eta_{int}$ and $P_{ni} + \eta_{ni}$, where η_{int} and η_{ni} are random variables unobserved to the econometrician, and are such that $corr\,\langle \eta_{int} - \eta_{ni}, \pi \rangle \neq 0$

environments with low investor protection (higher ϕ). Because of widespread input and market failures, firms in developing countries are often thought to be larger and more vertically integrated (see e.g. Palepu and Khanna (2000)). The model is consistent with this and other anecdotal evidence from business history. For example, Cull et al. (2005) describe the financing of small firms through the formation of partnerships during the industrial revolution and note that typically the partners were individuals or firms with upstream or downstream connections. I am not aware of rigorous empirical work testing the connection between access to finance and vertical integration at the firm level. Recent cross-country-industry empirical evidence on the institutional determinants of vertical integration however fails to find a strong empirical relationship between within industry levels of vertical integration and measures of financial development (see Acemoglu et al. (2005)). Macchiavello (2006) however provides evidence consistent with the idea that, because of industry equilibrium effects, the effects of financial development on the degree of vertical integration at the industry level might depend on other industry characteristics.

We have noted in the previous section that the inefficiency caused by asymmetric information in the bargaining game between the buyer and the seller is not necessarily a bad news from the point of view of external investors. A direct consequence of propositions 3 is the following corollary.

Corollary

Assume that the seller observes the realization of b. Under Assumption A1-A3 $P_i(0) < P_{ni}(0)$ *if and only if* $\pi < \frac{1}{\beta} \min\{\frac{b-\phi\Delta V}{(1-\phi)\overline{V}}, \sigma\}$.

The corollary clarifies that the cases in which inefficiencies in the bargaining process are likely to increase the pledgeable income of two non integrated firms are precisely the cases in which vertical integration becomes more likely. Despite the observation in the previous section, in a cross-section of firms, the model suggests the standard transaction costs prediction that higher inefficiencies associated with bargaining make vertical integration more likely.

Finally, the next proposition shows that the relationship between industry structure and vertical integration crucially depends on the distribution of bargaining power between the seller and the buyer.

Proposition 5 *Under Assumption A1-A3, higher competition in the upstream market (high* β*)*

makes vertical integration more likely if the seller has bargaining power ($\alpha = 0$) and less likely if the buyer has bargaining power ($\alpha = 1$).

This result shows that industry structure might affect vertical integration in the presence of financial constraints. I am not aware of empirical studies linking industry structure and vertical integration in the presence of financial constraints.[17]

5 Contractual Externalities

It has long been recognized by legal scholars (see e.g. Posner (1976)) that firms might improve on the contractual externalities (CE) associated with independent firms. In this section I focus on two particular sources of CE, and consider potential solutions. First, I ask whether joint liability contracts among two independent firms can remove CE that arise at the *input transaction stage*, from the fact that the investors do not sit at the bargaining table at which the input transaction is negotiated. Second, I consider a simple version of the model, in which multiple investors might fail to coordinate on financing independent firms and show that vertical integration is more likely to occur in the presence of CE among multiple investors at the *financing stage*.

5.1 CE at the Input Transaction Stage and Joint Liability Contracts

In the model in the previous section a firm is an organization with centralized ownership of physical assets and centralized (i.e. joint) financial liabilities. Non integrated firms suffer from the fact that in the bargaining process the price might be set at a level that induces either the seller or the buyer to default on the loan (by hiding the profits), i.e. parties are induced to bargain too aggressively from the point of view of external investors. Since the investor does not sit at the table at which the input transaction is negotiated, a particular form of contractual externalities which is detrimental to investor's interests might therefore arise reducing pledgeable income. This effect is absent in the case of a vertically integrated firm, since the manager is jointly responsible for both projects. [18]

[17] In a recent paper, Aghion et al (2006) reports the existence of a U-shaped relationship between vertical integration and the degree of product market competition, a result that could be consistent with an extensions of the current model that allow for separate agency conflict with the seller and considered also seller integration (see also Acemoglu et al. (2005)). Further theoretical and empirical work is needed on this subject.

[18] In an integrated firm the manager of the firm can not easily "move" money across units avoiding to repay the investors. Under non integration instead the outside options in the bargaining game are essentially given by the rents

While useful to analyze the dichotomy between "markets and hierarchies" from a theoretical perspective, the perfect correlation between centralized ownership and joint financial liabilities typical of the firm is far from being the only organizational form observed in practice. "Hybrid" organizational forms are indeed relatively common. In this subsection I ask whether the negative effect of non integration can be undone by imposing some form of joint liability for the two firms. In doing so I analyze the scope for an "hybrid" organizational form in which ownership of assets is decentralized, while financial liabilities are (partially) centralized.[19]

In particular, I assume that the financial contracts that the investor offers at the financing stage specify that if the upstream firm does not repay the debt B_u the downstream firm is liable for an amount L_d, and similarly if the downstream firm does not pay back the debt B_d, the upstream firm is liable for an amount B_u. Letting $\mathbf{B} = [B_d, B_u]$ and $\mathbf{L} = [L_d, L_u]$ be the vectors of debt and joint liability contracts respectively, the investor problem can be written as

$$\max_{B_d,B_u,L_d,L_u} \mathbf{\Sigma}_s \pi_s \left(\mathbf{\Sigma}_{j\in\{d,u\},j\neq j'} \mathbf{I}_j(s \mid \mathbf{B},\mathbf{L}) \cdot (B_j + L_j \cdot (1 - \mathbf{I}_{j'}(s \mid \mathbf{B},\mathbf{L}))) \right)$$

As for the case without joint liability contracts, the investor will have to trade-off a higher debt levels B_d and B_u, with a higher probability that the debt is repaid. The investor however has two additional instruments to extract money from the two firms. Joint liabilities make sure that if one of the two debts is not paid back, the investor still can potentially recover some money from the other firm. Joint liability becomes an important tool in the hands of the external investor to affect the bargaining process among the two units: by affecting the outside options it induces moderation in the bargaining game.

For the sake of simplicity, I focus on the case in which $\alpha = 1$. The following proposition derives the optimal contract when joint liability is allowed.

Proposition 6 *Assume $\alpha = 1$. Under Assumptions A1-A3 the optimal financial contract is given by:*

that can be gained by hiding profits. Note that the contractual externalities we are describing are similar to a form of collusion between the buyer and the seller against the investors (see Macchiavello (2005)).

[19] An interesting example is given by micro-finance contracts. In micro-finance arrangements however, financed projects are rarely vertically related with each other. We conjecture that needs for diversification is at the heart of this practice.

$$\mathbf{B}^* = \begin{bmatrix} B_u^* \\ B_d^* \end{bmatrix} = \begin{bmatrix} (1-\phi)(1-\beta)\overline{V} \\ (1-\phi)\beta\overline{V} \end{bmatrix}, \quad \textit{and } \mathbf{L}^* = \begin{bmatrix} L_u^* \\ L_d^* \end{bmatrix} = \begin{bmatrix} (1-\phi)\beta\overline{V} \\ \varkappa \end{bmatrix},$$

$$\textit{with } \varkappa = \begin{cases} (1-\phi)(1-\beta)\Delta V & \textit{if} (1-\sigma) > \frac{\Delta V + \beta(1-\phi)\underline{V} - b}{(1-\phi)(\Delta V + \beta \underline{V})} \\ (1-\beta(1-\phi))\Delta V - b & \textit{otherwise} \end{cases}.$$

The maximum pledgeable income of two non integrated firms under joint liability $P_{ni}^{JL}(1)$ *is given by*

$$P_{ni}^{JL}(1) = (1-\phi)(1-\pi)\overline{V} + \pi \max\{(1-\phi)(1-\sigma)(\Delta V + \beta\underline{V}), \Delta V + \beta(1-\phi)\underline{V} - b\}$$

The optimal contract displays positive levels of joint liability. The two joint liability amounts L_d and L_u play very different roles. L_d^* is used to increase repayments in those states in which the upstream firm defaults. Those states occur with probability π. From the point of view of the investor it is better to let the upstream firm defaults in those states, as having both firms repaying their debts multiplies the rents that have to be left, as the price P has to compensate more than proportionally the debt level B_u. L_u^* is instead used to decrease the value of default for the downstream seller. By increasing L_u^* the investor makes default for the downstream firm extremely costly, as the price offer P has to compensate for the joint liability amount L_u^*.[20]

The proposition implies the following corollary.

Corollary

1. *The optimal joint liability contract displays* $B_d^* \geq L_u^* > 0$ *and* $B_u^* > L_d^* > 0$.
2. *There exist a unique* $\pi^{JL} \in (\pi^*, 1)$ *such that* $P_{ni}^{JL}(1) > P_{ni}(1)$ *if and only if* $\pi < \pi^{JL}$.

