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PRUDENCE IN BARGAINING: THE EFFECT OF UNCERTAINTY ON BARGAINING OUTCOMES

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ABSTRACT

Prudence in Bargaining: The Effect of Uncertainty on Bargaining Outcomes*

We investigate the outcome of bargaining when a player's pay-off from agreement is risky. We find that a risk-averse player typically *increases* his equilibrium receipts when his pay-off is made risky. This is because the presence of risk makes individuals behave 'more patiently' in bargaining. Strong analogies are drawn to the precautionary saving literature. We show that the effect of risk on receipts can be sufficiently strong that a decreasingly risk-averse player may be *better off* receiving a risky pay-off than a certain pay-off.

JEL Classification: C71, C72, C78 and D80
Keywords: Nash bargaining, prudence, risk aversion and Rubinstein bargaining

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> "[M]ore importantly, people rarely have *precise* expectations at all. They do not expect that the price at which they will be able to sell a particular output in a particular future week will be just so-and-so much; there will be a certain figure, or range of figures, which they consider most probable, but deviations from this most probable figure on either side are considered to be more or less possible. This is a complication which deserves very serious attention."

John Hicks, "Value and Capital" (Oxford University Press, 1939) p.125.

1. INTRODUCTION

It is probably no exaggeration to say that the majority of the world's output is allocated through bargaining. In most western economies the labour share in national income is typically 60-70 per cent - and upwards of two thirds of the workforce is covered by collective agreements negotiated between unions and firms. Moreover, non-unionized workers further up the employment ladder commonly have their salaries set by individual negotiation, this being most clearly the case for managerial compensation. Much of the capital share in the economy is also allocated through bargaining. For example, firms negotiate over how to split the profits from a joint venture; and buyers and sellers negotiate over the price of a product. In developed economies, the importance of bargaining in the economy is a reflection of the importance of relationship-specific investments.[1]

To ensure mathematical tractability, most economic models of bargaining assume certainty of outcomes or risk-neutrality of negotiators.[2] Yet it is typically difficult or impossible to fully insure such that the influence of random events on the value of a bargained agreement becomes pay-off irrelevant. It is noticeable that all of the bargaining situations mentioned above involve pay-off uncertainty. Nominal wage bargaining between firms and unions is subject to inflation uncertainty. Managerial compensation typically involves a performance-related component. The returns to a joint venture are invariably risky. And buyer-seller negotiations often take place when the cost of production is uncertain. In fact, risky outcomes will arise whenever the bargained contract is unable to pin down receipts exactly. This may be often be due to contractual incompleteness or bounded rationality of the players as to future contingencies. But residual uncertainty can also remain in a complete contracts setting where it is optimal to offer only partial insurance (for reasons of moral hazard or adverse selection).

In this paper we take the existence of risk in bargained agreements seriously, and investigate its impact on the outcome of negotiations. In particular, we investigate the effect of pay-off uncertainty on the equilibrium outcome of Rubinstein's (1982) alternating-offer bargaining game. In Rubinstein's original model, two players divide a pie of *known size*; once they have agreed, they are *certain* what payments they will receive. We study the more realistic case where (at least) one of the parties faces a risky outcome. We provide detailed applications to all the examples mentioned above, and then prove results for a general model that includes these as special cases.

Our results are very surprising. Facing risk typically makes individuals more risk-averse (e.g. Eeckhoudt, Gollier and Schlesinger (1996)). So we might reasonably expect that individuals facing risk in a bargaining game would become more risk-averse and therefore do worse because risk aversion is a disadvantage in bargaining.[3] But in fact we show that on the contrary, under reasonable conditions *a risk-averse bargainer*

[1] In developing economies, where markets may be fragmented and uncompetitive, the fraction of resources allocated through bargaining is probably even higher than in developed countries. Moreover, in both developed and developing economies, resource allocations decided by the government (or by international agreements between governments) are likely to be strongly influenced by bargaining considerations.

[2] There are a few exceptions, including Riddell (1981), Roth and Rothblum (1982), and Yildiz (2002), but the focus of these papers is different from the question addressed here. The relation of this paper to the existing literature is discussed in section 6.

[3] See for example Telser (1990), p. 1251, who in his discussion of bargaining between baseball players and team owners

will increase his bargaining power when the agreement is risky, in the sense that he will increase the share of the pie which he receives. The reasons behind this have close parallels in the precautionary savings literature, so we call the effect *precautionary bargaining*. When income is risky, an extra dollar becomes more valuable, so a bargainer is more willing to hold out for an extra dollar in negotiation. Thus it is as if he had become "more patient", and so receives a larger share of the pie in equilibrium.

Even more surprisingly, we show that a prudent bargainer's strength can increase so much that he becomes *better off* as a result of having a risk added to his pay-off. This is because the rise in expected *marginal* utility of extra income which risk causes can persist even after the player has been fully compensated for the adverse effects of the risk on his *total* utility.[4]

Thus we argue that the existence of uncertainty has important distributional consequences. The party facing a risk will normally not bear the full cost of – and may even gain from - facing that risk, so that negotiators may have inadequate incentives to avoid making risky agreements. This may in turn help us to understand features of real world negotiated contracts, which are frequently very simple and do not involve complex contingent clauses.

The plan of this paper is as follows. Section 2 sets out the basic model, reviews existing comparative statics results and sets out some detailed examples and motivation. Further motivation for the form of preferences used is given in Appendix A. The main results of the paper are in sections 3 and 4, which show that risk increases receipts and welfare, respectively. In Section 5 we extend our results to the Nash bargaining case and offer some brief remarks on symmetric increases in uncertainty and the comparative statics of changes in risk aversion. Section 6 compares our results to related papers in the literature and section 7 offers some concluding remarks. Proofs are relegated to Appendix B.

2. THE BARGAINING GAME

2.1. The Basic Model

In order to set out the effects of risk most clearly, we will use the most familiar structure possible for the game. Thus we model two players involved in the alternating-offer bargaining game described by Rubinstein (1982).[5] In particular, the players have a pie of size one to divide between them; they take it in turns to make offers about the division of the pie, and neither can consume any of the pie until they have agreed on how to divide it. The difference from the standard game is that after players have agreed on an outcome, Player 1's receipts will be subject to a random shock with zero expectation (i.e. a Rothschild-Stiglitz (1970) increase in risk). Thus if the agreement is $(x, 1-x)$, Player 1 expects to receive a share x plus some noise z, where z is the particular mean-preserving spread to which his receipts are subjected. Henceforth, therefore, x will refer to Player 1's *expected* receipts. Players can only make offers about expected receipts; they cannot make contingent offers based on the realization of the noise z. We motivate this assumption further in section 2.3 below.

We denote the distribution of the mean preserving spread Player 1's receipts by $F(z;x)$ with density $f(z;x)$; thus we allow the size of the risk attached to Player 1's receipts to depend on the agreement reached. As will become clearer when we consider our motivating examples, we will be mainly interested in two particular cases. These are *additive risk*, where an agreement x results in a payoff $x+z$ with $F(z)$ independent of x, and *multiplicative risk*, where the agreement x results in payoff $x(1+\zeta)$ with ζ mean zero so that $z=\zeta x$. However, the existence proof below allows for more general forms of risk. There we require

writes: "If players have more risk aversion than owners, then uncertainty could push the split... in favor of the owner".

[4] This result was first noticed by Kimball (1990) in the context of additive risk. Here we generalize it to other types of risk.

[5] A discussion of the Nash Solution is given in section 5.

only monotonicity, i.e. that risk does not increase so quickly with x that more pie is worse.[6]

In deriving the main results of the paper (sections 3 and 4), we will follow the literature in performing comparative statics by focusing our attention on the effects of changes to only *one* player's environment, holding the other player's preferences and environment fixed. In particular, we assume that Player 1's environment changes by an the increase in the risk that he faces, and that Player 2's environment is fixed. Player 2's receipts may also be subject to risk, so that Player 2 receives $1 - x + z_2$ where the noise z_2 has a zero expectation, but if so, we assume that this noise is fixed as the noise associated with Player 1's receipts increases. For readers who find this formulation unnatural, one can alternatively suppose either that Player 2 is risk-neutral (so that his noise may increase with Player 1's but he is unaffected by it) or that Player 2 faces no noise; we motivate such assumptions further in section 2.3 below.[7],[8] Where it is important to highlight the noise faced by each player we will denote Player 1's noise by z_1 and Player 2's by z_2, but for ease of notation, we will for the most part suppress the noise faced by Player 2 and denote the noise faced by Player 1 simply by z.

While bargaining, both players are aware that Players' actual receipts may differ from the expected agreement x as a result of the realization of the noise, but neither can know the actual realization of the noise until after agreement is reached.[9] Thus information is entirely symmetric. The extensive form of the game is illustrated in Figure 1.

2.2. Existence of Equilibrium

We assume that each player i has a well-defined preference ordering $V_i(x, t, z)$ over agreements to consume x at time t with noise z. Under standard assumptions given in Appendix A, these preferences may without further loss of generality be represented by $\delta^t W(x, z) - ct$, with restrictions on the values of δ and c to ensure stationarity, that is to ensure that neither attitudes to risk within a given time period nor attitude to delay of a given consumption bundle across time periods vary over time. This class of preferences includes two special cases which have been of interest in the literature, the case of "pure discounting" ($c = 0, \delta < 1$) and the case of "pure fixed costs of bargaining" ($c > 0, \delta = 1$). Though $W(x, z)$ need only satisfy stationarity of risk attitude for the existence of a unique equilibrium, we will nevertheless impose the stronger expected utility representation $W(x, z) = E_z U(x + z)$ because this allows the tractable derivation of risk premia which will be useful in deriving comparative statics. Under these assumptions, it is possible to prove the following proposition.

PROPOSITION 1 *Suppose that the above assumptions A1-A6 hold for Player 1's preferences over the set $X \times T \times z_1$, and also for Player 2's preferences over the set $X \times T \times z_2$. Then the Rubinstein (1982) bargaining game where Player 1's receipts are subjected to a random shock z_1 and Player 2's receipts are subjected to random shock z_2 has a unique subgame perfect equilibrium.*[10]

[6] The monotonicity assumption will also rule out extreme cases of increasing absolute risk aversion, where an increase in wealth makes the existing risk so undesirable that the increment to wealth is itself undesirable. More detail on assumptions on preferences is given in Appendix A.

[7] In section 5 we will return briefly to the case where the change in risk is symmetric. In other words, denoting by $G(z_2)$ the mean-preserving spread which may affect Player 2's risk, we examine the case where $F(z_1) \equiv G(z_2)$.

[8] Note that we do not restrict the noise associated with players' receipts to be perfectly negatively correlated. That is, we do not require that what one player gains, the other loses, so that the size of the pie is constant. This may be an important special case, but the correlation between players' payoffs is irrelevant so long as (i) players care only about their own receipts, and (ii) the possibility of players' co-insuring is ruled out.

[9] We assume that once shocks are revealed it is impossible to reopen the bargaining. Since one player will always lose from reopening the bargaining, all that is required here is that players cannot commit to reopening the bargaining after shocks are realised, and that it requires mutual agreement to renegotiate ex post.

[10] Notice that we have imposed the important restriction that the noise associated with bargained outcome is independent of the time at which agreement is reached. One could allow uncertainty to vary with calendar time, but this would violate stationarity and thus render the game much more difficult to solve (uniqueness may be sacrificed in some cases (Binmore

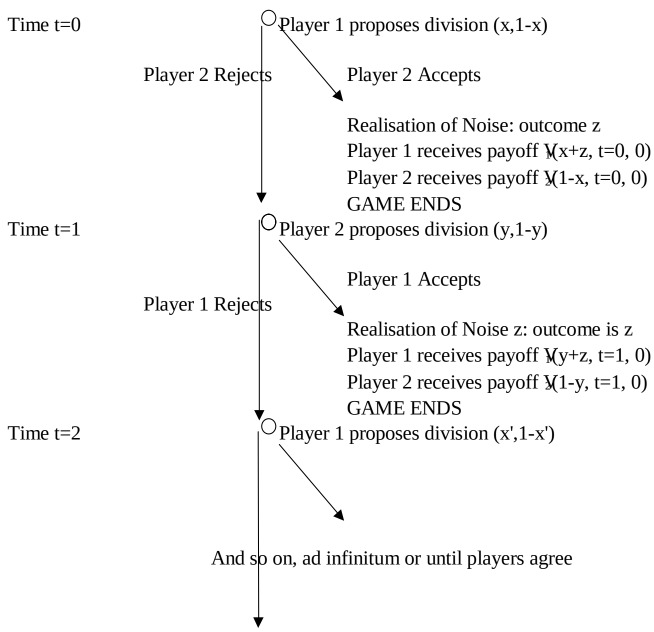

Figure 1: The Noisy Rubinstein Game

The proof, due to Rubinstein (1982),[11] is well known, and so is omitted. As will be familiar, the equilibrium is such that when it is Player 1's turn to offer he always offers Player 2 the agreement $\mathbf{x}^* = (x_1^*, x_2^*)$ which makes Player 2 just indifferent[12] between agreeing and waiting until next period for his own turn to offer (anything more and Player 2 strictly prefers to agree, so Player 1 could have offered less). Similarly Player 2 always offers the agreement $\mathbf{y}^* = (y_1^*, y_2^*)$ which leaves Player 1 just indifferent between agreeing now and waiting.[13] Player 1 accepts only offers with $y_1 \geq y_1^*$ and Player 2 accepts only offers with $x_2 \geq x_2^*$. It follows that x^* and y^* are given by:

$$\begin{aligned} x_2^* = 1 - x_1^* &= p_2(y_2^*, 1; z_2) \\ y_1^* &= 1 - y_2^* = p_1(x_1^*, 1; z_1) \end{aligned}$$

where the notation $p_i(x, t; z_i)$ is used to denote the "present value" to Player i of share x with noise z_i at time t in terms of equally risky pie that would give the same expected utility if received at time 0. It can be seen that the equilibrium is in fact the solution to a pair of indifference relations. An example of such a pair is illustrated in figure 2. Because Player 1 is impatient, his 'indifference curve', as we will describe the graph plotting the amount y to be received now which is indifferent to receiving x next period, always lies below the 45 degree line; strict discounting or strict risk aversion ($U'' < 0$) guarantee that its slope is always

1987)). Moreover, in many applications, it is not obvious whether the value of agreement should become more or less uncertain as agreement is postponed, so constant uncertainty provides a reasonable benchmark.

