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## ABSTRACT

### Merger Negotiations and Ex-Post Regret\*

We consider a setting in which two potential merger partners each possess private information pertaining both to the profitability of the merged entity and to stand-alone profits, and investigate the extent to which this private information makes ex-post regret an unavoidable phenomenon in merger negotiations. To this end, we consider ex-post mechanisms, which use both players' reports to determine whether or not a merger will take place and what each player will earn in each case. When the outside option of at least one player is known, the efficient merger decision can be implemented by such a mechanism under plausible budget-balance requirements. When neither outside option is known, we show that the potential for regret-free implementation is much more limited, unless the budget balance condition is relaxed to permit money-burning in the case of false reports.

JEL Classification: D82, G34 and L10

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## 1 Introduction

Mergers and acquisitions occur frequently in the corporate landscape. Nevertheless, it often appears that at least one party regrets the outcome of the transaction with the benefit of hindsight. Given the tremendous stakes in many merger decisions and the correspondingly large opportunity cost from accepting a merger at conditions that turn out to be unfavorable ex-post, it seems natural to ask whether such problems are inevitable.

Of course, regret in merger transactions may simply result from bad luck in situations of symmetric uncertainty: Mergers that are profitable in expectation may turn out to be unprofitable in the event of bad states—there is little scope for preventing this problem. Beyond that, internal agency conflicts may hinder regret-free outcomes as, for instance, self-interested managers engage in unprofitable mergers to expand their empire.

In this paper, we investigate an alternative and arguably more basic source of regret: the misalignment of interests between merging parties. To isolate this issue, we confine ourselves to situations where ownership and control in each firm coincide, and where any information that is relevant to the merger decision is present with at least one of the two firms. However, we let parties possess different *pieces* of this information. We then ask: Is it possible to structure the merger negotiations and the decision process in a fashion such that no party regrets its behavior afterwards?

To illustrate our setting, consider the following two examples. First, suppose two firms are privately informed about their own productivity. Typically, both a firm's stand-alone profits and the post-merger profits of an entity to which it belongs should be higher for more productive types. Moreover, if firms compete in the same market, each firm's productivity will usually have a negative impact on the competitor's stand-alone profits. As a second example, suppose two firms have been competing in a stable market environment over an extended period of time. Then, they may be expected to be reasonably well informed about those characteristics of the competitor that pertain to their stand-alone performance. However, they may have very limited knowledge about how they would fit together in the event of a merger, that is, how well the organizational cultures match. Thus, one might still expect substantial private information concerning post-merger profits, whereas—in contrast to the previous example—firms' outside options are common knowledge. This difference between the two examples (i.e., whether outside options are common knowledge or not) will turn out to be crucial to whether regret is avoidable.

In the setting described, merger negotiations have the function of assembling this privately held information, reaching a (preferably efficient) merger decision, and determining how profits are to be split among parties in the event of a merger. Using a mechanism-design approach, we formalize this negotiation process in terms of *merger mechanisms*, where agents (simultaneously) report their information (i.e.,

their ‘type’), and each of the above decisions is made contingent on these reports.

Motivated by our initial question, we restrict attention to merger mechanisms which result in no ex-post regret, that is, mechanisms such that truthful reporting is optimal for each party given any type of the other party and given truthful reporting on the other party’s behalf. In addition, we impose natural budget-balance and individual-rationality conditions. By the former, agents’ payoffs must sum to jointly realized profits for any outcome of the mechanism. By the latter, agents must receive at least their stand-alone profit for any equilibrium outcome of the mechanism.

We derive our first result for settings in which (at least) one party’s stand-alone profits are commonly known. We show that a simple mechanism where, in the event of a merger, the party with commonly known outside options is bought out by the other firm achieves efficient implementation, that is, mergers take place if and only if they increase joint profits.

In the remainder of the paper, however, we show that the scope for extending this positive result to more general settings is extremely limited: For a large class of settings, not only is implementation of the *efficient* merger decision impossible, but in fact *no* non-trivial merger decision can avoid ex-post regret. Specifically, we derive this result for settings such that both players’ stand-alone profits depend positively on own types, but negatively on the competitor’s type, whereas the payoffs of the merged firm depend positively on both types.

An important ingredient to this result is the aforementioned notion of budget balance, by which the mechanism must balance also *off equilibrium*, that is, if agents misreport their type. This property creates a problem of separating type profiles with identical merger profits, so that agents’ equilibrium payoffs cannot differ between such type profiles. Intuitively, on the one hand, as payoffs can be conditioned on actual types only through total merger profits, agents cannot be punished selectively in the event that observed joint mergers are inconsistent with individual reports. On the other hand, budget balance prohibits collective punishment as it requires that merger profits are fully distributed to the merger parties.

This latter point leads us to ask whether the scope for implementation can be increased by allowing money to be burnt in the event of off-equilibrium reports. It turns out that, in this case, the efficient merger decision can always be reached in a regret-free and individually rational way. The practical relevance of this result hinges, however, on arguably rather strong assumptions concerning agents’ ability to commit.

From a mechanism-design perspective, the problem considered in this paper has several distinguishing features. First, parties’ payments are conditionable on ex-post information: A merger mechanism must not only produce a merger decision, but also prescribe how joint profits are to be shared in the event of a merger. In the simplest case, this can be done by allocating fixed profit-shares in any merged entity. More sophisticated financial arrangements allow conditioning agents’ payoffs on reported

types and realized merger profits in an essentially arbitrary manner.<sup>1</sup> As has previously been noted, this ‘contingent-payment’ feature gives the mechanism designer additional degrees of freedom.<sup>2</sup> Second, the problem displays interdependent valuations in that agents’ valuations over outcomes depend on each other’s type. Hence, resorting to standard Vickrey-Clarke-Groves mechanisms is not possible. Third, requiring that merger negotiations always result in a budget-balanced outcome is natural, as there is no obvious candidate for a third-party residual claimant. Fourth and finally, as noted above, we require the mechanism to be regret-free.<sup>3</sup> Eventually, the combination of these features makes it impossible to directly relate our problem to the previous mechanism-design literature.<sup>4</sup>

In a more immediately related paper, Brusco et al. (2005) also investigate the role of asymmetric information in merger negotiations. In contrast to our paper, they search for *efficient* (Bayesian) rather than regret-free mechanisms. Differences in the set-up aside,<sup>5</sup> our findings complement each other by revealing a strong overlap in the attainability of these two goals: First, both papers show that when private knowledge does not pertain to outside options, both efficiency and regret-freeness are attainable. Second, for a large class of remaining settings, our papers combine to show that *neither* goal is attainable.

Interestingly, there has recently been quite some investigation into the efficient

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<sup>1</sup>Practical examples include collars (Officer, 2004), which use changes in stock prices to determine the partners remuneration, and contingent value rights (Hietala et al., 2003), where sellers obtain put options on the shares of the new entity.

<sup>2</sup>Hansen (1985) (see also Crémer, 1987) and Riley (1988) show that, in an auction setting, conditioning payments on ex-post information allows the auctioneer to improve his expected revenues. Mezzetti (2004) demonstrates in a more general setting that the use of ex-post information on payoffs can aid implementation of efficient decision rules.

<sup>3</sup>More generally, one may want to consider Bayesian-Nash implementation, that is, merger mechanisms such that truth-telling is optimal in *expected* terms, with expectations taken over the other party’s possible type. Our restriction to regret-free mechanisms is immediately driven by the question at hand. Nonetheless, we should note that the recent literature provides several independent arguments for this restriction based on robustness concerns. Specifically, these concern the robustness of Bayesian mechanism to players’ having incorrect beliefs about types and other players’ beliefs (see Bergemann and Morris, 2005, Chung and Ely, 2003, and the survey in Jehiel et al., 2006), as well as concerns about the necessity of *simultaneous* information revelation in Bayesian Mechanisms and the related problem of espionage (see Miller, 2005).

<sup>4</sup>For instance, Jehiel et al. (2006) show that non-trivial regret-free implementation is possible only for a very small, degenerate set of mechanism design problems with *multivariate* private information and interdependent valuations. On the other hand, there are many instances not only of non-trivial, but of *efficient* regret-free implementation in more specific settings with interdependent valuations and *univariate* private information, particularly in the context of auctions (see, for instance, Crémer and McLean, 1985; Maskin, 1992; Dasgupta and Maskin, 2000; Esó and Maskin, 2000; Jehiel and Moldovanu, 2001; Bergemann and Välimäki, 2002; Perry and Reny, 2002; Krishna, 2003; Ausubel, 2004). While none of these settings impose any budget-balance requirements, they also lack the contingent-payment feature of our problem, rendering direct comparisons impossible.

<sup>5</sup>On the one hand, Brusco et al.’s set-up allows merger negotiations taking place with more than one potential partner. On the other hand, their setting is in some ways more restrictive regarding the structure imposed on profit functions’ dependence on private information.

*dissolution* of partnerships under interdependent valuations, which in some ways represents the inverse to the problem considered in this paper. This literature works out conditions under which it is possible to efficiently dissolve joint ownership of a firm when valuations for that firm are interdependent.<sup>6</sup> In contrast to our paper, whether it is efficient to dissolve or not is generally not an issue: The social value of the partnership is simply the share-weighted average of agents' individual valuations, so dissolution will be efficient whenever agents' valuations differ. Rather, the question is only one of allocating the single indivisible item owned by the partnership to the agent with the highest valuation. Thus, the problem is strongly related to auction design, with some additional difficulties implied by budget-balance requirements. In our setting, on the other hand, the main question is whether the partnership should be formed or not. In contrast, how shares in the partnership are to be allocated in the event of a merger does not affect efficiency, as agents have pure common valuations concerning the partnership itself.

The paper is organized as follows. Section 2 introduces the basic model, describing both the merger environment and the mechanisms to be considered. Section 3 shows that if at least one party's outside option is common knowledge, efficient implementation is possible. Section 4 presents this paper's main result by showing that, for more generic environments, not only efficient but in fact any kind of non-trivial implementation is impossible. Providing a closer look at budget balance, Section 5 shows that the possibility of "burning money" off equilibrium reverses the largely negative results of Sections 4. Finally, Section 6 concludes.

## 2 The Model

This section presents the basis for our analysis of merger mechanisms. Section 2.1 introduces the general setup and basic terminology. We motivate our framework with specific examples in Section 2.2.

### 2.1 Merger Environment and Mechanism

We consider merger mechanisms in an environment of the following type:

**Definition 2.1.** A *merger environment*  $\mathcal{E}$  is a tuple  $(T_1, T_2, \pi_1, \pi_2, \pi^M)$  with the following components:

- (i)  $T_i = [0, 1]$ ,  $i = 1, 2$ , is the scalar-valued *type space* for firm  $i$ ;
- (ii)  $\pi_i : T_1 \times T_2 \rightarrow \mathbb{R}_{\geq 0}$  is the *stand-alone profit function* for firm  $i$ ;
- (iii)  $\pi^M : T_1 \times T_2 \rightarrow \mathbb{R}_{\geq 0}$  is the *merger profit function*.

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<sup>6</sup>See Fieseler et al. (2003), Jehiel and Paudner (2006), Ornelas and Turner (2004) and the survey in Moldovanu (2002).

