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ABSTRACT

Coarse Matching and Price Discrimination

We study two-sided markets with heterogeneous, privately informed agents who gain from being matched with better partners from the other side. Agents are matched through an intermediary. Our main results quantify the relative attractiveness of a coarse matching scheme consisting of two classes of agents on each side, in terms of matching surplus (output), the intermediary's revenue, and the agents' welfare (defined by the total surplus minus payments to the intermediary). In a nutshell, our philosophy is that, if the worst-case scenario under coarse matching is not too bad relative to what is achievable by more complex, finer schemes, a coarse matching scheme will turn out to be preferable once the various transaction costs associated with fine schemes are taken into account. Similarly, coarse matching schemes can be significantly better than completely random matching, requiring only a minimal amount of information.

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1 Introduction

We study two-sided markets with heterogeneous agents who gain from being matched with a better partner (or product) from the other side. If there is private information about attributes, as we assume here, agents may be unable to identify the quality of their potential matching partners, nor are their claims about own quality necessarily credible. In such markets intermediaries can play a crucial role. Examples are employment and recruiting agencies, dating and marriage matchmakers, real estate brokers, technology and business brokers.[1] Also, multi-product firms act as intermediaries by matching customers with heterogeneous tastes to products of different qualities. Intermediated exchange that consists of two price schedules, one for each side of the market, and a matching scheme, can induce agents to reveal their private information (or parts of it), thus allowing the implementation of allocations that satisfy some optimality criterion with respect to the underlying preferences, e.g., implementation of assortative matching that calls for pairing the best agent from one side with the best agent from the other side, the second-best with the second-best, and so forth.

Obviously, achieving optimality when faced with a diversity of attributes or preferences will require complex price and matching schedules. But, as Chao and Wilson (1987), Wilson (1989) and McAfee (2002) have aptly noted, it is rare to see a large number of categories of services or products to be offered. Instead, what one observes are coarse matching schemes where agents are partitioned into a small number of classes, and are then matched "randomly" within each class. Familiar examples from the realm of price discrimination are business and tourist class in aeroplanes and trains, standard and "professional" versions of software.

Our main purpose here is to study the relative attractiveness of a very coarse matching scheme consisting of only two classes of agents in terms of matching surplus (output), the intermediary's revenue, and the agents' welfare (defined by the total surplus minus payments to the intermediary). For coarse matching with two classes, the intermediary charges a fixed fee to each side of the market. An agent who did pay the fee is matched with a random partner from the other side of the market who also did pay the respective fee, while an agent who did not pay is randomly matched with an agent who also did not pay.[2] We only consider a very crude form of coarse matching where the cutoff between the upper and lower classes is determined by the mean of one of the distributions rather than by some maximization exercise.[3]

[1] For other examples, see Spulber's (2005) recent survey of the literature on two-sided markets.

[2] In order to keep complexity at a minimum, we abstract here from the possibility of exclusion.

[3] Obviously if the cutoff is optimally chosen to maximize output or revenue, the bounds obtained in this paper for

If agents' attributes are complementary, coarse matching creates an inefficiency in terms of output since the efficient matching is assortative, and thus much finer. Our main purpose here is to quantify this inefficiency and the associated sub-optimal outcomes in terms of the revenue to the intermediary and the agents' welfare. Our analysis is in terms of worst-case scenarios within large classes of distributions of the agents' attributes.

In a nutshell, our philosophy is that, if the worst-case scenario is not too bad relative to what is achievable by more complex, finer schemes, the very simple coarse matching scheme will turn out to be preferable once the various transaction costs associated with fine schemes are taken into account. These transaction cost may take the form of communication and complexity (or menu) costs for the intermediary, evaluation costs for the intermediary (who needs more detailed information about the environment in order to implement a fine scheme) and for the agents (who need precise information about their own and others' attributes in order to optimally respond to a fine scheme), or production costs for firms offering different qualities. In addition, coarse matching schemes have the advantage that agents reveal only partial information about themselves, thus avoiding, at least to some extent, exploitation in the future.

The literature on coarse matching is rather thin.[4] Chao and Wilson (1987) and Wilson (1989) consider a monopolist seller who matches a continuum of customers who differ in their valuations for service quality to a given feasible set of qualities. Besides a characterization of price schedules that lead to full information revelation by consumers, a main result is that the efficiency loss due to the usage of a coarse matching scheme using n classes converges to zero at a rate proportional to $1/n^2$, as n tends to infinity. This elegant result, however, is not very informative about the relative performance of coarse matching with a small number of classes. McAfee (2002) studies a matching model with a continuum of types and complete information on both sides. His remarkable result is that, for a broad class of distribution of types, a two-class scheme achieves at least as much surplus as the average of efficient assortative matching and random matching. In other words, the surplus loss associated with coarse matching with two classes may, in fact, be rather modest, and is always less than half total achievable surplus. We will both repeatedly use this result in our analysis, and extend it in several directions. Damiano and Li (2004) allow for two-sided incomplete information in a model with a continuum of types on each side, generalizing the type of analysis performed on

either criteria will continue to hold.

[4] In a different vein from the one considered here, Burdett and Cole (1997) study a dynamic matching framework where heterogeneous agents meet randomly over time. If two agents accept a match, they leave the market, otherwise they continue searching for a better partner. In any steady state equilibrium agents partition themselves into a finite number of classes according to their "quality" levels.

one side by Wilson. They characterize incentive-compatible pricing schedules as leading to coarse matching or, in the limit, to assortative matching. Their main result offers conditions under which an intermediary achieves a higher revenue by using the finest scheme (assortative matching) rather than some other coarser scheme. Thus, their interesting result is orthogonal to the type of analysis performed here. Most of the papers in the literature assume that the value of a match is the product of the types of the agents, and we will also use this specification.

The present paper is organized as follows:

In Section 2 we describe the two-sided matching model with heterogeneous, and privately informed agents on both sides of the market. We assume there, and in most of the sequel, that matched agents share the surplus equally. (Our analysis easily extends to other fixed sharing rules - see also Section 5 where we illustrate a one-sided model). In Subsection 2.1 we present several definitions and results from mathematical statistics that will be used for our approximations. Roughly speaking, these results offer bounds on the variability and on the values of distribution functions at certain points in their range for large, non-parametric classes of distributions functions. Our results about coarse matching will use only a minimal fom of information about the variability of the involved distributions.

In Section 3 we consider the incentive compatible price schedules that lead to assortative matching, and establish their relation to the unique stable payoff vector in the core of the market.

Section 4 contains the main results that provide a comparison of assortative matching and coarse matching involving two classes in terms of total surplus, the intermediary's revenue, and the agents' welfare.

In Subsection 4.1 we focus on total surplus. We first show that the loss of surplus form coarse matching is less than one-quarter of total surplus for the class of distributions considered by McAfee. We also extend his result to other classes of distributions, and we provide additional lower bounds on the surplus from coarse matching in terms of a coefficient measuring the degree of heterogeneity in the populations. Our analysis suggests that coarse matching performs relatively well in terms of total surplus if the distribution functions are logconcave or concave and have a low coefficient of covariation, thus offering some intuition for the rather "magic" analysis performed by McAfee.

In Subsection 4.2 we analyze the effects of coarse matching on the intermediary's revenue, and find a counterpart to McAfee's result for surplus (which was obtained for a model with complete information), albeit under more restrictive conditions. Roughly speaking, these conditions ensure that the expected utility of agents in the upper class from being randomly matched with agents in the lower class is sufficiently small. As a consequence, agents in the upper class have a relatively high willingness to pay for being matched with partners in the upper class. We also analyze under which circumstances one side of the market pays more to the intermediary in coarse matching than the other - a question intensely discussed in the literature on two-sided markets, which tends to focus on random matching among homogenous agents and on network externalities.[5] In contrast, our explanation is based on heterogeneity differentials.

In Subsection 4.3 we show that the two-class scheme may perform surprisingly well in terms of the agents' welfare. In particular, we identify conditions under which the agents' welfare under coarse matching exceeds the one under assortative matching.[6]

In Section 5 we illustrate how the previous results can be applied to a model of price discrimination by a monopolist that produces several levels of quality in a market with heterogeneous customers. Section 6 concludes. All proofs are contained in an Appendix.

2 The model

There are two groups of agents, "men" and "women." Each man is characterized by an attribute x, each woman by an attribute y. Agent's i attribute is private information to i. Attributes are distributed according to distributions F (men) and G (women) over the intervals $[0, \tau_F]$ and $[0, \tau_G]$, $\tau_F, \tau_G \leq \infty$, respectively. The distributions F and G are atomless and have continuous densities, $f > 0$ and $g > 0$, respectively. We assume here that the two groups have the same measure, normalized to be one.

