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ABSTRACT

How Market Fragmentation Can Facilitate Collusion*

When regulated markets are liberalized, economists always stress the benefits of fragmenting existing capacities among more firms. This is because oligopoly models typically imply that a larger number of firms generates stronger competition. I show in this paper that this intuition may fail under collusion. When individual firms are capacity constrained relative to total demand, the fragmentation of capacity facilitates collusion and increases the highest sustainable collusive price. This result can explain the finding in Sweeting (2005) that dramatic fragmentation of generation capacity in the English electricity industry led to increasing price cost margins.

JEL Classification: J1 and J11
Keywords: Bertrand-Edgeworth competition, collusion, industry restructuring and market fragmentation

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1 Introduction

One of the most robust results in economic theory is that market power tends to be reduced when the supply side of a market becomes fragmented. In static homogeneous goods models a replication of the industry will lead to price moving closer to marginal cost, reaching marginal cost in the limit of infinite replication. As long as splitting up given capacity among more firms does not have efficiency effects we should therefore expect lower prices when an industry becomes more fragmented[1] Following Scherer and Ross (1990) market fragmentation is also taken as the most important factor undermining the ability of firms to collude. A recent antitrust textbook (Motta 2004, page 142) states unambiguously: "Other things being equal, collusion is the more likely the smaller the number of firms in the industry."

Such strong predictions of theory invariably find their expression in policy advice. Especially in industries that have recently undergone regulatory reform and liberalization it has been a mantra of economists to call for the breakup of firms before liberalization. One example is the liberalization of electricity generation. In the early 1980s economists in the US advocated a cautious approach to electricity deregulation pointing to persistent high effective concentration in electricity generation (see Golub and Schmalensee 1984). The Spanish electricity liberalization 1998 was severely criticized because the government allowed considerable consolidation by a state owned generation company before privatization and liberalization occurred (see Kühn and Regibeau 1998; Arocena, Kühn, and Regibeau 1999). Similar criticism was directed at the pioneering British electricity liberalization experiment (see Henney 1987, Sykes and Robinson 1987). In calibrations of supply function equilibrium in the British electricity spot market Green and Newbery (1992) concluded that the creation of a fifth competitor would have significantly lowered price cost margins and noticeably enhanced welfare (Green 1995). These criticisms together with evidence for persistent market power finally made the electricity regulator for England and Wales force divestitures on the two largest firms in the industry. They led to the progressive fragmentation of electricity generation in England and Wales between 1995 and 2000. The industry saw the effective entry of three new significant players into the market based on diversified assets and a dramatic fall in the Herfindahl-Hirshman Index from 2264 to 1000. Surprisingly, these developments did not have the expected impact on price cost margins. Sweeting (2005) shows that market power measured in terms of price-cost margins did not fall, but slightly increased, during this period. He conjectures that one possible reason for this effect is the impact of tacit collusion.[2]

In this paper I resolve this apparent puzzle by re-examining homogeneous goods models with capacity constraints. I show that in markets with capacity constraints collusion is typically facilitated when a given amount of capacity is divided among a larger number of firms. In fact, under the most realistic assumptions on the relationship between capacity and demand, the highest achievable price will monotonically rise as the industry becomes more fragmented.

We derive this result for a set of simple models with n symmetric firms, each owning $\frac{1}{n}$th of the total capacity K in the market. Firms compete in prices but may be constrained by their capacity so they cannot serve all of the demand. The intuition for the result that increased market fragmentation at given total capacity enhances collusion is relatively simple: Suppose no firm in the industry can serve

[1] Replicating both the demand and the production side of a market is equivalent in our setting to keeping the demand side constant and splitting up the existing assets over more firms.

[2] Dissatisfaction with the performance of the industry has led to futher reform. In 2001 England and Wales spot market was replaced by New Electricity Trading Arrangement (NETA), which was hoped to produce more competitive outcomes.

all the demand at the monopoly price. Then any firm deviating from a collusive price will be capacity constrained. The gain from deviation in a symmetric model is then proportional to the capacity share of the firm. If firms can engineer some punishment that does not disappear at a rate at least as fast as $\frac{1}{n}$ in the number of firms, the benefits of deviating from collusion decrease relative to the losses from punishment for deviations.

In the collusion literature punishments for deviations are generated endogenously by inducing more competitor behavior after deviations from collusion. The standard model for collusion with capacity constraints is an infinitely repeated Bertrand-Edgeworth game, in which customers that cannot be served by a firm are rationed according to some rationing rule. (see Brock and Scheinkman 1985, Lambson 1987, Davidson and Deneckere 1990). In section 3, we show that in such models punishments from deviations always have the property that they becomes small at a rate less than $\frac{1}{n}$. The reason is that competition is intensified in the punishment equilibrium when n is higher: when a smaller firm deviates from punishments a larger proportion of the total capital K will be fully utilized. Hence, punishment profits fall more than proportionately as n increases and market fragmentation strictly facilitates collusion. The highest sustainable collusive price will therefore increase as markets become more fragmented as long as there is no firm that can serve the whole market at the monopoly price.

Capacity fragmentation will reduce the ability to collude as long as a single firm can serve the full monopoly output. However, when fragmentation is increased sufficiently, collusion will eventually become easier because small enough firm s will always be constrained relative to market demand. In fact, in many cases collusion in a market with very fragmented capacity will be easier than in a homogeneous goods duopoly model of collusion without capacity constraints.

In section 4, I demonstrate that these basic results are extremely robust to model specification. It is preserved for a wide range of rationing schemes and carries over to all types of cost functions for which the distribution of capacity between firms does not affect the efficiency of production. I also show that the basic insight continues to hold for asymmetric capacity holdings. In that case, fragmentation of the largest firm in the industry will always facilitate collusion as long as the largest firm could not serve the market at the monopoly price. This result is closely related to the analysis of coordinated effects of mergers in Compte, Rey, and Jenny (2002). In a model that allows for asymmetries in capacity holdings Compte et al. demonstrate that more symmetric capacity holdings tend to favor collusion. Our paper shows that the effect in Compte, Rey and Jenny (2002) should not necessarily be attributed to asymmetry of capacities: Even if complete symmetry is preserved, the splitting up of capacity among more firms will tend to facilitate collusion. It is the impact of fragmentation on collusion that makes mergers of the largest firm in the industry desirable. This is in contrast to other models of coordinated effects as that of Kühn (2004), where assets are modelled as product lines. When products are symmetrically distributed among more and more firms, Kühn and Rimler (2006) have shown that collusion gets more difficult - in contrast to the capacity case studied here. This is true although, in an asymmetric setting, mergers by the largest firm still make collusion harder in Kühn (2004). This effect is due purely to increased asymmetry between firms.

We complete section 4 with a discussion of how the analysis can be directly applied to electricity markets and the result of Sweeting (2006). In electricity markets competition can be approximated by competition in supply functions. We discuss, that our results directly carry over to that case. In fact, under supply function competition the results may hold even when firms are not capacity constrained in all states of the world.

