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ABSTRACT

Redistributive Politics with Distortionary Taxation

We extend the discussion of redistributive politics across electoral systems to allow for taxation to be distortionary. We allow politicians to choose any tax rate between zero and unity and then redistribute the money collected. We build on the model put forward by Myerson (1993) and Lizzeri and Persico (2001 and 2005) to show that the use of distortionary taxation can be understood as an analysis of the trade-off between efficiency and targetability. We derive the equilibrium taxes and redistribution schemes with distortions. We show that the presence of distortions makes full taxation unattractive. We also derive the size of the government, the deadweight loss and inequality as a function of distortions.

JEL Classification: D72, D78, H23 and H31

Keywords: distortionary taxation and redistributive politics

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1 Introduction

The question of the size of government and of the tax level is central to the study of government. However, the economic literature on taxes and redistribution is more developed on the normative side, that is the analysis of the optimal tax structure, than on the positive side, that is the study of the tax structure that would arise as an equilibrium of the political process. The main reason is that voting over taxes and redistribution is an example of a vote on a (possibly) multidimensional space, in which case it is not possible to use single dimensional voting equilibrium concepts, such as the median voter theorem.

The positive literature on taxation, starting with Meltzer and Richards (1981), has circumvented this difficulty by reducing the problem to an environment in which the median voter theorem applies. They analyze an environment in which the choice of tax is reduced to a country-wide tax rate¹ that applies to everybody and the taxes are redistributed uniformly across all citizens. In this environment, the median income citizen is pivotal and chooses the tax rates that he prefers.

More than the restriction on the instruments on taxation, the main restriction of this type of analysis is that redistribution is made without any strategic view. Yet, since the main insight of the positive theory of income taxation is to show how parties use taxes and redistribution to improve their electoral success, it seems odd to leave strategic redistribution completely out of the picture.

Whereas the literature on tax policy as an instrument to win elections is scant,² the incentives to redistribute to gain electoral success have been studied in some depth. In particular, two strands of the literature have emerged following the seminal contributions of Lindbeck and Weibull (1987)³ on the one hand and Myerson (1993) on the other. In the first strand of the literature, the so-called probabilistic voting models, some specific type of heterogeneity in voter preferences are imposed so that the mapping from policy proposals to vote shares is continuous and differentiable (at least around the equilibrium point). This allows for the equilibrium to be in pure strategies.

The contributions that start with Myerson (1993) do not introduce any type of

¹The main results of Meltzer and Richards have been extended since then to somewhat more realistic settings in which the choice of tax instruments is wider. See Section 1.2 .

²Recent contributions include Gouveia and David (1996) and Carbonell-Nicolau and Ok (in press).

³The probabilistic voting model was first introduced by Hinich, Ledyard and Ordershook (1972).

voter heterogeneity. Instead, in these models voters are all ex-ante identical. This allows to disentangle the welfare effects of strategic policy promises from those that derive from any differences in preferences among voters. The counterpart is that the equilibrium is typically in terms of mixed strategies.

This paper analyzes an electoral competition game in which politicians compete through binding promises to voters in terms of both taxes and targeted transfers. Because we are interested in the relationship between the efficiency of the tax system and politicians' choices, our model builds on Myerson (1993). To study the question we are interested in, we allow for taxes to be distortionary. Indeed, without distortions, it is optimal to tax all the money in the economy in order to maximize the amount of strategic redistribution. The presence of distortions implies that the amount of money available for redistribution is lower than the total of taxes collected. This introduces incentives for candidates to leave voters with some of their income. Viewing the choice of the level of taxation as an index of government size, our analysis also sheds some light on the relationship between the distortionary cost of taxation and the size of government.

Our model is centered therefore around a trade-off between efficiency and targetability. Taxes are inefficient but they give candidates the possibility of targeting some voters to win their votes. This defining trade-off is closely related to that between public good provision and targeted redistribution analyzed by Lizzeri and Persico (2001 and 2005). They focus on the trade-off between a public project that is efficient (in the sense that its return is greater than unity) but that cannot be targeted to specific voters and pure, targeted, redistribution, whose return is equal to unity. They analyze the inefficiency that arises when redistributive policies targeted to particular subsets of the population are overprovided at the expense of efficient, generalized public spending. The key determinant of the overprovision of targeted redistribution is that the targetability of redistribution is valuable to politicians whose objective is to win the election.

The analysis in our paper can be seen as complementary to that of Lizzeri and Persico. Given the inefficiency of taxation, no taxation is the efficient policy (it is similar to providing a public good; a policy that benefits everybody but that is not targetable) while taxation is inefficient but enables politicians to have a budget from which they can make targeted promises.

The indivisible aspect of public good provision leads in Lizzeri and Persico's analysis to the natural assumption that politicians face a binary choice between full

redistribution and provision of the public good. Yet, in our context, for the effect of distortionary taxation to be analyzed, we need to allow politicians more than two choices. Distortionary taxation is, in this respect, different from public good provision. Hence we solve for the equilibrium of the game with a continuous choice set: politicians can choose any tax rate between zero and 100%.

1.1 Results

We develop a model of electoral competition with two candidates. There is a continuum of voters. The two candidates compete to win the voters' support by simultaneously offering them individual, binding, credible campaign promises in terms of the level of taxation and a targeted transfer. Taxation is distortionary in the sense that the budget politicians have at their disposal to make transfers is lower than the total of income that is taxed away.

