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ABSTRACT

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JEL Classification: D43, L12 and L13
Keywords: monopolistic competition, multiproduct firms, product diversity and spatial models

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Abstract

The goal of this paper is to examine the role of multiproduct firms in the market provision of product diversity. The analysis is conducted within the spatial model of nonlocalized competition proposed by Chen and Riordan (2006). It turns out that the effect of multiproduct firms on product diversity depends on the size distribution of firms. Under duopoly, product diversity is lower than under monopolistic competition. Nevertheless, the number of varieties can still be socially excessive. In contrast, if a large multiproduct firm (incumbent monopolist) competes against a large number of small (single product) firms then product diversity is higher than under monopolistic competition. Moreover, the incumbent firm is unable to monopolize the market and deter entry.

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1 Introduction

Does the market provide too many or too few varieties? The economic analysis of this important question has typically focused on the case of single prod-

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uct firms. However, in many markets individual firms produce a significant fraction of all varieties. Those multiproduct firms choose their product range as an additional strategic tool, which may potentially affect the overall provision of variety. Does the presence of multiproduct firms reduce or expand product variety with respect to the case of single product firms? And with respect to the first best level of variety? Can an incumbent firm use product proliferation to monopolize the market and prevent entry?

In this paper I address these issues by introducing multiproduct firms into the spatial model of nonlocalized competition (spokes model) proposed by Chen and Riordan (2006). There are several reasons that justify the choice of model; in particular, the spokes model provides a tractable framework to examine both single product, monopolistically competitive firms and multiproduct firms. Moreover, results are very intuitive and transparent.

Spatial models similar to those proposed by Hotelling (1929) and Salop (1979) have played a prominent role in the field of Industrial Organization and in other areas. A crucial characteristic of these setups is that neighboring effects are essential (competition is localized). Thus, each firm is not negligible, in the sense that it cannot ignore its impact on, and hence reactions from, neighbor firms. Therefore, they are not an adequate representation of monopolistic competition, at least according to Chamberlin's (1933) characterization.

In most standard models of monopolistic competition, like Spence (1976) and Dixit and Stiglitz (1977) (SDS) neighboring effects are absent and each firm competes against 'the market'. Unfortunately, interpretation of some important results is not always straightforward. For instance, these models provide little guidance regarding the set of circumstances under which one should expect that, from a social point of view, equilibria involve excessive or insufficient product variety (See, for instance, Mankiw and Whinston, 1986). A notable exception is Ottaviano and Thisse (1999), which I discuss in more detail below.

It is also well known that standard spatial models do not provide a satisfactory framework to deal with multiproduct firms. In contrast, some recent papers, including Ottaviano and Thisse (1999) and Allanson and Montagna (2005) have put forward alternative approaches to modelling multiproduct firms in the SDS framework. Similarly, Anderson and de Palma (2006) have studied competition among multiproduct firms using a nested demand system.

The spokes model (Chen and Riordan, 2006) extends the Hotelling model to an arbitrary number of varieties and firms and provides a useful setup to study nonlocalized oligopolistic competition. The preference space consists of N spokes that start from the same central point. The producer of each potential variety is located in the extreme end of a different spoke. As the number of potential varieties goes to infinity, and if each variety must be produced by a different firm, then the model becomes an adequate representation of monopolistic competition. This setup provides new insights into the old question of "too many or too few products". Introduction of a new variety involves a fixed cost but it also has two positive effects: it increases aggregate demand and improves the matching between consumer preferences and the available product range. The relative weight of the aggregate demand effect decreases with the number of active varieties. This is precisely the main driving force behind the main welfare results for the case of monopolistic competition under free entry. If the fixed cost is relatively high then in equilibrium the number of active varieties is small, and smaller than in the social optimum. The reason is that firms cannot capture all the surplus generated by the increase in aggregate demand. In contrast, if the fixed cost is relatively low then in equilibrium the number of varieties is large, larger than in the social optimum. In this case, most of the revenues raised by a new entrant come from stealing the business of existing firms (matching effect) and private incentives to enter are superior to the social benefits generated.

In this paper I take the spokes model one step further by considering multiproduct firms. It turns out that the model can accommodate multiproduct firms as easily as the SDS model and, moreover, it brings about new insights and useful welfare results. Tractability is considerably enhanced by assuming that the number of varieties is sufficiently large. Thus, the product range of a multiproduct firm can be treated as a continuous variable. It turns out that the effect of multiproduct firms on product diversity depends on the size distribution of firms. In order to illustrate this point I restrict attention to two different (and somewhat extreme) market structures: (i) symmetric duopoly and (ii) a large (multiproduct) firm competing against a "competitive" fringe (a large number of single product firms). These market structures can be motivated by the existence of economies of scope and first mover advantages.

In the next section I present the finite model and formulate the continuous approximation.

In Section 3, I study the benchmark case (already studied in Chen and Riordan, 2006) in which each potential variety is produced by a different firm (monopolistic competition). By adding a (mild) restriction on parameter values, it is straightforward to represent in the same graph both the optimal and the free entry equilibrium value of the fraction of active varieties. The relationship between the size of the fixed cost, the fraction of active varieties, and whether the latter is insufficient or excessive from a social welfare perspective, can be illustrated in a very transparent way.

In Section 4, I move away from the one-to–one relationship between firms and varieties and consider the case that all potential varieties can be produced only by two firms. The obvious justification is the existence of economies of scope but for simplicity I take the number of firms as exogenously given. I model duopolistic competition as a two-stage game. In the first stage firms simultaneously choose which varieties to activate (and pay the fixed cost associated to each variety). In the second stage they simultaneously choose prices. Under duopoly product diversity is lower than in the case of monopolistic competition. One reason is that a new variety steals consumers from other varieties, including those varieties produced by the same firm (cannibalization effect). Thus, duopolists partially internalize the business stealing effect. The second reason is that by reducing the number of varieties duopolists are able to relax price competition.[1] That is, if a firm chooses to produce a lower number of varieties this induces its rivals to set a higher price (strategic price effect). In any case, it is important to note that despite the dampening effect of multiproduct duopolists on the number of varieties, and typically for intermediate values of the fixed cost, product diversity can still be socially excessive.

Finally, in Section 5, I study the interaction between a single (large) multiproduct firm and a large number of single product (small) monopolistically competitive firms. One possible justification is that the large firm enjoys a first mover advantage (incumbent monopolist). Firstly, the presence of the large firm creates a positive externality to small firms because it relaxes price competition. As a result, the incentives to enter are larger than in the absence of a large firm, and as a result product diversity is higher than under monopolistic competition. Secondly, the large firm is unable to monopolize the market, producing a large fraction of potential varieties in order to deter entry. The reason is that a successful monopolist would be setting very high prices, which in turn would exacerbate entry incentives of small firms.

These results contrast with those obtained in standard spatial models

[1] Such effect on prices requires that the fraction of active varieties be sufficiently large

(See, for instance, Schmalensee, 1978; and Bonanno, 1987). In these models, an incumbent firm may find it profitable to monopolize the market by crowding the product space or by choosing the appropriate location. Instead, the current set up suggests that the presence of neighboring effects was crucial. In fact, in the absence of neighboring effects proliferation cannot be an effective entry deterrence mechanism.

It is also worthwhile comparing the effect of multiproduct firms in this model with those obtained in the SDS and its close relatives (Ottaviano and Thisse, 1999; Allanson and Montagna 2005; and Anderson and de Palma, 2006). A non-desirable feature of the framework used in Ottaviano and Thisse (1999)[2] is that the optimal prices of a multiproduct firm are independent of the number of varieties produced by the firm. In other words, in their set up a firm cannot take advantage of producing a significant fraction of all potential varieties in order to raise its prices. In that model multiproduct firms generate less product diversity than in the case of monopolistic competition, simply because of the cannibalization effect[3], which is roughly the same for all parameter values. As a result, under multiproduct firms product diversity is socially excessive only if the fixed cost is sufficiently low (the threshold is lower than under monopolistic competition). In contrast, in the current set up when the fixed cost is low duopolists find it optimal (in the adequate parameter range) to restrict their product range in order to relax price competition, which results in insufficient product variety. In fact, product variety is socially excessive for intermediate values of the fixed cost.

