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ABSTRACT

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We consider a financing game with costly enforcement based on Townsend (1979), but where monitoring is non-contractible and allowed to be stochastic. Debt is the optimal contract. Moreover, the debt contract induces creditor leniency and strategic defaults by the borrower on the equilibrium path, consistent with empirical evidence on repayment and monitoring behaviour in credit markets.

JEL Classification: D02, D82, G21 and G33
Keywords: costly state verification, debt contract, priority violation and strategic defaults

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Optimal Debt Contracts under Costly Enforcement*

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December 5, 2006

Abstract

We consider a financing game with costly enforcement based on Townsend (1979), but where monitoring is non-contractible and allowed to be stochastic. Debt is the optimal contract. Moreover, the debt contract induces creditor leniency and strategic defaults by the borrower on the equilibrium path, consistent with empirical evidence on repayment and monitoring behavior in credit markets.

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1 Introduction

Financial contracts typically do not specify repayments to investors as a detailed function of all payoff relevant variables. For example, debt contracts put some easily describable liability on the firm through a fixed repayment. A celebrated approach in the literature that attempts to model this feature of financial contracts is the Costly State Verification (CSV) approach. The core of this approach is that the manager has superior information

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to investors about the true cash flow of the firm and may therefore attempt to divert part of it from investors. As a protection against this cash diversion problem, investors may verify the firm's income by taking it to bankruptcy court, but this is costly. Classic papers by Townsend (1979) and Gale & Hellwig (1985) derive debt as the optimal contract under such circumstances.[1] Under the optimal contract the manager-borrower pays in full what is owed if he has sufficient cash to do so, and defaults otherwise. In the event of a default, the investor-creditor verifies with probability one, and hence default and bankruptcy are synonymous.

The debt result of Townsend (1979) and Gale & Hellwig (1985) has been criticized on particular two accounts that we address in the present paper. One is the reliance on an assumption that the investor-creditor is able to commit to the optimal ex ante verification rule even though verification may not be optimal ex post. The second is their restriction to deterministic verification. (For concise discussions of these assumptions and their effects on the optimal contract, see Hart (1995) and Tirole (2006)). We depart from Townsend and Gale & Hellwig on both accounts. That is, we do not assume that the investor is able to commit to the ex ante optimal monitoring rule, and we do not constrain the analysis to deterministic monitoring. Yet, we derive debt as the optimal contract. Under the optimal contract, verification (monitoring) is stochastic and equilibrium repayment behavior yields strategic defaults and absolute priority violations, features consistent with repayment behavior observed empirically. In other words, by relaxing two assumptions viewed as critical in delivering debt as the optimal contract, we retain debt as the optimal contract. Moreover, the debt contract generates repayment and monitoring behavior much closer to that observed in actual debt markets than the original contract.

The importance of the assumption that the creditor is able to commit to a particular monitoring rule can be illustrated by the following example, based loosely on Hart (1995). Let the borrower's cash flow be equal to x with probability one, and suppose that the borrower owes an amount d and further that the cost of verifying x (and hence enforce his debt claim in court) equals to c. Suppose finally that $x - c < d < x$, which implies that the borrower has sufficient cash to repay in full what is owed, while at the same time

[1] See also Diamond (1984), Williamson (1986), and Winton (1995).

there is insufficient cash for the creditor to make full recovery of his claim in bankruptcy. As a result, the borrower may want to try to renegotiate his claim by offering the creditor an amount slightly larger $x - c$ and hence less than d, which will be optimal ex post for the creditor to accept. Thus, without a commitment by the creditor to reject such offers the debt claim as originally issued cannot be enforced. As noted by Hart (1995) matters become more complicated if x is uncertain since then the offer made by the borrower becomes a signal of the true x, in which case the "analysis of optimal ex ante contracts quickly becomes complicated" (p. 124). We solve for the optimal ex ante contract for the case in which x is uncertain and show that the optimal contract is a debt contract. Under the optimal contract, the borrower offers the creditor a payment $\min[d, x - c]$ to which the creditor responds by accepting offers equal to d with probability one, and accepting offers below d with a probability that is increasing in the size of the offer.

The idea that the optimality of debt relies critically on a restriction to deterministic verification schemes comes from results by Border & Sobel (1987), Mookherjee & Png (1989), and Boyd & Smith (1994), who derive random verification without a fixed payment as optimal. In other words, they find that debt is no longer optimal once the restriction to deterministic verification is relaxed. While these papers assume that the creditor is able to commit to the ex ante optimal verification rule, we assume no such commitment and show that the optimal contract will contain a fixed payment, and that verification under the optimal contract will be random. Interestingly enough, this result is consistent with a conjecture made by Townsend (1979) that "based on the results for deterministic verification, [...] the probability of verification should be a nonincreasing function of [the realized cash flow] and perhaps be zero in states with high realizations" (p. 278), which is what we find.

In a related paper, Krasa and Villamil (2000) derive debt as the optimal contract in a similar setting as ours in which monitoring is non-contractible. In the equilibrium that they consider, the borrower offers the creditor either full repayment (if the firm has sufficient liquidity to do so) or defaults by offering a zero repayment (if the firm has insufficient liquidity to satisfy the full repayment). As a result, a default is uninformative (beyond informing the creditor that the borrower has insufficient cash to avoid a default) and thus the expected payoff from verifying will be positive (given appropriate parameter restrictions), which in turn implies that verification will be optimal ex post. In other words, their equilibrium does not allow for defaults other than a zero repayment, and the creditor finds it optimal ex post to verify whenever faced with a default.[2] While both papers derive debt as the optimal contract, they generate quite different predictions regarding repayment behavior and contract enforcement. In particular, our setup predicts both absolute priority violations and strategic defaults, in contrast to that of Krasa & Villamil where a default is synonymous with bankruptcy.

Our finding that debt incurs strategic defaults is consistent with a large theoretical literature building on Hart and Moore (1989, 1998) that considers optimal contracting under symmetric and unverifiable information. It is also consistent with an emerging empirical corporate finance literature on repayment behavior in credit markets such as Brown, Ciochetti, and Riddiough (2003) and Davydenko (2005). For example, in a broad sample of firms, Davydenko (2005) finds that about 70% of defaulting firms are not liquidated.