We first solve for the equilibrium when distortions are linear. We show that in this equilibrium, politicians randomize across all tax rates. Distortions have an impact on the distribution function used to randomize between tax rates. The higher the distortions, the more weight politicians put on low tax rates. The promises made for a given tax rate are simple: they make no additional promise to half of the voters – these voters are thus taxed and receive no transfers – and promise the same after-tax transfer to the other half – these voters are also taxed but receive the full proceeds of taxation.

Turning to the relationship between the economy's welfare and the size of the distortion, we show that it is U-shaped. Thus, starting from a position where taxes are very distortionary, increasing the efficiency of taxation implies counter-intuitively that the welfare of the economy may decrease. The main reason for this is that as the efficiency of taxation increases, so does the politician's probability of selecting a high tax rate, and the increase in distortions due to this latter effect may more than counterbalance the positive effect of increased efficiency. Remark that this contrasts with the model of Lizzeri and Persico (2001), in which an increase in the value of the public good leads always to an increase in welfare. The reason is that, in their model, an increase in the value of the public good leads to an increase in the use of the efficient tool, while in our model, an increase in the value of the efficiency of taxes leads to an increase in the use of the inefficient options.

We then study to the relationship between the efficiency of taxation and inequality. Our results show that inequality is increasing in the efficiency of taxation. The

main intuition behind this result is twofold: an increase in the efficiency of taxation widens the gap between the after-tax income of the voters that receive transfers and those that do not; and increased efficiency also implies that politicians will select higher tax rates more often. It is the second effect that distinguishes our results from those that are obtainable in an extended version of Myerson (1993): allowing for distortionary taxation in his model, because all the money in the economy is taxed away by assumption, the only reason why inequality is positively related to the efficiency of taxes is because this increases the size of the support of the distribution of transfers.

We then allow for convex distortions. Our results show that introducing convex distortions does not modify how politicians redistribute across voters but that it does lead politicians to refrain from choosing too high a tax rate.

1.2 Related literature

1.2.1 Models of redistributive politics

The present paper belongs to the current strand of literature of positive models of redistributive politics. This literature starts with Myerson (1993).

He models redistributive politics as an electoral game between two candidates that make simultaneous, independent and binding redistribution promises to voters. This game is very similar to the well-known Colonel Blotto game⁴, that is a game between two players that have to decide simultaneously how to divide their troops among n battle fields. Myerson simplified the analysis by allowing for an infinite number of voters. This simplification made it possible to address the effect of electoral rules on redistribution and inequality. Following Myerson, Lizzeri (1999) used a similar model to explain the persistence of budget deficits. Sahuguet and Persico (2006) and Kovenock and Roberson (2006) built on this model of pure redistribution to analyze situation in which voters' loyalties vary across parties. Laslier and Picard (2002) and Roberson (2006a) study in depth the same game but let number of voters be finite.

Lizzeri and Persico (2001 and 2005) extend Myerson's model of redistributive politics to give politicians the possibility of using the taxed income to provide an economy-wide public good. When the public good is more efficient than redistribution, there exists a trade-off between efficiency and targetability. Roberson (2006b)

⁴The classic on the Colonel Blotto game is Gross and Wagner (1950).

develops a similar model in the context of a federal economy with a finite number of voters. In all these models, there is a binary choice between redistribution and public good provisions. We thus extend these models to a more general set-up that gives politicians a continuum of policy choices.

Another related paper is Dekel, Jackson, Wolinsky (2006a)⁵. They introduce a dynamic, alternating offers electoral game between two parties and a population of voters which may have a preference for one of the two parties. The two candidates compete for votes by making alternative public offers. Each candidate can make offers only up to the budget it has at its disposal. The main difference is thus that promises are not made in a simultaneous ways. They show that the outcome of the game involves substantial spending by parties and that this outcome is affected by the voters' preferences. The key behind the derivation of this equilibrium is that each candidate, when called to make a new offer, is constrained to make an offer that is higher or equal to the one he made in his previous turn. This immediately implies that the extension of their game to one like the one analyzed in this paper, in which parties compete in terms of both spending promises and tax rates is problematic, as the authors acknowledge themselves (Dekel et al. 2006a, p. 15)

1.2.2 The efficiency of taxation and the size of government

Our paper also contributes to the strand of the literature that analyzes the relationship between the size of government and the efficiency of taxation. As we said above, this strand is relatively narrow. The literature on the determinants and the composition of government spending is relatively larger⁶, but has typically relegated to the sidelines the effects of changes in the efficiency of the government's instruments. Very few papers focus explicitly on the relationship between efficiency and the characteristics of government.

Becker and Mulligan (2003) provide a model that shows that increases in the efficiency of taxation lead to less pressure against the growth of government. Our result about the positive relationship between the efficiency of taxation and the probability that candidates select higher tax rates is consistent with their view. More importantly, Becker and Mulligan use a political economy model with pressure groups to derive the result that an increase in the efficiency of taxation may be

⁵See also Dekel, Jackson and Wolinsky (2006b)

⁶Papers in this area include Kau and Rubin (1981), Grossman (1987), Wilson (1990), Persson and Tabellini (1999), Persson, Roland and Tabellini (2000) and Milesi-Ferretti, Rostagno and Perrotti (2002).

welfare reducing (at least for those taxpayers that are unorganized). We do not need to assume the existence of two different types of voters to obtain that increases in the efficiency of taxation are welfare-reducing.