[11] See also Osborne and Rubinstein (1990) and Binmore, Rubinstein and Wolinsky (1986) for useful discussions.

[12] For ease of expression, in performing comparative statics, we will ignore corner solutions where a restriction to non-negative offers is binding. These will not be relevant unless there are some fixed costs to bargaining - but it is in any case obvious how the argument would generalize to cover corner solutions.

[13] We will always denote Player 1's offers with x's and Player 2's with y's, as is standard in the literature.

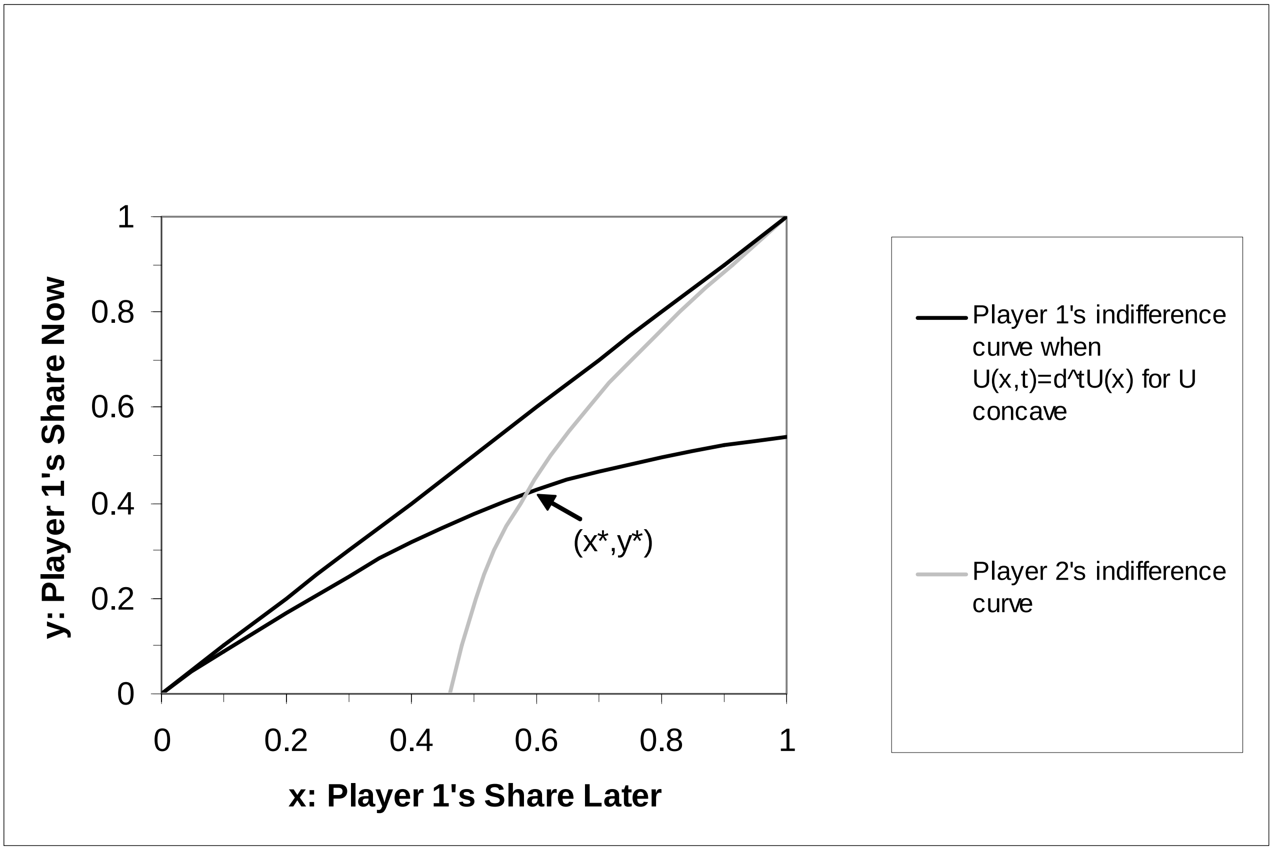

Figure 2:

less than one. Analogous arguments can be made for Player 2, whose indifference curve when drawn on the same axes also falls below the 45 degree line and has slope exceeding one. As we will see below, considering how changes in parameters affect the positioning of indifference curves is a simple way to do comparative statics exercises in what would otherwise be a relatively complex model of non-linear simultaneous equations. An example is given in figure 3. If Player 1's impatience (δ) or risk aversion ($-U''/U'$) is reduced in the standard model with certain receipts,[14] his indifference curve will shift outwards and the new equilibrium (x^{**}, y^{**}) will involve both a larger x_1 and a larger y_1 relative to the old equilibrium (x^*, y^*). That such an outwards shift results in an increase in equilibrium receipts for Player 1 will prove very useful in sections 3 and 4 below. Before that, however, we provide some further motivation for the particular problem that we will study.

2.3. Some Motivating Examples

In this subsection we provide some motivation for bargaining problem outlined above where one party must face a risk if agreement is reached. We begin with some examples of the type of bargaining situation which might be envisaged and postpone a more general discussion to the end of the subsection.

[14] For reasons which we discuss briefly in subsection 5.2, it is not always straightforward to distinguish between risk aversion and impatience in a model without risk. We do not wish to enter into this debate here, we simply illustrate how to perform comparative statics exercises in this bargaining model.

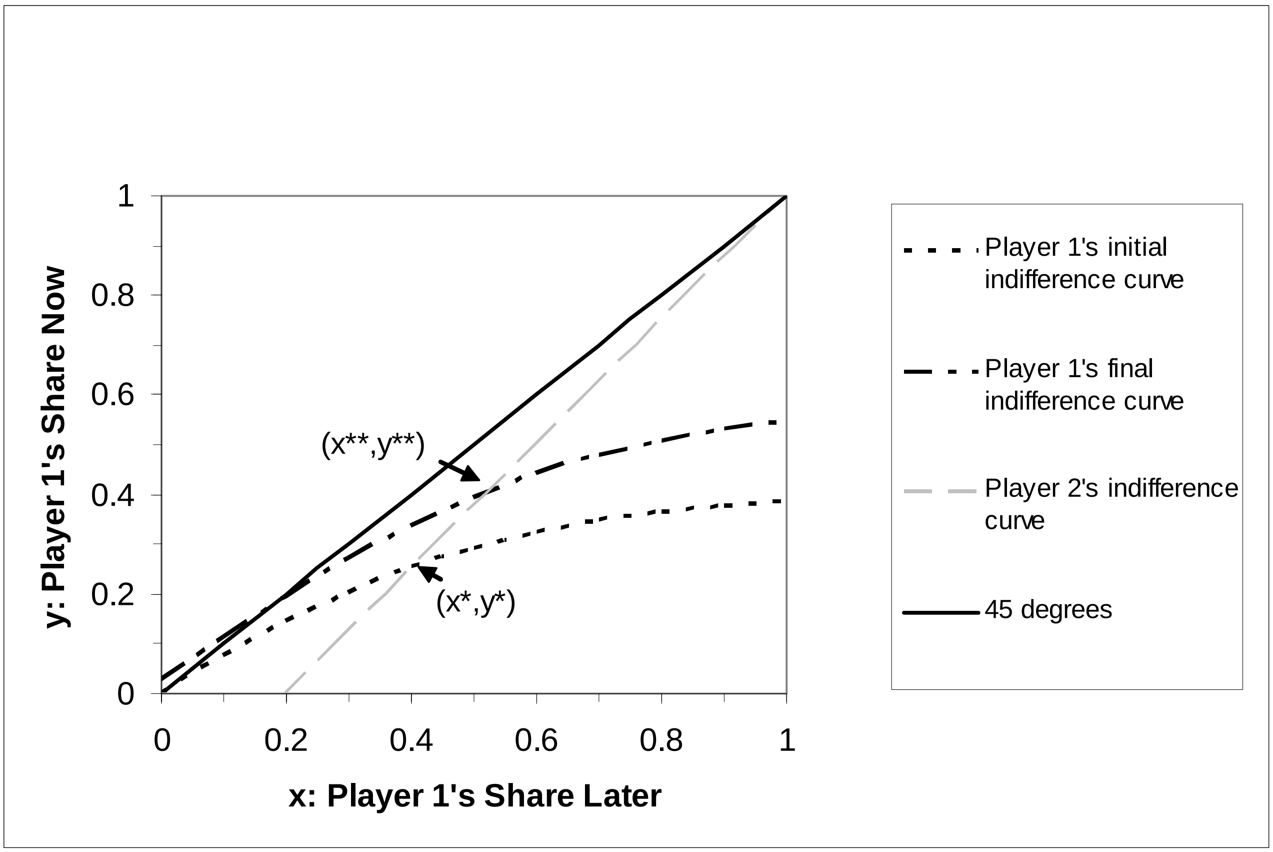
1
0.8
0.6
0.4
0.2
0
y: Player 1's Share Now
(x**,y**)
(x*,y*)
0
0.2
0.4
0.6
0.8
1
x: Player 1's Share Later
Player 1's initial indifference curve
Player 1's final indifference curve
Player 2's indifference curve
45 degrees

Figure 3:

(iii) Buyer-Seller Negotiations

To fix ideas, one might consider the following simple example of the type of scenario we envisage. A buyer (Player 2) wants a seller (Player 1) to manufacture a single object with known value v, and they bargain over the price p at which the object will be transferred after it has been made. However, the seller's cost of producing the object will not be known by either party until after the object has been produced. Both parties are aware of the expected cost of the object, which is c. They are also aware that they cannot perfectly foresee the future and that because of various uncertainties - for example, surrounding exact amount of labour time it will take to produce the object, or unexpected changes in the cost of raw materials - the cost will be hit by a random shock z, with mean zero and some distribution $F(z)$ independently of the price agreed. Thus Player 1's payoff is uncertain and given by the realization $p-(c+z)$; Player 2's is certain and given by $v-p$. If we normalize $v=1$ and $c=0$, then we have a model where players make offers $x \in [0,1]$ over a pie of size 1 as above, and only Player 1 faces a risk. In this paper we will investigate what happens when the risk faced by the seller increases.

We can also extend this example to allow for elastic demand, i.e. let the quantity of objects demanded by the buyer to depend on the unit price that is fixed in negotiations. The production cost of each object is a random variable which may be correlated across objects. It should be obvious that in this complete information bargaining game the outcome of bargaining will be efficient. Therefore if it is possible for the parties to use a two part tariff, the unit price will be set equal to expected marginal cost to maximize the size of the surplus; the fixed fee will then be used to divide this surplus according to relative bargaining powers. Thus this example is isomorphic to the previous unit demand one because quantity chosen (and hence risk to the seller) will be independent of the division of the surplus.[15]

(iii) Negotiation over Salaries: A Principal-Agent Model with Moral Hazard

As a second example, consider a standard principal-agent problem where the agent must make unverifiable effort e to produce verifiable risky output x for a principal, with $x = e+\varepsilon$, where the mean-zero risk ε can be taken to be either stochastic production or measurement error. The principal is risk-neutral but the agent is risk-averse; the agent's expected utility is given by $EU(w-c(e))$, where w is the payment he receives and $c(e)$ is the (monetary) cost of effort. Now consider negotiation over the agent's compensation, where, for familiar reasons we focus on linear contracts, so the agent receives $\alpha+\beta x$ (Holmstrom and Milgrom 1987).[16] Clearly, under standard assumptions, the agent must agree to bear some risk or else no effort will be exerted. If we take the slope of the incentive contract β as given outside the model (e.g., previously negotiated) bargaining over the fixed payment α is a bargaining game with additive risk.

Moreover, if the whole salary package (β and α) is negotiated at once, it is not hard to show that the standard second-best slope for the incentive contract will be agreed upon (again, because under complete information, bargaining outcomes will be efficient).[17] When the agent displays constant absolute risk aversion, β will thus be independent of the agent's bargaining power, so again we have a bargaining problem with additive risk.

On the other hand, we can consider the fixed component α as given and analyse negotiations over the level

[15] Notice that if two-part tariffs are for some reason infeasible, then the seller will try to extract surplus by setting a higher unit price, implying a lower quantity. Here the amount of risk borne is actually decreasing in the size of the bargaining outcome. We do not treat this case explicitly in the sequel, because any results which hold for the additive case will also hold strictly here, and it will be seen that the conditions for the additive case are not especially restrictive.

[16] The important problem of negotiation over the agent's salary has not, to my knowledge, been studied before - the literature typically sets either the principal or the agent to their respective reservation utilities.

[17] This holds as long as there is no additional limited liability constraint for the agent, or the agent's reservation payoff is large enough to ensure $\alpha \geq 0$.

of bonuses β. In this case, if effort is unresponsive to the share of output β then (normalising expected output to one) the agent's payoff from agreement will be $\alpha+\beta(1+z)$, which is to say that risk is *multiplicative* in the bargaining outcome attained. On the other hand, if effort is responsive to profit sharing, risk can generally be expected to increase less quickly than this payoff, i.e. the outcome will lie between the multiplicative and additive cases. For example, if the cost of effort is $c(e)=\frac{1}{2}e^2$, then (substituting in the first-order condition for optimal effort choice) the agent's monetary payoff will be $\frac{1}{2}\beta^2-\beta z$. In this case, the risk premium will increase proportionally to the monetary payoff, as compared to the multiplicative case where (we will see) it increases proportionally to the square of the monetary payoff and the additive case where it is constant. In order to reduce the number of cases to consider in the sequel, we will not treat this 'half-way house' separately, though this would certainly be possible. It should be understood that the conditions for risk to increase expected payments and welfare in this case will be weaker than in the multiplicative case and stronger than in the additive case.

Notice in addition that with some modification this example of principal-agent bargaining could also be interpreted as a model of Player 1 bargaining with his insurer over the premium and extent of coverage for a risk of accident, when the insuree can reduce his risk-exposure through costly effort.