Thus, an environment describes how each firm's stand-alone profits as well as joint profits in the event of a merger depend on firms' private information, defined over the joint type-space  $\mathbf{T} \equiv T_1 \times T_2$ . We will assume that  $\pi_1$ ,  $\pi_2$  and  $\pi^M$  are continuous.<sup>7</sup>

For such an environment, we investigate mechanisms which use a system of transfers and—provided that a merger occurs—allocation of the merged entity's profits. Anticipating the usual revelation argument, we restrict our attention to *direct* merger mechanisms, for which each agent's report is restricted to his type space:<sup>8</sup>

**Definition 2.2.** A (*direct*) *merger mechanism*  $\mathcal{M}$  is a tuple  $(m, \hat{\pi}^M, p^0)$  consisting of a merger decision function  $m$ , merger-profit sharing rules  $\hat{\pi}^M = (\hat{\pi}_1^M, \hat{\pi}_2^M)$ , and transfer functions  $p^0 = (p_1^0, p_2^0)$ , which are defined as follows:

- (i) The *merger decision function*  $m : \mathbf{T} \rightarrow \{0, 1\}$  maps a combination of reports  $\tilde{\mathbf{t}} = (\tilde{t}_1, \tilde{t}_2)$  by players 1 and 2 about their type into a merger decision, with  $m(\tilde{\mathbf{t}}) = 1$  if and only if the merger takes place as a result of the reports. We let  $M^0 \equiv m^{-1}(0)$  and  $M^1 \equiv m^{-1}(1)$ .
- (ii) The *transfer functions*  $p_i^0 : M^0 \rightarrow \mathbb{R}$ ,  $i \in \{1, 2\}$ , assign a transfer payment from player  $i$  to the mechanism operator for reported types  $\tilde{\mathbf{t}} \in M^0$ .
- (iii) The *merger-profit sharing rules*  $\hat{\pi}_i^M : M^1 \times \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}$ ,  $i \in \{1, 2\}$ , map reports  $\tilde{\mathbf{t}} \in M^1$  and realized merger profits  $\pi^M(\mathbf{t})$  into a payoff to firm  $i$ .

Thus, a merger mechanism's merger decision  $m$  produces a partition of type space  $\mathbf{T}$  into type profiles  $M^0$  such that no merger occurs and type profiles  $M^1$  such that a merger occurs. For simplicity, we consider only merger mechanisms that are *well-behaved* in the sense that (i) the boundary between  $M^0$  and  $M^1$  is almost everywhere smooth, and (ii) for every  $\mathbf{t} \in M^1$  and every  $\varepsilon$ -neighborhood  $U_\varepsilon(\mathbf{t})$ , there exists a point  $\mathbf{t}' \in U_\varepsilon(\mathbf{t})$  in the interior of  $M^1$ .<sup>9</sup>

Letting  $u_i(\tilde{\mathbf{t}}; \mathbf{t})$  denote agent  $i$ 's payoffs for any combination of reported and true types  $\tilde{\mathbf{t}}, \mathbf{t} \in \mathbf{T}$ ,

$$u_i(\tilde{\mathbf{t}}; \mathbf{t}) = \begin{cases} \pi_i(\mathbf{t}) - p_i^0(\tilde{\mathbf{t}}), & \text{if } \tilde{\mathbf{t}} \in M^0, \\ \hat{\pi}_i^M[\tilde{\mathbf{t}}; \pi^M(\mathbf{t})], & \text{if } \tilde{\mathbf{t}} \in M^1. \end{cases} \quad (1)$$

<sup>7</sup>Moreover, note that the above definition assumes that each agent's private information is univariate. Given the largely negative results concerning efficient implementation under interdependent valuations with *multi-dimensional* as opposed to *univariate* information in auction settings (Jehiel and Moldovanu, 2001) and in more general settings (Jehiel et al., 2006), scalar-valued information should improve the scope for efficient implementation in our merger setting.

<sup>8</sup>Importantly, we assume that the merger decision does not reveal any information that influences the strategic interaction between the two parties after the merger. Otherwise the merger process would have what Jehiel and Moldovanu (2006) call 'informational externalities' on the ensuing interaction, and the revelation principle will no longer apply. Hviid and Prendergast (1993) investigate such informational externalities in the context of mergers with *one-sided* asymmetric information.

<sup>9</sup>This will, for instance, exclude merger sets that contain isolated lines or isolated points.

In the simplest case, agents' payoffs over  $M^1$ ,  $\hat{\pi}_i^M(\cdot)$ , may be thought of as arising through (report-dependent) allocations of shares in the merged entity and accompanying payments. More generally, the use of more sophisticated securities will permit an arbitrary conditioning of payoffs on reports and true post-merger profits.<sup>10</sup>

For reasons given in the introduction, we shall confine ourselves to finding *ex-post* implementable mechanisms, that is, mechanisms for which truthful reporting is a best response to any given type of the competitor, *given* truthful reporting by the other type. Formally, letting  $U_i(\tilde{t}_i; t_i, t_j) = u_i(\tilde{t}_i, t_j; t_i, t_j)$  denote firm  $i$ 's payoff given truthful reporting by the other firm, the requirement of *ex-post* incentive compatibility can be compactly formulated as follows:

**Definition 2.3.** A merger mechanism  $\mathcal{M}$  is (*ex-post*) *incentive compatible (IC)* if

$$U_i(t_i; t_i, t_j) \geq U_i(\tilde{t}_i; t_i, t_j), \quad \text{for all } i \in \{1, 2\}, \tilde{t}_i, t_i \in T_i, t_j \in T_j. \quad (2)$$

We shall call a merger decision function  $m$  *implementable* if there exist  $\hat{\pi}^M$  and  $p^0$  such that the mechanism  $(m, \hat{\pi}^M, p^0)$  is incentive compatible.

Efficiency of a merger decision function is defined as follows:

**Definition 2.4.** A merger decision function  $m$  is *efficient* if, for all  $\mathbf{t} \in \mathbf{T}$ ,

$$m(\mathbf{t}) = \begin{cases} 0, & \text{if } \pi^M(\mathbf{t}) < \pi_1(\mathbf{t}) + \pi_2(\mathbf{t}), \\ 1, & \text{if } \pi^M(\mathbf{t}) > \pi_1(\mathbf{t}) + \pi_2(\mathbf{t}). \end{cases} \quad (3)$$

We call a merger mechanism  $\mathcal{M} = (m, \hat{\pi}^M, p^0)$  *efficient* if and only if  $m$  is efficient.

Since it is natural to assume that no third party should benefit from or subsidize the mechanism, we introduce the following condition:

**Definition 2.5.** A merger mechanism is *budget balanced (BB)* if, for all  $\tilde{\mathbf{t}}, \mathbf{t} \in \mathbf{T}$ ,

$$u_1(\tilde{\mathbf{t}}; \mathbf{t}) + u_2(\tilde{\mathbf{t}}; \mathbf{t}) = \begin{cases} \pi_1(\mathbf{t}) + \pi_2(\mathbf{t}), & \text{if } \mathbf{t} \in M^0, \\ \pi^M(\mathbf{t}), & \text{if } \mathbf{t} \in M^1. \end{cases} \quad (4)$$

Importantly, this budget balance condition is required to hold not only for truthful reports.<sup>11</sup>

Finally, letting  $V_i(t_i, t_j) \equiv U_i(t_i; t_i, t_j)$  denote agent  $i$ 's *equilibrium payoff function* (or *value function*), mechanisms in which agents will voluntarily participate must satisfy the following requirement:

**Definition 2.6.** A merger mechanism is *individually rational (IR)* if

$$V_i(\mathbf{t}) \geq \pi_i(\mathbf{t}), \quad \text{for all } i \in \{1, 2\}, \mathbf{t} \in \mathbf{T}. \quad (5)$$

<sup>10</sup>Observe that the transfer function  $p_i^0$  is only defined over  $M^0$  since over  $M^1$ , transfers may be included w.l.o.g. in each merger-profit sharing rule  $\hat{\pi}^M$ .

<sup>11</sup>Such a condition has typically been invoked also in the aforementioned literature on partnership dissolution (cf. Jehiel and Pauzner, 2006; Ornelas and Turner, 2004).

Consistent with our notion of *ex-post* incentive compatibility, note that (IR) requires *ex-post* rationality in that agents' equilibrium payoffs must exceed their outside option for any possible type of the other player.

Finally, observe that there is one particular mechanism which—albeit typically being highly inefficient—satisfies all of the above constraints, namely the mechanism which reproduces the status quo by prescribing (i) never merge, and (ii) always require zero payments. Such a mechanism satisfies (IC), (BB), and (IR).

## 2.2 Examples of Merger Environments

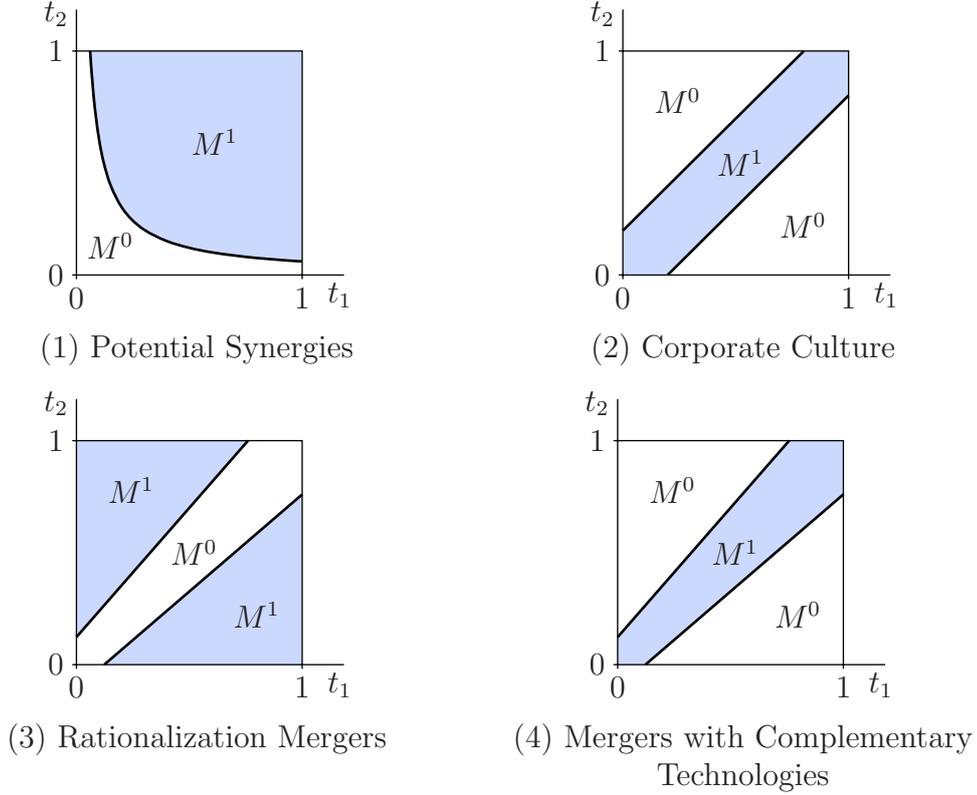
We illustrate our set-up with a few exemplary merger environments. The examples offer different instances of what the relevant private information  $t_1, t_2$  consists in and, correspondingly, how it relates to stand-alone and post-merger profits.