1.3 Outline of the paper

Section 2 describes the model. Section 3 solves for the equilibrium and shows how distortions influence the equilibrium choice of taxes and of redistribution. Section 4 relates the equilibrium of the model with the size of the government spending and the welfare of voters as a function of tax distortions. Section 5 discusses several extensions of the base-line model in several directions: non-linear distortions, comparative politics, and heterogeneous income distribution.

2 The Model

We analyze a model of redistributive politics based on Myerson (1993). Candidates make promises to voters to gain their votes. Those promises are not restricted in any way except that they must satisfy a budget constraint, that comes from the taxation of voters income. The main originality of the model is the addition of the fact that taxation is distortionary and thus that candidates can only redistribute a share of the income they taxed.

2.1 Economy and players

There are two candidates, 1 and 2. The electorate is made of a continuum E of total mass 1. Each voter is endowed with one unit of money. Candidates can tax voters' endowment and then make promises. These promises are subject to an economy-wide budget constraint: candidates must make balanced-budget policy pledges. These promises are binding.

The basic premises of the model are twofold. First, a candidate has to tax everybody the same way (by setting a nation-wide tax rate) but can use the money collected to make individual promises. Second, taxation is distortionary. There is a cost to collect taxes: only part of the taxed income is available for redistribution. When a candidate chooses a tax rate t , every voter is left with $(1 - t)$ and the budget for redistribution is $\lambda t \leq t^7$

⁷ λ can be interpreted as collection costs. Other distortions due to taxation, such as those arising from incentive problems impacting individual labor supply decisions, are also of interest. In section

The timing of the game is as follows:

1. Candidates, simultaneously and independently, choose a tax rate and make binding and credible promises to voters with the money collected;
2. After observing the two candidates' offers, voters cast their ballot for the candidate that has offered them the highest utility;
3. Vote shares determine the electoral outcome and payoffs are realized.

We use a reduced-form mapping from the legislature to the executive: the probability that the policy chosen by a candidate is the implemented one is an increasing function of the vote share of that candidate. This justifies in turn the fact that each voter votes sincerely, that is, casts his ballot in favor of the candidate that promises him the greatest utility. We solve the game under proportional representation (PR): candidates maximize the share of the total votes.

2.2 Game and candidates' strategies

A pure strategy for a candidate specifies the tax rate he chooses, and in the event he chooses a positive tax rate, it also specifies a promise of a transfer to each voter. Formally a pure strategy is a tax rate t and a function $X : E \rightarrow [0, +\infty)$, where $X(e)$ represents the consumption promised to voter e . The function X must satisfy the following balanced budget condition. $\int_e X(e) de = (1 - t) + \lambda t$ and $X(e) \geq 1 - t$, (in that case, $X(e) - (1 - t)$ represents the transfer promised to voter v , after taxes).

We focus on the case of distortions that are not too large, i.e. $\frac{1}{2} < \lambda \leq 1$. Indeed, for $\lambda \leq 1/2$, politicians would not tax voters. For $\lambda \geq 1$, politicians would tax all of the voters' income and we would be back in the framework of Myerson (1993). For the distortions that we are interested in, i.e. $\lambda \in (\frac{1}{2}, 1)$, there is no equilibrium in pure strategies.⁸ The intuition for this is as follows. Suppose one candidate were to play a pure strategy, that is, select a tax rate and an associated redistribution plan with probability 1. Then the other candidate, knowing this, could easily device a plan (that is, select a tax rate and an associated redistribution

5, we look at more general distortion functions.

⁸Whereas candidates randomize in equilibrium, voters observe their *realized* promise by each candidate. Voter e thus votes for candidate i if and only if

$$X_i(e) > X_{-i}(e).$$

plan) that gives him more than half of the vote, which is the equilibrium vote share of each candidate. In equilibrium candidates must therefore be randomizing over tax rate and the associated redistribution plans.

In equilibrium, both candidates are thus using mixed strategies. We study only symmetric mixed strategies in which candidates choose the tax rate according to the distribution function $\Upsilon(t)$, and then redistributes the money collected, net of distortions. The offer made to voter e , $X(e)$ is the realization from a common random variable with cdf $F_t : R^+ \rightarrow [0, 1]$, that depends on the chosen tax rate. F_t represents the empirical distribution of net transfers by candidate i to voters. The tax rate is thus the same for all voters, and voters are getting on average the same amount of money. This does not mean however that all the voters get the same amount of money ex-post: individual promises depend on the realization of an individual random draw from the distribution F_t .

3 Equilibrium

In this section, we solve for the equilibrium of the taxation and redistribution game. We show that candidates randomize between tax rates according to the continuous distribution function $\Upsilon(t) = t^{\frac{2\lambda-1}{\lambda}}$.

The redistribution function F_t turns out to be very simple: its support contains only two points. For a given tax rate and a corresponding budget, a candidate promises no additional transfer to 50% of the voters (they are thus promised $1 - t$) and promises to the other 50% twice the money collected per capita (these voters are promised $1 - t + 2\lambda t$).

Proposition 1 *Assume that candidates can choose any tax rate $t \in [0, 1]$, and then redistribute the revenue from taxation λt among the voters. Then the following strategies constitute an equilibrium of the electoral game:*

Candidates randomize across tax rates using the distribution function $\Upsilon(t) = t^{\frac{2\lambda-1}{\lambda}}$.