The analysis in this paper suggest a reassessment of the empirical literature that has attempted to identify industry features that facilitate collusion from cross-sectional studies of cartels.[3] The results on the impact of the number of firms in the industry on collusion are surprisingly mixed. Posner (1970) and Dick (1996) found a positive relationship between the ability to collude (measured in cartel duration) and the number of firms in the industry. Remarkably, Posner found that among cartels with large numbers of members the percentage of cartels surviving for more than 6 years was larger. Other studies (Hay and Kelley, Asch and Seneca, and Suslow 2005) fail to find any statistically significant relationship between cartel duration and number of firms in the industry, while Frass and Greer (1977) find that a larger number of firms reduces the likelihood of collusion. Levenstein and Suslow (2006) suggest that large cartels with more than 10 members tend to survive only when there is active involvement of a trade association. While this is plausible given that the coordination problems for a large cartel are difficult to resolve, the presence of a trade association would not explain why so many very large cartels are successful. The results from collusion model with homogeneous goods but without capacity constrained would make such outcolmes appear highly unlikely. Our analysis suggests that the number of firms is a bad measure for a cross-sectional study. What really matters is to what extent the largest firm is constrained by its capacity to serve the market and to what extent overall capacity is sufficient to drive down the price.

In section 2, I use the simplest conceivable framework of a monopoly model to give an intuition for the result that capacity fragmentation will lead to more collusive outcomes. Section 3, specifically looks at a standard Bertrand-Edgeworth model with infinite horizon to derive complete comparative statics results for prices with respect to capacity fragmentation. Section 4 shows that this analysis is robust to a wide range of alternative model formulations. Section 5 concludes.

2 How Capacity Fragmentation Facilitates Collusion

Consider a single homogeneous good with a market demand function $D(p)$, which is strictly decreasing, log-concave, and has $\lim_{p\to\infty} D(p) = 0$. For the production of the homogeneous good there is a total amount of K units of capital available. There are n firms in the industry each owning $\frac{K}{n}$ units of capital. Each firm has the same cost function with constant marginal cost c up to capacity $\frac{K}{n}$. The unconstrained monopoly price in the industry, p^m, solves $\max_p \pi(p) = \max_p (p-c)D(p)$. To make the problem non-trivial we assume that there is sufficient capacity in the market to serve demand at the monopoly price: $D(p^m) < K$.

We consider Bertrand-Edgeworth games in which each firm i simultaneously sets a price p_i. The firm is committed to serve all customers up to its capacity constraint at price p_i. Buyers purchase from the firm with the lowest price first. If the demand for firm i exceeds capacity, buyers at that firm are rationed according to some rationing rule (e.g. efficient rationing or proportional rationing). If several firms set the lowest price, total demand is allocated equally among the firms charging that price.

We first consider a very stylized model of collusion in which the game just described is played only

[3] Levenstein and Suslow (2006) provide a recent survey.

once. However it is assumed that firms commit to some punishment $F(n)$, which is imposed on a firm that deviates from the collusive price p^c. If capacity constraints are never binding (in the sense that a single firm can serve the whole market at the collusive price for all n), a firm makes profits $\frac{1}{n}\pi(p^c)$ when setting p^c and earns up to $\pi(p^c) - F(n)$ if it undercuts p^c. Hence, collusion is sustainable if and only if:

$$\frac{n-1}{n}\pi(p^c) \leq F(n) \tag{1}$$

The gains from deviating and winning over the market share of all other competitors, $\frac{n-1}{n}$, has to be smaller than the punishment $F(n)$ in order to sustain collusion. Note that $\frac{n-1}{n}$ is increasing in n. The more firms in the market, the smaller the initial market share of a single firm and the greater the market share that is gained from a deviation. The smallest fine that would prevent deviation from a collusive outcome, $\underline{F}(n)$, is therefore *increasing* in n. Conversely, for any given fixed fine $F < \pi(p^m)$, collusion at p^m breaks down for a sufficiently fragmented market structure. Since $\pi(p^c)$ falls on $(0, p^m)$, the most profitable sustainable collusive price falls in n, once p^m is not sustainable anymore.

This fragmentation result is reversed when individual firms are capacity constrained relative to market demand at the collusive price. Instead of $\pi(p^c) - F(n)$ a deviating can make a profit of at most $(p^c - c)\frac{K}{n} - F(n)$. Hence, the incentive constraint becomes:

$$\frac{1}{n}(p^c - c)K - \frac{1}{n}\pi(p^c) \leq F(n) \tag{2}$$

The incentive to deviate from collusion is now proportional to $\frac{1}{n}$. This means the incentive to deviate from collusion falls as the market becomes more fragmented. The lowest fine at which collusion at p^m can be sustained, $\underline{F}(n)$, *decreases* in n. In fact $n\underline{F}(n)$ is constant. Conversely, for any fixed fine F, $\frac{1}{2}\left[(p^m - c)K - \pi(p^m)\right] > F > 0$, collusion is impossible for $n = 2$ but becomes possible as the market becomes fragmented enough. This means that the most profitable collusive price is initially strictly increasing in n until p^m is reached. To see this, consider the left hand side of incentive condition (2). Differentiating in p^c yields:

$$\frac{1}{n}\left[K - D(p^c) - (p^c - c)D'(p^c)\right], \tag{3}$$

which is strictly positive unless the firms are capacity constrained at p^c. Hence, for a given punishment F, the incentive constraint can be strictly relaxed by lowering the collusive price.

Why is there such a stark difference between binding and non-binding capacity constraints? In both models a firm that undercuts attracts the customers from all other firms. However, with capacity constraints the deviating firm can only serve a fixed number of consumers $\frac{K}{n}$. When n increases, total capacity K is distributed among more and more firms. Each firm becomes smaller relative to the market. But at the same time a firm can also only serve a smaller and smaller fraction of the customers. A deviation becomes less attractive in face of a fixed fine or even a fine that declines less than proportionately in $\frac{1}{n}$. Intuitively, capacity constraints mean that a single firm becomes small not only when playing the collusive strategy but also when deviating from it.

Most plausible modelling assumptions on the fine imply that $nF(n)$ is non-decreasing, so that collusion under capacity constraints becomes (weakly) easier as n increases. One form of collusive contract modeled above would be that each firms posts a bond that is forfeited after a deviation from the collusive price. For example, if the firm can borrow up to expected future earnings, the maximal bond that could be posted would be constrained by future earnings under collusion: $\frac{1}{n}\pi(p^c)$, so that $nF(n) = \pi(p^c)$. If the firm could borrow against an asset with fixed value only, the fine would be independent of n. There are other realistic scenarios in which we would expect $nF(n)$ to be approximately constant. For example, if all that can be seized from a firm after a deviation are its assets, the liquidation value of a single firm should be expected to be approximately $1/n$ of the total liquidation value of all assets in the industry.

However, in most realistic collusion scenarios, punishments for deviation from collusion has to be imposed by aggressive pricing behavior on the market. This is due to the prohibition of collusive contracts by antitrust legislation. In collusion models based on infinitely repeated non-cooperative games, the punishments $F(n)$ are endogenously determined by the loss incurred when switching from an equilibrium that yields a share in monopoly profits to the lowest value continuation equilibrium. In this case we have:

$$F(n) = \frac{\delta}{1-\delta}\left[\frac{1}{n}\pi(p^c) - \underline{v}(n)\right] \tag{4}$$

where $\underline{v}(n)$ is the average per period payoff in the worst continuation equilibrium for the deviating firm and δ is the discount factor for future profits.[4] Clearly the behavior of $nF(n)$ depends only on $n\underline{v}(n)$. In the capacity unconstrained case $\underline{v}(n) = 0$ so that $nF(n)$ is independent of n. We will show in the rest of the paper that $n\underline{v}(n)$ is generally (weakly) decreasing in all homogeneous goods models with capacity constraints typically considered in the literature. Intuitively this is the case because in static Nash equilibrium payoffs will fall more than proportionately as the number of players is increased. Since punishments are at least as severe as a reversion to static Nash equilibrium play, punishment profits $\underline{v}(n)$ tend to inherit this property. This result on the behavior of punishment profits then immediately leads to our main result: in industries in which single firms are capacity constrained, the best sustainable collusive price typically rises as capacities are more and more fragmented.