Other related papers include Hvide & Leite (2005) who exogenously assume the existence of two securities, debt and equity, and derive the optimal mix of debt and equity. They do not derive optimal contracts but show that pure debt financing dominates pure equity financing if the verification costs are equal, where their pure debt equilibrium has the same structure as the equilibrium in the present paper. Gale and Hellwig (1989) analyze a similar repayment game to that in the present paper and derive necessary conditions for the existence of signalling equilibria that are broadly consistent with the equilibria of the present paper, but they do not specifically consider debt contracts nor do they derive optimal contracts.

The rest of the paper is structured as follows. In Section 2, we present the model. Section 3 contains the results, and Section 4 concludes the paper.

[2] By arguments from Gale & Hellwig (1989), reviewed in Hvide & Leite (2005), the pooling equilibrium imposes unrealistic off-equilibrium-path beliefs of the creditor. In the current setting, we rule out pooling equilibria by requiring that the repayment function be absolutely continuous.

2 Model

There are two risk-neutral agents, an entrepreneur and an investor. The penniless entrepreneur is endowed with a project that requires I units of funding to yield the cash flow x. The cash flow is stochastic with density $h(.)$ defined on $X = [x_L, x_H]$. In return for providing I, the investor gets a claim on x. This claim is a function $f : X \rightarrow \Re$. We make the feasibility restriction $f(x) \leq x$, $\forall x \in X$, and denote the set of contracts satisfying it for F. After being funded, x is generated and observed only by the entrepreneur. The entrepreneur makes a payment offer r to the investor, where the payment function $r(x)$ is a mapping $r : X \rightarrow \Re$ with the restriction $r \leq x$. We consider deterministic[3] and absolutely continuous payment functions $r(x)$, which implies that its derivative $r'(x)$ exists almost everywhere. The set of payment functions satisfying these criteria is denoted by R. The investor accepts or rejects the offer r based on his posterior beliefs h'. If the investor accepts, he receives r, and the manager gets the residual $x - r$. If the investor rejects/monitors, he receives a payoff y according to the written contract, i.e., $y = \min[f(x), x - c]$, and the manager gets the residual. Note that implicit in this formulation is that the cost of monitoring c is taken from the firm's cash flow (our results do not depend upon this assumption). The investor's accept probability function is a mapping $P : \Re \rightarrow [0, 1]$. To ensure sufficient liquidity to cover the monitoring cost, we assume that $c \leq x_L$. To make the problem interesting, we finally assume that an $r(x)$ that gives a constant payout on X falls short of making the investor willing to participate.

Let $e = 1$ if the investor rejects/monitors and $e = 0$ if the investor accepts an offer. The payoff functions π_i, where $i = I, E$ are then given by,

$$\begin{aligned} \pi_E &= (1-e)(x-r) + e(x-y) = x - (1-e)r - ey \\ \pi_I &= (1-e)r + ey \end{aligned} \tag{1}$$

for, respectively, the entrepreneur and the investor. For a given strategy tuple $\langle r(x), P(r) \rangle$

[3] There are technical problems in defining mixed strategies for a continuous type space. Barring such problems, we conjecture that a mixed repayment strategy is not consistent with equilibrium (in contrast to in Persons, 1997, who operates with a finite type space). The intuition is that a continuous X pins down a unique accept probability function $P(.)$, which in turn makes only one repayment offer optimal for given $\langle f(x), x \rangle$. Martimort & Stole (2002) makes a similar observation in a different context.

the expected payoffs are given by,

$$\begin{aligned} E\pi_E &= \int_X [P(r(x))(x-r) + (1-P(r(x))(x-y-c)]dH \\ E\pi_I &= \int_X [P(r(x))r + (1-P(r(x))y]dH \end{aligned} \tag{2}$$

The investor's participation constraint emerges from setting $E\pi_I = I$. The basic trade-offs are as follows. The entrepreneur makes a payment offer to the investor trading off the gains from cash diversion with cost of an increased probability of monitoring (and hence reducing the net payoff via reducing the cash flow). The investor follows a monitoring strategy that balances off the cost of monitoring against the possible gain from detecting a diversion attempt by the manager. We focus on Perfect Bayesian equilibria (PBE) of the payment game. A tuple $\langle r(x), P(r), h, h' \rangle$ is a PBE if a)$P(r)$ is optimal play by the investor given his posterior beliefs h', b)The entrepreneur anticipates the investor's behavior and chooses r to maximize his payoff, and c)The investor's posterior beliefs are formed using Bayes' rule whenever possible.

The implementation problem can be formulated as,

Problem 1 (3)

$$Max_{\langle r(.), P(.) \rangle} E\pi_E$$

$$s.t.\ E\pi_I = I$$

$$r(x) \in R$$

$$f(x) \in F$$

Strategies and beliefs are PBE

Problem 1 amounts to finding the payment function and monitoring probabilities that maximize the expected utility of the entrepreneur given the incentive constraints. Problem 1 is equivalent to finding a contract $f(x)$ that minimizes the expected monitoring (verification) cost $V = \int_X (1 - P(.))dH$ subject to the incentive compatibility constraints and the investor's participation constraint.

Let us define a debt contract as,

$$f^D(x) = \min(x, d) \tag{4}$$

This contract entitles the investor a constant payment d, and the full cash flow x for cash flow realizations below this point.

3 Analysis

The main result of the paper is as follows.

Theorem 1 *The optimal contract is a debt contract. Under the optimal contract, the borrower's repayment function equals* $r^*(x) = \min[x - c, d]$ *and the creditor accepts offers along* $r^*(x)$ *with probability* $P^*(r) = \min[1, \exp[-(d - r)/c]]$.