To capture production complementarities in the simplest way we assume that if man x and woman y are matched, they generate an output (or matching surplus) $u(x, y) = xy$.[7] We also assume that surplus is shared according to a fixed rule that does not react to market conditions: specifically, if matched, man x receives αxy and woman y receives $(1 - \alpha) xy$, where $\alpha \in [0, 1]$. Below we compare this setting to the benchmark where agents share output in a way that guarantees payoffs in the core of the matching market.

An intermediary who cannot observe the agent's types offers contracts that are characterized by a matching rule ϕ, a set-valued function that maps any $x \in [0, \tau_F]$ to a subset $\phi(x) \subseteq [0, \tau_G]$, and

[5] See, e.g., Rochet and Tirole (2005) and the literature cited in this paper.

[6] This is related to a result by Hoppe, Moldovanu and Sela (2006) in a framework where partners use wasteful signals in order to match. They describe circumstances where agents may be better off under random matching (without any waste) than under assortative matching which needs to be sustained by wasteful signaling.

[7] All the results about assortative matching can be generalized to the case of a general supermodular matching surplus functions $u(x, y)$, as we indicate below.

by price schedules, $p_m : [0, \tau_F] \to \mathbb{R}$ for men, and $p_w : [0, \tau_G] \to \mathbb{R}$ for women. For any $A \subseteq [0, \tau_F]$ let $\nu_m(A)$ the measure of men announcing types in A. Similarly, define $\nu_w(A)$ for women. The matching rule ϕ is *feasible* if for any $A \subseteq [0, \tau_F]$, $\nu_m(A) = \nu_w(\phi(A))$. That is, each subset of men is matched to a subset of women of equal measure.

As shown by Damiano and Li (2005), a feasible and incentive-compatible matching rule partitions the sets of men and women, respectively, into n corresponding subsets, matches the elements of this partition in a positively assortative way, and then, within each matched partition, matches agents randomly.

In this paper, we restrict attention to three special matching rules. First, we consider the limit case where the number of classes n goes to infinity - this is called *assortative matching*. Second, we consider the case of $n = 2$, i.e., *coarse matching* with two classes. Finally, we also consider the case $n = 1$, which yields completely *random* matching.

Throughout the paper we assume that those agents who are not served by the intermediary will be randomly matched with each other.

2.1 Failure rates and covariance

We briefly survey below several definitions and results from mathematical statistics that will be used in the analysis:

Definition 1 *1) A distribution function F is said to have an increasing failure rate - IFR (decreasing failure rate - DFR) if $f(t)/[1 - F(t)]$ is increasing (decreasing) on $[0, \tau_F)$, $\tau_F \leq \infty$.*[8]

2) A distribution function F is said to have an increasing reversed failure rate - $IRFR$ (decreasing reversed failure rate - $DRFR$) if $f(t)/F(t)$ is increasing (decreasing) on $[0, \tau_F)$, $\tau_F \leq \infty$.

3) A distribution function F is said to be new better than used in expectation - $NBUE$(new worse than used in expectation - $NWUE$) if

$$\int_t^{\tau} (1 - F(x))dx \leq (\geq) EX(1 - F(t)),\ t \geq 0$$

where EX denotes the mean of F.

Definition 2 *Let X, Y be non-negative random variables on $[0, \tau_F]$ and $[0, \tau_G]$, $\tau_F, \tau_G \leq \infty$, respectively. The coefficient of covariation of X and Y is given by*

$$CCV(X, Y)) \equiv \sqrt{Cov(X, Y)/EXEY}$$

[8] For example, the exponential, uniform, normal, power (for $\alpha \geq 1$), Weibull (for $\alpha \geq 1$), gamma (for $\alpha \geq 1$) distributions are *IFR*. The exponential, Weibull (for $0 < \alpha \leq 1$), gamma (for $0 < \alpha \leq 1$) are *DFR*. See Barlow and Proschan (1975) who use the terminology in the context of reliability theory. Other authors refer to "hazard rates".

In particular, note that $E(XY) = (1 + CCV^2(X,Y))EXEY$ *for any two non-negative random variables.*

By the integral form of Chebychevs's inequality (see Theorem 236 in Hardy et al., 1934), $Cov(X, h(X)) \geq 0$ for any increasing function h. Hence the coefficient of covariation is well defined for any X non-negative random variable and for any increasing function h. The coefficient of covariation reduces to the more common coefficient of variation $CV(X) \equiv \sqrt{Var(X)}/EX$ when h is the identity function. The latter is a dimensionless measure of variability relative to the value of the mean, and is a minimal amount of information about the respective distribution.

The next lemma gathers the main relations among the concepts defined above. For the less obvious implications, see Barlow and Proschan (1975).

Lemma 1

1. *Any* IFR *distribution is* $NBUE$*.*
2. *A distribution* F *is* IFR *if and only if its survivor function* $(1 - F)$ *is logconcave.*
3. *Any* DFR *distribution is* $NWUE$*.*
4. *Any convex distribution is* IFR
5. *Any* DFR *distribution is concave.*
6. *Any concave distribution is* $DRFR$*.*
7. *A distribution* F *is* $DRFR$ *if and only if* F *is logconcave.*
8. $CV^2(X) \leq (\geq)\ 1$ *if* F*, the distribution of* X*, is* $NBUE$ $(NWUE)$*.*
9. $CV^2(X) \leq (\geq)\ 1/3$ *if* F*, the distribution of* X*, is convex (concave).*
10. $CCV^2(X, G^{-1}F(X)) \leq (\geq)\ 1$ *if both* F *and* G *are* IFR (DFR) *distributions.*
11. *Let* h, k *be two increasing functions such that: a)* $h(0) = k(0) = 0$*; b)* $Eh(X) = Ek(X)$*; c)* $k^{-1}h$ *is convex. Then* $CCV(X, h(X)) \leq CCV(X, k(X))$*.*

The above properties are of interest since they describe large, non-parametric classes of distribution functions for which various upper or lower bounds on the values of distributions at various points in the range hold, as the following lemma shows.

Lemma 2

1. $F(EX)(1-F(EX)) \leq \frac{1}{4}$ *for all* F.

2. $F(EX) \leq (\geq)\frac{1}{2}$ if F *is convex (concave).*

3. $F(t) \leq (\geq)1-e^{-\frac{t}{EX}}$ $,t < EX$ $(t \leq EX)$ if F *is* IFR (DFR).

4. $\int_0^{\tau}(1-F(x))dx = EX$ for all F

5. $\int_0^{t} F(x)dx \leq (\geq)t - EX(1-e^{-\frac{t}{EX}})$ *if* F *is* $NBUE$ $(NWUE)$.

6. $\int_0^{EX} F(x))dx \leq (\geq)$ EXe^{-1} if F *is* $NBUE$ $(NWUE)$.

7. $F(EX)(1-F(EX)) \leq \frac{e-1}{e^2} \approx 0.23254$ *if* F *is* DFR.

3 Assortative matching

Under *assortative matching* each man x is matched with a woman $\psi(x)$, where $\psi : [0,\tau] \to [0,\tau]$ is implicitly defined by $F(x) = G(\psi(x))$. If the matching surplus function $u(x,y)$ is supermodular[9], as considered here, assortative matching yields the efficient outcome in terms of total output.

Under the assortative matching rule ψ, the price schedules p_m and p_w need to satisfy the following incentive-compatibility constraints:

$$\alpha x\psi(x) - p_m(x) \geq \alpha x\psi(\hat{x}) - p_m(\hat{x}) \tag{1}$$

for all $x, \hat{x} \in [0,\tau]$, and

$$(1-\alpha)\psi^{-1}(y)\,y - p_w(y) \geq (1-\alpha)\psi^{-1}(\hat{y})\,y - p_w(\hat{y}) \tag{2}$$

for all $y, \hat{y} \in [0,\tau]$.

Using the envelope theorem, we obtain

$$\alpha x\psi'(x) - \frac{dp_m(x)}{dx} = 0.$$

Since the man with the lowest type will be matched for sure with the woman with the lowest type, the willingness to pay of this type is always zero, which yields the boundary condition $p_m(0) = 0$. Hence, we have

$$p_m(x) = \int_0^x \alpha z\psi'(z)\,dz. \tag{3}$$

[9] A matching surplus is $u(x,y)$ is supermodular if $u_1 > 0, u_2 > 0$,and $u_{12} > 0$ where u_i, $i = 1,2$ denotes the derivative with respect to the i'th coordinate, and u_{12} denotes the mixed derivative.

