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ABSTRACT

National Labour Markets, International Factor Mobility and Macroeconomic Instability*

We analyze how global economic integration of factor markets affects the stability of the macroeconomy, with respect to expectations-driven fluctuations, when countries differ in their labour market institutions. It is shown that, due to the occurrence of equilibrium indeterminacy, liberalization of capital movements is likely to be accompanied by persistent fluctuations at the world level, while allowing also for labour movements may bring macroeconomic stability. Whether this also implies higher welfare in the long run depends on differentials in average firm size across countries. If the average firm size in a country operating under perfect competition and full employment is small relative to a country with rigid wages and unemployment, then free migration reduces unemployment, narrows wage differentials and expands world output.

JEL Classification: E24, E32, F15 and F20
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1 Introduction

The question of migration is taking centre stage in the debate about globalization, however, the existing literature has not, as yet, properly addressed the implications of international labor movements on macroeconomic stability.[1] In this paper we study whether, in a world where goods and capital markets are highly integrated, the liberalization of labor movements between countries that differ in their labor market institutions may stabilize the economies (with respect to expectation driven fluctuations) and, in addition, raise welfare.

Over the last decade, much research has been devoted to the labor market effects of globalization and, particularly, to the correlation between unemployment and globalization.[2] A relatively more recent concern among researchers is also that the observed instability of world markets may be linked to the increase in international integration of capital markets.[3] In standard general equilibrium (neoclassical) trade theory, where international capital mobility, free migration and free trade in goods can be lumped together via the factor price equalization theorem, the importance of international flows of input factors is largely overlooked, since free trade (or alternatively, free factor mobility) is enough to level out divergences in factor endowments across economies. In reality, however, countries' differences go far beyond differences in factor endowments. Differences in the labor market structure, for instance, may account for divergences in the wage levels and for the existence of migration flows, even when product and capital markets are integrated.[4] On the other hand, the effect of increased integration of world

[1] At the micro level, the literature has focused, traditionally, on the relationship between trade and migration or on the labor-market-adjustment to international labor movements. At the macro level, the literature has focused on the relationship between migration, growth and development.

[2] Although the evidence on the incidence of increased international economic integration on labor markets in developed countries is by no means conclusive, many researchers share the view that labor market rigidities, coupled with trade integration, contributes to the rise in unemployment. For an extensive coverage of the literature on these issues, see the volumes edited by Greenway and Nelson (2001).

[3] See, e.g., Azariadis and Pissarides (2005), Bhagwati (1998), Krugman (1999), Prasad et al. (2003), Rodrik (1997), Stiglitz (1999).

[4] Indeed, a relevant and intensively debated source of difference across countries lies precisely in labor market institutions. Take the case of Europe, with rigid wages and unemployment, versus the US, with more flexible labor markets and higher levels of employment. There is an academic debate about whether Europe should move towards a North American labor market structure to achieve higher employment, and to reduce the destabilizing effects of globalization in product and capital markets. See, for instance,

capital markets on macroeconomic fluctuations depends on the structural characteristics of countries and on the nature of shocks (for a survey see Obstfeld and Taylor, 2004). As suggested by Obstfeld and Taylor themselves, the often unpredictable direction of capital flows in international markets, points towards expectation driven shocks. Accordingly, in the present paper we build a model in which endogenous fluctuations in economic variables are driven by self-fulfilling changes in expectations, and we interpret macroeconomic instability (stability) as local dynamic indeterminacy of the steady state.[5]

A well established result, at least in the closed economy literature, is that the existence of distortions, in otherwise standard dynamic general equilibrium models, can induce indeterminacy (see Farmer, 1999). Recently, authors like Coimbra et al. (2005) and Pintus (1997) have shown that labor market distortions, such as unions or efficiency wages, can induce indeterminacy. The present paper, extends this type of analysis to the case of open economies. Specifically, we develop a two-country dynamic (Overlapping Generations) model, in which countries produce one identical good, output and capital markets are perfectly competitive, and the economies differ only in their labor market structure. One country has perfectly competitive labor markets, full employment and the autarkic equilibrium converges to a unique (determinate) steady state; while the other country is characterized by efficiency wages and involuntary unemployment. The latter, under not particularly stringent parameter restrictions, displays indeterminacy at the autarkic equilibrium and thereby endogenous fluctuations in output, employment, wages and interest rates may emerge. We then analyze how international factor movements, in the presence of differences in labor market institutions across countries, affect the local stability properties of the world economy and the steady state levels of unemployment, wages and output.[6]

We show that opening up the economy to free capital movements enlarges the scope for indeterminacy in the rigid wage country and is likely to bring indeterminacy to world markets. Thereby, the country with perfectly

Freeman (1998) and Bertola and Boeri (2002).

[5]The occurrence of *local indeterminacy* implies that there is a continuum of deterministic trajectories all converging to the steady state. In this case there are also infinitely many stochastic fluctuations with rational expectations, close to the steady state, driven by self fulfilling volatile expectations (see Guesnerie and Woodford, 1992). For this reason, the occurence of local indeterminacy is frequently associated to macroeconomic instability.

[6]Bertocchi (2003) also considers differential labor market structures within a dynamic, general equilibrium. However, the focus in her paper is on small open economies and the analysis is concerned with the impact of capital market liberalization and unionization on cross-country income convergence and distribution.

competitive labor markets too experiences output fluctuations and, depending on expectations of future interest rates, capital flows reversals across countries can be observed. Allowing also for free labor mobility, though, may eliminate indeterminacy and, therefore, may have stabilizing effects at the global macroeconomic level. From a steady state point of view, world economic integration does not affect wage and employment levels nor output in the rigid wage country, while it affects its unemployment level. Whether labor movements liberalization also implies higher welfare in the long run, however, crucially depends on the gap in employment per firm (that is in average firm size) between the two countries under autarky. In some cases labor mobility helps in achieving both stability and efficiency. In particular, this happens whenever the level of employment per firm in the fully employed competitive country fall sufficiently short of the corresponding level in the rigid wage country. In this case, under free labor movements there is upward pressure on the competitive wage, workers migrate towards the competitive country, unemployment and the wage gap between the two countries decrease, and world output expands. In other cases, however, a trade off between macroeconomic stability and efficiency may arise. Specifically, this happens whenever the competitive country has larger levels of employment per firm than the rigid wage country. In this case, free labor movements imply downward pressure on the competitive wage, the rigid wage country experiences net inflows of workers and higher unemployment, and world output shrinks.

The outcome that liberalization of capital movements may imply volatile capital flows is not entirely surprising and is coherent with earlier works by Lahiri (2001), Sakuragawa and Hamada (2001), Weder (2001) and Meng and Velasco (2003) among others. These authors, however, disregard distortions in labor markets.[7]

The results on labor mobility, on the other hand, provide new insights on the long run welfare effects of workers' migration and on its implications for macroeconomic instability linked to globalization of capital markets. If workers are free to move, workers' movements follow the direction of capital

[7] The papers by Weder and by Meng and Velasco are both extensions of Benhabib and Farmer (1996), and study how moving from a closed to a *small* open economy with free capital movements renders the condition for indeterminacy easier to obtain. Lahiri (2001) also focuses on the effects of capital mobility on equilibrium indeterminacy in a small open economy. In Sakuragawa and Hamada (2001), countries differ because of asymmetric information in financial markets and initial level of wealth. They show that, depending on the country's stage of development, capital market integration may lead to perverse movements in capital flows, which, eventually, may drive the developing country in a poverty trap state.

movements, which weakens the conditions under which expectation driven fluctuations may occur; therefore, we should expect less variability in macro-aggregates linked to changes in expectations. In the long run, net migration flows can go both ways, depending on initial differentials in average firm size. As a result, free migration does not necessarily exacerbate unemployment in the rigid wage country.

These steady state results are also of relevance to dual-economy models à la Harris and Todaro, since it is demonstrated that liberalizing labor movements, under internationally (or sectorally) mobile capital, may actually reduce unemployment and narrow the wage differentials between competitive and rigid wage countries (or sectors).[8]

Finally, the outcome that the perfectly competitive wage economy may be penalized under full integration is in sharp contrast with Davis (1998). He too considers a two-country model where differences across countries rely on labor market institutions. His analysis, however, uses a static Heckscher-Olin framework and focuses on autarkic versus free trade equilibria.[9] We, on the contrary, do not assume any trade based on comparative advantages and focus on the role of factor movements on both steady state and equilibrium dynamic properties of the system.

The remainder of the paper is as follows. In Section 2 we present the structure of the model and obtain the perfect foresight equilibrium for the closed economies. Section 3 focuses on the equilibrium of the world economy under free capital movements, while equilibrium under both capital and labor mobility is analysed in Section 4. Section 5 concludes.

2 Autarky

We consider a world made up of two countries, A and B, that share the same consumption and production structure, but differ in the functioning of the labor market.[10] In country A there are efficiency wages and unemployment, while in country B the labor market is perfectly competitive and the

[8]In their pioneering work on rural-urban migration, Harris and Todaro (1970) show that allowing for labor movements, between a rigid wage urban sector and a flexible wage rural sector, exacerbates urban unemployment and, possibly, widens the wage differentials across sectors.

[9]In Davis paper the source of labor market imperfections is a minimum wage and, by use of comparative static analysis, it is shown that a move from autarky to free trade doubles unemployment in the rigid-wage country (Europe) while the flexible-wage country (America) suffers no losses in employment and enjoy gains in wage income.

[10]Since we focus on fluctuations driven by volatile expectations, we assume that preferences and technologies are time invariant.

economy operates at full employment. All agents have rational expectations and, in what follows, we study the perfect foresight intertemporal equilibria.

We first present the technology and preferences, together with consumption decisions, which are analogous in both countries. Then, for each country, we lay out the model according to the labor market structure considered, derive the equilibrium system under perfect foresight, obtain the steady state equilibria and, finally, study the conditions for the occurrence of local indeterminacy. Before proceeding further it should be stressed that, although we focus on efficiency wages as the source of rigidity in country A labor market, analogous results would apply if we consider monopoly unions or search generated unemployment.