In this respect the results of Anderson and de Palma (2006) are closer to those of the present paper. In particular, in their model a broader product range also induces more aggressive pricing behavior, although the reasons are very different. In their set up consumers have preferences for both firms and varieties. Consumers' decisions are taken in two steps. First, they choose which firm to purchase from. Second, they choose which products to buy from the selected firm. As a result, a firm with a broader product range becomes more attractive to consumers, which provokes a more aggressive price response. In the free entry equilibrium there are too many firms, each one producing too narrow a product range.[4] In contrast (and consistent

[2] See also Ottaviano et al (2002).

[3] In fact, in their set up the strategic price effect induces firms to select a broader product range, although the latter is more than compensated for by the cannibalization effect..

[4] Allanson and Montagna (2005) obtain a similar result in the context of the SDS

with Ottaviano and Thisse, 1999), I consider markets where consumers have preferences only for varieties and hence I can discuss whether overall product variety under multiproduct firms is excessive or insufficient from a social point of view.

2 The spokes model

2.1 Finite number of varieties (Chen and Riordan, 2006)

Let us consider a market where there are N potential varieties, indexed by i, $i = 1, ..., N$. A particular variety may or may not be supplied. The preference space can be described in a way similar to the standard spatial models. There are N spokes of length $\frac{1}{2}$, also indexed by i, $i = 1, ..., N$, which start from the same central point. The producer of variety i is located in the extreme end of spoke i.

Demand is perfectly symmetric. There is a continuum of consumers with mass $\frac{N}{2}$ uniformly distributed over the N spokes. Each consumer has a taste for two varieties only and the pair of selected varieties differ across consumers. In fact, consumers are uniformly distributed over the $\frac{N(N-1)}{2}$ possible pairs. Thus, the mass of consumers who have a taste for an arbitrary pair is $\frac{1}{N-1}$ and since there are $N - 1$ pairs that contain a particular variety, the mass of consumers who have a taste for variety i is 1.

Consumers with a taste for varieties i and j, $i, j = 1, ..., N$, $j \neq i$, are uniformly distributed over the union of spokes i and j. Each consumer demands one unit of the good. As usual consumer location represents the relative valuation of the two varieties. In particular, a consumer who has a taste for varieties i and j and is located at a distance x, $x \in \left[0, \frac{1}{2}\right]$, from the extreme of the ith spoke, if she chooses to consume one unit of variety i then she obtains a utility equal to $R - tx$. Alternatively if she chooses to consume one unit of variety j then she obtains $R - t(1 - x)$. A maintained hypothesis is $R > 2t$. Under this condition, for any given number of active varieties, firms always have incentives to serve as many consumers as possible, which simplifies the presentation.[5]

framework with a nested CES utility function, with the degree of substitutability between the varieties produced by any one firm being higher than that between varieties produced by different firms.

[5] Chen and Riordan (2006) consider a larger parameter space, $R > t$.

Suppose all N varieties are active, each one supplied by an independent firm. If $N = 2$ then this is the standard Hotelling model. If $N > 2$ firm i competes symmetrically with the other $N - 1$ firms. If N is very large the model captures Chamberlain's definition of monopolistic competition, in the sense that each firm: (i) enjoys some market power, i.e., faces a downward sloping demand function, and (ii) is negligible, if ejected from the market then no other firm is significantly affected.

Supplying a variety involves a fixed cost, F, and, for simplicity, zero marginal costs.

2.2 The continuous approximation

It will be very convenient to treat the number of active varieties as a continuous variable. In particular, let us denote by γ, $\gamma \in [0, 1]$ the fraction of active varieties. If $0 < \gamma < 1$, consumers can be classified in three different groups. Some consumers will have access to the two varieties they have a taste for, some other consumers will only be able to buy one of the varieties, and finally the third group of consumers will drop out of the market since neither of the two selected varieties is available.

We can treat γ as a continuous variable by considering the limit model as N goes to infinity and expressing all relevant variables relative to the mass of consumers. In particular, for a given value of γ and N, the number of pairs of varieties for which two suppliers are active is:

$$\frac{\gamma N (\gamma N - 1)}{2}$$

Since the fraction of consumers who have a taste for a particular pair is:

$$\frac{2}{N (N - 1)}$$

then the fraction of consumers who have access to two varieties is:

$$\frac{\gamma N (\gamma N - 1)}{N (N - 1)}$$

If we take the limit as N goes to infinity the fraction of consumers with access to two varieties is γ^2.

Similarly, the fraction of consumers that have access to neither of the two preferred varieties is $(1-\gamma)^2$. Finally, the fraction of consumers with access to only one variety is $2\gamma(1-\gamma)$.

The total amount of fixed costs per consumer is:

$$\frac{\gamma NF}{\frac{N}{2}} = 2\gamma F$$

From the point of view of supplier i, the fraction of consumers that demand variety i and have the opportunity of choosing between product i and their other selected variety is:

$$\frac{\gamma N - 1}{N - 1}$$

Hence, as N goes to infinity this fraction is equal to γ. Similarly, in the limit as N goes to infinity, the fraction of consumers that demand variety i and do not have access to the other selected variety is $1-\gamma$.

2.3 The first best

What is the fraction of active varieties that maximizes total surplus? For any fraction, γ, it is efficient to allocate those consumers who have access to their two selected varieties to the closest supplier. Thus, the average surplus obtained by consumers with access to two varieties is $R-\frac{t}{4}$, since the average transportation cost is $\frac{t}{4}$. Those consumers that only have access to one variety will incur on average higher transportation costs. Specifically, their surplus will be $R-\frac{t}{2}$. Finally, those consumers without access to any of their selected varieties will receive zero surplus. As mentioned above the amount of fixed costs per consumer is $2\gamma F$. Therefore, we can write total surplus as follows:

$$W(\gamma) = \gamma^2\left(R-\frac{t}{4}\right) + 2\gamma(1-\gamma)\left(R-\frac{t}{2}\right) - 2\gamma F$$

Since this function is concave the optimal value of γ, denoted by γ^*, can be computed directly from the first order condition:

$$\frac{dW}{d\gamma} = 2\gamma\left(R-\frac{t}{4}\right) + 2(1-2\gamma)\left(R-\frac{t}{2}\right) - 2F = 0$$

which implies that (See Graph 1)

$$\gamma^* = \begin{cases} 0 \text{ if } F \geq R - \frac{t}{2} \\ \frac{R-\frac{t}{2}-F}{R-\frac{3}{4}t} \text{ if } \frac{t}{2} \leq F \leq R - \frac{t}{2} \\ 1 \text{ if } F \leq \frac{t}{4} \end{cases}$$

In order to obtain some intuition we can rewrite the first order condition as follows:

$$\frac{dW}{d\gamma} = 2\left\{\gamma\frac{t}{4} + (1-\gamma)\left(R - \frac{t}{2}\right) - F\right\} = 0$$

The first term represent the preference matching effect. A fraction γ of consumers who have a taste for the new variety already had access to one of their selected varieties and they will enjoy lower average transportation costs $\left(\frac{t}{4}\right)$, i.e., there is a better matching between available products and consumer preferences. The second term is the market expansion or aggregate demand effect. A fraction $1-\gamma$ of consumers did not have access to any of their selected varieties and once the new variety is introduced they will purchase one unit of the good and enjoy an average surplus of $\left(R - \frac{t}{2}\right)$ .The relative weights of these two effects depend on the fraction of existing varieties. Since $R - \frac{t}{2} > \frac{t}{4}$ then total surplus created by an additional variety decreases with γ. Finally, the third term is the cost of entry.

3 The benchmark: Monopolistic competition

Let us consider the case that the number of potential firms is equal to the number of potential varieties, N. Each firm is able to produce a single variety. If firm i decides to enter the market then it pays a fixed cost, F, and chooses a price, p_i. Firms maximize profits and enter the market only if net profits are positive. Since each firm is negligible and there is no uncertainty, then it does not matter whether entry and price decisions are taken sequentially or simultaneously. We focus exclusively on symmetric equilibria of this free entry game.

