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THE ROLE OF BARGAINING
POWER AND THE EVOLUTION
OF INFORMATIONAL ASYMMETRY**

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ABSTRACT

Venture Capital Financing: The Role of Bargaining Power and the Evolution of Informational Asymmetry*

We model a situation where the entrepreneur has an informational advantage during the early stages of an investment project while the venture capitalist has the informational advantage during the later stages. We examine how this evolution of informational asymmetry affects venture investment and the nature of financing contracts under two different scenarios with regard to the distribution of bargaining power between the venture capitalist and entrepreneur: when the venture capitalist has the bargaining advantage and when the entrepreneur has the bargaining advantage. Our results demonstrate that the distribution of bargaining power has a profound influence both on the terms of contracts and on investments in venture-backed projects. Changes in bargaining power can completely alter the payoff sensitivity of contracts offered to entrepreneurs, and, as witnessed in the recent past, when entrepreneurs hold the bargaining advantage, venture capitalists may acquiesce to excessive investments in early stages of projects and subsequently terminate a larger number of projects.

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1 Introduction

In its survey, “Venture Capital: Money to burn,” *The Economist* (May 27, 2000) stated that “It is clear that venture capital has been prone to periods of extreme boom and bust.” The next survey on the venture capital industry (April 3, 2004) “After the drought – Venture Capital” claimed that “the money available for investments in start-up companies slowed to a trickle after the bubble burst.” These articles, together with a plethora of anecdotal evidence, suggest that venture financing is cyclical with periods where entrepreneurs are readily able to have their projects financed followed by periods where venture financing is hard to come by.

Such fluctuations in market conditions, because they alter the balance in bargaining power between entrepreneurs and venture capitalists, are likely to influence the outcome of contract negotiations between them. Further, several recent empirical studies suggest that fluctuations in venture capital market conditions also profoundly impact the willingness of venture capitalists to finance projects, their willingness to continue financing projects after their initial investments, and the terms of their financing contracts. For example, Gompers and Lerner (2000) document the “money chasing deals” phenomenon that gives entrepreneurs increased share of cash flows through increased valuations when a lot of capital flows into venture funds.

Cross-sectional studies of venture capital financing provide further evidence on the possible linkages between the relative bargaining power of venture capitalists *vis a vis* entrepreneurs and their investment and financing decisions. It is likely that more experienced or reputable venture capitalists have greater bargaining power when negotiating with entrepreneurs. Further, Hsu (2004) shows that high-reputation venture capitalists are able to command better terms in exchange for financing projects, and Kaplan and Schoar (2005) provide evidence of a positive relationship between venture capitalist experience and the return on investments accruing to the venture capitalist.

Despite this evidence supporting a linkage between venture capitalists’ bargaining power with their financing, contracting and investment decisions, there is little theoretical analysis that can provide insights into these linkages and guide future research in this area¹. Therefore, in this paper, we show how variations in the relative bargaining position between an entrepreneur and a venture capitalist – either driven by financing cycles in the venture capital market or the venture capitalist’s experience or reputation – can explain the

¹Inderst and Mueller (2004) is the notable exception.

systematic variation in venture capitalist financing and investment decisions documented by researchers. We model the two extremes of the negotiations between an entrepreneur and venture capitalist when they first consider the financing of a venture. In the first situation, the venture capitalist has the bargaining advantage over the entrepreneur, and in the second the entrepreneur commands the bargaining advantage. Negotiations between the entrepreneur and the venture capitalist are complicated by the fact that they may be asymmetrically informed about the project. We model what we believe is the natural evolution of informational asymmetries over the life of a project. When the venture capitalist is first approached by the entrepreneur, the entrepreneur is better informed regarding the project. As the project approaches completion, as marked by an initial public offering or an industry sale, the venture capitalist develops an informational advantage over the entrepreneur.² The entrepreneur's initial informational advantage may arise because she is likely to be better informed regarding the technology employed in the venture and its likelihood of successful scalability from a technological perspective.³ As the project matures, management, marketing and financial know-how become increasingly important and it is likely that the venture capitalist can better assess the prospects of the project along these dimensions. This evolution of information asymmetry appears plausible because one of the cornerstones of venture capital financing is that venture capitalists work closely with the companies they are financing, for example, by serving as board members (see, e.g., Lerner, 1995).⁴

Our main results confirm that both the evolution of the informational asymmetry regarding projects and the bargaining position of the venture capitalist can profoundly influence the amount invested in projects and the nature of contracts that dictate the division of cash flows between the entrepreneur and the venture capitalist.

²This evolution of information asymmetries is consistent with Kaplan and Strömberg's (2004) assertion that the entrepreneur has an informational advantage regarding "internal factors" while venture capitalists may be better informed about "external factors."

³This assumption is common in the venture capital literature (see, for example, Trester, 1998; Dessein, 2005) and finds support in the empirical literature. For example, Kaplan and Strömberg (2003) find evidence consistent with the notion that control structures in ventures are engineered to limit problems arising from the informational advantage of entrepreneurs.

⁴There is also considerable support for this assumption. For example, Kaplan and Strömberg (2004) claim that "for some of these [external] risks, the VCs may even be better informed." This idea that venture capitalists are informed investors is consistent with Sahlman's (1990) evidence that venture capitalists specialize in a small number of industries and thus gain a deep understanding of those industries. Further, Megginson and Weiss (1990) show that venture backed IPOs are less underpriced than similar IPOs without venture capitalists backing. Also consistent with this idea that venture capitalists are informed investors who add value beyond just providing financing is that companies backed by venture capitalists are able to introduce products to the market faster than other companies (Hellman and Puri, 2000).

First, when the entrepreneur commands the bargaining advantage, the endogenous cost of screening out poor projects may be sufficiently high to prompt venture capitalists to avoid screening projects and finance even poor projects. Thus, few projects will be turned down by venture capitalists. Consistent with this result, *The Economist* (May 27, 2000) reported that the venture capital industry “has become less cautious about valuations and has financed too many competing companies with dubious business plans.” When the entrepreneur’s bargaining advantage is eroded either because of a reversal in market conditions or an increase in the venture capitalist’s experience, the endogenous cost of screening out poor projects is relatively low and thus only positive net present value (NPV) projects obtain financing. This result is consistent with the evidence in Hochberg, Ljunqvist, and Lu (2006), and Sorensen (2006) who document that venture capitalists with greater experience make better investments and are less likely to finance poor projects than venture capitalist with less experience.

Second, a consequence of the excessive financing of projects when entrepreneurs have the bargaining advantage is that there will be a higher incidence of project terminations when projects are reviewed at later stages of financing. Again, there is anecdotal evidence consistent with this result. *The Wall Street Journal* (October 1, 2002) reported that “With the amount invested since 1999 in start-ups that are no longer operational at \$15.3 billion, and with customer spending weak and the paths to liquidity still closed, few VCs are eager to throw good money after bad – *despite lower valuations and onerous follow-on terms favorable to VCs.*” Because of effective screening during the initial stages, project terminations will be less frequent when the venture capitalist has the bargaining advantage. These results are also consistent with Hochberg, Ljunqvist, and Lu’s (2006) evidence on the higher survival rates for firms backed by well-connected venture capitalists during later rounds of financing.

Third, and most obviously, when the venture capitalist has the bargaining advantage, he is able to appropriate much of the surplus from projects that are financed.⁵ The situation is reversed when there is excess supply of venture capital financing. Consistent with this result, Kaplan and Schoar (2005) show that aggregate returns for venture capital funds are lower after booms, which are characterized by a large increase in venture financing. The result is also consistent with Gompers and Lerner’s (2000) conclusion that entrepreneurs

⁵Consistent with this prediction, an article in *Business Week* (May 27, 2002) stated that “Richard LaPierre is a frustrated man It’s not that venture capitalists aren’t interested. It’s just that they’ll only write a check if LaPierre agrees to terms so onerous that he and his team would get scant compensation for all the work they’ve put into building a business from scratch...There would be little likelihood of a big payday unless the company achieved all but impossible growth targets.”

get higher cash flow shares and projects receive higher valuations when more capital flows into venture funds.⁶

Fourth, when the venture capitalist commands the bargaining advantage, as the project approaches maturity, venture capitalists will hold claims whose payoffs are more sensitive to the project's performance than entrepreneurs' claims. Call options on the project's cash flow or claims that are convertible into common stocks of the project are means of generating this sort of cash flow pattern. Once again, the situation is reversed when the locus of bargaining advantage moves to the entrepreneur. Venture capitalists will receive claims that are relatively insensitive to the project's cash flows while entrepreneurs receive claims that are highly sensitive to project performance. Together, these results suggest that there will be a greater tendency to finance venture projects with a mix of options and convertible claims under conditions of excess demand for venture financing or when the venture capitalist has a high reputation.

Finally, the level of investment in projects is also sensitive to the locus of the bargaining advantage. As was the case during the recent technology bubble, when the entrepreneur has the bargaining advantage, there is a tendency to overinvest in projects in the initial stages. In contrast, a switch to conditions that bestow the venture capitalist with the bargaining advantage results in overinvestment later in the life of the project.

The influence that the identity of the party with the bargaining advantage exerts on the characteristics of venture capital contracts and on investment in projects is rather subtle but intuitive. Because venture capitalists gain an informational advantage during the later stages of projects and because contract terms evolve in response to the change in the locus of the informational advantage, the terms of contracts that determine the actual sharing of cash flows at project maturity (the terminal contract) are strongly influenced by the venture capitalists' informational advantage. When venture capitalists command the bargaining advantage, they have the upper hand in negotiations and can extract the surplus from the project by restricting the payoffs to entrepreneurs. To maximize their share of the surplus, venture capitalists have to minimize the mispricing of the terminal contracts by the informationally disadvantaged entrepreneurs. The optimal contract minimizes the sensitivity of the entrepreneurs' payoffs to the venture capitalists' private information, i.e., the entrepreneur

⁶There is also anecdotal evidence supporting this prediction. For example, The Wall Street Journal (June 24, 2004) wrote that "...venture capitalists say that for now deals are more competitive, and closing at a faster clip," and "... we are seeing behavior that we'd associate with the bubble applied to a different cycle of the market. That won't lead to good returns."

receives a claim that is closest to a debt claim. Further, venture capitalists may resort to costly overinvestment during the later stages of a project to signal favorable information.

Reversing the negotiating positions completely alters the dynamic. Now entrepreneurs are in a position to extract the surplus from projects. However, the terminal contract is structured when venture capitalists have the informational advantage. The entrepreneurs' ability to extract surplus is contingent on their ability to elicit the private information from the venture capitalists. The most efficient way to do so and to limit the venture capitalists' potential gain from misrepresenting their information is to offer venture capitalists contracts that are relatively insensitive to their private information, i.e., offer the venture capitalists contracts that are as close as possible to debt contracts. When project profitability is relatively low, lowering the level of investment in projects during the later stages of financing also serves the same purpose because it reduces the sensitivity of contracts to the venture capitalists' private information.

Distortions in investments in the early stages of ventures are dictated by a desire to elicit information from entrepreneurs when they first approach venture capitalists and by a desire to elicit information from venture capitalists in the later stages of projects. We show that when information revelation by entrepreneurs is especially important, as is the case when entrepreneurs have the bargaining advantage, there is a tendency to overinvest in the initial stages of a project so as to increase the entrepreneurs' cost from project failure and thus limit their incentives to inflate a poor project's prospects⁷. When these endogenous costs of screening are relatively large, it is optimal to avoid screening projects. Because of the excessive financing of projects, overinvestment occurs⁸.

One model of venture capital financing that addresses the role of market conditions is Inderst and Mueller (2004). They demonstrate how the shift in bargaining power between the entrepreneur and the venture capitalist influences the ownership shares of the two parties and the valuation of the project. They also examine the long-run relation between the profitability of investments, the entry of venture capitalists into the market, and, thus the aggregate level of investment in new ventures. Unlike Inderst and Mueller, we do not focus on the dynamics of the venture capital market. Instead, we examine how conditions in the

⁷This result is similar to that of De Meza and Webb (1987) who show that borrowers who have good prospects may signal their type by overinvesting. In contrast, the seminal papers of Stiglitz and Weiss (1981) and Myers and Majluf (1984) demonstrate that asymmetric information leads to underinvestment.

⁸Manove, Padilla and Pagano (2001) obtain a similar result that negative NPV projects may get financing when credit markets are competitive (i.e. when borrowers have the bargaining advantage). When the banks have the bargaining advantage, they screen optimally and no negative NPV projects get financed. Manove et al. only consider debt contracts with collateral as financing instruments.

market can influence the scale of investment and the incentive to under or over invest in individual projects, and the nature of the financing contracts that are associated with these investments.

Several studies have examined the role of adverse selection problems in venture financing⁹. Some of this literature on adverse selection problems has focused on situations where the entrepreneur is better informed than the venture capitalist regarding the quality of the project being financed. For example, Dessein (2005) and Trester (1998) attempt to identify the optimal financing contract for a project that requires a fixed level of investment when the entrepreneur has both a bargaining and an informational advantage over the venture capitalist. In contrast, other studies have examined situations where this assumption regarding the identity of the party with the informational advantage is weakened or reversed. For example, Garmaise (2002) examines the design of the optimal financing contract when an entrepreneur needs financing for a project with a fixed level of investment. In his analysis, however, the entrepreneur has the bargaining advantage, but the venture capitalist has the informational advantage. Our analysis adds to these three studies by also examining optimal financing contracts when the venture capitalist has the bargaining advantage over entrepreneurs, and by allowing for two-sided adverse selection, thus enabling us to model the evolution of venture capital contracts¹⁰.

In a multi-stage setting like ours, Admati and Pfleiderer (1994) examine both the optimal investment and financing contract for a startup. Unlike our model, however, in their setting, a venture capitalist cannot gain an informational advantage over the entrepreneur, but both can be better informed than other investors. Ueda (2004) makes a similar assumption regarding the venture capitalist's information in her study of the choice between financing through an uninformed bank and a venture capitalist who can become informed about the project but may steal the entrepreneur's idea.

Other papers have adopted the moral hazard paradigm to examine the venture capital financing contracts. Casamatta (2003) studies the situation where both the entrepreneur and the venture capitalist have to exert effort simultaneously in order for the venture to be successful. In her model, the venture capitalist is given common stock if the investment is low and convertible securities if the investment is high. Schmidt (2003) examines a

⁹In a security design setting, Nachman and Noe (1994) show that when a privately informed firm is seeking financing from a competitive market, debt financing helps to mitigate the adverse selection problem.

¹⁰Inderst and Mueller (2006) model a situation where the financier is assumed to be the informed party and where the bargaining advantage varies. Inderst and Mueller show that when the borrower has the bargaining advantage, debt is optimal. Conversely, when the financier has the bargaining advantage, levered equity is optimal. In their model, the borrower does not have any private information and the investment size is fixed.

model where the entrepreneur and the venture capitalist exert effort sequentially. In his model, convertible securities solve the incentive problems so that first-best level of effort is achieved. Cornelli and Yosha (2003) show that convertible securities can be used to mitigate the entrepreneur's incentive to "window dress" or exaggerate the prospects of the new venture, and as a result, venture capitalists are able to make better investment decisions.¹¹

It is well documented that funds raised by venture capitalists and investments made by them into portfolio companies vary considerably over time (Gompers and Lerner, 2004). An excess supply situation in the venture capital market is most likely to occur when project profitability is expected to be high and venture capitalists are making initial large investments in projects. Conversely, an excess demand situation should be correlated with lower initial investments in projects. Our analysis suggests that the swings in investment may be larger than are justified by changes in expectations of project profitability alone. The reason is that in the early stages of projects, there will be a tendency to overinvest when the venture capital markets experience conditions consistent with an excess supply.

Our analysis also provides several novel, empirically testable hypotheses. For example, we show that the likelihood that poor quality projects receive financing will be higher when there are large inflows of funds into the venture capital market or when entrepreneurs approach less-established venture capitalists. Consequently, there will also be a higher incidence of projects' termination at later stages of financing in markets characterized by conditions of excess supply and for projects financed by less-established venture capitalists. Our analysis also demonstrates that established venture capitalists and tight conditions in the market for venture financing will result in venture capitalists receiving contracts that pay disproportionately large sums contingent on project success. Conversely, in markets characterized by an excess supply of venture capital or when financed by less established venture capitalists, entrepreneurs will capture a disproportionately large fraction of project payoffs when projects succeed. Thus, we expect that the propensity of finance projects with claims such as convertible debt and convertible preferred stock will increase as demand for venture capital financing increases. Further, our results also suggest that an increased propensity to finance projects with convertible claims will be correlated with smaller initial investments in projects.

¹¹Repullo and Suarez (2004) solve for the optimal securities, when there are multiple investment stages and the entrepreneur and the venture capitalist both have to exert effort. They show that when the interim milestone is verifiable, the optimal security is an equity-like contract. If the interim milestone is not verifiable, the optimal contract gives the venture capitalist zero payoff when the profitability of the new venture is low, and an increasing share of the payoffs, when the profitability is high.

The remainder of this paper is organized as follows: In Section 2, we describe our model and present details of the informational structure, agent payoffs, and the major assumptions. Section 3 contains an analysis of the optimal cash flow sharing rules and investment under conditions of excess demand. Section 4 is devoted to an analysis under conditions of excess supply. In Section 5, we extend our results to a situation where entrepreneurs' reservation wages are correlated with their private information. Section 6, contains a summary of our analysis and some concluding remarks. Proofs of all results are presented in the Appendix.

2 The model

Consider a three date model. All agents are risk neutral, and the risk-free rate is normalized to 0. At date 0, an entrepreneur approaches a venture capitalist for financing for a project. If the venture capitalist agrees to provide funding, an investment I_0 is made at date 0. At the next date, date 1, the two parties make another investment, I_1 , in the project. The entrepreneur has no capital at dates 0 or 1. Thus, the entire amount of the investments I_0 and I_1 are provided by the venture capitalist. Together these two investments generate a random cash flow X at date 2, the terminal date.¹² This cash flow has a two point support $X \in \{\underline{X}, \bar{X}\}$, where $0 \leq \underline{X} < \bar{X}$. If the venture capitalist chooses not to finance the project or if it is abandoned at date 1, i.e., either $I_0 = 0$ or $I_1 = 0$ respectively, the project generates a cash flow of 0, the entrepreneur obtains employment elsewhere and earns her reservation wage. For simplicity, we assume that the entrepreneur's reservation wage for the first period, from date 0 to date 1, is 0, and her reservation wage during the second period (date 1 to date 2) is w .