The incentive-compatible price schedule for women is analogously derived. Letting $\varphi = \psi^{-1}$, we have

$$p_w(y) = \int_0^y (1-\alpha) z \varphi'(z) \, dz. \tag{4}$$

Consider now the case where agents share their matching surplus in a way that guarantees core-stability: Given a matching rule ϕ, the sharing of the surplus is called *stable* if:

$$\begin{aligned} \forall x, \ \delta(x) + \rho(\phi(x)) &= x\phi(x) && (5) \\ \forall x, y, \quad \delta(x) + \rho(y) &\geq xy && (6) \end{aligned}$$

As efficiency is a prerequisite of stability, the only feasible rule for which the surplus can be shared in a stable way is assortative matching ψ. It is well-known and straightforward to show that the stable shares are:

$$\begin{aligned} \delta(x) &= \int_0^x \psi(z) \, dz && (7) \\ \rho(y) &= \int_0^y \varphi(z) \, dz && (8) \end{aligned}$$

The following result establishes the relation between stable shares and the incentive-compatible price schedules for any fixed sharing rule.

Proposition 1 *1. The incentive compatible price schedules for the fixed sharing rule satisfy*

$$\begin{aligned} p_m(x) &= \alpha\rho(\psi(x)) && (9) \\ p_w(y) &= (1-\alpha)\delta(\varphi(y)), && (10) \end{aligned}$$

Consequently, the net utilities of any agents x and y are $\alpha\delta(x)$ and $(1-\alpha)\rho(y)$, respectively.

2. The intermediary's revenue from each matched pair satisfies:

$$\min(\alpha, 1-\alpha) x\psi(x) \leq p_m(x) + p_w(\psi(x)) \leq \max(\alpha, 1-\alpha) x\psi(x)$$

3. In particular, the revenue from each matched pair is exactly half the matching surplus if $\alpha = 1/2$.[10]

[10] The proposition continues to hold for any supermodular surplus function.

The above result shows that the price schedules used by the intermediary correct for the induced distortion of incentives caused by the fixed sharing that does not respond to outside options: the net utilities of the agents constitute a stable sharing of the output remaining after payments to the intermediary are made.

For future use, we also note here that total surplus and the intermediary's revenue, respectively, are given by:

$$\begin{aligned} U^a &= \int_0^{\tau_F} x\psi(x)\, dF(x) \\ R^a_\alpha &= \alpha \int_0^{\tau_F} \psi(x) \left[x - \frac{1-F(x)}{f(x)} \right] dF(x) \\ &\quad + (1-\alpha) \int_0^{\tau_F} x \left[\psi(x) - \psi'(x) \frac{1-F(x)}{f(x)} \right] dF(x) \end{aligned}$$

Until the application performed in Section 5, we always assume that surplus is shared equally, i.e., $\alpha = 1/2$, and we therefore omit this index.

4 Coarse matching

We now turn our attention to "very coarse" matching that involves only two categories on each side of the market. We investigate how the two-class matching scheme fares against assortative matching in terms of total surplus, total revenue, and the agents' total welfare. We focus on the case where output is shared equally among matched partners[11], i.e., $\alpha = 1/2$.

Under coarse matching, the intermediary sets only two prices: p^c_m, p^c_w. Men that are willing (not willing) to pay p^c_m are randomly matched with women that are willing (not willing) to pay p^c_w. Denote by $\hat{x}$ ($\hat{y}$) the lowest type of men (women) who is willing to pay p^c_m (p^c_w). By the assumptions on the match surplus and utility functions these types are well defined. Such a pricing scheme is incentive compatible if and only if the following equations are satisfied:[12]

$$\alpha \int_0^{\hat{y}} \frac{\hat{x}y}{G(\hat{y})} dG(y) = \alpha \int_{\hat{y}}^{\tau_G} \frac{\hat{x}y}{1-G(\hat{y})} dG(y) - p^c_m \tag{11}$$

$$(1-\alpha) \int_0^{\hat{x}} \frac{x\hat{y}}{F(\hat{x})} dF(x) = (1-\alpha) \int_{\hat{x}}^{\tau_F} \frac{x\hat{y}}{1-F(\hat{x})} dF(x) - p^c_w \tag{12}$$

$$\hat{y} = \psi(\hat{x}) \tag{13}$$

[11] We provide a discussion of the case of $\alpha = 1$, i.e., when one side receives the whole surplus of the match, in Section 5.

[12] See also Damiano and Li (2005).

The prices p_m^c p_w^c are such that the cutoff types $\hat{x}$ and $\psi(\hat{x})$ are indifferent between between being randomly matched in the high class (while paying the price) and being randomly matched (for free) in the low class. Note also that $\hat{y}$ needs to be $\hat{x}$'s partner in assortative matching in order to ensure feasibility.

Let U^{EX} be the total surplus from coarse matching with two categories where the cutoff point $\hat{x} = EX$, i.e., the mean of F. Furthermore, let EX_L be the mean x-type of the low class, and EY_L the mean y-type of the low class when using the cutoffs $\hat{x} = EX$ and $\hat{y} = \psi(EX)$. We have:

$$EX_L = \frac{\int_0^{EX} xf(x)dx}{F(EX)} = EX - \frac{\int_0^{EX} F(x)dx}{F(EX)}$$

and similarly:

$$EY_L = \frac{\int_0^{\psi(EX)} yg(y)dy}{G(\psi(EX))} = \psi(EX) - \frac{\int_0^{\psi(EX)} G(x)dx}{G(\psi(EX))}$$

Equivalently, we have:

$$\begin{aligned} EX - EX_L &= \frac{\int_0^{EX} F(x)dx}{F(EX)}; \\ \psi(EX) - EY_L &= \frac{\int_0^{\psi(EX)} G(x)dx}{G(\psi(EX))} \end{aligned}$$

Using these definitions, we write the total surplus as:

$$\begin{aligned} U^{EX} &= \int_0^{EX} \int_0^{\psi(EX)} \frac{xy}{F(EX)} g(y) f(x) \, dy dx \\ &\quad + \int_{EX}^{\tau_F} \int_{\psi(EX)}^{\tau_G} \frac{xy}{1 - F(EX)} g(y) f(x) \, dy dx \qquad (14) \\ &= EXEY + \frac{F(EX)}{1 - F(EX)}(EX - EX_L)(EY - EY_L) \qquad (15) \end{aligned}$$

Similarly, we denote the respective revenue obtained with cutoff $\hat{x} = EX$ by R^{EX}. For $\alpha = 1/2$, we obtain:

$$\begin{aligned} R^{EX} &= \frac{1}{2} EX \left[\int_{\psi(EX)}^{\tau_G} ydG(y) - \frac{1 - G(\psi(EX))}{G(\psi(EX))} \int_0^{\psi(EX)} ydG(y) \right] \\ &\quad + \frac{1}{2} \psi(EX) \left[\int_{EX}^{\tau_F} xdF(x) - \frac{1 - F(EX)}{F(EX)} \int_0^{EX} xdF(x) \right] \\ &= \frac{1}{2} [EX(EY - EY_L) + \psi(EX)(EX - EX_L)] \qquad (16) \end{aligned}$$

Besides Lemma 2, the main working horse for our analysis is:

Lemma 3

1. $EX - EX_L = \frac{\int_0^{EX} F(x)dx}{F(EX)} \leq (\geq) \frac{1}{2}EX$ *if* F *is convex (concave).*

2. $EX - EX_L \leq (\geq) \frac{EXe^{-1}}{F(EX)}$ *if* F *is* $NBUE$ $(NWUE)$.

3. $EY - EY_L = EY - \psi(EX) + \frac{\int_0^{\psi(EX)} G(x)dx}{G(\psi(EX))} \leq (\geq) \frac{1}{2}EY$ *if* G *is convex (concave) and* ψ *is concave (convex).*

4. $EY - EY_L \geq (\leq) EX - EX_L$ *if* ψ *is convex (concave) and if* $EX \geq (\leq) EY$.

4.1 Total surplus

Chao and Wilson (1987) and Wilson (1989) have shown that the total surplus loss due to the usage of coarse matching with n categories is of order $1/n^2$. That is , $U_n \geq U^a - O\left(1/n^2\right)$, where U_n denotes the maximal total surplus from coarse matching with n classes. Of course, the order of magnitude in Chao and Wilson's analysis may still mean that the surplus from coarse matching with only a few categories is very small when compared to that in assortative matching. Under several technical conditions, we are able here to establish a tighter, explicit link between the surplus in assortative matching and the surplus in coarse matching with only two classes. We employ an elegant result due to McAfee (2002):

Proposition 2 *(McAfee, 2002)Let the distributions* F *and* G *be both* IFR *and* $DRFR$. *Then*

$$U^{EX} \geq \frac{U^a + U^r}{2} \tag{17}$$

where U^r *denotes the total surplus from random matching.*

Using the above result and Lemma 1-8 we obtain:

Corollary 1 *Under the conditions of the above Proposition, we have*

$$U^{EX} \geq \frac{2 + CCV^2(X, \psi(X))}{2(1 + CCV^2(X, \psi(X))}U^a \geq \frac{3}{4}U^a.$$

Since all convex distributions are IFR, the previous corollary and Lemma 1-9 and -11 imply:

Corollary 2 *Let* F *be convex and* $DRFR$, ψ *be convex, and* $EX = EY$, *then*

$$U^{EX} \geq \frac{7}{8}U^a$$

Example 1 *Assume that* $F = x^2$, $G = y^2$ *on* $[0, 1]$. *This yields* $U^{EX} = \frac{392}{405} U^a > \frac{7}{8} U^a$.