**Example 1 (Potential Synergies).** Suppose each firm holds private information which concerns the synergy potentials in the event of a merger, but which is relevant to neither firm's stand-alone operations. This information may relate to the ease with which each firm's production equipment, its workforce, or its organizational structure (including sales operations, etc.) can be integrated into a joint entity. More generally, it may relate to the profitability of some new business venture which is feasible only to a merged entity. If a higher value of  $t_i$  signals higher synergy potentials, efficiency requires that some set of types above a downward-sloping line engages in a merger, as illustrated in panel (1) of Figure 1.

**Example 2 (Corporate Culture).** As in the first example, suppose firms' private information is only relevant to post-merger profits. Suppose, however, that post-merger profits depend positively only on how *similar* firms' privately known types are. For instance, types could represent different patterns of corporate culture, all of which perform equally well in autarky, but where merging more dissimilar ones involves higher frictional losses.<sup>12</sup> Then the efficient set will typically contain points sufficiently close to the diagonal, as in panel (2) of Figure 1.

Examples 1 and 2 above have both assumed private information to be relevant only in the event of a merger. However, if a firms' private information concerns factors such as production costs, workforce quality, financial health, demand forecasts, etc., this information will typically be relevant both to profits in the event of a merger *and* to the firm's stand-alone profits if no merger occurs. Moreover, provided some form of interaction between the firms—such as by firms operating on related markets—each firm's private information is relevant also to the competitor's profit if no merger occurs. Adding mild further conditions on the *direction* in which

<sup>12</sup>Many case studies attribute failed mergers to incompatible corporate cultures of the merging firms; see Larsson and Finkelstein (1999).



**Figure 1:** Efficient Merger Decisions for Examples of Section 2.2.

information affects each profit function, we shall call such a merger environment ‘competitive:’

**Definition 2.7.** A merger environment is *competitive* if it is such that (i)  $\partial\pi_i/\partial t_i > 0$ , (ii)  $\partial\pi_i/\partial t_j \leq 0$ , and (iii)  $\partial\pi^M/\partial t_i > 0$  for all  $i, j \in \{1, 2\}$ ,  $i \neq j$ .

Thus, in a competitive merger environment, any good news held by a firm concerning its stand-alone profits translates into good news concerning the prospects of a merged firm, but will be detrimental to the competitor if no merger occurs. In spite of this common structure, competitive merger environments can still produce very different efficient merger sets, as the following two examples illustrate:

**Example 3 (Rationalization Mergers).** Consider a competitive merger environment in which a firm’s private information concerns its technological know-how. If this known-how is easily transferrable to the technologically inferior firm’s production process in the event of a merger, then merging will typically be more efficient for more unequal types, where this ‘rationalization-effect’ is largest. Hence, the efficient set will typically consist of points away from the diagonal, as in panel (3) of Figure 1.

**Example 4 (Mergers with Complementary Technologies).** Assume next that, in contrast to Example 3, the more efficient firm's advantage is *not* transferrable in the event of a merger (such as if its advantage derives, for instance, from a better-trained workforce). At the extreme, if the *inferior* firm's technology is decisive to the merged entity's profitability, which may happen when the assets brought into the relationship are highly complementary, merging will typically be more efficient when the difference in firms' efficiency levels is small, so that the efficiency loss incurred by the more efficient firm is small. As illustrated in panel (4) of Figure 1, qualitatively, the efficient merger decision will correspond to the inverse of that under rationalization. Finally, scenarios in between these two polar cases will produce essentially arbitrary other partitions of type-space.

### 3 Implementation with a Known Outside Option

In this section, we restrict attention to environments where the outside option of at least one player is commonly known. Clearly, this setting contains Examples 1 and 2 of the last section as special cases, where both outside options were commonly known. The main result shows that efficient implementation is possible in such a setting:

**Proposition 3.1.** *If the merger environment is such that neither agent holds any private information on some agent  $j$ 's stand-alone profits, so  $\pi_j$  is constant in  $\mathbf{t}$  for some  $j \in \{1, 2\}$ , then any efficient merger decision can be implemented by a mechanism satisfying (BB) and (IR) by agent  $i \neq j$  obtaining the full merger profits in the event of a merger and paying  $\pi_j$  to agent  $j$ , and zero transfers if no merger occurs.*

Under such a mechanism, agent  $j$  will always obtain a payoff of  $U_j(\tilde{t}_j; \mathbf{t}) = \pi_j$  no matter what signal  $\tilde{t}_j$  he sends, so that truth-telling will always be weakly optimal. Concerning agent  $i$ , observe that transfers and shares are unaffected by any deviations which leave the merger decision unaffected, so that we only need to show incentive compatibility for deviations which affect the merger decision. Given any  $\mathbf{t}$ , agent  $i$ 's payoff will be  $\pi^M(\mathbf{t}) - \pi_j$  from merging, and  $\pi_i(\mathbf{t})$  from not merging. Hence, agent  $i$ 's gains to merging are  $\pi^M(\mathbf{t}) - \pi_i(\mathbf{t}) - \pi_j$ , which will be positive for any type constellation such that merging is efficient, and negative for any type constellation such that merging is inefficient. Therefore, given efficiency of the merger decision function  $m$ , a type such that merging is efficient (given the other type) will lose from deviating from truth-telling, and so will a type such that no merger occurs. It is trivially checked that the described mechanism satisfies (BB) and (IR).

Intuitively, the mechanism employed in Proposition 3.1 makes the party whose outside option is private knowledge the residual claimant, thereby perfectly aligning this party's incentives with the efficiency goal. The other party in turn will obtain

a payoff equal to its known stand-alone profit *irrespective* of reports and merger decision, and will therefore have no incentive to misreport.<sup>13</sup>

A trivial corollary of Proposition 3.1 is that implementation of the efficient merger decision is also possible if *both* agents' stand-alone profits are common knowledge, that is, if each agent's private information pertains only to profits under a merger.<sup>14</sup> Thus, both the 'Potential Synergy'- and 'Corporate Culture'-examples discussed in Section 2.2 permit implementation of the efficient decision.

A weak point of the mechanism in Proposition 3.1 is that truth-telling will be only *weakly* optimal for the party whose stand-alone profits are known. This is clearly and undesirable property, but not uncommon in related problems (see e.g. Mezzetti, 2004). However, if *both* parties' stand-alone profits are known (as in Examples 1 and 2 above)—so private information concerns only merger profits  $\pi^M$ —then efficiency can be achieved by means of a mechanism satisfying (IC), (BB) and (IR) for which truth-telling is *strictly* optimal. The simplest way to achieve this is to split merger profits in proportion to firms' (constant) stand-alone profits, so that  $\hat{\pi}_i^M(\tilde{\mathbf{t}}; \mathbf{t}) = [\pi_i / (\pi_1 + \pi_2)] \cdot \pi^M(\mathbf{t})$ . More generally, by complementary use of transfers over  $M^1$ , arbitrary divisions of merger profits are implementable along with an efficient merger decision. As long as each agent receives a strictly positive share, truth-telling will be strictly optimal for such mechanisms.

## 4 Implementation with Unknown Outside Options

Contrasting Proposition 3.1, we will now show that, within the class of competitive merger environments specified in Definition 2.7, not only is implementing the efficient merger decision generally impossible, but that any kind of second-best solution can only rather trivial merger decision functions.

We derive this result by identifying restrictions which (IC), (IR) and (BB) impose on agents' value functions  $V_i$ . Sections 4.1 and 4.2 identify restrictions which apply over  $M^0$  and  $M^1$ , respectively, in order to prevent deviations from truth-telling which leave the merger decision unaffected. Section 4.3 in turn identifies restrictions which need to be met in order to prevent deviations in reporting that *alter* the merger decision, which results in properties to be met by the valuation functions  $V_i$  on the boundary between  $M^0$  and  $M^1$ . Ultimately, Section 4.4 assembles these restrictions to formulate this section's main result concerning the limited scope for

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<sup>13</sup>This essentially corresponds to the mechanism proposed by Crémer (1987) in a setting where a 'target' firm with known outside option is up for sale to a number of potential acquirers whose outside option is also known, but who possess private information concerning joint profits. Crémer shows that—in contrast to a pure cash auction—the target can come arbitrarily close to full surplus extraction by a contingent value auction such that the target receives essentially full residual claims to the merged entity and buys out the winning bidder by a cash transfer.

<sup>14</sup>This result parallels Proposition 2 in Brusco et al. (2005), who fittingly use the term 'acquisition mechanisms' for merger mechanisms of this specific type.

implementation in competitive merger environments.

#### 4.1 Restrictions on the Mechanism over $M^0$

Local incentive constraints over  $M^0$  have a simple implication: For any  $\tilde{\mathbf{t}}, \tilde{\mathbf{t}}'' \in M^0$  with  $\tilde{t}_j = \tilde{t}''_j$ , (IC) for agent  $i \neq j$  immediately implies  $p_i^0(\tilde{\mathbf{t}}) = p_i^0(\tilde{\mathbf{t}}'')$ , so that transfers  $p_i^0$  must be independent of the own report. The simple intuition for this is that over  $M^0$ , the returns to changing one's report are independent of the true type profile  $\mathbf{t}$  due to the additive separability of agents' payoff functions in true and reported types. Moreover, (BB) then immediately implies also  $p_j^0(\tilde{\mathbf{t}}) = p_j^0(\tilde{\mathbf{t}}'')$ , that is, the other agent's transfer must also be independent of the own report. Iterated application of this argument immediately leads to the insight that, over connected subsets of  $M^0$ , each  $p_i^0$  must be constant.<sup>15</sup> Moreover, it is straightforward to see that requiring (IR) in addition immediately implies that transfers are zero. The following lemma collects these results:

**Lemma 4.1.** *For any mechanism satisfying (IC) and (BB),*

- (a) *transfers  $p_i^0$  must be constant in reports over any open connected subset of  $M^0$ ;*
- (b)  *$p_i^0(\tilde{\mathbf{t}}) = 0$  for any  $\tilde{\mathbf{t}} \in M^0$  if (IR) is required in addition.*

Cast in terms of restrictions on agents' valuation functions  $V_i$ , (IC) and (BB) thus imply by part (a) Lemma 4.1 that for any agent  $i$ , over any open connected subset of  $M^0$ ,  $V_i$  equals  $\pi_i$  up to a constant. By part (b), imposing (IR) in addition implies equality also in the constant term.

#### 4.2 Restrictions on the Mechanism over $M^1$

Next, we consider restrictions imposed by (IC) and (BB) by considering constellations in which both true and reported types lie in  $M^1$ . This leads to the following key result, to be developed below:

**Proposition 4.2.** *Any mechanism satisfying (IC) and (BB) must be such that, over the interior of  $M^1$ , each agent  $i$ 's value function  $V_i$  is constant on each connected component of the intersection of any level set of  $\pi^M$  and the interior of  $M^1$ .*

Graphically speaking, each agent's value function  $V_i$  must thus be constant as we move along any  $\pi^M$ -level curve in  $M^1$ .