3 Fragmentation facilitates collusion in the standard model

3.1 The structure of optimal punishments

We now model collusion in the standard framework of an infinitely repeated game. In every period of the game firms first observe the realization of a public signal $\sigma \in [0, 1]$ and then play the Bertrand-Edgeworth game described in the previous section.[5] To facilitate the exposition we assume that there is efficient rationing when a firm is capacity constrained at the lowest price.[6]

A lower bound to the payoffs that a firm can obtain on average in this game can be derived by assuming that all rivals sell all of their capacity into the market. Under efficient rationing, the firm would then simply face the residual demand function after the $\frac{n-1}{n}K$ highest valuation customers have been served, i.e. $\max\{0, D(p) - \frac{n-1}{n}K\}$. This is the worst possible scenario for any firm. The firm can

[4] We follow the convention in the repeated games literature to express all terms in "average values". Dividing average values by $(1-\delta)$ yields the present value of a continuation equilibrium.

[5] I assume a public randomization device purely for convenience. It guarantees that the equilibrium value set is convex, which facilitates the exposition of the results.

[6] We discuss the extension of the results to more general rationing schemes in section 4.1.

therefore never make lower profits than:

$$\pi^{\min} = \max_p (p-c) \min\{\max\{0, D(p) - \frac{n-1}{n}K\}, \frac{K}{n}\} \tag{5}$$

If $D(c) \leq \frac{n-1}{n}K$, then residual demand is zero and $\pi^{\min} = 0$ because the firm cannot generate positive profits at a price exceeding marginal cost. In this case there exists a unique Nash equilibrium in the one shot game with $p^N = c$. Now suppose $D(c) > \frac{n-1}{n}K$ and let p^* maximize $(p-c)(D(p) - \frac{n-1}{n}K)$. If $D(p^*) - \frac{n-1}{n}K \geq \frac{1}{n}K$, firm i could increase the price up to $\bar{p}$ defined by $D(\bar{p}) = K$ without reducing sales. In this case $p^N = \bar{p}$ is the unique Nash equilibrium price of the one shot game. In both of these cases the Nash equilibrium coincides with the perfectly competitive outcome.

The most interesting case arises when $D(p^*) - \frac{n-1}{n}K < \frac{1}{n}K$. In this case the firm makes a profit $(p^* - c)\left[D(p^*) - \frac{n-1}{n}\right]$, which is strictly higher than the profit under perfect competition, $(\bar{p} - c)\frac{K}{n}$, since the firm could always have chosen $\bar{p} < p^*$ and sold all of its capacity. It is well known that for the parameter region in which $D(p^*) - \frac{n-1}{n}K < \frac{1}{n}K$, there does not exist a pure strategy Nash equilibrium of the one shot game. However, p^* corresponds to the upper bound of the support of the mixed strategy equilibrium. The expected profit in the mixed strategy equilibrium therefore coincides with the lower bound on profits $\pi^{\min}$. The lower bound of profits $\pi^{\min}$ can therefore be obtained as an average value of the infinitely repeated game by reverting to the one shot Nash equilibrium of the static game forever. Hence, $\underline{v}(n) = \pi^{\min}$. Given this simple characterization of average payoffs in the worst equilibrium it becomes immediate that the industry profits in the worst equilibrium decline as capacity is divided among more firms:

Proposition 1 *Total industry profit in the lowest profit equilibrium, $n\underline{v}(n)$, are declining in n. It is strictly declining if $n\underline{v}(n)$ strictly exceeds the competitive profits in the industry:* $\min\{(\bar{p} - c)K, 0\}$. *As n becomes arbitrarily large, $n\underline{v}(n)$ converges to the competitive profits.*

Proof. First note that, for any given K, there exists $n^+ \geq 2$ such that $\frac{n-1}{n}K < D(c)$ for all $n < n^+$ and $\frac{n-1}{n}K \geq D(c)$ if $n \geq n^+$. ($n^+ = \infty$ is allowed). Suppose $n < n^+$, then there is positive demand for some $p > c$. The price p^* that maximizes (5) will be at least as large as $\bar{p}$ so that $D(p^*) \leq D(\bar{p}) = K$. Differentiating

$$n\,(p^* - c)\left[D(p^*) - \frac{n-1}{n}K\right]$$

with respect to n yields by the envelope theorem:

$$(p^* - c)\left[D(p^*) - K\right] \leq 0,$$

where the inequality is strict unless $p^* = \bar{p}$. Hence, when $n < n^+$, industry profits $n\underline{v}(n)$ are declining in n. They are strictly declining when industry profits strictly exceed competitive profits, i.e. $p^* > \bar{p}$. Now suppose $K < D(c)$, so that $n^+ = \infty$. Clearly,

$$\begin{aligned} & \lim_{n\to\infty} \{\max_{p \geq \bar{p}} (p-c)\left[D(p) - \frac{n-1}{n}K\right]\} \\ = \; & \max_{p \geq \bar{p}} (p-c)\left[D(p) - K\right] \end{aligned}$$

Since $p^* \geq \bar{p}$, but $D(p) - K < 0$ for all $p > \bar{p}$ it follows that $\lim_{n\to\infty} p^* = \bar{p}$. This proves the last part of the proposition for $K < D(c)$. For $K > D(c)$, we have $n^+ < \infty$. At $n < n^+$, $p^* > c \geq \bar{p}$. For $n \geq n^+$, demand is zero and $n\underline{v}(n) = 0$. Hence, the competitive profits are attained at $n \geq n^+$. It follows that the proposition holds for all cases. ■

This proposition shows that industry profits in the worst equilibrium of the infinitely repeated game will strictly decrease in the number of firms unless the competitive equilibrium relative to $D(p)$ and K is achieved as the worst equilibrium. Hence, punishments can be made more severe the more fragmented capacity is in the industry. This insight is central to the main message of the paper. Essentially, in capacity constrained Bertrand-Edgeworth oligopoly the punishment effect is the only effect at work when firms are sufficiently capacity constrained. The increased capability of punishing deviators in a more fragmented industry assures that collusion is strictly easier with a more fragmented industry structure.

3.2 The main results

We now derive the implications for the comparative statics of the highest sustainable collusive price p^c. It turns out to be easiest to approach the comparative statics in price indirectly by first characterizing the lowest discount factor $\bar{\delta}(n)$ at which p^m can be sustained as a collusive price. We then go on to characterize $\underline{\delta}(n)$, the lowest discount factor such that for every $\delta > \underline{\delta}(n)$ an average profit strictly exceeding that in the one-shot game can be sustained. These two critical discount factors allow a precise characterization of the response of the highest collusive price when capacity is split among a greater number of firms.