We now consider concave distributions, and, in particular, DFR distribution functions about which McAfee's result is silent. The next proposition establishes lower bounds on the total surplus from two-class coarse matching, U^{EX}, expressed as a mark-up on the random output U^r. They suggest that coarse matching (that utilizes only a minimal amount of information) can be significantly better than random matching (for which no information is needed at all). Since $U^r = U^a / \left[1 + CCV^2(X, \psi(X))\right]$, these bounds can be translated into bounds involving the coefficient of covariation and the surplus in assortative matching U^a.

Proposition 3 *1) Let* F *be concave and let* ψ *be convex. Then*

$$U^{EX} \geq \frac{5}{4} U^r$$

2) Let F *be* DFR, *and let* ψ *be convex. Then*

$$U^{EX} \geq \frac{3}{2} U^r$$

3) Let F *be* DFR, *let* ψ *be convex, and assume that* $EX \geq EY$. *Then*

$$U^{EX} \geq \frac{e}{e-1} U^r \simeq 1.582 U^r$$

4)Let ψ *be concave, and switch the role of* F *and* G. *Then all above results for* U^{EY}.

Remark 1 *It is instructive to compare the above result*[13] *with McAfee's for those distributions where both apply. Consider first* $F = G = 1 - e^{-t}$ *which is* IFR, DFR, *and* $DRFR$. *We have* $CCV(X, X) = 1$ *and* $U^a = 2U^r$ *in this case. Our estimate gives* $U^{EX} = \frac{e}{e-1} U^r = \frac{1}{2} U^r + \frac{2e-1}{2e-2} U^r = \frac{1}{2} U^r + \frac{2e-1}{4e-4} U^a \simeq \frac{1}{2} U^r + 0.64 U^a > \frac{1}{2} U^r + \frac{1}{2} U^a$. *By continuity, we obtain a better estimate for* IFR *distributions that are not too convex with respect to the exponential. Next, consider the class of concave and* IFR *distributions. Then our estimate gives* $U^{EX} \geq \frac{5}{4} U^r = \frac{1}{2} U^r + \frac{3}{4(1+CCV^2(X,\psi(X))} U^a \geq (\leq) \frac{1}{2} U^r + \frac{1}{2} U^a$ *if* $CCV^2(X, \psi(X)) \leq (\geq) \frac{1}{2}$. *For example, we obtain* $U^{EX} \geq \frac{1}{2} U^r + \frac{9}{16} U^a > \frac{1}{2} U^r + \frac{1}{2} U^a$ *for* $F(t) = G(t) = t$, *where* $CCV^2(X, X) = \frac{1}{3}$.

The following corollary provides bounds on U^{EX} in terms of the surplus in assortative matching, U^a, for specific ranges of the coefficient of variation.

[13] The estimates are tight. The estimate in part 3 holds with equality for $F(t) = G(t) = 1 - e^{-t}$. The estimate in part 1 holds with equality for $F(t) = G(t) = t$. The estimate in part 2 cannot be improved without further knowledge about the involved functions. This follows by the same logic as in part 1, but in relation to the exponential distribution rather than the uniform.

Corollary 3 *1. Let F be concave, ψ convex, $EX = EY$, and $CV^2(X) \leq \frac{2}{3}$. Then $U^{EX} \geq \frac{3}{4}U^a$.*

2. Let F be concave, ψ convex, $EX = EY$ and $\frac{2}{3} \leq CV^2(X) \leq 1$. Then $U^{EX} \geq \frac{5}{8}U^a$.

3. Let F be DFR, ψ convex, $EX = EY$, and $1 \leq CV^2(X) \leq 2$. Then $U^{EX} \geq \frac{1}{2}U^a$.

Finally, Proposition 3-3 can be used to extend McAfee's result beyond a subclass of IFR distributions.

Proposition 4 *Let F be DFR, ψ be convex, $EX = EY$, and $CV^2(X) \leq \frac{2}{e-1}$.*[14] *Then*

$$U^{EX} \geq \frac{U^a + U^r}{2}$$

Example 2 *Assume that F and G are Weibull: $F(x) = 1 - e^{-x^{\frac{19}{20}}}$, $G(y) = 1 - e^{-y^{\frac{19}{20}}}$ on $[0, \infty)$. Then F is DFR, and satisfies $CV^2(X) \leq \frac{2}{e-1}$. We have: $U^{EX} \simeq 1.712$, $[U^a + U^r]/2 \simeq 1.628$, and thus $U^{EX} > [U^a + U^r]/2$.*

The main insight emerging from the above analysis is that coarse matching with only two classes may achieve a substantial fraction of the efficient (assortative) surplus for a wide range of distribution functions: if the distributions are IFR and $DRFR$, as considered by McAfee, the two-class scheme realizes more than 75% of U^a (Corollary 1); if the distributions are concave [DFR] and the coefficient of covariation is below 2/3 [between 1 and 2], the two-class scheme realizes more than 75% [50%] of U^a (Corollary 3). Furthermore, $U^{EX} \geq [U^a + U^r]/2$ holds for distributions that are IFR and $DRFR$ (McAfee's Theorem), and for DFR distributions with $CCV^2(X, \psi(X)) \leq 2/(e-1)$ (Proposition 4).[15]

The intuitive reason behind all these findings is due to the fact that $DRFR$ (and hence DFR distributions) are logconcave. Similarly to concavity, this property allows us to derive an upper bound on the survivor function of the distribution (see Sengupta and Nanda, 1999) which implies an upper bound on the mass of agents with types above the mean. If the mass of the high-type agents is relatively small, the loss from matching them randomly instead of assortatively is relatively small, which makes coarse matching more attractive. However, the surplus in random matching may be very small compared to the one in assortative matching. Note though that the surplus in random matching approaches the surplus in assortative matching if the coefficient of covariation goes to zero. Therefore, for the two-class scheme to perform sufficiently well, the properties of

[14] Alternatively, one could assume that F is DFR, ψ is convex, $EX \geq EY$, and $1 \leq CCV^2(X, \psi(X)) \leq \frac{2}{e-1}$.

[15] Proposition 4 requires that ψ is convex. Note that ψ can be concave, while exchanging the roles of F and G.

logconcavity or concavity have to be combined with specific bounds on the coefficient of variation. For example, logconcavity of the survivor function, which is equivalent to IFR, ensures that the coefficient of variation is less than 1. Roughly speaking, if the distribution is IFR, properly normalized differences between expected values of attributes in successive quantiles are decreasing. This implies an upper bound on the surplus gains from matching agents assortatively instead of randomly.

4.2 The intermediary's revenue

If the intermediary's goal is to maximize revenue, the decision to employ a coarse matching scheme will depend on how well it performs relatively to other schemes. Comparing assortative matching and coarse matching with n classes, Damiano and Li (2005) find that the former outperforms the latter for any $n < \infty$ if some conditions on the virtual valuations are satisfied. For the symmetric setting where $F = G$, these conditions boil down to the virtual valuation $x - [1 - F(x)] / f(x)$ being nondecreasing. In particular, F being IFR is a sufficient condition.

We take here a somewhat different perspective, which takes into account that employing a larger number of classes will usually be associated with high transaction costs (or production costs - see the Application in Section 5). Thus, two classes may be superior to infinitely many classes if this simple scheme yields a sufficiently high fraction of the revenue achievable from assortative matching. Furthermore, given that in many cases only limited information about the distribution functions is available, we evaluate only the profitability of coarse matching that uses the mean of one of the distributions as the cutoff separating the high class from the low class[16], i.e., $\hat{x} = EX$ and $\hat{y} = \psi(EX)$.

As a preliminary step, we relate the total revenue from coarse matching, R^{EX}, to the total surplus generated by this scheme, U^{EX}.

Proposition 5 *1) Let F be concave (convex), ψ be convex (concave), and $\alpha = 1/2$. Then*

$$R^{EX} \leq (\geq) \frac{1}{2}(U^{EX} - EX_L EY_L)$$

2) Let F and G be convex, and $\alpha = 1/2$. Then

$$R^{EX} < \frac{1}{2} U^{EX}$$

[16] In the symmetric setting where $F = G$ it is straightforward to check how revenue can be increased by using a slightly higher (lower) cutoff value. Taking the derivative of $R^{\hat{x}}$ with respect to $\hat{x}$ we obtain: $\left.\frac{\partial R^{\hat{x}}}{\partial \hat{x}}\right|_{\hat{x}=EX} \gtreqless 0 \Leftrightarrow \frac{F(EX)}{f(EX)} \gtreqless EX$. For example, if F is convex (concave), the revenue is increased by moving to a cutoff value below (above) EX.