(iii) Splitting a Risky Portfolio or Joint Venture

Bargaining to divide a risky portfolio, or to share the profits from a joint venture, is clearly an example where the risk borne is *multiplicative* in the share received. Normalise the expected return of the total portfolio to one, and denote the realised return on the portfolio by $(1+\zeta)$, where ζ is the deviation from expectation. Let Player 2 be risk-neutral and let him bargain to share this portfolio with risk-averse Player 1. If Player 1 receives a share x of the total portfolio, then the outcome on his share of the assets (which are assumed to be homogenous and perfectly divisible) will be $x(1+\zeta)$. A common example of bargaining in this context is the negotiations which go on between the various claimants on a firm when a firm enters bankruptcy. Here we can interpret player 1 as representing the interests of debt-holders in such negotiations: their concave claim makes them naturally risk-averse; whereas player 2 represents equity-holders whose convex claim tends to make them risk-loving when the value of debt is close to the value of assets. We can interpret our results below as showing that debt-holders will tend to receive a relatively larger share of the firm in such negotiations when the firm is riskier.

(iii) Bargaining over Nominal Wages

Probably the most economically important example of bargaining with multiplicative uncertainty is the case of wage bargaining in a situation where the future price level is uncertain and wages are set in nominal terms. The firm and the trade union negotiate over wages w, but the actual consumption value of the wage to union members – which is what the union cares about - will be $w(1/p)$, where p is the realisation of the uncertain price level. The value of agreement to the trade union is $EU(w/p)$, which if we normalise $E(1/p)=1$ is $EU(w+wz)$ where z represents the appreciation in the purchasing power of the currency relative to its expectation: $z=(1/p)-1$.[18]

(iii) General Discussion

In summary, buyer-seller negotiations are an example of additive risk, bargaining over nominal wages and division of a portfolio provide examples of multiplicative risk; negotiation over the fixed and variable compo-

[18] The impact of inflation uncertainty on wage bargaining is treated in much more detail in Voth and White (2003), where it is shown empirically that greater inflation uncertainty leads to a larger labour share.

nents of managerial salaries fall into the former and latter categories respectively. A common feature of all the above examples is that players do not share the risk in the efficient way. In cases where there is moral hazard, such as managerial compensation, the reasons for this are quite straightforward and consistent with a complete contracting approach. In other cases, the inability to contract on the sharing of the risk is a form of contractual incompleteness and thus requires more justification. The strongest reason for making risk an *exogenous* feature of agreement in these cases is methodological: this assumption will greatly simplify the analysis of the comparative statics of risk in bargaining agreements relative to allowing players to negotiate over sharing any changes in risk (see the further discussion in section 6). Moreover, if parties were to consider negotiating over sharing the risk, the outcome where they fail to agree to share the risk might well function as a disagreement point, so the analysis undertaken here would form a starting point for the solution of that more complex problem.

The second justification for such a study is empirical. It is a fact that many contracts are simple and just do not share risk efficiently. Nominal wage bargaining provides a very important example in this respect; wages are commonly not indexed to the price level even though this would provide risk-averse workers with greater insurance. Although economists do not fully understand the causes this empirical regularity, its consequences for wage negotiation certainly bear investigation.

Thirdly, in other cases the contractual obstacles to risk-sharing are much more obvious. In the type of buyer-seller negotiation described in the example above and envisaged by the incomplete contracts literature, it is quite natural to suppose that in some cases there are considerable transactions costs involved in writing a contingent contract which shares risk efficiently. It is clear that factors which affect the seller's cost may be difficult to describe or even foresee in advance of realizing the project which is to provide the players with a pie to divide.

In this type of situation players might consider *waiting* until after the outcome of the uncertainty is known before reaching an agreement as an alternative to writing contingent contracts. In this way, the realized agreement would implicitly become contingent on the outcome of uncertainty, though risk-sharing will not be optimal. We have ruled this out in the examples above, and assume instead that they must agree before the uncertainty resolves. Aside from the potential delay costs which might be involved in waiting for uncertainty to resolve, this assumption is reasonable in cases where later agreements would cause other problems such as hold-up or delays from informational asymmetries. For example, if the seller waited until after he had produced the object and so learned the cost of production, he would be vulnerable to hold-up by the buyer since his cost would then be sunk, so he might be unable to recover it in bargaining. If in addition the realization of uncertainty is learnt by only one player (e.g. only the seller learns his cost), this informational asymmetry would cause wasteful delay if bargaining occurred ex post. Nevertheless, though outside the scope of this paper, it is an interesting general problem to investigate under what circumstances one or both parties would prefer to wait until the outcome of uncertainty is known before bargaining; this is left for future research.

Finally, whilst the riskiness of receipts is entirely exogenous in this paper, we will see that the results may shed some light on the poorly understood issue of *why* a bargaining player may prefer to bargain over a simple contract than a complex contingent one which provides him with more insurance. In the next two sections we will show that a risk-averse player will typically be at least partially compensated for – and may even be better off as a result of - an increase in the riskiness of his bargained payoff. This may suggest a partial explanation for the fact that the contracts that embody bargained agreements are often vague and "Businessmen prefer to rely on...a brief letter [rather than a detailed contract]...even when the transaction involves exposure to serious risks," (MaCauley 1963, p58). Businessmen may feel that they are able to negotiate a better deal without resorting to detailed contracts if this exposure to risk improves their bargaining position.

3. THE EFFECT OF UNCERTAINTY ON BARGAINING OUTCOMES

This section shows under what restrictions on the utility function expected receipts from bargaining will increase when risk increases.

3.1. Conditions for Risk to Raise Receipts

We state at once the main proposition.

PROPOSITION 2 *Consider initial subgame perfect equilibrium of the standard riskless Rubinstein game, $(x^{\#}, y^{\#})$, and now add risk to Player 1's payoff. Either of the following conditions is sufficient for Player 1 to increase his expected receipts when his payoff becomes uncertain:*

CONDITION 1 *The ratio of risk premia $\frac{\pi(y^{\#},z)}{\pi(x^{\#},z)}$ at the initial equilibrium exceeds one.*

CONDITION 2 *The ratio of risk premia $\frac{\pi(y^{\#},z)}{\pi(x^{\#},z)}$ at the initial equilibrium exceeds the slope of Player 1's indifference curve at that point, $\frac{\delta U'(x^{\#})}{U'(y^{\#})}$ and this indifference curve is concave.*

The formal proof is given in Appendix B. But the idea behind the proof can be given a simple diagrammatic interpretation. Effectively the proof proceeds by showing that *if (x, y) is a point on Player 1's indifference curve under certainty, then $(x+\pi(x,z), y+\pi(y,z)))$ is a point on Player 1's indifference curve when receipts have noise z associated with them*, where $\pi(x,z)$ $[\pi(y,z)]$ is the compensating risk premium associated with risk z at wealth level x $[y]$.[19] Consider an arbitrary point (x, y) on Player 1's indifference curve over bargained agreements under certainty shown in figure 4. At such a point, receiving certain receipts y now is indifferent to receiving certain receipts x one period later, $V_1(x, t+1, 0) = V_1(y, t, 0)$. But we know that Player 1 is also indifferent between receiving certain receipts y now and receipts $y + \pi(y, z)$ plus noise z now, $V(y + \pi(y, z), t, z) = V(y, t, 0)$ and similarly for x: $V_1(x + \pi(x, z), t + 1, z) = V_1(x, t + 1, 0)$. Therefore by transitivity of indifference, $V_1(x + \pi(x, z), t + 1, z) = V_1(y + \pi(y, z), t, z)$: Player 1 is indifferent between receiving risky receipts $y + \pi(y, z)$ in period t, and $x + \pi(x, z)$ in period $t + 1$. Thus $(x + \pi(x, z), y + \pi(y, z))$ is a point on Player 1's indifference curve when the uncertainty associated with bargained outcomes is z. Since the point (x, y) was chosen arbitrarily subject to being on Player 1's indifference curve under certainty, it must be the case that for every such point (x, y) there is a point $(x + \pi(x, z_1), y + \pi(y, z_1))$ on Player 1's indifference curve with noise z_1.[20] We can thus trace out Player 1's indifference curve when the noise associated with bargained outcomes is z_1 for all points such that $x' \geq y'$.

It follows that the effect of adding uncertainty is to shift each point (x, y) on the indifference curve out at an angle $\pi(y, z)/\pi(x, z)$. Whether this corresponds to an outwards or an inwards shift of Player 1's indifference curve depends on the slope of the indifference curve itself. However, our assumptions on preferences ensure that the slope is always strictly less than one.[21] So since with non-increasing absolute risk aversion and additive risk $\pi(y, z)/\pi(x, z) > 1$ (as $x > y$ and z is independent of x, y) in the case of additive risk, we clearly have an outwards shift, as stated in the corollary below. (See figure 4, which illustrates the special case of constant absolute risk aversion.) When absolute risk aversion is increasing,

[19] For a more detailed discussion of the properties of the compensating risk premium, see Kimball (1990).

[20] Although it is not true that all such points will lie inside the set of feasible outcomes (e.g. some may have $x + \pi(x, z)$ greater than one).

[21] This comes in particular from the assumption of increasing loss to delay - A6 in Appendix A - which is embodied in the concavity of $U(.)$.

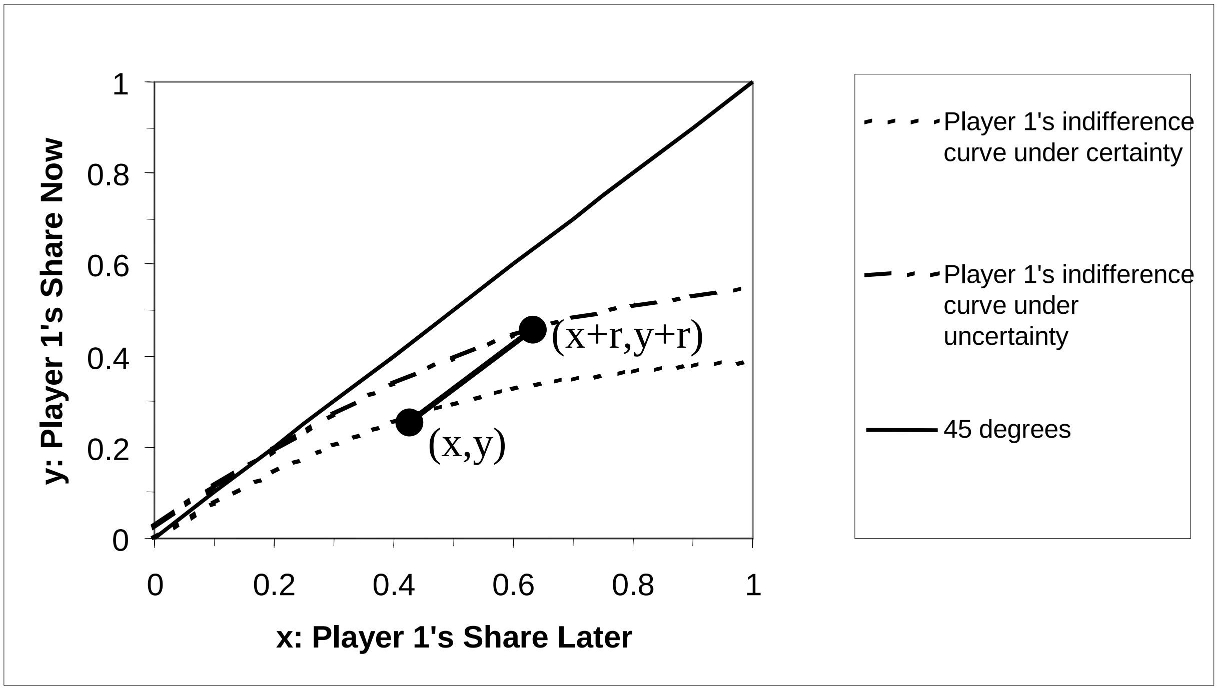

Figure 4: The effect of risk on the indifference curve (CARA case)

both the slope of the indifference curve and the ratio of risk premia are less than one, so one needs to find an expression for the latter to proceed further. This can be done using the Arrow-Pratt approximation under the expected utility hypothesis,[22] and yields the following:[23]

Corollary 1 *Additive Risk*

1. *(i) When his payoff becomes uncertain, Player 1 will increase his expected receipts if he exhibits non-increasing absolute risk aversion.*

   (ii) A sufficient condition for a small additive risk to increase receipts is:

Condition 3 *$U''' \geq 0$ and $\delta \leq 1$, with at least one inequality strict.*

Remark 1 *For the case of quadratic utility $U(x) = ax - bx^2$ (which exhibits increasing absolute risk aversion and $U''' = 0$) second-order Taylor Series Expressions for risk premia are of course exact, so in this case one can solve for the indifference curve explicitly to show that risk will affect the outcome of the bargaining game exactly to the extent that δ is strictly less than one. Notice, however, that if "discounting" is strict ($\delta < 1$), we do not actually require $U''' \geq 0$, but rather U''' not too negative.*[24]

[22] Thus the expected utility hypothesis greatly simplifies the analysis, though we do not actually require anything as strong as global EU maximization, as opposed to "locally smooth" preferences under uncertainty, along the lines of Machina (1982). Such preferences would give the same expression for local risk premia, and we require only a local result: an outwards shift at the initial equilibrium point.

[23] The argument given in the text might appear to rely on a first order approximation; the demonstration that it does not falls into two parts. For non-increasing absolute risk aversion, the increasing compensation assumption implies that the slope of the indifferene curve under certainty (which may be convex) is always less than one, so it cannot "catch up" with the outwards shift of the curve. For increasing absolute risk aversion, it can be shown that the indifference curve is always concave and thus that the indifference curve under certainty cannot rise back above the line along which the curve shifts out.

[24] This is confirmed by the analysis of the Nash solution in section 5 below, where the role of the discount factor is implicit.

Thus additive risk will lead to an increase in expected receipts if risk aversion does not increase too quickly. This is of course the usual case since one would normally expect decreasing absolute risk aversion (DARA), i.e. rich people to be less averse to a given risk than poor people, in which case additive risk will raise receipts. In fact, as we will see below, intuitions about risk aversion may be slightly misleading in this context. Instead it is the role of the sign of U''' – which determines whether or not the individual displays 'prudent' behavior, such as precautionary saving, in the face of risk – which will turn out to be important. But before discussing this let us we state the result for the case of multiplicative risk.