For a tax rate t , the candidate promises $(1 - t)$ to 50% of the voters and $(1 - t) + 2\lambda t$ to 50% of the voters.

$$F_t(x) = \begin{cases} 0 & \text{for } x < 1 - t \\ 1/2 & \text{for } 1 - t \leq x < 1 - t + 2\lambda t \\ 1 & \text{for } x \geq 1 - t + 2\lambda t. \end{cases}$$

Proof

The proof is by construction.

We first check that if the other candidate uses the proposed strategy, the vote share associated with any tax rate and its equilibrium redistribution plan is $1/2$. The redistribution plan associated with tax rate t promises a utility of $(1 - t)$ to 50% of the voters. They vote for this candidate when the other candidate proposes a higher tax rate and they get no additional promise - this happens with probability $\frac{1}{2}(1 - \Upsilon(t))$. The plan also promises $(1 - t) + 2\lambda t$ to the other 50% of the voters. They vote for this candidate if the other candidate makes no additional promises - this happens with probability $\frac{1}{2}$ because, given that $1 - t + 2\lambda t > 1$, $1 - t + 2\lambda t > 1 - \tilde{t}$ for any $\tilde{t} \in [0, 1]$ - or if the other candidate makes additional promises but has a lower tax rate - this happens with probability $\frac{1}{2}\Upsilon(t)$. The total vote share of a candidate using a tax rate t and the associated redistribution plan is thus

$$\frac{1}{2} \left(\frac{1}{2} (1 - \Upsilon(t)) \right) + \frac{1}{2} \frac{1}{2} (1 + \Upsilon(t)) = \frac{1}{2},$$

as claimed.

To complete the proof, we still have to prove that a candidate can not improve on the redistribution plan prescribed by the equilibrium strategy.

Let $W^*(x)$ denote the equilibrium probability of winning a vote with an offer of x . This function summarizes all the information about the promises made by the other candidates to voters. For a given tax rate, say t , a candidate leaves to everybody $(1 - t)$ units of money and has a budget, net of distortions, of λt to distribute.

This candidate makes transfers to voters to maximize his vote share subject to the budget constraint:

$$\text{Max}_{F_t} \int_{1-t}^{+\infty} W^*(x) dF_t(x) \text{ s.t. } \int_{1-t}^{+\infty} (x - (1 - t)) dF_t(x) = \lambda t$$

The Lagrangian associated to this problem is:

$$\mathcal{L} = \int_{1-t}^{+\infty} \{W^*(x) + \gamma[\lambda t + (1 - t) - x]\} dF_t(x).$$

To prove that the equilibrium redistribution is optimal, we use this Lagrangian

in two different ways. We first argue that the support of F_t must be such that all the promises in this support maximize \mathcal{L} . This defines a linear relation between the promises used in equilibrium and the probability of winning a vote associated with this promise. We then explicitly calculate $W^*(x)$ from the strategy used by the other player. Putting together these two pieces of information, we conclude that a tangency condition between $W^*(x)$ and the linear function defined above characterize the optimal promises. We then check that this condition leads to the proposed equilibrium redistribution.

Let γ^* be the equilibrium Lagrange multiplier. Then we must have that $W^*(x) - \gamma^*x$ is maximal and constant on the support of F_t . This immediately implies that there must be a linear relation between x and $W^*(x)$ on the support of F_t . The intuition is simple and follows Lizzeri and Persico (2005): $W^*(x)$ represents the expected benefits of making a promise of x dollars (for a given tax rate t). At an optimum, this benefit must be equal to the shadow cost of the budget constraint, which corresponds to the opportunity cost of a dollar. This opportunity cost is linear in x ; therefore W^* also needs to be linear in x on the support and must lie below this line outside of the support.

Consider now one of the two candidates. Suppose he chose tax rate t . The candidate has λt to make promises to voters in order to maximize his vote share. Let us now derive the winning function $W^*(x)$ associated with the equilibrium strategy used by the other player. It represents the probability of winning the vote of a given voter when you make him a promise of x .

$$\begin{aligned}
 W^*(x) &= \frac{1}{2} \left(1 - (1-x)^{\frac{2\lambda-1}{\lambda}} \right) \text{ when } x \leq 1 \\
 W^*(x) &= \frac{1}{2} \left(1 + \left(\frac{x-1}{2\lambda-1} \right)^{\frac{2\lambda-1}{\lambda}} \right) \text{ when } 1 \leq x \leq 2\lambda.
 \end{aligned} \tag{1}$$

This probability of winning function is convex for $x \leq 1$ and concave for $x \geq 1$. See figure 2 for an example of such a function.

Given the observation that when a politician optimizes his promises, the probability to win a vote with a given promise must be linear in the transfer. This may seem inconsistent to ask that this function to be linear and to be convex then concave as defined in Eq. 1. In fact, it implies that the optimal promises must at the same time belong to the W^* curve and be on this line. This defines a tangency

condition that characterizes the optimal promises.

In equilibrium, a candidate who chooses a tax rate of t , chooses to make no additional transfer to some of the voters. To see this, it is enough to note that, if the candidate would choose to make promises of at least $1 - t + \varepsilon$ to all voters, it would be better to choose a lower tax rate that leaves $1 - t + \varepsilon$ to everybody and since a lower tax rate leads to less distortions, this would be more efficient.

The best way to redistribute money is thus found by drawing a line starting at $(1 - t, W^*(1 - t))$ and ending at a point on the W^* curve and choosing the line with the largest slope – because this maximizes the efficiency of redistribution. Intuitively, the slope represents “the bang for the buck” of a given promise. By construction, there is no way to use money more efficiently than by randomizing between promises on this line.