The first part of the corollary implies that the pledgeable income of two non integrated firms always increases when joint liability contracts are allowed. A richer model would consider a setting

[20] On the ground that the state is not verifiable by third parties, we assumed that agents are restricted to contracts that allocates decisions rights over the optimal action to be taken, i.e. we assumed that the feasible contracts are highly incomplete. In principle however parties could design an ex-ante mechanism in which they report to the mechanism designer messages on the realization of the state of the world, and actions are taken accordingly to the reported messages. In Macchiavello (2005) I consider a similar model in which $\phi = 0$ but in which also the seller can derive private benefits from the low joint profit action. I show that under Nash implementation, Coalitional Rationality, and Limited Liability, the optimal mechanism is equivalent to a simple allocation of control right and joint liability.

in which joint liability contracts also brings costs. One obvious limitation of joint liability contracts is that, in the event of default of one firm, incentives to improve performance are reduced for the second firm. In other words, default of one firm might trigger default of other firms.[21]

The second part of the corollary instead implies that joint liability is not sufficient to make non integration always preferred to integration. It shows that, because of the possibility of hiding profits, joint liability contracts are not sufficient to create a complete *contractual centralization* effect under non-integration. This happens because the price P has to compensate more than proportionally the debt B_u for the joint liability L_d to be repaid. In environments with high external investors protection, ϕ is close to zero and non-integration always achieves higher pledgeable income than integration. The source of the *contractual centralization* effect is therefore the combination of poor investor protection and joint liability typical of an integrated firm. In summary, the intuition and results of the previous section continue to hold true even when joint liability contracts between two non integrated firms are allowed.

It is possible to imagine the symmetric hybrid organizational form, in which the two units are jointly managed but not jointly liable from the financial point of view. This seems to be the case in business groups and pyramids (see Bianco and Nicodano (2002) and Almeida and Wolfenzon (2006) for recent theoretical treatment of these hybrids from a financial perspective). Other hybrid organizational forms arise in common situations in which the input transaction between a buyer and a seller is linked to a financial transaction (see e.g. the extensive literature on trade credit (for example Burkart and Ellingsen (2004)) and putting out systems in manufacturing (for example Kranton (2006))).

5.2 CE at the Financing Stage

In section 3, I have solved the model assuming that investors maximize the sum of the pledgeable incomes of the two separate firms, thus removing all contractual externalities that could originate at the financing stage. For the case of non-integration we have shown that the decision on the optimal debt levels B_d and B_u are separable (see proof of Proposition 3). However, we have not shown that two independent investors would finance the two firms whenever this is profitable. Motivated by the insights offered by legal scholars (Kraakman and Hausmann (2001), Cheung (1983)) as well

[21] For a similar point in the context of microfinance literature see, among others, Besley and Coate (1995).

as business historians (see e.g. Lamoreaux (1998)) and economists (Holmstrom (1999)), in this subsection I explore the consequences of contractual externalities at the financing stage among two separate and independent investors.

Before I describe how to modify the basic model to allow for multiple investors, I illustrate the fundamental source of externalities across the to projects. When the upstream firm does not exist, the net present value of the downstream firm is given by

$$\mathcal{V}_d^0 = (1-\pi)\beta\overline{V} + \pi\beta\underline{V} - k_d$$

while the net present value when the upstream firm is financed is given by

$$\mathcal{V}_d = (1-\pi)\beta\overline{V} + \pi\alpha\Delta V + \beta\underline{V} - k_d$$

implying that the financing of the upstream firm increases the value of downstream firm of an amount equal to $\Delta\mathcal{V}_d = \mathcal{V}_d - \mathcal{V}_d^0 = \alpha\pi\Delta V$. Obviously, a similar reasoning applies for the positive externality that the financing of the downstream firm has on the net present value of the upstream firm. It is obvious that $\Delta\mathcal{V}_u = \mathcal{V}_u - \mathcal{V}_u^0 = (1-\alpha)\pi\Delta V$, where the notation is adapted from the previous expressions to the case of the upstream firm. It is thus possible to have parameters configurations in which none of the two projects has positive present value, unless the other project is also financed. As the previous expression shows, this is more likely to happen when π is high, i.e. when the two assets are highly complementary.

To explore how contractual externalities interact with the decision to finance an integrated firm as opposed to two non-integrated firms, consider the following modification to the model presented in section 2. Suppose that there are two investors, $\mathcal{D}$ and $\mathcal{U}$. I assume that $\mathcal{D}$ can only finance the operations of the buyer, i.e. downstream firm, while $\mathcal{U}$ can only finance the operations of the seller. By this I mean that whenever the buyer has control over the firm, only $\mathcal{D}$ can recover a positive fraction of the profits. Similarly, when the seller has control over the upstream firm, only $\mathcal{U}$ can recover a positive fraction of the upstream firm's profits. I formalize this intuition assuming that the parameter ϕ is specific to the investor-borrower relationship. In particular I assume $\phi(\mathcal{U}, u) = \phi(\mathcal{D}, d) = \phi$ as in Assumption A2, while $\phi(\mathcal{U}, d) = \phi(\mathcal{D}, u) = 1$. Although this assumption is presented under an extreme form, specific human capital in the monitoring of certain kinds of economic activities, past business relationships, reputation, belonging to a common network, and other factors imply that investors may not be perfect substitutes in lending relationships.

This assumption implies that $\mathcal{D}$ can finance two kind of projects: a small (non-integrated) buyer or a large vertically integrated firm managed by the buyer, that produces its own inputs. $\mathcal{U}$ instead can only finance a (non-integrated) seller. To keep the analysis simple, I assume that the two investors get a payoff equal to zero if they do not invest and that if $\mathcal{U}$ invests when $\mathcal{D}$ finances a vertically integrated firm the nature of competition in the industry implies negative returns L on $\mathcal{U}$'s investment, while $\mathcal{D}$ receive a fraction δ of the returns to vertical integration such that $\delta P_{int} - K \geq 0$.[22]

Finally, I assume that the two investors $\mathcal{D}$ and $\mathcal{U}$ takes their financing decisions independently and non cooperatively. As a justification for this assumption, we can think that, for instance, at the financing stage investors do not know the realization of the specific match between the buyer and the seller. If initial contracts are hard to renegotiate, multilateral contracts will not be effectively signed. The two investors $\mathcal{D}$ and $\mathcal{U}$ simultaneously decide whether to finance a firm, and in the case of $\mathcal{D}$ whether to finance a large vertically integrated firm, or a smaller non integrated firm. They also choose the appropriate debt levels that maximizes their expected returns. I focus on pure strategy Nash equilibria of this game.[23]

Since this analysis is meant to be purely illustrative, the next proposition presents results for a particular parametric configuration of the model presented above. To draw a starker connection between contractual externalities and integration I focus on the particular case in which the pledgeable income of vertical integration is minimal.

Proposition 7 *Assume $\alpha = \frac{1}{2}$, $\sigma \to 1$, $b = \beta \Delta V$ and $\pi < \frac{\Delta V}{V}$. Under Assumption A1-A3,*

1. $\max\{P_{ni}, K\} > P_i$ *is a necessary but not sufficient condition for non-integration to be a Nash equilibrium of the financing game, and*

2. $P_{ni} < \max\{P_i, K\}$ *is a sufficient condition for integration to be the unique Nash equilibrium of the financing game.*

[22] This could be the case if an additional entry in the upstream industry significantly lowers the profits of non-integrated firms, and if the seller can be replaced without incurring eccessive costs.

[23] Formally, a strategy for investor $\mathcal{D}$ is a two dimensional vector: an action $a_{\mathcal{D}} \in \mathcal{A}_{\mathcal{D}} = \{0, \mathbf{D}, \mathbf{I}\}$ and a debt level $B_{\mathcal{D}} \times \Re^{+}$, where $\mathbf{0}, \mathbf{D}, \mathbf{I}$ denote respectively no financing, financiang of a (small) downstream firm, and financing of an integrated firms. Similarly for investor $\mathcal{U}$ is a two dimensional vector: an action $a_{\mathcal{U}} \in \mathcal{A}_{\mathcal{U}} = \{0, \mathbf{B}\}$.

As emphasized above, the source of the externality is that the financing of the downstream firm improves the net present value of the upstream firm and viceversa. When the two investors $\mathcal{D}$ and $\mathcal{U}$ simultaneously decide whether to finance a firm they do not take into account these effects. Instead they focus on the individual returns of each firm, and as a result vertical integration becomes more likely. First of all, it could be that $\mathcal{D}$ prefers to finance an integrated firm than a smaller downstream firm, even if $\mathcal{U}$ finances an upstream firm. This in turn induces $\mathcal{U}$ to abstain from investing in the first place. Note that this can happen even if the joint financial return of two non-integrated firms is higher than the one offered by an integrated firm. Under these circumstances however, vertical integration is not necessarily inefficient (in fact, under the assumptions in Proposition 8, vertical integration increases efficiency). Integration simply increases the rents of the coalition formed by the buyer and $\mathcal{D}$, at the expenses of the seller and $\mathcal{U}$. Another scenario happens when the financial requirements of the two units are highly asymmetric (k_u is high relative to $(1-\beta)$, while k_d is low relative to β), in which case either vertical integration or underinvestment will occur. Underinvestment in turn, can take two forms: no firms being financed, or only a small, non integrated firm, being financed.

Note that the contractual externalities emphasized here operate at the moment of setting up the two firms. While I have shown that those contractual externalities make integration more likely, it is clear from the preceding analysis that integration does not remove those contractual externalities. As shown in the previous subsection however, integration could instead remove contractual externalities that arise once the two projects have been financed. In the previous subsection however I have focussed on CE between the coalition formed by the buyer and the seller, vis à vis a single external investor. Vertical integration however could remove other forms of CE between the coalition formed by the buyer (or the seller) and her investors, vis à vis the seller (or the buyer) and her respective investors. In a very interesting paper, Titman (1984) discusses the role of capital structure in mitigating CE originating from the fact that the liquidation decision of a firm typically does not take into account losses for customers and specialized suppliers. It would be interesting to consider richer environments in which external investors can seize control over firm's assets and even force liquidation in certain states of the world as in the models by Aghion and Bolton (1992) and Bolton and Scharfstein (1992)). We conjecture that in the presence of costly renegotiation, integration is less likely to lead to excessive liquidation, since the value of the entire supply chain as opposed to the value of a single firm should be taken more easily into account by external investors.