Combining (2) and (4), and slightly rearranging terms we can formulate the incentive compatibility condition for the highest best sustainable price p^c as:

$$(p^c - c)D(p^c) - (1-\delta)(p^c - c)\min\{nD(p^c), K\} \geq \delta n\underline{v}(n) \tag{6}$$

where the inequality is binding when $p^c < p^m$ or when $p^c = p^m$ and $\delta = \bar{\delta}(n)$. Equation (6) reveals that only the punishment profits determine the comparative statics in n, when the capacity of a firm is small enough so that it cannot serve all the demand $D(p^m)$ at the monopoly price. This point must necessarily be reached when capacity is sufficiently divided among enough firms, namely for $n > \hat{n} = \frac{K}{D(p^m)}$. For $n > \hat{n}$ a firm will not be able to serve the market at the monopoly price or at any lower price. Evaluating (6) at p^m and solving for $\bar{\delta}(n)$, yields:

$$\bar{\delta}(n) = \begin{cases} \frac{n-1}{n}\left[1 - \frac{n\underline{v}(n)}{\pi(p^m)}\right]^{-1} & \text{if} \quad n \leq \hat{n} \\ \frac{\hat{n}-1}{\hat{n}}\left[1 - \frac{n\underline{v}(n)}{(p^m-c)K}\right]^{-1} & \text{if} \quad n > \hat{n} \end{cases} \tag{7}$$

The analysis of the comparative statics of $\bar{\delta}(n)$ in n is easy when $n \geq \hat{n}$. Then, the number of firms, n, affects the incentive condition only through $n\underline{v}(n)$, which by proposition 1 is decreasing in n. Hence, the incentive constraint is (weakly) relaxed and $\bar{\delta}(n)$ weakly falls for all $n \geq \hat{n}$. This means that whenever a single firm cannot serve all the demand at the monopoly price, collusion becomes easier when a given level of capacity is distributed over more firms.

When $n < \hat{n}$, a firm deviating from the monopoly price is not capacity constrained. It serves all the demand at a price arbitrarily close to the collusive price. If punishments are pushed all the way to zero, the critical discount factor is as in a homogeneous goods model without capacity constraints: $\frac{n-1}{n}$. Otherwise, the discount factor is higher because punishments are less severe. For a given punishment profit $n\underline{v}(n)$ the critical discount factor $\bar{\delta}(n)$ would rise faster in n than in the capacity unconstrained model. However, the fact that $n\underline{v}(n)$ is decreasing in n still creates a countervailing effect. We show in the proof of proposition 2 that the net effect is for collusion to get more difficult when $n < \hat{n}$. Hence, $\bar{\delta}(n)$ is increasing on $[2, \hat{n})$ and decreasing on $[\hat{n}, \infty)$ as shown in Figure 1 below. This is the result of proposition 2a.

In a homogeneous goods model without capacity constraints the critical discount factor $\bar{\delta}(n)$ would fully characterize the scope for collusion in the model. In such a setting a price $p^c < p^m$ can never be sustained in a collusive equilibrium if p^m cannot be sustained.[7] However, this is feature does not carry over to a model with capacity constraints. We can therefore meaningfully ask the question: Is there a range of discount factors for which average equilibrium profits in the best equilibrium exceed profits in the one shot Nash equilibrium?

To make this idea formal, let $\underline{\delta}(n)$ denote the lowest discount factor such that for every $\delta > \underline{\delta}$, the highest average equilibrium profit strictly exceeds $\pi^N(n)$. For this discussion it is useful to distinguish between markets in which a single firm has enough capacity to serve demand at marginal costs and markets where this is not the case. For that purpose define $n_c \equiv \frac{K}{D(c)} < \hat{n} = \frac{K}{D(p^m)} < \infty$. If $n \leq n^c$ a single firm can serve the whole market when pricing at $p = c$. Otherwise a single firm cannot serve the market at marginal cost c. Suppose first that $n \leq n_c$. Then, $n\underline{v}(n) = 0$ since the one shot equilibrium involves marginal cost pricing in this case. Since every firm can serve the whole market at price $p = c$, it can also serve the whole market at any higher price. Hence, the capacity constraint is never binding on any deviating firm. The incentive constraint for collusion at any price p^c is thus given by:

$$\pi(p^c)\left[1 - (1-\delta)n\right] \geq 0 \tag{8}$$

which is satisfied independently of p^c. Hence, collusion at $p^c < p^m$ can only be sustained if it can also be sustained at p^m. Hence, for $n \leq n^c$, $\underline{\delta}(n) = \bar{\delta}(n) = \frac{n-1}{n}$ as in the case without capacity constraints.

Now consider the case $n > n^c$ so that there is no single firm that can serve the market at price $p = c$. If $n \leq \hat{n}$ it is still the case that each firm can serve the whole capacity at the monopoly price p^m. But for prices p^c just below p^m it is harder to sustain collusion, since $\pi(p^c)[1 - (1-\delta)n]$ falls when p^c is decreased. This is analogous to the capacity unconstrained case. However, when $n > n_c$, the price can always be lowered far enough so that an undercutting firm becomes capacity constrained at some price $p > c$. Let $\bar{n}$ be the smallest n such that $p^*(n) = \max\{\bar{p}, c\}$ for all $n \geq \bar{n}$. When the market is at least as fragmented as $\bar{n}$ punishments cannot be increased by increasing n further and the incentive condition becomes independent of n.[8] Proposition 2b shows that, for $\delta > \max\{\frac{1}{n}, \frac{1}{\bar{n}}\}$, there exists prices strictly exceeding $p^*(n)$, that can be sustained in a collusive equilibrium. This means that profits strictly above the one shot Nash profits are sustainable.

Proposition 2 *For any n there exists*

[7] Note that this is different from the description of our simple static collusive model of section 2. The reason is that in the infinite horizon model effective punishments also depend on the best collusive price p^c.

[8] Clearly $\bar{n} = n_c$ if $K > D(c)$.

(a) a critical discount factor $\bar{\delta}(n) < 1$, such that for all $\delta \geq \bar{\delta}(n)$ collusion at the monopoly price p^m is feasible. The critical discount factor $\bar{\delta}(n)$ is increasing on $[2, \hat{n})$ and decreasing on $(\hat{n}, \infty]$.

(b) a critical discount factor $\underline{\delta}(n) \leq \bar{\delta}(n)$ such that a collusive profit $\pi^c > \pi^N$ can be sustained if and only if $\delta > \underline{\delta}$. In particular,

$$\underline{\delta}(n) = \begin{cases} \frac{n-1}{n} & \text{if} \quad n \leq n_c \\ \max\{\frac{1}{n}, \frac{1}{\bar{n}}\} & \text{if} \quad n > n_c \end{cases}$$

Proof. See the Appendix. ■

Proposition 2 quite dramatically contradicts the common wisdom that market fragmentation facilitates collusion. If a duopolist cannot serve all of the demand at the monopoly price, i.e. $\hat{n} \leq 2$, then the scope for collusion uniformly decreases as capacity K is progressively distributed among more and more firms. This is true both in the sense that the critical discount factor for sustaining the monopoly price falls and in the sense that the lowest discount factor at which some collusion can be sustained falls. If, in addition, there is enough capacity in the market to serve all demand at marginal cost, i.e. $K > D(c)$, the result becomes even more dramatic: when the market becomes fragmented enough, collusion is easier in a market with $n > n_c$ firms than in a duopoly without a capacity constraint: $\bar{\delta}(n) = \frac{\hat{n}-1}{\hat{n}} = 1 - \frac{D(p^m)}{K} \leq \frac{1}{2}$. The inequality comes from the assumption that a single firm in a duopoly cannot serve the whole demand at the monopoly price. This means that collusion in a highly fragmented market is easier than in a duopoly without capacity constraints. The reason for this is very simple. Very fragmented markets allow for very severe punishments, while capacity constraints limit the incentives for deviation from a collusive price. Fragmentation therefore supports the ability to collude. This dramatic advantage of collusion in fragmented markets can also be observed in the critical discount factor $\underline{\delta}(n)$. When $K > D(c)$, $\lim_{n \to \infty} \underline{\delta}(n) = 0$, i.e. as the market becomes arbitrarily fragmented some collusion can be sustained at almost every discount factor.