The optimal repayment function is given by $r^*(x) = \min[x - c, d]$, which can be implemented by issuing a debt contract $f^D(x) = \min(x, d)$. Under this contract, the borrower repays d if $x \geq d + c$ and defaults whenever $x < d + c$ by offering the creditor a payment $x - c$. While defaults for $x \in [d, d+c)$ are purely strategic, defaults for $x < d$ are liquidity-based, but also strategic since the borrower offers $x - c$ rather than the amount x that the creditor is entitled to under his contract. In equilibrium, the creditor is indifferent between accepting and rejecting offers $r^*(x) < d$, accepting with a probability $P(.)$ that is increasing in the size of the offer.[4] A default, therefore, leads to bankruptcy with a probability $1 - P(.)$ that is decreasing in the size of the borrower's repayment offer.

The result that the creditor accepts offers below d with a positive probability implies a priority violation since it gives the borrower a positive payoff even though he fails to repay his debt in full. In other words, the optimal contract yields absolute priority violations as well as strategic defaults, both features observed in real credit markets.

[4]The type of equilibrium that we consider where the creditor is indifferent between verifying and not verifying is described by Diamond (2004) in the context of Krasa & Villamil (2000), although in their equilibrium the creditor strictly prefers to verify in the event of a default.

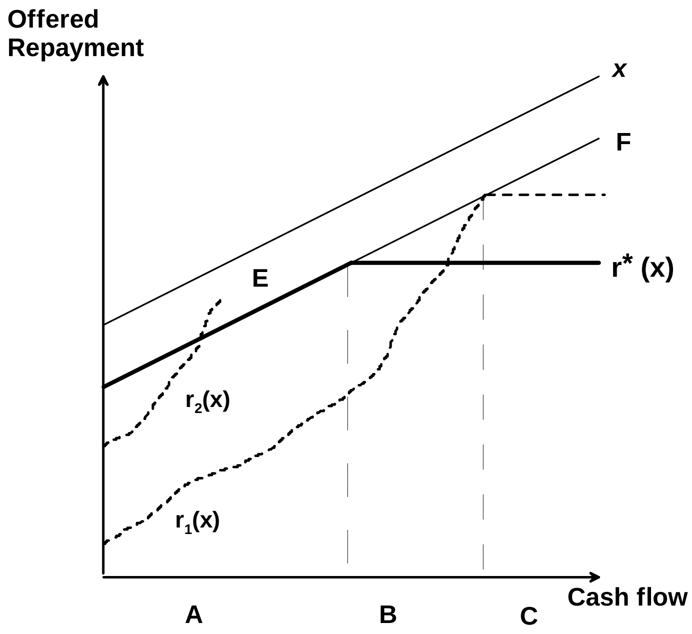

Figure 1

The intuition behind our result can be understood from Figure 1. The figure depicts the realized cash flow x on the horizontal axis and the dollar amount offered in payment to the investor on the vertical axis. The bold line depicts the optimal repayment function $r^*(x) = \min[d, x - c]$, which follows the feasibility barrier **F** for x in region **A** and gives a constant payout d in regions **B** and **C**. We now argue that $r^*(x)$ must be better than alternative payment functions, such as $r_1(x)$, by having lower monitoring costs, where the constant payout associated with $r_1(x)$ is an amount d' such that $d' > d$. To make the comparison between $r^*(x)$ and $r_1(x)$ interesting, assume that $r^*(x)$ and $r_1(x)$ both satisfy the investor's participation constraint. Note first that to induce any non-constant $r(x)$, the investor must be more likely to monitor the lower is the payment. At the level of the maximal payment, d and d', the monitoring probability is zero.

Given these observations, let us compare the monitoring costs for $r^*(x)$ and for $r_1(x)$ in the regions **A**, **B**, **C**. In region **C**, the investor receives his maximal payout under both $r^*(x)$ and $r_1(x)$ and does not have incentives to monitor in either case. In region **C**, therefore, $r^*(x)$ and $r_1(x)$ are equally good. In region **B**, $r^*(x)$ offers the maximal payout, and hence incurs no monitoring, while $r_1(x)$ pays less than its maximal payout and therefore must imply some monitoring by the investor (if not, the manager would never offer the maximal payout). Therefore, $r^*(x)$ yields lower monitoring costs compared to $r_1(x)$ in region **B**. This must also be the case in region **A**, because $r^*(x)$ offers more than $r_1(x)$ in this region. Thus $r^*(x)$ dominates $r_1(x)$ in all regions **A**, **B**, and **C**, and must therefore yield lower monitoring costs than $r_1(x)$. Now consider a payment scheme $r_2(x)$, which crosses the line **F**$= x - c$ and enters the area **E.** However, payments in **E** are not feasible. If the manager plays $r_2(x)$, it would be strictly optimal for the investor not to monitor following repayment offers in **E** since by monitoring he gets at most $x - c$, while by accepting the payment offer he gets more. But then an equilibrium with $r_2(x)$ would unravel, and therefore cannot exist.

The intuition for our result resembles the one associated with the standard debt contract derived by Gale and Hellwig (1985), which gives the creditor a fixed payment d if the firm is solvent, and maximum recovery (i.e., $x - c$) otherwise.[5] However, the predicted repayment behavior under the optimal contract differs. In particular, while under the standard debt contract the borrower defaults only if $x < d$ and receives a zero payment in this case, under the optimal contract of Theorem 1 the borrower defaults whenever $x < d + c$ and despite default receives a positive expected payment.

To illustrate the optimal contract and repayment behavior under the optimal contract, the following example is useful.

Example 1 *Let $c = 1$ and x be uniformly distributed on $[x_L, x_H]$=[1,2]*

The contract is $f^D(x) = \min(x, d)$. This contract implies that the manager plays $r^*(x) = \min(x - c, d)$. The manager defaults for $x \in [x_L, d + c)$, where defaults for $x \in [d, d + c)$ are purely strategic while defaults for $x \in [x_L, d)$ are partly liquidity-based and partly strategic (the latter since the borrower offers $x - c$ rather than x). The creditor monitors according to $P(r^*(x)) = \exp[-(d - r^*(x))/c] = \exp[-(d + c - x)/c]$ from which we observe that the greater is the size of the default $d - r^*(x)$ the higher is the probability $1 - P(.)$ of monitoring. It may also be shown that $P(r^*(x))$ is concave in the size of the

[5] The equivalent property in Townsend (1979), who assumes risk aversion, is that the borrower is left with a constant payoff in the verification region.

default.[6]

Given $r^*(x)$, the investor's participation constraint simplifies to $\int_{x_L}^{d+c}(x-c)dx + \int_{d+c}^{x_H} cdx = I$. Solving with respect to d and substituting in for $c = 1$, $x_L = 1$, and $x_H = 2$ gives $d = 1 - \sqrt{1-2I}$. We can note that d is increasing and convex in I, which implies that the interest rate, $d/I - 1$, as a spread over the riskfree rate increases in the funding requirement I.