6 Conclusions

In this paper I ask whether vertical integration reduces or increases transaction costs with external investors. I build a model in which a seller produces a good that can be used by a buyer, or sold on a spot market. The buyer and the seller have no cash, need to finance investments for production, and cannot foresee in advance whether the input is most efficiently traded on the spot market or among each other. I assume that ownership of physical assets gives control over contracting rights to those assets, that financial streams get transferred with ownership and that returns cannot be perfectly verified. The net balance of the costs and benefits of integration in terms of pledgeable income depends on the relative intensities of a positive "*contractual centralization*" effect against a negative "*de-monitoring*" effect. I find that larger projects, more specific assets, and low investors protection are determinants of vertical integration. I discuss various sources of contractual externalities and show that joint liability contracts between non integrated firms can mitigate contractual externalities at the input transaction stage, while contractual externalities at the financing lead to underinvestment and, conditional on investment, make vertical integration relatively more likely.

Several dimensions need further theoretical scrutiny. In particular the model could be extended to consider the role of trade credit and supply credit. Moreover, non contractible investments should be reintroduced in the model in order to make it more suitable for the analysis of industries in which (generic) skills and human capital are crucial elements. The ambiguous prediction on the role of the distribution of bargaining power and industry structure should motivate further theoretical as well as empirical effort. The exploration of these and other questions will be a stimulus for future research.

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7 Appendix: Proofs

Proof of Lemma 1

The case of integration is obvious since $b < \beta\Delta V$ and $\beta < 1$ imply $\overline{V} > \underline{V} + b$.

For the non integration case we have to analyze the bargaining game. With probability α the downstream manager makes a take-it-or-leave-it offer P for internal trade. Under non integration, with probability π internal trade generates high joint profits. She will choose P to maximize $\max\{\overline{V} - P, \beta\underline{V} + \mathbf{I}b\}$ subject to the constraint $P \geq (1-\beta)\underline{V}$, where $\mathbf{I} \in \{0, 1\}$ depending on whether the downstream manager has private benefits b from the low joint profits action. The solution to this program is $P = (1-\beta)\underline{V}$, and assumption 2 guarantees that $\max\{\overline{V} - P, \beta\underline{V}\} = \Delta V + \beta\underline{V} > \beta\underline{V} + \mathbf{I}b$ and hence internal trade takes place. The argument is similar for the case in which trade on the spot market generates high joint profits.

With probability $(1 - \alpha)$ the upstream manager makes a take-it-or-leave-it offer P. With probability π internal trade generates high joint profits. The upstream manager faces uncertainty with respect to the realization of b, and hence the offer P is made to maximize the expected profits $\max \mathbf{E}_\sigma \max\{P, (1-\beta)\underline{V}\}$. If $P \leq \overline{V} - \beta\underline{V} - b$ the offer is always accepted, if $P \in (\overline{V} - \beta\underline{V} - b, \overline{V} - \beta\underline{V}]$ the offer is accepted with probability $(1 - \sigma)$, and finally if $P > \overline{V} - \beta\underline{V}$ the offer is always rejected. It follows that the optimal P is $P \leq \overline{V} - \beta\underline{V} - b$ if $\overline{V} - \beta\underline{V} - b > (1 - \sigma)(\overline{V} - \beta\underline{V}) + \sigma(1 - \beta)\underline{V}$, i.e. if $b < \sigma\Delta V$ and $P = \overline{V} - \beta\underline{V}$ otherwise. In the latter case, the two firms trade on the market with probability σ and generate surplus equal to $\underline{V} + b$. With probability $1 - \pi$ trade on the market generates high joint profits. Even the most profitable offer $P = \underline{V} - \beta\overline{V} + b$ makes internal trade unprofitable for the upstream manager since $\underline{V} - \beta\overline{V} + b < (1 - \beta)\overline{V}$.[24]

Under non integration, the high joint profits trade configuration is implemented with probability $1 - \pi\sigma(1 - \alpha)$ while with the complementary probability the low joint profits trade is implemented and the realized surplus is given by $\underline{V} + b$.■

Proof of Proposition 1

The game has to be solved backward: for a given debt B I derive the states of the world in which the buyer repays debt. I then solve for the optimal level of debt. We have to distinguish 3 cases, depending on whether $B > (1 - \phi)\overline{V}$, $B \leq (1 - \phi)\underline{V}$ or $B \in ((1 - \phi)\underline{V}, (1 - \phi)\overline{V}]$.

[24] Since managers have deep pockets, P can be larger than $\underline{V}$.

First, conditional on implementing the action that yields high joint profits, the buyer will repay debt B if the payoff from repaying the debt $\max\{\overline{V} - B, 0\}$ is larger than the payoff from hiding profits $\overline{V}$ realizing private benefits of value $\phi\overline{V}$. In other words, the buyer repays debt if $B \leq (1-\phi)\overline{V}$. If this condition is violated, the buyer will never implement the high cash flow action whenever she derives private benefits from the low cash flow action, since A1 implies $\phi\underline{V} + b > \phi\overline{V}$.

Second, and similarly, conditional on implementing the action that yields low joint profits instead, she will repay debt B if and only if $B \leq (1-\phi)\underline{V}$. If this condition is satisfied however, she will never find it profitable to implement the low joint profits action since A1 implies $\overline{V} - B > \underline{V} - B + b$, i.e. $\Delta V > b$.

Finally, denoting by $\mathbf{I} \in \{0, 1\}$ an indicator function taking value equal to one when the buyer has private benefits b from the low cash flow action, and zero otherwise, if $B \in ((1-\phi)\underline{V}, (1-\phi)\overline{V})$ the buyer will implement the high profits action if and only if $\overline{V} - B > \mathbf{I}b + \phi\underline{V}$. This inequality is always satisfied if $\mathbf{I} = 0$. If instead $\mathbf{I} = 1$, the inequality is satisfied if and only if $B \leq \widetilde{B} = \overline{V} - b - \phi\underline{V}$, and $\widetilde{B} \in ((1-\phi)\underline{V}, (1-\phi)\overline{V}]$ since $b > \phi\Delta V$ by assumption.

We conclude that if $B \leq \overline{V} - b - \phi\underline{V}$ the buyer always repays debt, if $B \in (\widetilde{B}, (1-\phi)\overline{V}]$ the buyer repays debt with probability $(1-\sigma)$, and if $B > (1-\phi)\overline{V}$ the buyer never repays debt. The investors hence maximizes pledgeable income by trading off an higher debt B with a lower probability that the debt is paid back. Denoting by B_{int} the solution of the investor's problem, the linearity of the problem implies that the solution to the investor problem will either be $B_{int} = (1-\phi)\overline{V}$ or $B_{int} = \widetilde{B}$, which results in pledgeable income $P_{int} = \max\{\overline{V} - b - \phi\underline{V}, (1-\sigma)(1-\phi)\overline{V}\}$.■

Proof of Proposition 2

In order to prove proposition 2, we have to solve the game backward. In order to charcatrize the solution to the investor problem, I first analyze all possible bargaining scenarios as a function of debt levels B_d and B_u. I then show that the investor solves a linear programming problem, those solution is generically at a corner. I derive the candidate solutions, and show that the investor problem can be decomposed into two simpler independent problems.

The following Lemma finds the set of potential solutions (corners) to the investor's problem.

Lemma

Denote with B_d^* and B_u^* the solution to the investor's problem, then $\frac{B_d^*}{(1-\phi)} \in \{\beta\underline{V}; \beta\overline{V}; (\Delta V - b)(1-\phi) + \underline{V}; (\Delta V + \beta\underline{V})\}$ *and* $\frac{B_u^*}{(1-\phi)} \in \{(1-\beta)\underline{V}; (1-\beta)\overline{V}; (\overline{V} - b - \beta\underline{V}); (\overline{V} - \beta\underline{V})\}$

Proof:

Denote with V_i and V_m the joint profits realized with internal trade and with trade on the input market respectively, and let $\mathbf{I} \in \{0,1\}$ be an indicator of whether the buyer derives private benefits b from the trade configuration that yields low joint profits.

With probability α the buyer has the right to make a take-it-or-leave-it offer. We have to analyze four different cases, depending on the realizations of the private benefits and joint profits.

<u>Case 1.1</u>: With probability $\pi(1-\sigma)$ we have $V_i = \overline{V}$ and $\mathbf{I} = 0$.

Denote by $\widehat{D}$ the highest payoff the buyer can obtain with an offer P which is accepted. The offer P is accepted by the seller if and only if

$$\max\{P - B_u, \phi P\} \geq \max\{(1-\beta)\underline{V} - B_u, \phi(1-\beta)\underline{V}\} \tag{2}$$

$\widehat{D}$ is obtained as

$$\widehat{D} = \max_P\{\overline{V} - P - B_d, \phi(\overline{V} - P)\}$$

subject to 2. Denoting by $\widehat{P}$ the solution to the problem, the buyer makes offer $\widehat{P}$ if and only if the solution to the bargaining problem $\widehat{D}$ gives him a final payoff larger than her outside option $\overline{D} = \max\{\beta\underline{V} - B_d, \phi\beta\underline{V}\}$, otherwise she offers any $P < \widehat{P}$. The optimal $\widehat{P}$ depends on B_u : several cases have to be considered.