[Insert Figure 1]

The result of proposition 2 makes it easy to directly derive the comparative statics in prices. Essentially, this can be read off directly from Figure 1. Suppose first $\delta \geq \bar{\delta}(\hat{n})$. Then the monopoly price is sustainable for any n. Note that, despite the fact that the market becomes arbitrarily fragmented, the monopoly price can always be sustained. For $\delta \in [\bar{\delta}(2), \bar{\delta}(\hat{n})]$, the monopoly price can be sustained up to some n such that $\delta = \bar{\delta}(n)$. Since $\bar{\delta} > \underline{\delta}$, this means that there will also be some smaller profit $\pi(p^c) > \pi(p^N)$ sustainable for some $\delta \in (\underline{\delta}, \bar{\delta})$. However, there will exist $\varepsilon > 0$, such that $\pi(p^m) - \varepsilon$ is not sustainable at such a δ. Hence, at $\bar{\delta}$ the highest sustainable collusive price drops to some $p^c < p^m$. After that the price must strictly increase. To see that, note that at p^c the capacity constraint is strictly binding when a firm deviates from the collusive price p^c. Hence, the incentive constraint is only depends on $n\underline{v}(n)$, which is decreasing in n. This means that p^c can strictly be increased. These comparative statics are illustrated in Figure 2.

[Insert Figure 2]

If $\delta \in (\frac{1}{\bar{n}}, \bar{\delta}(2))$, collusion can be sustained at some price $p^c(n) < p^m$ and $p^c(n)$ will monotonically increase on $[2, \infty)$. However, when $\bar{\delta}(2) = \frac{1}{2}$, there will generally be a range of n such that no collusion is sustainable because $\delta < \frac{1}{n}$. Then there only exists the mixed strategy equilibrium of the one shot game. The highest price in the support of the mixed strategy equilibrium will be $p^*(n)$. For some n we will have $\delta = \frac{1}{n}$. At this point $\lim_{n \downarrow \frac{1}{\delta}} p^c(n) = p^*(n)$ and then strictly increases until $p^c(\bar{n})$ as n is increased. With the exception of the range $(\bar{\delta}(2), \bar{\delta}(\hat{n}))$, price will always be weakly increasing. In fact, for $\frac{K}{2} < D(p^m)$, i.e. $\hat{n} < 2$, this range does not exist. Under realistic conditions, our model will therefore generate a highest sustainable collusive price that increases as the fragmentation of the industry is increased. We summarize these observations in proposition 3:

Proposition 3 *Suppose that*

(a) $\delta \geq \bar{\delta}(\hat{n})$*. Then* $p^c(n) = p^m$ *for all* $n \geq 2$*.*

(b) $\bar{\delta}(2) \leq \delta < \bar{\delta}(\hat{n})$*. Then there exists* $2 \leq n(\delta) < \hat{n}$*, such that* $p^c(n) = p^m$ *for* $n \leq n(\delta)$*,* $\lim_{n \downarrow n(\delta)} p^c(n) < p^m$*, and* $p^c(n)$ *is strictly increasing on* $(n(\delta), \bar{n})$*, and constant thereafter.*

(c) $\frac{1}{\bar{n}} < \delta < \bar{\delta}(2)$*. Then there exists* $2 \leq n(\delta) \leq \bar{n}$*, such that no collusion is sustainable for* $n < n(\delta)$ *and* $p^c(n)$ *is strictly increasing on* $(n(\delta), \bar{n})$ *and constant thereafter.*

Proof. Follows directly from the discussion in the text. ■

Proposition 2 implies that we should see price cost margins increase as the market is fragmented as long as initially no single firm could serve the whole market at the prevailing collusive price. This is the case for the electricity industry example of Sweeting (2005). At market prices before fragmentation no firm could have served the whole market. Fragmentation led to an increase in price cost margins. This is precisely what the theory would predict. What we have thus shown is that in industries in which collusion is a major concern, policies to fragment capacity in the market will be counterproductive as a measure to make collusion more difficult. In the rest of this paper we explore the robustness of this result.

4 Extensions to the Basic Model

In this section we show that the results derived in section 2 are robust to many variations in the model: Changes in the rationing scheme, different specifications of the cost function, and asymmetric initial capacity distributions. We show how our result sheds further light on the literature on coordinated effects of mergers. We also discuss that our results will hold for extensions of the model to supply function equilibrium.

4.1 General Rationing Schemes

We first clarify that the basic insights do not depend on the rationing scheme assumed. In the spirit of Lambson (1997) we encompass a range of different rationing schemes by imposing assumptions directly on the residual demand function of the individual firm. These will be consistent with a wide range of rationing schemes, in particular with proportional rationing. Given that we are interested in symmetric optimal punishment equilibria it suffices to make assumptions on residual demand only for the case where all competitors of firm i set the same price p_j. Residual demand $d(p, p_j, K_{-i})$ then depends on the price of firm i, p, the price of all other firm, p_j, and the total capacity of firms other than i, K_{-i}. Obviously $d(p, p_j, K_{-i}) = D(p)$ if $p < p_j$ and $d(p, p_j, K_{-i}) = \frac{1}{n}D(p)$ if $p = p_j$. For $p > p_j$ residual demand is zero if $D(p_j) \leq K_{-i}$. Otherwise, firms other then i are capacity constrained and residual demand depends on the rationing scheme assumed. In particular, we assume that $d(p, p_j, K_{-i})$ is strictly decreasing and log-concave in p on $(p_j, \bar{p})$, $\bar{p} = \inf\{p : d(p, p_j, K_{-i}) = 0\}$, continuously decreasing in p_j on $(0, p)$, and $-\frac{d(p_i,p,K_{-i})}{K/n} \geq d_{K_{-i}}(p_i, p, K_{-i})$. All three properties are satisfied both for efficient rationing and proportionate rationing. The property that residual demand is decreasing in the price of others may be surprising at first but is natural for rationing schemes. To see this consider proportional rationing. If p_j is lowered, firms other than i cannot expand their production because they are already selling K_{-i}. Hence, with proportional rationing, there will be a greater number of customers left for firm i at any price. Hence, demand increases at every price p_i.[9]

Given these assumptions on residual demand Lambson (1997) has shown that the worst equilibrium outcome sustainable in a symmetric optimal punishment equilibrium is again the security level payoff. This means it is the smallest payoff a player can guarantee himself regardless of the strategies of others. Note that, for any price p_L that rivals charge, a firm i can guarantee itself almost $(p_L - c)\min\{D(p_L), \frac{K}{n}\}$ by undercutting p_L and it can obtain $sup_{p>p_L}(p-c)d(p, p_L, \frac{n-1}{n}K)$ by raising the price optimally above p_L. To minimize the benefits from undercutting, p_L should be set as low as possible. Note first that p_L must always be low enough to leave an undercutting firm capacity constrained. If not, setting a price $p > p_L$ would yield zero demand, so that p_L can be lowered without creating any incentive to raise the price above p_L. As p_L is lowered, $(p_L - c)\frac{K}{n}$ is strictly decreasing while $sup_{p>p_L}(p-c)d(p, p_L, \frac{n-1}{n}K)$ becomes strictly increasing when $D(p_L) = \frac{n-1}{n}K$. The benefits from deviating upwards and downwards are exactly the same when:

$$(p_L - c)\,\frac{K}{n} = \sup_{p>p_L}(p-c)\min\{d(p, p_L, \frac{n-1}{n}K), \frac{K}{n}\} \tag{9}$$