This means that the optimal way to use the funds is to choose the promise that maximizes the probability of winning per dollar promised. To find this, it is enough to find the line that starts at $((1 - t), W^*(1 - t))$ and that is tangent to the function $W^*(x)$ for $x \geq 1$. Since by construction any other promise $(x, W^*(x))$ would be below that tangent, it would not be optimal to use it. To conclude the proof, we need to show that the promise $(1 - t) + 2\lambda t$ satisfies this tangency condition.

The slope of the function W^* at x is $\frac{1}{2\lambda} \left(\frac{x-1}{2\lambda-1}\right)^{\frac{\lambda-1}{\lambda}}$. Hence

$$W^{*'}((1 - t) + 2\lambda t) = \frac{1}{2\lambda} t^{\frac{\lambda-1}{\lambda}}.$$

The slope of the line going from $((1 - t), W(1 - t))$ to $((1 - t) + 2\lambda t, W((1 - t) + 2\lambda t))$, is

$$\frac{W((1 - t) + 2\lambda t) - W(1 - t)}{(1 - t) + 2\lambda t - (1 - t)} = \frac{t^{\frac{2\lambda-1}{\lambda}}}{2\lambda t} = \frac{1}{2\lambda} t^{\frac{\lambda-1}{\lambda}}.$$

This completes the proof. ■

Note that when λ is close to $1/2$, the distribution function $\Upsilon(t)$ assigns most of the probability mass to tax rates which are very close to 0. As λ increases, the concavity of Υ decreases until it becomes a straight line, for $\lambda = 1$. Thus, when there are no collection costs, the two candidates randomize between tax rates according to a Uniform distribution on $[0, 1]$.

Figure 1 plots Υ for three levels of distortions. The most concave curve represent the case of large distortions, only $\lambda = 55\%$ of tax collected can be used for redistrib-

ution.. The intermediate curve represents intermediate distortions ($\lambda = 70\%$). The straight line represents the case when there are no distortions, $\lambda = 100\%$ of the tax collected can be redistributed.

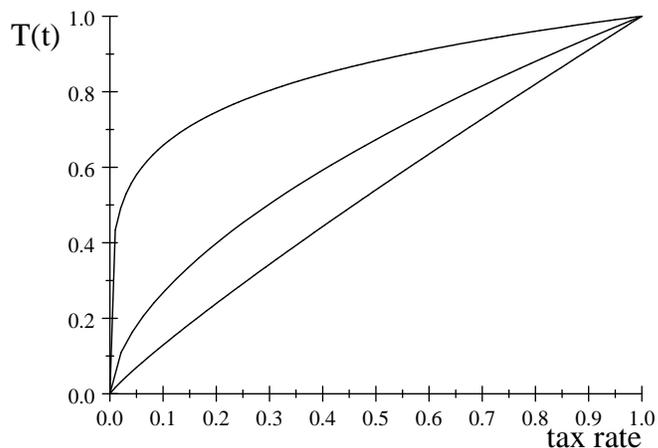


Figure 1: CDF of tax rates choices for 3 levels of distortions.

To illustrate the equilibrium, let us look at a concrete example.

Assume the distortions are $\lambda = 0.8$ (only 80% of the tax money collected can be used for redistribution, 20% is wasted in the collection process)

The equilibrium distribution of tax rates follows $\Upsilon(t) = t^{\frac{3}{4}}$.

The winning function W is

$$W^*(x) = \frac{1}{2} \left(1 - (1 - x)^{\frac{3}{4}} \right) \text{ when } x \leq 1$$

$$W^*(x) = \frac{1}{2} \left(1 + \left(\frac{x - 1}{0.6} \right)^{\frac{3}{4}} \right) \text{ when } 1 \leq x \leq 1.6$$

Figure 2 depicts the winning function W^* , and shows the tangency condition for $t = 0.6$ and $t = 0.2$.

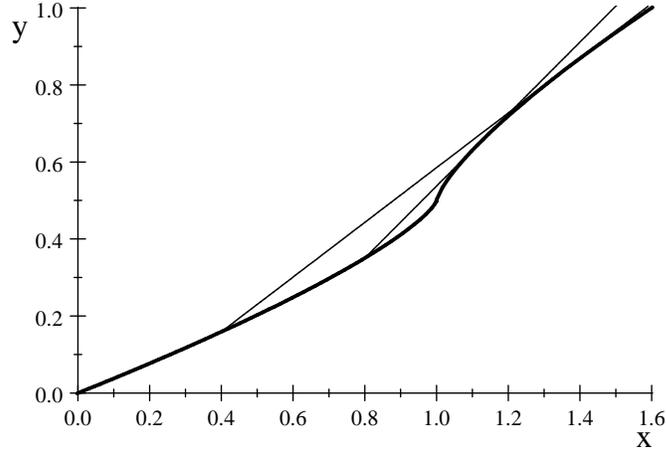


Figure 2: example with $\lambda = 0.8$

4 Size of government – welfare – inequality

4.1 Welfare and the size of government

Given the above equilibrium, it is interesting to examine how the efficiency of taxation impacts the politicians' strategic use of taxes and the voters' ex-ante welfare.