Assume first that $B_u \leq (1-\phi)(1-\beta)\underline{V}$. The outside option of the upstream firm is to trade on the input market realizing profits $\overline{U} = (1-\beta)\underline{V} - B_d$. The best price the buyer can offer is hence $\widehat{P} = (1-\beta)\underline{V}$, realizing profits $\widehat{D} = \max\{\Delta V + \beta\underline{V} - B_d, \phi(\Delta V + \beta\underline{V})\}$. If $B_d < (1-\phi)\beta\underline{V}$ then A1 implies $\widehat{D} > \overline{D}$. The offer is made and accepted in equilibrium, the two firms trade with each other, $V_i = \overline{V}$ is realized and debts B_u and B_d are fully repaid. Suppose instead that $B_d > (1-\phi)\beta\underline{V}$. Then again $\widehat{D} > \overline{D}$, the offer is made and accepted, firms trade with each other, B_u is fully repaid, while B_d is repaid if and only if $B_d < (1-\phi)(\Delta V + \beta\underline{V})$.

Assume instead that $B_u > (1-\phi)(1-\beta)\underline{V}$. Take $\varepsilon_d \geq 0$ and $\varepsilon_u \geq 0$ such that $B_u = (1-\phi)(1-\beta)\underline{V} + \varepsilon_u$ and $B_d = (1-\phi)(\Delta V + \beta\underline{V}) - \varepsilon_d$. If $P = \frac{B_u}{1-\phi}$ the offer is accepted and B_u is repaid, and the buyer gets payoff $\widehat{D} = \phi(\Delta V + \beta\underline{V}) + \varepsilon_d - \frac{\varepsilon_u}{1-\phi}$ and repays the debt. However $\widehat{D} < \phi(\Delta V + \beta\underline{V}) + \varepsilon_d$, which is the payoff she can guarantee herself offering $P = (1-\beta)\underline{V}$ and repaying her debt. The two firms hence trade with each other, however B_u is not repaid and B_d is repaid as long as $\varepsilon_d \geq 0$, i.e. if and only if $B_d < (1-\phi)(\Delta V + \beta \underline{V})$.

Case 1.2: With probability $(1-\pi)(1-\sigma)$ we have $V_m = \overline{V}$ and $\mathbf{I} = 0$.

For the two firms to trade with each other, the offer P must be such that

$$\max\{P - B_u, \phi P\} > \max\{(1-\beta)\overline{V} - B_u, \phi(1-\beta)\overline{V}\} \tag{3}$$

and moreover

$$P \leq \underline{V} \tag{4}$$

where the second constraint comes from the fact that the two managers have no money. Suppose first that $B_u \leq (1-\phi)(1-\beta)\overline{V}$, then it must be $P \geq (1-\beta)\overline{V}$ which is impossible because of A1 implies $(1-\beta)\overline{V} < \underline{V}$. Assume instead that $B_u > (1-\phi)(1-\beta)\overline{V}$, then again $P \geq (1-\beta)\overline{V}$, which is impossible. We conclude that the two firms always trade on the input market, realizes joint profits $V_m = \overline{V}$, debt B_u is repaid if $B_u \leq (1-\phi)(1-\beta)\overline{V}$and B_d is repaid if $B_d \leq (1-\phi)\beta\overline{V}$.

Case 1.3: With probability $\pi\sigma$ we have $V_i = \overline{V}$ and $\mathbf{I} = 1$.

Adopting notation as in Case 1.1, the buyer makes an offer P in order to

$$\widehat{D} = \max_P\{\overline{V} - P - B_d, \phi(\overline{V} - P)\}$$

subject to

$$\max\{P - B_u, \phi P\} \geq \max\{(1-\beta)\underline{V} - B_u, \phi(1-\beta)\underline{V}\} \tag{5}$$

Denoting with $\widehat{P}$ the solution to this program, the buyer makes offer $\widehat{P}$ if and only if $\widehat{D} \geq \overline{D} = \max\{\beta\underline{V} - B_d, \phi\beta\underline{V}\} + b$, otherwise she offers any $P < \widehat{P}$, the offer is rejected and trade takes place on the market. As in case1, the solution $\widehat{P}$ depends on debt B_u, and several cases have to be considered.

Assume first that $B_u \leq (1-\phi)(1-\beta)\underline{V}$. The outside option of the upstream firm is to trade on the input market realizing profits $\overline{U} = (1-\beta)\underline{V} - B_d$. The best price the buyer can offer is hence $P = (1-\beta)\underline{V}$, realizing profits $\widehat{D} = \max\{\Delta V + \beta\underline{V} - B_d, \phi(\Delta V + \beta\underline{V})\}$. Analogously to Case 1, it is easy to see that the buyer repays her debt if and only if $B_d < \Delta V + \beta(1-\phi)\underline{V} - b$.

Assume instead that $B_u > (1-\phi)(1-\beta)\underline{V}$. Take $\varepsilon_d \geq 0$ and $\varepsilon_u \geq 0$ such that $B_u = (1-\phi)(1-\beta)\underline{V} + \varepsilon_u$ and $B_d = \Delta V - b + \beta(1-\phi)\underline{V} - \varepsilon_d$. If $P = \frac{B_u}{1-\phi}$ the offer is accepted and B_u is repaid, and the buyer gets payoff $\widehat{D} = \phi\beta\underline{V} + b + \varepsilon_d - \frac{\varepsilon_u}{1-\phi}$ and repays the debt. However this proposal is always dominated by offering $P = (1-\beta)\underline{V}$ and repaying her debt, obtaining a payoff equal to $\beta\phi\underline{V} + b + \varepsilon_d$ which is also larger than the payoff she obtains from trading on the outside market. As before, she offers $P = (1-\beta)\underline{V}$, the offer is accepted, the two firms trade with each other and realize joint profits $V_i = \overline{V}$, B_u is not repaid and B_d is repaid as long as $\varepsilon_d \geq 0$, i.e. if and only if $B_d = \Delta V - b + \beta(1-\phi)\underline{V}$.

Case 1.4: With probability $(1-\pi)\sigma$ we have $V_m = \overline{V}$ and $\mathbf{I} = 1$

Analogously to Case 1.2, the two firms always trade on the input market, realizes joint profits $V_m = \overline{V}$, debt B_u is repaid if $B_u \leq (1-\phi)(1-\beta)\overline{V}$and B_d is repaid if $B_d \leq (1-\phi)\beta\overline{V}$.

With probability $1-\alpha$ the seller has the right to make a take-it-or-leave-it offer. We have to analyze two different scenarios, depending on the whether she observes $V_i = \overline{V}$ or not.

Case 2.1: With probability $(1-\pi)$ we have $V_m = \overline{V}$.

Analogously to Cases 1.2 and 1.4 above, A1 ensures that there does not exists a price $P \leq \underline{V}$ that makes both the buyer and the seller willing to trade with each other.

Case 2.2: With probability π we have $V_i = \overline{V}$.

I first prove the following claim

Claim: *Regardless of* B_d, *the seller offers a price* $P \in \{\overline{V} - \beta\underline{V}, \overline{V} - \beta\underline{V} - b\}$.

Proof: If $B_d \leq \beta\underline{V}$ and $P \leq \overline{V} - \frac{B_d}{1-\phi}$, depending on the realization of b, the buyer accepts an offer $P = \overline{V} - \beta\underline{V} - \mathbf{I}b$. If instead $P > \overline{V} - \frac{B_d}{1-\phi}$, the buyer accept the offer if and only if $P \leq \overline{V} - \frac{\beta\underline{V} - B_d}{\phi} - \frac{\mathbf{I}b}{\phi} \leq \overline{V} - \beta\underline{V} - \frac{\mathbf{I}b}{\phi} \leq \overline{V} - \beta\underline{V} - \mathbf{I}b$. If instead $B_d > \beta\underline{V}$ and $P > \overline{V} - \frac{B_d}{1-\phi}$, depending on the realization of b, the buyer accepts an offer $P = \overline{V} - \beta\underline{V} - \mathbf{I}b$. Instead if $P \leq \overline{V} - \frac{B_d}{1-\phi}$ the buyer accepts the offer if and only if $P \leq \overline{V} - B_d - \phi\beta\underline{V} - \mathbf{I}b \leq \overline{V} - \beta\underline{V} - \mathbf{I}b$. This concludes the proof of the Claim|

The claim established that the bargaining strategy of the seller does not depend on whether $B_d \gtrless \beta\underline{V}$. Given this result, the seller set $P = \overline{V} - \beta\underline{V} - \widehat{\mathbf{I}}b$, with $\widehat{\mathbf{I}} = 1$ if

$$\begin{aligned} & \max\{\overline{V} - \beta\underline{V} - b - B_u, \phi(\overline{V} - \beta\underline{V} - b)\} \\ \geq\ & (1-\sigma)\max\{\overline{V} - \beta\underline{V} - B_u, \phi(\overline{V} - \beta\underline{V})\} + \sigma\max\{(1-\beta)\underline{V} - B_u, \phi(1-\beta)\underline{V} \end{aligned}$$

It follows that if $B_u \leq (1-\phi)(1-\beta)\underline{V}$ or $B_u > (1-\phi)(\overline{V}-\beta\underline{V})$, $\widehat{\mathbf{I}} = 1$ if and only if $\sigma\Delta V \geq b$, if instead $B_u \in ((1-\phi)(\overline{V}-\beta\underline{V}-b),(1-\phi)(\overline{V}-\beta\underline{V})]$ then $\widehat{\mathbf{I}} = 0$, and finally if $B_u \in ((1-\phi)(1-\beta)\underline{V},(1-\phi)(\overline{V}-\beta\underline{V}-b)]$ then $\widehat{\mathbf{I}} = 1$.