Before approaching the venture capitalist at date 0, the entrepreneur observes a private signal t that informs her of the quality of her project. The realization of this signal can either be G or B , where signal G indicates that the project is "good" and signal B indicates that it is "bad." The ex ante probability of the entrepreneur observing a signal G is π . At date 1, before making the follow-on investment decision, the venture capitalist observes a private signal $j \in \{L, H\}$. The signal H is realized with probability ϕ and indicates that the

¹²We examine the implications of allowing for a cash flow at date 1 in Section 5 of the paper. Note, however, that the model described here is consistent with the stylized facts about new venture financing. New ventures tend not to generate much in the way of operating profits and the primary incentive that venture capitalists have to finance such ventures is to profit from the sale of the ventures rather than from capturing operating cash flows.

project has a “high” likelihood of success while the signal L indicates that it has a “low” likelihood of success.

The cash flow from the project is jointly determined by the investment at date 0, the investment at date 1, the entrepreneur’s private signal and the venture capitalist’s private signal. The cash flow \bar{X} is realized with probability $P_t(I_0)P_j(I_1)$ and the cash flow \underline{X} is realized with probability $1 - P_t(I_0)P_j(I_1)$ where $t \in \{G, B\}$ and $j \in \{L, H\}$. We assume that $P_t \in [0, 1]$ for all I_0 , $P_j \in [0, 1]$ for all I_1 , and that both sets of functions are increasing and concave in the amount invested: $P'_t > 0$, $P''_t < 0$, $P'_j > 0$, and $P''_j < 0$.

The project is more profitable if the entrepreneur observes signal G than if she observes signal B , that is, $P_G > P_B$. Similarly, $P_H > P_L$, which implies that the project is more profitable if the venture capitalist observes signal H than if he observes signal L . We also assume that a signal G increases the likelihood of realizing the cash flow \bar{X} , i.e.,

$$\frac{P'_G(I_0)}{P_G(I_0)} > \frac{P'_B(I_0)}{P_B(I_0)} \text{ for all } I_0. \quad (1)$$

Similarly, to capture that the notion that a signal H increases the likelihood of the project being successful, we assume that

$$\frac{P'_H(I_1)}{P_H(I_1)} > \frac{P'_L(I_1)}{P_L(I_1)} \text{ for all } I_1. \quad (2)$$

These assumptions are similar to the single crossing property employed in much of the adverse selection literature (see Riley, 2001).

To eliminate the uninteresting case where risk-free contracts are feasible, we assume that $w > \underline{X}$ and the initial investment $I_0 \geq I_{0\min} > \underline{X}$. To ensure that there exist internal optimal investment levels for the date 1 investment, we assume that $P'_j(0) = \infty$ and $P'_j(\infty) = 0$ for $j \in \{L, H\}$. Similarly, we assume that $P'_G(I_{0\min}) = \infty$ and $P'_G(\infty) = 0$. To ensure that the project is a positive NPV undertaking from both date 0 and date 1 perspectives if the entrepreneur observes signal G , we assume that there exist I_0 , $I_{1,H}$ and $I_{1,L}$ such that

$$\underline{X} + \phi [P_G(I_0)P_H(I_{1,H})\Delta X - I_{1,H}] + (1 - \phi) [P_G(I_0)P_L(I_{1,L})\Delta X - I_{1,L}] - I_0 - w > 0 \quad (3)$$

$$\underline{X} + P_G(I_0)P_L(I_{1,L})\Delta X - I_{1,L} - w > 0. \quad (4)$$

Finally, we assume that the project has a negative NPV ex ante, that is, for any I_0 , $I_{1,H}$ and $I_{1,L}$,

$$\begin{aligned} & \pi \{ \underline{X} + \phi [P_G(I_0)P_H(I_{1,H})\Delta X - I_{1,H}] + (1 - \phi) [P_G(I_0)P_L(I_{1,L})\Delta X - I_{1,L}] - I_0 \} \\ & + (1 - \pi) \{ \underline{X} + \phi [P_B(I_0)P_H(I_{1,H})\Delta X - I_{1,H}] + (1 - \phi) [P_B(I_0)P_L(I_{1,L})\Delta X - I_{1,L}] - I_0 \} \\ & - w < 0. \end{aligned} \quad (5)$$

This assumption ensures that NPV is negative if the entrepreneur observes signal B. Further, it rules out any pooling equilibrium where the project is financed regardless of the entrepreneur's signal. We consider the effects of loosening this restriction in Section 5 below.

At time 0, if the project is financed by the venture capitalist, both the entrepreneur and the venture capitalist agree on a set of contracts that share the entire date 2 cash flow between the two parties. Let $s(X)$ denote the amount the venture capitalist receives when the period 2 cash flow is X . Because of limited liability and because the entrepreneur has no wealth, the venture capitalist's payment $s(X)$ has to satisfy $0 \leq s(X) \leq X$. The entrepreneur receives $X - s(X)$, because the final period cash flow is fully shared by the two parties. Given that the cash flow has a two-point support, $X \in \{\underline{X}, \bar{X}\}$, we employ a simple characterization of the contracts as linear functions of the project's cash flow, where each contract is described by the slope and intercept terms of its relationship with the underlying cash flow. Define α and γ as

$$\alpha \equiv \frac{s(\underline{X})}{\underline{X}} \quad (6)$$

$$\gamma \equiv \frac{s(\bar{X}) - s(\underline{X})}{\bar{X} - \underline{X}}. \quad (7)$$

The term α represents the proportion of the low cash flow that the venture capitalist receives and $1 - \alpha$ is the proportion of the low cash accruing to the entrepreneur. The term γ is the slope of the financial contract with respect to the project's cash flow and thus captures the sensitivity of the venture capitalist's payoff to the project's cash flow. Similarly, $1 - \gamma$ captures the sensitivity of the entrepreneur's payoff to the project's cash flow. Note that because $w > \underline{X}$ and $I_0 \geq I_{0\min} > \underline{X}$ $0 < \gamma_j < 1$ for $j \in \{L, H\}$.

The initial contract calls for the venture capitalist to invest I_0 at date 0 if the entrepreneur has observed G . In exchange for the investment the venture capitalist receives a contract (α_0, γ_0) that provides him with a payoff of $\alpha_0 \underline{X}$ if the cash flow \underline{X} is realized at date 2, and a cash flow of $\alpha_0 \underline{X} + \gamma_0 (\bar{X} - \underline{X})$ if the cash flow \bar{X} is realized. One interpretation of this contract is that the venture capitalist receives a debt payment with the face value of $\alpha_0 \underline{X}$ at date 2 regardless of the project's outcome and receives equity payment of $\gamma_0 (\bar{X} - \underline{X})$ if \bar{X} is realized. Because all cash flows are shared by the venture capitalist and the entrepreneur, it follows that the entrepreneur's share of the date 2 cash flow is $(1 - \alpha_0) \underline{X}$ regardless of the outcome in addition to a variable compensation of $(1 - \gamma_0) \Delta X$, where $\Delta X = \bar{X} - \underline{X}$, if cash flow \bar{X} is realized. It follows that both the venture capitalist's and the entrepreneur's contracts can be expressed by the investment and cash flow sharing rule triple $(I_0, \alpha_0, \gamma_0)$.

In addition to settling on a date 0 contract (α_0, γ_0) , the venture capitalist and entrepreneur agree on a mechanism for renegotiating this contract at date 1. The renegotiation begins after the venture capitalist observes and reports his private signal regarding project quality. First, the agent who has the bargaining advantage offers the other agent a contract from a menu of contracts whose composition is agreed upon at date 0, when the initial contract is agreed upon. If the contract that is offered does not belong to the initially agreed-upon menu, the negotiations are assumed to break down, resulting in the eventual liquidation of the venture. To keep things simple, we assume that the assets yield a value of zero if the venture is liquidated. The agent receiving the offer has to choose from three possible responses: accept the new contract being offered, retain the original contract, or opt out of the venture. If the agent chooses to opt out of the venture, the venture is liquidated, once again resulting in a zero payoff to both the entrepreneur and the venture capitalist. Note that under this negotiation structure, both the venture capitalist and the entrepreneur have the ability to walk away from the venture by either proposing a contract that is not part of the agreed-upon menu or by rejecting the contract that is offered during the negotiations.

Let the set of menus from which future contracts can be chosen consist of two elements, and let these two contracts and their associated investment levels be represented by $(I_{1,H}, \alpha_H, \gamma_H)$ and $(I_{1,L}, \alpha_L, \gamma_L)$, where the subscript H denotes the contract-investment level pair that the venture capitalist is offered if he claims to have observed the signal H , and the contract-investment level pair with the subscript L denotes the contract the venture capitalist receives if he claims to have observed the signal L . In the appendix, we show that as long as the party with the bargaining advantage sets the payoff on the initial date 0 contract, (α_0, γ_0) , sufficiently low, both parties will agree to change to a date 1 contract from this menu. Thus, to simplify the exposition and concentrate on the more interesting contracting issues, in the following analysis, we limit our analysis to the design of the date 1 cash flow sharing contracts, (α_H, γ_H) and (α_L, γ_L) .

3 The venture capitalist has the bargaining advantage

In this section, we examine the nature of the optimal contract when the venture capitalist has the bargaining advantage during the life of the project. We begin by characterizing the optimal cash flow sharing rule. Then we characterize the optimal investments at date 0 and date 1. Finally we employ a numerical example to provide insights into the nature of the investment distortions embodied in date 0 and date 1 investments.

First, define the payoff function for the entrepreneur as $U_i^j(\alpha, \gamma, I_0, I_1)$ when the venture capitalist is of type i and the entrepreneur is of type j , that is $U_i^j(\alpha, \gamma, I_0, I_1) \equiv (1 - \alpha)\underline{X} + (1 - \gamma)P_j(I_0)P_i(I_1)\Delta X$ where $i \in \{H, L\}$ and $j \in \{G, B\}$. Similarly, let $V_i^j(\alpha, \gamma, I_0, I_1)$ represent the payoff to the venture capitalist when the venture capitalist is of type i and the entrepreneur is of type j , that is $V_i^j(\alpha, \gamma, I_0, I_1) \equiv \alpha\underline{X} + \gamma P_j(I_0)P_i(I_1)\Delta X$, where $i \in \{H, L\}$ and $j \in \{G, B\}$.

Given that the venture capitalist has all the bargaining power, he will design contracts that minimize the entrepreneur's share of the cash flows. However, he has to pay the entrepreneur at least her reservation wage to induce her to participate in the project. Further, since project NPV is negative if the entrepreneur is of type B and the entrepreneur is paid at least her reservation wage, the venture capitalist has no incentive to undertake the project with a type B entrepreneur.¹³ Thus, the venture capitalist will design contracts that are acceptable to the entrepreneur only if she is of type G by ensuring that the expected payoff from the contracts is lower than the reservation wage w if the entrepreneur is of type B. Assuming that the venture capitalist credibly conveys his date 1 private information through his contract choice, the contract has to satisfy the following conditions:

$$\phi \left[U_H^G(\alpha_H, \gamma_H, I_0, I_{1,H}) \right] + (1 - \phi) \left[U_L^G(\alpha_L, \gamma_L, I_0, I_{1,L}) \right] \geq w \quad (8)$$

$$U_H^G(\alpha_H, \gamma_H, I_0, I_{1,H}) \geq w \quad (9)$$

$$U_L^G(\alpha_L, \gamma_L, I_0, I_{1,L}) \geq w \quad (10)$$

$$\phi \left[U_H^B(\alpha_H, \gamma_H, I_0, I_{1,H}) \right] + (1 - \phi) \left[U_L^B(\alpha_L, \gamma_L, I_0, I_{1,L}) \right] \leq w \quad (11)$$

Condition (8) ensures that a type G entrepreneur is willing to have her project financed by the venture capitalist because, from a date 0 perspective, the entrepreneur's expected payoff is higher than her expected reservation wage. Similarly (9) and (10) ensure that a type G will be willing to continue with the venture at date 1 after the venture capitalist reports observing signal realization H and L , respectively. Finally, (11) ensures that the expected payoff from entering the venture is no higher than the reservation wage if the entrepreneur is of type B.

Note that the above constraints on the optimal contract design require that the entrepreneur believes that the venture capitalist is of type H when the date 1 contract that is agreed upon is $(I_{1,H}, \alpha_H, \gamma_H)$, and believes that the venture capitalist is of type L when

¹³Note that condition (5) ensures that there exists no pooling equilibrium where the venture capitalist finances the entrepreneur regardless of her signal. In Section 5 we formally establish that no pooling equilibria exist even when (5) is weakened to allow for the possibility that ex ante project NPV is positive.

the date 1 contract that is agreed upon is $(I_{1,L}, \alpha_L, \gamma_L)$. This belief is possible only if the date 1 contracts enable the venture capitalist to credibly signal his type. Therefore, the optimal contracts must also satisfy the conditions

$$V_H^G(\alpha_H, \gamma_H, I_0, I_{1,H}) - I_H \geq V_H^G(\alpha_L, \gamma_L, I_0, I_{1,L}) - I_L \quad (12)$$

$$V_L^G(\alpha_L, \gamma_L, I_0, I_{1,L}) - I_L \geq V_L^G(\alpha_H, \gamma_H, I_0, I_{1,H}) - I_H \quad (13)$$

Finally, to ensure that the venture capitalist is willing to participate in the project, it must be the case that his expected payoff from both a date 0 perspective and from a date 1 perspective is greater than 0, i.e.,

$$V_H^G(\alpha_H, \gamma_H, I_0, I_{1,H}) - I_H \geq 0 \quad (14)$$

$$V_L^G(\alpha_L, \gamma_L, I_0, I_{1,L}) - I_L \geq 0 \quad (15)$$

$$\phi \left[V_H^G(\alpha_H, \gamma_H, I_0, I_{1,H}) - I_{1,H} \right] + (1 - \phi) \left[V_L^G(\alpha_L, \gamma_L, I_0, I_{1,L}) - I_{1,L} \right] - I_0 \geq 0 \quad (16)$$

Because the venture capitalist has the bargaining advantage, he selects a contract that maximizes his expected payoff subject to the conditions described above. That is, the venture capitalist solves the following problem:

$$\max_{\alpha_H, \alpha_L, \gamma_H, \gamma_L, I_0, I_{1,H}, I_{1,L}} \phi \left[V_H^G(\alpha_H, \gamma_H, I_0, I_{1,H}) - I_{1,H} \right] + (1 - \phi) \left[V_L^G(\alpha_L, \gamma_L, I_0, I_{1,L}) - I_{1,L} \right] - I_0 \quad (17)$$

subject to constraints (8) through (16) and

$$\alpha_H \in [0, 1], \alpha_L \in [0, 1], \gamma_H \in [0, 1], \gamma_L \in [0, 1]$$

We now characterize the cash flow sharing contracts (α_H, γ_H) and (α_L, γ_L) that solve this problem. First note that conditions (8) through (10) are the only conditions that place a lower limit on the expected value of the contract that the venture capitalist can offer to the entrepreneur. Further, if the venture capitalist can design contracts that satisfy the entrepreneur's date 1 participation conditions (9) and (10), he automatically satisfies condition (8), the entrepreneur's date 0 participation constraint. Now, note that, because cash flows with a type B entrepreneur are first order stochastically dominated by cash flows with a type G entrepreneur, if the contracts selected by the venture capitalist satisfy (9) and (10) as equalities, they should satisfy condition (11). Thus, by selecting contracts that satisfy (9) and (10) as equalities and by forcing the type G entrepreneur to her reservation wage w the venture capitalist is able to effectively maximize his share of the cash flow and deter the entrepreneur from seeking financing if she is of type B.

Lemma 1 *If there exists a solution to the venture capitalist's problem, there exists a solution $(I_0^*, \alpha_H^*, \gamma_H^*, I_{1,H}^*, \alpha_L^*, \gamma_L^*, I_{1,L}^*)$, where*

$$U_H^G(\alpha_H^*, \gamma_H^*, I_0^*, I_{1,H}^*) = w \quad (18)$$

$$U_L^G(\alpha_L^*, \gamma_L^*, I_0^*, I_{1,L}^*) = w. \quad (19)$$

The above lemma suggests that the venture capitalist is able to screen entrepreneurs by simply ensuring that the expected cash flow from any date 1 contract that is offered to the entrepreneur provides an expected payoff of w to the entrepreneur if she is of type G. This result affords the venture capitalist considerable flexibility in the design of the contracts that also have to credibly convey his date 1 private information.

At date 1, the venture capitalist's problem is to convince the entrepreneur that the contract is worth w . Because the cash flows under a type L venture capitalist are first order stochastically dominated by cash flows under a type H venture capitalist, any contract offered by a type L venture capitalist would be worth less to the entrepreneur than the same contract offered by a type H venture capitalist. Thus, the entrepreneur recognizes that a type L venture capitalist has an incentive to misrepresent his type to the entrepreneur and try to induce her to accept a contract that is actually worth less than w . It follows that if the venture capitalist is of type H, to credibly convince the entrepreneur that he has observed the signal H , he has to ensure that the contract terms prevent mimicry by a type L venture capitalist. This problem is similar to that dealt with in much of the literature on contract design in the presence of adverse selection and consequently has a similar solution—the type H venture capitalist is able to credibly convey his private information to the entrepreneur by assuming a contract that pays him disproportionately more when higher cash flows are realized. In our context, this solution means that the contract only pays the venture capitalist if the high cash flow is realized at date 2. This minimizes mimicry incentives for a type L venture capitalist because the contractual payoffs are concentrated on cash flow that a type L venture capitalist is least likely to generate but a type H venture capitalist is most likely to realize. Because there exists no incentive for a type H venture capitalist to mimic a type L venture capitalist, in equilibrium a type L venture capitalist can offer any contract to the entrepreneur so long as it satisfies the condition in the Lemma 1 above. This result is presented in the following proposition:

Proposition 2 *If there exists a solution to the problem, there exists a solution $(I_0^*, \alpha_H^*, \gamma_H^*, I_{1,H}^*, \alpha_L^*, \gamma_L^*, I_{1,L}^*)$, where $\alpha_H^* = \alpha_L^* = 0$.*

This proposition shows that the venture capitalist only receives a payoff when cash flow \bar{X} is realized. It follows that the venture capitalist's payoff is highly sensitive to project performance when the venture capitalist has the bargaining advantage. The entrepreneur, on the other hand, captures all of \underline{X} and obtains a slightly larger payoff when \bar{X} is realized. This sharing rule ensures that the entrepreneur's payoff is relatively insensitive to project performance. Given the above proposition and Lemma 1, it can also be shown that $\gamma_H > \gamma_L$, that is, the sensitivity of the venture capitalist's payoff to the project's performance is even greater when the project is expected to be more profitable.