2.1 Production and consumption

In each period $t = 1, ..., \infty$, a single output, used as consumption or as capital good, is produced under perfect competition. Within each country there is a given number m_i, $i = A, B$, of identical profit maximizing firms. Output is taken as the numeraire. The technology consists of a constant returns to scale Cobb-Douglas private production function and a multiplicative labor externality in production.[11] The production function of a typical firm in country i is then given by

$$\mathcal{A}\bar{l}_{i,t}^{\nu} k_{i,t}^{\theta} l_{i,t}^{1-\theta}\ ,\ \ 0 < \theta < 1$$

where $\mathcal{A}$ is a scale parameter, $l_{i,t}$ represents the number of units of effective labor employed by each firm, $k_{i,t}$ is the amount of capital available for production at the outset of period t, and rented by each firm at the rate $r_{i,t}$, $\bar{l}_{i,t}$ is the average level of employment in the country (which is taken as given by each firm), and v is the degree of labor externalities ($1+v$ measuring the degree of social returns to scale).[12]

The market for capital services is perfectly competitive. Hence, for a profit maximizing firm, $k_{i,t}$ must be such that the marginal productivity

[11] The existence of social increasing returns to scale, due either to capital or/and labor externalities, is needed to ensure that the steady state is well defined. This is a feature which is not peculiar to our set up. See, for instance, Barinci and Chéron (2001), Coimbra et al. (2005) and Lloyd Barga et al. (2006). As in Lloyd Barga et al. (2006), we use (country specific) labor externalities to allow for the existence of indeterminacy with a positive interest rate. Labor external effects can be interpreted as coming from thick labor market effects or from knowledge spillovers.

[12] Note that, at a symmetric equilibrium (where $\bar{l}_{i,t} = l_{i,t}$), the aggregate marginal productivity of labor curve is downward sloping when $v < \theta$.

of capital is identical to the rental rate $r_{i,t}$. At a symmetric equilibrium $\bar{l}_{i,t} = l_{i,t}$ and we have,

$$\theta \mathcal{A} k_{i,t}^{\theta-1} l_{i,t}^{1-\theta+\nu} = r_{i,t}. \tag{1}$$

Population is constant over time and agents live for two periods. In each period $t = 1, ..., \infty$, there is a continuum of identical young agents, with a constant given mass N_i, $i = A, B$, native from each country. Preferences of a typical individual born at t are described by the following lifetime utility function

$$c_{i,t}^{\alpha} c_{i,t+1}^{1-\alpha} - a e_{i,t}, \tag{2}$$

where $0 < \alpha < 1$, and $c_{i,t}$ and $c_{i,t+1}$ are consumption in the young and old age, respectively. Agents work only when young and $e_{i,t} \in \{0, 1\}$ represents the number of units of effective labor (or effort) supplied. A worker contributes with one unit of effective labor (i.e., $e_{i,t} = 1$) if he/she is employed (and does not shirk).

A young employed worker, who receives a wage $w_{i,t}$, saves through investment in productive capital goods, $k_{i,t+1}^h$, which are rented to firms in the next period and used for consumption in the old age. We assume that capital is totally depreciated in one period, so that $r_{i,t}$ is also the interest factor. Accordingly, he/she chooses $k_{i,t+1}^h$, $c_{i,t}$, $c_{i,t+1}$ to maximize his/her expected lifetime utility subject to the following constraints: $k_{i,t+1}^h = w_{i,t} - c_{i,t}$ and $c_{i,t+1} = r_{i,t+1} k_{i,t+1}^h$. Hence, the amount of capital goods, $k_{i,t+1}^h$, bought in period t by each employed worker, and made available for production at the outset of period $t + 1$, is given by,

$$k_{i,t+1}^h = (1 - \alpha) w_{i,t}. \tag{3}$$

Consumption when young and when old, of each employed individual, amounts, respectively, to

$$\begin{aligned} c_{i,t} &= \alpha w_{i,t} \\ c_{i,t+1} &= r_{i,t+1}(1 - \alpha) w_{i,t}, \end{aligned} \tag{4}$$

where r_{t+1} is the expected value, evaluated at period t, of the interest rate in the next period, which under perfect foresight is identical to its realized value.

2.2 Equilibrium with efficiency wages and unemployment: Country A

2.2.1 The labor market. For country A, we use a simplified version of the Shapiro-Stiglitz (1984) efficiency wage model, which accounts for the existence of unemployment. If a worker employed in a firm is caught shirking (i.e., $e_t = 0$) he/she is fired, that is the firm does not pay the wage, and he/she is forced to enter the unemployment pool. However, employers can only imperfectly monitor workers. The monitoring technology is not made explicit, we simply assume that the ex-ante probability of shirking and being caught is given by $0 < \lambda < 1$. A young agent faces three possibilities: either he/she is employed and does not shirk, or he/she is employed and shirks, or he/she is unemployed. Combining (2) and (4) the indirect utility of a worker in each of these alternatives is, accordingly, given as follows,

$$V = \begin{array}{cc} \alpha^{\alpha}(1-\alpha)^{1-\alpha} r_{A,t+1}^{1-\alpha} w_{A,t} - a & if\ not\ shirking \\ (1-\lambda)\alpha^{\alpha}(1-\alpha)^{1-\alpha} r_{A,t+1}^{1-\alpha} w_{A,t} & if\ shirking \\ 0 & if\ unemployed \end{array} \tag{5}$$

Firms choose w_A, l_A and k_A such that profits are maximized. Since the output of a worker who shirks is zero, at equilibrium the positive wage payed by firms should be such that it induces workers not to shirk, that is each employed worker supply $e_t = 1$. Using (5) above, it can be easily seen that employed workers have no incentives to shirk when $\lambda\alpha^{\alpha}(1-\alpha)^{1-\alpha} r_{A,t+1}^{1-\alpha} w_{A,t} \geq a$. Accordingly, the problem solved by the firm is the following

$$\underset{w_{At}, l_{At}, k_{At} \in \Re^3_{++}}{Max} \left(\mathcal{A}\bar{l}^{v}_{A,t} k^{\theta}_{A,t} l^{1-\theta}_{A,t} - w_{A,t} l_{A,t} - r_{A,t} k_{A,t} \right),$$

$$s.t.\ \lambda\alpha^{\alpha}(1-\alpha)^{1-\alpha} r_{A,t+1}^{1-\alpha} w_{A,t} \geq a.$$

Obviously the incentive compatibility constraint is binding and, therefore, from the first order conditions we obtain, at a symmetric equilibrium, expression (1) and

$$(1-\theta)\,\mathcal{A} k^{\theta}_{A,t} l^{-\theta+v}_{A,t} = w_{A,t} \tag{6}$$

$$\frac{\bar{w}_{A,t}}{\lambda} = w_{A,t} \tag{7}$$

where,

$$\bar{w}_{A,t} \equiv \frac{a}{\alpha^{\alpha}(1-\alpha)^{1-\alpha} r_{A,t+1}^{1-\alpha}}. \tag{8}$$

Expression (8) defines the reservation wage $\bar{w}_{A,t}$, that is the level of wage at which an employed (not shirking) worker is as well off as an unemployed worker. Due to the imperfect monitoring technology ($0 < \lambda < 1$) wages are set as a mark up ($1/\lambda$) over the reservation wage (see 7). Expressions (6) and (1) show, respectively, that the level of employment and the demand for capital services are such that the wage and the rental rate of capital are identical to their respective marginal products, as in perfectly competitive markets.[13] Denoting by n_A^R the mass of young agents (per firm) resident in country A, a symmetric equilibrium with unemployment requires $l_{A,t} < n_{A,t}^R$.[14] Hence, using (6)-(7) we obtain the following Lemma.

Lemma 1 . *A symmetric equilibrium in the labor market with unemployment in country A is characterized by*

$l_{A,t} = \left(\frac{\bar{w}_{A,t}}{\lambda(1-\theta)\mathcal{A}k_{A,t}^{\theta}} \right)^{\frac{1}{-\theta+v}} < n_{A,t}^R$,

$w_{A,t} = (1-\theta)\,\mathcal{A}k_{A,t}^{\theta} l_{A,t}^{-\theta+v} > \bar{w}_{A,t}$, *where* $\bar{w}_{A,t}$ *is given by (8).*

As long as international labor movements are not allowed, all young residents in country A are native from A and, therefore, in Lemma 1 we have $n_{A,t}^R = N_A/m_A \equiv n_A$.

Note that, since $\bar{w}_{A,t}$ depends on $r_{A,t+1}$ (see 8), the equilibrium level of wages and employment is influenced by expectations of future interest rates. For instance, when expected future interest rate decreases, the reservation wage and wages increase, leading to a decrease in employment per firm if $\nu < \theta$. As we shall see, this opens the door to fluctuations in wages and employment driven by self-fulfilling volatile expectations (endogenous fluctuations).

Endogenous fluctuations may have relevant welfare implications. To see how, consider an equilibrium with volatile expectations and fluctuations

[13] Accordingly, given the constant returns assumption at the private level, profits are zero.

[14] Indeed, under efficiency wages, unemployment is the meaningfull outcome. If that was not the case a worker that was caught shirking, and thereby fired from a firm, could find employment in some other firm, so that workers would always have incentive to shirk and production would not take place.

around a steady state. In a period where the expected future interest rate is low (relative to its steady state level) any employed worker benefits from higher wages and higher current consumption levels, although workers face a higher unemployment rate; whereas, in a period where the expected future interest rate is high the reverse would happen.[15] Since it is likely that some generations benefit while others are harmed by fluctuations, we cannot *a priori* establish whether an equilibrium with fluctuations driven by expectations is welfare improving or not for the economy as a whole; it depends on the social welfare function. If the latter is sufficiently concave in utility of different generations, equilibrium fluctuations may become quite costly from an inter-generational equity point of view.

Moreover notice that, since in any period t wages are set as a mark up over the reservation wage, employed workers are better off than unemployed workers, implying that employment fluctuations affect not only inter-generational but also intra-generational equity.