We are now ready to establish the maximum pledgeable income when the two units are independently owned and managed. Let $\mathbf{I}_d(s \mid B_d, B_u) \in \{\mathbf{0},\mathbf{1}\}$ and $\mathbf{I}_u(s \mid B_d, B_u) \in \{\mathbf{0},\mathbf{1}\}$ be two indicator functions taking values equal to one if in state s, and given B_u and B_d the outcome of the bargaining process is such that B_d and B_u respectively are repaid. Let π_s be the probability that state s is realized. The investor solves

$$\max_{B_d,B_u} \mathbf{\Sigma}_s \pi_s \left(\mathbf{\Sigma}_{j\in\{d,u\}} B_j \cdot \mathbf{I}_j(s \mid B_d, B_u)\right)$$

From the analysis of the different cases 1.4 to 2.2, it is clear that the indicators $\mathbf{I}_j(s \mid B_d, B_u)$ are piecewise linear in B_d and B_u. Hence the investor solves a linear programming problem, and, as it is well known, the solution of the program will (generically) be at a corner. It follows immediately from the analysis of cases 1.4 to 2.2 that the corners are as stated in the Lemma. This concludes the Proof of the Lemma. ‖

Remark:

The proof of the previous Lemma implies that in each state s, the repayment of B_d does not depend on B_u and viceversa, i.e. $\mathbf{I}_j(s \mid B_d, B_u)$ can be rewritten as $\mathbf{I}_j(s \mid B_j)$. The investor's problem can thus be decomposed into two simpler problems: for $j \in \{d, u\}$,

$$\max_{B_j} \mathbf{\Sigma}_s \pi_s B_j \cdot \mathbf{I}_j(s \mid B_j)$$

Denote with $P^j(\alpha)$ the solution to this problem, and hence $P_{ni}(\alpha) = P^d(\alpha) + P^u(\alpha)$. From the previous Lemma we have

$$\begin{aligned} P^d(\alpha) &= (1-\phi)\max\{\beta\underline{V}, \beta\overline{V}(1-\pi(1-\alpha)), \\ &\quad (\Delta V + \beta\underline{V})\alpha\pi(1-\sigma), (\frac{\Delta V - b}{1-\phi} + \beta\underline{V})\alpha\pi\} \end{aligned}$$

and

$$
\begin{aligned}
P^u(\alpha) &= (1-\phi)\max\{(1-\beta)\underline{V}, (1-\beta)\overline{V}(1-\pi\alpha), \\
&\quad (\overline{V}-\beta\underline{V})(1-\alpha)\pi(1-\sigma), (\overline{V}-\beta\underline{V}-b)(1-\alpha)\pi\}
\end{aligned}
$$

Consider first $\alpha = 1$. To determine $P^d(1)$, note that assumption A1 implies $\frac{\Delta V - b}{1-\phi} + \beta\underline{V} \leq \Delta V + \beta\underline{V}$ and assumption A3 implies $\beta\overline{V} \geq (\Delta V + \beta\underline{V})\pi$. Since $(\Delta V + \beta\underline{V}) \geq \max\{(\Delta V + \beta\underline{V})(1-\sigma), (\frac{\Delta V - b}{1-\phi} + \beta\underline{V})\}$ we conclude that $P^d(1) = \beta\overline{V}$.It is instead obvious that $P^u(1) = (1-\phi)(1-\beta)\max\{\underline{V}, \overline{V}(1-\pi)\}$, and hence

$$
P_{ni}(1) = (1-\phi)\left[\beta\overline{V} + (1-\beta)\max\{\underline{V}, \overline{V}(1-\pi)\}\right]
$$

Consider now $\alpha = 0$. It is obvious that $P^d(0) = (1-\phi)\beta\max\{\underline{V}, \overline{V}(1-\pi)\}$. To determine $P^u(0)$, note that assumption A2 implies $\overline{V} - \beta\underline{V} - b \leq \overline{V} - \beta\underline{V} - \phi\Delta V$, and assumptions A2 and A4 imply $(1-\beta)\overline{V} \geq \left(\overline{V} - \beta\underline{V} - \phi\Delta V\right)\pi$. We conclude that $P^u(0) = (1-\phi)\max\{(\overline{V} - \beta\underline{V})\pi(1-\sigma), (1-\beta)\overline{V}\}$ and hence

$$
P_{ni}(0) = (1-\phi)\left[\beta\max\{\underline{V}, \overline{V}(1-\pi)\} + \max\{(\overline{V}-\beta\underline{V})\pi(1-\sigma), (1-\beta)\overline{V}\}\right]
$$

This concludes the Proof of Proposition 2.■

Proof of Proposition 3

I consider the two cases $\alpha = 1$ and $\alpha = 0$ separatedly. Consider first the case $\alpha = 1$. We have to prove that $P_i > P_{ni}(1)$ if and only if $\pi > \pi^*(1, \beta, \phi, \sigma, V, b) = \frac{1}{(1-\beta)}\min\{\frac{b - \phi\Delta V}{(1-\phi)\overline{V}}, \sigma\}$.

Suppose first that $\pi \geq \frac{\Delta V}{\overline{V}}$. Then $P_{ni}(1) = (1-\phi)(\underline{V} + \beta\Delta V)$. But than note that $P_i(1) \geq \overline{V} - b - \phi\underline{V} \geq P_{ni}(1)$ as $b \leq (1 - \beta(1-\phi))\Delta V$ is implied by A1. Suppose instead $\pi < \frac{\Delta V}{\overline{V}}$. Then $P_{ni}(1) = (1-\phi)\overline{V}(1 - \pi(1-\beta))$. If $\pi > \frac{\sigma}{1-\beta}$, then $P_i(1) \geq (1-\phi)\overline{V}(1-\sigma) \geq P_{ni}(1)$. If instead $\pi < \frac{\sigma}{1-\beta}$, then $P_{ni}(1) > P_i(1)$ if $(1-\phi)\overline{V}(1-\pi(1-\beta)) > \overline{V} - b - \phi\underline{V}$, i.e. if $\frac{1}{(1-\beta)}\frac{b+\phi\Delta V}{(1-\phi)\overline{V}} > \pi$. Assumption A1 implies that $\overline{V} - b - \phi\underline{V} \geq (1-\beta)\overline{V} + (\beta-\phi)\underline{V}$, and hence $\pi > \frac{(\beta-\phi)}{(1-\phi)(1-\beta)}\frac{\Delta V}{\overline{V}}$ implies $P_{ni}(1) < P_i(1)$. Since A1 implies $\frac{(\beta-\phi)}{(1-\phi)(1-\beta)} < 1$, rearranging terms we conclude that $P_i > P_{ni}(1)$ if and only if $\pi > \frac{1}{(1-\beta)}\min\{\frac{b-\phi\Delta V}{(1-\phi)\overline{V}}, \sigma\}$ as stated in the proposition.

Consider now the case $\alpha = 0$. We have to prove that $P_i > P_{ni}(0)$ if and only if $\pi > \pi^*(\alpha, \beta, \phi, \sigma, V, b) = \frac{1}{\beta}\min\{\frac{b-\phi\Delta V}{(1-\phi)\overline{V}}, \sigma, \frac{\beta(1-\beta)\overline{V}}{(1-\sigma)(\overline{V}-\beta\underline{V})}\}$

I consider two separate cases, depending on whether $\beta \lessgtr \frac{1}{2}$. Consider first $\beta < \frac{1}{2}$. Note that $(\overline{V} - \beta\underline{V})\pi > (1-\beta)\overline{V}$ if $\pi > \frac{(1-\beta)\overline{V}}{(\overline{V}-\beta\underline{V})}$, however $\beta < \frac{1}{2}$ implies $\frac{(1-\beta)\overline{V}}{(\overline{V}-\beta\underline{V})} > \frac{\beta\overline{V}}{\Delta V+\beta\underline{V}}$ and hence $(\overline{V} - \beta\underline{V})\pi < (1-\beta)\overline{V}$. If $\pi > \frac{\Delta V}{\overline{V}}$ then $P_{ni}(0) = (1-\phi)[\beta\underline{V} + (1-\beta)\overline{V}]$. However assumption A1 implies $P_i(0) \geq (1-\beta)\overline{V} + (\beta-\phi)\underline{V}$, and $\Delta V > 0$ implies $(1-\beta)\overline{V} + (\beta-\phi)\underline{V} > P_{ni}(0)$. Suppose instead $\pi < \frac{\Delta V}{\overline{V}}$, then $P_{ni}(0) = (1-\phi)(1-\beta\pi)\overline{V}$. The results follows as in the proof of proposition 3, since assumption A1 implies $\frac{b-\phi\Delta V}{(1-\phi)\overline{V}} < \frac{\Delta V}{\overline{V}}$.