The maximum fundable amount is obtained for $d = x_H - c = 2 - 1 = 1$, in which case the investor's payoff becomes $\int_X (x-c)dH = \int_1^2 (x-1)dx = 1/2$. Hence, in this example, any $I \in [0, .5]$ is obtainable from the investor. The literature on bankruptcy costs (e.g., Andrade & Kaplan, 1998) finds that bankruptcy costs are about 10-30% of bankrupt firms' value. Interpreting c as a bankruptcy cost, the example generates expected bankruptcy costs $E(c|e=1)$ within these bounds for $I \in [0.15, 0.37]$.

We can calculate the gain in utility for the manager from defaulting strategically by playing $r^*(x)$ rather than adhering to the written contract by playing $r(x) = \min(x, d)$. For $x \in [x_L, d)$ the gain equals $(r(x) - r^*(x))P(r^*(x)) = c\exp[(x-d-c)/c]$, which increases in x. For $x \in [d, d+c)$ the gain equals $(d-x+c)P(r^*(x)) = (d-x+c)\exp[(x-d-c)/c]$ which decreases in x. Therefore the expected gain for the manager is concave in x and is maximized for $x = d$. The economic implication is that under the optimal contract, the expected priority violation is maximized when the manager is closer to solvency. This implication is consistent with empirical evidence from Betker (1995), who find priority violations to be higher for distressed firms that are closer to solvency.

Consider now the effect of a change in the riskiness of the cash flow in the form of a mean-preserving-shift. In particular, augment the original density $h(x) = 1$ with masspoints at $x_L = 1$ and $x_H = 2$, each with probability mass $p/2 > 0$ to create a new density $h_p(x) = 1 - p$ on $x \in (x_L, x_H)$. This new density with masspoints yields an expected cash flow equal to that implied by $h(x) = 1$, and hence an increase in p yields higher risk by a mean preserving spread. The debt payment d_p under $h_p(x)$ is determined from the investor's participation constraint $\int_1^{d_p+1}(x-1)(1-p)dx + d_p\int_{d_p+1}^{2}(1-p)dx = I$,

[6] The intuition for concavity is that when r is low then $P(.)$ is low and the gains from cheating is small simply because the probability of getting away with it is low. On the other, hand the loss from cheating is proportional in $P'(.)$. The only way to induce adherence to $r^*(x)$ is therefore for the cheating deterrence to be stronger the higher r, or in other words for $P'(.)$ to be higher for higher r.

which yields $d_p = 1 - \frac{\sqrt{(1-p)(1-p-2I)}}{1-p}$. Differentiating d_p with respect to p gives

$$\frac{\partial d_p}{\partial p} = \frac{I}{\sqrt{(1-p)(1-p-2I)}(1-p)} > 0,$$

which says that the interest rate, $d/I - 1$, as a spread over the riskfree rate is increasing in the riskiness of the firm's cash flow (this result does not depend upon the distribution being uniform).

4 Conclusion

Townsend (1979), Gale and Hellwig (1985) and others derive standard debt as the optimal contract from first principles using costly state verification as the relevant contracting friction. Their result has been criticized on account that it relies on a restriction of the contracting space to deterministic verification schemes, and that it relies on the ability of the investor to commit to the optimal ex ante verification rule. Indeed, results by Border & Sobel (1987), Mookherjee & Png (1989) and others show that retaining the commitment assumption while relaxing the restriction to deterministic verification gives random verification schemes without a fixed payment as optimal.

We relax both assumptions and yet derive debt as the optimal contract. Under the optimal contract, the borrower defaults strategically and the creditor is lenient towards defaults by accepting offers from the borrower below the full debt payment with a positive probability. In other words, by relaxing assumptions frequently viewed as critical to debt as the optimal contract under costly state verification, not only do we retain debt as the optimal contract but we obtain as optimal a debt contract with repayment and monitoring behavior consistent with what is observed empirically, such as strategic defaults and absolute priority violations. This is unlike the standard debt contract derived by Townsend (1979), Gale & Hellwig (1985), and Krasa & Villamil (2000) which does not distinguish between default and bankruptcy and hence rules out strategic defaults and priority violations.

Our result that the optimal contract implies strategic defaults and absolute priority violations is consistent with the corporate finance literature on repayment behavior of debt in real financial markets. In fact, Anderson & Sundaresan (1996) and Mella-Barall & Perraudin (1997) use strategic defaults to explain why observed risk premia on debt exceeds that implied by the Merton (1974) debt valuation model. It is also consistent with a large literature building on Hart & Moore (1989, 1998) that considers optimal contracting under symmetric and unverifiable information. This literature generally obtains debt as the optimal contract but, in equilibrium, yields neither strategic defaults, priority violations, nor liquidations, unlike our approach based on asymmetric information.

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6 Appendix

We prove Theorem 1 in several steps. Since the investor cannot precommit to a monitoring strategy, the revelation principle does not hold, and we have to apply a more indirect method of proof.[7] The strategy of the proof is to solve a simplified version of Problem 1 and then to show that this solution also solves Problem 1.

Let $\Gamma(f)$ be the set of PBE induced by a contract $f(x) \in F$. We say that the payment function $r(x)$ is *inducable* (implementable) if there exists $f(x) \in F$ such that $r(x)$ is contained in $\Gamma(f)$. Now a key definition. Let $\tilde{x}$ be an arbitrary constant on X and denote by M (for monotonicity) the set of payment functions satisfying (i)$r'(x) > 0$ for $x \in [x_L, \tilde{x}]$ and (ii)$r'(x) = 0$. Informally speaking, M contains all payment functions that are either strictly increasing on X or initially strictly increasing and then flat. $M \subset R$ since R also includes non-monotonic payment functions.