2.2.2 Equilibrium dynamic system. To derive the equilibrium dynamic system under autarky, we proceed as follows. At equilibrium the aggregate demand for capital services, $m_A k_{t+1}$, must be identical to its aggregate supply, $k^h_{t+1} m_A l_{A,t}$. Hence, using (3) we have that $k_{t+1} = (1-\alpha) l_t w_t$. Combining the latter and expression (6), we obtain the following capital accumulation equation for country A

$$k_{A,t+1} = \gamma k^{\theta}_{A,t} l^{1-\theta+\nu}_{A,t}, \tag{9}$$

where $\gamma \equiv (1-\alpha)(1-\theta)\mathcal{A}$. Combining (1), (6), (7) and (9), we obtain

$$l^{(1-\alpha)(1-\theta+\nu)}_{A,t+1} = \delta k^{-\theta(1-(1-\alpha)(1-\theta))}_{A,t} l^{(1-\theta+\nu)(1-\theta)(1-\alpha)+\theta-\nu}_{A,t}, \tag{10}$$

where $\delta \equiv \frac{a(1-\alpha)(1-\theta)^{1-\alpha}}{\lambda\alpha^{\alpha}\theta^{1-\alpha}\gamma^{1+\theta(1-\alpha)}}$.

Equations (9) and (10) define a two dimensional dynamic model, and characterize the equilibrium in terms of the two state variables (k_A, l_A).

[15] Note that, by the use of (5) and (6)-(8), along any deterministic equilibrium trajectory (with perfect foresight, that is with point expectations), utility of an employed worker is constant and identical to $a(1-\lambda)/\lambda$. However, under equilibria exhibiting stochastic endogenous fluctuations (sunspot equilibria), utility of an employed worker will fluctuate, depending on how realized future interest rates (that influence future consumption) match expectations (that influence real wages); that is utility is equal to $a[(r_{A,t+1}/\hat{r}_{A,t+1}) - \lambda]/\lambda$, where $\hat{r}_{A,t+1}$ is the interest rate in period $t+1$ expected at time t, and $r_{A,t+1}$ is the realized interest rate at $t+1$.

Note that, in period t, k_A is a predetermined variable whose value is given by past saving. By contrast, l_A is a non predetermined variable whose value, in period t, is influenced by expectations of future interest rates via the workers' reservation wage.

Definition 1 . *An intertemporal perfect foresight equilibrium under autarky for country A is a sequence* $(k_{A,t},\ l_{A,t}) \in \Re^2_{++}$, $t = 1, ...\infty$, *such that (9) and (10) are satisfied, where* $0 < \lambda < 1$ *is the ex-ante probability of shirking and being caught.*

2.2.3 Steady state. In country A, the interior steady state $(k_A, l_A) \in \Re^2_{++}$, verifying the dynamic system (9)-(10), with $k_A = k_{A,t} = k_{A,t+1} > 0$ and $l_A = l_{A,t} = l_{A,t+1} > 0$ satisfies

$$k_A = \gamma^{\frac{1}{1-\theta}} l_A^{\frac{1-\theta+\nu}{1-\theta}}, \tag{11}$$

$$l_A = \delta^{\frac{1-\theta}{\nu}} \gamma^{-\frac{\theta(1-(1-\alpha)(1-\theta))}{\nu}}. \tag{12}$$

Note that equation (12) gives a unique solution for the steady state value l_A.[16] Substituting the latter into (11) we obtain the steady state value of k_A.

The steady state values of the wage and interest rate are, accordingly, given by: $w_A = \left[(1-\alpha)^{\theta} (1-\theta) \mathcal{A} l_A^{\nu}\right]^{\frac{1}{1-\theta}}$ and $r_A = \theta/(1-\alpha)(1-\theta)$. It is worth noticing that this economy has a positive interest rate at the steady state equilibrium, i.e., $r > 1$, if and only if the propensity to consume when young satisfies the following restriction, $\alpha > (1-2\theta)/(1-\theta)$. To ensure this, and other results to follow, from now on we impose the following restrictions on parameter values.

$$\frac{1+2\theta}{1+3\theta} > \alpha > Max\left\{\frac{1-2\theta}{1-\theta}, \frac{1}{2}\right\}, \tag{13}$$

$$0 < \theta < 1/2 . \tag{14}$$

[16] By Lemma 1, at equilibrium $l_A < n_A$. In order for this condition to be fulfilled at the steady state, the parameter a has to satisfy the following restriction: $0 < a < \lambda n_A^{\frac{\nu}{1-\theta}} \theta^{1-\alpha} \alpha^{\alpha} (1-\alpha)^{\frac{\theta}{1-\theta}} \mathcal{A}^{\frac{1}{1-\theta}} (1-\theta)^{\frac{\theta+\alpha(1-\theta)}{1-\theta}}$. This restriction ensures that, at the steady state defined in (11)-(12), there is unemployment, i.e. $l_A < n_A$, and it also ensures that $l_{A,t} < n_A$ along trajectories sufficiently close to the steady state.

These restrictions cover most empirically relevant values of θ and α.[17]

2.2.4 Equilibrium dynamics and indeterminacy. Local indeterminacy occurs when the number of stable eigenvalues is higher than the number of predetermined variables. The system (9) and (10) is loglinear and the associated 2x2 Jacobian matrix is provided in Appendix B. Since in our model there is only one predetermined variable, k_A, indeterminacy arises when both eigenvalues (in absolute value) are lower than 1. The proposition below dictates parameter conditions under which local indeterminacy occurs.

Proposition 1 . *Assume that (13)-(14) are satisfied, and define* $\underline{v}_{au} \equiv (\alpha(1-\theta)-(1-2\theta))/(1-\alpha)$ *and* $\bar{v}_{au} \equiv 2(1-\alpha(1-\theta))/(2\alpha-1)$*. Then, country A exhibits indeterminacy under autarky if and only if* $\underline{v}_{au} < v < \bar{v}_{au}$*.*

Proof. *See Appendix B.* ■

Note that assuming $\alpha > \frac{1-2\theta}{1-\theta}$ implies that $\underline{v}_{au} > 0$. Therefore indeterminacy, with a positive interest rate and capital accumulation, requires a minimum degree of labor externalities bounded away from zero.[18] Nevertheless, indeterminacy still occurs with a sufficiently small degree of externalities, consistent with empirical evidence[19] and with a standard negatively sloped (aggregate) labor demand curve, i.e., $v < \theta$. For instance, indeterminacy prevails when $\alpha = 0.6$, $\theta = 1/3$ and $v = 0.18$.[20] This means that, under empirically relevant values of the propensity to consume, capital share in output and externalities, there are stochastic endogenous fluctuations, whereby employment and wages fluctuate due to self-fulfilling volatile expectations.

To gain an intuition of why indeterminacy requires a lower bound on externalities, consider the case of an economy that, at period t, is at its steady state for some time and that, for some reason, experiences an increase in the *expected* future interest rate. Then, the reservation wage, $\bar{w}_{A,t}$, will decrease (see 8) and (see Lemma 1) the current level of employment per firm, $l_{A,t}$, will increase (assuming $\nu < \theta$), leading to an increase in capital

[17]Estimates from national accounting for OECD countries are usually in accordance with values of the capital share of output, θ, that belong to the 0.25-0.4 range, and to values of the propensity to consume when young, α, usually higher than 0.5 .

[18]Lloyd-Braga et al. (2006) discuss this property at length. Note also that $\alpha < \frac{1+2\theta}{1+3\theta}$ ensures $\bar{\nu}_{au} > \underline{v}_{au}$, and that the same restriction on α applies in Lloyd-Braga et al. (2006) for an infinetely elastic labor supply.

[19]The degree of externalities found in empirical works is quite small, usually below 0.3. See, Basu and Fernald (1997) and Burnside (1996).

[20]For $\alpha = 0.6$ and $\theta = 1/3$ we obtain $\underline{v}_{au} = 0.17$ and $\bar{v}_{au} = 6$.

accumulation $k_{A,t+1}$ (driven by the wage bill). This, by itself, would produce a tendency for a decrease in the interest rate at $t+1$. However, the increase in $k_{A,t+1}$ increases the marginal productivity at $t+1$, inducing *per se* an increase in employment per firm at $t+1$ (since $\partial \log l_{t+1}/\partial \log k_{t+1} = \theta/(\theta - v)$). Higher employment triggers in turn an increase in the interest rate at $t+1$. If employment per firm at $t+1$ rises by a sufficient amount, that is if v is sufficiently high, its positive effect on the realized interest rate at $t+1$ will off set the negative effect due to the increase in $k_{A,t+1}$. As a result, the initial expectation of an increase in future interest rates can become self-fulfilled. Indeterminacy also implies that equilibrium trajectories will eventually return back to the steady state. In our case, we should observe a reversal in the future capital stock, that is the future wage bill should decrease. The latter will only be possible if $l_{A,t+1}$ does not increase too much. Note, however, that the required increase in $l_{A,t+1}$ needed for $r_{A,t+1}$ to rise is lower the higher the labor externality. Accordingly, a necessary condition for indeterminacy to occur is the existence of a lower bound on labor externalities (i.e., $\nu > \underline{\nu}_{au}$).

2.3 Equilibrium with a perfectly competitive labor market and full employment: Country *B*

2.3.1 The labor market. In country B, we consider a perfectly competitive labor market with full employment[21] (where shirking is not possible). The firm labor demand in each period t is, therefore, determined by the identity between wages and the marginal productivity of labor, $(1-\theta)\mathcal{A}\bar{l}^{v}_{B,t}k^{\theta}_{B,t}l^{-\theta}_{B,t}$. Using (4), the indirect utility of an employed worker is $\alpha^{\alpha}(1-\alpha)^{1-\alpha}r^{1-\alpha}_{B,t+1}\left(w_{B,t}-\bar{w}_{B,t}\right)$, where $\bar{w}_{B,t} \equiv \frac{a}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}r^{1-\alpha}_{B,t+1}}$ is the reservation wage. Hence, when $w_{B,t} = \bar{w}_{B,t}$ a young agent is indifferent between being employed or unemployed, all young agents being willing to become employed when $w_{B,t} > \bar{w}_{B,t}$. Therefore, denoting by n^{R}_{B} the mass of young agents per firm resident in country B, it is straightforward to obtain the following Lemma.

Lemma 2 . *A symmetric equilibrium in the labor market with full employment in country B is characterized by*

$l_{B,t} = n^{R}_{B}$,

$w_{B,t} = (1-\theta)\,\mathcal{A}k^{\theta}_{B,t}l^{\nu-\theta}_{B,t} \geq \bar{w}_{B,t}$, *where* $\bar{w}_{B,t} \equiv \frac{a}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}r^{1-\alpha}_{B,t+1}}$.

[21]For now, note that full employment in country B is needed to ensure the existence of a two country equilibrium when both capital and labor are free to move. Later on, in Section 4, we provide a comprehensive discussion of the issue.