Let us first compute the symmetric equilibrium price, for a given γ. In those market segments where consumers have access to two varieties, consumers will choose supplier exactly as in the Hotelling model. That is, the fraction of consumers that choose firm i is given by:

$$\frac{1}{2}+\frac{\overline{p}-p_i}{2t}$$

where $\overline{p}$ is the price charged by rival firms. In those segments where firm $i's$ product is the consumers' only choice total demand is 1, provided all consumers obtain a positive surplus, i.e., $p_1 + t \leq R$. In fact, under the maintained hypothesis ($R > 2t$) firms never find it optimal to set a price above $R-t$, which is the monopoly price (that which maximizes total profits). Hence, firms i's optimization problem consists of choosing p_i in order to maximize:

$$\pi_i\left(\gamma, p_i, \overline{p}\right)=\left[\gamma\left(\frac{1}{2}+\frac{\overline{p}-p_i}{2t}\right)+1-\gamma\right]p_i-F \tag{1}$$

subject to $p_i \leq R-t$. If this constraint is not binding then the optimal price is given by:

$$p_i=\frac{t\left(1-\gamma\right)}{2\gamma}+\frac{\overline{p}}{2}$$

Hence, the symmetric equilibrium price, $p_i=\overline{p}\equiv p^{MC}$, is given by:

$$p^{MC}=t\frac{2-\gamma}{\gamma} \tag{2}$$

As γ increases competition intensifies and the equilibrium price falls. If all potential varieties are active ($\gamma = 1$) then $\overline{p} = t$, as in the standard Hotelling model. Equilibrium price is given by equation (2) provided its value is not above $R-t$, which implies that $\gamma \geq \frac{2t}{R}$. Hence, the fraction of varieties in equilibrium, γ^{MC}, will be given by the zero profit condition, provided it lies in the interval $\left[\frac{2t}{R}, 1\right]$:

$$\pi_i\left(\gamma^{MC}, p^{MC}, p^{MC}\right)=\frac{t}{2}\frac{\left(2-\gamma^{MC}\right)^2}{\gamma^{MC}}-F=0$$

i.e., if $F \in \left[\frac{t}{2}, \frac{(R-t)^2}{R}\right]$ then

$$\gamma^{MC}=1+\frac{F}{t}-\sqrt{\left(1+\frac{F}{t}\right)^2-2} \tag{3}$$

and if $F \leq \frac{t}{2}$ then $\gamma^{MC}=1$.

If $\gamma < \frac{2t}{R}$, which occurs whenever $F \geq \frac{(R-t)^2}{R}$, then each individual firm faces little competition and finds it optimal to set the monopoly price, $R-t$, and serve all consumers that have no other choice. In this case, the zero profit condition is:

$$\pi_i\left(\gamma^{MC}, R-t, R-t\right) = \frac{2-\gamma^{MC}}{2}(R-t) - F = 0$$

Hence, if $F \in \left[\frac{(R-t)^2}{R}, R-t\right]$ then

$$\gamma^{MC} = 2\frac{R-t-F}{R-t} \tag{4}$$

and if $F \geq R-t$ then $\gamma^{MC} = 0$.

We can compare product diversity under monopolistic competition with the first best. A crucial threshold value is $F^{MC} \equiv \frac{(R-t)^2}{R-\frac{t}{2}}$, $\frac{t}{2} < F^{MC} < R-t$. The results are depicted in Graph 1 and can be written as follows (See Appendix):

Proposition 1 *Under monopolistic competition: (i) if* $F \in \left(\frac{t}{4}, F^{MC}\right)$ *there is excessive entry, i.e.,* $\gamma^{MC} > \gamma^*$, *and (ii) if* $F \in \left(F^{MC}, R-\frac{t}{2}\right)$ *there is insufficient entry, i.e.,* $\gamma^{MC} < \gamma^*$.

If a new firm decides to enter the market, then its revenues come from two sources. First, from consumers who did not have any option in the absence of the firm (expansion of aggregate demand) and, second, from consumers who would purchase an already existing variety in the absence of the firm (business stealing). The relative weight of the first effect decreases with γ. If γ is sufficiently small, then expansion of aggregate demand is strong. However, firms cannot appropriate all the surplus generated and hence, entry is insufficient. In contrast, if γ is large, then the business stealing effect dominates and the rent captured by the entrant exceeds the social surplus generated. As a result, there is excessive entry.

Proposition 1 can be interpreted as a smooth generalization of results obtained in standard spatial models. For instance, in Salop's circular model entry by an additional firm typically involves either an expansion effect (high transportation costs) or a business stealing effect (low transportation costs). Both effects take place at the same time only for very specific parameter values. In contrast, in the current model both effects are always present and their relative weight varies continuously with γ, and ultimately with F.

Obviously, these results are equivalent to those obtained in Chen and Riordan (2006). However, working directly in the limiting case of a large number of firms, plus focusing on a slightly smaller parameter space ($R > 2t$), simplifies the presentation significantly (compare, for instance, Figure 3 in Chen and Riordan, 2006, with Figure 1 in the present paper) and, more importantly, sets the stage for the introduction of multiproduct firms.

4 Multiproduct duopoly

In the presence of economies of scope each firm produces a significant fraction of the total number of varieties. Those multiproduct firms are large relative to the market and hence the market structure is oligopolistic. In order to simplify the presentation I focus on the duopoly case.[6] The aim of this section is to investigate how the strategic incentives of large multiproduct firms affect prices and product diversity.

Let us consider the following game. There are two firms, A and B. In the first stage firms simultaneously choose the fraction of potential varieties they wish to supply, γ_A and $\gamma_{B.}$ In the second stage, after observing γ_A and γ_B, firms simultaneously set prices for all the active varieties. We focus on symmetric subgame perfect equilibria, where $\gamma_A = \gamma_B = \frac{1}{2}\gamma^D$, and all varieties are sold at the same price.

In the second stage, given γ_A and γ_B, a fraction γ_A^2 of consumers will have access to two varieties supplied by firm A, a fraction $2\gamma_A (1 - \gamma_A - \gamma_B)$ will have access only to one of the varieties supplied by firm A, and a fraction $2\gamma_A\gamma_B$ will have access to one variety supplied by firm A and one variety supplied by firm B. Hence, firm A enjoys absolute monopoly power with the first two groups of consumers and competes with firm B for the third group.

Under the maintained assumptions, and like in the previous section, no firm has incentives to set a price above $R - t$. Hence, individual firms' optimization problem can be written as follows. Firm A chooses the price of its varieties, p_A, in order to maximize:

[6] I could have followed Ottaviano and Thisse (1999) or Anderson and de Palma (2006) and assumed that there is an arbitrary number of potential firms, each one of them must pay an entry cost, G, and a fixed cost $F\gamma$, which is proportional to the fraction of varieties produced by the firm, γ. Clearly, there would be values of G for which the equilibrium configuration is a duopoly.

$$\pi_A = \left[\gamma_A^2 + 2\gamma_A (1-\gamma_A-\gamma_B) + 2\gamma_A\gamma_B\left(\frac{1}{2}+\frac{p_B-p_A}{2t}\right)\right] p_A - 2\gamma_A F$$

subject to $p_A + t \leq R$.

For the moment, let us consider the case that the constraint is not binding. Then, firm $A's$ reaction function is:

$$p_A = \frac{t}{2\gamma_B}(2-\gamma_A-\gamma_B) + \frac{p_B}{2}$$

As usual reaction functions are upward sloping (prices are strategic complements). More interesting is the effect of the fraction of varieties supplied by each firm on the optimal price. First, p_A decreases with $\gamma_{A.}$ The reason is that the fraction of total consumers that have to choose between two varieties supplied by different firms, $\frac{2\gamma_A\gamma_B}{\gamma_A^2+2\gamma_A(1-\gamma_A-\gamma_B)+2\gamma_A\gamma_B}$, increases with γ_A. Second, p_A decreases with γ_B. A higher γ_B reduces the fraction of consumers that can only buy from firm A and raises the fraction of consumers that have two options.

Firm $B's$ reaction function is symmetric. The equilibrium firm A 's price for a given pair (γ_A, γ_B) is:

$$p_A = \frac{t}{3}\frac{(2-\gamma_A-\gamma_B)(2\gamma_A+\gamma_B)}{\gamma_A\gamma_B} \tag{5}$$

The properties of equilibrium prices warrant several comments. First, if $\gamma_A = \gamma_B = \frac{1}{2}$, then $p_A = p_B = 2t$. That is, if all potential varieties are supplied (and firms hold a symmetric position), then prices are higher under duopoly $(2t)$ than under monopolistic competition (t). In fact, a duopolistic firm can be interpreted as a coalition of one half of monopolistically competitive firms. By raising the price above the monopolistically competitive level a duopolistic firm can raise its profits, since two thirds of its potential consumers are able to choose between two varieties supplied by different firms, but one third are trapped and can only choose between two varieties supplied by the same firm. Second, $p_A > p_B$ if and only if $\gamma_A > \gamma_B$. Thus, a firm with a higher number of varieties is able to charge a higher price. The reason is that an increase in γ_B has a larger impact on p_A than on p_B. Third, p_A decreases with both γ_A and γ_B. That is, the higher the fraction of varieties supplied by either firm the lower the prices.