Define Problem 1' as Problem 1 except that $r(x) \in R$ in the third line of (3) is replaced by $r(x) \in M$. In Lemma 1-4 we show that (a)$\langle r^*(x), P^*(r)\rangle$ is the solution to Problem 1' and that (b)the debt contract $f^D(x)$ implements $\langle r^*(x), P^*(r)\rangle$. In Lemma 5 we show that the solution to Problem 1' generalizes to Problem 1. Informally speaking, Lemma 1 shows that to be inducable a repayment function must lie (weakly) to the south of **F** in Figure 1. Lemma 2, which is the key lemma, shows how to minimize expected verification cost

[7] Bester & Strausz (2001) show that a modified version of the revelation principle holds under limited commitment. Since we operate in a setting with a continous type space, their results do not immediately apply.

when implementing an arbitrary $r(x)$ and solves for the associated monitoring behavior by the creditor. Lemma 3-4 use Lemma 1-2 to prove (a) and (b).

Lemma 1 *For any $r(x) \in M$ to be inducable, it must satisfy $r(x) \leq x - c$, $\forall x \in X$.*

Proof. The proof is by contradiction. Let $r(x) \in M$ and furthermore assume for convenience that $r(x)$ is strictly increasing. Suppose that there exists a contract $\hat{f}(x) \in F$ that induces $r(x) > x - c$ on some interval $X' = [x_1, x_2]$. Since $\hat{f}(x) \leq x$ by feasibility, we must then have that $r(x) > \hat{f}(x) - c$ on X'. But then in a PBE the investor must accept offers on $[r(x_1), r(x_2)]$ with probability 1. But then the entrepreneur never offers more than $r(x_1)$ on X', which contradicts the assumption that $r(x)$ is strictly increasing. To extend the proof to the case where $r(x)$ is only weakly increasing is straightforward and omitted. ■

Informally speaking, Lemma 1 limits the set of inducable payment functions in M to lie weakly below the line **F** in Figure 1.

Lemma 2 *For any inducable $r(x) \in M$, (i)The contract $f^*(x) = r(x) + c$ induces $r(x)$. (ii)The associated accept probability function of the creditor is $P(r) = \exp\left[\frac{r - r(x_H)}{c}\right]$. (iii)$f^*(x)$ is the cheapest way (i.e., it minimizes expected verification costs) to induce $r(x)$.*

Proof. Fix an inducable $r(x) \in M$ and suppose that the contract is $f^*(x) = r(x) + c$. We first show that $f^*(x)$ induces $r(x)$. Note that if the manager adheres to $r(x)$, the investor is indifferent between monitoring and not since the payment offer equals what the investor would get if he verifies (i.e., $r = y$) Given that the manager plays $r(x)$, any $P(r)$ is therefore consistent with optimal play by the investor (given h' appropriately defined). We now pin down $P(r)$ with the requirement that the manager does not have incentives to deviate from $r(x)$. Since the ensuing $P(r)$ is unique, we thereby prove both (i) and (ii). For given x and $P(r)$ the expected payoff for the manager from offering r equals,

$$U_E(r) = P(r)(x - r) + (1 - P(r))(x - f^*(x)) \tag{5}$$

Differentiating with respect to r, we get

$$\begin{aligned} U_E'(r) &= P'(r)(x-r) - P - P'(r)(x - f^*(x)) \\ &= P'(r)(f^*(x) - r) - P \end{aligned} \quad (6)$$

For $r(x)$ to be optimal play by the manager, it must be a local maximum for all x,

$$P'(r)c - P(r) = 0 \quad (7)$$

The unique solution to this differential equation (barring the trivial solution $P(x) = 0$) is $P(r) = K\exp(r/c)$, where K is an integration constant. Invoking the corner condition $P(r(x_H)) = 1$ (the investor accepts the maximal offer with probability 1) pins down K, and we get,

$$P(r) = \exp\left[\frac{r - r(x_H)}{c}\right] \quad (8)$$

$P(r)$ is increasing and convex in r. Note that the associated monitoring probability $1 - P$ lies between zero and one for all r. To show that adhering to $r(x)$ is a global optimum for the manager, observe that $P'(r) = \frac{P(r)}{c}$. Substituting into $U_E'(r)$,

$$\begin{aligned} U_E'(r) &= P'(r)(f^*(x) - r) - P \\ &= P(r)[\frac{f^*(x) - r}{c} - 1] \\ &= P(r)[f^*(x) - c - r]/c \end{aligned} \quad (9)$$

This expression is negative for $r > f^*(x) - c$ and positive for $r < f^*(x) - c$. Hence $r(x) = f^*(x) - c$ is a global optimum for the manager. To complete the proof of (i) and (ii), we need to construct beliefs that support the equilibrium. The prior of the investor is that x follows $h(x)$. For an offer r on the equilibrium path, the investor's posterior beliefs h' are degenerate at $r + c$ for $r < r(x_H)$, and unrestricted for $r = r(x_H)$. These posterior beliefs are obviously consistent with the manager's strategy. We do not need to restrict the investor's posterior beliefs for offers outside the interval $[r(x_L), r(x_H)]$; for any posterior beliefs with support on X it will be optimal for the investor to accept $r > r(x_H)$ and optimal to reject $r < r(x_L)$. We have then proved (i) and (ii).