As long as international labor movements are not allowed, all young residents in country B are native from B and, therefore, in Lemma 2 we have $n_B^R = N_B/m_B \equiv n_B$.

2.3.2 The equilibrium dynamic system. In country B the equilibrium dynamics is summarized by a first-order difference equation; namely, the capital accumulation equation, $k_{B,t+1} = (1-\alpha)w_{B,t}n_B$. Using Lemma 2, we derive a one dimensional dynamic equilibrium model in k_B, i.e.,

$$k_{B,t+1} = \gamma k_{B,t}^{\theta} n_B^{1-\theta+\nu}. \tag{15}$$

Therefore,

Definition 2 . *An intertemporal perfect foresight equilibrium under autarky for country B is a sequence $k_{B,t} > 0$, such that (15) is satisfied.*

2.3.4 Steady state. The steady state, k_B, verifying the dynamic equation (15), where $k_B = k_{B,t} = k_{B,t+1} > 0$, is given by,

$$k_B = \gamma^{\frac{1}{1-\theta}} n_B^{1-\theta+\nu}. \tag{16}$$

Using (1) and Lemma 2, the steady state values of the interest rate and wage are, accordingly, given by $r_B = \theta/(1-\alpha)\,(1-\theta)$ and $w_B = \left[(1-\alpha)^{\theta}\,(1-\theta)\,\mathcal{A}n_B^{\nu}\right]^{\frac{1}{1-\theta}}$.[22]

It is interesting to note that at the steady state interest rates in both countries are identical and independent of the levels of capital and employment per firm.[23] This feature, however, does not prevent the existence of capital movements driven by changes in expectations of future interest rates, as we shall see in next section.

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[22] We assume that $0 < a < \theta^{1-\alpha}\alpha^{\alpha}(1-\alpha)^{\frac{\theta}{1-\theta}}\mathcal{A}^{\frac{1}{1-\theta}}\,(1-\theta)^{\frac{\theta+\alpha(1-\theta)}{1-\theta}}\,n_B^{\frac{\nu}{1-\theta}}$. This restriction implies that $w_B > \bar{w}_B$ at the steady state and therefore $w_{B,t} \geq \bar{w}_{B,t}$ close to the steady state, as required by Lemma 2. Note that the above restriction is equivalent to $n_B > \lambda^{\frac{1-\theta}{\upsilon}}.l_A$, where l_A is given by (12).

[23] This occurs because, at steady state, the interest rate depends only on the propensity to save and on capital and labor share of output, which are fixed and identical in both countries due to the assumptions of Cobb Douglas technology and Cobb Douglas preference over consumption. Formally, $r = \theta\frac{y}{k}$ where θ is the capital share in output, y. Also, savings through investment in capital goods are given by $k = (1-\alpha)wl$ where $(1-\alpha)$ is the marginal propensity to save. Hence, $r = \frac{\theta}{1-\alpha}\frac{1}{wl/y}$. Since the labor share in output is $1-\theta$, we obtain that $r = \theta/(1-\alpha)(1-\theta)$.

2.3.4 Equilibrium dynamics and (in)determinacy. Since there is full employment and capital is a predetermined variable, there is no indeterminacy at the autarkic equilibrium. Moreover, since the equilibrium dynamics, given by (15), is loglinear and $\theta < 1$, all equilibrium trajectories converge to the steady state. The following proposition restates the result.

Proposition 2 . *Under autarky, country B has a (stable) determinate steady state.*

3 Free capital mobility

In this section we first study the equilibrium dynamic system for the case of free international capital mobility. We then show that a steady state exists, study the occurrence of local indeterminacy, and derive relevant comparative static results.

3.1 Equilibrium dynamic system

Liberalization of capital movements between both countries implies a no arbitrage condition in the world capital market, that is interest rates, given in each country by (1), must be identical in every period. Hence, the equilibrium world capital stock (K_t), available for production in every period t, must be distributed across firms of both countries in a way such that

$$K_t = m_A k_{A,t} + m_B k_{B,t} \tag{17}$$

$$r_{A,t} = r_{B,t} \,. \tag{18}$$

These equations, together with (1), can be used to obtain the level of capital rented by a representative firm in each country, that is $k_{A,t}$ and $k_{B,t}$, as a function of K_t and of the level of employment per firm in each country

$$k_{A,t} = (K_t/m_A)\,\frac{1}{1+z_t} \tag{19}$$

$$k_{B,t} = (K_t/m_B)\,\frac{z_t}{1+z_t} \tag{20}$$

where,

$$z_t = \frac{m_B}{m_A}\left(\frac{l_{B,t}}{l_{A,t}}\right)^{\frac{1-\theta+\nu}{1-\theta}}. \tag{21}$$

Capital accumulation in the world is driven by the sum of saving (i.e. labor income) in both countries,

$$K_{t+1} = (1-\alpha)\left(w_{A,t} m_A l_{A,t} + w_{B,t} m_B l_{B,t}\right). \tag{22}$$

From this equation, substituting the expressions for w_A and w_B as given in Lemma 1 and 2, and using (19)-(20), we obtain the following dynamic equation

$$K_{t+1}/m_A = \gamma \left(K_t/m_A\right)^{\theta} l_{A,t}^{1-\theta+\nu} H_t^{1-\theta}, \tag{23}$$

where,

$$H_t \equiv H\left(l_{A,t}\right) = 1 + z_t, \;\; z_t \text{satisfying (21) with } l_{B,t} = n_B. \tag{24}$$

Combining (1), (19), (24), (23) and Lemma 1, we obtain the other dynamic equation governing employment in country A,

$$l_{A,t+1}^{(1-\alpha)(1-\theta+\nu)} H_{t+1}^{(1-\alpha)(1-\theta)} = \delta \left(K_t/m_A\right)^{-\theta(1-(1-\alpha)(1-\theta))} l_{A,t}^{(1-\theta+\nu)(1-\theta)(1-\alpha)+\theta-\nu} H_t^{(1-\alpha)(1-\theta)^2+\theta}. \tag{25}$$

Equations (23) and (25) define the equilibrium dynamic system written in terms of two variables: K, whose value is determined by the world past savings, and l_A, whose value is influenced by current expectations of future rental rates.[24]

3.2 Steady state

A steady state, (K, l_A), for the system (23) and (25) is a solution of the following system of equations

[24]Note that, although K is predetermined, k_A and k_B are non-pretermined variables. The values of k_A and k_B, given in (19)-(20) with z as given in (24), are influenced by employment per firm in country A, which depends on the reservation wage and thereby on expectations of future interest rates.

$$K/m_A = (1+z)\,\gamma^{\frac{1}{1-\theta}} l_A^{\frac{1-\theta+\nu}{1-\theta}}\text{, with } z = (m_B/m_A)\,(n_B/l_A)^{\frac{1-\theta+\nu}{1-\theta}} \tag{26}$$

$$l_A = \delta^{\frac{1-\theta}{\nu}} \gamma^{-\frac{\theta(1-(1-\alpha)(1-\theta))}{\nu}}. \tag{27}$$

Note that steady state level of employment per firm in country A, as given in (27), is identical to the steady state level of employment per firm under autarky, as given in (12). By use of (27), (26), (19) and (20), it can be checked that the steady state values k_A and k_B are the same as in the autarkic equilibrium. Hence, wages and interest rates at the steady state are also the same. Accordingly we have,

Proposition 3 . *Under free capital mobility the steady state values of the interest rate, wages, employment and capital per firm in both countries are unchanged with respect to the case of autarky.*

To gain an intuition of why steady state values are unchanged note that, by use of (20), country B capital share of world capital, evaluated at the steady state, is given by $s_B^k \equiv m_B k_B/K = z/(1+z)$. While, by use of Lemma 1 and 2 and (19)-(21), country B saving share of world savings, evaluated at the steady state, is given by $s_B^s \equiv w_B n_B/(w_A l_{A+} w_B n_B) = z/(1+z)$. Therefore, at the steady state, the amount of capital goods used in production in country B is equal to investment in capital goods through savings ($s_B \equiv s_B^k = s_B^s$); the same applying to country A ($s_A^k = s_A^s \equiv 1 - s_B$). This means that, at the steady state, there are no net exports or imports of capital services between countries, and the values of l_A, k_A and k_B are the same irrespective of capital mobility. However, as discussed in the following section, indeterminacy may occur and in this case there are stochastic equilibrium trajectories, driven by self-fulfilling volatile expectations, along which net capital flows between the two countries are observed.

3.3 Local Equilibrium dynamics and indeterminacy

The dynamic system (23)-(25) implicitly defines a two dimensional non linear map G, such that $(K_{t+1}/m_A, l_{A,t+1}) = G\ (K_t/m_A, l_{A,t})$ around the steady state. We follow the usual procedure of (log)linearizing around the steady state and studying the eigenvalues of the associated 2x2 Jacobian matrix evaluated at the steady state. Details are in Appendix B. The following proposition dictates parameter conditions under which indeterminacy occurs.

Proposition 4 . *Assume that (13)-(14) are satisfied, and define* $\underline{v}_k \equiv \frac{(\alpha(1-\theta)-(1-2\theta))(1-s_B)}{(1-\alpha)+s_B(\alpha-(1-2\theta)/(1-\theta))}$ *and* $\bar{v}_k \equiv \frac{2(1-\alpha(1-\theta))(1-s_B)}{(2\alpha-1)+2s_B(1/(1-\theta)-\alpha)}$. *Then, the world economy exhibits local indeterminacy under free capital mobility if and only if* $\underline{v}_k < v < \bar{v}_k$.

Proof. *See Appendix B.* ■

Proposition 4 implies that, when free capital movements are allowed, a country with unemployment may 'export' instability to the rest of the world and to an otherwise determinate (stable) economy (country B). Liberalizing capital movements entails that 'local' shocks to expectations may now affect other countries and render the latter also susceptible to equilibrium fluctuations driven by self-fulfilling volatile expectations. Indeed, changes (an increase, for instance) in expected future interest rates induce changes (an increase, if $v < \theta$) in the current level of employment per firm in country A and, thereby, changes (an increase) in its current interest rate, $r_{A,t}$, which, becoming different (higher) from $r_{B,t}$, induces capital flows between (from) country B and (to) country A, leading to fluctuations in wages and output in country B as well.