It follows that $n\underline{v}(n) = (p_L(n) - c)K$. The punishment profit is therefore decreasing in n if $p_L(n)$ decreases in n. But this is the case under our assumptions. The result is trivial when the firm is capacity constrained at the security level because then the competitive profit is obtained. Otherwise the firm is not capacity constrained at the security profit and we can totally differentiating (9) in p_L and n to yield:

$$\frac{dp_L}{dn} = \frac{(p^* - c)\left[d(p, p_L, \frac{n-1}{n}K) - d_{K_{-i}}(p, p_L, \frac{n-1}{n}K)\frac{K}{n}\right]}{\frac{K}{n} - (p^* - c)\frac{\partial d(p, p_L, \frac{n-1}{n}K)}{\partial p_L}} \tag{10}$$

Since the denominator is positive by the assumption that $\frac{\partial d(p,p_L,\frac{n-1}{n}K)}{\partial p_L}$ and the numerator negative by the assumption $-\frac{d}{K/n} > d_{K_{-i}}$, it follows directly that means that $n\underline{v}(n)$ is decreasing in n. Given

[9] There exist rationing schemes that do not have this property. For example, suppose rationing switches from efficient rationing to efficient rationing if p is increased. But any such scheme would involve a change in the relative degree of rationing of higher and lower parts of the demand function as p is increased. Rationing schemes of such a form seem hard to justify.

this feature of punishment profits all of the results for efficient rationing carry over to other forms of rationing including proportional rationing.

4.2 Constant Returns to Scale

We now show that we can also extend the analysis to other cost functions. In this section we return to the efficient rationing assumption but assume that the production function has constant returns to scale in all inputs. This implies a cost function that is homogeneous of degree 1 in $\frac{K}{n}$: $C(q, \frac{K}{n}) = \frac{K}{n} C(\frac{nq}{K}, 1)$. Marginal cost is given by $c(\frac{nq}{K}) = C'(\frac{nq}{K}, 1)$ and is strictly increasing in $\frac{nq}{K}$. This assumption implies that for a given level of total production, nq, and total capacity K, total costs and marginal costs are independent of n. If this were not the case, redistributions of given capacities across firms could have productive efficiency effects unrelated to the incentives to collude. This is therefore the most general cost model for which capacity redistributions between firms do not have efficiency effects.

The monopoly price in the market is given by $p^m = \arg\max_p \{pD(p) - KC(\frac{D(p)}{K}, 1)\}$. Call $q_K(p^c, n) = c^{-1}(p^c)\frac{K}{n}$ the capacity of a firm at price p^c. This plays the same role as K for the incentives to deviate from the monopoly price. We maintain the Bertrand-Edgeworth assumption that firms can turn customers away. Hence, a firm that faces demand exceeding $q_K(p^c, n)$ would turn these customers away because they would cause losses at the margin. Clearly if a firm rations consumers at price p^m will ration them at any lower price. It is then obvious that there is some smallest $\hat{n}$ such that $q_K(p^m, n) < D(p^m)$ for all $n > \hat{n}$. To simplify the argument, we will discuss our results only for $n \geq \hat{n}$. The incentive compatibility constraint for a collusive price p^c to be incentive compatible then becomes:

$$(p^c - c)\, D(p^c) = (1-\delta)(p^c - c)c^{-1}(p^c)K + \delta n \underline{v}(n) \tag{11}$$

As in our base model the incentive compatibility condition depends on n only through $n\underline{v}(n)$. To show that the same qualitative comparative statics will hold we only need to show that $n\underline{v}(n)$ is declining in n.

To do this denote the highest production that can ever be obtained by a firm as $\bar{q}_K = \lim_{p\to\infty} c^{-1}(p^c)\frac{K}{n}$. Any production level below $\bar{q}_K$ can be obtained from a firm as long as the price in the market is high enough. But production beyond $\bar{q}_K$ is physically impossible. To see that $\bar{q}_K$ is of great importance for punishment profits, first consider the case of, for example, the quadratic cost function, which has $\bar{q}_K = \infty$. Since under punishments a firm commits to sell to any buyer at the posted price, punishment profits can always be reduced to zero. Set p_L such that

$$(1-\delta)\left[p_L \frac{D(p_L)}{n} - C(\frac{D(p_L)}{K}, 1)\right] + \delta \frac{\pi^m}{n} = 0$$

Since production of $\frac{D(p_L)}{n-1}$ is feasible for each firm, firms will produce this amount when some firm deviates to a price $p > p_L$. But then demand at any price above p_L is zero. It follows that $n\underline{v}(n) = 0$ for all n and $\bar{\delta}(n) = \frac{\hat{n}-1}{\hat{n}}$ for all $n \geq \hat{n}$. In this case collusion does not get easier as capital is split over more and more firms, but it also does not get more difficult. In fact, when $\hat{n} \leq 2$, the ease of collusion is independent of the number of firms.

When $\bar{q}_K < \infty$, firms will again be capacity constrained when a single firm deviates from punishments as long as $D(p_L) > (n-1)\bar{q}_K = \phi \frac{n-1}{n} K$, where $\phi \equiv \lim_{p\to\infty} c^{-1}(p)$. The deviating firm then faces a residual demand function given by:

$$\min[\max\{0, D(p) - \phi\frac{n-1}{n}K\}, \bar{q}_K]$$

With this modification the analysis of section 2 can be repeated. Clearly for $n \geq \hat{n}$ we again obtain the result that collusion gets easier as n increases.

4.3 Asymmetric capacity holdings and the coordinated effects of mergers

The basic insight of the model also carries over to asymmetric collusive schemes. I show this here considering only quasi-symmetric equilibria (as in Compte, Jenny, and Rey 2002) in which all firms set the same price, but share the sales in proportion to their share in total capacity. Let us assume that at current distribution of capacities no firm can serve the whole market at monopoly prices. If a firm i has capacity share s_i its incentive constraint becomes:

$$s_i(p^m - c)D(p^m) \geq (1-\delta)(p^m - c)s_iK + \delta(p^* - c)[D(p^*) - (1-s_i)K]$$

Dividing by s_i shows directly that the incentive constraint can only be binding for the largest firm because

$$\begin{aligned} & \frac{\partial}{\partial s_i}\frac{(p^*-c)[D(p^*)-(1-s_i)K]}{s_i} \\ = & \frac{\partial}{\partial s_i}\frac{(p^*-c)[D(p^*)-K]}{s_i} = \frac{(p^*-c)[K-D(p^*)]}{s_i^2} > 0 \end{aligned}$$

At the same time this means that collusion gets easier if the market share of the largest firm in the market becomes more fragmented. For merger analysis this implies the result that collusion becomes more difficult when the largest firm in the market merges with a smaller firm.

Compte, Jenny, and Rey (2002) have attributed this result to asymmetries between the firms. However, our results for the symmetric model suggests that it is the fragmentation of market share itself that facilitates collusion in this case and makes mergers by the largest firm undermine collusion.