We now need to show (iii) that there are no cheaper ways to induce an arbitrary $r(x) \in M$ than through $f^*(x)$. We show that a contract $\hat{f}(x)$ that induces $r(x)$ with $\hat{f}(x) \neq r(x)+c$ for some interval(s) on X must be suboptimal. We suppose for convenience that $r(x)$ is strictly increasing on X (the proof is readily extendable to the case where $r(x)$ is flat at the top). Now, since $\hat{f}(x) \neq r(x) + c$ for some interval(s) on X, there must exist constants x_1 and x_2 such that $\hat{f}(x) > r(x)+c$ or $\hat{f}(x) < r(x)+c$ for $x \in X' = [x_1, x_2] \subset X$. For convenience assume that $\hat{f}(x) = f^*(x)$ for $x \notin X'$ (the logic of the proof is the same if this condition does not hold). First suppose that $\hat{f}(x) < r(x) + c$ on X'. This implies that $r(x) > \hat{f}(x) - c$ on X' and the investor would set $P(r) = 1$ for $r \in [r(x_1), r(x_2)]$. But in that case the manager would offer $r(x_1)$ for all $x \in X'$, which is inconsistent with $r(x)$ being strictly increasing. Now suppose $\hat{f}(x) > r(x) + c$ on X'. Then the investor would set $P(r) = 0$ for $r \in [r(x_1), r(x_2)]$, since $y > r$. For the manager to have incentives to follow $r(x)$ for $x \in [x_L, x_1]$ it follows immediately that $P(r) = 0$ for $r \in [r(x_L), r(x_1)]$, which cannot be cheaper than $f^*(x)$. Now consider the interval $[x_2, x_H]$. Since $\hat{f}(x) = f^*(x)$ for $x \in [x_2, x_H]$, then by the same argument as in the first part of the proof we must have that $P(x) = \exp[r(x) - r(x_H)]$ for $x \in [x_2, x_H]$. Let us now compare the monitoring costs from $\hat{f}(x)$ with the monitoring costs from $f^*(x)$, assuming that $\hat{f}(x)$ induces $r^*(x)$. For $x \in [x_2, x_H]$, the accept probability is the same for every x, and the expected monitoring cost of $\hat{f}(x)$ and $f^*(x)$ on $[x_2, x_H]$ must be the same. For $x \in [x_L, x_2]$, however, the monitoring costs from $\hat{f}(x)$ must be strictly higher than the monitoring costs from $f^*(x)$, since $f^*(x)$ induces investor lenience while under $\hat{f}(x)$ the investor monitors with probability 1 for $x \in [x_L, x_2]$. It follows immediately that $f^*(x)$ dominates $\hat{f}(x)$, and consequently that $f^*(x)$ is the optimal contract to induce $r(x)$. ■

We have shown that $f^*(x) = r(x) + c$ is the optimal contract to induce an $r(x) \in M$ that is inducable by Lemma 1. Equipped with Lemma 1 and Lemma 2 we can replace Problem 1' with the equivalent and more manageable Problem 1". Let us denote by M'

the set of payment functions in M that is inducable by Lemma 1.

$$\textbf{Problem 1''} \tag{10}$$

$$\text{Max}_{\langle r(.)\rangle} \int \mathbf{e}^{r(x)-r(x_H)} dH$$

$$s.t.\ E\pi_I = I$$

$$r(x) \in M'$$

$$f(x) \in F$$

$$\text{Strategies and beliefs are PBE}$$

To obtain Problem 1" from Problem 1', we have substituted in $r(x) \in M'$ for $r(x) \in M$ by Lemma 1, and $P(r) = \exp\left[\frac{r-r(x_H)}{c}\right]$ by Lemma 2. Moreover, since Lemma 2 enables us to map $r(x)$ into $P(r)$, observe that we now maximize over only $r(x)$ instead of over $\langle r(.), P(.)\rangle$. Informally speaking, Problem 1" is the problem depicted in Figure 1. Now define $D = [x_L, m+c)$ and $E = [m+c, x_H]$. Clearly $D \cup E = X$ and $D \cap E = \varnothing$.

Lemma 3 *The solution to Problem 1" is* $r^*(x)$, *where*

$$r^*(x) = \begin{cases} x - c & \quad x \in D \\ & \textit{for} \\ m & \quad x \in E \end{cases} \tag{11}$$

Proof. $r^*(x)$ follows the feasibility barrier $r(x) \leq x - c$ and then becomes flat for $x = m+c$ (as depicted in Figure 1). To prove (i), let $\hat{r}(x) \in M'$ be an arbitrary alternative payment function in M' that raises the same amount as r^*, i.e., $\int_X \hat{r}(x)dH = \int_X r^*(x)dH$. Recall that for an arbitrary $\hat{r}(x)$, its expected monitoring cost equals $\int_X c[1-P(\hat{r}(x))]dH$. Let $\hat{V}$ be the expected monitoring cost of $\hat{r}(x)$ and V^* be the expected monitoring cost of $r^*(x)$. To show that $r^*(x)$ solves Problem 1" is equivalent to showing that $\hat{V} > V^*$. In the following we show that $\hat{V} \geq V^*$. To extend the proof to holding for strict inequality is straightforward and omitted. Denote the expected monitoring cost of $r^*(x)$ on D (E) for V_D^* (V) and the expected monitoring cost of $\hat{r}(x)$ on D (E) for $\hat{V}_D$ ($\hat{V}_E$). By definition, $\hat{V}_D + \hat{V}_E = \hat{V}$ and $V_D^* + V_E^* = V^*$. $r^*(x) = r(x_H)$ for $x \in E$ implies $V_E^* = 0$ and therefore $V_E^* \leq \hat{V}_E$. It therefore suffices to prove that $V_D^* \leq \hat{V}_D$. Since $r^*(x) = x - c$

for $x \in D$, Lemma 1 implies $r^*(x) \geq \hat{r}(x)$ for $x \in D$. Recall from Lemma 2 that for an arbitrary $\hat{r}(x)$ we have $P(\hat{r}(x)) = \exp\left[\frac{\hat{r}(x)-\hat{r}(x_H)}{c}\right]$. To show that $V_D^* \leq \hat{V}_D$ it is therefore sufficient to show that $\hat{r}(x_H) \geq r^*(x_H)$. Since $r^*(x) \geq \hat{r}(x)$ for $x \in D$, we have that $\int_D r^*(x)dH \geq \int_D \hat{r}(x)dH$. Therefore $\int_E \hat{r}(x)dH \geq \int_E r^*(x)dH$ must hold for the investor to be indifferent between $r^*(x)$ and $\hat{r}(x)$. But since $r^{*\prime}(x) = 0$ for $x \in E$, there must exist a constant $\tilde{x} \in E$ such that $\hat{r}(x) \geq (\leq) r^*(x)$ for $x > (<) \tilde{x}$. Therefore $\hat{r}(x_H) \geq r^*(x_H)$. Note finally that by adjusting m we can satisfy any feasible investor participation constraint (it is easy to show that $r^*(x)$ maximizes the range of fundable projects). That completes the proof. ■

We have shown that $r^*(x)$ solves Problem 1" and now show that $f^D(x)$ is the optimal contract inducing $r^*(x)$.