The positive literature on taxation is interested in the size of government and the amount of loss in the economy due to the distortions of the tax system. Ex-ante, the expected size of tax revenues is:

$$\begin{aligned}
 T(\lambda) &= \int_0^1 t d\Upsilon t = \int_0^1 t \left(\frac{2\lambda - 1}{\lambda} \right) t^{\frac{\lambda-1}{\lambda}} dt \\
 &= \frac{2\lambda - 1}{3\lambda - 1}
 \end{aligned}$$

whereas the expected size of redistribution is

$$\begin{aligned}
 R(\lambda) &= \lambda T(\lambda) \\
 R(\lambda) &= \lambda \cdot \frac{2\lambda - 1}{3\lambda - 1}
 \end{aligned}$$

We thus have:

Proposition 2 *For $1/2 < \lambda \leq 1$, the expected size of redistribution and the expected amount of taxes collected increase with efficiency of taxes. They are equal to 0 when*

$\lambda = 1/2$ and increase to $1/2$ when $\lambda = 1$.

We can also derive the expected welfare in this economy. The welfare measure that we use is the amount of money that is owned by voters after the political process, $V(\lambda)$:

$$\begin{aligned} V(\lambda) &= 1 - T(\lambda) + R(\lambda) = \left(1 - \frac{2\lambda - 1}{3\lambda - 1}\right) + \lambda \frac{2\lambda - 1}{3\lambda - 1} \\ &= \frac{2\lambda^2}{3\lambda - 1} \end{aligned}$$

Turning to the expected deadweight loss due to the political process, this is given by:

$$D(\lambda) = 1 - V(\lambda) = 1 - \frac{2\lambda^2}{3\lambda - 1}$$

We thus have:

Proposition 3 *For $1/2 < \lambda \leq 1$, welfare is U-shaped. It is maximum for $\lambda = 1/2$ or $\lambda = 1$. For small λ , an increase in the efficiency of taxation decreases the welfare, for larger values of λ , an increase in the efficiency of taxation increases the welfare. The minimum welfare is reached for $\lambda = 2/3$.*

Deadweight costs behave the opposite way as welfare. They are hump-shaped and reach their maximum at $\lambda = 2/3$.

Figure 3 depicts the size of taxes, size of redistribution, the welfare and the deadweight costs.

The comparative statics on the size of redistribution are not very surprising and are in line with the previous literature of positive theories of taxation. The result about the welfare is more interesting. We show that an increase in the efficiency of tax collection can have the perverse effect that it makes redistribution a more attractive tool. This can lead candidates to use it more aggressively (in the sense that the probability that they select a higher tax rate is higher) even though it is inefficient.

This effect is reminiscent of the idea developed by Becker and Mulligan (2003) in a very different framework. The case for collecting taxes in the most efficient way seems intuitive and rather obvious. In particular, for a given total government spending, the welfare in the population is maximized when the deadweight

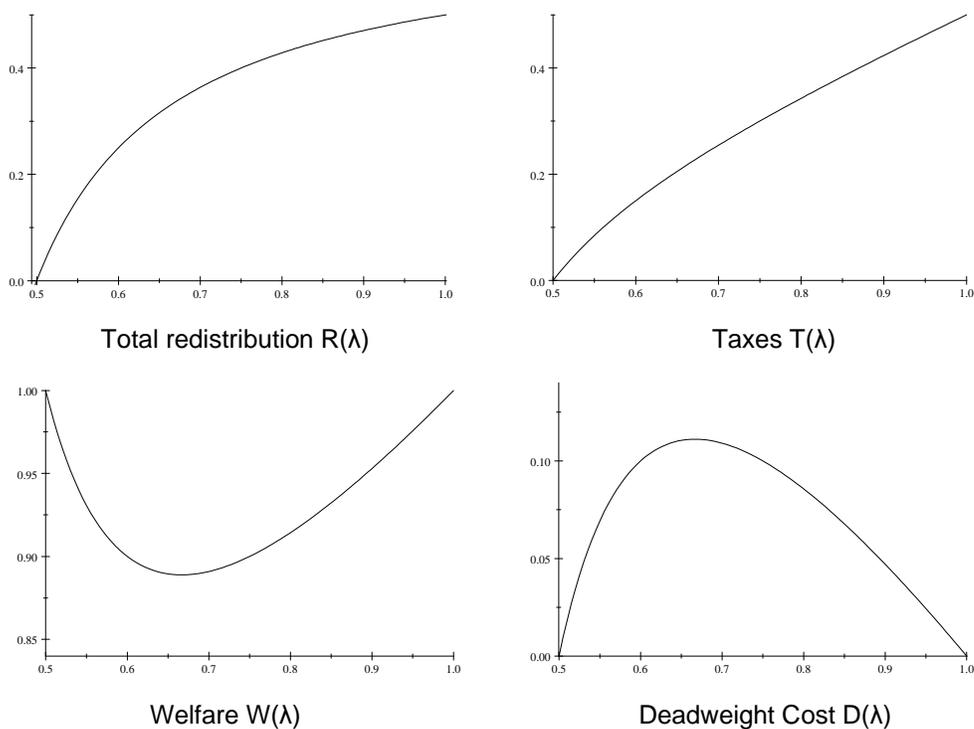


Figure 3: Revenues, redistribution and welfare as a function of λ

loss coming from tax collection is minimized. However, when government spending is endogenously determined (as in our model) as a consequence of electoral competition, the effect of a more efficient tax system is ambiguous. Our model makes this point in a very simple way.

It is also interesting to note than in the model of Lizzeri and Persico (2001), an increase in the value of the public good would always lead to an increase in welfare. The reason is that, in their model, an increase in the value of the public good leads to an increase in the use of the efficient tool, while in our model, an increase in the value of the efficiency of taxes leads to an increase in the use of the inefficient options.