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<sup>15</sup>Obviously, constance holds more generally also for *disconnected* subsets  $A, B \subseteq M^0$  provided there exist  $\tilde{\mathbf{t}}' \in A$  and  $\tilde{\mathbf{t}}'' \in B$  with  $\tilde{t}'_j = \tilde{t}''_j$  for some  $j \in \{1, 2\}$ . However, the weaker version will suffice for the line of arguments developed below.

### 4.2.1 A Simple Case

At the heart of Proposition 4.2 lies a basic separation problem concerning type profiles  $\mathbf{t}$  which yield identical merger profits  $\pi^M(\mathbf{t})$ . We first state a simple version of this result which applies only to a special case, but nevertheless provides valuable intuition:

**Lemma 4.3.** *Under any mechanism satisfying (IC), for any  $\mathbf{t}' = (t'_1, t'_2)$  and  $\mathbf{t}'' = (t''_1, t''_2)$  such that (i)  $\pi^M(\mathbf{t}') = \pi^M(\mathbf{t}'')$ , (ii)  $\mathbf{t}', \mathbf{t}'' \in M^1$ , and (iii)  $t'_j = t''_j$  for some  $j \in \{1, 2\}$  ( $i \neq j$ ), we must have  $V_i(\mathbf{t}') = V_i(\mathbf{t}'')$ .*

In words, if any two type profiles  $\mathbf{t}', \mathbf{t}''$  (i) both lead to a merger under truth-telling, (ii) both yield the same merger profits  $\pi^M$ , and (iii) differ only in one agent  $i$ 's type, then agent  $i$ 's equilibrium payoff  $V_i$  must be the same in  $\mathbf{t}'$  and  $\mathbf{t}''$ .

The reason for this result is simple: By construction, the mechanism cannot distinguish the outcome from truthful reporting given types  $\mathbf{t}''$  from the outcome if agent  $i$  falsely reports  $t''_i$  given types  $\mathbf{t}'$ , that is, both outcomes of the revelation game must result in the same payoffs. Thus, preventing such a false report requires

$$V_i(\mathbf{t}') \geq \hat{\pi}_i^M[t''_i, t'_j; \pi^M(\mathbf{t}')] = V_i(\mathbf{t}''). \quad (6)$$

By the same token,  $V_i(\mathbf{t}'') \geq V_i(\mathbf{t}')$ , which immediately implies Lemma 4.3.<sup>16</sup>

### 4.2.2 Proving Proposition 4.2

Lemma 4.3 is useful only in a limited class of merger environments: For instance, it provides no implication at all if  $\pi^M$  is strictly increasing in types. However, invoking (BB) provides an extension of Lemma 4.3 to situations in which the  $\mathbf{t}', \mathbf{t}''$  under consideration may indeed differ in *both* agents' types. Formally, this generalization consists in replacing the requirement that  $t'_j = t''_j$  for some  $j \in \{1, 2\}$  in Lemma 4.3 by the requirement that  $(t'_1, t'_2), (t''_1, t''_2) \in M^1$ . To develop this result, we first extend condition (6) above to this situation:

**Lemma 4.4.** *Under any mechanism satisfying (IC) and (BB), for any  $i \neq j \in \{1, 2\}$  and  $\mathbf{t}' \equiv (t'_i, t'_j)$ ,  $\mathbf{t}'' \equiv (t''_i, t''_j) \in \mathbf{T}$  such that  $\pi^M(\mathbf{t}') = \pi^M(\mathbf{t}'')$  and  $(t''_i, t'_j) \in M^1$ ,*

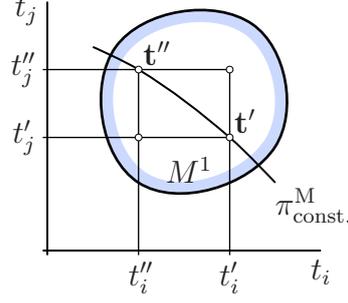
- (a)  $V_i(\mathbf{t}') + V_j(\mathbf{t}'') \geq \pi^M(\mathbf{t}')$ , and
- (b)  $V_i(\mathbf{t}') \geq V_i(\mathbf{t}'')$ , if  $\mathbf{t}'' \in M^1$ , in addition.

The proof is provided in the Appendix.

To understand this result, consider the visualization provided in Figure 2, and

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<sup>16</sup>This result is immediately related to the Spence-Mirrlees condition guaranteeing separability of types (see Spence, 1973, and Mirrlees, 1971): Since  $\pi^M(\mathbf{t}') = \pi^M(\mathbf{t}'')$ , the difference in agent  $i$ 's payoff from reporting  $t'_i$  rather than  $t''_i$  is independent of whether true types are  $\mathbf{t}'$  or  $\mathbf{t}''$  (given truthful reporting by agent  $j$ ), implying that there is no way to 'separate'  $t'_i$ - from  $t''_i$ -types when agent  $j$  is of type  $t'_j = t''_j$ . Indeed, any environment for which Lemma 4.3 is applicable for some  $\mathbf{t}', \mathbf{t}''$  violates the Spence-Mirrlees sorting condition.



**Figure 2:** Illustration of Lemma 4.4 and Lemma 4.3'.

let  $\tilde{\pi}^M \equiv \pi^M(\mathbf{t}') = \pi^M(\mathbf{t}'')$ . Now assume that the revelation game has resulted in reports  $(t''_i, t'_j)$  and merger profits of  $\tilde{\pi}^M$ . This outcome may either be the result of agent  $i$  having provided a false report given types  $\mathbf{t}'$ , or of agent  $j$  having provided a false report given types  $\mathbf{t}''$ .<sup>17</sup> Because the mechanism can condition payoffs only on reports and observed merger profits, either situation must yield agents the same payoff. Moreover, by (BB), agents' payoffs must sum to  $\tilde{\pi}^M$ . Incentive compatibility (particularly, keeping  $i$  from reporting  $t''_i$  when true types are  $\mathbf{t}'$  and agent  $j$  from reporting  $t'_j$  when true types are  $\mathbf{t}''$ ) then immediately leads to Lemma 4.4(a). If  $\mathbf{t}'' \in M^1$  in addition, then payoffs from truthful reporting in  $\mathbf{t}''$  must sum to  $\pi^M(\mathbf{t}'')$  as well. Thus, whatever agent  $j$  gets less from unilaterally misreporting  $t'_j$  when true types are  $\mathbf{t}''$ , agent  $j$  gets more, implying that agent  $i$ 's payoff for reports  $(t''_i, t'_j)$  and true types  $\mathbf{t}''$  must be at least  $V_i(\mathbf{t}'')$ , which produces part (b).

More intuitively, Lemma 4.4 rests on two key restrictions concerning the mechanism's response to deviations from truth-telling. *First*, as payoffs can be conditioned on true types only through  $\pi^M$ , outcomes which constitute an apparent deviation from truth-telling (such as reports  $(t''_i, t'_j)$  and merger profits  $\tilde{\pi}^M$  above) cannot entail a *selective* punishment of the party responsible for the deviation. *Second*, *collective* punishments for such inconsistent outcomes are precluded by (BB), which requires that the full merger profits  $\pi^M$  be distributed to the merging parties, even if reports are inconsistent with the observed  $\pi^M$  (we will return to this last point in Section 5 below).

The next result is an immediate implication of Lemma 4.4 to situations where  $\mathbf{t}'$ ,  $\mathbf{t}''$ ,  $(t''_1, t'_2)$  and  $(t'_1, t''_2)$  all lie in  $M^1$ :

**Lemma 4.3'.** *Under any mechanism satisfying (IC) and (BB), for any  $\mathbf{t}' = (t'_1, t'_2)$  and  $\mathbf{t}'' = (t''_1, t''_2)$  such that (i)  $\pi^M(\mathbf{t}') = \pi^M(\mathbf{t}'')$  and (ii)  $\mathbf{t}', \mathbf{t}'', (t''_1, t'_2), (t'_1, t''_2) \in M^1$ , we must have  $V_i(\mathbf{t}') = V_i(\mathbf{t}'')$ ,  $i = 1, 2$ .*

<sup>17</sup>Generally, such an outcome may of course also result from both agents *simultaneously* deviating from truth-telling. By the Nash-nature of *ex-post* incentive compatibility, however, only *unilateral* deviations from truth-telling must be made unprofitable.

Finally, Proposition 4.2, stated above, represents a straightforward implication of Lemma 4.3': For any two points  $\mathbf{t}', \mathbf{t}''$  connected by a  $\pi^M$  level-curve lying in the interior of  $M^1$ , a sequence of type profiles  $\mathbf{t}^1, \dots, \mathbf{t}^n$  can be found such that Lemma 4.3' is pairwise applicable to  $(\mathbf{t}', \mathbf{t}^1), (\mathbf{t}^1, \mathbf{t}^2), \dots, (\mathbf{t}^{n-1}, \mathbf{t}^n), (\mathbf{t}^n, \mathbf{t}'')$ , implying  $V_i(\mathbf{t}') = V_i(\mathbf{t}'')$ ,  $i = 1, 2$ .

### 4.3 Restrictions at the Boundary between $M^0$ and $M^1$

We next consider agents' incentives to misreport their types in the neighborhood of the merger boundary  $\overline{M^0} \cap \overline{M^1}$ , using the following terminology:

**Definition 4.5.** Agent  $i$ 's type is *locally pivotal* to the merger decision  $m$  at  $\mathbf{t} = (t_1, t_2) \in \mathbf{T}$  if and only if, for any  $\varepsilon > 0$ , there exists a  $t'_i$  with  $|t_i - t'_i| < \varepsilon$  such that  $m(\mathbf{t}') \neq m(\mathbf{t})$ .

At any point on the boundary, at least *one* agent's type must be locally pivotal. Graphically, any parts of the boundary over which *only* agent  $i$ 's type is locally pivotal will be perpendicular to the  $t_i$ -axis (i.e., either horizontal or vertical).

The two key restrictions to be developed in this section are (i) a continuity-property concerning agents' valuation functions across the merger boundary, and (ii) a condition ensuring that the change in the merger decision at the boundary is aligned with agents' fundamental preferences.

#### 4.3.1 A Simple Case

As in Section 4.2, it is instructive to first develop a weaker version of these results in a simpler setting.

**Proposition 4.6.** *For any merger mechanism satisfying (IC), let  $\hat{\mathbf{t}} = (\hat{t}_1, \hat{t}_2)$  denote any interior type profile such that some agent  $i$ 's type is locally pivotal. Then, if  $\pi^M$  is constant in  $t_i$  over some neighborhood of  $\hat{\mathbf{t}}$ , the following must hold:*

- (i)  $V_i$  is continuous in  $t_i$  at  $\hat{\mathbf{t}}$ ;
- (ii)  $\pi_i$  is weakly decreasing (increasing) in  $t_i$  at  $\hat{\mathbf{t}}$  if  $m$  is increasing (decreasing).