Corollary 2 *Multiplicative Risk*

A sufficient condition for a small multiplicative risk to strictly increase expected receipts is:

Condition 4 $-xU'''(x)/U''(x) \geq 2$ *and* $\delta \leq 1$*, with at least one inequality strict.*

Remark 2 *This condition may seem hard to interpret, so the reader may find it useful to note that familiar utility functions which satisfy these criteria are the negative exponential (CARA) utility function for the range where relative risk aversion exceeds two; and the log utility function* $U(x) = ln(x)$*.*[25] *It will always be satisfied by any utility function satisfying decreasing relative risk aversion with parameter greater than one. For example, the decreasing relative risk aversion utility function* $U(x) = (x - \gamma)^{(1-\alpha)}/(1-\alpha)$ *with* $\gamma > 0$*,* $0 < \alpha < 1$ *satisfies these conditions provided that wealth* x *is close enough to the subsistence level:* $x < 2\gamma/(1-a)$*.*

The intuition behind – and thus the reason for – the conditions in corollaries 1 and 2 will become clear very shortly. Before moving on to this, let us briefly discuss the plausibility of the conditions. Decreasing relative risk aversion (DRRA) implies that rich people hold a larger fraction of their portfolio in risky assets; an implication which seems to be verified by introspection and is indeed backed up by plenty of empirical evidence (Kessler and Wolff 1991, Rosenzweig and Binswanger 1993, Guiso et al 1996, Ogaki and Zhang 2000). The question of whether the coefficient of relative risk aversion (CRRA) exceeds one is more controversial – together with the expected utility hypothesis it implies a strong preference for smooth consumption over time (it is equivalent to an elasticity of intertemporal substitution < 1). Under this hypothesis, saving responds negatively to the real interest rates, for example (the income effect dominates the substitution effect). This seems to accord with most (though not all) empirical evidence on the subject (Deaton 1992, Muellbauer 1994). Importantly in our context, the joint hypotheses of decreasing relative risk aversion with parameter greater than one imply that real interest rate uncertainty (e.g. because inflation is uncertain) leads to precautionary saving. This is certainly consistent with the savings boom of the inflationary 1970s, though of course that may have had many other causes.

3.2. Discussion and Interpretation

In this section we interpret the conditions on the utility functions derived in corollaries 1 and 2 above. In fact it can be shown that:

Proposition 3 *With* $\delta = 1$*, risk will increase risk-averse Player 1's expected receipts if they raise his expected marginal utility of pie. In other words:*

(i) The sufficient condition 3 in corollary 1, $U''' > 0$ *, is equivalent to the condition that expected marginal utility increase when income becomes additively risky.*

[25] Note that for the log utility function to make sense here, x must be renormalized, e.g. to run between 1 and 2 (since ln(0) is undefined).

(ii) Condition 4, $\frac{-xU'''}{U''} \geqslant 2$, *is approximately equivalent to the condition that expected marginal utility increase when income becomes multiplicatively risky.*

Part (i) of this proposition is well-known since Leland (1968), Sandmo (1971), and Drèze and Mogdigliani (1972). If an individual is *prudent*, i.e. $U''' > 0$ (Kimball 1990), marginal utility is convex, so facing a risk raises expected marginal utility. In the savings context, this means that if future income becomes risky, expected future marginal utility rises, so income should be transferred from now to the future via *precautionary saving*. In a bargaining context, the phenomenon is the same: if agreement becomes risky, the expected marginal utility of an extra dollar from agreement goes up, so Player 1 will strictly prefer to hold out to make his own offer, whereas at the old equilibrium he was indifferent to doing so. An increase in marginal utility is beneficial to a bargaining agent: it is as if Player 1 became more patient as a result of bearing a risk, and this improves his bargaining outcome in equilibrium.

Part (ii) of the proposition is less obvious. The equivalent expression is more complex since multiplicative risk has two conflicting effects. First there is the same effect as for additive risk: when pie becomes risky, $U''' > 0$ implies increased marginal utility of pie. Second, (and unlike in the additive case), when one receives more pie, one is receiving proportionally more risk, so the marginal utility of increments tends to be reduced by risk. It can be shown by Taylor Series expansion[26] that the balance of these two effects is an increase in marginal utility provided the condition $-xU'''/U'' > 2$ holds. This is the same condition as would be required for precautionary saving to occur in a world where income risk is proportional to rather than independent of income (see Eeckhoudt and Gollier (1992), Laffont (1989)).

In the light of the above proposition we can see the theory set out here as a theory of precautionary or prudent behavior in bargaining, or "bargaining for a rainy day". The reason why prudent behavior is a sufficient and not a necessary and sufficient condition here (whereas it is both necessary and sufficient in the precautionary saving literature) is that with bargaining there is an additional effect if $\delta < 1$. Then, delaying agreement delays bearing a risk and so reduces impact of bearing the risk on expected utility (think of delaying the payment of the risk premium). This effect does not arise in the savings problem, but it clearly reinforces players' preference for waiting and thus their equilibrium payoff.

4. THE EFFECT OF RISK ON WELFARE

In the previous section we showed when the noise with which Player 1 believes his payoffs to be associated increases, risk-averse Player 1's indifference curve between agreements now and later "typically" shifts upwards, making Player 1 behave more "patiently" in bargaining.[27] Other things being equal, (and in particular, either the uncertainty associated with Player 2's receipts remaining unchanged or Player 2 being risk-neutral) this will increase Player 1's receipts in any bargained agreement. This does not necessarily make Player 1 better off since the increase in his receipts may not be sufficient to cover the losses due to the increased uncertainty associated with them. In this section we show that there are conditions under which a risk-averse player's utility actually increases when the uncertainty associated with his bargained outcomes is increased.

4.1. Conditions for Risk to Raise Welfare

The main result is the following:

[26] For decreasing relative risk aversion with parameter greater than one, the approximation is exact.

[27] This is in direct contrast to the effect of increasing uncertainty about whether the game will continue in the "exogenous probability of breakdown" interpretation of the Rubinstein game due to Binmore (1987). The reason for the difference is that risk-averse players can avoid uncertainty about breakdown by agreeing now (so it makes them more impatient); they cannot avoid the uncertainty about payoffs in my model in the same way. I thank Alexander Ljundqvist for this remark.

PROPOSITION 4 *(i) In equilibrium, if Player 1's risk aversion decreases sufficiently quickly, he may be able to extract* ***more*** *than the compensation needed to make him just as well off as he was in equilibrium before the risk associated with his payoff increased. Consequently in equilibrium he becomes* ***better off*** *when his pay-off becomes risky.*

(ii) An approximate condition for Player 1 to become better off than at the initial equilibrium $(x^{\#}, y^{\#})$ *when a risk* z *is added to his payoff is:*

$$p_2'(1-y^{\#},1) > \frac{\pi(x^{\#}, z(x^{\#}))}{\pi(y^{\#}, z(y^{\#}))}$$

Once again, the proof has a relatively straightforward diagrammatic interpretation. As outlined above, Player 1's indifference curve shifts out at an angle $\pi(y^{\#}, z)/\pi(x^{\#}, z)$ when risk is added. If this angle is steeper than the slope of Player 2's indifference curve ($= 1/p_2'$), the new equilibrium must lie to the right of the point $(x^{\#} + \pi(x^{\#}, z), y^{\#} + \pi(y^{\#}, z))$, meaning that Player 1 is better off than if he were only just compensated for the increase in risk.

Interpreting these conditions for the specific cases of additive and multiplicative risk, we have:

COROLLARY 3 *(i) Additive Risk. If risk is additive, and Player 2 has pure fixed costs of bargaining, (strict) decreasing absolute risk aversion (DARA) is necessary and sufficient for Player 1 to be (strictly) better off negotiating over a risky than a certain agreement.*

(ii) Multiplicative Risk. If risk is multiplicative, and Player 2 has pure fixed costs of bargaining, a necessary and sufficient condition Player 1 to be (strictly) better off negotiating over a risky than a certain agreement is $x.CRRA(x)$ *(strictly) decreasing, which is equivalent to*

$$x(\frac{U''}{U'} - \frac{U'''}{U''}) > 2.$$

(iii) If Player 2 is risk-averse or strictly discounts the future, these conditions are necessary but not sufficient.

REMARK 3 *The condition for multiplicative risk to raise welfare is again satisfied for the utility function* $U(x) = (x-\gamma)^{(1-\alpha)}/(1-\alpha)$ *with* $\gamma > 0$, $0 < \alpha < 1$ *provided that wealth* x *is close enough to the subsistence level:* $x < 2\gamma$.

4.2. Discussion and Interpretation

We can also interpret this result in the light of the precautionary saving literature:

PROPOSITION 5 *The conditions given in Corollary 3 above are approximately equivalent to the precautionary premium exceeding the risk premium.*

For an additive risk the compensating *risk premium* - the amount of extra income required to compensate *total utility* for an increase in risk - is $\frac{1}{2}\sigma^2 A(x)$, where we abbreviate the coefficient of absolute risk aversion $A(x) = -U''(x)/U'(x)$. The *precautionary premium* - the amount of extra income required to compensate *marginal utility* for an increase in risk – is $\frac{1}{2}\sigma^2 P(x)$ where $P(x)$, the coefficient of absolute prudence, is $-U'''(x)/U''(x)$. Kimball (1990) showed for an additive risk that decreasing absolute risk aversion is equivalent to the precautionary premium exceeding the risk premium.

The equivalent conditions for multiplicative risk have not – to my knowledge – been previously set down. It turns out that the precautionary premium in this case is given by $\frac{1}{2}x^2\sigma^2 P(x) - x\sigma^2$, where the extra term derives from the counteracting influence noted above that with multiplicative risk income is bundled with risk. The risk premium for multiplicative risk is given by $\frac{1}{2}x^2\sigma^2 A(w)$; combining the two expressions gives the condition in corollary 3(ii).

To understand the significance of this result recall that as discussed above, increased marginal utility of income in a bargaining game, will, other things being equal, translate into an increased willingness to wait for a higher payoff: hence greater "patience" and higher receipts in equilibrium. The point to note here is that when the precautionary premium exceeds the risk premium, an individual will still have higher marginal utility of income even *after* he has been fully compensated for the effects of risk on his expected utility, so the increased patience effect continues to operate even after this point. Thus it is quite possible for Player 1 to gain more than his risk premium (but less than his precautionary premium) in bargaining. Of course his ability to do so will be limited by the fact that in general Player 2's marginal utility of income will typically also rise as a result of giving up the compensating risk premium to Player 1.[28]

5. SOME EXTENSIONS

5.1. Remarks on Symmetric Increases in Uncertainty

In the preceding sections, we derived conditions for increases in receipts and welfare assuming that either (i) the noise associated with Player 2's payoff is unchanged when that associated with Player 1's increases, i.e. $\partial G(z_2)/\partial F(z_1) = 0$ or (ii) Player 2 is risk-neutral or risk-loving, i.e. his utility increases when $G(z_2)$ is subjected to a mean-preserving spread where the subscript 2 indicates Player 2.[29] Both of these conditions allow us to avoid the additional complications raised by the possibility that Player 2's indifference curve may also shift. We now consider informally the effect of introducing such complications.

The introduction of uncertainty makes (not too increasingly) risk-averse agents behave "more patiently" in a bargaining game. It follows from this that if uncertainty is increased symmetrically between symmetric players, it will tend to have the effect of reducing first mover advantage. (Simple diagrammatic inspection confirms this - see figure 5). Leaving aside the first mover advantage,[30] if uncertainty is increased symmetrically between equally patient players, the equilibrium division of the pie is tipped in favour of the more risk-averse player (who has a disadvantage under certainty), since other things being equal, this player will have larger compensating risk premia, so that his indifference curve will shift out further.[31] Conversely, when players are equally risk-averse, increasing uncertainty tends to reduce the disadvantage to the less patient player.[32] One way to see this is to note that Player 1's increase in receipts from an increase in risk will be smaller if Player 2's indifference curve is flatter. Thus I conjecture that symmetric increases in uncertainty will generally result in more equal outcomes.

5.2. Remarks on the Comparative Statics of Risk Aversion

Results on the comparative statics of changes in risk aversion are beset by a degree of controversy regarding whether or not an increase in risk aversion ought to leave preferences over the timing of certain prospects unchanged (see Binmore et al 1986). Consider the model set out in sections 1-4 above. If preferences over certain outcomes are held constant, whilst an increase in risk-aversion makes risky prospects less attractive,

[28] As before, when there is strict discounting, there is an additional effect: Player 2's decision to wait depends not only on the derivative of his period utility function $U(.)$, but on how much this will be worth to him next period, which depends on δ_2.

[29] In the case where Player 2 is risk-loving, an increase in the risk he faces will tend to shift his "indifference curve" inwards, so that the comparative statics of changes in Player 1's risk will still be as described in sections 3 and 4 above (only the size of the effects will be larger).

[30] For example, by taking the Nash solution.

[31] For more on the comparative statics of risk aversion, see section 5.2 below.

[32] Unlike in the risk aversion case, the least patient player's relative disadvantage cannot be completely reversed by risk: impatience remains a disadvantage under risk.