The analysis to prove the case $\beta > \frac{1}{2}$ is more involved, as $P_{ni}(0)$ is non-monotonic in π, and hence several cases have to be considered.

Case 1: Suppose first that $\pi < \min\{\frac{\Delta V}{\overline{V}}, \frac{(1-\beta)\overline{V}}{(\overline{V}-\beta\underline{V})(1-\sigma)}\}$, then $P_{ni}(0) = (1-\phi)(1-\beta\pi)\overline{V}$ and the analysis is as before.

Case 2: Suppose instead $\pi > \max\{\frac{\Delta V}{\overline{V}}, \frac{(1-\beta)\overline{V}}{(\overline{V}-\beta\underline{V})(1-\sigma)}\}$, and hence $P_{ni}(0) = (1-\phi)[\beta\underline{V} + (\overline{V} - \beta\underline{V})\pi)(1-\sigma)]$. We know that $P_i(0) \geq \overline{V} - b - \phi\underline{V}$ and that $\frac{\partial P_i}{\partial(1-\sigma)} = (1-\phi)\overline{V}$ if $(1-\sigma) > \frac{\overline{V}-b-\phi\underline{V}}{(1-\phi)\overline{V}}$, while $\frac{\partial P_i}{\partial(1-\sigma)} = 0$ if $(1-\sigma) < \frac{\overline{V}-b-\phi\underline{V}}{(1-\phi)\overline{V}}$. Since $(1-\phi)\overline{V} > \frac{\partial P_{ni}}{\partial(1-\sigma)} = (1-\phi)(\overline{V} - \beta\underline{V})\pi > 0$, the condition

$$P_i(0)_{|(1-\sigma)=\frac{\overline{V}-b-\phi\underline{V}}{(1-\phi)\overline{V}}} > P_{ni}(0)_{|(1-\sigma)=\frac{\overline{V}-b-\phi\underline{V}}{(1-\phi)\overline{V}}} \tag{6}$$

is necessary and sufficient to prove $P_i(0) > P_{ni}(0)$ for all values of σ when $\pi > \max\{\frac{\Delta V}{\overline{V}}, \frac{(1-\beta)\overline{V}}{(\overline{V}-\beta\underline{V})(1-\sigma)}\}$. Let's substitute $(1-\sigma) = \frac{\overline{V}-b-\phi\underline{V}}{(1-\phi)\overline{V}}$ in $P_{ni}(0)$. After some tedious algebra we can show that condition 6 can be rewritten as $\pi < \frac{\overline{V}}{\overline{V}-\beta\underline{V}} \frac{\overline{V}-b-\phi\underline{V}-\beta(1-\phi)\underline{V}}{\overline{V}-b-\phi\underline{V}}$. The right hand side is decreasing in b and increasing in ϕ. Hence a sufficient condition is that the inequality is satisfied when $b = \beta\Delta V$, and when $\phi = \max\{2 - \frac{1}{\beta}, 0\} = 0$, where the last equality follows from $\beta > \frac{1}{2}$. After substituting $b = \beta\Delta V$ and $\phi = 0$, the inequality becomes $\pi < \frac{\overline{V}}{\overline{V}-\beta\underline{V}} \frac{\overline{V}(1-\beta}{\overline{V}(1-\beta+\beta\underline{V}}$. Assumption A3 ($\pi < \frac{\beta\overline{V}}{\Delta V+\beta\underline{V}}$) implies that the last inequality is always satisfied, since some algebra and $\beta > \frac{1}{2}$ imply $\frac{\overline{V}}{\overline{V}-\beta\underline{V}} \frac{\overline{V}(1-\beta}{\overline{V}(1-\beta+\beta\underline{V}} > \frac{\overline{V}}{\Delta V+\beta\underline{V}} > \frac{\beta\overline{V}}{\Delta V+\beta\underline{V}}$.

Case 3: Suppose that π belongs to the non empty interval $(\frac{\Delta V}{\overline{V}}, \frac{(1-\beta)\overline{V}}{(\overline{V}-\beta\underline{V})(1-\sigma)})$. In this case $P_{ni}(0) = (1-\phi)(\beta\underline{V} + (1-\beta)\overline{V})$ and it is easy to show that assumption A1 implies $P_i(0) \geq \overline{V}(1-\beta) + (\beta-\phi)\underline{V} > P_{ni}(0)$.

Case 4: Suppose that π belongs to the non empty interval $(\frac{(1-\beta)\overline{V}}{(\overline{V}-\beta\underline{V})(1-\sigma)}, \frac{\Delta V}{\overline{V}})$. In this case

$P_{ni}(0) = (1-\phi)(\beta\overline{V}(1-\pi)+\pi(1-\sigma)(\overline{V}-\beta\underline{V}))$. As in case 2, $0 < \frac{\partial P_i}{\partial(1-\sigma)} < \frac{\partial P_{ni}}{\partial(1-\sigma)}$ if $(1-\sigma) > \frac{\overline{V}-b-\phi\underline{V}}{(1-\phi)\overline{V}}$ and $\frac{\partial P_i}{\partial(1-\sigma)} > \frac{\partial P_{ni}}{\partial(1-\sigma)} = 0$ if $(1-\sigma) < \frac{\overline{V}-b-\phi\underline{V}}{(1-\phi)\overline{V}}$. To see whether $P_{ni}(0) \gtrless P_i(0)$ let's again consider $(1-\sigma) = \frac{\overline{V}-b-\phi\underline{V}}{(1-\phi)\overline{V}}$. Let $\Phi = \frac{\overline{V}-b-\phi\underline{V}}{(1-\phi)\overline{V}}$, condition 6 can be rewritten as

$$\Phi > \beta(1-\pi) + \pi\Phi\frac{\overline{V}-\beta\underline{V}}{\overline{V}}$$

which after some algebra becomes

$$\Phi > \frac{\beta(1-\pi)}{1-\pi\frac{\overline{V}-\beta\underline{V}}{\overline{V}}}$$

since $1-\pi\frac{\overline{V}-\beta\underline{V}}{\overline{V}}$ for all $\pi < 1$. The right hand side is decreasing in π, while the left hand side is decreasing in b. A sufficient condition for 6 to hold is to be verified at $\pi = \frac{\Delta V}{\overline{V}}$ and $b = \phi\Delta V$. Substituting for those values, condition 6 can be rewritten as $\frac{\overline{V}(1-\beta)+(\beta-\phi)\underline{V}}{(1-\phi)\overline{V}} > \beta$, which can be rewritten as $\overline{V}(1-\beta(2-\phi)) > -(\beta-\phi)\underline{V}$. Assumption A1 guarantees that this last inequality is always satisfied. We conclude that $P_i > P_{ni}(0)$ if and only if $\pi > \frac{1}{\beta}\min\{\frac{b-\phi\Delta V}{(1-\phi)\overline{V}}, \sigma, \frac{\beta(1-\beta)\overline{V}}{(1-\sigma)(\overline{V}-\beta\underline{V})}\}$ as stated in the proposition.■

Proof of Proposition 4

Proposition 4 performs comparative statics on the likelihood of vertical integration $L(\overline{\pi}, \alpha, \beta, \phi, \sigma, V, b)$ on parameters $\overline{\pi}, \phi, \sigma, V$ and b.

1. Straightofroward differentiation of 1 implies $\frac{\partial L(\cdot)}{\partial \overline{\pi}} = f\ (\cdot) > 0$

Comparative statics with respect to parameters $x = [\phi, \sigma, V, b]$ gives $sign\left|\frac{\partial L(\cdot)}{\partial x}\right| = -f\ (\cdot)sign\left|\frac{\partial \pi^*(\cdot)}{\partial x}\right|$.

2. Differentiation implies $\frac{\partial\left(\frac{b-\phi\Delta V}{(1-\phi)\overline{V}}\right)}{\partial\overline{V}} = \frac{1}{1-\phi}\frac{-\phi\overline{V}-(b-\phi\Delta V)}{\overline{V}^2} < 0$, $\frac{\partial\sigma}{\partial\overline{V}} = 0$, and $\frac{\partial\left(\frac{\beta(1-\beta)\overline{V}}{(1-\sigma)(\overline{V}-\beta\underline{V})}\right)}{\partial\overline{V}} = -\frac{\beta(1-\beta)}{(1-\sigma)}\frac{\beta\underline{V}}{(\overline{V}-\beta\underline{V})^2} < 0$ so that there exist a unique $\overline{V}^*$ such that $\frac{\partial\pi^*(\cdot)}{\partial\overline{V}} < 0$ if $\overline{V} > \overline{V}^*$ and $\frac{\partial\pi^*(\cdot)}{\partial\overline{V}} = 0$ if $\overline{V} \leq \overline{V}^*$. Conversely, $\frac{\partial\left(\frac{b-\phi\Delta V}{(1-\phi)\overline{V}}\right)}{\partial\underline{V}} = \frac{\phi}{(1-\phi)\overline{V}} > 0$, $\frac{\partial\sigma}{\partial\underline{V}} = 0$ and $\frac{\partial\left(\frac{\beta(1-\beta)\overline{V}}{(1-\sigma)(\overline{V}-\beta\underline{V})}\right)}{\partial\underline{V}} = \frac{\beta^2(1-\beta)\overline{V}}{(1-\sigma)}\frac{1}{(\overline{V}-\beta\underline{V})^2} > 0$ so that there exist a unique $\underline{V}^*$ such that $\frac{\partial\pi^*(\cdot)}{\partial\underline{V}} > 0$ if $\underline{V} < \underline{V}^*$ and $\frac{\partial\pi^*(\cdot)}{\partial\underline{V}} = 0$ if $\underline{V} \geq \underline{V}^*$.