To see (i), suppress  $\hat{t}_j$  and assume without loss of generality that  $m(t_i) = 1$  for  $t_i < \hat{t}_i$  and  $m(t_i) = 0$  for  $t_i > \hat{t}_i$  (i.e., that  $m$  is *decreasing* in  $t_i$ ). Then, by our previous results,  $V_i$  is constant (in  $t_i$ ) for  $t_i < \hat{t}_i$ , and  $p_i^0$  is constant for  $t_i > \hat{t}_i$ . Denote these values by  $\tilde{V}_i^1$  and  $\tilde{p}_i^0$ , respectively. Then, to prevent type  $t_i < \hat{t}_i$  from reporting any  $\tilde{t}_i > \hat{t}_i$  and thereby preventing the merger, (IC) requires that  $\tilde{V}_i \geq \pi_i(t_i) - \tilde{p}_i^0$  for all  $t_i < \hat{t}_i$ . Exploiting the assumed continuity of  $\pi_i$ , this implies  $\tilde{V}_i \geq \pi_i(\hat{t}_i) - \tilde{p}_i^0$  by letting  $t_i \nearrow \hat{t}_i$ . On the other hand, preventing agents of type  $t_i > \hat{t}_i$  from reporting any  $\tilde{t}_i < \hat{t}_i$  to induce a merger requires  $\pi_i(t_i) - \tilde{p}_i^0 \geq \hat{\pi}_i^M[\tilde{t}_i; \pi^M(t_i)] = \pi_i^M[\tilde{t}_i; \pi^M(\hat{t}_i)] = \tilde{V}_i$  for all  $t_i > \hat{t}_i$ , where we have used the presumption that  $\pi^M$  is constant in  $t_i$ .

Letting  $t_i \searrow \hat{t}_i$ , this implies  $\tilde{V}_i \leq \pi_i(\hat{t}_i) - \tilde{p}_i^0$ . Combining these insights implies that  $\tilde{V}_i = \pi_i(\hat{t}_i) - \tilde{p}_i^0$  and hence part (i).

To see part (ii), simply assume to the contrary that  $\pi_i$  is strictly decreasing at  $\hat{t}_i$  even though  $m$  is increasing. Then there exists a  $t_i > \hat{t}_i$  such that  $\pi_i(t_i) - \tilde{p}_i^0 < \pi_i(\hat{t}_i) - \tilde{p}_i^0$ , and hence, by continuity,  $\pi_i(t_i) - \tilde{p}_i^0 < \tilde{V}_i^1$ . For this  $t_i$ , in violation of (IC), it will then be profitable to report any  $\tilde{t}_i \in [0, \hat{t}_i)$  rather than the true type.<sup>18</sup>

### 4.3.2 A More General Result

Again, Proposition 4.6 in itself is of limited value as it is useless unless merger profits are independent of agent  $i$ 's type. To understand the problems involved in its generalization, note that if  $\pi^M$  were strictly monotone in  $t_i$  instead, then the payoff  $\hat{\pi}_i^M[\tilde{t}_i; \pi^M(t_i)]$  obtained by a type  $t_i \in (\hat{t}_i, 1]$  from reporting  $\tilde{t}_i \in [0, \hat{t}_i)$  so as to prevent the merger no longer relates to any of the incentive constraints for types  $t_i \in [0, \hat{t}_i)$  since those types' payoffs will all be conditioned on alternative merger payoffs  $\pi^M$ .

However, as above, a generalization can be obtained by instead appealing to (BB) to link these incentive constraints to incentive constraints faced by the *other* agent for type profiles which involve the *same* merger profits  $\pi^M$ . This leads to a continuity requirement which, instead, holds on  $\pi^M$ -level curves:

**Proposition 4.6'.** *Suppose  $\pi^M$  is strictly monotone in both types. Then, under any mechanism satisfying (IC) and (BB), at any point  $\hat{\mathbf{t}}$  on any smooth part of the boundary, the following must hold:*

- (a) *If agent  $i$ 's type is locally pivotal at  $\hat{\mathbf{t}}$ , then*
  - (ai)  *$V_i$  must be continuous at  $\hat{\mathbf{t}}$  along the level curve of  $\pi^M$  through  $\hat{\mathbf{t}}$ ;*
  - (aii)  *$\pi_i$  must be weakly increasing in the direction of the  $\pi^M$ -level curve through  $\hat{\mathbf{t}}$  in which  $m$  is decreasing;*
- (b)  *$\pi_1$  and  $\pi_2$  must be constant on any connected subset of the boundary along which  $\pi^M$  is constant.*

See the Appendix for the proof.

Part (a) extends the two insights from the simple case.<sup>19,20</sup> As described, it de-

<sup>18</sup>Part (ii) relates to the more general concept of 'Positive Association of Differences' (PAD) property (see Roberts, 1979): If a change in agent  $i$ 's signal from  $t_i$  to  $t_i'$  makes some alternative A relatively more profitable than some other alternative B (gross of transfers), then an incentive compatible mechanism must respect this by not choosing A at signal  $t_i$  and B at signal  $t_i'$ .

<sup>19</sup>The statements in Proposition 4.6' are restricted to smooth parts of the boundary to make the proof less tedious. This will be sufficient for our purposes, however, given that we have assumed the boundary to be almost everywhere smooth.

<sup>20</sup>Note that by Proposition 4.6, Proposition 4.6'(a) trivially extends also to merger environments where  $\pi^M$  is constant in *both* types. It runs into limitations, however, if  $\pi^M$  depends on *one* agent's type alone. To understand why, note that the underlying argument relies on the observed outcome

rives from combining agents' incentive constraints for different type profiles yielding the same merger payoffs, therefore producing a continuity requirement which holds on  $\pi^M$ -level curves. Part (b) can be understood as an extension of (aii) to a special case: If the merger boundary locally coincides with a level curve of  $\pi^M$ , then both agents' types will be locally pivotal, but the merger decision function  $m$  will generally be *constant* along this  $\pi^M$ -level curve. As part (b) shows, valuation functions must in this case also be constant along the relevant  $\pi^M$ -level curve (by the continuity-property (ai) and by transfers being constant over  $M^0$ , valuation functions being constant is equivalent to stand-alone profits being constant).

Proposition 4.6' has far-reaching consequences concerning the potential locations of merger boundaries in type space. To see this, simply note that at points where continuity in *both* agents' value functions  $V_i$  is required, their sum,  $V_1 + V_2$ , must be continuous as well. Since  $V_1 + V_2 \equiv \pi^M$  over  $M^1$  and  $V_1 + V_2 \equiv \pi_1 + \pi_2$  over  $M^0$  by (BB), this can only be the case at points  $\mathbf{t}$  where  $\pi^M(\mathbf{t}) = \pi_1(\mathbf{t}) + \pi_2(\mathbf{t})$ . Moreover, the required alignment of the merger decision with individual preferences immediately translates into an alignment of the merger decision with *social* objectives:

**Corollary 4.7.** *Under the prerequisites of Proposition 4.6', any smooth parts of the boundary at which both agents' types are locally pivotal must be contained in the set  $\{\mathbf{t} \in \mathbf{T} \mid \pi^M(\mathbf{t}) = \pi_1(\mathbf{t}) + \pi_2(\mathbf{t})\}$ . Moreover, at any such boundary point,  $m$  must be decreasing in any direction in which  $\pi^M - \pi_1 - \pi_2$  is strictly decreasing.*

See the Appendix for the proof.

Somewhat loosely speaking, except for horizontal or vertical parts of the boundary and parts where the boundary is not smooth, the boundaries of implementable merger decision functions must be efficient in a local sense.

#### 4.4 Implementation in a Competitive Merger Environment

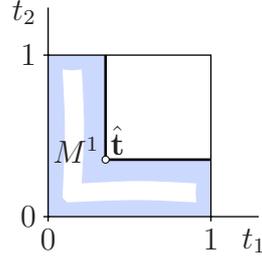
Building on our previous results, we now argue that, within the class of *competitive* merger environments (see Definition 2.7), only merger decisions of a rather trivial nature can be implemented. To formulate this result, we introduce the following definition:

**Definition 4.8.** A merger decision function  $m$  is *L-shaped* if, for some  $\hat{\mathbf{t}} = (\hat{t}_1, \hat{t}_2) \in \mathbf{T}$  and any  $\mathbf{t} = (t_1, t_2) \in \mathbf{T}$ , it satisfies

$$m(\mathbf{t}) = \begin{cases} 0, & t_1 > \hat{t}_1 \text{ and } t_2 > \hat{t}_2, \\ 1, & t_1 < \hat{t}_1 \text{ or } t_2 < \hat{t}_2. \end{cases} \quad (7)$$

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of the unilateral deviation by agent  $i$  being replicable by some unilateral deviation on behalf of the other agent  $j$  from some other type profile, which is not possible if  $\pi^M$  depends on  $t_i$  alone. In that case, punishing agent  $i$ 's deviation leads to no conflict with any of agent  $j$ 's incentive constraints. Continuity need therefore no longer hold for this agent's value function.



**Figure 3:** An L-Shaped Merger Decision Function.

Figure 3 provides an illustration of an L-shaped merger decision function. Special cases of an L-shaped merger decision function include mergers *never* occurring if  $\hat{\mathbf{t}} = (0, 0)$ , mergers *always* occurring if  $\hat{\mathbf{t}} = (1, 1)$ , and the merger decision globally depending on only one agent's report if  $\hat{t}_1 = 0$  or  $\hat{t}_2 = 0$ .

The characteristic features of an L-shaped merger decision function are the following: (i) the merger decision is non-increasing in each type (as required for competitive merger environments by Proposition 4.6'(aii)), and (ii) at each type profile in  $M^1$ , at most one agent can inhibit the merger by a false report. As the next result shows, any other merger decision function is not implementable under (BB):

**Proposition 4.9.** *In any competitive merger environment, under (BB), any merger decision function which is not L-shaped is not implementable.*

See the Appendix for the proof.

The key to this result lies in the simple insight that, in a competitive merger environment, agents' stand-alone profits  $\pi_i$  will always be strictly increasing in *opposite* directions along any  $\pi^M$ -level curve, making it impossible to implement merger decisions whose boundaries contain (smooth) parts where *both* agents' types are locally pivotal: To see this, simply note that at any such point, to ensure the alignment between merger decision and private objectives required by Proposition 4.6', both  $\pi_1$  and  $\pi_2$  would need to be non-increasing in the *same* direction on the  $\pi^M$ -level curve (namely, in *either* direction if  $\pi^M$ -level curve and boundary locally coincide, and in the direction in which  $m$  is increasing in all other cases). This insight immediately reduces the set of implementable merger decisions to those whose boundaries consist only of combinations of horizontal and vertical segments. The proof shows that, by a related argument, implementable merger decisions cannot contain points in  $M^1$  such that both agents can inhibit the merger. Combined with the opposite alignment between  $\pi_i$  and  $m$  required by Proposition 4.6'(aii), this leaves only merger decision functions which are L-shaped as candidates for implementation.<sup>21</sup>

<sup>21</sup>Conversely, however, L-shapedness is not a sufficient property to guarantee implementability

If we add (IR) to the requirements of Proposition 4.9, this immediately implies that  $\pi^M(\mathbf{t}) \geq \pi_1(\mathbf{t}) + \pi_2(\mathbf{t})$  for all  $\mathbf{t} \in M^1$ : If joint merger profits are lower than aggregate stand-alone payoffs, at least one agent must necessarily loose from the merger.<sup>22</sup> Combined with Proposition 4.9, the following result is immediate:

**Corollary 4.10.** *Unless a competitive merger environment is such that some agent  $i$ 's type being low enough is a sufficient indicator that a merger is efficient, only trivial decision of the type ‘never merge’ are implementable if the mechanism is to satisfy (IR) in addition to (BB).<sup>23</sup>*

Thus, in a wide class of competitive merger environments (including Examples 3 and 4 above), a merger mechanism satisfying (BB) and (IR) can achieve no more than reproduce the status quo.