In some instances, a type H venture capitalist may be unable to credibly convey his information to the entrepreneur by merely proposing a contract that only pays off if cash flow \bar{X} is realized, i.e., the mimicry incentive for a type L venture capitalist may be too strong to be deterred by the cash flow sharing rule alone. In these instances, as is the case in many signaling equilibria, the type H venture capitalist will have to resort to a costly signal to credibly convey his private information to the entrepreneur. In our setting, this signaling is achieved by distorting the date 1 investment level. Assumption (2) ensures that increased investment increases the probability that a type H venture capitalist will generate cash flow \bar{X} at a faster rate than increased investment increases the probability that a type L venture capitalist will generate cash flow \bar{X} . Thus, by resorting to overinvestment, a type H venture capitalist is able to increase the cost of mimicry by a type L venture capitalist. Consequently, as demonstrated in the following proposition, the optimal contract may call for excessive investment after the venture capitalist observes the signal H at date 1. Because a type H venture capitalist has no incentive to mimic a type L venture capitalist, the date 1 investment following the observation of the signal L by the venture capitalist does not distort investment.

Now consider the investment at date 0. As demonstrated above, when the venture capitalist has the bargaining advantage, the payoff to the entrepreneur is fixed at w . Thus, the venture capitalist captures the entire surplus (NPV) from the project. Because an investment distortion results in a reduction in total output and a reduction in the venture capitalist's payoff, the venture capitalist has an incentive to minimize investment distortions. As demonstrated above, the venture capitalist is always able to design a contract that screens out an entrepreneur who has observed signal B because the venture capitalist faces no constraint in lowering the entrepreneur's payoff to her reservation wage of w . Thus, the only reason for investment distortion is to enable the venture capitalist to credibly convey his private information after observing the signal H at date 1, i.e., to ensure that condition (13) is satisfied. Distorting the date 0 investment is a relatively costly and, thus, inefficient

means of facilitating the revelation of information at date 1. To see this result, note that any attempt to use changes in the date 0 investment to alter the right-hand side of (13) will also alter the left-hand side of the expression, limiting the benefit of investment distortion at date 0. This effect contrasts with the effect of changing the date 1 investment that can be employed to directly affect only one side of the incentive constraint. A change in the date 1 investment level following the revelation of signal H to the venture capitalist only affects the right-hand side of the expression and thus provides increased flexibility in eliminating mimicry incentives.

Not only is the distortion of the date 0 investment an inefficient way to facilitate the revelation of the venture capitalist's date 1 private information, but this investment distortion may in fact be counterproductive. For example, overinvestment at date 0 actually makes it more difficult to satisfy the crucial constraint (13). This result follows because increased date 0 investment increases the expected cash flow from the project and, because the entrepreneur is being held at her reservation wage, reduces the entrepreneur's share of the cash flow from the project. The latter effect is more marked if the entrepreneur believes that the venture capitalist has observed the signal H . Consequently, the venture capitalist has a greater incentive to misrepresent his private information if he observes the signal L . The following proposition formalizes these arguments.

Proposition 3 *Let the constrained Pareto optimal levels of investment $I_0^{CPO}(I_H, I_L)$, $I_H^{CPO}(I_0)$, and $I_L^{CPO}(I_0)$ be defined as the solutions to the following three equations, respectively:*

$$\phi P'_G(I_0^{CPO}) P_H(I_H) \Delta X + (1 - \phi) P'_G(I_0^{CPO}) P_L(I_L) \Delta X - 1 = 0 \quad (20)$$

$$P_G(I_0) P'_H(I_H^{CPO}) \Delta X - 1 = 0 \quad (21)$$

$$P_G(I_0) P'_L(I_L^{CPO}) \Delta X - 1 = 0 \quad (22)$$

In any solution to the venture capitalist's problem, the optimal investment levels I_0^ , I_H^* , and I_L^* satisfy*

$$a. I_0^* \leq I_0^{CPO}(I_H^*, I_L^*)$$

$$b. I_H^* \geq I_H^{CPO}(I_0^*)$$

$$c. I_L^* = I_L^{CPO}(I_0^*)$$

The above result implies that, in equilibrium, the marginal return from the date 0 investment exceeds the cost of the investment while holding the date 1 investment constant. In contrast, the marginal return from the date 1 investment after the receipt of the signal H is lower than the cost of investment and the marginal return from the date 1 investment following the receipt of the signal L equals the cost of investing. However, this proposition does not establish that the information problems facing the venture capitalist and the entrepreneur result in underinvestment relative to the *unconstrained* Pareto optimal level of investment at date 0 and overinvestment relative to the *unconstrained* Pareto optimal level at date 1 following the receipt of the signal H .

To allow for a comparison of investment decisions relative to unconstrained Pareto optimal levels of investment and to provide some insight into the comparative statics of the date 0 and date 1 investment decisions with respect to the average quality of projects (ϕ) and the uncertainty regarding project outcomes (ΔX), we have to resort to numerical techniques. We assume that $\underline{X} = 0.1$ and $w = 11$. Further, we assume that $P_G(I_0) = 1 - e^{-5I_0}$, $P_B(I_0) = \lambda_B(1 - e^{-10I_0})$, $P_H(I_1) = \frac{1}{10}I_1^{0.35}$, and $P_L(I_1) = \frac{1}{10}I_1^{0.35\lambda_L}$.¹⁴ The parameter λ_B (λ_L) is between 0 and 1, and captures the information asymmetry between type G and type B entrepreneurs (type H and type L investors).

Figure 1 illustrates how investment is affected by changes in the average quality of projects and the uncertainty regarding project outcomes. The figure contains four panels. Each panel illustrates optimal investment policies for various values of λ_B and λ_L and given values of ϕ and ΔX . As is clear from the figure, when the venture capitalist has the bargaining advantage, investment distortions only occur at date 1 following the observation of signal H by the venture capitalist. Otherwise only Pareto optimal investments are made in the project. Note that when ΔX is larger, the region where there is no investment distortion increases. The larger ΔX is, the larger the difference between the project's total expected cash flow across the two signals the venture capitalist can observe. Because this larger difference increases the sensitivity of the expected value of the contracts to the venture capitalist's signal, it is easier to use contract design to separate the two types of investors without resorting to investment distortion. Surprisingly, the quality of the project – i.e. the probability ϕ that the project is type H – does not affect the investment decision. This result follows primarily because the date 1 investment decision is made after the project quality is revealed to the venture capitalist and, thus, it is not a factor in his date 1 decision. Further, the venture capitalist's date 0 decision is not affected either because, for the parameter val-

¹⁴These functions are well-behaved probability functions and satisfy conditions (1) and (2) in the parameter value space that is graphed.

ues employed in the example, there appears to be no incentive to distort investment at date 0.

4 The Entrepreneur has the bargaining advantage

In the previous section, we assumed that the venture capitalist had a bargaining advantage that enabled him to capture all the surplus generated by the project. Now we examine the effects of reversing this assumption. Once again, we first examine the cash flow sharing rules that are part of the optimal contracts. Then we examine the optimal investment policies. As the following analysis demonstrates, the shift in bargaining power has a profound impact on both the optimal cash flow sharing rules and the nature of investment distortions.

Despite the shift in bargaining power to the entrepreneur the factors constraining the optimal contracts continue to be similar to those described in the previous section. The contracts have to provide the entrepreneur with payoffs that (i) make her willing to participate in the project and (ii) participate in the project only if she is of type G. That is, contracts have to continue to satisfy conditions (8) through (11). Similarly, the contracts have to satisfy condition (14) through (16) to ensure participation by the venture capitalist. Finally, the contracts have to provide the venture capitalist with the incentives to truthfully reveal his private information at date 1, i.e., the contracts have to satisfy conditions (12) and (13). It follows that the optimal contracts are the solution to the following problem:

$$\max_{\alpha_H, \alpha_L, \gamma_H, \gamma_L, I_0, I_{1,H}, I_{1,L}} \phi \left[U_H^G(\alpha_H, \gamma_H, I_0, I_{1,H}) \right] + (1 - \phi) \left[U_L^G(\alpha_L, \gamma_L, I_0, I_{1,L}) \right] \quad (23)$$

subject to the constraints (8) through (16) and must satisfy

$$\alpha_H \in [0, 1], \alpha_L \in [0, 1], \gamma_H \in [0, 1], \gamma_L \in [0, 1]$$

When the entrepreneur is endowed with the bargaining advantage she will attempt to restrict the cash flows captured by the venture capitalist to the extent possible. However, her ability to do so is limited by the venture capitalist's informational advantage at date 1. The contract designed by the entrepreneur has to provide the venture capitalist with the incentive to reveal his private information. As the following lemma demonstrates, an equilibrium contract may allow the venture capitalist to capture some of the surplus from the project.

Lemma 4 *If there exists a solution to the problem, there exists a solution, $(I_0^*, \alpha_H^*, \gamma_H^*, I_{1,H}^*, \alpha_L^*, \gamma_L^*, I_{1,L}^*)$, where*

$$V_L^G(\alpha_L^*, \gamma_L^*, I_0^*, I_{1,L}^*) - I_{1,L}^* \geq 0 \quad (24)$$

$$V_H^G(\alpha_H^*, \gamma_H^*, I_0^*, I_{1,H}^*) - I_{1,H}^* > 0 \quad (25)$$

and when I_0^* is sufficiently small

$$\phi \left[V_H^G(\alpha_H^*, \gamma_H^*, I_0^*, I_{1,H}^*) - I_{1,H}^* \right] + (1 - \phi) \left[V_L^G(\alpha_L^*, \gamma_L^*, I_0^*, I_{1,L}^*) - I_{1,L}^* \right] - I_0^* > 0 \quad (26)$$

The shift in the bargaining advantage to the entrepreneur completely alters the incentive structure for the venture capitalist at date 1 and, thus, alters the nature of the optimal cash flow sharing rules. As described in the previous section, when the venture capitalist has the bargaining advantage, the role of the optimal contract design is to limit the cash flow captured by the entrepreneur by credibly indicating when a type H venture capitalist is offering a contract. In the setting being examined here, the optimal contract design has to limit the venture capitalist's share of the cash flow. The entrepreneur has to provide the venture capitalist with a contract whose value compensates the venture capitalist for the investment he makes in the project. To capture a larger share of the cash flow, the venture capitalist has an incentive to systematically under-report the profitability of the project. Thus, the date 1 contract has to be designed to minimize misreporting incentives of the venture capitalist following receipt of the signal H . This design is achieved by setting $\alpha_L = 1$ so as to ensure that, to the extent possible, the payoffs to a venture capitalist who reports signal L are concentrated on cash flow \underline{X} , the cash flow that is more likely to be realized by a venture capitalist who actually observes signal L . Minimizing the sensitivity of the value of the venture capitalist's contract to signal realization restricts the gains to a type H venture capitalist and thus minimizes mimicry incentives. Because the venture capitalist has no incentive to mimic following the receipt of the signal L , the informational problem at date 1 does not provide any incentive to restrict the design of the contract offered to the venture capitalist after he reports receipt of the signal H . However, minimizing the sensitivity of the venture capitalist's payoff to project performance maximizes the sensitivity of the entrepreneur's payoff to project performance. This result helps deter mimicry by a type B entrepreneur at date 0, and, thus, in equilibrium α_H is set equal to 1.

Proposition 5 *If there exists a solution to the problem, there exists a solution, $(I_0^*, \alpha_H^*, \gamma_H^*, I_{1,H}^*, \alpha_L^*, \gamma_L^*, I_{1,L}^*)$, where $\alpha_L^* = \alpha_H^* = 1$.*

Next we examine the optimal level of investment in the project. First consider the date 1 investment decision. In instances when the cash flow sharing rules alone cannot eliminate mimicry incentives by a type H venture capitalist, the optimal contract calls for investment distortion. As demonstrated below, mimicry by a type H venture capitalist is limited by reducing the investment made following the receipt of the signal L by the venture capitalist below its Pareto Optimal level. This investment distortion is effective in deterring mimicry because the cash flow under a type H venture capitalist is more sensitive to investment than is cash flow under a type L venture capitalist. Thus, the reduced investment following the receipt of the signal L by a venture capitalist effectively generates a bigger reduction in cash flows if the venture capitalist observes signal H and misreports the signal. It follows that lowering the investment level associated with the signal L lowers the benefits from mimicry by a type H venture capitalist. Because a venture capitalist has no incentive to misreport his information after the receipt of signal L , it is not necessary to distort investment following receipt of the signal H .

Now consider the date 0 investment. Given that bargaining advantage belongs to the entrepreneur, she will attempt to restrict the venture capitalist's share of the cash flow. At the same time, she has to credibly convey to the venture capitalist that she is a type G entrepreneur. This attempt is only possible if the value of cash flows accruing to the entrepreneur is lower than w if the entrepreneur is of type B . This dual objective is easiest to achieve if the difference in the value of the entrepreneur's contract varies by a relatively large amount across the two entrepreneur types. Because increased investment magnifies this variation in value across entrepreneur types, there is an incentive for excessive investment at date 0. Excessive date 0 investment is also encouraged by the fact that it discourages mimicry by the venture capitalist if he observes the signal H at date 1. This beneficial side-effect of excessive date 0 investment results primarily because the relatively low date 1 investment following receipt of the signal L results in a large loss in total output, and thus, the venture capitalist's payoff if he is of type H and tries to misreport his information.

Proposition 6 *Let the constrained Pareto optimal levels of investment $I_0^{CPO}(I_H, I_L)$, $I_H^{CPO}(I_0)$, and $I_L^{CPO}(I_0)$ be defined as the solutions to the following equations, respectively:*

$$\phi P'_G \left(I_0^{CPO} \right) P_H(I_H) \Delta X + (1 - \phi) P'_G \left(I_0^{CPO} \right) P_L(I_L) \Delta X - 1 = 0 \quad (27)$$

$$P_G(I_0) P'_H \left(I_H^{CPO} \right) \Delta X - 1 = 0 \quad (28)$$

$$P_G(I_0) P'_L \left(I_L^{CPO} \right) \Delta X - 1 = 0 \quad (29)$$

In any solution to the entrepreneur's problem, the optimal investment levels I_0^* , I_H^* , and I_L^* satisfy

a. $I_0^* \geq I_0^{CPO}(I_H^*, I_L^*)$

b. $I_H^* = I_H^{CPO}(I_0^*)$

c. $I_L^* \leq I_L^{CPO}(I_0^*)$

Once again, the above results demonstrate that there will be excessive investment at date 0, whereas investment will be restricted at date 1 following receipt of the signal L by the venture capitalist. However, the investment distortions are not benchmarked relative to the *unconstrained* Pareto optimal investment levels. To allow for a comparison of optimal investment decisions relative to the unconstrained Pareto optimal levels of investment and to complete the characterization of the optimal contracts, we once again resort to numerical techniques.

Figure 2 presents the results of our numerical analysis. All parameter values employed to generate this figure are the same as employed in Figure 1. The figure contains four panels. Each panel illustrates optimal investment policies for various values of λ_B and λ_L and a given value of ϕ and ΔX . As is clear from the figure, when the entrepreneur has the bargaining advantage, the nature of investment distortions is much more varied than in the case where the bargaining advantage resides with the venture capitalist. Investment distortions are possible both at date 0 and at date 1. The figure confirms our earlier result that investment distortions at date 1 occur only after the venture capitalist observes signal L , and the investment distortion takes the form of underinvestment. In contrast, if there is any investment distortion at date 0, it takes the form of overinvestment.

Note that, once again, higher values of ΔX increase the size of the area where there is no investment distortion. The intuition is identical to that provided above: The larger ΔX is, the larger the difference between the project's total expected cash flow across the two signals the venture capitalist can observe. This larger difference, because it also increases the sensitivity of the expected value of the contracts to the venture capitalist's signal makes it easier to separate the two types of investors by using contract design and creates less need to resort to investment distortion. Unlike the case when venture capitalists have the bargaining power, the quality of the project – i.e. the probability ϕ that the project is type H – has a marked effect on investment decision. Higher quality leads to more investment distortion. This result follows because the higher the quality of the project, the greater the

gains to a type B entrepreneur from having a project financed, limiting the effectiveness of contract design alone in separating the two entrepreneur types. Thus, increased investment distortion is required to screen out type B entrepreneurs.

5 Extensions

In this section we consider the effects of modifying some of our assumption. First we consider the effect of loosening condition (5). Then we consider the effect of allowing for an intermediate date 1 cash flow.

5.1 Pooling equilibria

In this section we loosen the condition (5) by assuming that the ex-ante NPV is positive. That is, there exists I_0 , $I_{1,H}$ and $I_{1,L}$ such that

$$\begin{aligned} & \pi \{ \underline{X} + \phi [P_G(I_0) P_H(I_{1,H}) \Delta X - I_{1,H}] + (1 - \phi) [P_G(I_0) P_L(I_{1,L}) \Delta X - I_{1,L}] - I_0 \} \\ & + (1 - \pi) \{ \underline{X} + \phi [P_B(I_0) P_H(I_{1,H}) \Delta X - I_{1,H}] + (1 - \phi) [P_B(I_0) P_L(I_{1,L}) \Delta X - I_{1,L}] - I_0 \} \\ & - w \geq 0. \end{aligned} \tag{30}$$

In addition, we continue to assume that the project NPV is negative so long as the entrepreneur is of type B . That is for all I_0 , $I_{1,H}$ and $I_{1,L}$,

$$\underline{X} + \phi [P_B(I_0) P_H(I_{1,H}) \Delta X - I_{1,H}] + (1 - \phi) [P_B(I_0) P_L(I_{1,L}) \Delta X - I_{1,L}] - I_0 - w < 0. \tag{31}$$

We now consider the effect of replacing (5) with (30) and (31). First, we demonstrate that this change has no effect on our results in the case where the venture capitalist has the bargaining advantage. Then we examine how our analysis of the case where the entrepreneur has the bargaining advantage is altered by this modification.

5.1.1 The venture capitalist has the bargaining advantage

After we make this change in assumptions, the venture capitalist has a new way of dealing with the adverse selection problem when first approached by the entrepreneur. He can offer the entrepreneur a “pooling” contract. Thus, the venture capitalist can choose between

designing a contract and investment strategy that screens out the entrepreneur if she has observed the signal B , or financing the project regardless of the signal obtained by the entrepreneur.¹⁵ The venture capitalist's choice between these two alternatives is made based on a comparison of their costs.

First consider the case when the venture capitalist chooses to screen out a type B entrepreneur. As demonstrated earlier, the optimal contract always keeps the type G entrepreneur at her reservation wage. This result ensures that a type B entrepreneur would earn less than her reservation if she obtained financing for her project by mimicking a type G entrepreneur. Consequently, no investment distortion is necessary to dissuade the entrepreneur from obtaining financing if she observes the signal B . Now consider the cost of offering the entrepreneur a pooling contract. By offering a pooling contract and permitting the entrepreneur to obtain financing even after observing signal B , the venture capitalist lowers the profitability of the project. It follows that pooling is prohibitively costly when compared with the alternative of screening. This result is formalized in the following proposition.