By plugging equilibrium prices in the profit function we obtain firm A's payoff as seen from the first stage, i.e., profits as a function of γ_A and γ_B :

$$\pi_A(\gamma_A, \gamma_B) = \frac{t}{9\gamma_A\gamma_B}(2 - \gamma_A - \gamma_B)^2(2\gamma_A + \gamma_B)^2 - 2\gamma_A F$$

The first order condition evaluated at $\gamma_A = \gamma_B = \frac{1}{2}\gamma^D$ characterizes the fraction of active varieties in the symmetric equilibrium under duopoly:

$$\frac{(2 - \gamma^D)(1 - 2\gamma^D)}{\gamma^D} = \frac{3F}{2t} \tag{6}$$

Note that $\gamma^D < \frac{1}{2}$. Otherwise, the left hand side of equation (6) would be negative.

Equation (6) characterizes the equilibrium fraction of varieties under duopoly only if constraint $p_A + t \leq R$ is not binding. Using price equation (5), this condition can be written as:

$$\gamma^D \geq \frac{4t}{R + t} \tag{7}$$

If condition (7) fails then firms set their price equal to $R - t$, and profits are given by:

$$\pi_A = \gamma_A(2 - \gamma_A - \gamma_B)(R - t) - 2\gamma_A F$$

From the first order condition with respect to γ_A, evaluated at $\gamma_A = \gamma_B = \frac{1}{2}\gamma^D$:

$$\gamma^D = \frac{4}{3}\frac{R - t - F}{R - t} \tag{8}$$

Thus, we need to split the parameter space into three different regions.

Region A) If $R < 3t$ then condition (7) is never satisfied, since

$$\frac{4t}{R + t} > 1$$

Hence, γ^D is given by equation (8) (See Graph 2a and Appendix for details).

Region B) If $7t > R > 3t$, then $\frac{1}{2} < \frac{4t}{R+t} < 1$, and hence γ^D is given by equation (8) if $R - t \geq F \geq \frac{(R-t)(R-2t)}{R+t}$ and equal to $\frac{4t}{R+t}$ if $0 \leq F \leq .\frac{(R-t)(R-2t)}{R+t}$. See Graph 2b and Appendix for details.

Region C) If $R > 7t$ then $\frac{1}{2} > \frac{4t}{R+t}$ and hence the function $\gamma^D(F)$ has a different specification in each of the three regions. If $R - t \geq F \geq \frac{(R-t)(R-2t)}{R+t}$ then γ^D is given by equation (8), if $\frac{(R-t)(R-7t)}{3(R+t)} \leq F \leq . \frac{(R-t)(R-2t)}{R+t}$ then γ^D is equal to $\frac{4t}{R+t}$, and finally if $0 \leq F \leq \frac{(R-t)(R-7t)}{3(R+t)}$ then γ^D is given implicitly by equation (6). See Graph 2c.

If we compare γ^D with γ^{MC} then we obtain a very clear-cut result:

Proposition 2 *Under duopoly the fraction of varieties supplied in equilibrium is lower than under monopolistic competition. In fact, it is strictly lower for all values of F for which $\gamma^{MC} \in (0, 1)$.*

Under duopoly firms have incentives to introduce less products than under monopolistic competition for two reasons. First, if a multiproduct firm introduces a new variety it anticipates that some of the buyers are already customers of the firm (cannibalization effect). Second, beyond a certain threshold, a higher number of varieties triggers lower equilibrium prices and as a result lower profits. In other words, duopolistic firms restrain themselves from introducing too many varieties in order to relax price competition (price strategic effect).

Let us now compare γ^D and γ^* (See Graphs 3a, 3b, and 3c, and Appendix for details). It is convenient to introduce some notation for crucial threshold values of the fixed cost: $F_a^D = \frac{(R-t)\left(R-\frac{3}{2}t\right)}{R}$, $F_b^D = \frac{R^2-\frac{7}{2}tR+\frac{5}{2}t^2}{R+t}$.

Proposition 3 *Under duopoly: (i) If $R < 3t$, then there is excessive entry if $\frac{t}{4} < F < F_a^D$, and insufficient entry if $F_a^D < F < R - \frac{t}{2}$, and (ii) If $R > 3t$, then there is insufficient entry if $0 < F < F_b^D$ or if $F_a^D < F < R - \frac{t}{2}$, and excessive entry if $F_b^D < F < F_a^D$.*

Despite the dampening effect of multiproduct firms on product diversity, there are still parameter regions with excessive product diversity. If $R < 3t$ then excessive entry occurs for relatively low values of F. In this case the business stealing effect (γ^D is close to one) dominates the cannibalization effect, while the strategic price effect is irrelevant (equilibrium prices are equal to the monopoly level). In contrast, if $R > 3t$ excessive product diversity occurs for intermediate values of F. The reason is that for very low values of F the strategic price effect dominates. That is, if firms were to introduce additional varieties this would trigger more aggressive pricing behavior.

5 Entry deterrence and asymmetric competition

Can an incumbent firm crowd the product space and make further entry unprofitable? What is the outcome of such preemption efforts in terms of product selection and prices? This question has been examined in the context of standard spatial models (See Schmalensee, 1978; and Bonanno, 1987).

Consider the following three stage game. There is a large firm, L, who can supply an arbitrary number of varieties, and a large number of small firms, each one of them able to supply a single variety (a competitive fringe). For simplicity all firms face the same fixed cost F for each product and zero marginal costs.[7] In the first stage, firm L chooses γ_L. In the second stage, small firms decide whether or not to enter, and hence the mass of small active firms, γ_C, is determined. In the third stage, firms simultaneously set the prices for those varieties that have been activated.

In the third stage, given (γ_L, γ_C) and the price set by the competitive fringe, p_C, firm L chooses p_L in order to maximize (the problem is analogous to the pricing policy of a duopolist in the previous section):

$$\pi_L = \left[\gamma_L^2 + 2\gamma_L(1-\gamma_L-\gamma_C) + 2\gamma_L\gamma_C\left(\frac{1}{2}+\frac{p_C-p_L}{2t}\right)\right]p_L - 2\gamma_L F$$

subject to $p_L \leq R - t$. If the constraint is not binding then the reaction function is given by:

$$p_L = t\frac{2-\gamma_L-\gamma_C}{2\gamma_C} + \frac{p_C}{2} \tag{9}$$

A small firm i chooses p_i in order to maximize:

$$\pi_i = \left[1-\gamma_L-\gamma_C+\gamma_C\left(\frac{1}{2}+\frac{p_C-p_i}{2t}\right)+\gamma_L\left(\frac{1}{2}+\frac{p_L-p_i}{2t}\right)\right]p_i - F$$

[7] One possible justification is that there are no economies of scope, but there is a firm with a first-mover advantage (an incumbent monopolist). A second justification is that there are two possible production technologies. A firm can activate a single variety by paying a fixed cost F or, alternatively, a firm can activate any fraction of potential varieties, γ, by paying a fixed cost $G + f\gamma$, where $f < F$. There is a region of parameter values for which the equilibrium configuration is the one considered in this section.

subject to $p_L - t \leq p_i \leq R - t$. If these constraints are not binding then the reaction function is given by:

$$p_i = t\frac{2-\gamma_L-\gamma_C}{2(\gamma_L+\gamma_C)} + \frac{\gamma_L}{\gamma_L+\gamma_C}\frac{p_L}{2} + \frac{\gamma_C}{\gamma_L+\gamma_C}\frac{p_C}{2} \tag{10}$$

Let us consider the case that F takes values in the interval $\left[\frac{(R-t)^2}{R}, R-t\right]$. Under monopolistic competition equilibrium prices are equal to the monopoly level, $p^{MC} = R - t$, and the equilibrium value of γ, γ^{MC}, is given by the zero profit condition (4). In the current game, firm L does not find it profitable to set $\gamma_L > \gamma^{MC}$ since this implies that $\pi_L < 0$. For any $\gamma_L \leq \gamma^{MC}$ and assuming that all firms set prices equal to $R - t$, then the zero profit condition of a small firm is identical to the case of monopolistic competition. Therefore, $\gamma_C = \gamma^{MC} - \gamma_L$, and all firms set prices equal to $R-t$ and make zero profits in equilibrium. Hence, the total mass of varieties is the same as in the case of monopolistic competition. Moreover, the incumbent firm cannot exploit its size and first mover advantages.