Note, moreover, that the lower bound on externalities, needed for indeterminacy, is now smaller than that required for indeterminacy to prevail in country A under autarky, i.e., $\underline{v}_k < \underline{v}_{au}$ since $0 < s_B < 1$. For instance, when $\alpha = 0.6$, $\theta = 1/3$ and $s_B = 1/2$ indeterminacy prevails when $v = 0.09$.[25] Therefore, under capital mobility it is easier to obtain fluctuations driven by self-fulfilling expectations with small values of ν consistent with empirical evidence. To understand why capital mobility may induce indeterminacy at a lower value of ν, consider the sequence of events following an increase in the expected future interest rate analyzed under autarky. As referred above, the initial increase in $l_{A,t}$ will tend to trigger, in period t, inflows of capital from country B into country A; which, will further increase current saving in country A, while it will decrease current saving in country B. Hence, ceteris paribus, a differential between future returns in the two countries would arise, which cannot be sustained under perfect capital mobility. Indeed, the differential in future returns induces a reversal in capital movements from

[25] With $\theta = 1/3$, $\alpha = 0.6$ and $s_B = 1/2$ we have that $\underline{v}_k = 0.08$, while $\bar{v}_k = 0.55 > \theta$ (cf. footnote 20). Remark that $\bar{v}_k < \bar{v}_{au}$; however, this is of little relevance because $\bar{v}_k = 0.55$ is still well above empirically plausible values of ν.

country A to country B, which re-establish the no arbitrage condition in capital markets at $t+1$. The capital inflows in country B at $t+1$, off setting the initial outflow at t, help bringing the equilibrium trajectories back to the steady state, as required when indeterminacy occurs. Therefore, due to these additional effects linked to capital movements, externalities need not to be as high as under autarky.

4 Integrated equilibrium

In what follows, we presume the existence of free capital mobility between countries and allow for free international mobility of workers assuming that firms do not discriminate workers by their origin. Workers can seek employment in either country A or country B, and can only be employed in the country of residence. To simplify, we ignore the travel costs of migration. As in previous sections, we focus on (two-country) equilibria characterized by efficiency wages and unemployment in country A, and perfect competition and full employment in country B. Therefore, Lemma 1 and Lemma 2 still apply, where the mass of young resident per firm in each country, $n_{i,t}^R$, can now differ from the mass of young native per firm, $n_{i,t}$, $i = A, B$.

4.1 Equilibrium Dynamic System

Equilibrium in the world labor market is attained when, in each period t, neither workers in country A are willing to move to country B, nor workers in country B are willing to move to country A. Accordingly, the expected utility of working in country A should be identical to the expected utility of working in country B. By use of (2), (4) and (8) the indirect utility of workers living in country A is $(w_{A,t} - \bar{w}_{A,t})\,\alpha^\alpha(1-\alpha)^{1-\alpha}r_{A,t+1}^{1-\alpha}$ if employed ($e_{A,t} = 1$), and zero if unemployed ($e_{A,t} = w_{A,t} = 0$). Since the probability of being employed is $l_{A,t}/n_{A,t}^R$, the expected utility of a young agent resident in country A is $\frac{l_{A,t}}{n_{A,t}^R}(w_{A,t} - \bar{w}_{A,t})\,\alpha^\alpha(1-\alpha)^{1-\alpha}r_{A,t+1}^{1-\alpha}$. Similarly, using (2), (4) and the definition of $\bar{w}_{B,t}$ in Lemma 2, the indirect utility of workers in country B is given by $(w_{B,t} - \bar{w}_{B,t})\,\alpha^\alpha(1-\alpha)^{1-\alpha}r_{B,t+1}^{1-\alpha}$, which, with full employment, is identical to the expected utility of a young agent resident in country B. Under free capital movements (18) applies, so that $\bar{w}_{A,t} = \bar{w}_{B,t} = \bar{w}_t \equiv \frac{a}{\alpha^\alpha(1-\alpha)^{1-\alpha}r_{t+1}^{1-\alpha}}$. Then, the following no arbitrage condition in the world labor market must hold in every period,

$$w_{B,t} - \bar{w}_t = \frac{l_{A,t}}{n^R_{A,t}} \left(w_{A,t} - \bar{w}_t \right). \tag{28}$$

Notice that, from Lemma 1, we have $w_{A,t} > \bar{w}_t$ and $l_{A,t} < n^R_{A,t}$. Therefore, a two-country equilibrium in the world labor market implies that $w_{A,t} > w_{B,t} > \bar{w}_t$, that is wages do not equalize.[26]

Dividing both sides of (28) by $w_{A,t}$, and recalling that, by (7), $\bar{w}_t / w_{A,t} = \lambda$, the above no arbitrage condition can be re-written as,

$$\frac{w_{B,t}}{w_{A,t}} = \lambda + (1-\lambda) \frac{l_{A,t}}{n^R_{A,t}}. \tag{29}$$

From (29) it can be seen that, for a given $l_{A,t}$, the lower is net migration into country A, that is the lower is $n^R_{A,t} - n_A$, the lower is the wage gap ($w_{B,t}/w_{A,t}$ moves closer to 1). Combining Lemma 1, Lemma 2 and (19)-(21), equation (29) becomes,

$$n^R_{B,t} = l_{A,t} \left(\lambda + (1-\lambda) \frac{l_{A,t}}{n^R_{A,t}} \right)^{\frac{1-\theta}{v}}. \tag{30}$$

From Lemma 2 recall that $l_{B,t} = n^R_{B,t}$, hence $n^R_{B,t}$ also represents the level of employment per firm in Country B.

By definition, the world young population N satisfies,

$$N \equiv m_A n_A + m_B n_B = m_A n^R_{A,t} + m_B n^R_{B,t}. \tag{31}$$

Using (31), expression (30) can be re-written as,

$$n^R_{B,t} = C(l_{A,t}, n^R_{B,t}) \equiv l_{A,t} \left(\lambda + (1-\lambda) \frac{m_A l_{A,t}}{N - m_B n^R_{B,t}} \right)^{\frac{1-\theta}{v}}. \tag{32}$$

[26]Note that the no arbitrage condition in world labor market, requiring $w_{B,t} > \bar{w}_t$, is not compatible with an equilibrium where unemployment exists in country B. In fact, a two-country equilibrium would not be possible if unemployment prevailed in the perfectly competitive country B. In our model, if unemployment prevailed in country B, wages would be identical to the reservation wage ($w_{B,t} = \bar{w}_t$) and the expected utility of a worker living in B would be zero, which is identical to the utility of being unemployed. On the other hand, the expected utility derived by moving to country A would be positive (since $w_{A,t} > \bar{w}_t$); hence, no young agents will be willing to live and work in country B.

Combining Lemma 1, Lemma 2 and (22) - and recalling that the no arbitrage condition in the world capital market implies that (19)-(21) must be satisfied - the dynamic world capital accumulation equation and the dynamic equation for employment per firm in country A are given, respectively, by

$$K_{t+1}/m_A = \gamma \left(K_t/m_A\right)^{\theta} l_{A,t}^{1-\theta+\nu} H_t^{1-\theta}\,, \tag{33}$$

$$l_{A,t+1}^{(1-\alpha)(1-\theta+\nu)} H_{t+1}^{(1-\alpha)(1-\theta)} = \delta \left(K_t/m_A\right)^{-\theta(1-(1-\alpha)(1-\theta))} l_{A,t}^{(1-\alpha)(1-\theta)(1-\theta+\nu)+\theta-\nu} H_t^{(1-\alpha)(1-\theta)^2+\theta}, \tag{34}$$

where,

$$H_t \equiv \tilde{H}(l_{A,t}) = 1 + z_t,\ z_t \text{satisfying (21) with } l_{B,t} = n_{B,t}^R,\ n_{B,t}^R \text{ satisfying (32).} \tag{35}$$

Note that the only difference between this dynamic system and that with free capital mobility (23-25) lies in the expression for z_t, as employment per firm in country B is now affected by changes in employment per firm in country A through (32).

4.2 Steady state

The steady state levels of K, l_A and n_B^R must satisfy

$$K/m_A = (1+z)\,\gamma^{\frac{1}{1-\theta}} l_A^{\frac{1-\theta+\nu}{1-\theta}},\ \text{with } z = (m_B/m_A)\left(n_B^R/l_A\right)^{\frac{1-\theta+\nu}{1-\theta}} \tag{36}$$

$$l_A = \delta^{\frac{1-\theta}{\nu}} \gamma^{-\frac{\theta(1-(1-\alpha)(1-\theta))}{\nu}} \tag{37}$$

$$n_B^R = \left(\lambda + (1-\lambda)\frac{l_A m_A}{N - m_B n_B^R}\right)^{\frac{1-\theta}{v}} l_A\,. \tag{38}$$

4.2.1 Existence. First note that equation (37) gives us the unique solution for the steady state value l_A. Then given l_A, and using (31), equation (38) can be re-written as an identity between two functions in n_A^R/l_A, i.e.,

$$LHS(\frac{n_A^R}{l_A}) \equiv \frac{N}{m_A l_A} - \frac{n_A^R}{l_A} = \frac{m_B}{m_A}\left(\lambda + (1-\lambda)\frac{l_A}{n_A^R}\right)^{\frac{1-\theta}{v}} \equiv RHS(\frac{n_A^R}{l_A}). \quad (39)$$

A steady state value for n_A^R/l_A is thus a solution of (39) and, by Lemma 1, it must also satisfy $n_A^R/l_A > 1$.

Since equation (39) is non linear multiple steady state may exist. In what follows we state necessary and sufficient conditions on the world level of young population, N, for the existence of a unique steady state $n_A^R/l_A > 1$.

Proposition 5 . *Assume that $v < 1-\theta$ and define $N_1 \equiv (m_A + m_B)\, l_A$, where l_A satisfies (37). Then, under Lemma 1, a unique steady state $n_A^R/l_A > 1$ exists if and only if $N > N_1$.*

Proof. *See Appendix A.* ■

Using the steady state values of l_A and n_A^R/l_A we can then determine the corresponding steady state value of n_A^R and of net migration into country A, i.e., $n_A^R - n_A$. The associated steady state level of employment per firm in country B is, by use of (31), $n_B^R = N/m_B - (m_A/m_B)\, n_A^R$. Finally, given l_A and n_B^R, the steady state value for K is obtained through (36).