Proposition 7 *When the venture capitalist has the bargaining advantage, there exist no pooling equilibria where the entrepreneur's project is financed regardless of her private information.*

The above result demonstrates that, when the venture capitalist has the bargaining advantage, the project is not financed when it has a negative NPV. Thus, even under the changed assumption, the primary driver of contract design and investment distortion is the venture capitalist's private information at date 1. This problem is essentially the same as the one examined earlier in Section 3. Thus, the loosening of assumption (5) has no qualitative effect on our predictions when the venture capitalist has the bargaining advantage.

5.1.2 The entrepreneur has the bargaining advantage

In contrast to the setting where the venture capitalist has the bargaining advantage, replacing (5) with (30) and (31), leads to significant changes in optimal contracts and investments. As demonstrated in Section 4, when the project is close to having a zero NPV under a type

¹⁵The pooling strategy was never optimal under our original assumption because the venture capitalist was always better off not financing the project than financing it regardless of the entrepreneur's private information.

B entrepreneur, the date 0 investment has to be distorted to prevent the entrepreneur from obtaining financing if she is of type *B*. This distortion is optimal so long as the cost of this investment distortion is small relative to the reduction in profitability from financing the project regardless of the entrepreneur's type. Otherwise, it is optimal to make no attempt to deter a type *B* entrepreneur from obtaining financing. Consequently, there will exist pooling equilibria where the entrepreneur obtains financing regardless of her private information and these equilibria will prevail when the ex ante probability of the entrepreneur observing signal *G*, π is relatively high. For low values of π , however, it is more cost effective to design contracts that deter the entrepreneur from obtaining financing after she observes the signal *B*.

Thus, when the project has a positive NPV from an ex ante perspective, an entrepreneur may obtain financing for her project even though she observes the signal *B* and knows that NPV is negative. This overinvestment gives rise to the possibility of reviewing the feasibility of the project at date 1 following the receipt of the venture capitalist's signal. When the following condition is satisfied

$$\pi [\underline{X} + P_G(I_0) P_L(I_{1,L}) \Delta X - I_{1,L}] + (1 - \pi) [\underline{X} + P_B(I_0) P_L(I_{1,L}) \Delta X - I_{1,L}] - w < 0, \quad (32)$$

it is not feasible to continue the project at date 1 if type *B* entrepreneurs received financing at date 0. However, because

$$\underline{X} + P_G(I_0) P_L(I_{1,L}) \Delta X - I_{1,L} - w \geq 0. \quad (33)$$

it is optimal to continue with the project conditional on the entrepreneur being type *G*. Consequently, when (5) is replaced with (30) and (31), there exist two types of pooling equilibria where the entrepreneur obtains financing at date 0. In both types of equilibria, the project is always continued if the venture capitalist observes the signal *H* at date 1. However, the equilibria differ in terms of the date 1 outcome conditional on the venture capitalist observing the signal *L*.

The first set of equilibria obtain when (32) is satisfied. In these equilibria, at date 1, following the receipt of the signal *L* by the venture capitalist, the contract obtained by the entrepreneur only satisfies her reservation wage if she is of type *G*, and thus the project is only continued if the entrepreneur is of type *G* and is terminated if the entrepreneur is of type *B*. Define the payoff of the venture capitalist given that both type *G* and type *B* entrepreneurs participate as

$$V_i^P(\alpha_i, \gamma_i, I_0, I_{1,i}) = \pi V_i^G(\alpha_i, \gamma_i, I_0, I_{1,i}) + (1 - \pi) V_i^B(\alpha_i, \gamma_i, I_0, I_{1,i}) \quad (34)$$

where $i \in \{H, L\}$. Then the optimal contracts and investments in these equilibria solve the following problem:

$$\max_{\alpha_H, \alpha_L, \gamma_H, \gamma_L, I_0, I_{1,H}, I_{1,L}} \phi \left[U_H^G(\alpha_H, \gamma_H, I_0, I_{1,H}) \right] + (1 - \phi) \left[U_L^G(\alpha_L, \gamma_L, I_0, I_{1,L}) \right] \quad (35)$$

subject to the entrepreneur's participation constraints at H ,

$$U_H^G(\alpha_H, \gamma_H, I_0, I_{1,H}) \geq w, \quad U_H^B(\alpha_H, \gamma_H, I_0, I_{1,H}) \geq w, \quad (36)$$

the entrepreneur's participation constraints at L ,

$$U_L^G(\alpha_L, \gamma_L, I_0, I_{1,L}) \geq w, \quad U_L^B(\alpha_L, \gamma_L, I_0, I_{1,L}) \leq w, \quad (37)$$

the venture capitalist's truth telling constraints at H and L ,

$$V_H^P(\alpha_H, \gamma_H, I_0, I_{1,H}) - I_H \geq V_H^P(\alpha_L, \gamma_L, I_0, I_{1,L}) - I_L \quad (38)$$

$$V_L^G(\alpha_L, \gamma_L, I_0, I_{1,L}) - I_L \geq V_L^G(\alpha_H, \gamma_H, I_0, I_{1,H}) - I_H \quad (39)$$

the venture capitalist's participation constraints,

$$V_H^P(\alpha_H, \gamma_H, I_0, I_{1,H}) - I_H \geq 0 \quad (40)$$

$$V_L^G(\alpha_L, \gamma_L, I_0, I_{1,L}) - I_L \geq 0 \quad (41)$$

$$\phi \left[V_H^P(\alpha_H, \gamma_H, I_0, I_{1,H}) - I_{1,H} \right] + (1 - \phi) \left[V_L^G(\alpha_L, \gamma_L, I_0, I_{1,L}) - I_{1,L} \right] - I_0 \geq 0 \quad (42)$$

and

$$\alpha_H \in [0, 1], \alpha_L \in [0, 1], \gamma_H \in [0, 1], \gamma_L \in [0, 1]$$

In the second set of pooling equilibria, which obtain when (32) is not satisfied, the project is never terminated. The optimal contracts and investments in these equilibria maximize the same objective function (35) subject to the same set of constraints except that the entrepreneur's participation constraint (37) becomes

$$U_L^G(\alpha_L, \gamma_L, I_0, I_{1,L}) \geq w, \quad U_L^B(\alpha_L, \gamma_L, I_0, I_{1,L}) \geq w, \quad (43)$$

and V_L^G is substituted by V_L^P in (39), (41), and (42).

The contract designs that solve both problems are identical to those described in our earlier analysis of the case where the entrepreneur has the bargaining power. Because entrepreneurs are being financed regardless of their type, the two incentive problems shaping contract designs are (i) the desire to minimize "mispricing" of the financing contract by

the venture capitalist and (ii) the desire to deter mimicry by a type H venture capitalist. The first problem is similar to that examined in much of the capital structure literature and calls for the venture capitalist to receive a relatively cashflow-insensitive claim, i.e., setting $\alpha = 1$. The solution to the second problem, as described earlier, also calls for a similar contract design. Thus, in general, entrepreneur's receive relatively more cashflow-sensitive claims when they have the bargaining advantage.

We employ numerical techniques to illustrate how replacing (5) with (30) and (31) changes the nature of equilibria. With one exception, the value of π , we maintain the same parameterizations as employed in Figures 1 and 2. Here π takes the value of either 0.5 or 0.7. Figure 3, presents the regions that support the three possible equilibria: (i) where screening out type B entrepreneurs is optimal, (ii) where financing both types of entrepreneurs and terminating the project if the venture capitalist observes signal L and the entrepreneur is of type B is optimal, and (iii) where financing both types of entrepreneurs without any date 1 termination is optimal. The figure demonstrates that, as we argue above, financing both types of entrepreneurs is optimal only when the profitability under a type B entrepreneur is relatively high. Further, the date 1 termination of the project is optimal only when profitability conditional on the signal L and entrepreneur being type B is relatively low.

Figure 4 graphs the distortion in the date 0 investment policy. As the figure demonstrates, the change in assumption has no qualitative impact on the nature of the date 0 investment distortion—investment distortion at date 0 continues to take the form of overinvestment. When it is optimal to deny financing to the entrepreneur after she observes signal B , the intuition behind this result is identical to that provided earlier. The cash flows are more sensitive to investment if the entrepreneur observes signal G . Thus, increasing investment following the receipt of signal G makes contract values more sensitive to the entrepreneur's type and thereby creates larger differences in contract values across entrepreneur types. This difference makes it easier to design a contract that enables a type G entrepreneur to capture a larger share of the surplus from the project while controlling mimicry incentives of a type B entrepreneur. In the pooling equilibria, the result is driven by two competing forces. There is a tendency toward overinvestment because type B entrepreneurs are being financed even though they would be denied financing in the absence of any information asymmetry. The financing of entrepreneurs regardless of type reduces ex ante profitability and consequently the level of investment in the project. However, the second factor is not sufficiently strong to offset the first. Overall, there is overinvestment.

Date 1 investment decisions are illustrated in Figures 5 and 6. Figure 5 focuses on investment decisions conditional on the venture capitalist observing signal H , whereas Figure 6 focuses on the case where the venture capitalist has observed signal L . In both cases, investment distortion takes the form of underinvestment when profitability under a type B entrepreneur is relatively low. Overinvestment occurs following both date 1 signal realizations when profitability under a type B entrepreneur is relatively high. Thus, there are two significant differences between our earlier analysis of investment when the entrepreneur has the bargaining advantage and our current results — there is investment distortion following signal realization H and there is the possibility that the firm might overinvest at date 1. To understand the cause of these differences, we need to first examine the forces shaping investment policy.

First, as described in our earlier analysis of investment when the entrepreneur has the bargaining advantage, there is an incentive to underinvest following the signal L so as to deter mimicry by a type H venture capitalist. Second, there is a tendency toward overinvestment, simply because the project is financed even when the entrepreneur is of type B . Finally, there is a tendency toward underinvestment because expected profitability is relatively low because there is the possibility that the entrepreneur may be of type B . When profitability under a type B entrepreneur is sufficiently low, optimal contracts screen out type B entrepreneurs and the latter two forces are absent, which results in underinvestment following the signal L . When profitability under a type B entrepreneur is sufficiently high, pooling contracts that provide entrepreneurs financing regardless of their type are optimal and the latter two forces come into play. The last force is relatively weak when profitability under a type B entrepreneur is relatively high and gathers strength as profitability under a type B entrepreneur falls. Thus, when profitability under a type B entrepreneur only slightly exceeds the profitability threshold for pooling equilibria, the venture capitalist underinvests. At higher levels of profitability under a type B entrepreneur the venture capitalist overinvests, because the last force is relatively weak.

5.2 Introduction of an intermediate cash flow

In our analysis, we consider a project that generates only a terminal cash flow because the major component of the return on venture investments arises from the sale of the venture to the public or to another company, which we view as the terminal date of our model. We now consider the implication of allowing for a risky cash flow at date 1. Let us consider the more interesting and likely case where the intermediate cash flow, while risky, is only

influenced by the entrepreneur's private information and the level of initial investment in the project.¹⁶ Further, let a favorable signal to the entrepreneur result in a higher likelihood of a high date 1 cash flow.

The sensitivity of the date 1 cash flow to the entrepreneur's private information can be exploited to alleviate some of the inefficiencies arising from the entrepreneur's initial informational advantage. By increasing the sensitivity of the entrepreneur's date 1 payment to date 1 cash flow, it is possible to ensure that the expected value of the entrepreneur's date 1 payoff will be more sensitive to the private signal. This result will help deter an entrepreneur who observes the signal B from approaching a venture capitalist for financing. Consequently, there will be less need for the use of date 0 investment distortions to screen out an entrepreneur who has observed signal B . By extension, an even greater reduction in the level and likelihood of date 0 investment distortions may be attained by tying the value of the entrepreneur's second period (date 2) cash flow to the date 1 cash flow realization. Thus, regardless of who has the bargaining advantage, the entrepreneur's payoff should be sensitive to date 1 cash flow realizations. Given that entrepreneur bears much of the risk of the date 1 cash flow, the venture capitalist's payoffs should be relatively insensitive to the date 1 cash flow. This sharing rule can be best achieved by financing projects by issuing claims that have debt-like payoffs with respect to the date 1 cash flow to venture capitalists, i.e., giving the venture capitalists "liquidation rights" over the project.

Together with our earlier results, this analysis suggests that the evolution of the sensitivity of venture capitalist's and entrepreneur's contracts to project performance will vary with the distribution of bargaining power between the entrepreneur and venture capitalist. When the venture capitalist has the bargaining advantage, he will receive contracts whose sensitivity to venture performance increases over time. This result is consistent with the pattern of decreasing liquidation rights of the venture capitalist described in Kaplan and Strömberg (2004). Conversely, when the entrepreneur has the bargaining advantage, the evolution in the sensitivity of venture capitalist's payoffs to project performance should be less marked.

¹⁶If the cash flow were also influenced by the venture capitalist's private information, the contract design problems would be minor extensions of those examined above. The only difference would be that the intermediate cash flow would also have to be apportioned by the contracts. Given that there is no change in the incentive structure, the solutions to the problems would involve using the same sort of cash flow sharing rule for the date 1 cash flow as is employed for the date 2 cash flow.

6 Conclusion

We have modeled a situation where an entrepreneur is seeking funds from a venture capitalist while both possess some information advantage. During the early stages the entrepreneur has better information regarding the prospects of the proposed new venture. Conversely during the later stages, the venture capitalist is better informed about the new firm's prospects. We also let the bargaining power of the two parties vary.

Our results suggest that both the evolution of the informational asymmetry regarding a project's prospects and the distribution of bargaining power between the entrepreneur and venture capitalist are important in determining the amount invested in projects and the nature of contracts that divide the cash flows between the entrepreneur and the venture capitalist. First, when the venture capitalist has the bargaining advantage, he is able to appropriate much of the surplus from projects; screening projects is always optimal; venture capitalists' claims are relatively sensitive to project payoffs; and there is little incentive to distort investment in the early stages of the project and a tendency to overinvest during later stages. The situation is reversed when the bargaining advantage lies with the entrepreneur. In this case, the entrepreneur is able to appropriate much of the surplus from projects; projects may not be screened; venture capitalists' claims are relatively insensitive to project payoffs; and there is a tendency to overinvest in the early stages of a project and a tendency to distort investment during the later stages as well. These results provide a rational explanation for the fact that investment in and financing of venture-backed projects vary considerably over time in a cyclical fashion and that returns to venture capital investments vary depending on the market conditions. Further, these results provide insight into the documented systematic relationship between a venture capitalist's experience/reputation and performance.

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Appendix

When the venture capitalist has the bargaining advantage, we first analyze a problem with a reduced number of constraints, and then we show that the obtained solution satisfies the remaining constraints.

Suppose the venture capitalist maximizes the objective function

$$\max_{\alpha_H, \alpha_L, \gamma_H, \gamma_L, I_{1,H}, I_{1,L}} \phi \left[V_H^G(\alpha_H, \gamma_H, I_0, I_{1,H}) - I_{1,H} \right] + (1 - \phi) \left[V_L^G(\alpha_L, \gamma_L, I_0, I_{1,L}) - I_{1,L} \right] - I_0 \quad (\text{A-1})$$

subject to the following constraints:

$$U_H^G(\alpha_H, \gamma_H, I_0, I_{1,H}) \geq w \quad (\text{A-2})$$

$$U_L^G(\alpha_L, \gamma_L, I_0, I_{1,L}) \geq w \quad (\text{A-3})$$

$$V_L^G(\alpha_L, \gamma_L, I_0, I_{1,L}) - I_L \geq V_L^G(\alpha_H, \gamma_H, I_0, I_{1,H}) - I_H \quad (\text{A-4})$$

$$\alpha_H \in (0, 1), \alpha_L \in (0, 1), \gamma_H \in (0, 1), \gamma_L \in (0, 1)$$

Let λ_H , λ_L , and λ_{VC} be the Lagrange multipliers for constraints (A-2), (A-3), and (A-4), respectively. The FOCs of the Lagrangian associated with this problem are

$$\frac{\partial L}{\partial \alpha_H} = \phi - \lambda_H - \lambda_{VC} \quad (\text{A-5})$$

$$\frac{\partial L}{\partial \alpha_L} = 1 - \phi - \lambda_L + \lambda_{VC} \quad (\text{A-6})$$

$$\frac{\partial L}{\partial \gamma_H} = \phi - \lambda_H - \lambda_{VC} \frac{P_L(I_H)}{P_H(I_H)} \quad (\text{A-7})$$

$$\frac{\partial L}{\partial \gamma_L} = 1 - \phi - \lambda_L + \lambda_{VC} \quad (\text{A-8})$$

$$\frac{\partial L}{\partial I_0} = \phi P'_G(I_0) P_H(I_H) \Delta X + (1 - \phi) P'_G(I_0) P_L(I_L) \Delta X - 1 + \lambda_{VC} P'_G(I_0) \Delta X [\gamma_L P_L(I_L) - \gamma_H P_L(I_H)] \quad (\text{A-9})$$

$$\begin{aligned} \frac{\partial L}{\partial I_H} &= -(\phi - \lambda_H - \lambda_{VC}) + \lambda_H [P_G(I_0) P'_H(I_H) \Delta X - 1] \\ &+ \lambda_{VC} \gamma_H P_G(I_0) P'_H(I_H) \Delta X \left(\frac{P_L(I_H)}{P_H(I_H)} - \frac{P'_L(I_H)}{P'_H(I_H)} \right) \end{aligned} \quad (\text{A-10})$$

$$\frac{\partial L}{\partial I_L} = (1 - \phi + \lambda_{VC}) [P_G(I_0) P'_L(I_L) \Delta X - 1] \quad (\text{A-11})$$

Lemma A.1 *In every solution to this problem, $\frac{\partial L}{\partial \gamma_H}$, $\frac{\partial L}{\partial \gamma_L}$, $\frac{\partial L}{\partial I_0}$, $\frac{\partial L}{\partial I_H}$, and $\frac{\partial L}{\partial I_L}$ must be zero.*

Proof. If the boundary conditions on γ_H , γ_L , I_0 , I_H , and I_L do not bind, then the partial derivatives are equal to zero by the Kuhn-Tucker conditions. The boundary conditions on I_0 , I_H , and I_L cannot bind because of our assumptions regarding the properties of the production functions P . If one of the constraints on γ_H or γ_L binds, then γ_H or γ_L is either 0 or 1. If γ_H or γ_L is 0, the venture capitalist receives the same cash flow in both states and the most he can receive is \underline{X} . This result contradicts our assumption that the project cannot be financed using riskless debt. If γ_H or γ_L is 1, the entrepreneur receives the same cash flow in both states, and the most she can receive is \underline{X} . This result contradicts our assumption that $w > \underline{X}$. Hence the γ_H and γ_L must line in the interval $(0, 1)$. ■