Thus, under the two-class coarse scheme, the intermediary extracts less than half the total surplus if F and G are both concave (Part 1) or convex (Part 2). This is in clear contrast to the case of assortative matching, where $R^a = \frac{1}{2}U^a$ for any distributions (recall Proposition 1). Since total surplus is maximal under assortative matching, the above result immediately implies the next corollary, which extends the analysis by Damiano and Li (2005).

Corollary 4 *Let either a) F be concave and ψ be convex, or b) F and G be convex. Then*

$$R^{EX} < R^a$$

For concave distribution functions, we now establish a lower bound for the revenue from coarse matching as a fraction of the revenue from assortative matching :

Proposition 6 *Let F and G be both concave and IFR. Then*[17]

$$R^{EX} \geq \frac{1}{2}R^a$$

Proposition 6 constitutes the analog, in terms of revenue, of McAfee's result about surplus: since the revenue from matching agents randomly is zero, $R^{EX} \geq \frac{1}{2}R^a \Leftrightarrow R^{EX} \geq [R^a + R^r]/2$. The message is that the conditions are more restrictive here than those required for the respective surplus result. That is, while concavity is needed here, the surplus result requires that distributions are $DRFR$, which is weaker. In fact, it can be easily verified that the revenue result may fail to hold if the distributions are $DRFR$ (but not concave) and IFR.[18] To intuitively understand why concavity plays an important role here, recall that payments are obtained only from agents in the high class. Their willingness to pay for being matched with a random partner from the high class on the other side of the market is determined by the value of the outside option, i.e. being randomly matched with a partner from the low class. As the distribution of types on the other market side becomes more concave, the mass of potential partners with very low types gets larger, reducing the value of the outside option. As a consequence, agents in the high class are willing to pay more, leading to a higher revenue.

Which side pays more? In practice, price schedules observed in two-sided markets are often uneven, with one side paying more than the other.[19] Answers to the question of which side pays

[17] For $F = 1 - e^{-x}$, $G = 1 - e^{-y}$, we have $R^{EX} = \frac{1}{e-1}R^a \simeq 0.582R^a$.

[18] Consider, for example, $F = x^3$, $G = y^3$ on $[0, 1]$, which are IFR and $DRFR$.This yields $R^{EX} < \frac{1}{2}R^a$.

[19] See, e.g., *The Economist*, "Matchmakers and trustbusters", p.84, Dec 10th, 2005.

more tend to focus on network externalities.[20] A result in Hoppe, Moldovanu, Sela (2006) can be adapted to show that men's total payment is larger (smaller) than women's total payment if the intermediary uses assortative matching, and if the matching function $\psi = G^{-1}F$ is convex (concave). In particular, agents are willing to pay more as the other side becomes more heterogeneous (since then the marginal gains in terms of winning a better matching partner are larger at the high end of the type range).

We inquire here whether this finding carries over to coarse matching. Let R_m^{EX} be the total payments obtained from men, and R_w^{EX} the total payments obtained from women under coarse matching with cutoff EX.

Proposition 7 *1) Assume that either (a) F is convex and G concave, or (b) F is IFR and G is DFR. Then ψ is convex and $R_m^{EX} \geq R_w^{EX}$.*

2) Assume that $EX \geq EY$ and that ψ is convex. Then $R_m^{EX} \geq R_w^{EX}$.

Part 2 of the proposition resembles the finding for assortative matching, but the intuition is slightly different. Here, the agents' willingness to pay is determined by the outside option of being randomly matched within the low class. If F and G have the same mean, but the distribution of women, G, has a higher variance, the chances for men who take the outside option to end up with a very low type partner are higher than for women. As a consequence, for types in the high class, men's willingness to pay is larger than women's[21].

4.3 The agents' welfare

Another important criterion for measuring the relative performance of coarse matching is the agents' welfare, defined as total surplus minus the payments to the intermediary. Let $W^a = U^a - R^a$ be the agents' welfare under assortative matching and $W^{EX} = U^{EX} - R^{EX}$ under coarse matching with cutoff EX.

Proposition 8 *1) Let F and G be concave and IFR, and let ψ be convex. Then*

$$W^{EX} \geq \frac{3}{4} W^a$$

2) Let F and G be convex and $DRFR$. Then

$$W^{EX} \geq W^a$$

[20] For an analysis of two-sided markets with network externalities, see, for instance, Rochet and Tirole (2003).

[21] In addition, there is a size effect (this is similar to assortative matching): if ψ is convex, the mass of men with types above a certain threshold is larger than the mass of women with types above that threshold.

Example 3 *1) Assume that* $F(x) = 1 - e^{-x}$, $G(y) = 1 - e^{-y}$ *on* $[0, \infty)$. *The distributions are concave and* IFR. *We have:* $W^{EX} = 1$, $W^a = 1$, *and thus* $W^{EX}/W^a = 1$.

2) Assume that F *and* G *are uniform on* $[0,1]$. *The distributions are convex and* $DRFR$. *We have:* $W^{EX} = 3/16$, $W^a = 1/6$, *and thus* $W^{EX}/W^a = 9/8$.

The result shows that the two-class scheme may perform surprisingly well in terms of agents' welfare. Recall that under the conditions of part 1, we have shown that $R^{EX} \geq \frac{1}{2}R^a = \frac{1}{4}U^a$, (Propositions 6 and 1), that $U^{EX} \geq \frac{5}{8}U^a$ (Corollary 3-2) and that $R^{EX} < \frac{1}{2}U^{EX}$ (Proposition 5). The combination of these results yield the exhibited bound.

The assumptions in part 2 describes a situation where coarse matching is relatively strong in terms of surplus (McAfee's Theorem), but tends to be weak in terms of revenue (Proposition 6). In this case the agents' welfare exceeds the one obtainable under assortative matching!

At least for the case covered by part 1 of the above result, our insights here together with Proposition 6 suggest that an intermediary who wishes to maximize the agents' welfare while respecting a budget constraint (analogous to the Ramsey-Boiteux problem in the theory of public finance) will find a coarse scheme to be quite attractive relative to assortative matching.

5 An application to price discrimination

Throughout the above analysis, we assumed that privately informed agents populate both market sides. Here we briefly illustrate how some of our previous analysis and results can be modified for a context where there are privately informed agents only on one side who get matched to observable items (or partners) on the other side. For instance, Wilson (1989) studies a multi-product seller in the electricity industry, seeking to match customers having heterogeneous valuations for power provision to different service qualities that represent different service probabilities. In this case, consumers naturally obtain the whole surplus from their purchase minus their payment to the seller. In other words the sharing rule corresponds to $\alpha = 1$, and we have to adjust our previous revenue and welfare results for this case. Total surplus is of course unaffected by the value of α.

For $\alpha = 1$, total revenue from assortative matching is given by

$$R_1^a = \int_0^{\tau_F} \psi(x) \left(x - \frac{1 - F(x)}{f(x)} \right) dF(x) \tag{18}$$

and the total revenue from coarse matching is given by

$$R_1^{EX} = EX(EY - EY_L) \tag{19}$$

Simple relations between revenues for $\alpha = 1$ and $\alpha = 1/2$ are:

Lemma 4 *If the assortative matching function ψ is convex (concave) then $R_1^a \geq (\leq) R_{1/2}^a$. If the assortative matching function ψ is convex (concave) and if $EX \geq (\leq) EY$ then $R_1^{EX} \geq (\leq) R_{1/2}^{EX}$.*

The next two results illustrate how our previous results can be adapted to this framework, and provide a comparison of the two matching rules in terms of revenue and agents' welfare:

Proposition 9 *Let F be IFR, G be IFR and concave. Then $R_1^{EX} \geq \frac{1}{4} R_1^a$. If, in addition, ψ is concave, then $R_1^{EX} \geq \frac{1}{2} R_1^a$.*

Proposition 10 *1) Let F and G be IFR and concave, and let ψ be convex. Then $W_1^{EX} \geq \frac{1}{2} W_1^a$. 2) Let F and G be convex and $DRFR$, and let ψ be convex. Then $W_1^{EX} \geq W_1^a$.*

In Wilson's model, the distributions of customers' valuations and the feasible distribution of service probabilities (and hence the assortative matching function ψ) are exogenously given[22]. Propositions 9 and 10 can therefore be directly used to identify the circumstances under which two service classes are relatively attractive in terms of revenue and/ or agents' welfare.