Lemma 4 *$f^D(x)$ induces $r^*(x)$ and is the cheapest way to induce it.*

Proof. That $f^D(x)$ induces $r^*(x)$ follows from Lemma 2, part (i). That $f^D(x)$ is optimal in inducing $r^*(x)$ follows from Lemma 2, part (iii). ■

We have showed that $r^*(x)$ is optimal in M and that a debt contract is optimal in inducing $r^*(x)$. It follows directly that the manager defaults strategically under the optimal contract since for $x \in [x_L, m+c]$ the contractual obligation is x, while the actual payment offer equals $x - c$. Also, the creditor is lenient towards defaults, since his monitoring probability $1 - P$ is less than unity for any equilibrium path default. We have therefore proven Theorem 1 under the limitation $r(x) \in M$. To complete the proof of the theorem we therefore need to prove the following.

Lemma 5 *Any $r(x) \in R$ must be dominated by $r^*(x)$.*

Proof. Since we have proved in Lemma 1-4 that $r^*(x)$ dominates all other $r(x) \in M$, it is sufficient to show that $r^*(x)$ dominates all $r(x) \in R$ that are not contained in M. We split the proof into two parts. First we show that (i)$r(x) \notin M$ with $r(x) \leq x - c$ must be dominated, and then that (ii)$r(x) \notin M$ with $r(x) > x - c$ for some interval on X must be dominated.

To prove (i), denote a candidate payment function by $\hat{r}(x)$. First suppose that $\hat{r}(x)$ is weakly increasing. By the same construction as in Lemma 1, $\hat{r}(x)$ should be implemented by $f(x) = \hat{r}(x) + c$, and the only accept probability function consistent with $\hat{r}(x)$ being part of a PBE is $P(x) = \exp\left[\frac{\hat{r}(x)-\hat{r}(x_H)}{c}\right]$. But then exactly the same dominance argument as in Lemma 1 shows that $r^*(x)$ dominates $\hat{r}(x)$. Suppose instead that $\hat{r}(x)$ is strictly decreasing on some interval(s). Again, by the same construction as in Lemma 1, the only accept probability function consistent with $\hat{r}(x)$ being part of a PBE is $\exp\left[\frac{\hat{r}(x)-\hat{r}(x_u)}{c}\right]$, where $x_u = \arg\max_{\{x\}} \hat{r}(x)$. If $x_u = x_H$, the proof goes through by the same dominance argument as before. Let us therefore suppose that $x_u < x_H$. We now construct an alternative payment function $\bar{r}(x)$ through modifying $\hat{r}(x)$ and show that $\bar{r}(x)$ constructed in a suitable manner dominates $\hat{r}(x)$. We assume for convenience that there exists x' so that $\bar{r}(x)$ reaches a local minimum for x'.[8] We construct $\bar{r}(x)$ in two steps. In step 1, let $\bar{r}(x) = r(x_u) - \delta$ in an ϵ-neighborhood of x_u, labeled X_A. ϵ is small enough to guarantee that $\bar{r}(x)$ pays less than $\hat{r}(x)$ in X_A, and δ defined to ensure continuity of $\bar{r}(x)$ in the endpoints of X_A. In step 2, perform a similar modification of $\bar{r}(x)$ in a neighborhood of x_u, but now "shave" from below so that $\bar{r}(x)$ raises more than $\hat{r}(x)$. Formally, let $\bar{r}(x) = \hat{r}(x_u) + \psi$ in an ϵ-neighborhood of x_u labeled by X_B. ψ is defined to ensure continuity in the endpoints of X_B. Let now ϵ be such that the investor is indifferent between $\bar{r}(x)$ and $\hat{r}(x)$ (by the continuity of $\bar{r}(x)$ and $\hat{r}(x)$ such ϵ exists). Let us now compare the expected monitoring costs for $\bar{r}(x)$ and $\hat{r}(x)$. In X_A, the expected monitoring cost for $\bar{r}(x)$ is zero, while greater than zero for $\hat{r}(x)$. In X_B, the monitoring cost for $\bar{r}(x)$ is lower than for $\hat{r}(x)$, since the payment is higher for $\bar{r}(x)$. Finally, outside X_A and X_B, $\bar{r}(x)$ must also have a lower monitoring cost than $\hat{r}(x)$, since the maximal payout is higher for $\bar{r}(x)$ than for $\hat{r}(x)$. Hence $\bar{r}(x)$ beats $\hat{r}(x)$ and consequently any $\hat{r}(x)$ strictly decreasing on some interval(s) cannot be optimal.

We now need to show that (ii)$r^*(x)$ beats any $r(x) \notin M$ with $r(x) > x - c$ for some interval on X. Denote a candidate payment function of this type by $\hat{r}(x)$ and the set of such functions by $\hat{R}$, where $\hat{R} \subset R$. The optimal payment function in $\hat{R}$ we denote by $\hat{r}^*(x)$. The strategy of the proof is to derive $\hat{r}^*(x)$ and then show that $r^*(x)$ beats $\hat{r}^*(x)$ by having lower monitoring costs. We first consider weakly increasing $\hat{r}(x)$ in steps 1-7.

[8] If such a local minimum does not exist, $\hat{r}(x)$ must either be weakly increasing (in which case it is covered in the previous paragraph) or reach a local minimum for $x = x_H$. The proof extends readily to the latter case.