Lemma A.2 *In every solution to this problem both participation constraints must bind, i.e. $\lambda_L^* > 0$ and $\lambda_H^* > 0$.*

Proof. Note that by Equation (A-5), $\frac{\partial L}{\partial \gamma_L} = 1 - \phi - \lambda_L^* + \lambda_{VC}^* = 0$. Because $1 - \phi > 0$, this implies that $\lambda_L^* > 0$. Now we demonstrate that $\lambda_H^* > 0$. If $\lambda_{VC}^* = 0$, then this follows directly because, by Equation (A-7), in any solution $\frac{\partial L}{\partial \gamma_H} = \phi - \lambda_H^* - \lambda_{VC}^* \frac{P_L(I_H^*)}{P_H(I_H^*)} = 0$. Now we consider the case where $\lambda_{VC}^* > 0$. First note that by assumption (A-10),

$$\begin{aligned} \frac{\partial L}{\partial I_H} = & -(\phi - \lambda_H^* - \lambda_{VC}^*) + \lambda_H^* [P_G(I_0^*) P_H'(I_H^*) \Delta X - 1] \\ & + \lambda_{VC}^* \gamma_H^* P_G(I_0^*) P_H'(I_H^*) \Delta X \left(\frac{P_L(I_H^*)}{P_H(I_H^*)} - \frac{P_L'(I_H^*)}{P_H'(I_H^*)} \right) = 0. \end{aligned} \quad (\text{A-12})$$

Now note that $\phi - \lambda_H^* - \lambda_{VC}^* < 0$ since $\frac{\partial L}{\partial \gamma_H} = 0$ and $\frac{P_L(I_H^*)}{P_H(I_H^*)} < 1$. Finally, note that by assumption (2) that $\frac{P_L(I_H^*)}{P_H(I_H^*)} > \frac{P_L'(I_H^*)}{P_H'(I_H^*)}$. It follows that

$$-(\phi - \lambda_H^* - \lambda_{VC}^*) + \lambda_{VC}^* \gamma_H^* P_G(I_0^*) P_H'(I_H^*) \Delta X \left(\frac{P_L(I_H^*)}{P_H(I_H^*)} - \frac{P_L'(I_H^*)}{P_H'(I_H^*)} \right) > 0.$$

For $\frac{\partial L}{\partial I_H} = 0$, it must be the case that $\lambda_H^* [P_G(I_0^*) P_H'(I_H^*) \Delta X - 1] < 0$. Hence $\lambda_H^* > 0$. ■

Lemma A.3 *Constraints 8, 11, 14, 15 and 16 do not bind.*

Proof. As proved in the previous lemma, constraints 9 and 10 always bind, The result for constraint 8 follows directly from that result. The result for constraint 11 follows by first

order stochastic dominance argument in addition to constraints 9 and 10 being binding. The result for constraints 14, 15 and 16 is implied by constraints 9 and 10 being binding and by the assumption that both investments are positive NPV investments. ■

Lemma A.4 (i) $P_G(I_0^*) P'_H(I_H^*) \Delta X - 1 \leq 0$, (ii) $I_H^* \geq I_H^{CPO}(I_0^*)$.

Proof. First note that because there exists an internal optimal solution level of investment,

$$\begin{aligned} \frac{\partial L}{\partial I_H} &= -(\phi - \lambda_H^* - \lambda_{VC}^*) + \lambda_H^* [P_G(I_0^*) P'_H(I_H^*) \Delta X - 1] \\ &\quad + \lambda_{VC}^* \gamma_H^* P_G(I_0^*) P'_H(I_H^*) \Delta X \left(\frac{P_L(I_H^*)}{P_H(I_H^*)} - \frac{P'_L(I_H^*)}{P'_H(I_H^*)} \right) = 0. \end{aligned}$$

From Lemma A.2, we know that $\lambda_H^* > 0$, $\phi - \lambda_H^* - \lambda_{VC}^* < 0$ and from assumption (2) we know that $\frac{P_L(I_H^*)}{P_H(I_H^*)} - \frac{P'_L(I_H^*)}{P'_H(I_H^*)} > 0$. Thus, the first order condition can only be satisfied if $P_G(I_0^*) P'_H(I_H^*) \Delta X - 1 \leq 0$.

Now note that by definition, $I_H^{CPO}(I_0^*)$, the constrained Pareto optimal satisfies the condition

$$P_G(I_0^*) P'_H(I_H^{CPO}(I_0^*)) \Delta X - 1 = 0.$$

The second claim follows directly from the previous result and by the assumption $P''_H < 0$. ■

Lemma A.5 (i) $P_G(I_0^*) P'_L(I_L^*) \Delta X - 1 = 0$, (ii) $I_L^* = I_L^{CPO}(I_0^*)$.

Proof. Note that

$$\frac{\partial L}{\partial I_L} = (1 - \phi + \lambda_{VC}^*) [P_G(I_0^*) P'_L(I_L^*) \Delta X - 1] = 0 \quad (\text{A-13})$$

given Equation (A-11). Since $1 - \phi > 0$ and $\lambda_{VC}^* \geq 0$, it must be that

$$P_G(I_0^*) P'_L(I_L^*) \Delta X - 1 = 0. \quad (\text{A-14})$$

Now note that by definition, $I_L^{CPO}(I_0^*)$, the constrained Pareto optimal satisfies the condition

$$P_G(I_0^*) P'_L(I_L^{CPO}(I_0^*)) \Delta X - 1 = 0.$$

The second claim follows directly by comparing this expression with the previous result. ■

Lemma A.6 *If there exists a solution to the problem (A-1), there exists a solution $(I_0^*, \alpha_H^*, \gamma_H^*, I_{1,H}^*, \alpha_L^*, \gamma_L^*, I_{1,L}^*)$, where $\alpha_H^* = \alpha_L^* = 0$.*

Proof. First assume $\lambda_{VC}^* = 0$. In this case, $\frac{\partial L}{\partial \alpha_H} = \frac{\partial L}{\partial \gamma_H} = \phi - \lambda_H^*$. If $\phi - \lambda_H^* \neq 0$, then both α_H^* and γ_H^* must be the corner solution, i.e., either $\alpha_H^* = \gamma_H^* = 0$ or $\alpha_H^* = \gamma_H^* = 1$. In either case, the entrepreneur's participation constraint (A-2) will not hold as an equality as shown in Lemma A.2. Hence, $\phi - \lambda_H^* = 0$. Then we can pick α_H^* as any number between 0 and 1 and adjust γ_H^* to satisfy the participation constraint. Thus we can set $\alpha_H^* = 0$.

Next assume $\lambda_{VC}^* > 0$. In this case, $\frac{\partial L}{\partial \alpha_H} = \phi - \lambda_H^* - \lambda_{VC}^*$ and $\frac{\partial L}{\partial \gamma_H} = \phi - \lambda_H^* - \lambda_{VC}^* \frac{P_L(I_H^*)}{P_H(I_H^*)}$. Given that and $\frac{P_L(I_H^*)}{P_H(I_H^*)} < 1$ and $\frac{\partial L}{\partial \gamma_H} = 0$, it must be the case that $\frac{\partial L}{\partial \alpha_H} < 0$. Hence $\alpha_H^* = 0$.

Finally note that $\frac{\partial L}{\partial \alpha_L} = \frac{\partial L}{\partial \gamma_H} = 1 - \phi - \lambda_L^* + \lambda_{VC}^*$. Using the similar argument for α_H^* in the case of $\lambda_{VC}^* = 0$, we can set $\alpha_L^* = 0$. ■

Lemma A.7 (i) $\phi P'_G(I_0^*) P_H(I_H^*) \Delta X + (1 - \phi) P'_G(I_0^*) P_L(I_L^*) \Delta X - 1 \geq 0$, (ii) $I_0^* \leq I_0^{CPO}(I_H^*, I_L^*)$.

Proof. First note that, given Equation (A-9),

$$\begin{aligned} \frac{\partial L}{\partial I_0} &= \phi P'_G(I_0^*) P_H(I_H^*) \Delta X + (1 - \phi) P'_G(I_0^*) P_L(I_L^*) \Delta X - 1 \\ &\quad + \lambda_{VC}^* [\gamma_L P_L(I_L^*) - \gamma_H P_L(I_H^*)] = 0. \end{aligned}$$

From Lemma A.6 and Lemma A.2, γ_H^* and γ_L^* must satisfy,

$$\begin{aligned} \underline{X} + (1 - \gamma_H) P_G(I_0) P_H(I_{1,H}) \Delta X &= w \\ \underline{X} + (1 - \gamma_L) P_G(I_0) P_L(I_{1,L}) \Delta X &= w. \end{aligned}$$

Because $P_H(I_{1,H}) > P_L(I_{1,L})$, $\gamma_H > \gamma_L$. Since $\lambda_{VC}^* \geq 0$ and $P_L(I_L^*) - P_L(I_H^*) < 0$,

$$\lambda_{VC}^* [\gamma_L P_L(I_L^*) - \gamma_H P_L(I_H^*)] < 0.$$

Then, it must be that

$$\phi P'_G(I_0^*) P_H(I_H^*) \Delta X + (1 - \phi) P'_G(I_0^*) P_L(I_L^*) \Delta X - 1 \geq 0 \quad (\text{A-15})$$

Next note that by definition, $I_0^{CPO}(I_H^*, I_L^*)$, the constrained Pareto optimal satisfies the condition

$$\phi P'_G(I_0^*) P_H(I_H^*) \Delta X + (1 - \phi) P'_G(I_0^*) P_L(I_L^*) \Delta X - 1 = 0.$$

The second claim follows directly from the previous result and by the assumption $P''_G < 0$. ■

Lemma A.8 $V_H^G(\alpha_H, \gamma_H, I_0, I_{1,H}) - I_H > V_H^G(\alpha_L, \gamma_L, I_0, I_{1,L}) - I_L$, i.e. Constraint (12) does not bind.

Proof. First assume that $\lambda_{VC}^* = 0$. This implies that $I_H^* = I_H^{PO}$. Substituting $\alpha_H^*, \alpha_L^*, \gamma_H^*, \gamma_L^*$ from the participation constraints A-2 and A-3, Constraint (12) can be expressed as

$$\underline{X} + P_G(I_0^*) P_H(I_H^{PO}) \Delta X - w - I_H^{PO} \geq \underline{X} + P_G(I_0^*) P_L(I_L^*) \Delta X - w - I_L^* + \gamma_L^* P_G(I_0^*) \Delta X [P_H(I_L^*) - P_L(I_L^*)].$$

It suffices to show that Constraint 12 strictly holds for $\gamma_L = 1$. Substituting $\gamma_L = 1$ into Constraint 12 the problem reduces to $P_G(I_0^*) P_H(I_H^{PO}) \Delta X - I_H^{PO} > P_G(I_0^*) P_L(I_L^*) \Delta X - I_L^*$, which holds for any I_L .

Next assume that $\lambda_{VC}^* > 0$. When $\lambda_{VC}^* > 0$, we know from Lemma A.6 that $\alpha_H^* = 0$. Now let $R := \alpha_L^* \underline{X} + (I_H^* - I_L^*) 0$. Because Constraint (A-4) binds, $R + \gamma_L^* P_G(I_0^*) P_L(I_L^*) \Delta X = \gamma_H^* P_G(I_0^*) P_L(I_H^*) \Delta X$. We can write Constraint (12) as

$$\gamma_H^* P_G(I_0^*) P_H(I_H^*) \Delta X \geq R + \gamma_L^* P_G(I_0^*) P_H(I_L^*) \Delta X.$$

Then

$$\frac{\gamma_H^* P_G(I_0^*) P_H(I_H^*) \Delta X}{\gamma_H^* P_G(I_0^*) P_L(I_H^*) \Delta X} = \frac{P_H(I_H^*)}{P_L(I_H^*)} \geq \frac{R + \gamma_L^* P_G(I_0^*) P_H(I_L^*) \Delta X}{R + \gamma_L^* P_G(I_0^*) P_L(I_L^*) \Delta X}. \quad (\text{A-16})$$

We can establish our result by showing that the inequality (A-16) is strict. Let $g(R) = \frac{R + \gamma_L^* P_G(I_0^*) P_H(I_L^*) \Delta X}{R + \gamma_L^* P_G(I_0^*) P_L(I_L^*) \Delta X}$. Now note that $\frac{dg(R)}{dR} < 0$, i.e. decreasing in R . It follows then that $\frac{P_H(I_L^*)}{P_L(I_L^*)} > \frac{R + \gamma_L^* P_G(I_0^*) P_H(I_L^*) \Delta X}{R + \gamma_L^* P_G(I_0^*) P_L(I_L^*) \Delta X}$. The proof is concluded by noting that, by assumption (2), $\frac{P_H(I_H^*)}{P_L(I_H^*)} > \frac{P_H(I_L^*)}{P_L(I_L^*)} > \frac{R + \gamma_L^* P_G(I_0^*) P_H(I_L^*) \Delta X}{R + \gamma_L^* P_G(I_0^*) P_L(I_L^*) \Delta X}$. ■

Lemma A.9 Let $(I_0^*, I_H^*, I_L^*, \alpha_H^*, \gamma_H^*, \alpha_L^*, \gamma_L^*)$ be the solution to problem (17). This solution is implementable if the venture capitalist proposes (I_0^*, a, b) , ($a \geq \max(\alpha_H^*, \alpha_L^*)$ and $b \geq \max(\gamma_H^*, \gamma_L^*)$), as the initial contract, and $(I_H^*, \alpha_H^*, \gamma_H^*)$ and $(I_L^*, \alpha_L^*, \gamma_L^*)$ as the menu of possible time 1 contracts.

Proof. Consider the following strategies and beliefs:

- At date 0, the venture capitalist proposes (I_0^*, a, b) as the initial contract, and $(I_H^*, \alpha_H^*, \gamma_H^*)$ and $(I_L^*, \alpha_L^*, \gamma_L^*)$ as the menu of possible date 1 contracts. At date 1, the venture capitalist proposes to switch to $(I_H^*, \alpha_H^*, \gamma_H^*)$ if he observes H , and to $(I_L^*, \alpha_L^*, \gamma_L^*)$ if he observes L .

- At date 0, if the entrepreneur is of type G, she accepts any menu of contracts that includes a date 0 investment of I_0^* and gives her at least the same payoff as in $(I_H^*, \alpha_H^*, \gamma_H^*)$ under state H and $(I_L^*, \alpha_L^*, \gamma_L^*)$ under state L. At date 1, a type G entrepreneur agrees to switch to either $(I_H^*, \alpha_H^*, \gamma_H^*)$ or $(I_L^*, \alpha_L^*, \gamma_L^*)$. A type B entrepreneur rejects the same contract.
- At date 0, the venture capitalist believes that the entrepreneur who accepts the menu of contracts is of type G, and otherwise the entrepreneur is of type B. At date 1, the entrepreneur believes that the venture capitalist who proposes to switch to $(I_H^*, \alpha_H^*, \gamma_H^*)$ has observed H and any other venture capitalist as observing L.

We need to show that such strategies and beliefs constitute a perfect Bayesian equilibrium in the bargaining game. At date 1, because the initial contract specifies that the venture capitalist gets more than what is specified in either H contract or L contract, the entrepreneur is willing to change to either contract. Because the new contract pays the entrepreneur her reservation wage, if the venture capitalist chooses to stay with the date 0 contract, the entrepreneur will be paid less than the reservation wage and will choose to leave the project. Hence the venture capitalist must switch to one of the two prespecified contracts. Any new proposal results in a negotiation breakdown and is suboptimal.

At date 0, a type G entrepreneur accepts a contract that pays her less than reservation wage because she knows that the venture capitalist has to switch to a different contract at date 1 that pays her the reservation wage. A type B entrepreneur rejects the contract because she is not going to attain reservation wage in any case. The venture capitalist proposes such a contract because it maximizes his expected payoff as the parameters solve problem (17). ■

Proof of Lemma 1, Proposition 2, and Proposition 3. Based on Lemmas A.3 and A.8, we show that the venture capitalist's maximization problem (A-1) is equivalent to the maximization problem (17). Thus, Lemma 1 is proved by Lemma A.2. Proposition 2 is proved by Lemma A.6. Proposition 3 is proved by Lemmas A.4, A.5, and A.7. ■

Next, we prove our results when the entrepreneur has the bargaining advantage. We will first analyze a problem with a reduced number of constraints, and then we will show that the solution obtained satisfies all the remaining constraints.