4.2.2 Welfare We now state welfare properties of the steady state, by analyzing how migration flows affect economic activity in both countries. From expression (37) it can be seen that the level of employment per firm in country A is identical to the level obtained under autarky or free capital mobility. Also by use of (1), (6), (36), (37) and (19), it can be checked that r, w_A and k_A remain the same, which leads to the following proposition.[27]

Proposition 6 . *Assume that the conditions stated in Proposition 5 hold. Then, migration flows do not affect the steady state world interest rate nor wages, capital and employnet per firm in country A; which, in a fully integrated economy, are unchanged with respect to the autarkic and free capital mobility steady states.*

Using Lemma 2, (20)-(21) and (36), and given the steady state value of n_B^R, we can derive the steady state level of capital per firm in country B, $k_B = \gamma^{\frac{1}{1-\theta}} n_B^{R\,\frac{1-\theta+\nu}{1-\theta}}$, and the steady state level of wages in country B, $w_B = \left[(1-\alpha)^{\theta}\,(1-\theta)\,\mathcal{A}(n_B^R)^{\nu}\right]^{\frac{1}{1-\theta}}$, which are both increasing in n_B^R. Hence,

[27] Note that, as in the case of perfect capital mobility, there are no net capital movements at the steady state (hence $s_B^s = s_B^k = s_B$).

using (31), it can be seen that the lower is n_A^R, the lower is net migration in country A, the higher is n_B^R and thereby k_B and w_B, and the lower is the wage differential between the two countries (see 29). Since wages, capital and employment per firm in country A do not vary with respect to migration flows, a steady state with a lower value for n_A^R is superior welfarewise than one with higher values of n_A^R, under a utilitarian social welfare function for the world. This result motivates the following proposition.

Proposition 7 . *Assume that conditions stated in Proposition 5 hold. Then, the steady state level of net migration into country A is negatively correlated with the wage, capital and employment per firm in country B, and with world welfare (according to a utilitarian social welfare function); and it is positively correlated with the wage gap between the two countries.*

We now analyze how unemployment in country A, wages, capital and employment per firm in country B, and world output compare with the corresponding steady state levels realized under autarky or free capital mobility. Note that, although the steady state level of l_A does not change with respect to the free capital or autarky case (see Proposition 6), unemployment may increase or decrease according to whether net migration into country A is positive or negative.

Indeed, if there are net migration flows into country A (i.e., $n_A^R - n_A > 0$) unemployment, measured by $m_A(n_A^R - l_A)$, will be higher than the corresponding level under autarky and free capital mobility, measured by $m_A\,(n_A - l_A)$. Since, from (31), the identity $m_A(n_A^R - n_A) = m_B(n_B - n_B^R)$ must hold, then, positive net migration in country A corresponds to a decrease in the level of employment per firm in country B relative to the cases of free capital mobility and autarky, i.e., $n_B^R < n_B$. This implies that w_B and k_B will also be lower, and the steady state level of output at the world level decreases too relative to the case of autarky or free capital mobility. Denoting by n_B^* the level of employment per firm in country B at which the return of working in country B is identical to that of working in country A at the steady state under autarky (and free capital mobility), then the conditions under which there is positive or negative net migration into country A can be summarized as follows.

Proposition 8 . *Assume that the conditions stated in Proposition 5 hold and define* $n_B^* \equiv (\lambda + (1-\lambda)(l_A/n_A))^{\frac{1-\theta}{v}}\, l_A$. *Then, compared to the steady state under autarky or free capital mobility, the steady state of the fully integrated economy exhibits:*

(i) Positive net migration and higher unemployment in country A, lower wages, capital and employment per firm in country B and lower output in the world, if and only if $n_B > n_B^*$.

(ii) Negative net migration and lower unemployment in country A, higher wages, capital and employment per firm in country B and higher output in the world, if and only if $n_B < n_B^*$.

Proof. *See Appendix A.* ■

Propositions 7 and 8(i) imply that eliminating barriers to international factor movements may induce a decrease in world output and welfare. Given that, here, we are analyzing economies operating under several market distortions, this result is consistent with the theory of the second best, according to which the correction of one market failure does not necessarily improve welfare.

Propositions 7 and 8 (ii) imply that low employment per firm in the perfectly competitive full employment economy relative to the rigid wage country, before integration, induce positive net migration into the competitive economy and creates a more efficient world. Indeed, this is the outcome one would expect in partial equilibrium, or in a static model, if both countries were characterized by perfectly competitive labor markets and full employment and factor movements were liberalized. In this case workers would move to the country offering a higher wage, that is from the labor abundant country to the labor scarce country, until real wages are equalized. As a result there would be a world redistribution of workers to the advantage of the more productive country and an expansion in world output. In our dynamic general equilibrium model, where one of the two countries is characterized by rigid wages and unemployment, however, the process is different. Recall that, irrespective of liberalization of factor movements, in our economy we have $w_i = \left[(1-\alpha)^{\theta}(1-\theta)\mathcal{A}l_i^{\nu}\right]$, $i = A, B$; that is wages at the steady state are positively related with employment per firm or, equivalently, with the average firm size.[28] Accordingly if, under autarky or free capital mobility, country B is sufficiently less labor abundant than country A, the wage in country B is much lower than in country A and expected income of working in B is relatively low (see 28). To ensure no arbitrage in the world labor market, then, the wage in country B has to be higher under

[28] This happens because in our economy capital, being driven by the wage bill, is increasing in employment per firm. Hence, when the latter increases, the level of capital per firm increases as well, which, by increasing the marginal productivity of labor, leads to an increase in wages at steady state. Note that, in our model, this is true even for very small levels of labor externalities.

full integration. As a result, wages and the average firm size (i.e, n_B^R) in the competitive country increase, unemployment in the rigid wage country (i.e., $n_A^R - l_A$) decreases and world output expands. Moreover, although wages do not equalize, the wage gap between countries is reduced (see 29).

4.3 Local dynamics and (in)determinacy

We now proceed to analyze the local stability properties of the steady state under full integration. Although the equilibrium dynamic equations under full integration (33-34) look identical to those under free capital mobility (23-25), we have to take into account that with migration the level of employment per firm in country B ($n_{B,t}^R$), instead of being fixed and identical to n_B, depends on $l_{A,t}$ through expression (32). Indeed, the conditions for indeterminacy now become also function of an additional parameter $\eta \in (1, +\infty)$, that represents the elasticity of employment per firm in country B with respect to employment per firm in country A evaluated at the unique steady state.[29]

The proposition below summarizes the conditions under which local indeterminacy prevails.

Proposition 9 . *Assume that the conditions in Proposition 5 hold and define* $\underline{v}_l \equiv \frac{(\alpha-(1-2\theta)/(1-\theta))(1-s_B(1-\eta))}{(1-\alpha)+(\alpha-(1-2\theta)/(1-\theta))s_B(1-\eta)}$, $\bar{v}_l \equiv \frac{2(1-\alpha(1-\theta))(1-s_B(1-\eta))}{(2\alpha-1)+2(1/(1-\theta)-\alpha)s_B(1-\eta)}$, $\eta^1 \equiv 1 + \frac{(1-\alpha)}{(\alpha-(1-2\theta)/(1-\theta))s_B}$ *and* $\eta^2 \equiv 1 + \frac{(2\alpha-1)}{2(1/(1-\theta)-\alpha)s_B}$. *Then, the world economy exhibits local indeterminacy under full integration if and only if one of the following set of conditions hold:*

(a) $\bar{v}_l > v > \underline{v}_l$ *and* $1 < \eta < \eta^2$, *or*

(b) $v > \underline{v}_l$ *and* $\eta^2 < \eta < \eta^1$

Proof. *See Appendix B.* ■

Notice that the minimum bound on labor externalities, $\underline{v}_l$, required for indeterminacy is higher than that required under autarky, that is $\underline{v}_l > \underline{v}_{au}$; while the upper bound is lower, $\bar{v}_l < \bar{v}_{au}$. Therefore, the range of values under which indeterminacy occurs is smaller in the integrated equilibrium than in autarky. Accordingly, indeterminacy is more difficult to obtain with free international movements of capital and labor. To explain why, we refer to the same example analyzed previously, where we considered an increase in the expected future interest rate. Earlier we saw that, following an increase in the expected future interest rate at time t, $l_{A,t}$ increases. Since the elasticity of employment per firm in country B with respect to employment

[29]See Appendix A, Lemma 4.

per firm in country A (i.e, η) is positive, higher $l_{A,t}$ implies migration into country B and an increase in current employment per firm in country B.[30] Moreover, capital now flows out of country A into country B. Indeed, since $\eta > 1$, $n^R_{B,t}/l_{A,t}$ and z_t increase with $l_{A,t}$ (see 21), therefore $k_{B,t} = K_t \frac{z_t}{1+z_t}$ also increases (see 20). This implies in turn that, for a given predetermined value K_t, capital flows from country A to country B. As a result, under free labor movements, current savings tend to increase in both countries and so does world capital at $t+1$.[31] The latter renders more difficult the occurrence of an increase in r_{t+1}, as initially expected at time t. Also, the increase in world capital tends to increase labor productivity and employment per firm, which induces a further increase in world savings at $t+1$, and thereby of world capital in $t+2$, reinforcing the initial swerving away from the steady state observed at time t. Hence the reversal of equilibrium trajectories, required for indeterminacy, is now more difficult to obtain. Finally, note that migration flows at t are higher the higher is the responsiveness of employment per firm in country B to changes in employment per firm in country A; therefore, for indeterminacy to occur the elasticity η needs to be bounded from above.

5 Conclusions

Although the issue of labor movements, in a world where goods and capital markets are highly integrated, is taking centre stage in the debate about globalization, the existing macroeconomic literature has been lacking behind. A number of authors have investigated the link between capital mobility and macroeconomic instability. Most recently, Azariadis and Pissarides (2005) have emphasized the role of capital movements in explaining unemployment volatility in a dynamic general equilibrium set up.[32] Also, as testified by the recent volume by Obstfeld and Taylor (2004), there is a growing interest in the literature for testing the impact of capital market liberalization on macroeconomic performance. None of this, however, addresses the macroeconomic implications of international labor movements.

A peculiarity of the recent wave of globalization is that increased integra-

[30] Note that, due to the existence of unemployment in country A, migration into country B does not imply changes in employment per firm in country A.