Step 1. A weakly increasing $\hat{r}(x) \in \hat{R}$ must have a constant payout for $X' = [x_L, t]$, where t is some constant, since the same contradiction argument as eliminating $r_2(x)$ in Figure 1 would otherwise apply. It follows that to find $\hat{r}^*(x)$ we can restrict attention to $\hat{r}(x)$ that are continuous approximations to $\hat{\rho}(x)$, where

$$\hat{\rho}(x) = \begin{cases} q & x \in [x_L, t] \\ x - c \quad \text{for} & x \in [t, m + c] \\ m & x \in [m + c, x_H] \end{cases} \tag{12}$$

$\hat{\rho}(x)$ has constant payout q on X', then follows $x - c$, and flattens at $x = m + c$.

Step 2. $\hat{r}^*(x)$ must induce the investor to monitor stochastically on X': if it is strictly optimal for the investor to accept q then $\hat{r}(x)$ cannot be an equilibrium,[9] and if it is strictly optimal for the investor to monitor with probability 1 then $\hat{r}(x)$ cannot be optimal. For the investor to monitor stochastically on X' we must have that,

$$\begin{aligned} \int_{X'} q dH &= \int_{X'} (f(x) - c) dH, \text{ which implies} \\ q(H(t) - H(x_L)) &= \int_{X'} f(x) dH - cH(t), \text{ which simplifies to} \\ q &= \int_{X'} f(x) dH / H(t) - c \end{aligned} \tag{13}$$

On the left hand side is what the investor gets if he accepts an offer q, and on the right hand side is what he expects to get if he monitors.

Step 3. For any choice of contract $f(x)$, equation (13) generates a function $q(t)$, where $q(x_L) = x_L - c$ by L'Hospitals rule. By a straightforward dominance argument, to find $\hat{r}^*(x)$ we want to pick the northernmost $q(t)$. This must arise from maximizing $\int_{X'} f(x) dH$ on X' with respect to $f(x)$, which is obtained by setting $f(x) = f^*(x) = x$ on X'.

[9] Recall the assumption that the scheme $r(x) = q$ for $x \in X$ does not satisfy the investor's participation constraint. Therefore, candidate schemes with payout q for $x \in X'$ must have a higher payout for $x \notin X'$. But if $P(q) = 1$, the manager would offer q also for $x \notin X'$.

Step 4. Substituting $f(x) = x$ back into (13), $\hat{r}^*(x)$ must satisfy

$$q = \int_{X'} x dH/H(t) - c \tag{14}$$

Note that $\int_{X'} xdH/H(t) = E(x|x \in X')$, where $E(x|x \in X')$ is the conditional mean of x on X'. (14) implies that $E(x|x \in X') = q + c$, a fact that will be used in Step 7.

Step 5. Since $\int_{X'} xdH/H(t) - c = \int_{X'}(x - c)dH/H(t) = \int_{X'} r^*(x)dH/H(t)$, equation (14) implies that $\hat{r}^*(x)$ and $r^*(x)$ gives the same aggregate investor payoff on X'. It follows directly from (12) that $\hat{r}^*(x)$ and $r^*(x)$ must be identical on $x \in X'_c$ (where $X'_c = X/X'$) i.e., $\hat{r}^*(x) = r^*(x)$ for $x \in X'_c$. By Lemma 1, the accept probability and monitoring costs of $\hat{r}^*(x)$ and $r^*(x)$ are therefore also identical for $x \in X'$. To show that $r^*(x)$ beats $\hat{r}^*(x)$ it is therefore sufficient to show that $r^*(x)$ has a lower monitoring cost than $\hat{r}(x)$ on X'. This is equivalent to showing that $r^*(x)$ has a higher average accept probability than $\hat{r}^*(x)$.

Step 6. The average accept probability for $r^*(x)$ on X' equals $\int_{X'} P^*(x)dH/H(t)$, where $P^*(x) = \exp\left[\frac{x-c-r^*(x_H)}{c}\right]$. Since $\hat{r}^*(x)$ has a constant payout on X', its average accept probability simply equals $P^*(q) = \exp\left[\frac{q-r^*(x_H)}{c}\right]$.[10] We therefore need to show that

$$P^*(q) \leq \int_{X'} P^*(x)dH/H(t) \tag{15}$$

Step 7. We now show that (15) holds strictly, except in the non-generic case where it holds with equality. Note that the left hand side of (15) is unaffected by $h(x)$. It is therefore sufficient to show that (15) holds for the $h(x)$ that maximizes the right hand side subject to (13). Since both $r^*(x)$ and $\hat{r}(x)$ are linear in x, (13) will hold for any distribution that keeps $E(x|x \in X')$ constant, or in other words for any mean-preserving shift of $h(x)$ on X'. Since $H(t)$ is constant through mean-preserving shifts and $P^*(x)$ is convex in x, the right hand side of (13) is maximized by minimizing risk, i.e., putting an

[10] This is where the requirement that $\hat{r}(x)$ is continuous bites. Allowing $\hat{r}(x)$ to be discontinuous in the point t would soften the incentive constraint of the manager, and decrease the monitoring probability for the offer q.

atom of the size $H(t)$ at the point $x = q + c$. Substituting into (15), we get the condition

$$\exp\left[\frac{q - r^*(x_H)}{c}\right] \leq H(t)\exp\left[\frac{q - r^*(x_H)}{c}\right]/H(t) = \exp\left[\frac{q - r^*(x_H)}{c}\right] \qquad (16)$$

which always holds. Hence we have shown that $r^*(x)$ dominates $\hat{r}(x)$ strictly except in the non-generic case where $h(x)$ is a non-generate distribution.

We finally need to eliminate $r(x)$ that have $r(x) > x - c$ on some interval *and* is strictly decreasing on some (possibly different) interval. But this follows from the same type of argument as in part (i): for any such decreasing $r(x)$ we can construct an alternative payment function which pays more than $r(x)$ in the region where $r(x)$ is strictly decreasing and less in a region around the point where $r(x)$ is maximized, and show that this alternative payment function must dominate $r(x)$. ■