The objective function is

$$\max_{\alpha_H, \alpha_L, \gamma_H, \gamma_L, I_0, I_{1,H}, I_{1,L}} \phi \left[U_H^G(\alpha_H, \gamma_H, I_0, I_{1,H}) \right] + (1 - \phi) \left[U_L^G(\alpha_L, \gamma_L, I_0, I_{1,L}) \right] \quad (\text{A-17})$$

subject to the following constraints:

$$\phi [U_H^B(\alpha_H, \gamma_H, I_0, I_{1,H})] + (1 - \phi) [U_L^B(\alpha_L, \gamma_L, I_0, I_{1,L})] \leq w \quad (\text{A-18})$$

$$\phi [V_H^G(\alpha_H, \gamma_H, I_0, I_{1,H}) - I_H] + (1 - \phi) [V_L^G(\alpha_L, \gamma_L, I_0, I_{1,L}) - I_L] - I_0 \geq 0 \quad (\text{A-19})$$

$$V_L^G(\alpha_L, \gamma_L, I_0, I_{1,L}) - I_L \geq 0 \quad (\text{A-20})$$

$$V_H^G(\alpha_H, \gamma_H, I_0, I_{1,H}) - I_H \geq V_L^G(\alpha_L, \gamma_L, I_0, I_{1,L}) - I_L \quad (\text{A-21})$$

$$V_L^G(\alpha_L, \gamma_L, I_0, I_{1,L}) - I_L \geq V_L^G(\alpha_H, \gamma_H, I_0, I_{1,H}) - I_H \quad (\text{A-22})$$

$$\alpha_H \in (0, 1), \alpha_L \in (0, 1), \gamma_H \in (0, 1), \gamma_L \in (0, 1)$$

Let λ_B , λ_{VC0} , λ_{L1} , λ_{ICH} , and λ_{ICL} be the Lagrange multipliers for constraints (A-18), (A-19), (A-20), (A-21) and (A-22), respectively. The FOCs of the associated Lagrangian are

$$\frac{\partial L}{\partial \alpha_H} = -\phi(1 - \lambda_B - \lambda_{VC0}) + \lambda_{ICH} - \lambda_{ICL} \quad (\text{A-23})$$

$$\frac{\partial L}{\partial \alpha_L} = -(1 - \phi)(1 - \lambda_B - \lambda_{VC0}) + \lambda_{L1} - \lambda_{ICH} + \lambda_{ICL} \quad (\text{A-24})$$

$$\frac{\partial L}{\partial \gamma_H} = -\phi \left(1 - \lambda_B \frac{P_B(I_0)}{P_G(I_0)} - \lambda_{VC0} \right) + \lambda_{ICH} - \lambda_{ICL} \frac{P_L(I_H)}{P_H(I_H)} \quad (\text{A-25})$$

$$\frac{\partial L}{\partial \gamma_L} = -(1 - \phi) \left(1 - \lambda_B \frac{P_B(I_0)}{P_G(I_0)} - \lambda_{VC0} \right) + \lambda_{L1} - \lambda_{ICH} \frac{P_H(I_L)}{P_L(I_L)} + \lambda_{ICL} \quad (\text{A-26})$$

$$\begin{aligned} \frac{\partial L}{\partial I_0} = & \left(1 - \lambda_B \frac{P_B'(I_0)}{P_G'(I_0)} \right) [\phi P_G'(I_0) P_H(I_H) \Delta X + (1 - \phi) P_G'(I_0) P_L(I_L) \Delta X - 1] \\ & - \lambda_B \left(\frac{P_B(I_0)}{P_G(I_0)} - \frac{P_B'(I_0)}{P_G'(I_0)} \right) [\phi \gamma_H P_G'(I_0) P_H(I_H) \Delta X + (1 - \phi) \gamma_L P_G'(I_0) P_L(I_L) \Delta X - 1] \\ & + \lambda_{L1} + \lambda_{ICH} \left(1 - \frac{P_H(I_L)}{P_L(I_L)} \right) + \lambda_{ICL} \left(1 - \frac{P_L(I_H)}{P_H(I_H)} \right) \end{aligned} \quad (\text{A-27})$$

$$\begin{aligned} \frac{\partial L}{\partial I_H} = & \phi \left(1 - \lambda_B \frac{P_B(I_0)}{P_G(I_0)} \right) [P_G(I_0) P_H'(I_H) \Delta X - 1] \\ & + \lambda_{ICL} \left[\left(1 - \frac{P_L(I_H)}{P_H(I_H)} \right) + \gamma_H P_G(I_0) P_H'(I_H) \Delta X \left(\frac{P_L(I_H)}{P_H(I_H)} - \frac{P_L'(I_H)}{P_H'(I_H)} \right) \right] \end{aligned} \quad (\text{A-28})$$

$$\begin{aligned} \frac{\partial L}{\partial I_L} = & (1 - \phi) \left(1 - \lambda_B \frac{P_B(I_0)}{P_G(I_0)} \right) [P_G(I_0) P_L'(I_L) \Delta X - 1] \\ & + \lambda_{ICH} \left[\left(1 - \frac{P_H(I_L)}{P_L(I_L)} \right) + \gamma_L P_G(I_0) P_L'(I_L) \Delta X \left(\frac{P_H(I_L)}{P_L(I_L)} - \frac{P_H'(I_L)}{P_L'(I_L)} \right) \right]. \end{aligned} \quad (\text{A-29})$$

Lemma A.10 In every solution to this problem, $\frac{\partial L}{\partial \gamma_H}$, $\frac{\partial L}{\partial \gamma_L}$, $\frac{\partial L}{\partial I_0}$, $\frac{\partial L}{\partial I_H}$, and $\frac{\partial L}{\partial I_L}$ must be zero.

Proof. Same as the proof for Lemma A.1. ■

Lemma A.11 $1 - \lambda_B^* \frac{P_B(I_0^*)}{P_G(I_0^*)} - \lambda_{VC0}^* > 0$

Proof. Assume that $1 - \lambda_B^* \frac{P_B(I_0^*)}{P_G(I_0^*)} - \lambda_{VC0}^* \leq 0$. From FOC (A-25)

$$\frac{\partial L}{\partial \gamma_H} = -\phi \left(1 - \lambda_B^* \frac{P_B(I_0^*)}{P_G(I_0^*)} - \lambda_{VC0}^* \right) + \lambda_{ICH}^* - \lambda_{ICL}^* \frac{P_L(I_H^*)}{P_H(I_H^*)} = 0$$

it must be that

$$\lambda_{ICH}^* \leq \lambda_{ICL}^* \frac{P_L(I_H^*)}{P_H(I_H^*)} < \lambda_{ICL}^*. \quad (\text{A-30})$$

Also from FOC (A-26)

$$\frac{\partial L}{\partial \gamma_L} = -(1 - \phi) \left(1 - \lambda_B^* \frac{P_B(I_0^*)}{P_G(I_0^*)} - \lambda_{VC0}^* \right) + \lambda_{L1}^* - \lambda_{ICH}^* \frac{P_H(I_L^*)}{P_L(I_L^*)} + \lambda_{ICL}^* = 0$$

it follows that

$$\lambda_{L1}^* + \lambda_{ICL}^* \leq \lambda_{ICH}^* \frac{P_H(I_L^*)}{P_L(I_L^*)}. \quad (\text{A-31})$$

Because $\lambda_{L1}^* \geq 0$ and $\frac{P_H(I_L^*)}{P_L(I_L^*)} > 0$, $\lambda_{ICL}^* \frac{P_H(I_L^*)}{P_L(I_L^*)} \leq \lambda_{ICH}^*$. Combined with (A-30), we have $\frac{P_L(I_H^*)}{P_H(I_H^*)} \geq \frac{P_L(I_L^*)}{P_H(I_L^*)}$. This is the contradiction because the function $z(I) = \frac{P_L(I)}{P_H(I)}$ is strictly decreasing in I . To see this result, note that

$$\frac{\partial z}{\partial I} = \frac{P'_L P_H - P'_H P_L}{P_H^2} = \frac{P_L}{P_H} \left(\frac{P'_L}{P_L} - \frac{P'_H}{P_H} \right),$$

which is negative given assumption 2. Hence it must be that $1 - \lambda_B^* \frac{P_B(I_0^*)}{P_G(I_0^*)} - \lambda_{VC0}^* > 0$. ■

Lemma A.12 If $1 - \lambda_B^* \frac{P_B(I_0^*)}{P_G(I_0^*)} - \lambda_{VC0}^* > 0$, then $\lambda_{ICH}^* > 0$.

Proof. Follows directly from $\frac{\partial L}{\partial \gamma_H} = 0$. ■

Lemma A.13 If $\lambda_{ICH}^* > 0$, then $\alpha_L^* = 1$.

Proof. From FOCs (A-24) and (A-26), we have in equilibrium

$$\frac{\partial L}{\partial \alpha_L} - \frac{\partial L}{\partial \gamma_L} = (1 - \phi) \lambda_B^* \left(1 - \frac{P_B(I_0^*)}{P_G(I_0^*)} \right) - \lambda_{ICH}^* \left(1 - \frac{P_H(I_L^*)}{P_L(I_L^*)} \right).$$

Given that $\frac{P_B(I_0^*)}{P_G(I_0^*)} < 1 < \frac{P_H(I_L^*)}{P_L(I_L^*)}$, and $\frac{\partial L}{\partial \gamma_L} = 0$, it follows that $\frac{\partial L}{\partial \alpha_L} > 0$. Thus $\alpha_L^* = 1$. ■

Lemma A.14 *If $\lambda_{ICH}^* > 0$, then $\lambda_{ICL}^* = 0$.*

Proof. From Lemma A.13 we know that in equilibrium $\alpha_H^* \leq \alpha_L^* = 1$. Let $R := \underline{X}(1 - \alpha_H^*) + (I_H^* - I_L^*) > 0$. Because $\lambda_{ICH}^* > 0$, constraint (A-21) binds, that is, $\gamma_H^* P_G(I_0^*) P_H(I_H^*) \Delta X = R + \gamma_L^* P_G(I_0^*) P_H(I_L^*) \Delta X$. We can write constraint (A-22) as $R + \gamma_L^* P_G(I_0^*) P_L(I_L^*) \Delta X \geq \gamma_H^* P_G(I_0^*) P_L(I_H^*) \Delta X$. Then

$$\frac{\gamma_H^* P_G(I_0^*) P_H(I_H^*) \Delta X}{\gamma_H^* P_G(I_0^*) P_L(I_H^*) \Delta X} = \frac{P_H(I_H^*)}{P_L(I_H^*)} \geq \frac{R + \gamma_L^* P_G(I_0^*) P_H(I_L^*) \Delta X}{R + \gamma_L^* P_G(I_0^*) P_L(I_L^*) \Delta X}. \quad (\text{A-32})$$

We can establish our result by showing that the inequality (A-32) is strict. Let $g(R) = \frac{R + \gamma_L^* P_G(I_0^*) P_H(I_L^*) \Delta X}{R + \gamma_L^* P_G(I_0^*) P_L(I_L^*) \Delta X}$. Now note that $\frac{dg(R)}{dR} < 0$, i.e. decreasing in R . It follows then that $\frac{P_H(I_L^*)}{P_L(I_L^*)} > \frac{R + \gamma_L^* P_G(I_0^*) P_H(I_L^*) \Delta X}{R + \gamma_L^* P_G(I_0^*) P_L(I_L^*) \Delta X}$. The proof is concluded by noting that, by assumption (2), $\frac{P_H(I_H^*)}{P_L(I_H^*)} > \frac{P_H(I_L^*)}{P_L(I_L^*)} > \frac{R + \gamma_L^* P_G(I_0^*) P_H(I_L^*) \Delta X}{R + \gamma_L^* P_G(I_0^*) P_L(I_L^*) \Delta X}$. ■

Lemma A.15 *If $\lambda_{ICH}^* > 0$, then there exists a solution such that $\alpha_H^* = 1$.*

Proof. From FOCs (A-23) and (A-25), we have in equilibrium

$$\frac{\partial L}{\partial \alpha_H} - \frac{\partial L}{\partial \gamma_H} = \phi \lambda_B^* \left(1 - \frac{P_B(I_0^*)}{P_G(I_0^*)} \right) - \lambda_{ICL}^* \left(1 - \frac{P_L(I_H^*)}{P_H(I_H^*)} \right).$$

From Lemma A.14, $\lambda_{ICL}^* = 0$. Given that $\frac{P_B(I_0^*)}{P_G(I_0^*)} < 1$ and $\frac{\partial L}{\partial \gamma_H} = 0$, it follows that $\frac{\partial L}{\partial \alpha_H} \geq 0$. If $\lambda_B^* > 0$, the inequality is strict and $\alpha_H^* = 1$. If $\lambda_B^* = 0$, α_H can be chosen as any number between 0 and 1 and there exists one equilibrium such that $\alpha_H^* = 1$. ■

Lemma A.16 *If $1 - \lambda_B^* \frac{P_B(I_0^*)}{P_G(I_0^*)} - \lambda_{VC0}^* > 0$, then $\lambda_{L1}^* > 0$.*

Proof. Follows directly from Lemmas A.13 and A.14 and

$$\frac{\partial L}{\partial \gamma_L} = -(1 - \phi) \left(1 - \lambda_B^* \frac{P_B(I_0^*)}{P_G(I_0^*)} - \lambda_{VC0}^* \right) + \lambda_{L1}^* - \lambda_{ICH}^* \frac{P_H(I_L^*)}{P_L(I_L^*)} + \lambda_{ICL}^* = 0. \quad (\text{A-33})$$

■

Lemma A.17 (i) $P_G(I_0^*) P'_H(I_H^*) \Delta X - 1 = 0$, (ii) $I_H^* = I_H^{CPO}(I_0^*)$.

Proof. Note that $\frac{\partial L}{\partial I_H} = \phi \left(1 - \lambda_B \frac{P_B(I_0^*)}{P_G(I_0^*)} \right) [P_G(I_0^*) P'_H(I_H^*) \Delta X - 1] = 0$ given the assumption (A-28). Further given $\frac{\partial L}{\partial \gamma_H} = -\phi \left(1 - \lambda_B \frac{P_B(I_0^*)}{P_G(I_0^*)} - \lambda_{VC0} \right) + \lambda_{ICH} = 0$ and $\lambda_{ICH} \geq 0$, it must be the case that $1 - \lambda_B \frac{P_B(I_0^*)}{P_G(I_0^*)} > 0$. It follows then that $P_G(I_0^*) P'_H(I_H^*) \Delta X - 1 = 0$.

Now note that by definition, $I_H^{CPO}(I_0^*)$, the constrained Pareto optimal satisfies the condition

$$P_G(I_0^*) P'_H(I_H^{CPO}(I_0^*)) \Delta X - 1 = 0.$$

The second claim follows directly by comparing this expression with the previous result.

■

Lemma A.18 (i) $P_G(I_0^*) P'_L(I_L^*) \Delta X - 1 > 0$. (ii) $I_L^* < I_L^{CPO}(I_0^*)$.

Proof. First note that, given FOC (A-29)

$$\begin{aligned} \frac{\partial L}{\partial I_L} &= (1 - \phi) \left(1 - \lambda_B \frac{P_B(I_0^*)}{P_G(I_0^*)} \right) [P_G(I_0^*) P'_L(I_L^*) \Delta X - 1] \\ &\quad + \lambda_{ICH} \left[\left(1 - \frac{P_H(I_L^*)}{P_L(I_L^*)} \right) + \gamma_L^* P_G(I_0^*) P'_L(I_L^*) \Delta X \left(\frac{P_H(I_L^*)}{P_L(I_L^*)} - \frac{P'_H(I_L^*)}{P'_L(I_L^*)} \right) \right] = 0. \end{aligned}$$

Next note that, by assumption, $\left(1 - \frac{P_H(I_L^*)}{P_L(I_L^*)} \right) < 0$ and $\left(\frac{P_H(I_L^*)}{P_L(I_L^*)} - \frac{P'_H(I_L^*)}{P'_L(I_L^*)} \right) < 0$. It follows that the first order condition can only be satisfied if

$$(1 - \phi) \left(1 - \lambda_B \frac{P_B(I_0^*)}{P_G(I_0^*)} \right) [P_G(I_0^*) P'_L(I_L^*) \Delta X - 1] > 0.$$

From the argument used in the previous result, we know that $\left(1 - \lambda_B \frac{P_B(I_0^*)}{P_G(I_0^*)} \right) > 0$. Thus, the necessary condition for a solution can only be satisfied if $P_G(I_0^*) P'_L(I_L^*) \Delta X - 1 > 0$.

Now note that by definition, $I_L^{CPO}(I_0^*)$, the constrained Pareto optimal satisfies the condition

$$P_G(I_0^*) P'_L(I_L^{CPO}(I_0^*)) \Delta X - 1 = 0.$$

The second claim follows directly from the previous result and by the assumption $P''_L < 0$.

■

Lemma A.19 (i) $\phi P'_G(I_0^*) P_H(I_H^*) \Delta X + (1 - \phi) P'_G(I_0^*) P_L(I_L^*) \Delta X - 1 \leq 0$, (ii) $I_0^* \geq I_0^{CPO}(I_H^*, I_L^*)$.

Proof. First note that, given assumption (A-27)

$$\begin{aligned} \frac{\partial L}{\partial I_0} &= \left(1 - \lambda_B \frac{P'_B(I_0^*)}{P'_G(I_0^*)}\right) [\phi P'_G(I_0^*) P_H(I_H^*) \Delta X + (1 - \phi) P'_G(I_0^*) P_L(I_L^*) \Delta X - 1] \\ &- \lambda_B \left(\frac{P_B(I_0^*)}{P_G(I_0^*)} - \frac{P'_B(I_0^*)}{P'_G(I_0^*)}\right) [\phi \gamma_H^* P'_G(I_0^*) P_H(I_H^*) \Delta X + (1 - \phi) \gamma_L^* P'_G(I_0^*) P_L(I_L^*) \Delta X - 1] \\ &\quad + \lambda_{L1} + \lambda_{ICH} \left(1 - \frac{P_H(I_L^*)}{P_L(I_L^*)}\right) = 0. \end{aligned}$$

From $\frac{\partial L}{\partial I_H} = 0$ we get that $\phi \left(1 - \lambda_B \frac{P_B(I_0^*)}{P_G(I_0^*)} - \lambda_{VC0}\right) = \lambda_{ICH}$. Now substitute that expression into $\frac{\partial L}{\partial I_0} = 0$ to get $-\left(1 - \lambda_B \frac{P_B(I_0^*)}{P_G(I_0^*)} - \lambda_{VC0}\right) + \lambda_{L1} + \lambda_{ICH} \left(1 - \frac{P_H(I_L^*)}{P_L(I_L^*)}\right) = 0$. Since from earlier results we know that $1 - \lambda_B \frac{P_B(I_0^*)}{P_G(I_0^*)} - \lambda_{VC0} > 0$ it follows that $\lambda_{L1} + \lambda_{ICH} \left(1 - \frac{P_H(I_L^*)}{P_L(I_L^*)}\right) > 0$. It follows that

$$\begin{aligned} &\left(1 - \lambda_B \frac{P'_B(I_0^*)}{P'_G(I_0^*)}\right) [\phi P'_G(I_0^*) P_H(I_H^*) \Delta X + (1 - \phi) P'_G(I_0^*) P_L(I_L^*) \Delta X - 1] \\ &- \lambda_B \left(\frac{P_B(I_0^*)}{P_G(I_0^*)} - \frac{P'_B(I_0^*)}{P'_G(I_0^*)}\right) [\phi \gamma_H^* P'_G(I_0^*) P_H(I_H^*) \Delta X + (1 - \phi) \gamma_L^* P'_G(I_0^*) P_L(I_L^*) \Delta X - 1] < 0 \end{aligned}$$

is a necessary condition for equilibrium. Now we establish by means of a contradiction that $\phi P'_G(I_0^*) P_H(I_H^*) \Delta X + (1 - \phi) P'_G(I_0^*) P_L(I_L^*) \Delta X - 1 < 0$. First assume that $\phi P'_G(I_0^*) P_H(I_H^*) \Delta X +$

$(1 - \phi) P'_G(I_0^*) P_L(I_L^*) \Delta X - 1 > 0$. Given $1 - \lambda_B \frac{P_B(I_0^*)}{P_G(I_0^*)} > 0$, it must be the case that $1 - \lambda_B \frac{P_B(I_0^*)}{P'_G(I_0^*)} > \lambda_B \left(\frac{P_B(I_0^*)}{P_G(I_0^*)} - \frac{P'_B(I_0^*)}{P'_G(I_0^*)} \right)$. It follows then that, given our assumption

$$\begin{aligned}
& \left(1 - \lambda_B \frac{P'_B(I_0^*)}{P'_G(I_0^*)} \right) [\phi P'_G(I_0^*) P_H(I_H^*) \Delta X + (1 - \phi) P'_G(I_0^*) P_L(I_L^*) \Delta X - 1] \\
& - \lambda_B \left(\frac{P_B(I_0^*)}{P_G(I_0^*)} - \frac{P'_B(I_0^*)}{P'_G(I_0^*)} \right) [\phi \gamma_H^* P'_G(I_0^*) P_H(I_H^*) \Delta X + (1 - \phi) \gamma_L^* P'_G(I_0^*) P_L(I_L^*) \Delta X - 1] \\
& > \lambda_B \left(\frac{P_B(I_0^*)}{P_G(I_0^*)} - \frac{P'_B(I_0^*)}{P'_G(I_0^*)} \right) [\phi P'_G(I_0^*) P_H(I_H^*) \Delta X + (1 - \phi) P'_G(I_0^*) P_L(I_L^*) \Delta X - 1] \\
& - \lambda_B \left(\frac{P_B(I_0^*)}{P_G(I_0^*)} - \frac{P'_B(I_0^*)}{P'_G(I_0^*)} \right) [\phi \gamma_H^* P'_G(I_0^*) P_H(I_H^*) \Delta X + (1 - \phi) \gamma_L^* P'_G(I_0^*) P_L(I_L^*) \Delta X - 1] \\
& = \lambda_B \left(\frac{P_B(I_0^*)}{P_G(I_0^*)} - \frac{P'_B(I_0^*)}{P'_G(I_0^*)} \right) [\phi (1 - \gamma_H^*) P'_G(I_0^*) P_H(I_H^*) \Delta X + (1 - \phi) (1 - \gamma_L^*) P'_G(I_0^*) P_L(I_L^*) \Delta X] > 0.
\end{aligned}$$

This is the desired contradiction.