Suppose now that $F \in \left(\frac{t}{2}, \frac{(R-t)^2}{R}\right)$. In this parameter range $1 > \gamma^{MC} > \frac{2t}{R}$ and $p^{MC} < R - t$. Let us consider the possibility that entry is effectively prevented, $\gamma_C = 0$. In this case, firm L sets the monopoly price in the third stage, $p_L = R - t$. However, such a pricing policy makes further entry more attractive. In fact, a new entrant is bound to make higher profits per product than firm L. In particular, firm L's profits are given by:

$$\pi_L = 2\gamma_L\left[\frac{2-\gamma_L}{2}(R-t) - F\right]$$

and firm i's profits:

$$\pi_i = \left[\frac{2-\gamma_L}{2} + \gamma_L\frac{R-t-p_i}{2t}\right]p_i - F$$

If $p_i = R - t$ then profits per product are the same across firms, $\pi_L = 2\gamma_L\pi_i$. In fact, firm i finds it optimal to set $p_i < R-t$ if and only if $\gamma_L > \frac{2t}{R.}$. If firm L sets $\gamma_L \leq \frac{2t}{R}$ then firm i's optimal price is $p_i = R - t$ and π_i is strictly positive. If $\gamma_L > \frac{2t}{R}$ then $p_i < R - t$ and $\pi_L < 2\gamma_L\pi_i$. Therefore, in this parameter range it is never profitable to prevent entry, $\gamma_C > 0$.

In fact, the total mass of varieties is higher than in the case of monopolistic competition. Note that π_i can be written as a function of the total mass of varieties, $\gamma_C + \gamma_L$, and the average price in the industry, $\widehat{p} = \frac{\gamma_C}{\gamma_L+\gamma_C} p_C + \frac{\gamma_L}{\gamma_L+\gamma_C} p_L$.

$$\pi_i = \left[\frac{2-(\gamma_C+\gamma_L)}{2} + \frac{\gamma_C+\gamma_L}{2t}(\widehat{p}-p_i)\right] p_i - F$$

The reader should remember that in the case of monopolistic competition, in equilibrium we have that $\widehat{p} = p_i$. In contrast, in the present model we have that $p_C = p_i$. Moreover, the higher $\widehat{p}$ the higher firm i's profits in equilibrium.

First of all, if $\gamma_L + \gamma_C \leq \frac{2t}{R}$ then $p_C = p_L = R - t$ and $\pi_i > 0$. Hence, in equilibrium $\gamma_L + \gamma_C > \frac{2t}{R}$. Suppose that (γ_L, γ_C) are such that $p_L = R - t$. In this case, the optimal price of a small firm is below $R - t$, and from the reaction function (10) we have that $p^{MC} < p_C < R - t$. Hence, the average price in the industry is higher than under monopolistic competition and hence, for the same mass of varieties a small firm's profits are higher and hence in equilibrium we have that $\gamma_C + \gamma_L > \gamma^{MC}$. Finally, suppose that (γ_L, γ_C) are such that $p_L < R - t$. Then, firm L's price is given by equation (9), and it can be immediately checked that again $p_L > p_C > p^{MC}$ and hence $\gamma_C + \gamma_L > \gamma^{MC}$.

This discussion is summarized in the following proposition.

Proposition 4 *If* $F \in \left[\frac{(R-t)^2}{R}, R - t\right]$ *then* $\gamma_C + \gamma_L = \gamma^{MC}$ *and the values of* γ_C *and* γ_L *are undetermined (all firms make zero profits). If* $F \in \left(\frac{t}{2}, \frac{(R-t)^2}{R}\right)$ *then* $\gamma_C + \gamma_L > \gamma^{MC}$ *and* $\gamma_C > 0$.

Remark 1 *The presence of a large multiproduct firm generates more product variety by raising average prices.*

Remark 2 *Monopolization is not possible. In particular, an incumbent multiproduct firm cannot make strictly positive profits by preventing further entry.*

In standard location models (see, in particular, Schamalensee, 1978, and Bonanno, 1987) it was shown that an incumbent monopolist may find it optimal to prevent further entry by either crowding out the product space or by choosing the right location pattern. In our model this is not possible. The reason is that in the current set up neighboring effects are absent. That is, in previous spatial models the firm producing the new brand competes only with one or two of the existing brands. Hence, the entrant correctly anticipates that the incumbent firm will react to the entry decision by cutting the price of the competing brands. In contrast, in our set up those neighboring effects are absent and a new brand does not change prices of existing brands, which implies that, for a given number of established brands, incentives to enter increase with concentration.

6 Concluding remarks

In this paper I have examined the role of multiproduct firms in the market provision of product variety. The spokes model provides a very useful set up to compare product diversity under both single product and multiproduct firms. Moreover, the main results can be easily interpreted.

The effect of multiproduct firms on product diversity depends on the size distribution of firms. This point has been illustrated by examining two somewhat extreme market structures. On the one hand, in the symmetric equilibrium of a duopoly game product variety is lower than in the case of single product firms (monopolistic competition). The reason is twofold. An oligopolistic firm has incentives to restrict its product range because of, firstly, the cannibalization effect (introduction of a new variety reduces the revenue obtained from other varieties produced by the same firm) and secondly, the strategic price effect (if a firm selects a broader product range then it induces its rivals to adopt a more aggressive price behavior). Nevertheless, under duopoly product diversity may still be excessive from a social point of view, which typically occurs for intermediate values of the fixed cost.

On the other hand, if a large multiproduct firm (incumbent monopolist) competes with a large number of single product firms (competitive fringe) then product diversity is higher than in the absence of multiproduct firms. The reason is that the presence of the large firm relaxes price competition and encourages entry by small, single product firms. Moreover, since competition is non-localized, the incumbent firm is unable to monopolize the market and deter entry.

These results contribute to a better understanding of the impact of multiproduct firms and nicely complement those obtained in the Spence-Dixit-Stiglitz framework. Moreover, on a more methodological spirit, the analysis indicates that the spokes setup is sufficiently flexible to accommodate multiproduct firms and hence it reinforces the idea that the spokes model proposed by Chen and Riordan (2006) is indeed a useful development within the family of spatial models.

The market structures considered in this paper are extreme and somewhat arbitrary. In some real world markets we do observe firms producing a broad product range competing with firms producing a much more limited product range. Equilibria with asymmetric firms may be caused by first mover advantages, but also by the existence of alternative technologies. Perhaps, firms must incur into a large sunk cost in order to reduce the fixed cost associated to the production of each variety. In any case, to endogenize the market structure seems a natural step forward. However, if we were to follow this avenue we would discussing not only the optimal number of varieties but also the optimal number of firms.

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8 Appendix

8.1 Proof of Proposition 1

Let us first consider the region where $F \in \left[\frac{(R-t)^2}{R}, R-t\right]$, i.e., $\gamma < \frac{2t}{R}$. We need to check that in this region the only symmetric equilibrium involves $p = R - t$. A representative firm i chooses p_i in order to maximize:

$$\pi_i = \left[\gamma\left(\frac{1}{2} + \frac{\overline{p} - p_1}{2t}\right) + (1-\gamma)\frac{R - p_i}{t}\right] p_i$$

subject to $p_i \geq R - t$. Suppose the constraint is not binding. Then, from the first order condition evaluated at $p_i = \overline{p}$, we can compute the price:

$$p(\gamma) = \frac{\gamma t + 2(1-\gamma) R}{4 - 3\gamma}$$

It turns out that $p(0) = \frac{R}{2} < R - t$, and $p'(\gamma) < 0$. We have reached a contradiction. Hence, the only symmetric equilibrium involves $p = R - t$, and the equilibrium value of γ is given by equation (4). Hence, in this region both γ^* and γ^{MC} are linear functions of F. Next, we compute the value of F, denoted F^{MC}, at which $\gamma^* = \gamma^{MC}$:

$$F^{MC} = \frac{(R-t)^2}{R - \frac{t}{2}}$$

Note that $\overline{F} < F^{MC} < R - t$.