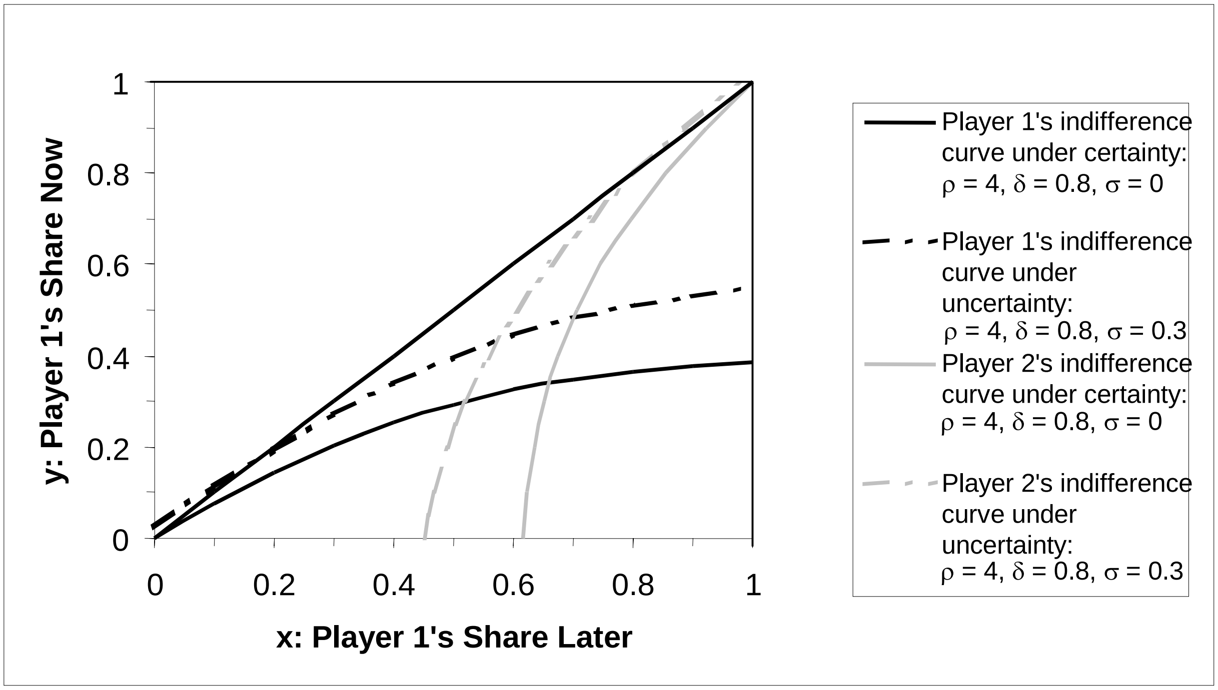

Figure 5: Symmetrically increasing uncertainty when player 2 is risk averse (CARA players)

then ceteris paribus the effect will be analogous to that of an increase in risk: the risk-premia will increase, and provided that risk-aversion is does not increase too quickly, the indifference curve will shift outwards. Thus given certain reasonable properties of the utility function, an increase in Player i's risk-aversion which preserves his preferences over the timing of certain outcomes will increase his share of a risky pie in a Rubinstein bargaining game. Then risk aversion would be an advantage in bargaining over risky outcomes, and have no effect in bargaining over certain outcomes.

The same effect - which we shall refer to as the *risk-premium effect* - occurs in the case where Player i's preferences over certain outcomes are not preserved when his risk-aversion is increased (e.g. Roth 1985). However, an additional complication - the "impatience effect" - arises due to the movement of the indifference curve under certainty.[33] Taking a concave transformation of the utility function which preserves the point of time-indifference (interpreted as increasing risk aversion in the certainty case) makes the indifference curve between payoffs now and payoffs later flatter for any given x, so pivoting it downwards, as shown in figure 6. Thus under certainty the indifference curve of a more risk-averse individual lies everywhere below that of a less risk-averse individual, and so the more risk-averse Player i receives less in equilibrium when bargaining with any given Player j. Thus Roth (1985) argues that risk aversion is a disadvantage in bargaining. When payoffs are risky, this *impatience effect* will be at least partially offset by the risk-premium effect as shown in figures 7 and 8; depending on the size of the risk, the indifference curve of the more risk-averse individual will lie partly or even wholly above that of the less risk-averse player. In general, for initial equilibrium payoffs close to zero, the *risk premium effect* dominates, whereas for large payoffs, the risk premium effect it is insufficient to offset the *impatience effect* of increased risk aversion.[34]

[33] This effect would also occur if one used a Rubinstein game with exogenous risk of breakdown in lieu of time preference. In a Nash bargaining framework, the lack of extensive form means that one must stipulate whether one is considering bargaining "as if" it were taking place over time, or "as if" it were subject to uncertainty: see Binmore, Rubinstein and Wolinsky (1986). Roth and Rothblum (1982) may be considered as implicitly assuming a Nash bargaining game of the latter type in deriving their result that risk aversion is sometimes disadvantageous.

[34] The basic reason for this is that we assume that in the absence of risk, risk-aversion does not affect the point of time

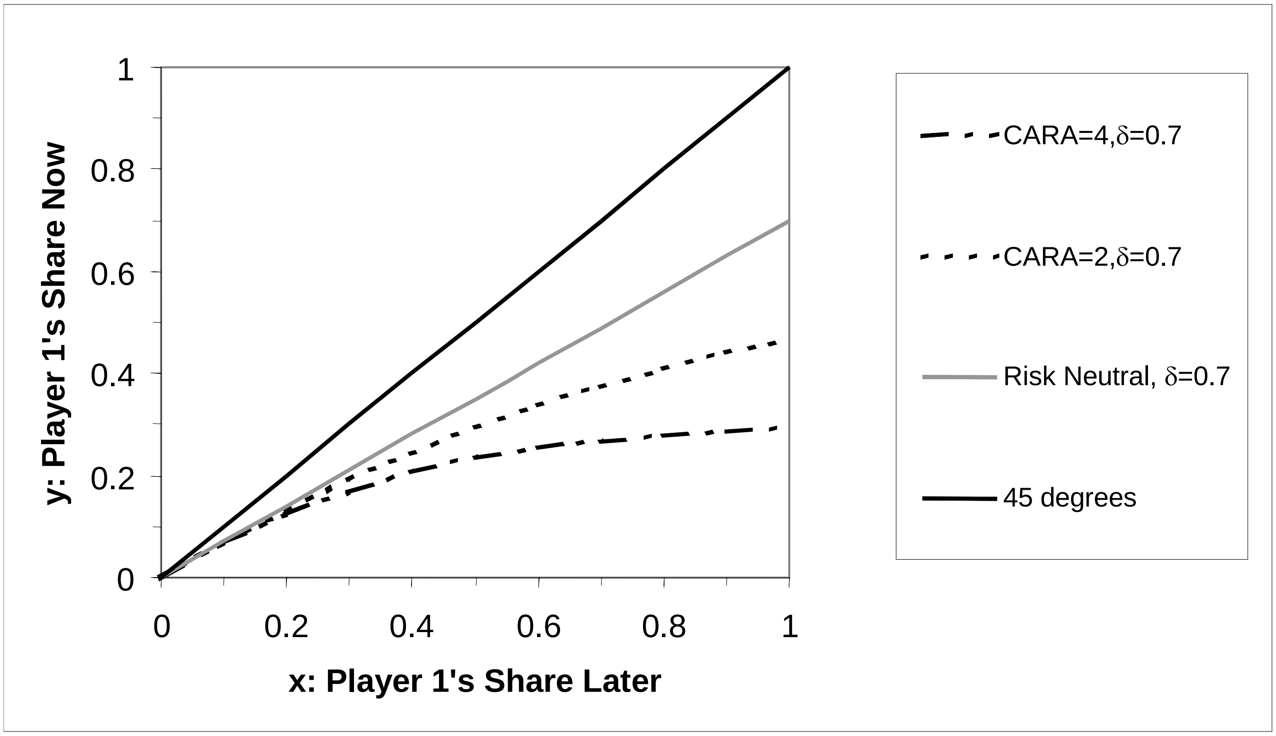

Figure 6: The effect of risk aversion under certainty

One cannot properly talk of bargainers becoming better off by an increase in risk-aversion when their payoff is risky since this involves an inter-personal comparison since the utility function has changed. But players certainly may have incentives to “pretend” that they are more risk-averse than they really are, or even to delegate to agents more risk-averse than themselves (particularly when attitudes towards certain outcomes are not adversely affected by the change). Surprisingly, the incentive to delegate to a risk-averse agent may be greater in more risky situations.

5.3. The Nash Bargaining Case

In remainder of this section we consider the effect of risk on the outcome of Nash bargaining. For ease of exposition we will consider only the case of additive risk. Since the Nash bargaining solution can be seen as the limit of the Rubinstein bargaining solution when players have symmetric discount rates and the time between offers becomes arbitrarily short (Binmore et al 1986), this section can be seen as a specialisation of the previous ones, allowing somewhat more precise results.[35] In this context, we interpret the Nash bargaining solution as implying that the division of the pie $(x, 1-x)$ is effected to maximize the product of players' expected utilities.[36] We have the following result:

Proposition 6 *A player in a Nash bargaining game whose receipts are exposed to an arbitrary mean-zero risk $z_1 \epsilon Z$ with support in $[a, b]$ will increase his receipts in bargaining relative to the riskless case if and only*

indifference. Indifference curves are closer together near this point, so the size of the risk premium effect needed to offset the impatience effect is smaller.

[35] This is in effect because the use of the Nash solution assumes that preferences have a particular form (see next footnote). This means that we only have to keep track of one equilibrium outcome, x, rather than two (x, y).

[36] This is the interpretation that follows from interpreting the Nash solution as the limit of the Rubinstein game with a zero disagreement point. (Thus, in applying it, we are in effect assuming that there are no fixed costs of bargaining.)

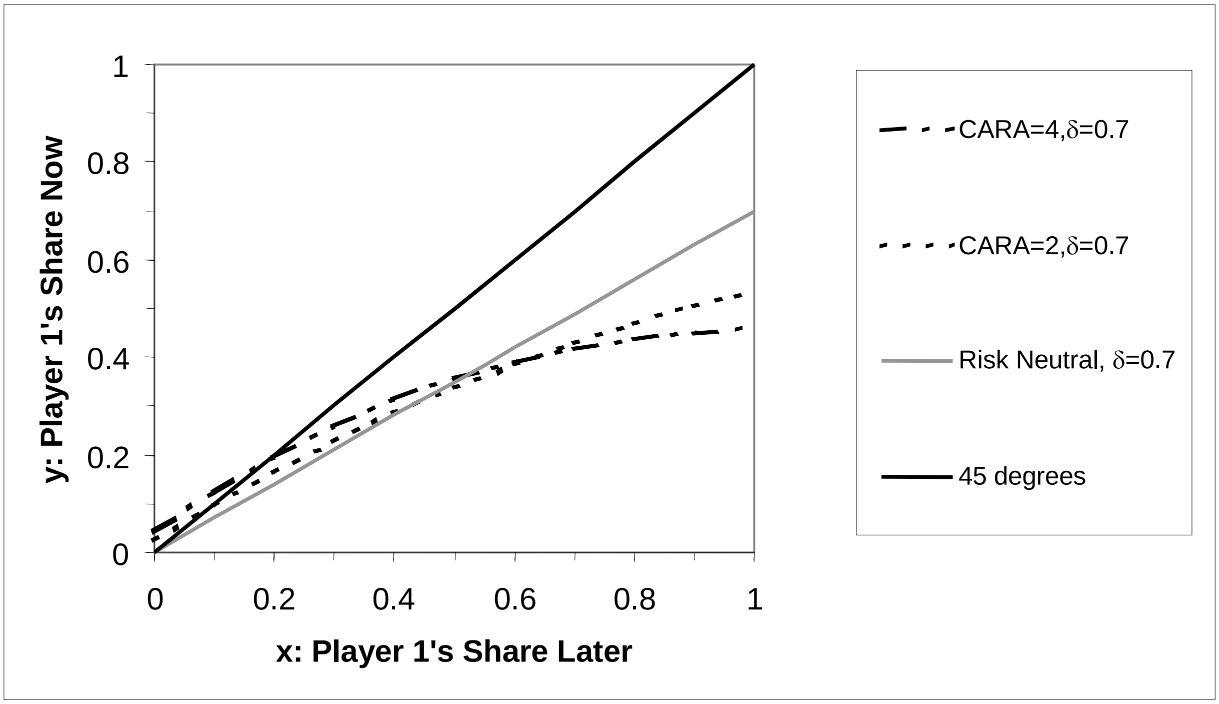

Figure 7: The effect of risk aversion under uncertainty, $\sigma = 0.3$.

if his utility function satisfies the following condition:[37]

Necessary and Sufficient Condition:

$$\forall z \epsilon [a,b] : U(x+z)/U(x) - U'(x+z)/U'(x) \leq \; z[U'(x)/U(x) - U''(x)/U'(x)]$$

Local Diffidence: If the risk z is small, then he will increase his receipts when $\frac{-U'''}{U''} > \frac{-U'}{U}$*, the coefficient of absolute prudence is greater than minus boldness.*

The necessary and sufficient condition is derived by applying the "diffidence theorem" of Gollier and Kimball (1995). Unfortunately - as is often the case - it is not easy to interpret. But the form of the "local diffidence" condition for small risks (which is derived from a Taylor Series expansion as in section 3) is not surprising: it says that receipts will increase as long as U''' is not too negative. As discussed in section 3, this condition will normally be satisfied since generally we assume $U'' \leq 0$ and $U''' \geq 0$. As in the strict discounting case, the necessary condition is weaker than $U''' \geq 0$, not surprisingly since the use of the Nash solution is based on the assumption of preferences of this type (see Binmore et al 1986). Although marginal utility falls with uncertainty when $U''' < 0$, as long as U'' is also negative, total utility falls as well, so the total effect on Player 1's boldness may still be positive. *If the proportionate change in marginal utility is more positive than the proportionate change in utility when risk is added, the overall effect of uncertainty on receipts will be positive.* The local diffidence condition clearly illustrates this trade-off since the former effect is governed by U'''/U', the latter by U''/U.

We now consider under what conditions Player 1 can be better off in equilibrium as a result of having risk added to his payoff in the Nash case. We find that:

PROPOSITION 7 *To a second-order approximation (i.e., for small risks), the welfare of a risk-averse player*

[37] If this condition holds for all $x \in [0,1]$, then it will hold for any possible opponent. Alternatively, one may consider whether the condition holds for all $z \in Z$ at the initial equilibrium point, x^*.