Next note that by definition, $I_0^{CPO}(I_H^*, I_L^*)$, the constrained Pareto optimal satisfies the condition

$$\phi P'_G(I_0^*) P_H(I_H^*) \Delta X + (1 - \phi) P'_G(I_0^*) P_L(I_L^*) \Delta X - 1 = 0$$

The second claim follows directly from the previous result, and note that by assumption $P''_G < 0$. ■

Lemma A.20 *If $\lambda_{L1}^* > 0$ and $\lambda_{ICH}^* > 0$, then constraints (8), (9) and (10) do not bind.*

Proof. If $\lambda_{L1}^* > 0$, then constraint (10) does not bind because investments are assumed to be positive NPV investments. Since $I_H^* = I_H^{PO}$, $U_H^G(\alpha_H^*, \gamma_H^*, I_0^*, I_{1,H}^*) = U_H^G(\alpha_H^*, \gamma_H^*, I_0^*, I_{1,H}^{PO})$. Now decompose

$$\begin{aligned}
U_H^G(\alpha_H^*, \gamma_H^*, I_0^*, I_{1,H}^{PO}) &= \underline{X} + P_G(I_0^*) P_H(I_{1,H}^{PO}) \Delta X - V_H^G(\alpha_H^*, \gamma_H^*, I_0^*, I_{1,H}^{PO}) \\
&= P_G(I_0^*) \Delta X [P_H(I_{1,H}^{PO}) - P_H(I_{1,L}^*)] + \underline{X} + P_G(I_0^*) P_H(I_{1,L}^*) \Delta X - V_H^G(\alpha_H^*, \gamma_H^*, I_0^*, I_{1,H}^{PO}).
\end{aligned}$$

By applying $V_H^G(\alpha_H^*, \gamma_H^*, I_0^*, I_{1,H}^{PO}) = V_H^G(\alpha_L^*, \gamma_L^*, I_0^*, I_{1,L}^*) - (I_{1,H}^{PO} - I_L^*)$, we are able to state that

$$U_H^G(\alpha_H^*, \gamma_H^*, I_0^*, I_{1,H}^{PO}) = P_G(I_0^*) \Delta X [P_H(I_{1,H}^{PO}) - P_H(I_{1,L}^*)] - (I_{1,H}^{PO} - I_L^*) + U_L^G(\alpha_L^*, \gamma_L^*, I_0^*, I_{1,L}^*) > w,$$

which proves that constraint (9) does not bind. Finally, because constraints (9) and (10) do not bind, constraint (8) does not bind. ■

Lemma A.21 Let $(I_0^*, I_H^*, I_L^*, \alpha_H^*, \gamma_H^*, \alpha_L^*, \gamma_L^*)$ be the solution to problem (23). This solution is implementable if the entrepreneur proposes (I_0^*, a, b) , ($a \leq \min(\alpha_H^*, \alpha_L^*)$ and $b \leq \min(\gamma_H^*, \gamma_L^*)$), as the initial contract, and $(I_H^*, \alpha_H^*, \gamma_H^*)$ and $(I_L^*, \alpha_L^*, \gamma_L^*)$ as the menu of possible time 1 contracts after observing G .

Proof. This proof is virtually identical to that of Lemma A.9. except for the obvious modifications to account for the shift in bargaining power. ■

Proof of Lemma 4, Proposition 5, and Proposition 6. Based on Lemma A.20, we show that the entrepreneur's maximization problem (A-17) is equivalent to the maximization problem (23). Thus, Lemma 4 is proved by Lemma A.12. Proposition 5 is proved by Lemmas A.13 and A.15. Proposition 6 is proved by Lemmas A.17, A.18, and A.19. ■

Lemma A.22 When the venture capitalist has the bargaining advantage, there does not exist an equilibrium $(I_0^*, \alpha_H^*, \gamma_H^*, I_{1,H}^*, \alpha_L^*, \gamma_L^*, I_{1,L}^*)$, where $I_{1,H}^* < I_{1,L}^*$.

Proof. Let the constrained Pareto optimal levels of investment $I_H^{CPO}(I_0)$ and $I_L^{CPO}(I_0)$ be defined as the solutions to the following two equations, respectively:

$$P_0(I_0) P_H' \left(I_H^{CPO} \right) \Delta X - 1 = 0 \quad (\text{A-34})$$

$$P_0(I_0) P_L' \left(I_L^{CPO} \right) \Delta X - 1 = 0. \quad (\text{A-35})$$

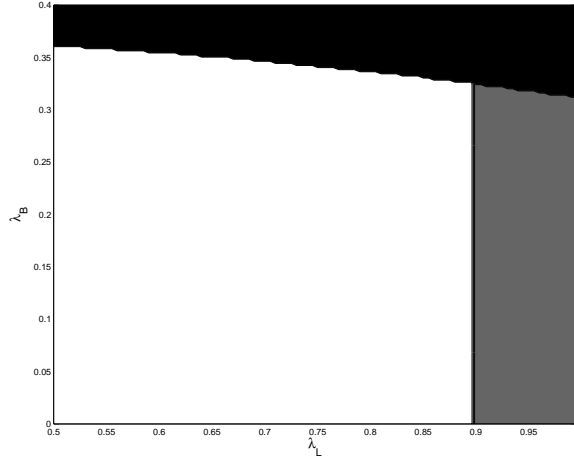
From assumption (2), we can show that $I_L^{CPO} < I_H^{CPO}$. Suppose there exists an equilibrium where $I_{1,H}^* < I_{1,L}^*$. If $I_{1,L}^* \leq I_H^{CPO}$, consider the following solution $(I_0^{**}, \alpha_H^{**}, \gamma_H^{**}, I_{1,H}^{**}, \alpha_L^{**}, \gamma_L^{**}, I_{1,L}^{**})$ where $I_{1,H}^{**} = I_{1,L}^{**} = I_{1,L}^*$, $\alpha_H^{**} = \alpha_L^{**} = \alpha$, and $\gamma_H^{**} = \gamma_L^{**} = \gamma$. In addition, choose α and γ such that the expected payoff to the entrepreneur in this new solution remains the same. Because the contracts in state H and L are the same, the venture capitalist's information revelation constraints are satisfied in the new solution. The entrepreneur's participation constraints are satisfied because she receives the same payoff. The venture capitalist's payoff is higher in this solution because $I_{1,H}^{**}$ is closer to I_H^{CPO} while other investment levels remain the same. This is the desired contradiction. If $I_{1,L}^* > I_H^{CPO}$, we can construct the new solution in a similar way except that $I_{1,H}^{**} = I_{1,L}^{**} = I_H^{CPO}$. The venture capitalist's payoff is higher because $I_{1,H}^{**} = I_H^{CPO}$ and $I_{1,L}^{**}$ is closer to I_L^{CPO} . Again, this new solution contradicts the equilibrium assumption. ■

Proof of Proposition 7. Let $(I_0^*, \alpha_H^*, \gamma_H^*, I_{1,H}^*, \alpha_L^*, \gamma_L^*, I_{1,L}^*)$ be the equilibrium where both type G and B entrepreneurs have their projects financed. Then this solution must satisfy the venture capitalist's truth telling constraints:

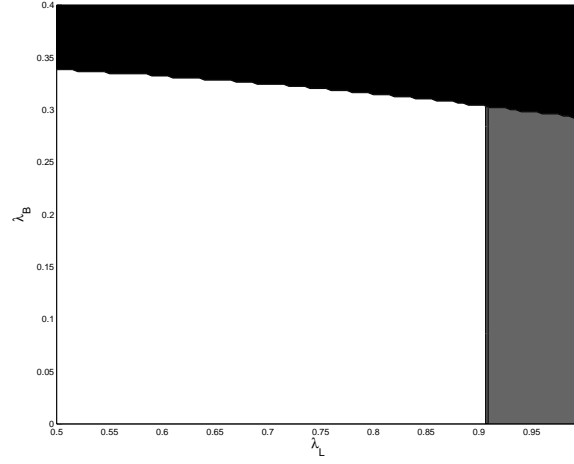
$$\alpha_H^* \underline{X} + \gamma_H^* P_0(I_0^*) P_H(I_{1,H}^*) \Delta X - I_{1,H}^* \geq \alpha_L^* \underline{X} + \gamma_L^* P_0(I_0^*) P_H(I_{1,L}^*) \Delta X - I_{1,L}^* \quad (\text{A-36})$$

$$\alpha_L^* \underline{X} + \gamma_L^* P_0(I_0^*) P_L(I_{1,L}^*) \Delta X - I_{1,L}^* \geq \alpha_H^* \underline{X} + \gamma_H^* P_0(I_0^*) P_L(I_{1,H}^*) \Delta X - I_{1,H}^*. \quad (\text{A-37})$$

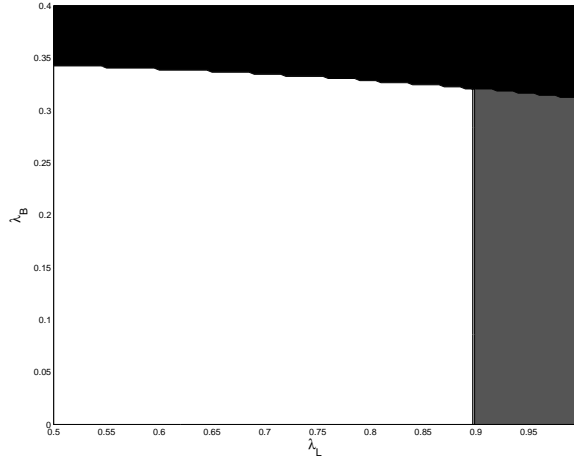
Consider a solution that is the same as the assumed equilibrium except that $\gamma_L^{**} = \gamma_L^* + \varepsilon$ and $\gamma_H^{**} = \gamma_H^* + \varepsilon \frac{P_L(I_L^*)}{P_L(I_H^*)}$, $\varepsilon > 0$. Because the same value of $\varepsilon P_0(I_0^*) P_L(I_L^*)$ is added to both sides of inequality A-37 if we substitute the new solution, this constraint is satisfied. If we plug the new solution into inequality A-36, the left-hand side adds $\varepsilon P_0(I_0^*) \frac{P_L(I_L^*)}{P_L(I_H^*)} P_H(I_H^*)$, and the right-hand side adds $\varepsilon P_0(I_0^*) P_H(I_L^*)$. It can be shown that $\frac{P_L(I_L^*)}{P_L(I_H^*)} P_H(I_H^*) > P_H(I_L^*)$ because $z(I) = \frac{P_L(I)}{P_H(I)}$ is strictly decreasing in I and $I_H^* > I_L^*$ from Lemma A.22. Hence the new solution satisfies both truth telling constraints of the venture capitalist. If the entrepreneur of type B is given more than her reservation wage in the assumed equilibrium, by choosing ε small enough, this new solution can increase the venture capitalist's payoff and satisfy the entrepreneur's participation constraints. This solution contradicts the assumption of the equilibrium. If the entrepreneur of type B is given her reservation wage in the assumed equilibrium, the payoff of the entrepreneur of type G must be strictly higher than her reservation wage. In the new solution, ε can be chosen such that type G entrepreneur's participation constraint is still satisfied while type B entrepreneur's participation constraint is not satisfied. Then the expected payoff of the venture capitalist in the new solution is even higher because the expected project payoff is higher if only a type G entrepreneur is financed. This result contradicts the equilibrium assumption. ■



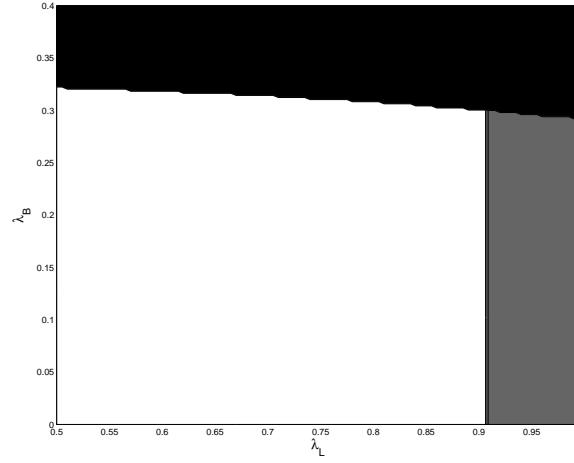
(a) $\phi = 0.3, \Delta X = 300$



(b) $\phi = 0.3, \Delta X = 320$

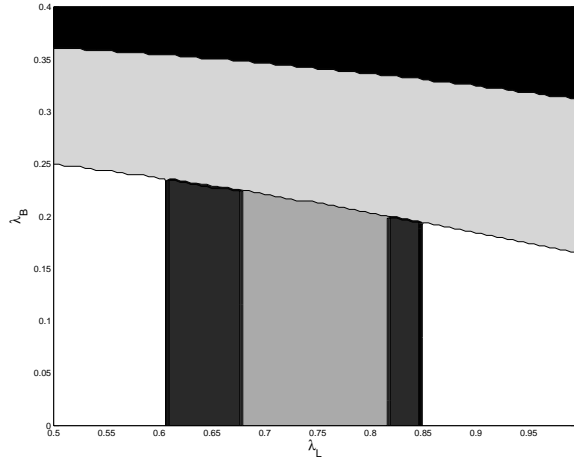


(c) $\phi = 0.5, \Delta X = 300$

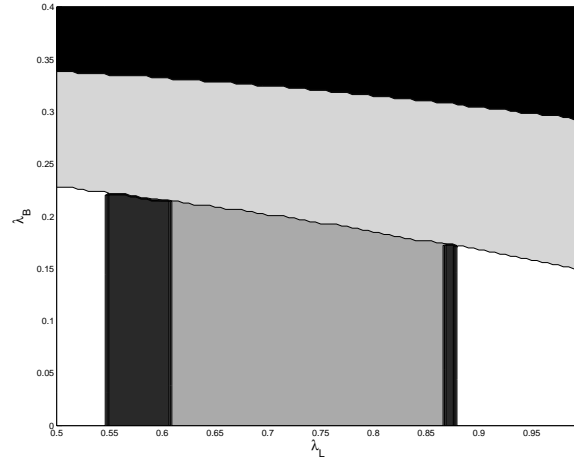


(d) $\phi = 0.5, \Delta X = 320$

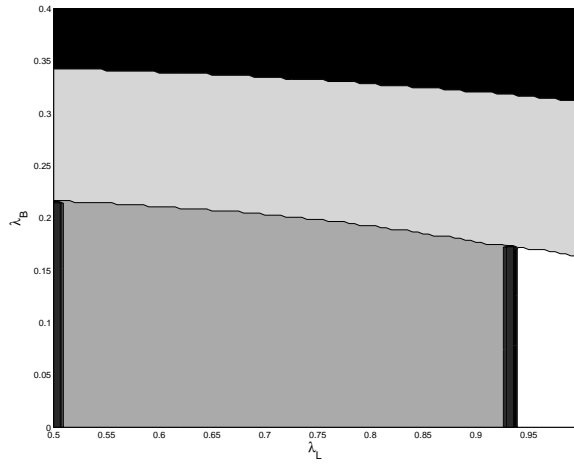
Figure 1: Investment distortion from Pareto optimal levels when the venture capitalist has the bargaining power. The three regions from the darkest to the lightest are: (1) negative NPV in state B not satisfied, (2) overinvesting in state H at time 1, and (3) no investment distortion. In this figure, we assign the following values to the variables: $\underline{X} = 0.1$ and $w = 11$. Further, we assume that $P_G(I_0) = 1 - e^{-5I_0}$, $P_B(I_0) = \lambda_B(1 - e^{-10I_0})$, $P_H(I_1) = \frac{1}{10}I_1^{0.35}$, and $P_L(I_1) = \frac{1}{10}I_1^{0.35\lambda_L}$.