4.2 Inequality

Finally, we wish to analyze how the efficiency of tax collection is linked to inequality in this model. In a model with distortionary taxation, the first-order effect of taxes comes from the distortions. However, it is still interesting to analyze the inequality that results from redistribution, even if it is only a second order effect.

Given that there are only two levels of after-tax incomes in equilibrium, we start by a very simple measure of inequality that is then easily adapted to compute the usual Gini coefficient. Let T_1 be the expected post-election net income to the group that does not receive any targeted transfers. Let T_2 be the expected post-election net income to the group that receives $2\lambda t$ after tax. Then:

$$\begin{aligned} T_1 &= \int_0^1 (1-t) d\Upsilon t = \int_0^1 (1-t) \left(\frac{2\lambda-1}{\lambda} \right) t^{\frac{\lambda-1}{\lambda}} dt \\ &= 1 - \frac{2\lambda-1}{3\lambda-1} \\ T_2 &= \int_0^1 (1-t+2\lambda t) d\Upsilon t = 1 + (2\lambda-1) \frac{2\lambda-1}{3\lambda-1} \end{aligned}$$

A cross-group measure of inequality is:

$$\Delta T = 2\lambda \frac{2\lambda-1}{3\lambda-1}$$

It is immediate to check that $\frac{\partial \Delta T}{\partial \lambda} > 0$ for all $\lambda > 1/2$. Thus inequality is always increasing in the efficiency of taxation.

If we use the standardized Gini coefficient, i.e. twice the area between the diagonal – normalized at $1/2$ – and the Lorenz curve computed using income shares – so that the area under the Lorenz curve is also between 0 and $1/2$ – inequality is given by:⁹

$$Gini = 2 \left(\frac{1}{2} - \frac{T_1}{T_1 + T_2} \right) = 1 - \frac{1}{2\lambda}$$

which is also increasing in λ .

We thus have:

Proposition 4 *Inequality increases with the efficiency of taxation, λ .*

Remark that this result is due to two effects. First of all, as taxation becomes less distortionary, inequality increases because the gap in after-tax income between the group that receives transfers and that does not increases. Secondly, inequality increases with improvements in the efficiency of taxes because this pushes politicians

⁹See Laslier and Picard (2002, p. 122)

to choose a mixed strategy that puts more weight on higher tax rates. It is the second effect that distinguishes our results from those that are obtainable in an extended version of Myerson (1993). Allowing for distortionary taxation in his model, because all the money in the economy is taxed away by assumption, the only reason why inequality is positively related to the efficiency of taxes is because this increases the size of the support of the distribution of transfers.

5 Extensions

5.1 Non-linear distortions

So far we assumed that the distortions due to taxation were linear in the taxes collected: $\beta(t) = t$. This is a reasonable assumption if we believe that this represents the costs of collecting taxes. However, distortionary taxation can also be caused by incentive issues. In particular, if income comes from a labor decision, the distortions will be non-linear: $\beta(t) \neq t$.

To solve for equilibrium, the tangency condition used in the proof of theorem 1 would now read:

$$\frac{\Upsilon(t)}{2\lambda\beta(t)} = \frac{1}{2} \frac{\tau(t)}{2\lambda(\partial\beta/\partial t) - 1}.$$

This leads to the following ordinary differential equation:

$$\frac{\tau(t)}{\Upsilon(t)} = \frac{2\lambda(\partial\beta/\partial t) - 1}{\lambda\beta(t)}.$$

The solution of this homogenous first-order ODE is :

$$\Upsilon(t) = C \exp \int_0^t \frac{2\lambda(\partial\beta/\partial t) - 1}{\lambda\beta(t)} dt.$$

Note that for $\Upsilon(\cdot)$ to be increasing, a necessary condition is that $2\lambda(\partial\beta/\partial t) \geq 1$. In case the distortions are too large locally, the tax rate associated would not be used in equilibrium. Note that this means that when distortions get more convex (such that the previous condition binds), politicians do not use the highest tax rates. We thus have:

Proposition 5 *The highest tax rate used with positive probability in equilibrium is decreasing in the degree of convexity of tax distortions, $\beta(t)$.*

We now turn to an example of convex taxation that we can solve explicitly.

Suppose for instance that a given citizen has to work to earn his income. A unit of time spent working leads to a unit of income produced. The opportunity cost of work (cost of effort or cost of foregone labor) is $C(x) = \frac{x^2}{2}$.

The citizen confronted with a tax rate of t solves :

$$Max_x (1-t)x - \frac{x^2}{2}$$

This leads to an income gross of tax of $x^* = (1-t)^a$. The tax collected would thus be $t(1-t)$. The distortion due to incentives is thus non-linear in the tax rate. If we keep the assumption of a collection cost, the budget to be redistributed would be $\beta(t) = \lambda t(1-t)$. We get:

$$\Upsilon(t) = C \exp \int_0^t \frac{2\lambda(1-2t) - 1}{\lambda t(1-t)} dt$$

$$\begin{aligned} \frac{2\lambda(1-2t) - 1}{\lambda t(1-t)} &= \frac{2}{t} - \frac{2\lambda t + 1}{\lambda t(1-t)} \\ &= \frac{2}{t} - \frac{2}{1-t} - \frac{1}{\lambda t(1-t)} \\ &= \frac{2}{t} - \frac{2}{1-t} - \frac{1-2t+2t}{\lambda t(1-t)} \\ &= \frac{2}{t} - \frac{2}{1-t} - \frac{1-2t}{\lambda t(1-t)} - \frac{2}{\lambda(1-t)} \end{aligned}$$