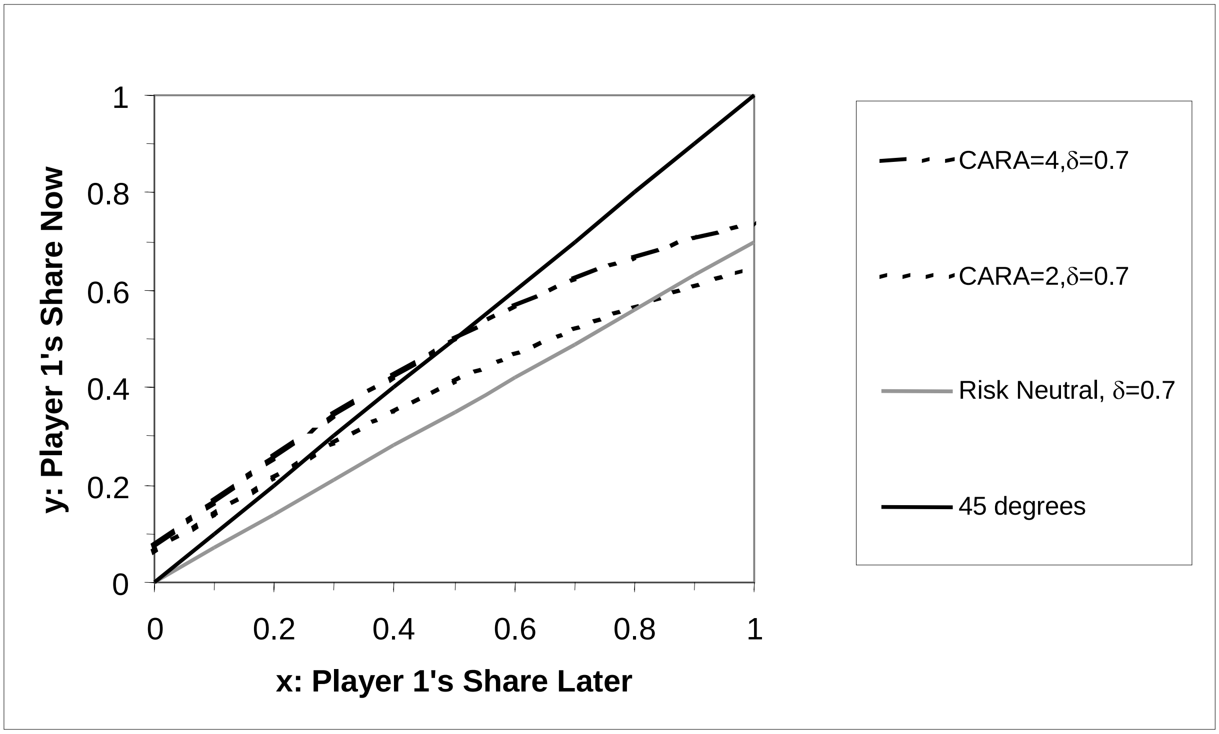

Figure 8: The effect of risk under uncertainty, $\sigma = 0.5$.

in a Nash bargaining game will rise when his receipts become uncertain if his risk-aversion decreases proportionately faster than his opponent's boldness increases. That is, Player 1's welfare is higher when he faces a small risk if and only if:

$$\begin{aligned} U_1''(x)/U_1'(x) - U_1'''(x)/U_1''(x) &\geq U_2'(1-x)/U_2(1-x) - U_2''(1-x)/U_2'(1-x) \\ -RiskAversionP1 + PrudenceP1 &\leq BoldnessP2 + RiskAversionP2 \end{aligned}$$

Or equivalently,

$$-d/dx\ log(-U_1''/U_1')^3 d/dx\ log(U_2'/U_2).$$

To gain some intuition for this result, consider the following. The expression on the LHS of the inequality is minus the logarithmic derivative of absolute risk aversion: if absolute risk aversion is decreasing, this will be positive. In this case, the Drèze-Modigliani substitution effect (the change in savings once the equivalent change in wealth has been accounted for) will be positive if absolute risk aversion is decreasing since then the effect of prudence on the marginal utility of income is more important than that of risk aversion (see Kimball (1990)). That is, the individual still has a higher marginal utility of income as a result of having risk imposed upon him, even after he has been fully compensated for the effect of that risk on his utility. Ceteris paribus he would therefore gain more in the Nash solution. However, Player 2's marginal utility of pie will also rise as pie is taken from him to compensate Player 1. The expression on the RHS is minus the logarithmic derivative of Player 2's boldness,[38] which will also be positive if Player 2 is risk-neutral or risk-averse.[39] Thus whether Player 1's utility increases as a result of an increase in the uncertainty of his

[38] For a discussion of the usefulness of the concept of boldness, see Roth (1989) and the references cited therein.

[39] A similar expression does not appear on the LHS, since Player 1 (unlike Player 2) has been compensated for the effects of the change on his utility through the addition of π to his payoff, so if it were not for the effects of prudence and risk aversion, Player 1's boldness would be the same as before.

receipts depends on whether his risk-aversion falls proportionately faster than Player 2's boldness rises as pie is transferred from Player 1 to Player 2.

To get some idea of when this condition is likely to hold, consider the following two examples. Firstly, suppose Players 1 and 2 are initially symmetrically placed, with the same utility function. Then in order for the player whose payoff is made more risky to become better off in the bargaining game between them, they must have absolute risk aversion decreasing proportionately faster than boldness. For the class of utility functions $U(x) = x^{\gamma}(0 < \gamma < 1)$ the two decrease at the same rate (i.e. the inequality holds with equality when $x = 1 - x$). So starting from a symmetric situation with two players with constant and equal relative risk aversion, to the second-order Player 1 will suffer no disutility resulting from any increase in the riskiness of his receipts: Player 2 will be forced to bear the whole cost of Player 1's compensating risk premium.[40] Secondly, if Player 1 again has constant relative risk aversion $U(x) = x^{\gamma}$ and is strictly risk-averse, whilst Player 2 is risk-neutral, Player 1 will indeed gain when a small risk is added to his pay-off.

6. RELATED LITERATURE

Given the practical importance of pay-off uncertainty as outlined in the introduction, it is surprising that there has not been more work in this area. Perhaps one reason for the relative neglect of this topic has been the (mistaken) belief that the comparative statics of risk in games will be analogous to the comparative statics of risk aversion. In fact, as we have seen, this is not the case. Risk aversion is a *disadvantage* in bargaining games with certain outcomes (Roth 1979, Roth 1985); Roth and Rothblum (1982) show that it is also disadvantageous in the Nash solution in risky settings if one's opponent's most preferred outcome is better than the disagreement outcome. In contrast, we have shown that facing risk is an *advantage* in bargaining games: the effect of adding risk to a risk-averse player's receipts is typically to *increase* his bargained payoff.

This paper provides the first results regarding the comparative statics of pure payoff risk in the Rubinstein game and the Nash Solution. Few previous studies even allow for pay-off risk in bargaining.[41] In this section we compare our paper with others that do so. Roth and Rothblum (1982) presented a Nash model with risky outcomes[42] - but their treatment of risk was designed to extend previous results on the comparative statics of risk aversion in bargaining (i.e., to show that it could sometimes be an advantage), rather than to judge the effects of risk *per se*. Indeed, it is difficult to gauge the effects of changes in risk in their model because the risks which players face are not zero mean and the level of risk borne by each player is endogenous to the outcome. By contrast we have here analysed the effect of exogenously imposing Rothschild-Stiglitz increases in risk on one or both players' payoffs, allowing us to derive clear results on the effect of uncertainty.

Avery and Zemsky (1995) consider a Rubinstein-style model where the future size of the pie is uncertain and evolves over time. However, they are interested in the option-value effect which the evolution of pie size causes and they explicitly assume risk neutrality for tractability. But we have shown that if at least one of the players is risk-averse, uncertainty will affect the bargaining outcome even when the pie is stationary and no new information arrives until after agreement.

Yildiz (2002) considers a model of bargaining between two agents each endowed with risky assets, where the *sole purpose* of negotiation is to share their risks. The main result is that the revelation of information over finite time motivates bargainers to agree immediately on a Pareto Optimal allocation. The approach

[40] To see this, solve the Nash problem under certainty to show that in the case $\gamma/x = 1/(1-x)$. Then show that the inequality in the proposition holds, provided $1/x > 1/(1-x)$. These two expressions imply that the inequality holds when $\gamma < 1$.

[41] We are not here concerned with risk arising from asymmetric information between parties, which has been extensively studied elsewhere (see e.g. Kennan and Wilson (1993) for a survey).

[42] Their model has subsequently been generalized to allow for a risky disagreement outcome. See Safra et al. (1990) and Chun and Thompson (1990).

is in a sense the polar opposite of the one adopted in the present paper, where the purpose of negotiation is to share a pie and bearing risk is the unfortunate side-effect of agreement. Yildiz does not consider the comparative statics of risk and can obtain the comparative statics of risk aversion only for a special case (constant absolute risk aversion). As mentioned in section 2, the benefit of our approach is that by abstracting from negotiation over risk-sharing itself we greatly simplify the analysis of the impact of risk.

The paper most closely related to this model is Riddell (1981). Riddell considers a variety of cooperative bargaining models (Nash, Harsanyi, Kalai-Smorodinsky and 'proportional') where players will have the opportunity to split different-sized pies depending on the realization of the state of nature. Before nature makes her move, they have the opportunity to bargain over making additional state-contingent payments to one another, which, by allowing risk-sharing, will be Pareto-improving if the two parties' risk aversion differs. The disagreement outcome for this initial negotiation is the expected utility of the vector of outcomes that will occur if they wait and bargain after uncertainty is realized. Riddell then considers a mean-preserving spread in the probabilities with which different states of nature occur. He shows that for the Harsanyi Solution, both players are made worse off. For the other three solutions, he shows that more uncertainty makes the outcome more favorable to 'the insurer' (the player whose ideal outcome becomes more variable after the increase in uncertainty) than 'the insuree' (the player whose ideal outcome becomes less variable), though it may be that both are worse off. To the extent that the more risk-averse player is the insuree, his results contrast with those presented here.[43] For example, if one player is risk-neutral, that player gains from increase in uncertainty at the expense of his risk-averse rival, whereas in our model the opposite occurs. The difference arises in part because in Riddell increasing the risk of agreement also implies increasing the risk of the disagreement outcome, and this latter tends to hurt a risk-averse player; though it is difficult to disentangle this effect from the effect of allowing for negotiation over risk-sharing.

The main advantage of the model presented here over previous work is that we isolate the effects of making an agreement risky from any other effects, thus allowing clear and tractable results. Moreover, as argued in section 2, the case where players bargain first and foremost to allocate a pie seems to be the empirically most relevant case; risk-sharing is often a secondary consideration. Changes in risk associated with the disagreement will not always be relevant to outcome of bargaining (Binmore et al 1986), but even when they are it is in many cases not obvious why they should be connected to changes in the risk associated with agreement.

7. CONCLUSIONS

We have investigated the effect of uncertainty about the value of receipts from agreement in Rubinstein and Nash bargaining games. We have found that if the risk associated with agreement is additive (i.e. independent of the agreement reached) then a risk-averse player will be (at least partially) compensated for any increase in the riskiness of his bargained payoff. This will be true unless risk aversion increases with wealth at an implausibly fast rate. If the risk associated with agreement is multiplicative then the conditions for an increase in receipts are more stringent but not implausible.

We demonstrated that with both additive and multiplicative risk, there are circumstances under which a decreasingly risk-averse player can extract more than his compensating risk premium in increased receipts. *He thus becomes better off when his payoff becomes risky.* Our results are important because nearly all real-world bargaining games involve some kind of ex post payoff uncertainty. And in many contexts, such as wage bargaining and negotiating over managerial compensation, risk aversion is likely to be significant.

[43] This is not straightforward, because who is the insuree and who is the insurer is actually endogenous to the anticipated bargaining after uncertainty is resolved, thought the modelling of this second stage bargaining is suppressed for tractability. Again, allowing for negotiation over risk-sharing makes comparative statics difficult to obtain.

We showed how our general model fitted these and several other specific examples. Our results imply that inflation uncertainty will tend to raise trade union power when bargaining over nominal wages. A seller may benefit from production cost uncertainty when negotiating with a buyer over the price of the object. Similarly, when a joint venture or portfolio has to be shared between two agents then the split will tend to favour the risk-averse negotiator if the portfolio is risky.

The examples which we have given are economically important in their own right. But they also highlight a more fundamental point. Since a bargaining player will not bear the full cost of any increase in the riskiness of his bargained payoff, he is in general likely to have insufficient incentives to reduce the risk that he will face if an agreement is reached. He may even want strategically to increase the risk he faces. Consider the implications of this point in the context of our example of negotiation over managerial compensation. Managers' total pay will tend to increase - and they may become better off - if their compensation is more risky. This suggests that managers may try to manipulate their bargaining situation by negotiating first on the bonus component of their salary and second on their fixed wage, because a high-powered (and hence more risky) bonus scheme will tend to make them tougher bargainers over the fixed component. Thus they may be better off at the shareholders' expense, but they are bearing too much risk from an efficiency point of view. A fuller investigation of this issue is left to further research.

The basic intuition behind our results is that *when payoffs are risky, a larger income becomes relatively more valuable.* Therefore a risk-averse player is prepared to wait longer to get a larger income in a risky situation: he acts as if he has become 'more patient'. As we have explained, the situation is very similar to one of precautionary saving. The prudence of the utility function ($U''' > 0$) plays a crucial role in driving the results both here and in the precautionary saving literature. Precautionary saving is an empirically well-documented phenomenon - so the conditions under which risk will increase a player's bargaining power are very likely to hold (because these are typically weaker than the conditions for precautionary saving to arise). The model presented here is clearly special (alternating-offer or Nash bargaining). But because the impact of risk is felt through increased marginal utility of income (and this is virtually always an advantage), the results are likely to extend to a wide variety of other bargaining frameworks, and indeed other strategic games (see Eső and White, forthcoming).

8. APPENDIX

8.1. Players' Preferences

Players bargain over a pie of size one, and the set of feasible offers (x_1, x_2) is $X = \{\mathbf{x} = (x_1, x_2) : x_1 \geq 0, x_2 \geq 0, x_1 + x_2 \leq 1\}$. We will denote the time at which agreement is reached by $t \epsilon T (T = \{0, 1, 2, ...\})$. We will make the standard assumptions on preferences necessary to guarantee that equilibrium exists in the Rubinstein game. However, because we wish to study the equilibrium effects of adding some risk $z \epsilon Z$, for some set of mean-preserving spreads Z to Player 1's payoff, we must ensure that equilibrium exists for all z in the set under consideration. We thus define a preference mapping $V : X \times T \times Z \to R$ over such certain receipts. We will use these preferences to draw indifference maps relating receipts now to receipts later, thence to infer what outcomes a player will be willing to accept given what he expects to receive next period. In order to ensure that we have indifference "curves" rather than regions, we need to make a series of assumptions on preferences that are familiar from consumer theory. Firstly, let us normalize $V(0, 0, 0) \equiv 0$. Then we will call any combination (x, z) with $V(x, 0, z) \geq 0$ "desirable." For all $(x, z) \epsilon X \times Z$ which the individual regards as desirable, we follow Rubinstein (1982) in assuming:

A1 Monotonicity: More pie is desirable. $V(x, t; z) > V(y, t; z)$ if and only if $x > y$.

A2 Impatience: Time is valuable. $V(x, t; z) \geq V(x, t+1; z)$ with strict inequality whenever $V(x, 0, z) >$

0.