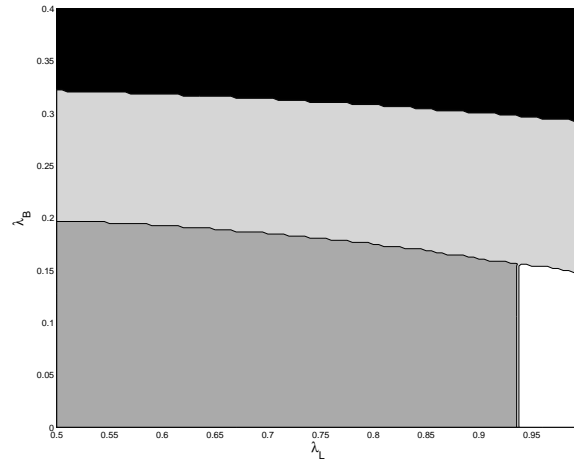
(a) $\phi = 0.3, \Delta X = 300$



(b) $\phi = 0.3, \Delta X = 320$

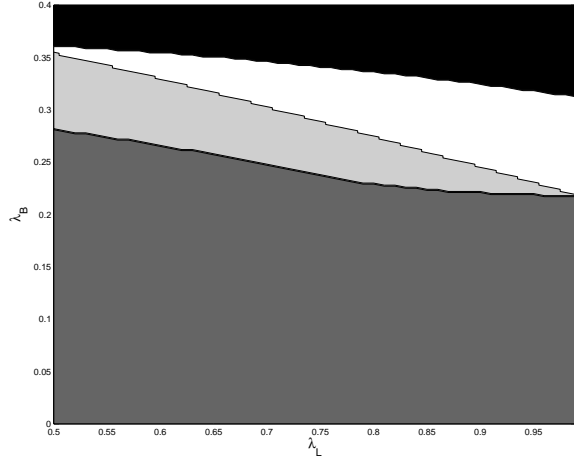


(c) $\phi = 0.5, \Delta X = 300$

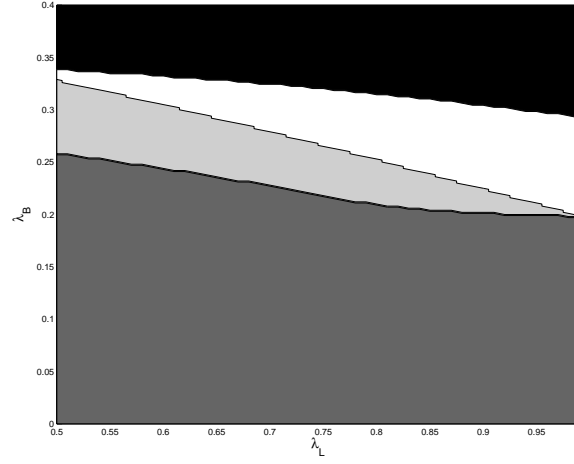


(d) $\phi = 0.5, \Delta X = 320$

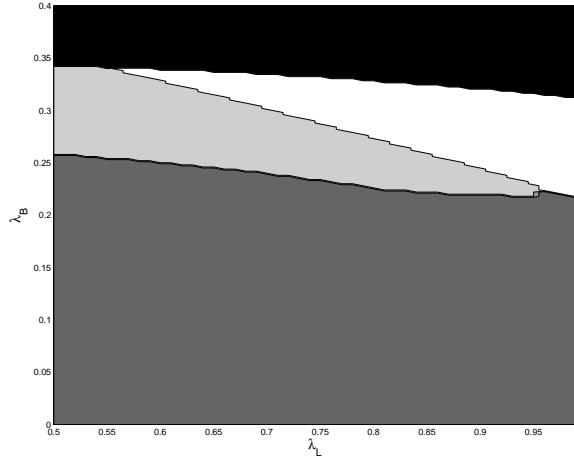
Figure 2: Investment distortion from Pareto optimal levels when the entrepreneur has the bargaining power. The five regions from the darkest to the lightest are: (1) negative NPV in state B not satisfied, (2) underinvesting in state L at time 1, (3) overinvesting at time 0 and underinvesting in state L at time 1, (4) overinvesting at time 0, and (5) no investment distortion. In this figure, we assign the following values to the variables: $\underline{X} = 0.1$ and $w = 11$. Further, we assume that $P_G(I_0) = 1 - e^{-5I_0}$, $P_B(I_0) = \lambda_B(1 - e^{-10I_0})$, $P_H(I_1) = \frac{1}{10}I_1^{0.35}$, and $P_L(I_1) = \frac{1}{10}I_1^{0.35}\lambda_L$.



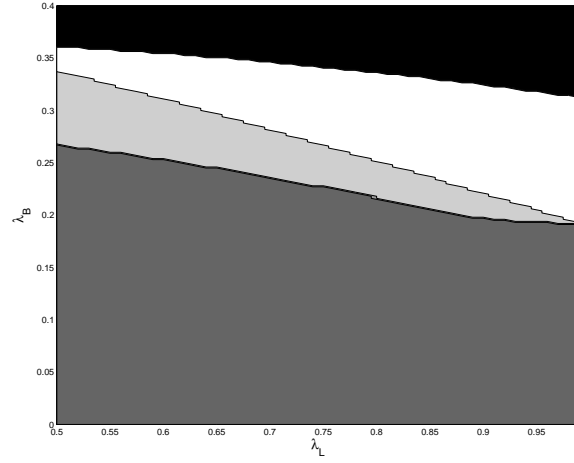
(a) $\phi = 0.3, \Delta X = 300, \pi = 0.5$



(b) $\phi = 0.3, \Delta X = 320, \pi = 0.5$

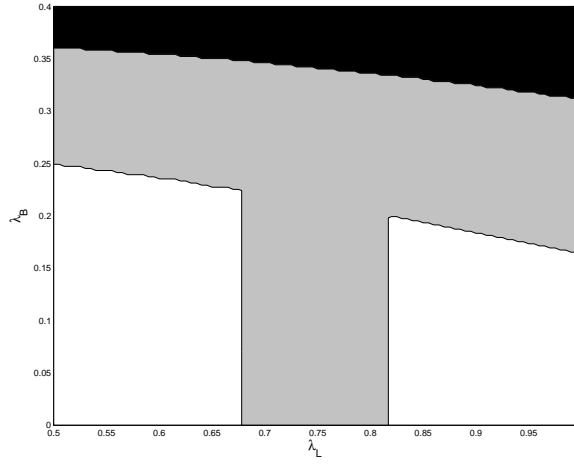


(c) $\phi = 0.5, \Delta X = 300, \pi = 0.5$

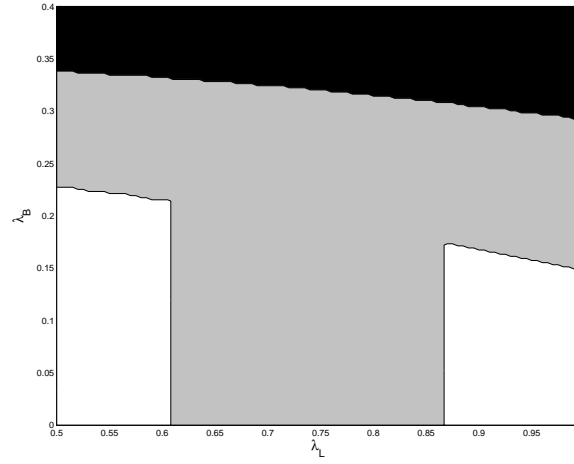


(d) $\phi = 0.3, \Delta X = 300, \pi = 0.7$

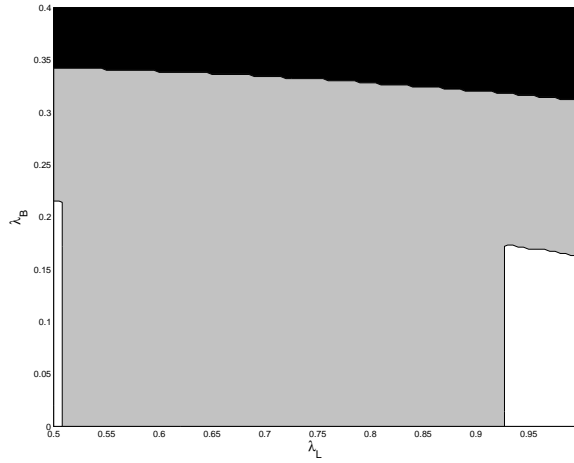
Figure 3: Equilibrium when the entrepreneur has the bargaining power. The four regions from the darkest to the lightest are: (1) negative NPV in state B not satisfied, (2) separating equilibrium, (3) pooling with B type project being shut down at L, and (4) pooling with no shutdown. In this figure, we assign the following values to the variables: $\underline{X} = 0.1$ and $w = 11$. Further, we assume that $P_G(I_0) = 1 - e^{-5I_0}$, $P_B(I_0) = \lambda_B(1 - e^{-10I_0})$, $P_H(I_1) = \frac{1}{10}I_1^{0.35}$, and $P_L(I_1) = \frac{1}{10}I_1^{0.35\lambda_L}$.



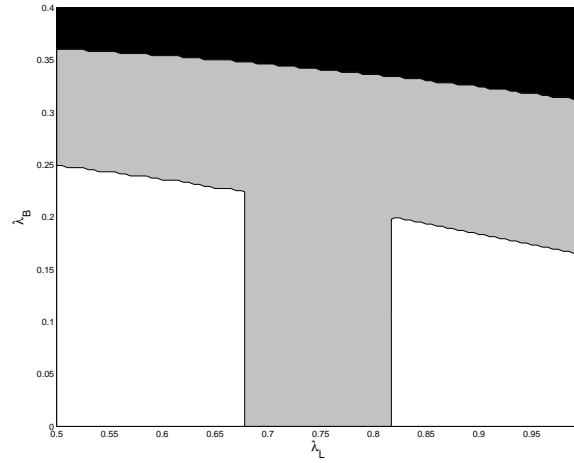
(a) $\phi = 0.3, \Delta X = 300, \pi = 0.5$



(b) $\phi = 0.3, \Delta X = 320, \pi = 0.5$

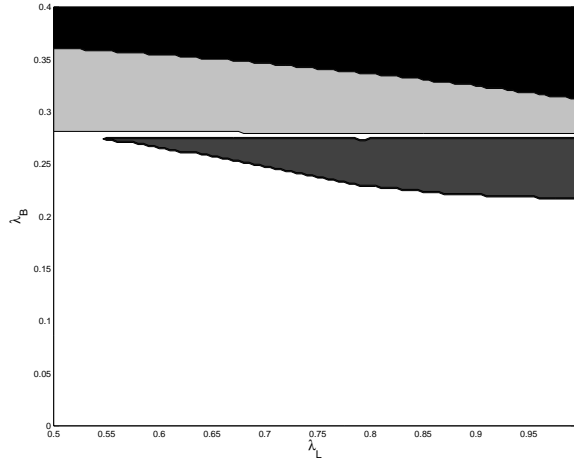


(c) $\phi = 0.5, \Delta X = 300, \pi = 0.5$

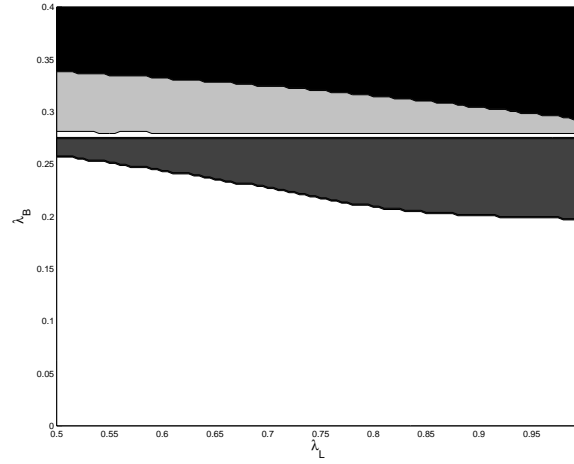


(d) $\phi = 0.3, \Delta X = 300, \pi = 0.7$

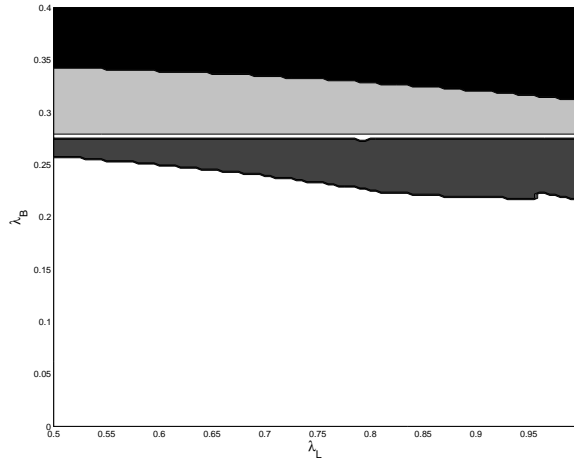
Figure 4: Investment distortion at time 0 from Pareto optimal levels when the entrepreneur has the bargaining power and there may be pooling equilibria. The three regions from the darkest to the lightest are: (1) negative NPV in state B not satisfied, (2) overinvesting, and (3) no investment distortion. In this figure, we assign the following values to the variables: $\underline{X} = 0.1$ and $w = 11$. Further, we assume that $P_G(I_0) = 1 - e^{-5I_0}$, $P_B(I_0) = \lambda_B(1 - e^{-10I_0})$, $P_H(I_1) = \frac{1}{10}I_1^{0.35}$, and $P_L(I_1) = \frac{1}{10}I_1^{0.35\lambda_L}$.



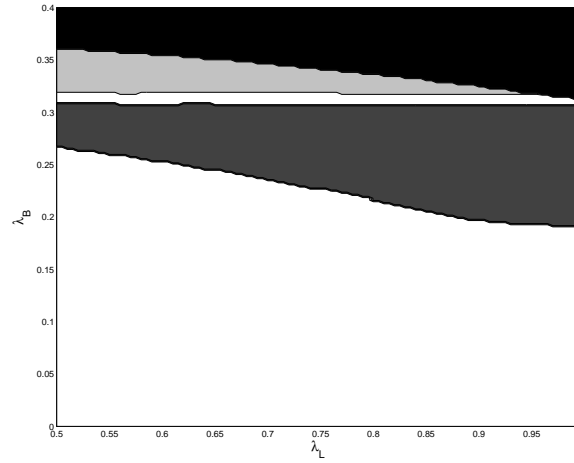
(a) $\phi = 0.3, \Delta X = 300, \pi = 0.5$



(b) $\phi = 0.3, \Delta X = 320, \pi = 0.5$

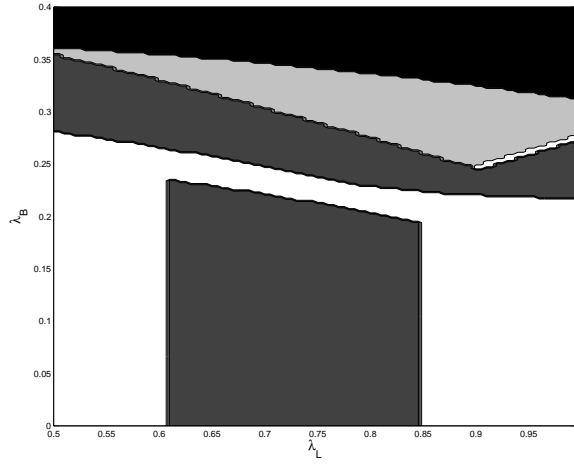


(c) $\phi = 0.5, \Delta X = 300, \pi = 0.5$

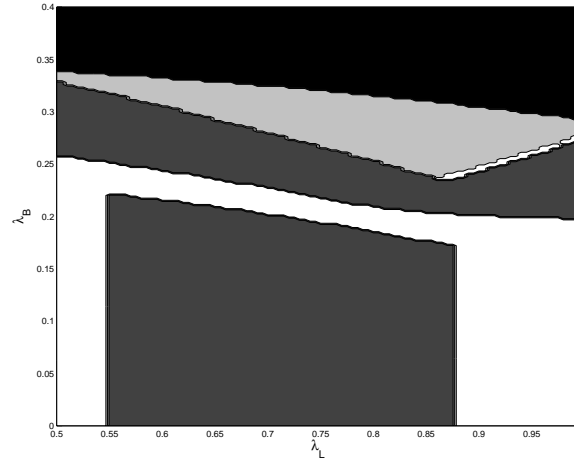


(d) $\phi = 0.3, \Delta X = 300, \pi = 0.7$

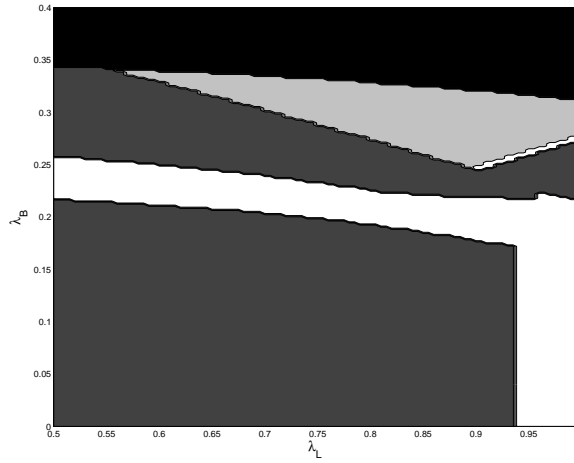
Figure 5: Investment distortion for state H at time 1 from Pareto optimal levels when the entrepreneur has the bargaining power and there may be pooling equilibria. The four regions from the darkest to the lightest are: (1) negative NPV in state B not satisfied, (2) underinvesting, (3) overinvesting, and (4) no investment distortion. In this figure, we assign the following values to the variables: $\underline{X} = 0.1$ and $w = 11$. Further, we assume that $P_G(I_0) = 1 - e^{-5I_0}$, $P_B(I_0) = \lambda_B(1 - e^{-10I_0})$, $P_H(I_1) = \frac{1}{10}I_1^{0.35}$, and $P_L(I_1) = \frac{1}{10}I_1^{0.35\lambda_L}$.



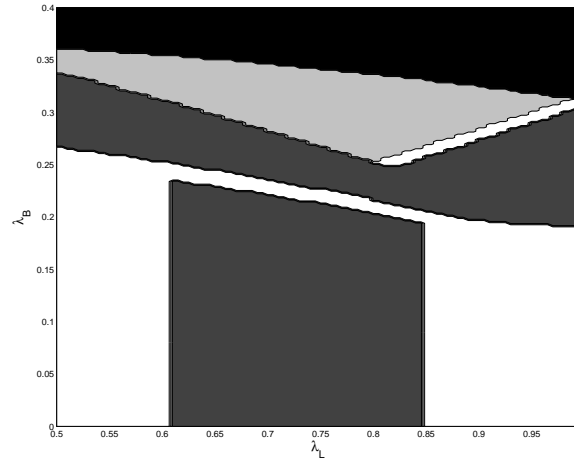
(a) $\phi = 0.3, \Delta X = 300, \pi = 0.5$



(b) $\phi = 0.3, \Delta X = 320, \pi = 0.5$



(c) $\phi = 0.5, \Delta X = 300, \pi = 0.5$



(d) $\phi = 0.3, \Delta X = 300, \pi = 0.7$

Figure 6: Investment distortion for state L at time 0 from Pareto optimal levels when the entrepreneur has the bargaining power and there may be pooling equilibria. The four regions from the darkest to the lightest are: (1) negative NPV in state B not satisfied, (2) underinvesting, (3) overinvesting, and (4) no investment distortion. In this figure, we assign the following values to the variables: $\underline{X} = 0.1$ and $w = 11$. Further, we assume that $P_G(I_0) = 1 - e^{-5I_0}$, $P_B(I_0) = \lambda_B(1 - e^{-10I_0})$, $P_H(I_1) = \frac{1}{10}I_1^{0.35}$, and $P_L(I_1) = \frac{1}{10}I_1^{0.35\lambda_L}$.