Let us now turn to the region where $\frac{t}{2} \leq F \leq \overline{F}$ and hence γ^{MC} is given by equation (3). As noted in the text $\gamma^{MC}\left(F = \frac{t}{2}\right) > \gamma^*\left(F = \frac{t}{2}\right)$. Also, $\gamma^{MC}\left(\overline{F}\right) > \gamma^*\left(\overline{F}\right)$. In fact, $\frac{d\gamma^{MC}}{dF} < 0$ and $\frac{d^2\gamma^{MC}}{dF^2} > 0$. Thus, in principle γ^{MC} and γ^* could cross twice in this region. In this case we would have parameter values with insufficient entry. However, this is not the case and γ^{MC} and γ^* do not cross in this region. The values of the crossing points are given by:

$$\gamma = \frac{R + \frac{t}{2} \pm \sqrt{\left(R + \frac{t}{2}\right)^2 - 4t\left(R - \frac{t}{4}\right)}}{2\left(R - \frac{t}{4}\right)}$$

It turns out that the lowest of these two values is always below $\frac{2t}{R}$.

8.2 Proof of Proposition 2

Let us first consider the region where $F \in \left[\frac{(R-t)^2}{R}, R - t\right]$,i.e., $\gamma < \frac{4t}{R+t}$. Note that $\frac{4t}{R+t} > 1$ if and only if $R > 3t$. We need to check that in this region the only symmetric equilibrium involves $p = R - t$. Firm A chooses p_A in order to maximize:

$$\pi_A = \left[\gamma_A^2 + 2\gamma_A\left(1 - \gamma_A - \gamma_B\right)\frac{R - p_A}{t} + 2\gamma_A\gamma_B\left(\frac{1}{2} + \frac{p_B - p_A}{2t}\right)\right]p_A$$

subject to $R - \frac{t}{2} \geq p_i \geq R - t$. Suppose the constraint is not binding. Then, from the first order condition evaluated at a $p_A = p_B$, and $\gamma_A = \gamma_B = \frac{1}{2}\gamma^D$, we can compute the price:

$$p\left(\gamma^D\right) = \frac{\gamma^D t + 2\left(1 - \gamma^D\right)R}{4 - \frac{7}{2}\gamma^D}$$

It turns out that $p(0) = \frac{R}{2} < R - t$, although $p'(\gamma)$ may be positive if $R < 4t$. In any case, $p(1) = 2t < R - t$. We have reached a contradiction. Hence, the only symmetric equilibrium involves $p = R - t$, and the equilibrium value of γ is given by equation (8).

In Region A we need to check that γ^D and γ^{MC} do not cross each other. First, in the interval $\gamma \in \left(0, \frac{2t}{R}\right)$ both functions are equal to 0 at $F = R - t$, and the slope of γ^D is $-\frac{4}{3}$ which is smaller in absolute value than the slope of γ^{MC} (-2). Second, if γ^D and γ^{MC} cross each other in the interval $\gamma \in \left(\frac{2t}{R}, 1\right)$ then the two candidates to be the crossing points are:

$$\gamma = \frac{2\left(R \pm \sqrt{R^2 - 3tR + t^2}\right)}{3R - t}$$

The highest value is higher than one, and the lowest is below $\frac{2t}{R}$. Hence, as depicted in Graph $2a$, in Region A γ^D is always below γ^{MC}.

In Region B obviously the flat portion of γ^D cannot cross γ^{MC}.

Finally, in Region C it is also straightforward to check that the piece of γ^D given by equation (7) cannot cross γ^{MC}. Note that $\gamma^D < \frac{1}{2}$. The value of F for which $\gamma^{MC} = \frac{1}{2}$ is $\frac{R}{2} - \frac{t}{8} > \frac{(R-t)(R-7t)}{3(R+t)}$.

8.3 Proof of Proposition 3

If $R < 3t$, then γ^* and γ^D are linear functions of F provided $\gamma^*, \gamma^D \in (0,1)$. These two straight lines cross at $F_a^D = \frac{(R-t)\left(R-\frac{3}{2}t\right)}{R}$. Since the slope of γ^D is in absolute value higher than the slope of γ^* the welfare result follows.

If $R > 3t$, then γ^* and γ^D cross not only at F_a^D but also the flat portion of γ^D cross γ^* at $F_b^D = \frac{R^2 - \frac{7}{2}tR + \frac{5}{2}t^2}{R+t}$.Note that, in case $R > 7t$, γ^D is given by equation (6) if $F < \frac{(R-t)(R-7t)}{3(R+t)} < F_b^D$. However, this portion cannot cross γ^*. The reason is that $\gamma^*(F=0) = 1$ and $\gamma^*(F)$ is a concave function, and $\gamma^D(F=0) = \frac{1}{2}$ and $\gamma^D(F)$ is convex. Given these observations, the welfare result follows.

FIGURE 1

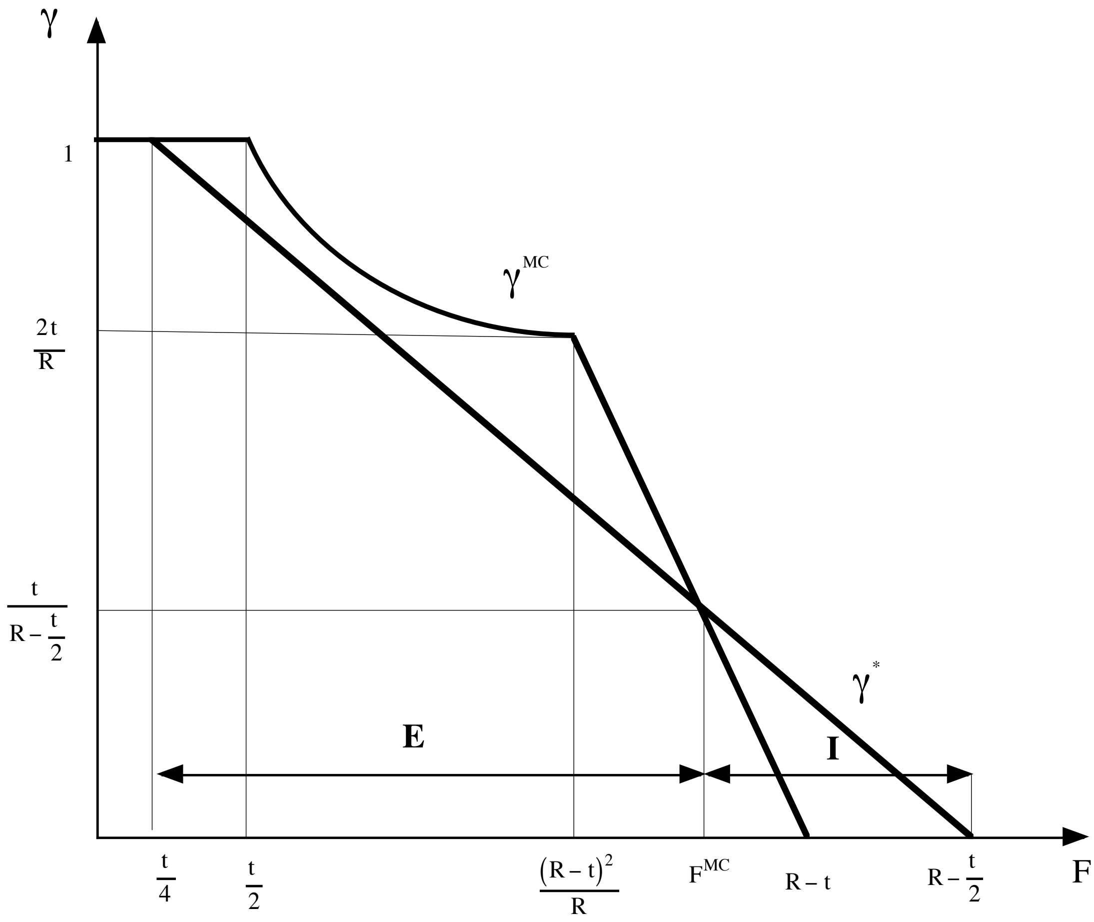

FIGURE 2 a
(R < 3t)