A3 Continuity: Preferences are continuous. If $\{x, t; z\}_{n=1}^{\infty}$ and $\{y, t; z\}_{n=1}^{\infty}$ are sequences of members of $X \times \{z\}$ for which $lim_{n\to\infty} x_n = x$ and $lim_{n\to\infty} y_n = y$, then $(x, t; z) \geq (y, t; z)$ whenever $(x_n, t; z) \geq (y_n, t; z) \forall n$.

These assumptions will be sufficient to allow us to construct indifference curves over bargained outcomes with a given level of uncertainty. Denote Player 1's preferences over such outcomes by $V_1 : X \times T \times Z \to R$, and define V_2 similarly for Player 2. In order to reduce notation we often drop the subscript where the identity of the player is not important.

It will also be convenient to use these assumptions to allow us to define the present value of receiving a share x with risk z at t to Player i: $p_i(x, t; z) = y$ if $(y, 0; z)\tilde{}_i(x, t; z)$. Note that $p_i(x, t; z)$ may be negative, if for example players have fixed per period costs of bargaining. We will refer to the curves $(x, y; z)$ such that $(y, 0; z)\tilde{}_1(x, 1; z)$ and $(x, 0; z)\tilde{}_2(y, 1; z)$ as respectively Player 1's and Player 2's "indifference curves".

The preferences defined by **A1-A3** are sufficient to allow us to draw indifference curves over receipts this period t and receipts next period $t + 1$ for a given level of risk z, but they do not guarantee that when the next period $t + 1$ arrives, preferences between receipts now (at $t + 1$) and receipts in the following period $(t + 2)$ will be in any way similar to the period t preferences between receipts now and receipts later. If these preferences are substantially different, the solution of the game is more complicated because though the structure of the game is essentially the same as that two periods before, to the players involved it looks different - they may not behave in a time consistent manner. Therefore, we make the following stationarity assumption on preferences:[44]

A4 Stationarity:

(i) For any $t \in T, x, y \in X, z \in Z$ we have $V(x, t; z) > V(y, t + 1; z)$ only if $V(x, 0; z) > V(y, 1; z)$.

(ii) For any $t \in T, x, y \in X, z \in Z$ we have $V(x, t; z) > V(y, t; z)$ only if $V(x, 0; z) > V(y, 0; z)$.

Part (ii) of the stationarity assumption - which says that attitudes to risk do not change purely with the passage of calendar time - is equivalent to the assumption that attitudes towards risk are *utility-independent* of time.[45] This in turn must imply that the value function for consumption at time t is a positive affine transformation of that for consumption at time zero: $V(x, t, z) = \alpha_t + \beta_t V(x, 0, z)$. When in addition a player's preferences satisfy the expected utility hypothesis, a player's attitude towards mean preserving spreads can be derived from his cardinal utility function for consumption bundles in period zero. This will allow us a useful simplification in notation. We will use $U_i(x) \equiv V_i(x, 0, 0)$ to describe Player i's utility function for certain bundles at time zero.

The assumption of stationarity of time preferences **A4**(i) also imposes some more familiar restrictions on the form of the value function; in particular on the form of the series α_t and β_t given in the expression $V(x, t, z) = \alpha_t + \beta_t V(x, 0, z)$. It can be shown that stationarity implies that $\beta_t = \beta_1^t$, and that $\alpha_t = \alpha_1 \Sigma_{s=1}^{t} \beta_1^{s-1}$. Monotonicity (**A1**) also implies that $\beta_1 > 0$; impatience (**A2**) implies that $\alpha_1 < 0$. Two special cases of such preferences have received special attention in the bargaining literature, and we will

[44] This is somewhat stronger than the analogous assumption made in the standard Rubinstein model, which amounts only to part (i) of the assumption. In order for preferences over risky bargains to be stationary, we require not only that "impatience" is stationary, but also that attitude to risk is stationary. In dealing with expected utility maximizers, (i) implies (ii), since an individual's "time adjustment calculus" must commute with the operation of taking its certainty equivalent in order for his preferences to be consistent in the Von Neumann-Morgernstern sense (Prakash 1977). As is familiar from Binmore (1987) (and more recently Merlo and Wilson 1995), as long as the pie shrinks over time, the game can be solved, without stationarity assumptions, as the limit of the corresponding n stage finite horizon bargaining game as $n \to \infty$. Thus, the basic results of the paper - that a risk-averse bargainer may increase his equilibrium receipts and utility will still hold without stationarity assumption (i), though this greatly simplifies the analysis by ensuring (in conjunction with the other assumptions) a unique equilibrium. If, however, aversion to risk, or the size of the risk associated with agreements, rises quickly enough over time, then the results presented here could be reversed, with risk-averse players becoming more impatient to agree now, and hence receiving a smaller share.

[45] For more detail on this, see Keeney and Raiffa (1993), Chapter 5, and also especially Chapter 9. Much of the original work was done by Koopmans (1972).

sometimes refer to these in providing examples:

Pure Fixed discount factors: $\alpha_1 = 0$ and $\beta_1 = \delta$ so that under the expected utility hypothesis $V(x,t,z) = \delta^t EU(x+z)$.

Pure Fixed costs of bargaining: $\alpha_1 = -c$ and $\beta_1 = 1$ so that $V(x,t,z) = EU(x+z) - ct$.

In general we abuse terminology somewhat and refer to preferences where $\alpha_1 \neq 0$ as preferences where there is a fixed cost of bargaining, and preferences where $\beta < 1$ as preferences where there is discounting. One thing to notice about the two special cases of pure discounting and pure fixed costs is that in each of them, and in contrast to the more general formulation with both "discounting" and "costs" of bargaining, attitudes towards time and risk are mutually utility independent, i.e. in addition to **A4**(ii) we have that time is utility independent of risk level. This may reduce the attractiveness of restricting preferences to these forms, but on the other hand it makes it all the more surprising that adding a mean preserving spread to payoffs will change the outcome even in these cases.

In order for the game to be interesting, we also require that there is some possible agreement which is better for players than the disagreement outcome. The game will also be simpler to solve if the disagreement outcome does not restrict the set of possible bargains under certainty.

A5: The Disagreement Outcome

(i) Disagreement is not the best outcome under uncertainty $z \in Z$. Assume that there exist $x \in X$ such that $V_i(x,0,z) > V_i(D)$ for $i = 1,2$ and $z \in Z$, where the subscripts 1 and 2 denote the utility functions of players 1 and 2 respectively and D denotes the disagreement outcome. That is, suppose that it is possible for players to make a mutually beneficial agreement, even given the risk that they must bear if they do so.[46]

(ii) Disagreement is the worst outcome under certainty. For all $(x,t,0) \in X \times T \times \{z \equiv 0\}$, $V_1(x,t) \geq V_1(D)$ and $V_2(x,t) \geq V_2(D)$. In particular, let $V_1(D) = V_2(D) = 0$.

The assumption $V_1(D) = V_2(D) = 0$, means that "disagreement" is effectively an outside option equivalent to receiving zero for certain. Whilst this assumption is restrictive and perhaps not very realistic when there are fixed costs to bargaining, it greatly simplifies the analysis. It means that we need not consider the possibility of agreements which give players negative net present outcomes.[47] Note that we also assume that the disagreement outcome is independent of the risk level associated with agreements. In our Rubinstein game without risk of breakdown this is harmless since the disagreement serves only to provide a lower-bound on payoffs, which with risk-averse players will not be effective provided that disagreement is the worst outcome under certainty, and there is some possible agreement under uncertainty preferred to disagreement (i.e. **A5** holds).

Finally, in order to ensure that the game has a unique equilibrium, we follow Rubinstein in making the following assumption on preferences:

A6: Increasing loss to delay. Players are assumed to require increasing compensation for delay, so that the difference $x - p_i(x,1;z)$ is an increasing function of x.

As Rubinstein emphasizes, this assumption is far from necessary to ensure uniqueness of equilibrium, but it is a convenient tool for doing so. In our context a sufficient condition for **A6** is that the function $\partial^2 V(x,0,z)/\partial x^2 \leq 0$ and that $\beta_1 \leq 1$, with at least one of the inequalities strict.[48] Since we are in any case

[46] Obviously, if the risk becomes too large, there will be situations where players would rather disagree than bear the risk. The assumption A5 is not intended to be plausible for all possible mean-preserving spreads, but may rather be thought of as a restriction on the set Z of mean-preserving spreads that we consider: the analysis of those which would result in infinite disagreement is not very interesting.

[47] By making appropriate assumptions about the preferences over negative net present value outcomes, one could carry out the same analysis without this assumption. However, little insight would be gained by the additional complication, since the main results would be unaffected.

[48] Let $V(y,z,0) = V(y+\varepsilon(y),z,1) = \alpha + \beta V(y,z,0)$. Then differentiating with respect to expected receipts y, we have $V'(y,z,0) = \beta V'(y+\varepsilon(y),1)(1+d\varepsilon/dy)$. Positive delay costs imply that $y+\varepsilon > y$, and strictly concave utility then implies that $V'(y,z,t) > V'(y+\varepsilon(y),z,t+1)$ (since $EU'(y+z,) > EU'(y+\varepsilon(y))$) which, provided $\beta \leq 1$, implies $d\varepsilon/dy > 0$, which

interested in risk-averse impatient players, we may hope that the assumption is not too restrictive.

In the absence of fixed costs of bargaining, the above assumptions on preferences are sufficient to prove proposition 1: the existence and uniqueness of equilibrium.[49] When there are fixed costs of bargaining, it is well-known that there can be corner solutions (there will not be multiple equilibria in the risk-averse case, however, and corner solutions are also less likely.) For simplicity, we simply ignore the corner solution case; the paper treats the case of interior solutions.

In order to derive expressions for risk premia where required, we also make the following assumption.

A7: Expected Utility. Players' preferences satisfy the expected utility hypothesis: $U(x+z)$ can be represented by $\int U(x+z)h(z)\,dz$ where $h(z)$ is the density of the random noise outcomes z.

8.2. PROOFS

PROOF OF PROPOSITION 1

The proof of the existence and uniqueness of equilibrium is absolutely standard. See Rubinstein (1982); also Osborne and Rubinstein (1990) and Binmore, Rubinstein and Wolinsky (1986) for useful discussions.

PROOF OF PROPOSITION 2

Part (i) Let $x^{\#}, y^{\#}$ denote the initial equilibrium solutions and $x^*, y*$ denote the final solutions after the increase in risk z. We will show that if $\pi(y^{\#}) > \pi(x^{\#})$ then $x^* > x^{\#}$, that is, that making Player 1's payoff risky increases his receipts in equilibrium.

Firstly note that since x^*,y^*,$x^{\#}$ and $y^{\#}$ are equilibrium solutions we have (by definition):

$$\begin{aligned} V_1(y^{\#},0,0) &= V_1(x^{\#},1,0) && \text{(1a)}\\ V_2(1-x^{\#},0,0) &= V_2(1-y^{\#},1,0) && \text{(1b)}\\ V_1(y^*,0,z) &= V_1(x^*,1,z) && \text{(1c)}\\ V_2(1-x^*,0,0) &= V_2(1-y^*,1,z). && \text{(1d)} \end{aligned}$$

Notice also that by definition of the (compensating) risk premium $\pi(.,.)$, we have:

$$\begin{aligned} V_1(y^{\#},0,0) &= V_1(y^{\#}+\pi(y^{\#},z),0,z) && \text{(1e)}\\ V_1(x^{\#},1,0) &= V_1(x^{\#}+\pi(x^{\#},z),1,z). && \text{(1f)} \end{aligned}$$

Substituting the first of these expressions into the second, we have:

$$V_1(y^{\#}+\pi(y^{\#},z),0,z) = V_1(x^{\#}+\pi(x^{\#},z),1,z). \tag{2}$$

Now suppose, to argue by contradiction, that $x^{\#} > x^*$. Then certainly $x^{\#}+\pi(x^{\#},z) > x^*$, so applying the increasing compensation for delay assumption (A6), for Player 1 (at risk level z) we have

$$x^{\#}+\pi(x^{\#},z)-[y^{\#}+\pi(y^{\#},z)] > x^*-y^* \tag{3}$$

and for Player 2 (at zero risk), we have

$$1-y^*-[1-x^*] > [1-y^{\#}]-[1-x^{\#}] \rightarrow\; x^{\#}-y^{\#} < x^*-y^*. \tag{4}$$

Substituting (4) into (3) yields: $\pi(y^{\#},z)\; -\pi(x^{\#},z) < 0$, which contradicts our initial hypothesis $\pi(y^{\#}) > \pi(x^{\#})$. Hence it must be the case that instead $x^* > x^{\#}$: the addition of the risk z to Player 1's payoff

means that compensation is increasing.

[49] The reason for this is that given our assumption that disagreement is not the best outcome, in the absence of fixed costs, players will always make an agreement which has positive expected utility to each of them (otherwise it would be impossible for both the indifference conditions to apply). Therefore, assumptions on negative expected utility (x,z) combinations are not necessary.

increases his equilibrium level of receipts. Notice that since the additional noise z was chosen arbitrarily, the same argument can be made for any increase in risk which satisfies the Rothschild-Stiglitz conditions.[50]

Part (ii) The proof of part (ii) follows the argument given in the text. If the outwards shift of points from the old to the new 'indifference curve' is at an angle $\frac{\pi(y^{\#})}{\pi(x^{\#})}$ greater than the slope of old indifference curve at that point, $\frac{\delta U'(x^{\#})}{U'(y^{\#})}$, the new indifference curve must lie above the old one if the old indifference curve is concave.

Proof of Corollary 2.1: Additive Risk

(i) If Player 1 exhibits non-increasing absolute risk aversion, then since $x > y$, with additive risk we have $\pi(y,z)/\pi(x,z) > 1$, fulfilling part (i) of Proposition 2.

(ii) The ratio of the risk premia is approximately $[\frac{1}{2}\delta^2 U_1''(y^{\#})]/[\frac{1}{2}\delta^2 U_1''(x^{\#})]$, which must be larger than the slope of the indifference curve $\delta U'(x^{\#})/U'(y^{\#})$. Simplifying this gives: $U_1''(y^{\#}) > \delta U_1''(x^{\#})$.This will be satisfied if $U'''(.) \geq 0$ and $\delta \leq 1$ with at least one inequality strict. We do not need to worry about possible convexity of the indifference curve here because the indifference curve can only be convex if risk-aversion is decreasing, in which case the proof can be done using part (i) of the corollary.