5.1 Price discrimination with quality costs

Here we briefly illustrate how Propositions 9 and 10 can be applied to the more realistic situation in which the monopolist takes production costs into account when determining the available qualities. In contrast to Wilson's model, the matching rule will be now endogenously determined.

We denote quality by q. There is a measure one of consumers each demanding a unit of the good. Consumers are distributed over $[0, 1]$ according to distribution F with $f = F' > 0$. The utility of type v from quality q is vq. The cost of producing y units of quality q is $c(q)\, y$ where $c(0) = c'(0) = 0$ and where , $c'(q) > 0$ and $c''(q) > 0$ for $q > 0$.

Consider first a monopolist who uses standard non-linear pricing. In this case the monopolist chooses a menu of prices and qualities from which the consumers can pick their preferred combination. We make the standard assumption that the virtual valuation $v - (1 - F(v))/f(v)$ is

[22] See McAfee (2002) for a mapping of Wilson's set-up into the matching framework used here (albeit with complete information).

increasing, which holds, for example, if F is IFR. Denote by $(p(v), q(v))$ the element that is chosen by type v. Using standard arguments, the monopolist's revenue and profit are given by:

$$\begin{aligned} R_1^a &= \int_0^1 q(v)\left(v - \frac{1-F(v)}{f(v)}\right) f(v)\,dv \\ \pi_1^a &= \int_0^1 \left[q(v)\left(v - \frac{1-F(v)}{f(v)}\right) - c(q(v))\right] f(v)\,dv. \end{aligned}$$

Let r be such that $r - (1-F(r))/f(r) = 0$. The function q that maximizes the above profit function is determined by and

$$\begin{aligned} q(v) &= 0 \ \text{ if } \ v \le r \\ c'(q(v)) &= v - \frac{1-F(v)}{f(v)} \ \text{ if } \ v \ge r \end{aligned}$$

The optimal menu contains a continuum of quality levels. Denote the distribution of quality levels by G, and note that $G(y) = F\left(q^{-1}(y)\right)$ for $y \in (0, q(1)]$. Thus, the function q plays the role of the matching function ψ[23].

We first compare the revenue from the optimal menu derived above with the revenue that the monopolist can achieve by producing only two quality levels and charging two prices. Suppose that customers with valuations below $EV = \int_0^1 v dF(v)$ are matched with the quality level $Q^L = \int_0^{EV} q(z)\,dF(z)/F(EV)$ and customers with valuations above this cutoff are matched with the quality level $Q^H = \int_{EV}^1 q(z)\,dF(z)/(1-F(EV)) = \frac{EQ - F(EV)Q^L}{1-F(EV)}$.

In order to be consistent with out general treatment, assume that the price paid by the members of the low class $p^L = 0$. The price paid by members of the high class is determined by

$$p^H = \frac{EV \int_{EV}^1 q(x)\,dF(x)}{1-F(EV)} - \frac{EV \int_0^{EV} q(x)\,dF(x)}{F(EV)} = EV\left(Q^H - Q^L\right).$$

The revenue from the coarse provision of quality is:

$$R_1^{EV} = EV\left(EQ - Q^L\right)$$

which is analogous to expression (19).

Example 4 *Suppose that* $c(q) = q^2$ *and that* v *is uniformly distributed over* $[0,1]$. *In this case,* $q(v) = 0$ *if* $v \le \frac{1}{2}$ *and* $q(v) = \frac{2v-1}{2}$ *if* $v > \frac{1}{2}$. *Thus* $G(y) = \frac{1+2y}{2}$ *for* $y \in \left[0, \frac{1}{2}\right]$, *which is concave*

[23] Although $q^{-1}(0)$ is not defined here, q can be approximated by an everywhere strictly increasing function, allowing us to use previous results. In fact, since the monopolist does not get revenue from agents that are not served, the results from the approximation underestimate the ratio between revenue in coarse matching versus revenue in assortative matching, since only a fraction of the agents are assortatively matched with a positive quality.

and IFR. Although $G(0) > 0$, $CCV^2(X, \psi(X)) \leq 1$, as is required for applying Proposition 9-2. Straightforward algebra yields $Q^H = \frac{1}{2}$, $R_1^a = \frac{1}{12}$, $R_1^{EV} = \frac{1}{16}$, and thus $R_1^{EV} = \frac{3}{4}R_1^a$. Furthermore, considering total profit (that takes into account the production cost) we get $\pi_1^a = \frac{1}{24}$, $\pi_1^{EV} = \frac{1}{32}$, and $\pi_1^{EV} = \frac{3}{4}\pi^a$. Moreover, the conditions of Proposition 10-1 are also satisfied. We can thus conclude that $W^{EV} \geq \frac{1}{2}W^a$. The numerical calculation yields: $W^{EV} = \frac{1}{32} > \frac{1}{48} = W^a$.

Two issues are worth pointing out:

1) The analysis considerably underestimates the revenue from coarse matching because we assumed that the monopolist produces two valuable qualities, but charges zero for the low class. This assumption was made in order to be consistent with our previous framework where there was no exclusion. A price discriminating monopolist can either set Q^L at zero or increase p^L to a positive value (thus excluding some customers) in order to increase revenue.

2) Proposition 9 deals only with revenue. What can be said about the profit comparison? Since costs are convex, the monopolist saves costs by producing the average quality levels Q^L and Q^H rather than producing the whole range of qualities. This does not immediately translate into a lower bound on profits from coarse provision of quality. Yet, if the cost function is sufficiently convex, the cost savings will be substantial and the bound on profits will match the bound on revenues (see example above).

Thus, when the transaction costs of providing different qualities are also taken into account, our analysis suggests that coarse provision of quality may well be the profit-maximizing choice of the monopolist for a broad range of customer type distributions and cost functions. Moreover, a public agency that is interested in the agents's welfare but has some revenue considerations (for budgetary reasons, etc...) will also have a strong incentive to offer coarse matching.

6 Conclusion

We have studied the performance of very coarse matching schemes and the associated price schedules in a two-sided market with heterogenous agents and with private information about attributes. In a variety of settings we have shown that such schemes are very effective. The type of analysis performed in this paper is not standard in the Economics literature. This literature is mostly fixated on the zero-one dichotomy between optimality versus suboptimality, and degrees of suboptimality are not quantified or compared. Several papers in mechanism design argue that only "simple" mechanisms are realistic[24]. But, a majority of these follow the above dichotomy, by completely specifying

[24] This is one argument in what is sometimes called the "Wilson doctrine" (see Wilson, 1987).

special settings where simple mechanisms are **fully** optimal[25]. Instead, we focus on clearly **suboptimal** mechanisms, while identifying settings where such mechanisms are very effective (and thus may become optimal once transaction costs associated with more complex mechanisms are taken into account)[26]. The scarcity of "worst-case" studies (or other quantifications of suboptimality) in the Economics literature should be contrasted to the wealth of papers following precisely this philosophy in the Operations Research/Computer Science literature[27]. We believe that both our understanding of existing trading institutions and our ability to design new effective institutions will profit from more studies in this vein.

7 Appendix

Proof of Lemma 2. 1) This is immediate by observing that on the interval $[0,1]$ the function $x(1-x)$ has a maximum at $x=\frac{1}{2}$.

2) Note that $F(X)$ is a uniformly distributed random variable on the interval $[0,1]$, and hence $E[F(X)]=\frac{1}{2}$. The claim follows then by Jensen's inequality since $F(EX)\leq(\geq)\ E[F(X)]=\frac{1}{2}$ if F is convex (concave).

3) The assertions are contained in Theorems 4.4 and 4.7 in Barlow and Proschan (1965).

4) This is immediate from integration by parts.

5) This follows by 4) and by rearrangement of terms from the fact that $\int_t^{\tau}(1-F(x))dx \leq (\geq$ $)\ EXe^{-\frac{t}{EX}}$ if F is $NBUE$ $(NWUE)$ (see Barlow and Proschan (1975), page 187)

6) This is just the instance of 6) for $t=EX$.

7) By 3) for $t=EX$, we know that $F(EX)\geq 1-e^{-1}$ for F DFR. Since $1-e^{-1}\geq\frac{1}{2}$, we obtain that the function $x(1-x)$ is decreasing for $x\geq 1-e^{-1}$. Thus, $F(EX)(1-F(EX))\leq (1-e^{-1})(e^{-1})=\frac{e-1}{e^2}$. ■

[25] E.g, Myerson (1981) shows that standard one-object auctions with a reserve price are revenue maximizing in the symmetric, independent private value environment. Holmstrom and Milgrom (1987) identify conditions where linear contracts are optimal for the provision of intertemporal incentives in a principal-agent relationship.