The effects of asset consolidation have not only been studied for capacity transfers but also for other assets like the product lines of firms (see Kühn 2004). This literature has similarly derived results that suggest that mergers by larger firms may reduce the scope for collusion. Kühn (2004) also attributes this result to the effects of asymmetries. Is it the case that these results also have to be attributed to market fragmentation effects on mergers rather than asymmetries between the firms? This question has been studied in Kühn and Rimler (2006) using a symmetric version of the model of Kühn (2004) in which firms' size is determined by the length of their product lines. In this framework it is easy to see that fragmenting the product lines so that there are more firm with shorter product lines has countervailing effects on collusion. On one hand, firms with shorter product lines have a greater incentive to deviate from collusion. On the other hand they can credibly threaten harsher equilibrium punishments. However, Kühn and Rimler (2006) show that the first effect dominates the second: Fragmentation makes collusion harder. In the case of the model of Kühn (2004) it is then indeed asymmetry between firms that drives the result that the merger of large firms may reduce collusion. The result that fragmentation of assets facilitates collusion is therefore very specific to the case of capacity constraints.

In light of these results it is not surprising that cross-sectional studies across different industries have not found a systematic impact of the number of firms in the industry on the ability to collude. Clearly the distribution of capacities has a very different effect than the distribution of other assets and this would have to be controlled for in empirical studies.

4.4 Supply Function Competition

Note that we have in this paper only considered Bertrand-Edgeworth strategies. But other forms of price-quantity strategies would allow one to derive similar results. For example, for collusion in supply function equilibria the same basic features apply: As long as a single firm cannot supply the whole market for any demand realization, the best deviation for a single firm is to its inverse marginal cost function for every realization of price. This is the equivalent of hitting the capacity constraint in our model with increasing marginal cost. Then again there would be no state of the world in which the deviation profits are affected by n other than proportionately. Conversely, as long as there is an absolute constraint on maximal production by each firm that diminishes proportionately in n as the number of firms grows, we will again obtain our general result of market fragmentation facilitating collusion. The insights of the model with increasing marginal costs are therefore applicable. This is the sense in which our paper is directly relevant to competition in the English electricity market as studied by Sweeting (2005).

5 Conclusion

The most immediate policy consequence of the results in this paper arise for industries in which restructuring of capacities is an option in the process of regulatory reform. The paper does not imply that such fragmentation should not be undertaken.[10] However, when there are serious concerns about the potential for collusion in an industry, changes in the market mechanism may be of greater importance to improve the performance of the industry than the fragmentation of assets. This appears to be a lesson learned from the electricity industry in England and Wales and our analysis gives a theoretical explanation for the failure of the divestment policy. It should also be noted that this paper does not imply that additional firms in an industry would not be desirable if one wishes to limit collusion. What drives the results in this paper is that *given capacity* is redistributed among firms. Policies that lead to increases of capacity overall will help mitigate the collusion problem.

[10] In fact, in the Spanish electricity industry, in which there is considerable vertical integration between activities upstream and downstream from the spot market, collusion may be a lesser concern. Then market fragmentation can greatly enhance the performance of the industry.

6 Appendix

Proof of Proposition 2:

Part (a): Clearly, for $p^c = p^m$, (6) is strictly satisfied at $\delta = 1$ and is not satisfied at $\delta = 0$. Since the incentive constraint is strictly relaxed by increasing δ there exists a unique $\bar{\delta}(n)$ at which (6) is just binding. It is given by:

$$\bar{\delta}(n) = \begin{cases} \frac{n-1}{n}\left[1 - \frac{n\underline{v}(n)}{\pi(p^m)}\right]^{-1} & \text{if} \quad n \leq \hat{n} \\ \frac{\hat{n}-1}{\hat{n}}\left[1 - \frac{n\underline{v}(n)}{(p^m-c)K}\right]^{-1} & \text{if} \quad n > \hat{n} \end{cases} \tag{12}$$

If $n > \hat{n}$, $\bar{\delta}(n)$ is decreasing from proposition 1 because $n\underline{v}(n)$ is decreasing. Suppose $n < \hat{n}$. If $p^*(n) = \max\{\bar{p}, c\}$, $\bar{\delta}(n)$ is increasing. Suppose $p^*(n) > \max\{\bar{p}, c\}$, then

$$\begin{aligned}
& \left.\frac{\partial}{\partial n}\right|_{\delta=\bar{\delta}(n)} [(p^m - c)D(p^m) - (1-\delta)(p^m - c)nD(p^m) - \delta n\underline{v}(n)] \\
= \quad & -(1-\bar{\delta}(n))(p^m - c)D(p^m) - \bar{\delta}(n)\left[\underline{v}(n) + n\underline{v}'(n)\right] \\
= \quad & -\frac{\pi(p^m)}{n} - \bar{\delta}(n)n\underline{v}'(n) \\
= \quad & -\frac{\pi(p^m)}{n}\left[1 - \frac{n-1}{n}\frac{(p^* - c)K}{\pi(p^m) - n\underline{v}(n)}\right] \\
= \quad & -\frac{\pi(p^m)}{n\left[\pi(p^m) - n\underline{v}(n)\right]}\left[\pi(p^m) - n\underline{v}(n) - \frac{n-1}{n}(p^* - c)K\right] \\
= \quad & -\frac{\pi(p^m)}{n\left[\pi(p^m) - n\underline{v}(n)\right]}\left[\pi(p^m) - n(p^* - c)\left[D(p^*) - \frac{n-1}{n}K\right] - \frac{n-1}{n}(p^* - c)K\right] \\
= \quad & -\frac{\pi(p^m)}{n\left[\pi(p^m) - n\underline{v}(n)\right]}\left[\pi(p^m) - \pi(p^*) - (n-1)\left[D(p^*) - K\right]\right] \\
< \quad & -\frac{\pi(p^m)}{n\left[\pi(p^m) - n\underline{v}(n)\right]}\left[\pi(p^m) - \pi(p^*)\right] < 0
\end{aligned}$$

where the second equality comes from substituting in from the incentive constraint (6) and the third equality comes from substituting in from (**??**) and factoring out $\frac{\pi(p^m)}{n}$. The first strict inequality follows from $D(p^*) < K$. This proves the part (a) of Proposition 2.

Part (b): Note that at $\bar{p}$ all firms are strictly capacity constrained when undercutting other firms. This implies that there is an interval of prices $(\bar{p}, D^{-1}(\frac{K}{n}))$, such that undercutting at this price would make a firm strictly capacity constrained. From (6) it follows that the collusive price that maximally relaxes the incentive constraint for a colluding firm solves :

$$\max_{p \in [\bar{p}, D^{-1}(\frac{K}{n})]} (p - c)\max\{[D(p) - (1-\delta)K], \frac{K}{n}\}$$

Note that for any $\mu \in (0,1)$, $p^*(\mu) = \arg\max_{p \in [\bar{p}, D^{-1}(\frac{K}{n})]}(p-c)\max\{[D(p) - \mu K], \frac{K}{n}\}$ is decreasing in μ with $\bar{\mu} < 1$ the smallest μ for which $p^*(\mu) = \bar{p}$. Then $\bar{n} \equiv \frac{1}{1-\bar{\mu}}$ is the smallest n such that, for all $n > \bar{n}$, $\bar{p} = \arg\max_{p \geq \bar{p}}\ (p-c)\max\{[D(p) - \frac{n-1}{n}K], 0\}$. By the envelope theorem $\max_{p \in [\bar{p}, D^{-1}(\frac{K}{n})]}(p -$