Proof of Corollary 2.2: Multiplicative Risk

If the ratio of risk premia exceeds one, then we have a sufficient condition under proposition 2.1 part (i). The conditions under which this holds are discussed in the proof of corollary 4.1. If this ratio is less than one then we need an explicit expression for the risk premium with mean-zero multiplicative risk. Following Pratt (1964),

$$\begin{aligned} U(x-\pi) &\equiv EU(x(1+z). && (5a)\\ U(x-\pi) &\approx U(x) - \pi U'(x) + \ldots && (5b)\\ EU(x(1+z)) &\approx E[U(x) + xzU'(x) + \frac{1}{2}x^2z^2U''(x) + \ldots], && (5c) \end{aligned}$$

So that the risk premium for small multiplicative risks is $-\frac{1}{2}x^2\delta^2U_1''(x)/U'(x)$. For the ratio of risk premia to exceed the slope of the indifference curve we require that: $[\frac{1}{2}y^2\delta^2U_1''(y)/U'(y)]/[\frac{1}{2}x^2\delta^2U_1''(x)/U'(x)] > \delta U'(x)/U'(y)$. Cancelling $U'(x)$ and $U'(y)$, in the case when $\delta = 1$, we thus need $-t^2U"(t)$ decreasing in t, which, differentiating yields $-\frac{U'''t}{U''} > 2$. When $\delta < 1$, this condition is sufficient but not necessary. This proves the corollary. To show that decreasing relative risk aversion with parameter greater than one implies this condition, differentiate the expression for relative risk aversion and substitute in a value of at least one to show that it must be that $-U'''x/U'' > 2$.

PROOF OF PROPOSITION 3

(i) This result is well-known (see Sandmo 1968, Leland 1971, Drèze-Mogdigliani 1972, Kimball 1990). If $U''' > 0$ then marginal utility is a convex function, so by Jensen's inequality adding a mean-zero risk will raise expected marginal utility.

(ii) We wish to show that to a second-order approximation, multiplicative zero-mean risk will raise expected marginal utility if $\frac{1}{2}xU'''(x) > -U''$. Note that :

$$\frac{d}{dx}EU(x(1+z)) = EU'((1+z)x) + EzU'((1+z)x),$$

and

$$EU'((1+z)x) \approx E[U'(x) + \frac{1}{2}z^2x^2U'''(x) + ...]$$

[50] Indeed, there is no special reason to begin with the indifference curve under certainty, except that we are sure that it exists: one could begin with an indifference curve in which payoffs are subject to noise and then increase the noise, deriving the new curve by reference to the increase in risk-compensation premium. We know from Ross (1981) that the property of decreasing risk aversion is preserved under expectations, and thus the proof would go through very much as before.

since $E[zxU''(x)] = 0$. Moreover,

$$EzU'((1+z)x) \approx \ E[z^2xU" + \frac{1}{2}z^3x^2U'''(x) + ...]$$

since $E[zU'(x)] = 0$ as $E[z] = 0$ by assumption.

Thus taking a second-order approximation (i.e. assuming that terms of order z^3 and above are small), and using the fact that $E[z^2] = \delta^2$ when $E[z] = 0$, where δ^2 denotes the variance of the mean-zero risk,

$$\frac{d}{dx}EU(x(1+z)) \approx U\prime(x) + \frac{1}{2}\delta^2x^2U'''(x) + \delta^2xU''(x).$$

This is increasing in δ^2, the variance of the multiplicative risk, if $\frac{1}{2}xU'''(x) > -U''(x)$.

PROOF OF PROPOSITION 4:

As before, denote the initial equilibrium $(x^{\#}, y^{\#})$ and the equilibrium after the increase in risk (x^*, y^*). As in the proof of proposition 2, we will first examine the consequences of strongly decreasing risk aversion, and then argue by contradiction that this is not consistent with a reduction in welfare. Strongly decreasing risk aversion implies that the compensating risk premium at $x^{\#}$ is much smaller than that at $y^{\#}$:

$$\pi(x^{\#}, z(x^{\#})) \ll \pi(y^{\#}, z(y^{\#})) \tag{6}$$

From now on we suppress the argument of z to reduce notation. If risk reduces his equilibrium welfare then the increase in receipts accompanying the increase in risk must be smaller than the compensating risk premium:

$$\begin{aligned} V_1(x^{\#}, 1, 0) &\geq V_1(x^*, 1, z) \\ V_1(x^{\#} + \pi(x^{\#}, z), 1, z) &\geq V_1(x^*, 1, z) \\ &\Rightarrow x^{\#} \geq x^* - \pi(x^{\#}, z) \end{aligned}$$

(by substituting for the compensating risk premium for z at x^* as before, and then using the motonicity of U_1). At the final equilibrium we have:

$$V_1(x^*, 1, z) = V_1(y^*, 0, z).$$

And from the initial equilibrium and the definition of the compensating risk premium $\pi(.,.)$ we have:

$$V_1(x^{\#}, 1, 0) = V_1(x^{\#} + \pi(x^{\#}, z), 1, z) = V_1(y^{\#}, 0, 0) = V_1(y^{\#} + \pi(y^{\#}, z), 0, z).$$

We have assumed that there is increasing compensation for delay at any given level of risk. So at any given level of risk, increasing receipts this period (y), means an increased difference $(x - y)$ between indifferent levels of receipts next period (x) and this period (y). Applying this axiom to Player 1's receipts at risk level z and to Player 2's we have given that $x^{\#} \geq x^* - \pi(x^{\#}, z)$, we have for Player 1:

$$x^{\#} - y^{\#} > x^* - \pi(x^{\#}, z) - [y^* - \pi(y^{\#}, z)] \tag{7}$$

and for Player 2:

$$[1 - (x^* - \pi(x^{\#}, z))] - p_2^{-1}[(1 - (x^* - \pi(x^{\#}, z))), 1] > [1 - y^{\#}] - [1 - x^{\#}] \tag{8a}$$

$$\Rightarrow x^{\#} - y^{\#} < x^* - \pi(x^{\#}, z) - [1 - p_2^{-1}(1 - (x^* - \pi(x^{\#}, z)), 1)]. \tag{8b}$$

Taking the implications of (7) and (8a) together, we have $p_2^{-1}(1 - [x^* - \pi(x^{\#}, z)], 0)] > 1 - [y^* - \pi(y^{\#}, z)]$, or inverting $p_2(.)$:

$$1 - [x^* - \pi(x^{\#}, z)] > p_2(1 - [y^* - \pi(y^{\#}, z)]) \tag{9}$$

Because x^*, y^* is an equilibrium, we know that $1 - x^* = p_2(1 - y^*, 1)$. Subtracting this from both sides we have:

$$1 - [x^* - \pi(x^\#, z)] - [1 - x^*] \quad > \quad p_2(1 - [y^* - \pi(y^\#, z)]) - p_2(1 - y^*, 1) \tag{10a}$$

$$\Rightarrow \quad \pi(x^\#, z) > p_2((1 - y^*) + \pi(y^\#, z)], 1) - p_2(1 - y^*, 1) \tag{10b}$$

But this is inconsistent with our assumption of strongly decreasing risk aversion $\pi(x^\#, z) << \pi(y^\#, z)$ (consider for instance the case $\pi(x^\#, z) = 0, \pi(y^\#, z) > 0$). Therefore it must be the case that if risk aversion decreases quickly enough, Player 1's welfare increases as a result of an increase in the riskiness of his receipts.

We can use a Taylor series expansion to determine approximately how quickly Player 1's risk aversion must fall for Player 1 to be better off as a result of the increase in risk. We require:

$$x^\# + \pi(x^\#, z) \quad < \quad 1 - p_2((1 - y^\#) + \pi(y^\#, z)], 1) \approx \; 1 - \pi(y^\#, z)p_2\prime(1 - y^\#, 1) + ... \tag{10c}$$

$$\Rightarrow \quad p_2'(1 - y^\#, 1) > \pi(x^\#, z)/\pi(y^\#, z) \tag{10d}$$

since $x^\# = 1 - p_2(1 - y^\#)$.

Proof of Corollary 4.1

If Player 2 is risk-neutral with fixed costs of bargaining then $p_2'(.) = 1$ as long as $(1 - y > c_2)$. Thus from proposition 4 above, a necessary and sufficient condition for an increase in welfare is $\pi(y^\#, z)/\pi(x^\#, z) > 1$. This is equivalent to decreasing absolute risk aversion if z is independent of the agreement x (or y), i.e. for the case of additive risk. For multiplicative risk we need a further expression for the ratio of risk premia, which will be $yR(y)/xR(x)$, where $R(.)$ is the coefficient of relative risk aversion. We then require that $xR(x)$ be decreasing in its argument, which by differentiation gives the expression in the text.

PROOF OF PROPOSITION 5:

1. Differentiating the natural logarithm of the coefficient of absolute risk aversion shows that absolute risk aversion can be (strictly) decreasing if and only if the coefficient of absolute prudence (strictly) exceeds the coefficient of absolute risk aversion. This result is due to Kimball (1990).
2. The precautionary premium ψ for multiplicative risk is given by:

$$U'(x - \psi) \equiv d/dxEU(x(1 + z)).$$

Ignoring terms in ψ^2 (as is conventional in deriving risk premia for small risks), we have:

$$U'(x - \psi) \approx U'(x) - \psi U''(x),$$

and from the proof of proposition 3(ii) we have

$$(d/dx)EU(x(1 + z)) \approx U'(x) + \frac{1}{2}\sigma^2 x^2 U'''(x) + \sigma^2 x U''(x).$$

It follows that $\psi \approx -\frac{1}{2}\sigma^2 x^2 U'''(x)/U''(x) - \sigma^2 x$, and thus that $\psi > \pi$ (the risk premium – see proof of corollary 2.2) is approximately equivalent to:

$$-\frac{1}{2}\sigma^2 x^2 U'''(x)/U''(x) - \sigma^2 x > -\frac{1}{2}\sigma^2 x^2 U''(x)/U'(x),$$

which on simplifying gives the condition given in corollary 4.1: $x(U''(x)/U'(x) - U'''(x)/U''(x)) > 2$.

PROOF OF PROPOSITION 6:

Here we prove the case for small risks, since it is quite simple and intuitive. The necessary and sufficient condition is obtained as a direct application of the "Diffidence Theorem" of Gollier and Kimball (1995), which also yields the condition given for small risks as a "local diffidence" result. The reader is referred to their paper for these proofs.[51]

The maximization of the product of utilities in the certain case yields the following first order condition:

$$U_1'(x)/U_1(x) = U_2'(1-x)/U_2(1-x). \tag{11}$$

Subjecting Player 1's payoff to zero mean uncertainty z, we have the condition:

$$EU_1'(x+z)/EU_1(x+z) = U_2'(1-x)/U_2(1-x). \tag{12}$$

Thus for uncertainty about his payoff to increase (strictly) Player 1's share of the pie, from (11) and (12) we require

$$EU_1'(x+z)/EU_1(x+z) > U_1'(x)/U_1(x), \tag{13}$$

since a player's "boldness" (that is, the ratio of the derivative of utility to the value of utility) is a decreasing function (Roth (1989)). Now, when uncertainty is added to a risk-averse player's payoff, his utility will fall, so the denominator of the LHS of (20) is smaller than the denominator of the RHS. And marginal expected utility will rise when uncertainty increases only if $U''' > 0$, in which case the numerator of the LHS of (13) will be larger than the numerator of the RHS. Thus we can generally expect condition (13) to hold. Rearranging (13) we have:

$$EU_1'(x+z)/U_1'(x) > EU_1(x+z)/U_1(x). \tag{14}$$

Using Taylor Series approximations,

$$\begin{aligned} EU_1'(x+z)/U_1'(x) &\approx [U_1'(x) + \frac{1}{2}\sigma^2 U_1'''(x)]/U_1'(x) = 1 + \frac{1}{2}\sigma^2 U_1'''(x)/U_1'(x) \\ EU_1(x+z)/U_1(x) &\approx /[U_1(x) + \frac{1}{2}\sigma^2 U_1''(x)]/U_1(x) = 1 + \frac{1}{2}\sigma^2 U_1''(x)/U_1(x). \end{aligned}$$

Hence, to a second-order approximation, (14) is satisfied if $U_1'''(x)/U_1'(x) > U_1''(x)/U_1(x)$, or $-U'''/U" > -U'/U$, the coefficient of absolute prudence is greater than minus boldness.

PROOF OF PROPOSITION 7:

In order for Player 1 to be better off as a result of uncertainty being added to his payoff, it must be the case that he is able to extract at least his compensating risk premium p from Player 2. Thus as a necessary and sufficient condition, we have:

$$EU_1'(x+\pi+z)/EU_1(x+\pi+z) \geq U_2'(1-\pi-x)/U_2(1-\pi-x). \tag{15}$$

By definition of π, we have $EU_1(x+\pi+z) = U_1(x)$. But $EU_1'(x+\pi+z) > U_1'(x)$ if $U_1(x)$ exhibits decreasing absolute risk aversion. However, taking pie from Player 2 tends to increase his boldness, so this effect means that Player 1 could be worse off despite the increase in his boldness. Employing Taylor Series approximations,

$$\begin{aligned} EU_1'(x+\pi+z)/U_1(x) &\approx [U_1'(x) + \pi U_1''(x) + \frac{1}{2}\sigma^2 U_1'''(x)]/U_1(x) \\ U_2'(1-x-\pi)/U_2(1-x-\pi) &\approx [U_2'(1-x) - \pi U_2''(1-x)]/[U_2(1-x) - \pi U_2'(1-x)]. \end{aligned}$$

Rearranging and substituting in the expression for π shows that Player 1 will be no worse off as a result of introducing a small risk to his payoff if and only if:

[51] Gollier and Kimball (1995) also give simpler sufficient conditions for a case analogous to this one - see section 3.2.1 in their paper.

$$\begin{aligned} U_1''(x)/U_1'(x) - U'''(x)/U''(x) &\geq U_2'(1-x)/U_2(1-x) - U_2''(1-x)/U_2'(1-x) \\ -RiskAversionP1 + PrudenceP1 &\geq BoldnessP2 + RiskAversionP2 \\ -\frac{d}{dx}log(-\frac{U''}{U'}) &\geq \frac{d}{dx}log(\frac{U_2'}{U_2}). \end{aligned}$$

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