[31] It should be stressed that the current capital outflow from country A may partially damp the increase in savings due to the rise in employment $l_{A,t}$.

[32] Azariadis and Pissarides (2005) consider a dynamic OLG model of a *small* open economy where unemployment exists at equilibrium due to search costs, and equilibrium fluctuations around the steady state are generated through *exogenous* productivity shocks.

tion in goods and capital markets is accompanied by increased restrictions in labor movements. Is such an asymmetric process of integration of world markets beneficial or harmful for macroeconomic stability and efficiency? As regard stability, one major concern is that increased financial openness can produce unwanted disturbances to economies in so far as domestic capital becomes more responsive to expected future changes in international prices and, correspondingly, magnifies the amplitude of fluctuations in real wages or employment levels. As regard efficiency, one major concern is that labor movement liberalization, between countries with different labor market institutions, may exacerbate unemployment and reduce world output.

In this paper we have shown that labor movement liberalization, between economies with integrated capital markets and different labor market structures, helps to achieve higher aggregate stability by reducing the scope for expectation driven fluctuations. Our results also suggest that, if the competitive country becomes a net importer of workers, then moving to a fully integrated world economy bring about both macroeconomic stability and efficiency. If instead, as a result of labor movement liberalization, the rigid wage country becomes a net importer of workers, then the world economy faces a trade off between efficiency and stability. Whether the rigid wage country becomes a net importer or exporter of workers depends on how large is the gap in the average firm size between the rigid wage and competitive countries before integration.

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Appendix A: Steady State Integrated Equilibrium - Proofs

Proof of Proposition 5 *(Existence and uniqueness of the steady state).* In figure 1 we plot the functions $LHS(\frac{n_A^R}{l_A}) \equiv \frac{N}{m_A l_A} - \frac{n_A^R}{l_A}$ and $RHS(\frac{n_A^R}{l_A}) \equiv \frac{m_B}{m_A}\left[\lambda + (1-\lambda)\frac{l_A}{n_A^R}\right]^{\frac{1-\theta}{v}}$, for $\frac{n_A^R}{l_A} \in (0,\infty)$, considering l_A fixed and given by (37). The first function is represented by the line LHS, which has a slope identical to -1, i.e., $LHS'\left(n_A^R/l_A\right) = -1$. See Figure 1, where three different lines represent the $LHS(\frac{n_A^R}{l_A})$ for three different values of N. The second function is represented by the curve RHS. Assuming that $v < 1-\theta$, $RHS(\frac{n_A^R}{l_A})$ defines a convex negatively sloped function, with a slope identical to,

$$\text{RHS}'\left(n_A^R/l_A\right) = -\frac{1-\theta}{v}(1\text{-}\lambda)\frac{m_B}{m_A}\left[\lambda + (1-\lambda)\frac{l_A}{n_A^R}\right]^{\frac{1-\theta}{v}-1}\left(\frac{l_A}{n_A^R}\right)^2 < 0. \quad \text{(A1)}$$

The RHS tends to infinite as n_A^R/l_A tends to 0, goes through m_A/m_B for $n_A^R/l_A = 1$ with a slope $RHS'(1) = -\frac{1-\theta}{v}(1-\lambda)\frac{m_B}{m_A}$, and tends to $(m_B/m_A)\,\lambda^{(1-\theta)/\nu}$ as n_A^R/l_A tends to ∞. See Figure 1, where $\left(n_A^R/l_A\right)^*$ is such that $RHS'\left(\left(n_A^R/l_A\right)^*\right) = -1$, and where we have represented the case where $RHS'(1) > -1$, i.e., $\left(n_A^R/l_A\right)^* < 1$; N_1 is the value of N at which $LHS(1) = RHS(1)$, and a is such that $LHS(a) = RHS(a)$ for $N = N_1$.

[Figure 1]

A steady state value n_A^R/l_A is a value for $n_A^R/l_A > 1$ at which the two curves, LHS and RHS, cross each other, i.e., (39) is satisfied. We can immediately see that, in the case represented in Figure 1 ($\left(n_A^R/l_A\right)^* < 1$), existence of a steady state $n_A^R/l_A > 1$ requires $LHS(1) > RHS(1)$, i.e., $N > (1 + m_B/m_A)\, m_A l_A \equiv N_1$, as represented by line LHS_3. Otherwise, if the curve RHS crossed the line LHS, it would do it twice but for values $n_A^R/l_A < 1$. In the case represented in Figure 1, the condition $LHS(1) > RHS(1) \Leftrightarrow N > N_1$ also ensures uniqueness of a steady state value $n_A^R/l_A > 1$. The reader may easily check that in the other case, where $1 < \left(n_A^R/l_A\right)^* \Leftrightarrow RHS'(1) < -1$ (i.e., $a = 1, b > 1$), the condition $LHS(1) > RHS(1) \Leftrightarrow N > N_1$, although not required for the existence of a steady state value $n_A^R/l_A > 1$, it is required for its unniqueness. Existence of a steady state would only require $N > N_2$, and two steady states would exist for $N_2 < N < N_1$.■

We now set some other related results, which are important for proofs of Proposition 8 and Proposition 9. We highlight them in two Lemmas.

Lemma 3 *Under the conditions stated in Proposition 5 and given (A1), the unique steady state satisfies* $0 > RHS'\left(\left(n_A^R/l_A\right)^{ss}\right) > -1$.

Proof. From observation of Figure 1, notice that when $N > N_1$, the unique steady state must satisfy $\left(n_A^R/l_A\right)^{ss} > \left(n_A^R/l_A\right)^*$, where $\left(n_A^R/l_A\right)^*$ is such that $RHS'\left(\left(n_A^R/l_A\right)^*\right) = -1$. Given convexity of the $RHS(n_A^R/l_A)$, this implies that $RHS'\left(\left(n_A^R/l_A\right)^{ss}\right) > -1$. ■

The other result of relevance is linked to the domain of η, that is the elasticity of employment per firm in country B with respect to employment per firm in country A, in accordance with (32), and evaluated at the steady state.

Lemma 4 *Assume that Lemma 3 holds and define* $\eta \equiv \left[\frac{dn_{B,t}^R}{dl_{A,t}}\frac{l_{A,t}}{n_{B,t}^R}\right]_{n_B^R,l_A}$, *then* $\eta > 1$ *at the unique steady state* $\left(n_A^R/l_A\right)^{ss} > 1$.

Proof. Using expression (32) we obtain $\eta \equiv \left[\frac{dn_{B,t}^R}{dl_{A,t}}\frac{l_{A,t}}{n_{B,t}^R}\right]_{n_B^R,l_A} = \frac{1+\frac{1-\theta}{\nu}(1-\lambda)\frac{s_P(1-s_B)}{s_B}}{1-\frac{1-\theta}{\nu}(1-\lambda)\frac{s_P^2(1-s_B)}{s_B}}$, where $s_P \equiv n_B^R m_B/n_A^R m_A > 0$ represents the ratio of residents in country B and in country A, evaluated at the steady state under analysis, and $s_B \equiv z/\left(1+z\right) \in (0,1)$ represents, as under perfect capital mobility, country B share of world capital where now $z \equiv \left(m_B/m_A\right)\left(n_B^R/l_A\right)^{\frac{1-\theta+\nu}{1-\theta}} > 0$. Using (A1) we can rewrite $\eta = \frac{1-RHS'\left(n_A^R/l_A\right)^{ss}\frac{1}{s_P}}{1+RHS'\left(\left(n_A^R/l_A\right)^{ss}\right)}$. By Lemma 3, we then have that, under Proposition 5, $\eta > 1$ at the unique steady state $\left(n_A^R/l_A\right)^{ss}$. ■

Proof of Proposition 8 *(Net migration)* To prove the result of Proposition 8, we just have to show that under condition (i) there is negative net migration into country B and that under condition (ii) there is positive net migration into country B. Note that, using (31), expression (38) can be rewritten as,

$$n_B^R = \left(\lambda + (1-\lambda)\frac{l_A}{n_A + n_B\frac{m_B}{m_A} - \frac{m_B}{m_A}n_B^R}\right)^{\frac{1-\theta}{v}} l_A\,. \qquad \text{(A2)}$$

From (A2) it can be seen that if $n_B = n_B^* \equiv \left(\lambda + (1-\lambda)\frac{l_A}{n_A}\right)^{\frac{1-\theta}{v}} l_A$, then net migration is zero, since $n_B^R = n_B$.

Differentiating (A2) with respect to n_B^R and n_B, we obtain that $\frac{\partial n_B^R}{\partial n_B} = \frac{RHS'\left(n_A^R/l_A\right)}{1+RHS'\left(n_A^R/l_A\right)}$, where $RHS'\left(n_A^R/l_A\right)$ is given by (A1). Since, by Lemma 3, $0 > RHS'\left(\left(n_A^R/l_A\right)^{ss}\right) > -1$ at the unique steady state under Proposition 5, we obtain that $\frac{\partial n_B^R}{\partial n_B} < 0$. Therefore, if n_B decreases from $\left(\lambda + (1-\lambda)\frac{l_A}{n_A}\right)^{\frac{1-\theta}{v}} l_A$ then n_B^R increases from $\left(\lambda + (1-\lambda)\frac{l_A}{n_A}\right)^{\frac{1-\theta}{v}} l_A$, so that $n_B^R - n_B > 0$, that is positive net migration into country B. This proves (ii) of Proposition 8. A symmetric argument proves (i). ■

Appendix B: Local Indeterminacy - Proofs

Proof of Proposition 1 *(Autarky)***.** The system (9) and (10) is log-linear and deviations from the steady state, $dLogK_{A,t+1} = LogK_{A,t+1} - LogK_A$, and $dLogl_{A,t+1} = Logl_{A,t+1} - Logl_A$ are determined as follows

$$\begin{bmatrix} dLogk_{A,t+1} \\ dLogl_{A,t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \theta & 1-\theta+v \\ \frac{-\theta(1-(1-\alpha)(1-\theta))}{(1-\alpha)(1-\theta+\nu)} & \frac{((1-\theta+\nu)(1-\theta)(1-\alpha)+\theta-\nu)}{(1-\alpha)(1-\theta+\nu)} \end{bmatrix}}_{J} \begin{bmatrix} dLogk_{A,t} \\ dLogl_{A,t} \end{bmatrix},$$

the trace, T, and the determinant, D, of the associated Jacobian matrix, J, being given, respectively, by

$$\begin{aligned} T &= \frac{(1-\theta)-\alpha(1-\theta+\nu)}{(1-\alpha)(1-\theta)(1-\theta+\nu)} \\ D &= \frac{\theta(1-\theta)}{(1-\alpha)(1-\theta)(1-\theta+\nu)}. \end{aligned}$$

The eigenvalues of the 2x2 Jacobian matrix J are the roots of the characteristic polynomial $P(\lambda) \equiv \lambda^2 - T\lambda + D$. Since there is only one predetermined variable (capital), indeterminacy arises when both eigenvalues (in absolute value) are lower than 1. This case will be obtained when, simultaneously, $D > T - 1$, $D < 1$ and $D > -T - 1$. The condition $D > T - 1 \Leftrightarrow \nu(1-\theta) > 0$, is always verified given that $\nu > 0$ and $\theta < 1$. $D < 1 \Leftrightarrow (1-\theta)(\alpha(1-\theta) - (1-2\theta)) < \nu(1-\theta)(1-\alpha)$. Since

$\alpha(1-\theta) > (1-2\theta)$ under (13)-(14), both left and right hand side of the latter inequality are positive. Therefore this inequality is satisfied iff $v > \frac{(\alpha(1-\theta)-(1-2\theta))}{(1-\alpha)} \equiv \underline{v}_{au}$.