3. Differentiation implies $\frac{\partial\left(\frac{b-\phi\Delta V}{(1-\phi)\overline{V}}\right)}{\partial\phi} = \frac{1}{\overline{V}}\frac{b-\Delta V}{(1-\phi)^2} < 0$, $\frac{\partial\sigma}{\partial\phi} = 0$, and $\frac{\partial\left(\frac{\beta(1-\beta)\overline{V}}{(1-\sigma)(\overline{V}-\beta\underline{V})}\right)}{\partial\phi} = 0$ so that there exists a unique ϕ^* such that $\frac{\partial\pi^*(\cdot)}{\partial\phi} < 0$ if $\phi > \phi^*$ and $\frac{\partial\pi^*(\cdot)}{\partial\phi} = 0$ if $\phi \leq \phi^*$

4. Differentiation implies $\frac{\partial\left(\frac{b-\phi\Delta V}{(1-\phi)\overline{V}}\right)}{\partial b} = \frac{1}{(1-\phi)\overline{V}} > 0$, $\frac{\partial\sigma}{\partial b} = 0$, and $\frac{\partial\left(\frac{\beta(1-\beta)\overline{V}}{(1-\sigma)(\overline{V}-\beta\underline{V})}\right)}{\partial b} = 0$ so that there exists a unique b^* such that $\frac{\partial\pi^*(\cdot)}{\partial b} > 0$ if $b < b^*$ and $\frac{\partial\pi^*(\cdot)}{\partial b} = 0$ if $b \geq b^*$. Similarly, $\frac{\partial\left(\frac{b-\phi\Delta V}{(1-\phi)\overline{V}}\right)}{\partial\sigma} =$

$0, \frac{\partial \sigma}{\partial \sigma} = 1$, and $\frac{\partial \left(\frac{\beta(1-\beta)\overline{V}}{(1-\sigma)(\overline{V}-\beta \underline{V})} \right)}{\partial \phi} = \frac{\beta(1-\beta)\overline{V}}{\overline{V}-\beta \underline{V}} \frac{1}{(1-\sigma)^2} > 0$ so that there exists a unique σ^* such that $\frac{\partial \pi^*(\cdot)}{\partial \sigma} > 0$ if $\sigma < \sigma^*$ and $\frac{\partial \pi^*(\cdot)}{\partial \sigma} = 0$ if $\sigma \geq \sigma^*$.■

Proof of Proposition 5

The proof of Proposition 5 is similar to the Proof of Proposition 4. When $\alpha = 1$, we have $\pi^*(1, \beta, \phi, \sigma, V, b) = \frac{1}{(1-\beta)} \min\{\frac{b-\phi \Delta V}{(1-\phi)\overline{V}}, \sigma\}$, and straightforward differentiation implies $\frac{\partial \pi^*(1,\cdot)}{\partial \beta} > 0$. When instead $\alpha = 0$ we have $\pi^*(0, \beta, \phi, \sigma, V, b) = \frac{1}{\beta} \min\{\frac{b-\phi \Delta V}{(1-\phi)\overline{V}}, \sigma, \frac{\beta(1-\beta)\overline{V}}{(1-\sigma)(\overline{V}-\beta \underline{V})}\}$ and since $\frac{\partial \left(\frac{(1-\beta)\overline{V}}{(\overline{V}-\beta \underline{V})} \right)}{\partial \beta} = -\frac{\overline{V}\Delta V}{(\overline{V}-\beta \underline{V})^2} < 0$, it follows that $\frac{\partial \pi^*(0,\cdot)}{\partial \beta} < 0$.■

Proof of Proposition 6

The proof of proposition 6 mimics the proof of proposition 3. I first derive the outcome of the bargaining process in the four states of nature, and then analyze the optimal contract. With a minor abuse of notation, I denote with $\mathbf{I}_j$ an indicator function taking value equal to 1 if firm $j \in \{d, u\}$ defaults. I denote with $V_{j,t}^{d_j,d_z}(\mathbf{B}, \mathbf{L})$ the payoff of firm $j \in \{d, u\}$ when trade $t \in \{i, m\}$ takes place and firms j (resp. $z \neq j$) defaults ($d_j = 1$, (resp. $d_z = 1$)) or not ($d_j = 0$, (resp. $d_z = 0$)).

Consider first the case $V_m = \overline{V}$, which happens with probability $(1 - \pi)$. Analogously to the proof in proposition 3, since $\beta \overline{V} < \Delta V$ trade has to take place on the spot input market, and debts are repaid as long as $B_u \leq (1 - \phi)(1 - \beta)\overline{V}$ and $B_d \leq (1 - \phi)\beta \overline{V}$. Note that in these states of the world it is not impssible to do better using joint liability. To see this, suppose the buyer defaults, and $L_u > 0$. The seller will not default if and only if $B_u + L_u \leq (1 - \phi)(1 - \beta)\overline{V}$.

Let us now consider the case in which $V_i = \overline{V}$ and $\mathbf{I}_b = 0$, which happens with probability $\pi(1 - \sigma)$.

Suppose, without loss of generality, $B_u > (1 - \phi)(1 - \beta)\underline{V}$ and $B_d > (1 - \phi)\beta \underline{V}$. This implies that if trade takes place on the input market, the upstream firm does not pay the debt B_u. Suppose the downstream firm offers a price P. If $P < (1 - \beta)\underline{V}$, the offer is rejected, and trade takes place on the input market. The payoff of the dowsntream firm is then given by $V_{d,m}^{\cdot,0} = \max\{\beta \underline{V} - B_d - L_d, \phi \beta \underline{V}\} = \phi \beta \underline{V}$. If instead $P \in \{(1 - \beta)\underline{V}, \frac{B_u + \mathbf{I}_d L_u}{(1-\phi)})$, the seller accepts the offer, but still defaults on her loan. The payoff of the downstream firm is gven by $V_{d,i}^{\cdot,0} = \max\{\overline{V} - P - B_d - L_d, \phi \left(\overline{V} - P\right)\}$.

Since in this case the optimal P is equal to $(1-\beta)\underline{V}$, we have $V_{d,i}^{\cdot,0} > V_{d,m}^{\cdot,0}$. This ensures that the downstream always makes an offer that is accepted. Finally, if $P \geq \frac{B_u + \mathbf{I}_d L_u}{(1-\phi)}$, the payoff of the downstream firm is given by $V_{d,i}^{\cdot,1} = \max\{\overline{V} - P^1 - B_d, \phi(\overline{V} - P^0)\}$, where P^0 is the price that has to be paid if $\mathbf{I}_d = 1$ and P^1 if $\mathbf{I}_d = 0$. Clearly, if $\mathbf{I}_d = 0$ we have $P^1 = \frac{B_u}{(1-\phi)}$ and the downstream firm repays the debt if $\overline{V} - \frac{B_u}{(1-\phi)} - B_d > \phi(\overline{V} - \frac{B_u + L_u}{(1-\phi)})$. By setting $L_u = (1-\phi)\overline{V} - B_u$ the right hand side becomes equal to zero, and the two debts are repaid if $\overline{V} - \frac{B_u}{(1-\phi)} - B_d > \overline{V} - (1-\beta)\underline{V} - B_d - L_d$. In an optimal contract, the right hand side is equal to $\phi(\Delta V + \beta\underline{V})$. It follows that if $\frac{B_u}{(1-\phi)} + B_d \leq \overline{V}(1-\phi) + \phi(1-\beta)\underline{V}$. If the latter inequality is not satisfied instead, the downstream firm repays debt B_d and liability L_d if and only if $B_d + L_d \leq (1-\phi)(\Delta V + \beta\underline{V})$. Since the problem is linear, we know that the solution will be at a corner. We start by proving the following Lemma.

Lemma: *The optimum is given by* $\frac{B_d^*}{1-\phi} = \beta\overline{V}$, $\frac{B_u^*}{1-\phi} = (1-\beta)\overline{V}$.