## 5 Relaxing Budget Balance

Our discussion in Section 4 has pointed out two important ingredients to the impossibility result developed there: *First*, that the mechanism typically cannot distinguish unilateral deviations by an agent over  $M^1$  from deviations by the other, which precludes a selective punishment for deviations from truth-telling. *Second*, since (BB) requires budget balance also off-equilibrium, a collective punishment for outcomes which constitute an apparent deviation on some parties' behalf are also precluded.

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of a merger decision function under (BB). However, it is easily shown that a weaker version holds: L-shaped merger decision functions for which  $\hat{t}_i = 0$  for some  $i \in \{1, 2\}$  are always implementable under (BB). These are degenerate instances of L-shaped merger decision functions, however, as at least one agent's report will be altogether irrelevant to the merger decision. More generally, it can be shown that an L-shaped merger decision function with vertex  $\hat{\mathbf{t}} = (\hat{t}_1, \hat{t}_2)$ ,  $\hat{t}_1, \hat{t}_2 > 0$ , is implementable under (BB) if and only if for any  $\mathbf{t}', \mathbf{t}'' \in M^1$  such that (i)  $\pi^M(\mathbf{t}') = \pi^M(\mathbf{t}'')$ , and (ii)  $t'_1 = \hat{t}_1$  and  $t''_2 = \hat{t}_2$ , it holds that  $\pi^M(\mathbf{t}') \geq \max\{\pi_1(\mathbf{t}') + \pi_2(\mathbf{t}''), \pi_1(\mathbf{t}'') + \pi_2(\mathbf{t}')\}$ .

<sup>22</sup>There is another argument in favor of *a priori* focussing attention to mechanisms which satisfy this latter condition, based on a renegotiation-proofness argument: Any time a mechanism produces an inefficient merger decision and parties have somehow revealed their type through the mechanism, there is ample scope for renegotiation. The applicability of any inefficient mechanism therefore relies crucially on the mechanism operator's ability to commit. However, standard reputation arguments to justify the credibility of such a commitment are likely to fail in the merger context, where the problem to be solved by the mechanism is of an inherent one-shot nature.

<sup>23</sup>We should point out, however, that adding the requirement  $\pi^M(\mathbf{t}) \geq \pi_1(\mathbf{t}) + \pi_2(\mathbf{t})$  for all  $\mathbf{t} \in M^1$  to the conditions imposed on  $m$  in Proposition 4.9 is *not* sufficient to ensure that  $m$  is implementable under (IC), (IR) and (BB). This has to do with the fact that, whereas equilibrium utilities  $V_i$  must be constant along  $\pi^M$ -level curves in  $M^1$ , the outside options  $\pi_1$  and  $\pi_2$  will vary. More specifically, for any  $m$  described in Proposition 4.9 and any attainable level of merger profits  $\tilde{\pi}^M$ , let  $\mathbf{A}(\tilde{\pi}^M) \equiv \{\mathbf{t} \in M^1 \mid \pi^M(\mathbf{t}) = \tilde{\pi}^M\}$  denote the intersection of the corresponding level curve with  $M^1$ , and let  $\tilde{V}_1(\tilde{\pi}^M), \tilde{V}_2(\tilde{\pi}^M)$  denote the (by Proposition 4.2 constant) equilibrium utility levels on  $\mathbf{A}(\tilde{\pi}^M)$ . Then (IR) requires  $\tilde{V}_i \geq \max_{\mathbf{t} \in \mathbf{A}(\tilde{\pi}^M)} \pi_i(\mathbf{t})$ ,  $i = 1, 2$ , and hence  $\tilde{\pi}^M \geq \max_{\mathbf{t} \in \mathbf{A}(\tilde{\pi}^M)} \pi_1(\mathbf{t}) + \max_{\mathbf{t} \in \mathbf{A}(\tilde{\pi}^M)} \pi_2(\mathbf{t})$ . It is quickly checked that this results in an upward boundary on  $\hat{t}_i$  (intuitively, so that  $\pi_1$  and  $\pi_2$ 's variability over  $M^1$  is limited).

This section picks up on the latter point by showing that the possibility of “burning money” off-equilibrium indeed makes efficient implementation possible in a large class of environments. To this end, we replace (BB), the budget balance concept used hitherto, with the following, weaker concept:

**Definition 5.1.** A merger mechanism satisfies *equilibrium budget balance (EBB)* if, for all  $\mathbf{t} \in \mathbf{T}$ ,

$$V_1(\mathbf{t}) + V_2(\mathbf{t}) = \begin{cases} \pi_1(\mathbf{t}) + \pi_2(\mathbf{t}), & \text{if } \mathbf{t} \in M^0, \\ \pi^M(\mathbf{t}), & \text{if } \mathbf{t} \in M^1. \end{cases} \quad (8)$$

In contrast to (BB), (EBB) allows the budget to be broken *off equilibrium*, that is, when agents misreport their types. This opens up the way for efficient implementation in a large class of environments:

**Proposition 5.2.** *In any merger environment where  $\pi^M$  is strictly monotone in each type, any efficient merger decision can be implemented by a merger mechanism satisfying (IC), (IR), and (EBB).*

The formal proof, provided in the Appendix, confirms the intuition that imposing a sufficiently harsh *collective* punishment for any outcomes involving inconsistencies in reports and realized profits will permit implementation of the efficient merger decision with essentially arbitrary profit-sharing rules.<sup>24,25</sup>

Proposition 5.2’s scope is of course limited by the extent to which “off-equilibrium money burning” poses a realistic possibility in merger negotiations. Depending on the merger environment, the amount of wealth destroyed can be severe, and absent an adequate commitment device, parties will have strong incentives to renege on

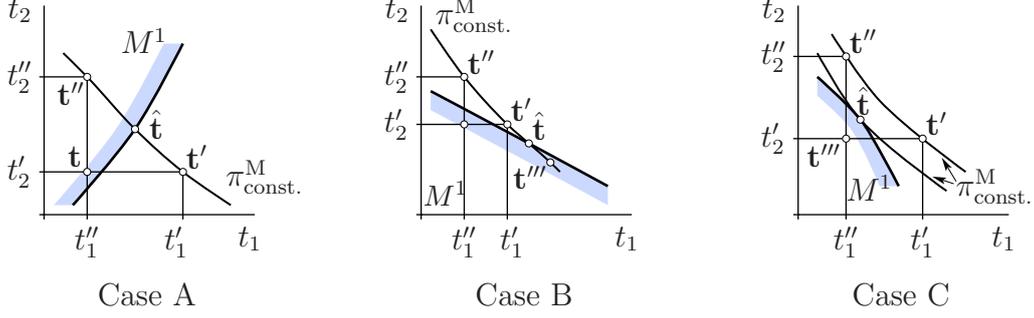
<sup>24</sup>Indeed, in keeping with this intuition, the mechanism employed in the proof of Proposition 5.2 employs a sufficiently large *uniform* punishment  $\alpha_i$  for any merger outcome involving reports that are incompatible with observed profits (requirement (b) in the proof). However, a more differentiated punishment can reduce the extent of “off-equilibrium money-burning”. To see this, note that for any possible off-equilibrium merger outcome consisting of reports  $\tilde{\mathbf{t}} = (\tilde{t}_1, \tilde{t}_2)$  and observed profits  $\pi^M \neq \pi^M(\tilde{\mathbf{t}})$ , the assumed monotonicity of  $\pi^M$  implies that there exists at most one  $\hat{t}_i \in T_i$  such that  $\pi^M(\hat{t}_i, \tilde{t}_j) = \pi^M$ , that is, such that the observed outcome is attainable by agent  $i$  unilaterally deviating from truthful reporting. If such a  $\hat{t}_i$  exists, (IC) can thus be ensured by setting  $\hat{\pi}_i^M(\tilde{\mathbf{t}}, \pi^M) = \pi_i(\hat{t}_i, \tilde{t}_j)$  for any such outcome (which constitutes the maximum off-equilibrium payment which ensures (IC), i.e. the minimal extent of “money burning”); if no such  $\hat{t}_i$  exists,  $\hat{\pi}_i^M(\tilde{\mathbf{t}}, \pi^M)$  may be set arbitrarily for this outcome.

<sup>25</sup>The monotonicity requirement in Proposition 5.2 is related to the separation problem discussed in Section 4 (see Footnote 16, in particular), which arises even in the absence of off-equilibrium budget balance when  $\pi^M$  is not monotone. To understand this, suppose there exist  $\mathbf{t}' = (t'_i, t'_j)$  and  $\mathbf{t}'' = (t''_i, t''_j)$  with  $t'_j = t''_j$ ,  $t'_i \neq t''_i$ ,  $\mathbf{t}', \mathbf{t}'' \in M^1$  and  $\pi^M(\mathbf{t}') = \pi^M(\mathbf{t}'')$ . Then (IC) requires  $V_i(\mathbf{t}') = V_i(\mathbf{t}'')$  (see Lemma 4.3). This additional restriction will generally prohibit choosing  $\hat{\pi}^M$  such that  $V_1(\mathbf{t}) \geq \pi_1(\mathbf{t})$  and  $V_2(\mathbf{t}) \geq \pi_2(\mathbf{t})$  for  $\mathbf{t} = \mathbf{t}', \mathbf{t}''$ —at least so long as  $\pi_1(\mathbf{t}') \neq \pi_1(\mathbf{t}'')$  and  $\pi_2(\mathbf{t}') \neq \pi_2(\mathbf{t}'')$ . Moreover, preventing false reports which lead to inefficient mergers requires that  $V_i(\mathbf{t}'') \leq \pi_i(\mathbf{t}')$  for any  $\mathbf{t}' \in M^0$  and  $\mathbf{t}'' \in M^1$  (the rest as above), which collides with  $V_i(\mathbf{t}'') \geq \pi_i(\mathbf{t}'')$  whenever  $\pi_i(\mathbf{t}') \leq \pi_i(\mathbf{t}'')$ .

such outcomes. On the other hand, given that possibilities are very limited *without* money burning by our results in Section 4, parties should have a strong interest to find a device which enforces such punishments.

## 6 Conclusion

This paper has shown that the potential for implementing merger decisions which avoid ex-post regret depends in an important way on the information concerning player's outside options. If at least one player's outside option is subject to neither party's private information, then efficient implementation is possible by means of a simple buyout of this player by the other party. When private information concerns both party's outside options, however, the scope for regret-free implementation of any non-trivial merger decisions is extremely limited, at least as long as budget balance is required also off equilibrium. Restricting the budget-balance requirement to hold only under truth-telling, however, restores efficient implementation quite generally, but requires a third-party intermediary appropriating parties' wealth in off-equilibrium outcomes. In sum, these findings indicate that in a wide range of situations, merger negotiations under private information necessarily involve a gamble.