[26] Our analysis is similar in spirit to Neeman's (2003) study about the effectiveness of the English auction in environments where this auction is not revenue maximizing.

[27] Koutsoupias and Papadimitriou (1999) and Roughgarden and Tardos (2004) are excellent examples since they also combine game-theoretic reasoning. These authors study the "price of anarchy" in various network routing games, which is defined as the ratio between the welfare in the worst Nash equilibrium and the welfare in the Pareto-optimal allocation.

Proof of Proposition 1. To derive the net utilities of matched agents x and $\psi(x)$, note that

$$u(x,\psi(x)) = \int_0^x \psi(z)\,dz + \int_0^x z\psi'(z)\,dz$$

Thus, the net utilities of agents x and $\psi(x)$ from contracting with the intermediary (and being matched accordingly with each other) can be written, respectively, as

$$\begin{aligned} \alpha x\psi(x) - p_m(x) &= \alpha \int_0^x \psi(z)\,dz \\ (1-\alpha)\,x\psi(x) - p_w(\psi(x)) &= (1-\alpha)\int_0^y \varphi(z)\,dz \end{aligned}$$

■

Proof of Lemma 3. 1) Geometrically, the term $\left[\int_0^{EX} F(x)dx\right] / [EXF(EX)]$ is the ratio among the area below F up to the mean and the area of the rectangle with sides of EX and $F(EX)$, respectively. Note that any chord to the graph of a continuous convex (concave) function lies entirely above (below) the graph. Thus $\int_0^{EX} F(x)dx$ covers less (more) than $\frac{1}{2}$ of the area of the rectangle when F is convex (concave). Observe that the equality $EY - EY_L = \frac{1}{2}EY$ is indeed obtained for the uniform distribution (on any interval), which is both convex and concave.

2) This follows immediately from Lemma 2 -6.

3) By the same geometric argument as in 1), we get $\psi(EX) - EY_L = \frac{\int_0^{\psi(EX)} G(x)dx}{G(\psi(EX))} \leq (\geq)\frac{1}{2}\psi(EX)$ if G is convex (concave). Thus $EY_L \geq (\leq)\frac{1}{2}\psi(EX)$ if G is convex (concave) and ψ is concave (convex), and the result follows because $\frac{1}{2}\psi(EX) \geq (\leq)\frac{1}{2}EY$ if ψ is concave (convex) by Jensen's inequality.

4) Assume first that ψ is convex. If $F = G$, the result is obvious. Thus, assume $F \neq G$. By Theorem 6.2 in Barlow and Proschan (1981) $F(t) \leq G(t)$ for $t < EX$. In other words, the unique crossing of F and G (which must exists in this case) cannot occur below $t = EX$. Since $F \neq G$, we obtain $\psi(EX) < E(\psi X) = EY \leq EX$. Thus

$$F(t) \leq G(t) \text{ for } t \leq \psi(EX)$$

This yields the following chain:

$$
\begin{aligned}
\frac{\int_0^{\psi(EX)} G(t)dt}{G(\psi(EX))} &\geq \frac{\int_0^{\psi(EX)} F(t)dt}{F(EX)} \Leftrightarrow \\
\frac{\int_0^{\psi(EX)} G(t)dt}{G(\psi(EX))} + \frac{\int_{\psi(EX)}^{EX} F(EX)dt}{G(\psi(EX))} &\geq \frac{\int_0^{\psi(EX)} F(t)dt}{F(EX)} + \frac{\int_{\psi(EX)}^{EX} F(EX)dt}{G(\psi(EX))} \Leftrightarrow \\
\frac{\int_0^{\psi(EX)} G(t)dt}{G(\psi(EX))} + \frac{\int_{\psi(EX)}^{EX} F(EX)dt}{G(\psi(EX))} &\geq \frac{\int_0^{\psi(EX)} F(t)dt}{F(EX)} + \frac{\int_{\psi(EX)}^{EX} F(t)dt}{G(\psi(EX))} \Leftrightarrow \\
EX - \psi(EX) + \frac{\int_0^{\psi(EX)} G(t)dt}{G(\psi(EX))} &\geq \frac{\int_0^{EX} F(t)dt}{F(EX)} \Leftrightarrow \\
EY - (\psi(EX) - \frac{\int_0^{\psi(EX)} G(t)dt}{G(\psi(EX))}) &\geq \frac{\int_0^{EX} F(t)dt}{F(EX)} \Leftrightarrow \\
EY - EY_L &\geq EX - EX_L
\end{aligned}
$$

The first inequality holds by the above observation and because $G(\psi(EX)) = F(EX)$. The third holds because F is increasing.

If ψ is concave, then ψ^{-1} is convex, and the argument holds with reversed roles. ■

Proof of Corollary 1. The result follows because $U^r = U^a / \left[1 + CCV^2(X, \psi(X)\right]$ and because $CCV^2(X, \psi(X)) \leq 1$ if F and G are IFR. ■

Proof of Proposition 3. 1) By Lemma 2-2, $F(EX) \geq \frac{1}{2}$ and hence $\frac{F(EX)}{1-F(EX)} \geq 1$. The result follows then by Lemma 3-1,3.

2) Since F is DFR and ψ is convex, G must be DFR and hence concave. Since ψ is convex, we get that

$$
\begin{aligned}
EY_L &\leq \frac{1}{2}\psi(EX) \leq \frac{1}{2}EY \Leftrightarrow \\
EY - EY_L &\geq \frac{1}{2}EY
\end{aligned}
$$

Since F is DFR , and hence $NWUE$, we know by Lemma 3-2 that

$$EX - EX_L = \frac{\int_0^{EX} F(t)dt}{F(EX)} \geq \frac{EX}{eF(EX)}$$

Combining the above insights:

$$\begin{aligned}
U^{EX} &= EXEY + \frac{F(EX)}{1-F(EX)}(EX-EX_L)(EY-EY_L) \\
&\geq EXEY + \frac{F(EX)}{(1-F(EX)}\frac{EX}{eF(EX)}\frac{EY}{2} \\
&= EXEY + \frac{EXEY}{2e(1-F(EX))} \\
&\geq EXEY + \frac{eEXEY}{2e} = \frac{3}{2}EXEY = \frac{3}{2}U^r
\end{aligned}$$

3) By Lemma 3 - 4 we know that $EY - EY_L \geq EX - EX_L$. Since F is DFR we obtain:

$$\begin{aligned}
U^{EX} &= EXEY + \frac{F(EX)}{1-F(EX)}(EX-EX_L)(EY-EY_L) \\
&\geq EXEY + \frac{F(EX)}{1-F(EX)}(\frac{EX}{eF(EX)})^2 \\
&= EXEY + \frac{EXEY}{e^2F(EX)(1-F(EX)} \\
&\geq EXEY + \frac{e^2EXEY}{(e-1)e^2} = \frac{e}{e-1}U^r
\end{aligned}$$

The last inequality follows by Lemma 2-7.

4) This is obvious by the above calculations. ■

Proof of Proposition 4. Assume that F be DFR, ψ be convex, and that $EX \geq EY$. We need to show that $\left[U^{EX} - U^r\right] / \left[U^a - U^r\right] \geq \frac{1}{2}$. Note that $U^a - U^r = CCV^2(X, \psi(X))U^r$, and $U^{EX} - U^r \geq \frac{1}{e-1}U^r$ by Proposition 3-(3). Solving for $CCV^2(X, \psi(X))$ and using Lemma 1-11 yields the result as stated. ■

Proof of Proposition 5. For the first part, consider the following chain that holds for ψ convex and F concave (the other direction is analogous):

$$\begin{aligned}
U^{EX} &= EXEY + \frac{F(EX)}{1-F(EX)}(EX-EX_L)(EY-EY_L) \\
&\geq EXEY + (EX-EX_L)(EY-EY_L) \\
&= 2EXEY - EXEY_L - EYEX_L + EX_LEY_L \\
&= 2[\frac{1}{2}(EX(EY-EY_L) + EY(EX-EX_L))] + EX_LEY_L \\
&\geq 2[\frac{1}{2}(EX(EY-EY_L) + \psi(EX)(EX-EX_L))] + EX_LEY_L \\
&= 2R^{EX} + EX_LEY_L
\end{aligned}$$

The first inequality follows from Lemma 2 -1 and the second inequality holds since $EY \geq \psi(EX)$ for ψ convex. The last equality made use of the formula (16).