$c) \max\{[D(p)-(1-\delta)K], \frac{K}{n}\}$ strictly exceeds $n\underline{v}(n) = \max_{p \geq \bar{p}}(p-c)\max\{[D(p)-\frac{n-1}{n}K], 0\}$ if and only if $\delta > \max\{\frac{1}{n}, \frac{1}{\bar{n}}\}$. It follows that for $\delta \leq \max\{\frac{1}{n}, \frac{1}{\bar{n}}\}$ no collusive profits above $\pi^N(n)$ can be obtained. We now show that for $\delta > \max\{\frac{1}{n}, \frac{1}{\bar{n}}\}$, a collusive profit is sustainable. For that purpose note that, for any $\delta > \max\{\frac{1}{n}, \frac{1}{\bar{n}}\}$,

$$p(\delta) = \arg\max(p-c)[D(p)-(1-\delta)K] > p^*(n)$$

Hence:

$$\begin{aligned} & \frac{\partial}{\partial \delta}\left[\max_p (p-c)[D(p)-(1-\delta)K] - \delta n\underline{v}(n)\right] \\ = \;& (p(\delta)-c)K - n\,(p^*(n)-c)\,[D(p^*(n)) - \frac{n-1}{n}K] \\ \geq \;& (p(\delta)-p^*(n))K > 0, \end{aligned}$$

where the first inequality arises because $[D(p^*(n)) - \frac{n-1}{n}K] \leq \frac{K}{n}$. Hence, for $\delta > \max\{\frac{1}{n}, \frac{1}{\bar{n}}\}$, there is a price $p^c > p(\delta)$ that can be sustained even when the monopoly price is not sustainable. The result follows by concavity of the profit function.

References

[1] Arocena, P., K.-U. Kühn, and P. Regibeau (1999), "Regulatory Reform in the Spanish electricity industry: a missed opportunity for competition", *Energy Policy* 22, pp. 387-399.

[2] Asch, P. and J. Seneca (1975), "Characteristics of collusive firms", *Journal of Industrial Economics*, 23, pp. 223-237.

[3] Brock, W.A. and J. Scheinkman (1985), "Price Setting Supergames with Capacity Constraints", *Review of Economic Studies*, 52, pp. 371-382.

[4] Compte, O., P. Rey, and F. Jenny (2002), "Capacity constraints, mergers, and collusion", *European Economic Review*, 46, pp. 1-29.

[5] Davidson, C. and R.J. Deneckere (1990), "Excess Capacity and Collusion", *International Economic Review*, 31, pp. 521-541.

[6] Dick, A. (1996), "When are cartels stable outcomes", *Journal of Law and Economics*, 39, pp. 241-83.

[7] Frass, A. and D. Greer (1977), "Market structure and price collusion: an empirical analysis", *Journal of Industrial Economics*, 26, pp. 21-44.

[8] Green, R. (1996), "Increasing Competition in the British Electricity Spot Market", *Journal of Industrial Economics*, 44, pp. 205-216.

[9] Green, R. J. and D. M. Newbery (1992), "Competition in the British Electricity Spot Market", *Journal of Political Economy*, 100, pp. 929-953.

[10] Hay, G. and D. Kelley (1974), "An empirical survey of price-fixing conspiracies", *Journal of Law and Economics*, 17, 13-38.

[11] Henney, A. (1987), "Privatize Power: Restructuring the Electricity Supply Industry", Policy Study 83, Center Policy Studies, London.

[12] Kühn, K.-U. (2004), "The Coordinated Effects of Mergers in Differentiated Goods Markets", CEPR Discussion Paper 4769, London.

[13] Kühn, K.-U. and P. Regibeau (1998), "¿Ha llegado la competencia? Un análisis económico de la reforma de la regulación del sector eléctrico en España", *Informes del IAE*, Instituto de Análisis Económico, Barcelona.

[14] Kühn, K.-U. and M.S. Rimler (2006), "The Comparative Statics of Collusion Models", CEPR Discussion Paper 5742, London.

[15] Lambson, V.E. (1987), "Optimal Penal Codes in Price Setting Supergames with Capacity Constraints", *Review of Economic Studies*, 54, pp. 385-397.

[16] Levenstein, M. and V. Suslow (2006), "What Determines Cartel Success", *Journal of Economic Literature* 44(1), pp. 43-95.

[17] Motta, M. (2004), “Competition Policy: Theory and Practice”, Cambridge University Press, Cambridge.

[18] Posner, R. (1970), “A Statistical Study of Antitrust Enforcement”, *Journal of Law and Economics*, 13, pp. 365-419.

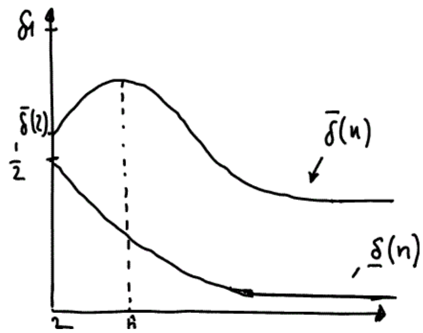
[19] Scherer, F.M. and D. Ross (1990), “Industrial Market Structure and Economic Performance”, 3rd edition, Houghton Mifflin Company, Boston.

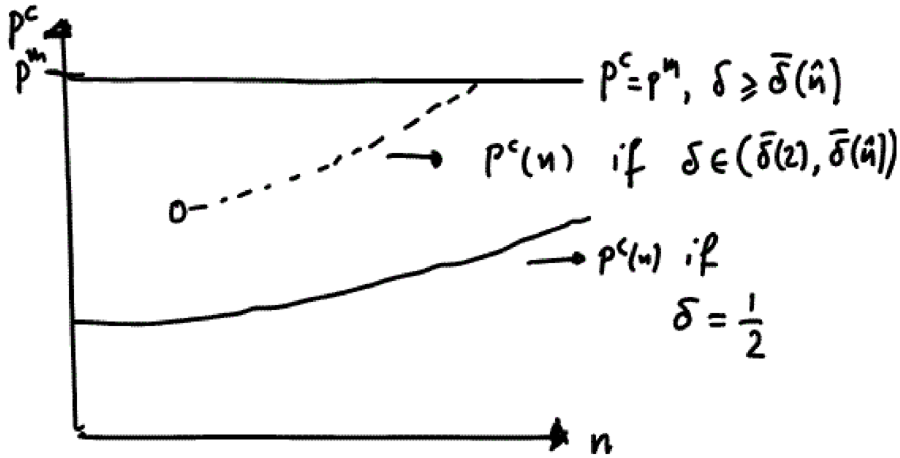
[20] Schmalensee, R. and B.W. Golub (1984), “Estimating Effective Concentration in Deregulated Wholesale Electricity Markets”, *Rand Journal of Economics*, 15, pp. 12-26.

[21] Suslow, V. Y. (2005), “Cartel contract duration: empirical evidence from inter-war international cartels”, *Industrial and Corporate Change*, 14, pp. 705-744

[22] Sweeting, A. (2006), “Market Power in the England and Wales Wholesale Electricity Market 1995-2000”, forthcoming in *Economic Journal*.

[23] Sykes, A. and C. Robinson (1987), “Current Choices: Good Ways and Bad to Privatise Electricity”, Policy Study 87, Center Policy Studies, London.





p^c
p^m
$p^c = p^m,\ \delta \geq \bar{\delta}(\hat{n})$
$p^c(n)$ if $\delta \in (\bar{\delta}(2), \bar{\delta}(\hat{n}))$
$p^c(n)$ if
$\delta = \frac{1}{2}$
n