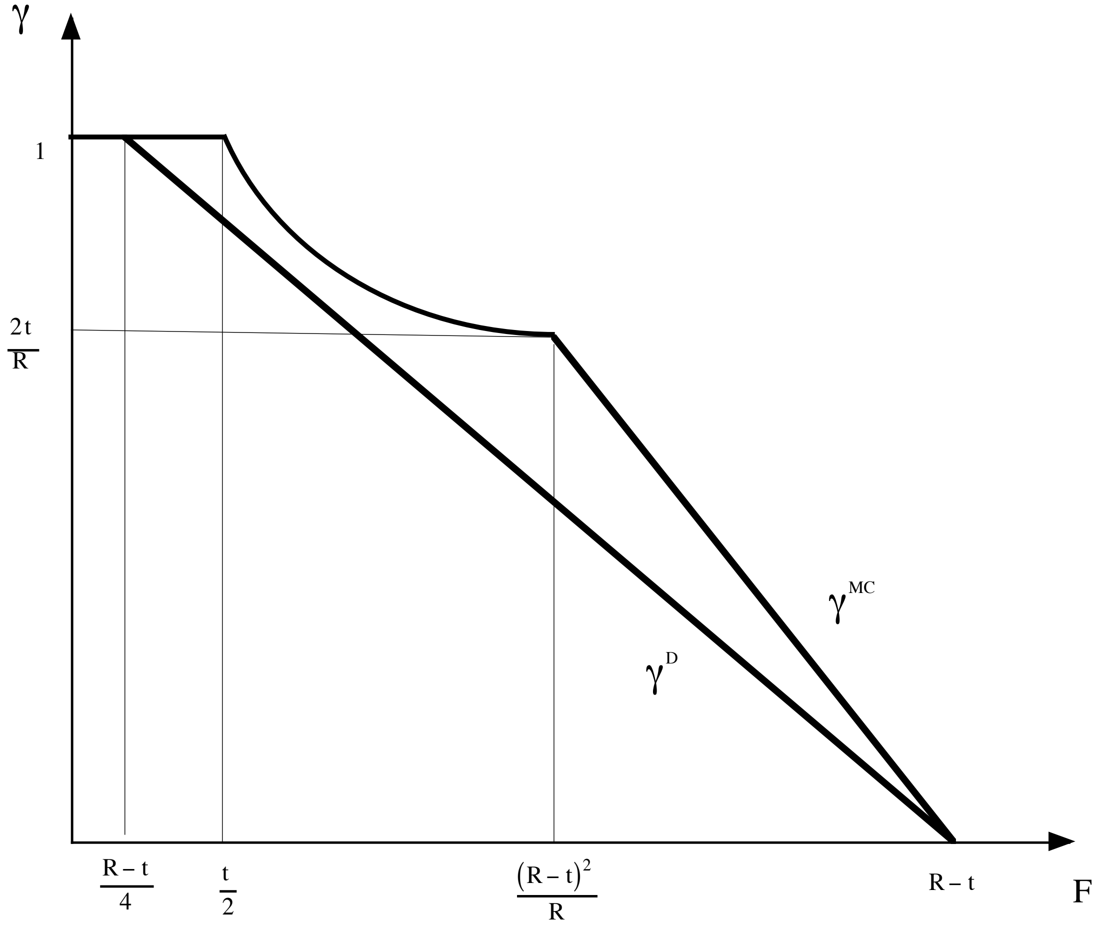

FIGURE 2 b

(7t > R > 3t)

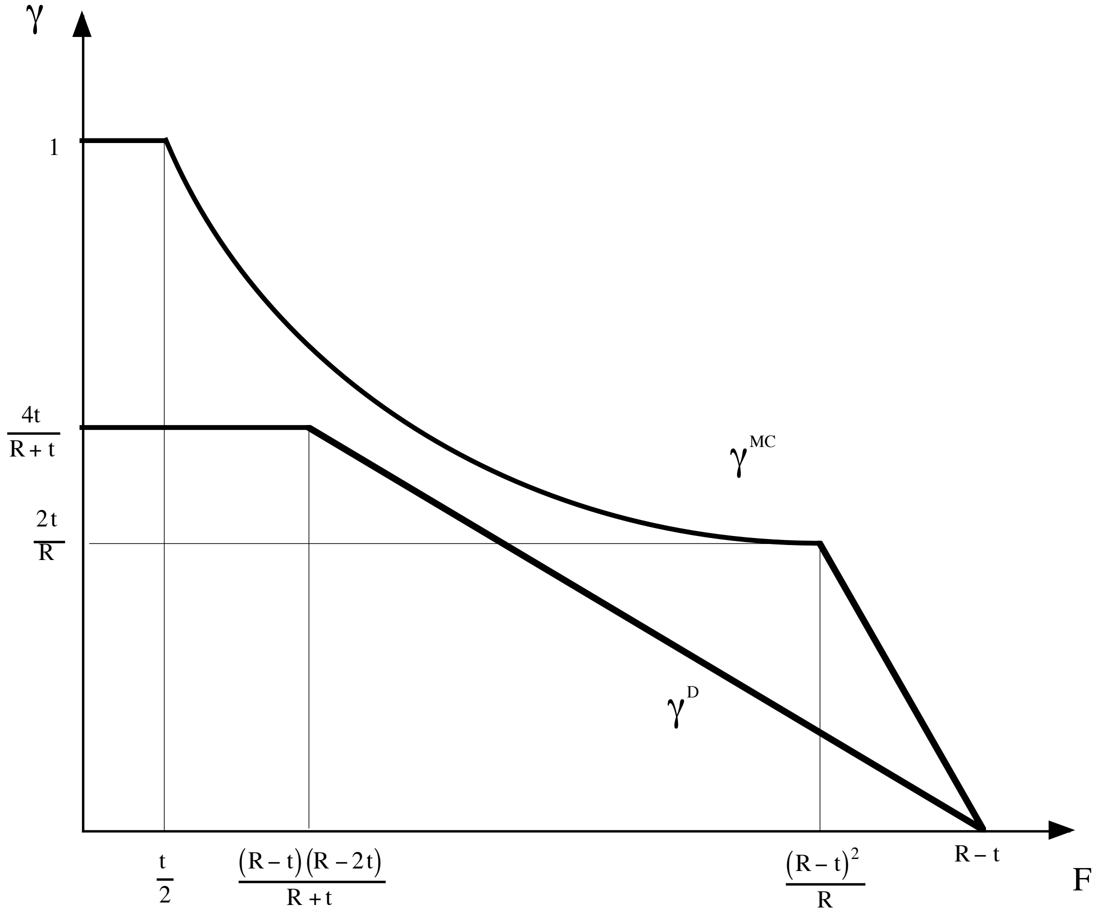

FIGURE 2 c
(R > 7t)