**Figure A.1:** Illustrations Accompanying Proof of Proposition 4.6<sup>7</sup>.

## Appendix

*Proof of Lemma 4.4.* (a) To keep agent  $i$  from reporting  $t''_i$  when true types are  $\mathbf{t}'$  (and agent  $j$  reports truthfully), (IC) requires

$$V_i(\mathbf{t}') \geq \hat{\pi}_i^M[t''_i, t'_j; \pi^M(\mathbf{t}')]. \quad (\text{A.1})$$

Similarly, to keep agent  $j$  from reporting  $t''_j$  when true types are  $\mathbf{t}''$  (and agent  $i$  reports truthfully),

$$V_j(\mathbf{t}'') \geq \hat{\pi}_j^M[t'_i, t''_j; \pi^M(\mathbf{t}'')]. \quad (\text{A.2})$$

Since  $\pi^M(\mathbf{t}') = \pi^M(\mathbf{t}'')$  by presumption, the right-hand sides of (A.1) and (A.2) sum to  $\pi^M(\mathbf{t}'')$  by (BB), which proves the claim.

(b) If  $\mathbf{t}'' \in M^1$ , then  $V_j(\mathbf{t}'') + V_i(\mathbf{t}'') = \pi^M(\mathbf{t}'')$  by (BB). Since the right-hand sides of (A.1) and (A.2) add to  $\pi^M(\mathbf{t}'')$  as well, (A.2) may be rewritten as  $V_i(\mathbf{t}'') \leq \hat{\pi}_i^M[t''_i, t'_j; \pi^M(\mathbf{t}'')]$ , which together with (A.1) implies part (b).  $\square$

*Proof of Proposition 4.6<sup>7</sup>.* Without loss of generality, we let  $\pi^M$  be strictly *increasing* in both types, so that the  $\pi^M$ -level curves are downward-sloping in  $t_1/t_2$ -space.<sup>26</sup> Further, due to symmetry, it suffices to prove the claim for agent 1.

*Case A:* Suppose first that the  $\pi^M$ -level curve and boundary intersect at  $\hat{\mathbf{t}}$  and that the boundary has a non-negative slope, as illustrated in Figure A.1(a). Then, any neighborhood of  $\hat{\mathbf{t}}$  will contain  $\mathbf{t}' \in M^0$ ,  $\mathbf{t}'' \in M^1$  such that (i)  $\pi^M(\mathbf{t}') = \pi^M(\mathbf{t}'') = \pi^M(\hat{\mathbf{t}})$ , (ii)  $(t'_1, t'_2) \in M^1$ .<sup>27</sup> Moreover, for any sufficiently small neighborhood of  $\hat{\mathbf{t}}$ ,  $V_1$  will be constant on the part of the  $\pi^M$ -level curve lying in  $M^1$  by Proposition 4.2, and  $p_1^0$  will be constant over  $M^0$  by Lemma 4.1(a).<sup>28</sup> Restricting attention to such sufficiently small

<sup>26</sup>The other cases are obtained by a simple re-normalization of agents' type spaces.

<sup>27</sup>While Figure A.1(a) illustrates this for a merger decision function which is locally decreasing in agent 1's type, this is quickly seen to be true also in the locally decreasing case.

<sup>28</sup>Larger neighborhoods might, for instance, contain parts of the level curve that belong to different connected components of the intersection with the interior of  $M^1$ .

neighborhoods of  $\hat{\mathbf{t}}$  and denoting these values by  $\tilde{V}_1^1$  and  $\tilde{p}_1^0$ , respectively, Lemma 4.4(b) immediately implies  $\pi_1(\mathbf{t}') - \tilde{p}_1^0 \geq \tilde{V}_1^1$ . Continuity of  $\pi_1$  thus implies

$$\pi_1(\hat{\mathbf{t}}) - \tilde{p}_1^0 \geq \tilde{V}_1^1. \quad (\text{A.3})$$

Moreover, any such neighborhood of  $\hat{\mathbf{t}}$  will contain a  $\mathbf{t} \in M^1$  with  $\pi^M(\mathbf{t}) = \pi^M(\hat{\mathbf{t}})$  such that agent 1 can inhibit the merger and obtain a payoff of  $\pi_1(\mathbf{t}) - \tilde{p}_1^0$  (such as type profile  $\mathbf{t}''$  in Figure A.1(a)), implying  $\tilde{V}_1 \geq \pi_1(\mathbf{t}) - \tilde{p}_1^0$ , and therefore

$$\tilde{V}_1 \geq \pi_1(\hat{\mathbf{t}}) - \tilde{p}_1^0. \quad (\text{A.4})$$

(A.3) and (A.4) imply part (ai).

To see that part (aia) holds in this scenario, assume to the contrary that  $\pi_1$  is *strictly decreasing* at  $\hat{\mathbf{t}}$  in the direction along the part of the  $\pi^M$ -level curve running into  $M^0$ . Then, given the constance of  $V_1$  over the part of the level curve lying in  $M^1$  and the proven continuity result, there necessarily exist  $\mathbf{t}' \in M^0$ ,  $\mathbf{t}'' \in M^1$  with the same properties as above and such that  $\pi_1(\mathbf{t}') - \tilde{p}_1^0 < \tilde{V}_1^1$ , in contradiction to Lemma 4.4(b). Note that the above arguments carry through also if agent 2's report is not locally pivotal (i.e., if the boundary in Figure A.1(a) is vertical).

*Case B:* Next, consider a point of intersection  $\hat{\mathbf{t}}$  where the boundary has a strictly negative slope, as in Figure A.1(b). Any neighborhood of  $\hat{\mathbf{t}}$  will now contain  $\mathbf{t}', \mathbf{t}'' \in M^0$  such that (i)  $\pi^M(\mathbf{t}') = \pi^M(\mathbf{t}'') = \pi^M(\hat{\mathbf{t}})$  and (ii) either  $(t'_1, t'_2) \in M^1$  or  $(t''_1, t''_2) \in M^1$ . Then Lemma 4.4(a) implies  $\pi_1(\mathbf{t}') + \pi_2(\mathbf{t}'') \geq \pi^M(\hat{\mathbf{t}})$ . Again, since this is true of any sufficiently small neighborhood of  $\hat{\mathbf{t}}$  and profit functions are continuous, it follows that  $\pi_1(\hat{\mathbf{t}}) + \pi_2(\hat{\mathbf{t}}) \geq \pi^M(\hat{\mathbf{t}})$ . On the other hand, for any  $\mathbf{t} \in M^1$  sufficiently close to  $\hat{\mathbf{t}}$  and such that  $\pi^M(\mathbf{t}) = \pi^M(\hat{\mathbf{t}})$  (such as  $\mathbf{t}'''$  in Figure A.1(b)), either agent can inhibit the merger decision. Again restricting attention to small enough neighborhoods to let us use the notation  $\tilde{V}_i^1$  and  $\tilde{p}_i^0$  employed above, incentive compatibility requires that for any such  $\mathbf{t}$  and any  $i \in \{1, 2\}$ ,

$$\tilde{V}_i \geq \pi_i(\mathbf{t}) - \tilde{p}_i^0. \quad (\text{A.5})$$

Therefore, summing the two incentive constraints,  $\pi^M(\hat{\mathbf{t}}) \geq \pi_1(\mathbf{t}) + \pi_2(\mathbf{t})$ . Hence,  $\pi^M(\hat{\mathbf{t}}) \geq \pi_1(\hat{\mathbf{t}}) + \pi_2(\hat{\mathbf{t}})$ , which, combined with our above result, immediately implies  $\pi^M(\hat{\mathbf{t}}) = \pi_1(\hat{\mathbf{t}}) + \pi_2(\hat{\mathbf{t}})$ . Thus,  $V_1 + V_2$  is continuous along the  $\pi^M$ -level curve.

By a limit argument, (A.5) must apply also at  $\mathbf{t} = \hat{\mathbf{t}}$ . However, given that  $\pi^M(\hat{\mathbf{t}}) = \pi_1(\hat{\mathbf{t}}) + \pi_2(\hat{\mathbf{t}})$ , this condition can be met for both  $i \in \{1, 2\}$  only if both conditions bind, which implies continuity of each  $V_i$  along the  $\pi^M$ -level curve. Proving part (aia) is straightforward by an argument as in Case A.

*Case C:* It thus remains to consider boundary points where boundary and  $\pi^M$ -level curve are tangent. Using a similar argument as in Case B,  $\pi^M(\hat{\mathbf{t}}) = \pi_1(\hat{\mathbf{t}}) + \pi_2(\hat{\mathbf{t}})$  at such a tangency point: Any neighborhood of  $\hat{\mathbf{t}}$  contains  $\mathbf{t}', \mathbf{t}'' \in M^0$  such that  $\pi^M(\mathbf{t}') = \pi^M(\mathbf{t}'')$  and such that  $(t'_1, t'_2) \in M^1$ , implying  $\pi^M(\hat{\mathbf{t}}) \leq \pi_1(\hat{\mathbf{t}}) + \pi_2(\hat{\mathbf{t}})$  by Lemma 4.4(a) and a simple limit argument. Moreover, any neighborhood of  $\hat{\mathbf{t}}$  contains a  $\mathbf{t} \in M^1$  such that either agent can inhibit the merger with a unilateral misreport, implying by (BB) that  $\pi^M(\hat{\mathbf{t}}) \geq \pi_1(\hat{\mathbf{t}}) + \pi_2(\hat{\mathbf{t}})$ .

Now let  $(\mathbf{t}_n)$  denote any infinite sequence in  $M^1$  which converges to  $\hat{\mathbf{t}}$ . Then, for  $n$  large enough (so that either agent can inhibit the merger), incentive compatibility combined with (BB) implies

$$\pi_1(\mathbf{t}_n) - \tilde{p}_1^0 \leq V_1(\mathbf{t}_n) \leq \pi^M(\mathbf{t}_n) - \pi_2(\mathbf{t}_n) - \tilde{p}_1^0. \quad (\text{A.6})$$

As  $n \rightarrow \infty$ , the left- and right-hand sides of (A.6) converge to  $\pi_1(\hat{\mathbf{t}}) - \tilde{p}_1^0$  since  $\pi^M(\hat{\mathbf{t}}) = \pi_1(\hat{\mathbf{t}}) + \pi_2(\hat{\mathbf{t}})$  by our previous argument. Thus,  $V_1$  is continuous at  $\hat{\mathbf{t}}$ , which completes the proof of part (a).