For the second case where F, G are both convex, we use (16) to obtain:

$$
\begin{aligned}
R^{EX} &= \frac{1}{2}[EX(EY - EY_L) + \psi(EX)(EX - EX_L)] \\
&\leq \frac{1}{2}[EX(EY - \frac{1}{2}\psi(EX)) + \psi(EX)\frac{1}{2}EX] \\
&= \frac{1}{2}[EXEY - \frac{1}{2}EX\psi(EX) + \frac{1}{2}\psi(EX)EX] \\
&= \frac{1}{2}U^r < \frac{1}{2}U^{EX}
\end{aligned}
$$

where the first inequality follows from Lemma 3. ■

Proof of Proposition 6. 1) By Lemma 3, we know that:

1. $EX - EX_L \geq \frac{1}{2}EX$, and
2. $EY_L \leq \frac{1}{2}\psi(EX)$, and hence $EY - EY_L \geq EY - \frac{1}{2}\psi(EX)$.

This yields:

$$
\begin{aligned}
R^{EX} &= \frac{1}{2}[EX(EY - EY_L) + \psi(EX)(EX - EX_L)] \\
&\geq \frac{1}{2}[EX(EY - \frac{1}{2}\psi(EX)) + \psi(EX)\frac{1}{2}EX] \\
&= \frac{1}{2}[EXEY - \frac{1}{2}EX\psi(EX) + \frac{1}{2}\psi(EX)EX] \\
&= \frac{1}{2}U^r = \frac{1}{2}\left(\frac{E(XY)}{1 + CCV^2(X, \psi(X))}\right) \\
&= \frac{1}{1 + CCV^2(X, \psi(X))}R^a
\end{aligned}
$$

2) This follows from $CCV^2(X, \psi(X)) \leq 1$ if F and G are NBUE. ■

Proof of Proposition 7. 1) a) By the concavity of G, we get that $R_m^{EX} = \frac{1}{2}EX(EY - EY_L) \geq \frac{1}{4}EXEY$. By the convexity of F and ψ, we get $R_w^{EX} = \frac{1}{2}\psi(EX)(EX - EX_L) \leq \frac{1}{2}EY(EX - EX_L) \leq \frac{1}{4}EXEY$. The result follows.

b) Because G is DFR and ψ is convex we obtain the following chain:

$$\begin{aligned}
EY - EY_L &= EY - \psi(EX) + \frac{\int_0^{\psi(EX)} G(t)dt}{G(\psi(EX))} \\
&\geq EY - \psi(EX) + \frac{\int_0^{\psi(EX)} (1 - e^{-\frac{t}{EY}})dt}{(G\psi(EX))} \\
&= EY - \psi(EX) + \frac{\psi(EX) - EY + EY e^{-\frac{\psi(EX)}{EY}}}{G(\psi(EX))} \\
&\geq \frac{(EY - \psi(EX))(G(\psi(EX)) + \psi(EX) - EY + EY(1 - G(\psi(EX)))}{G(\psi(EX)} \\
&= \frac{\psi(EX)(1 - G(\psi(EX))}{G(\psi(EX))}
\end{aligned}$$

This yields the following chain:

$$\begin{aligned}
R_m^{EX} &= \frac{1}{2} EX (EY - EY_L) \geq \frac{EX\psi(EX)(1 - G(\psi(EX))}{2G(\psi(EX))} \\
&= \frac{EX\psi(EX)(1 - F(EX)}{2F(EX))} \geq \frac{1}{2}\psi(EX)(EX - EX_L) = R_w^{EX}
\end{aligned}$$

where the last inequality follows because F is IFR.

2) From Lemma 3 -4 we know that $EY - EY_L \geq EX - EX_L$. Since $EX = EY \geq \psi(EX)$, we obtain $R_m^{EX} = EX(EY - EY_L) \geq \psi(EX)(EX - EX_L) = R_w^{EX}$. ■

Proof of Proposition 8. For the first part, we know from Proposition 5 that $R^{EX} < \frac{1}{2}U^{EX.}$. Hence:

$$\begin{aligned}
W^{EX} &= U^{EX} - R^{EX} > \frac{1}{2}U^{EX.} \geq \frac{1}{4}(U^a + U^r) \\
&\geq \frac{1}{4}(U^a + \frac{1}{2}U^a) = \frac{3}{8}U^a = \frac{3}{4}W^a
\end{aligned}$$

The first inequality follows from $R^{EX} < \frac{1}{2}U^{EX.}$. The second by McAfee's result (recall that any concave distribution is $DRFR$). The last equality follows by Proposition 1.

For the second part, we know from the proof of Proposition 5 that, for F and G convex:

$$R^{EX} \leq \frac{1}{2}U^r$$

This gives:

$$W^{EX} = U^{EX} - R^{EX} \geq \frac{1}{2}(U^a + U^r) - \frac{1}{2}U^r = \frac{1}{2}U^a = W^a$$

where the first inequality follows from $R^{EX} \leq \frac{1}{2}U^r$ together with McAfee's result (recall that convex distributions are IFR), and the last equality follows by Proposition 1. ■

Proof of Lemma 4. The claim for $\alpha = 1$ follows by observing that $\frac{dR^a}{d\alpha} = \int_0^1 \left(x\psi'(x) - \psi(z)\right)(1 - F(z))\, dz >$ $(<)\, 0$ if ψ is convex (concave). The claim for $\alpha = 1/2$ follows by noting that $R_1^{EX} = EX(EY - EY_L) \geq \frac{1}{2}[EX(EY - EY_L) + \psi(EX)(EX - EX_L)] = R_{1/2}^{EX}$ follows by the same argument as the one used in the proof Proposition 7.

Proof of Proposition 9. By Lemma 3, we know that $EY - EY_L \geq \frac{1}{2}EY$ if G is concave. This yields:

$$\begin{aligned} R_1^{EX} &= EX(EY - EY_L) \\ &\geq \frac{1}{2}EXEY \\ &= \frac{1}{2}U^r = \frac{1}{2}\left(\frac{E(XY)}{1 + CCV^2(X, \psi(X))}\right) \\ &= \frac{1}{2}\frac{1}{1 + CCV^2(X, \psi(X))}U^a \\ &\geq \frac{1}{2}\frac{1}{1 + CCV^2(X, \psi(X))}R_1^a \end{aligned}$$

By Lemma 4, $R_1^a < R_{1/2}^a$ if ψ is concave. By Proposition 1, $U^a = 2R_{1/2}^a$. Since U^a remains unchanged as a function of the sharing rule, it follows that $U^a > 2R_1^a$ if ψ is concave. This implies

$$\begin{aligned} R^{EX} &= \frac{1}{2}\frac{1}{1 + CCV^2(X, \psi(X))}U^a \\ &\geq \frac{1}{1 + CCV^2(X, \psi(X))}R_1^a \end{aligned}$$

if ψ is concave. The result follows then by noting that $CCV^2(X, \psi(X)) \leq 1$ if F and G are both IFR. ■

Proof of Proposition 10. **1)** We have the following chain:

$$\begin{aligned} U^{EX} &= EXEY + \frac{F(EX)}{1 - F(EX)}(EX - EX_L)(EY - EY_L) \\ &\geq EXEY + (EX - EX_L)(EY - EY_L) \\ &= EXEY + R_1^{EX} - EX_L EY + EY_L EX_L \\ &= R_1^{EX} + EY(EX - EX_L) + EY_L EX_L \\ &\geq R_1^{EX} + \frac{1}{2}EXEY + EY_L EX_L \\ &= R_1^{EX} + \frac{1}{2}EXEY - \frac{1}{2}EXEY_L + \frac{1}{2}EXEY_L + EY_L EX_L \\ &= \frac{3}{2}R_1^{EX} + \frac{1}{2}EXEY_L + EY_L EX_L \end{aligned}$$

The second line follows from Lemma 2-2 since F is concave, the third line is due to $R_1^{EX} = EX(EY - EY_L)$, the fifth line follows from Lemma 3-1. This implies that

$$\begin{aligned} W_1^{EX} &= U^{EX} - R_1^{EX} \geq \frac{1}{3}U^{EX.} \geq \frac{1}{6}(U^a + U^r) \\ &\geq \frac{1}{6}(U^a + \frac{1}{2}U^a) = \frac{1}{4}U^a \geq \frac{1}{2}W_{1/2}^a \end{aligned}$$

where the second inequality follows from McAfee's result (recall the concave distributions are $DRFR$), the third inequality is due to the fact that $U^r \geq \frac{1}{2}U^a$ if F and G are both IFR, and the last inequality is due to the fact that $U^a < 2R_1^a$ if ψ is convex.

2) If F, G are both convex, we use Lemma 3. to obtain:

$$R_1^{EX} = EX(EY - EY_L) \leq \frac{1}{2}U^r < \frac{1}{2}U^{EX}$$

This implies that

$$W_1^{EX} = U^{EX} - R_1^{EX} \geq \frac{1}{2}(U^a + U^r) - \frac{1}{2}U^r = \frac{1}{2}U^a \geq W_1^a$$

where the second inequality follows from McAfee's result since convex distributions are IFR. The last inequality follows from the same argument as the one at the end of part 1. ■

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