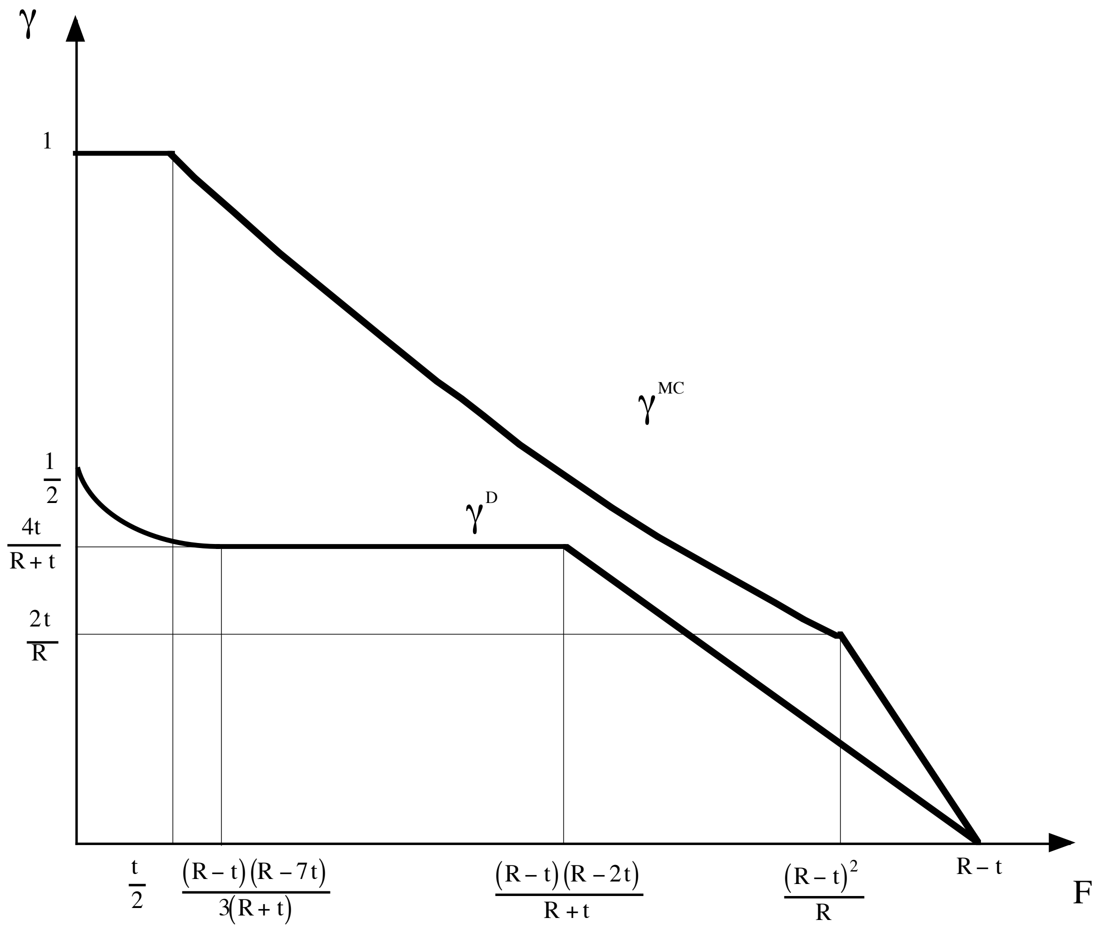

FIGURE 3 a
(R < 3t)

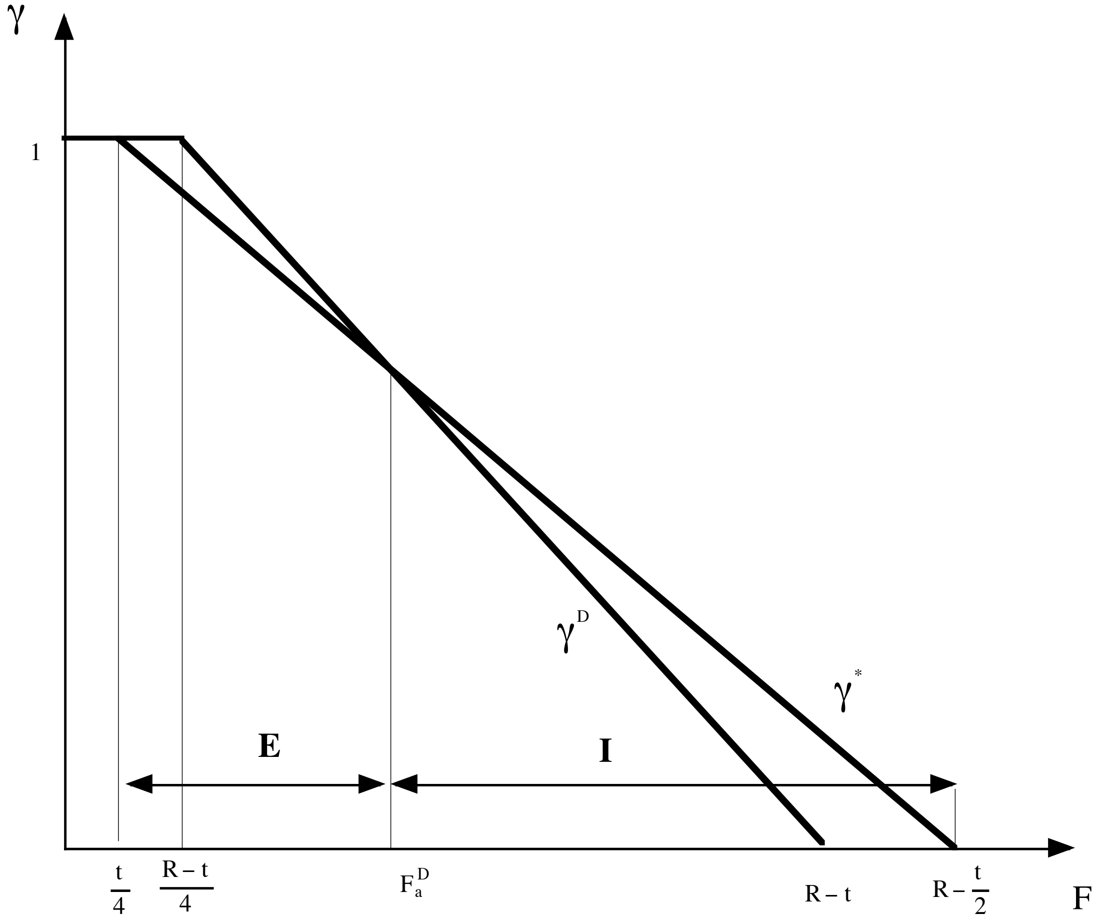

FIGURE 3 b

(7t > R > 3t)

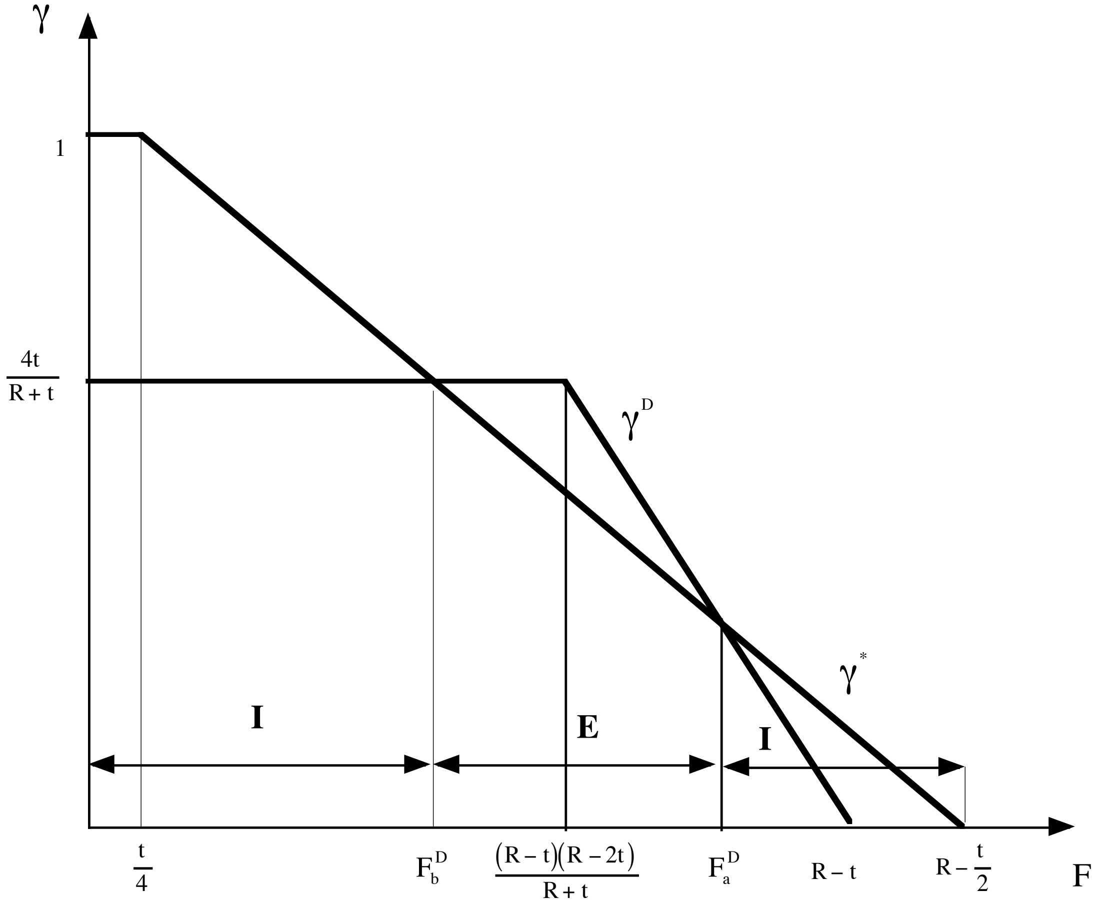

FIGURE 3 c
(R > 7t)