To prove part (b), simply note that for any two tangency points  $\hat{\mathbf{t}}$  and  $\hat{\mathbf{t}}'$  lying in a connected subset of the boundary on which  $\pi^M$  is constant,  $\hat{V}_1^1(\hat{\mathbf{t}}) = \hat{V}_1^1(\hat{\mathbf{t}}')$ : For any sequence  $(\mathbf{t}_n)$  in  $M^1$  converging to  $\hat{\mathbf{t}}$ , we can find a sequence  $(\mathbf{t}'_n)$  in  $M^1$  which converges to  $\hat{\mathbf{t}}'$  such that  $\pi^M(\mathbf{t}_n) = \pi^M(\mathbf{t}'_n)$  for every  $n$  and hence, by the constance of value functions over  $\pi^M$  level sets in  $M^1$ ,  $V_1(\mathbf{t}_n) = V_1(\mathbf{t}'_n)$  for every  $n$ . Since  $\hat{V}_1^1(\hat{\mathbf{t}}) = \pi_1(\hat{\mathbf{t}}) - \tilde{p}_1^0$  and  $\hat{V}_1^1(\hat{\mathbf{t}}') = \pi_1(\hat{\mathbf{t}}') - \tilde{p}_1^0$ , this immediately delivers  $\pi_1(\hat{\mathbf{t}}) = \pi_1(\hat{\mathbf{t}}')$ , as claimed.  $\pi_2(\hat{\mathbf{t}}) = \pi_2(\hat{\mathbf{t}}')$  follows by symmetry.  $\square$

*Proof of Corollary 4.7.* The argument proving the first part of Corollary 4.7 is provided in the text. To see the second part, suppose to the contrary that there exists some direction in which  $\pi^M - \pi_1 - \pi_2$  is strictly decreasing but  $m$  is increasing at  $\hat{\mathbf{t}}$ . Then, any neighborhood of  $\hat{\mathbf{t}}$  will contain a  $\mathbf{t} \in M^1$  such that  $\pi^M(\mathbf{t}) < \pi_1(\mathbf{t}) + \pi_2(\mathbf{t})$  and such that both agents can inhibit the merger with some false report contained in this neighborhood. For sufficiently small neighborhoods, transfers are constant over  $M^0$  in this neighborhood. Denoting these values by  $\tilde{p}_i^0$ , incentive compatibility requires  $V_i(\mathbf{t}) \geq \pi_i(\mathbf{t}) - \tilde{p}_i^0$  and hence, by (BB),  $V_1(\mathbf{t}) + V_2(\mathbf{t}) \geq \pi_1(\mathbf{t}) + \pi_2(\mathbf{t})$ —a contradiction.  $\square$

*Proof of Proposition 4.9. Step 1:* Combining Proposition 4.6'(ai) with the properties of competitive merger environments, it is immediately obvious that any smooth parts of the boundary must be either horizontal or vertical (see the argument in the main text). Thus, the merger boundary can consist only of combinations of horizontal and vertical segments.

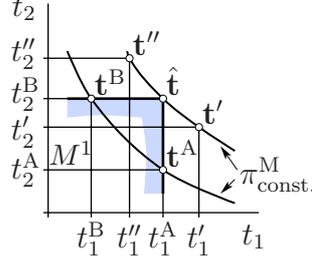
*Step 2:* The next result restricts these cases further, leaving only the merger decision functions described in the proposition as potential candidates for implementation:

**Lemma A.1.** *In any competitive merger environment, any merger mechanism satisfying (IC) and (BB) must be such that,*

- (a) *at any smooth part of the merger boundary where only agent  $i$ 's type is locally pivotal, the merger decision function  $m$  must be decreasing in  $\tilde{t}_i$ ;*
- (b) *the merger decision function exhibits no  $\mathbf{t} \in M^1$  such that both agents have the possibility of inhibiting the merger with some unilateral misreport.*

Part (a) is an immediate consequence of Proposition 4.6'(aai): By the properties of competitive merger environments,  $\pi_i$  is strictly increasing on any  $\pi^M$ -level curve in the direction in which  $t_i$  is increasing, so that  $m$  must be decreasing in that direction.

To see part (b), observe first that, given step 1 of the argument, a merger set  $M^1$  containing type profiles  $\mathbf{t}$  such that *both* agents have the possibility of inhibiting the merger must (at least over some subset of type space) look as depicted in Figure A.2:



**Figure A.2:** Illustration Accompanying Proof of Proposition 4.9.

an inverted L-shape with vertex  $\hat{\mathbf{t}} = (\hat{t}_1, \hat{t}_2)$ , where mergers occur if  $\tilde{t}_1 < \hat{t}_1$  and  $\tilde{t}_2 < \hat{t}_2$ , and where no mergers occur if either  $\tilde{t}_1 > \hat{t}_1$  or  $\tilde{t}_2 > \hat{t}_2$ . To see that such a merger decision function cannot be implementable, observe first that, as shown in the figure, any neighborhood of  $\hat{\mathbf{t}}$  will contain  $\mathbf{t}', \mathbf{t}'' \in M^0$  lying on opposite sides of the  $\pi^M$ -level curve through  $\hat{\mathbf{t}}$  such that  $(t'_1, t'_2) \in M^1$ . Lemma 4.4(b) then implies  $\pi^M(\hat{\mathbf{t}}) \leq \pi_1(\mathbf{t}') + \pi_2(\mathbf{t}'')$ , and hence, by a simple limit argument,

$$\pi^M(\hat{\mathbf{t}}) \leq \pi_1(\hat{\mathbf{t}}) + \pi_2(\hat{\mathbf{t}}). \quad (\text{A.7})$$

Moreover, as illustrated in Figure A.2, there exist  $\mathbf{t}^A, \mathbf{t}^B$  on the boundary such that  $t_1^A = \hat{t}_1$ ,  $t_2^A < \hat{t}_2$ ,  $t_2^B = \hat{t}_2$ ,  $t_1^B < \hat{t}_1$ ,  $\pi^M(\mathbf{t}^A) = \pi^M(\mathbf{t}^B)$ , and such that, by Proposition 4.2, value functions are constant on the  $\pi^M$ -level curve between  $\mathbf{t}^A$  and  $\mathbf{t}^B$ . Denoting the latter values by  $\tilde{V}_1$  and  $\tilde{V}_2$ , respectively, incentive compatibility for agent 1 at  $\mathbf{t}'$  requires  $\tilde{V}_1 \geq \pi_1(\mathbf{t}^A) - \tilde{p}_1^0$ , whereas at  $\mathbf{t}^B$ , it requires  $\tilde{V}_2 \geq \pi_2(\mathbf{t}^B) - \tilde{p}_2^0$  for agent 2.<sup>29</sup> Thus,

$$\tilde{V}_1 + \tilde{V}_2 \geq \pi_1(\mathbf{t}^A) + \pi_2(\mathbf{t}^B). \quad (\text{A.8})$$

But for competitive merger environments,  $\pi_1(\mathbf{t}^A) \geq \pi_1(\hat{\mathbf{t}})$  and  $\pi_2(\mathbf{t}^B) \geq \pi_2(\hat{\mathbf{t}})$  and  $\tilde{V}_1 + \tilde{V}_2 < \pi^M(\hat{\mathbf{t}})$  (due to  $\pi^M$  being increasing in both types and because of (BB)), implying

$$\pi^M(\hat{\mathbf{t}}) > \pi_1(\hat{\mathbf{t}}) + \pi_2(\hat{\mathbf{t}}), \quad (\text{A.9})$$

which contradicts (A.7) and thereby completes the proof of Lemma A.1.

Finally, given steps 1 and 2, only L-shaped merger decisions are left as possible candidates for implementation, which concludes the proof of Proposition 4.9.  $\square$

*Proof of Proposition 5.2.* Choose the merger mechanism  $(m, \hat{\pi}^M, p_i^0)$  such that  $m$  is efficient, and such that

- (a)  $p_i^0(\tilde{\mathbf{t}}) = 0$  for  $\tilde{\mathbf{t}} \in M^0$ ;
- (b)  $\hat{\pi}_i^M(\tilde{\mathbf{t}}, \pi^M) \equiv \alpha_i$ ,  $i = 1, 2$ , for all  $\tilde{\mathbf{t}}, \pi^M$  such that  $\pi^M(\tilde{\mathbf{t}}) \neq \pi^M$ , where  $\alpha_i$  is a real number such that  $\alpha_i < \inf_{\mathbf{t} \in \mathbf{T}} \pi_i(\mathbf{t})$ ;

<sup>29</sup>In the by now familiar fashion, we restrict attention to sufficiently small neighborhoods of  $\hat{\mathbf{t}}$  such that  $p_i^0(\mathbf{t}) = \tilde{p}_i^0$  for all  $\mathbf{t} \in M^0$  over this neighborhood. Moreover, note that strictly speaking, the above incentive constraints obtain directly only if the boundary points  $\mathbf{t}^A, \mathbf{t}^B$  are contained in  $M^1$ . However, if this is not the case, the same restrictions obtain by a simple limit argument.

- (c) for  $\tilde{\mathbf{t}}$  and  $\pi^M$  such that  $\pi^M(\tilde{\mathbf{t}}) = \pi^M$ ,  $\hat{\pi}_i^M$  satisfies
- (c1)  $\hat{\pi}_i^M(\tilde{\mathbf{t}}, \pi^M) \geq \pi_i(\tilde{\mathbf{t}})$  for  $i = 1, 2$ , and
  - (c2)  $\hat{\pi}_1^M(\tilde{\mathbf{t}}, \pi^M) + \hat{\pi}_2^M(\tilde{\mathbf{t}}, \pi^M) = \pi^M(\tilde{\mathbf{t}})$  ( $= \pi^M$ ).

That is: a no-merger outcome involves no transfers by (a), type reports incompatible with observed profits under a merger entail sufficiently harsh punishments for both parties by (b), whereas equilibrium payoffs under a merger exceed standalone-profits for both parties by (c1) and add up to joint merger profits by (c2). Note that requirements (c1) and (c2) can jointly be met if the merger decision is efficient (more generally, if  $M^1 \subseteq M^{1*}$ ).<sup>30</sup>

By requirements (a) and (c1), the mechanism satisfies (IR). (EBB) is trivially implied by (a) and (c2). As to (IC), consider first any  $\mathbf{t}' = (t'_i, t'_j) \in M^0$  and consider  $i$ 's incentives to report any  $t''_i \neq t'_i$ , leading to reports  $\mathbf{t}'' \equiv (t''_i, t'_j)$ . Since  $p_i^0$  is constant in reports by requirement (a), such a deviation can only be profitable if  $\mathbf{t}'' \in M^1$ , that is, if it induces a merger. By requirement (b), however, this can only be profitable if  $\pi^M(\mathbf{t}') = \pi^M(\mathbf{t}'')$  (i.e., if the false report is compatible with realized merger profits), which in turn is precluded by joint merger profits  $\pi^M$  being strictly monotone in both types.

Next, consider truthful reports  $\mathbf{t}' \in M^1$  and any unilateral deviation by agent  $i$  leading to reports  $\mathbf{t}''$ . Since  $V_i(\mathbf{t}') = \hat{\pi}_i^M[\mathbf{t}', \pi^M(\mathbf{t}')] \geq \pi_i(\mathbf{t}')$  by (c1) and  $p_i^0(\mathbf{t}'') = 0$  by (a), any such deviation involving  $\mathbf{t}'' \in M^0$  cannot be profitable. For  $\mathbf{t}'' \in M^1$  such that  $\pi^M(\mathbf{t}') \neq \pi^M(\mathbf{t}'')$ , requirements (b) and (c1) together imply  $V_i(\mathbf{t}') \geq \hat{\pi}_i^M[\mathbf{t}'', \pi^M(\mathbf{t}')] = 0$ . A profitable deviation would thus need to involve  $\pi^M(\mathbf{t}') = \pi^M(\mathbf{t}'')$ , which is again precluded by  $\pi^M$  being strictly monotone in both types.  $\square$

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<sup>30</sup>Also, note that the surplus generated by the merger may be split in any way.

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