First note that the two candidate solution on the line $\frac{B_u}{(1-\phi)} + B_d = \overline{V}(1-\phi) + \phi(1-\beta)\underline{V}$ must have either $\frac{B_u^*}{1-\phi} = (1-\beta)\overline{V}$ or $\frac{B_u^*}{1-\phi} = (1-\beta)\underline{V}$. In the first case, debts are always repaid, which gives a total return to the investor equal to $(1-\phi)\overline{V} - (1-\beta)\phi\Delta V$. This return is always smaller than $(1-\pi)(1-\phi)\overline{V} + \pi(1-\phi)(\Delta V + \beta\underline{V})$ which can be obtained with a contract $\frac{B_d^*}{1-\phi} = \beta\overline{V}$, $\frac{B_u^*}{1-\phi} = (1-\beta)\overline{V}$ and $L_d = (1-\phi)(1-\beta)\Delta V$. To see this, note that the returns associated with this last contract are decreasing in π, and are equal to $(1-\phi)\left[\frac{\Delta V}{\Delta V + \beta\underline{V}}(1-\beta)\overline{V}) + \beta\overline{V}\right]$ at their minimum, i.e. when $\pi = \frac{\beta\overline{V}}{\Delta V + \beta\underline{V}}$. In the second case, the debts are repaid with probability π, and with probability $(1-\pi)$ only the upstream firm repays at most $(1-\phi)(1-\beta)\overline{V}$. Returns are thus given by $(1-\phi)\left[\pi\overline{V} + (1-\beta)(1-\pi)\overline{V}\right]$. Note that the inequality

$$\pi\overline{V} + (1-\beta)(1-\pi)\overline{V} < (1-\pi)\overline{V} + \pi(\Delta V + \beta\underline{V})$$

is always satisfied since the RHS is decreasing in π, the LHS is increasing in π and the inequality is satisfied at $\pi = \frac{\beta\overline{V}}{\Delta V + \beta\underline{V}}$ This implies that the highest repayment that the investor can get out of the two firms is given by the contract The optimum is given by $\frac{B_d^*}{1-\phi} = \beta\overline{V}$, $\frac{B_u^*}{1-\phi} = (1-\beta)\overline{V}$ and $L_d = (1-\phi)(1-\beta)\Delta V$ ‖

Remark: The proof for the case in which $V_i = \overline{V}$ and $\mathbf{I}_b = 1$ is completely analogous, and is therefore omitted. In this case, the highest financial return that can be extracted from the two firms is given by $B_d + L_d = \overline{V} - b - \phi\underline{V} - (1-\beta)(1-\phi)\underline{V} > (1-\phi)\beta\overline{V}$. The last inequality implies that

$L_d^* > 0$ and $B_d^* = (1-\phi)\beta\overline{V}$. B_u is set at the highest level consistent with repayment in states in which $V_m = \overline{V}$, i.e. $B_u^* = (1-\phi)(1-\beta)\overline{V}$. As shown above $L_u^* = (1-\phi)\overline{V} - B_u^* = (1-\phi)\beta\overline{V} = B_d^*$.

The highest pledgeable income immediately follows. If $(1-\sigma) > \frac{\Delta V + \beta(1-\phi)\underline{V} - b}{(1-\phi)(\Delta V + \beta\underline{V})}$ it is optimal to set $B_d^* + L_d^* = (1-\phi)(\Delta V + \beta\underline{V})$, otherwise it is optimal to choose $B_d^* + L_d^* = \Delta V + \beta(1-\phi)\underline{V} - b$. With probability $1-\pi$ the firms jointly repay $B_u^* + B_d^*$, while with probability π they repay $L_d^* + B_d^*$. This completes the proof.■

Proof of Proposition 7

To proove the proposition, we start by prooving the followig Lemma:

Lemma: *Assume* $\alpha = \frac{1}{2}$, $\sigma \to 1$, $b = \Delta V$ *and* $\pi < \frac{\Delta V}{\overline{V}}$. *Under Assumption A2-A4* $P_{ni} = (1-\phi)\overline{V}(1-\frac{\pi}{2})$.

Proof. Since, conditional on the two firms having been separatedly financed, the optimal level of debts are determined separatedly, the result in the Lemma presented in the proof of proposition 3 holds. Consider the detrmination of B_u^*. First,$\pi < \frac{\Delta V}{\overline{V}}$ implies $\frac{\pi}{2} < \frac{\Delta V}{\overline{V}}$, ruling out $B_u^* = (1-\phi)(1-\beta)\underline{V}$. Also, $\sigma \to 1$ rules out $B_u^* = (1-\phi)(\overline{V} - \beta\underline{V})$ (which is repaid with probability $\alpha\pi(1-\sigma)$). Hence $B_u^* \in \{(1-\phi)(1-\beta)\overline{V}, (\overline{V} - b - \beta\underline{V})\}$. The first term is repaid with probability $(1-\alpha\pi) = 1 - \frac{\pi}{2}$, the second with probability $\alpha\pi = \frac{\pi}{2}$. Moreover, $b = \beta\Delta V$ implies that $\overline{V} - b - \beta\underline{V} = (1-\beta)\overline{V}$, and since $\pi < 1$, we have $1 - \frac{\pi}{2} > \frac{\pi}{2}$. Clearly, the returns on the upsream firm are given by $P^u = (1-\phi)(1-\beta)(1-\frac{\pi}{2})\overline{V}$.

Consider now the downstream firm. An analogous proof shows that $P^d = (1-\phi)\beta(1-\frac{\pi}{2})\overline{V}$ if, when $b = \beta\Delta V$, the inequality $\left(\overline{V} - b - \phi\underline{V} - (1-\beta)\underline{V}\right)\frac{\pi}{2} < (1-\phi)\beta\overline{V}(1-\frac{\pi}{2})$ is verified. To see that this is the case, take the highest possible $\pi = \frac{\Delta V}{\overline{V}} < \frac{\beta\overline{V}}{\Delta V + \beta\underline{V}}$ (this last inequality implies $\frac{\Delta V^2}{\Delta V^2 + \overline{V}\underline{V}} < \beta < \frac{\Delta V}{\overline{V}}$). The inequality can be rewritten as $2\beta\overline{V}\underline{V}(1-\phi) > (1-\beta(2-\phi))\Delta V^2$. If $\phi \to 0$ this expression is always satisfied. If instead $\phi \to \beta$ the inequality becomes $\frac{\Delta V^2}{\overline{V}\underline{V}} < 2\frac{\beta}{1-\beta}$. The RHS is increasing in β, and the inequality is always satisfied since it is true when $\beta = \frac{\Delta V^2}{\Delta V^2 + \overline{V}\underline{V}}$. This completes the proof of the Lemma.■

Denote by P_d^1 and P_u^1 the returns that investors $\mathcal{D}$ and $\mathcal{U}$ obtain from financing the non integrated buyer and seller respectively when the other investor has also financed the respective non integrated firm. The previous Lemma implies $P_d^1 = (1-\phi)(1-\beta)(1-\frac{\pi}{2})\overline{V}$ and $P_d^1 = (1-\phi)\beta(1-\frac{\pi}{2})\overline{V}$. Similarly, denote by P_d^0 and P_u^0 the returns that investors $\mathcal{D}$ and $\mathcal{U}$ obtain from financing the non integrated buyer and seller respectively when the other investor has not financed the respective non integrated firm.

To proove the first part of the proposition, let's start by considering the case of investor $\mathcal{U}$. With probability π the firm realizes profits $(1-\beta)\underline{V}$, while with probability $1-\pi$ the upstream firm realizes profits $(1-\beta)\overline{V}$. Since $\pi < \frac{\Delta V}{\overline{V}}$, the optimal amount of debt in this case is $(1-\phi)(1-\beta)\overline{V}$ which is repaid with probability $1-\pi$. It follows that $P_u^0 = (1-\phi)(1-\beta)(1-\pi)\overline{V}$ and (symmetrically) $P_d^0 = (1-\phi)\beta(1-\pi)\overline{V}$. Hence $P_j^1 > P_j^0$ for $j \in \{d, u\}$. The fact that non-integration is a Nash equilibrium implies that $P_u^1 > k_u$ (otherwise $\mathcal{U}$ best reply would be to not invest), and $P_d^1 > \max\{k_d, P_{int} - k_u\}$ (if $P_d^1 < k_d$ $\mathcal{D}$ could ensure a higher payoff by not investing, while if $P_d^1 < P_{int} - k_u$ she would improve her payoff by financing an integrated firm). The second inequality implies $k_u > P_{int} - P_d^1$, and hence the first inequality implies $P_u^1 > P_{int} - P_d^1$, which rearranged yields $P_{int} < P_d^1 + P_u^1 = P_{ni}$. This complete the first part of the proof.

Remark 1: To see why $P_{int} < P_{ni}$ is not sufficient to ensure non-integration, consider the case in which $k_d < P_d^1 < P_{int}$ (which happens if β is small), and $k_u \to 0$. Then $\mathcal{D}$ improve her payoff by financing an integrated firm. If she does so, the best response for $\mathcal{U}$ is to not invest as $L < 0$. But $P_{int} > P_d^1$ implies $P_{int} > P_d^0$, and hence the action profile in which $\mathcal{D}$ finances a vertically integrated firm while $\mathcal{U}$ does not invest is an equilibrium.

To proove the second part of the proposition, it is sufficient to prove that $P_{int} > P_{ni} = P_u^1 + P_d^1$ implies that $P_{int} - K < P_d^1 - k_d$ (the best scenario under which $\mathcal{D}$ chooses not to finance a vertically integrated firm) leads to a contradiction. To see this, Suppse that $P_{int} - K < P_d^1 - k_d$, i.e. that $P_{int} < P_d^1 + k_u$. Combining this inequality with $P_{int} > P_u^1 + P_d^1$ implies $k_u > P_u^1$, which means that $\mathcal{U}$ prefers not to invest. A contradiction of the equilibrium with non-integration.■

Remark 2: Suppose instead $P_{int} < K$, and $P_d^1 < k_d$. Than if $k_u \in [P_u^0, P_u^1)$ the only equilibrium is that no firm is financed, despite the fact that $P_d^1 < k_d$ does not rule out $P_{ni} > K$, since $P_u^1 > k_u$.