Hence, $\Upsilon(t) = C(t^{\frac{2\lambda-1}{\lambda}}(1-t)^{\frac{2\lambda+1}{\lambda}})$. Given that $2\lambda(1-t) - 1 = 0$ when $t \geq \frac{1}{2} - \frac{1}{4\lambda}$, we get the boundary condition: $F(1/2 - 1/(4\lambda)) = 1$. We can then get the constant C to obtain:

$$\Upsilon(t) = \frac{256t^{\frac{2\lambda-1}{\lambda}}(1-t)^{\frac{2\lambda+1}{\lambda}}}{\left(\frac{1}{\lambda}(2\lambda-1)\right)^{\frac{1}{\lambda}(2\lambda-1)} \left(\frac{1}{\lambda}(2\lambda+1)\right)^{\frac{1}{\lambda}(2\lambda+1)}}$$

When the distortions come only from incentives reasons, ($\lambda = 1$), the tax rates used are $t \in [0, 1/4]$ and $\Upsilon(t) = \frac{256}{27}t(1-t)^3$. The maximum tax rate used is 25%. This value does not come from a Laffer curve type of argument but from a comparison at the margin with the lower tax rates.

5.2 Comparative politics

The main result of the paper by Lizzeri and Persico (2001) is to show that different electoral systems give different incentives to candidates who are facing the trade-off between the targetability and the efficiency of various policies (redistribution or public good) in their framework. Their insight has been confirmed in another context by Persson and Tabellini (1999) who also give some empirical support to this notion. Given that in our model, the same trade-off between targetability and efficiency emerges in the analysis of distortionary taxation, it would seem natural that the same comparative politics exercise could be made.

However, when the policy choice is continuous (any tax rate can be chosen), plurality rule leads to non-existence of equilibrium. (in fact, it is enough to have 3 choices of tax to get this result) It is therefore not possible to extend the analysis of Lizzeri and Persico (2001) to a model with more than two options.

5.3 Heterogeneous income distribution

In the previous models of redistributive politics, on which we are building, the issue of heterogeneous initial income distribution was never important. In Myerson's and in Lizzeri and Persico's models, candidates have the incentive to tax all the money in the economy to use it to get elected. In Myerson's model, he uses all this budget to make transfer to buy votes. In Lizzeri and Persico, similarly, whether a candidate decides to offer the public good, or to make targeted promises, it is optimal to first tax every body at a rate of 100%.

In our model, the initial distribution of income would play an important role. First, in our model, the efficient option is the status-quo, $t = 0$. Since the status-quo would be compared to the equilibrium distribution of promises, the initial income distribution would matter. Some issues would come into consideration. First, the tax systems that could be used could, in principle, be very complicated. In particular, the assumption of a unique tax rate, used in the present model, would not be natural. Progressive or regressive tax schemes could be used and would complicate the analysis. Carbonell-Nicolau and Ok (2006) analyze a model along those lines. Taxation is not distortionary and redistribution issues are left aside. Their focus is on how to collect a given government budget by competing with income tax schemes proposed to voters with heterogeneous incomes. This is not a very tractable model and their main result is on the existence of equilibrium rather than on the properties

of the equilibrium. Adding redistribution and distortionary taxation looks like a difficult problem.

Another issue that arises with heterogeneous income distribution is the issue of information. Do candidates know the income of each voter and can target the promises perfectly as a function of income or do candidates only know the distribution of income? In particular, if they knew only the distribution, that would make the status-quo much more attractive since it would not only represent an efficient option but would also be difficult to defeat through targeted promises, since the mixed strategy nature of the distribution of promise would also be present. The analysis of such a model is beyond the scope of the present paper and is left for future research.

6 Conclusion

In this paper, we analyzed a model of redistributive politics with distortionary taxation. The first contribution of the paper was to show how the strategic use of distortionary taxation can be understood along an efficiency/targetability trade-off. We show that, even though distortions are increasing in the level of taxation and therefore high taxes are an inefficient instrument of government policy, high taxes are still used by politicians in equilibrium because it gives them the opportunity to redistribute the collected budget to target promises to voters for electoral success.

We show that the welfare of society is not a monotonic function of the efficiency of the tax system. Increasing the efficiency of the tax system by reducing the distortions can have perverse effects since a more efficient tax system gives incentives to politicians to use higher taxes more often.

The above result also provide a rationale for why we should expect the size of government to be positively correlated with the efficiency of the tax system. Notice that our rationale does not rely on there being any ex-ante heterogeneity in the polity, contrary to previous contributions that have analyzed this issue, such as Becker and Mulligan (2003).

From a theoretical perspective, our model has extended both the model of pure redistribution of Myerson (1993) and the model of Lizzeri and Persico (2001). We generalized the trade-off between targetability and efficiency of public good provision and monetary promise to the case of distortionary taxation. It would be interesting to see how the comparative politics results derived in these models would be changed

in the context of distortionary taxation. In particular, understanding how the number of parties and the electoral rules influence the size of government, the amount of distortions and the general efficiency of the economy would be useful extensions. The question of the number of parties seems promising but is left for future research. The question of comparing proportional representation and a winner-take-all system seems more complicated. There is no equilibrium in the game where getting one more vote is enough to get elected. It is thus necessary to deal first with the question of the modelling a majoritarian system in such a context.

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