Finally, $D > -T-1 \Leftrightarrow 2(1-\theta)(1-\alpha(1-\theta)) > \nu(1-\theta)(2\alpha-1)$. The left and right hand side of this inequality are both positive, since $2\alpha - 1 > 0$ under (13). Hence the latter inequality is verified iff $\nu < \frac{2(1-\alpha(1-\theta))}{(2\alpha-1)} \equiv \bar{\nu}_{au}$, with $\bar{\nu}_{au} > \underline{v}_{au}$ if and only if $\alpha < \frac{1+2\theta}{1+3\theta}$, the latter inequality being satisfied under (13).■

Proof of Proposition 4 *(Capital mobility)***.** Under perfect capital mobility, the Jacobian matrix, associated with the linearised system (23)-(25) around the steady state, is given by

$$\left[\begin{array}{cc} \theta & (1-\theta+\nu)+(1-\theta)\frac{d\ln H}{d\ln l_A} \\ \frac{-\theta(1-(1-\alpha)(1-\theta))}{(1-\alpha)(1-\theta+\nu)+(1-\alpha)(1-\theta)\frac{d\ln H}{d\ln l_A}} & \frac{((1-\alpha)(1-\theta)(1-\theta+\nu)+\theta-\nu)+\left(\theta+(1-\alpha)(1-\theta)^2\right)\frac{d\ln H}{d\ln l_A}}{(1-\alpha)(1-\theta+\nu)+(1-\alpha)(1-\theta)\frac{d\ln H}{d\ln l_A}} \end{array}\right], \tag{B1}$$

where, by use of (24),

$$\frac{d\ln H}{d\ln l_A} = -\frac{1-\theta+\nu}{1-\theta}s_B, \text{ with } s_B = z/(1+z) \in (0,1). \tag{B2}$$

Using (B2), the trace and determinant of the Jacobian matrix (B1) are, respectively, equal to

$$T = \frac{(1-\theta)-\alpha(1-\theta+\nu)(1-x)(1-\theta)-(1-\theta+\nu)x}{(1-\alpha)(1-\theta)(1-\theta+\nu)(1-x)} \tag{B3}$$

$$D = \frac{\theta(1-\theta)-\theta(1-\theta+\nu)x}{(1-\alpha)(1-\theta)(1-\theta+\nu)(1-x)}, \tag{B4}$$

where $x \equiv s^B \in (0,1)$.

Since $1-x > 0$, the denominator of both T and D is positive. The condition $D > T-1 \Leftrightarrow \nu(1-\theta) > 0$, is always verified given that $\nu > 0$ and $\theta < 1$. $D < 1 \Leftrightarrow (1-\theta)\left(1-s^B\right)(\alpha(1-\theta)-(1-2\theta)) < \nu(1-\alpha)(1-\theta) + s^B(\alpha(1-\theta)-(1-2\theta))$. Since $\alpha(1-\theta) > (1-2\theta)$ under (13)-(14), both left and right hand side of the latter inequality are positive. Therefore, this inequality is satisfied iff $v > \frac{(\alpha(1-\theta)-(1-2\theta))(1-s^B)}{(1-\alpha)+s^B(\alpha-(1-2\theta)/(1-\theta))} \equiv \underline{v}_k$. Finally, $D > -T-$

$1 \Leftrightarrow (1-s^B)(1-\theta)2\,(1-\alpha(1-\theta)) > \nu\left((1-\theta)\,(2\alpha-1)+2s^B\,(1-\alpha(1-\theta))\right)$. Both left and right hand sides of this inequality are positive, since $2\alpha-1>0$ under (13). Hence the latter inequality is verified iff $\nu < \frac{2(1-\alpha(1-\theta))(1-s^B)}{(2\alpha-1)+2s^B(1/(1-\theta)-\alpha)} \equiv \bar{\nu}_k$, with $\bar{\nu}_k > \underline{\nu}_k$ if and only if $\alpha < \frac{1+2\theta}{1+3\theta}$, the latter inequality being satisfied under (13).■

Proof of Proposition 9 *(Capital and labor mobility)***.** Since, equations (33)-(34) are analogous to (23) and (25), the matrix given in (B1) still represents the Jacobian matrix, associated with the linearized system (33)-(34). However, using (35), the elasticity $\frac{d\ln H}{d\ln l_A}$, as given in (B2), must now be substituted by $\frac{d\ln \tilde{H}}{d\ln l_A}$, that is by the elasticiy of $\tilde{H}$ with respect to l_A evaluated at the steady state. Differentiating (35) and (32) we obtain

$$\frac{d\ln \tilde{H}}{d\ln l_A} = -\frac{1-\theta+\nu}{1-\theta}s^B(1-\eta) \qquad \text{(B5)}$$

By Lemma 4, $\eta > 1$, hence $s^B(1-\eta) < 0$. Using (B1) and (B5) it can be checked that, under labor mobility, the trace and determinant of the Jacobian matrix are given by (B3) and (B4) with $x \equiv s^B(1-\eta) < 0$.

Since $x \in (-\infty, \mathbf{0})$, $1-x>0$ and the denominator of both T and D is positive. Hence, $D > T-1 \Leftrightarrow \nu(1-\theta) > 0$, which is always verified. Also $D<1$ $\Leftrightarrow (\alpha-(1-2\theta)/(1-\theta))\,(1-x) < \nu\left((1-\alpha)+(\alpha-(1-2\theta)/(1-\theta))\,x\right)$. Since $x = s^B(1-\eta) < 0$ and $\alpha(1-\theta) > (1-2\theta)$ under (13)-(**??**), the left hand side of the latter inequality is always positive. If $s^B(1-\eta) < -\frac{(1-\alpha)(1-\theta)}{\alpha(1-\theta)-(1-2\theta)} \Leftrightarrow \eta > \eta^1 \equiv 1+\frac{(1-\alpha)}{(\alpha-(1-2\theta)/(1-\theta))s_B}$, the right hand side is negative and $D>1$. If $\eta < \eta^1$, then $D<1$ for $v > \frac{(\alpha-(1-2\theta)/(1-\theta))\left(1-s^B(1-\eta)\right)}{(1-\alpha)+(\alpha-(1-2\theta)/(1-\theta))s^B(1-\eta)} \equiv \underline{\nu}_l$. Turning to the condition $D > -T-1$, we have $D > -T-1 \Leftrightarrow 2\,(1-\alpha\,(1-\theta))\,(1-x) > \nu\,((2\alpha-1)+2x\,(1/\,(1-\theta)-\alpha))$. The left hand side of this inequality is always positive. Since, under (13), $2\alpha-1>0$ the right hand side can be negative or positive. If $s^B\,(1-\eta) > -\frac{(2\alpha-1)}{2[1/(1-\theta)-\alpha]} \Leftrightarrow \eta < \eta^2 \equiv 1+\frac{(2\alpha-1)}{2(1/(1-\theta)-\alpha)s_B}$, the right hand side is positive. Hence, if $\eta < \eta^2$ and $\nu < \frac{2(1-\alpha(1-\theta))\left(1-s^B(1-\eta)\right)}{(2\alpha-1)+2(1/(1-\theta)-\alpha)s^B(1-\eta)} \equiv \bar{\nu}_l$, then $D > -T-1$. Also if $\eta > \eta^2$, then $D > -T-1$.

Finally note that $\eta^1 > \eta^2 \Leftrightarrow \alpha < \frac{1+2\theta}{1+3\theta}$, the latter inequality being verified under (13). Moreover, when $\alpha < \frac{1+2\theta}{1+3\theta}$ and $\eta < \eta^2$, then $\bar{\nu}_l > \underline{\nu}_l$. Accordingly, the conditions for indeterminacy, $D > T-1$, $D<1$ and $D > -T-1$, are simulataneously satisfied if and only if one of the conditions stated in Proposition 9 is verified.■

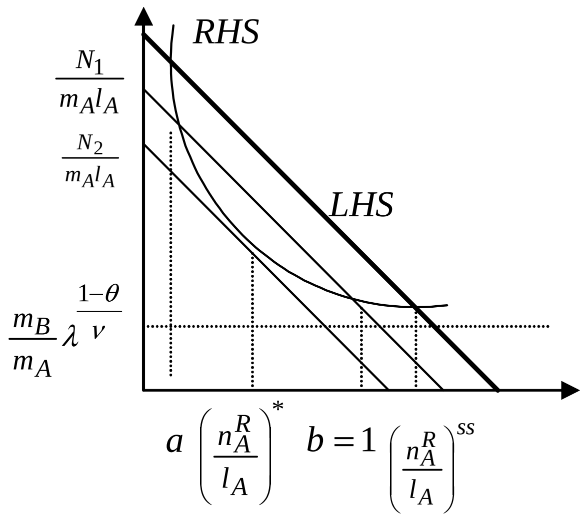

Figure 1: The case of $RHS'(1) > 1 \Longleftrightarrow (n_A^R/l_A)^* < 1$