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RATE REGIMES FOR STABILIZATION  
POLICY**

Bernardino Adao, Isabel Correia  
and Pedro Teles

***INTERNATIONAL MACROECONOMICS***



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# ON THE RELEVANCE OF EXCHANGE RATE REGIMES FOR STABILIZATION POLICY

**Bernardino Adao**, Banco de Portugal

**Isabel Correia**, Banco de Portugal and Universidade Catolica Portuguesa and  
CEPR

**Pedro Teles**, Banco de Portugal and Universidade Catolica Portuguesa and CEPR

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Centre for Economic Policy Research

90–98 Goswell Rd, London EC1V 7RR, UK

Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999

Email: [cepr@cepr.org](mailto:cepr@cepr.org), Website: [www.cepr.org](http://www.cepr.org)

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## ABSTRACT

### On the Relevance of Exchange Rate Regimes for Stabilization Policy

This paper assesses the relevance of the exchange rate regime for stabilization policy. This regime question cannot be dealt with independently of other institutions, in particular how fiscal policy is designed. We show that once fiscal policy is taken into account, the exchange rate regime is irrelevant. This is the case independently of the severity of price rigidities, independently of asymmetries across countries in shocks and transmission mechanisms and regardless of the incompleteness of international financial markets. The only relevant condition is labour mobility. The immobility of labour across countries is a necessary condition for our results.

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Bernardino Adao  
Research Department  
Banco de Portugal  
Av Almirante Reis 71  
1150 Lisboa  
PORTUGAL  
Email: badao@bportugal.pt

Isabel Correia  
Departamento de Estudos  
Económicos,  
Banco de Portugal  
R. Francisco Ribeiro  
P-1100 Lisboa  
PORTUGAL  
Email:  
Maria.Isabel.Horta.Carvalho@bportugal.pt

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Pedro Teles  
Departamento de Estudos  
Económicos,  
Banco de Portugal  
R. Francisco Ribeiro  
P-1100 Lisboa  
PORTUGAL  
Email: pteles@fcee.ucp.pt

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## I. Introduction

This paper revisits the issues in the optimal currency area literature, as in Mundell (1961) and a more recent literature, on the optimal choice of an exchange rate regime. What are the costs of a fixed exchange rate regime when there is a role for stabilization policy? We address this question allowing for heterogeneity in the shocks and the response to them, restrictions on the mobility of factors and incompleteness of asset markets, as is standard in the optimal currency area literature. The main difference of our approach is that we take into account that fiscal instruments may be used fully for stabilization policy, in the absence of independent monetary.

When different shocks hit different countries or when there are differences across countries in the effects of shocks, monetary policy, that has a stabilization role because of some form of nominal rigidity, may have to react differently in the different countries. Because of this heterogeneity it is common to infer that there are costs of coordinating monetary policies, either through a fixed exchange rate regime or a monetary union. In the literature, these costs are taken to be higher the stronger are the asymmetries, the more severe are the nominal rigidities, the more pronounced is the incompleteness of international asset markets, the less mobile is labor, and, finally, the less able is fiscal policy in effectively stabilizing the national economies (Corsetti, 2005).

We show that when both fiscal and monetary policies are considered jointly with the same flexibility in response to shocks, the loss of the country specific monetary tool is of no cost. This is true irrespective of the asymmetry in shocks or response to these and the severity of the nominal rigidities. The elements that are crucial in assessing the costs of a single monetary policy are the two last ones in the list by Corsetti above, but labor mobility works in the opposite way to the conventional wisdom. Fiscal and monetary policy are able to eliminate the costs of a monetary union only if labor is not mobile across countries.

We consider a standard two country model. Each country specializes in the production of a composite tradeable good, which aggregates a continuum of goods produced using labor only. Labor is not mobile across countries. Money is used for transactions according to a cash-in-advance constraint on the purchases of the two composite goods by the households of each country. The government of each country must finance exogenous expenditures on the good produced at home with distortionary taxes and seigniorage. The tax instruments are standard state-contingent labor income and consumption taxes. There is state-contingent private debt inside each country in zero net supply and noncontingent nominal public debt in each currency that can be traded internationally.

We start by analyzing a benchmark economy where prices are flexible (sections 2 and 3). We show, in Section 3, that any equilibrium allocation in the flexible price, flexible exchange rates, economy can be implemented with fiscal and monetary policies that induce stable producer prices and fixed exchange rates. This result has implications for economies with nominal rigidities and fixed exchange rates (Section 4). For those policies, that under flexible prices keep prices constant, if firms were restricted in the setting of prices such as in Calvo (1983), those restrictions would be irrelevant and the same allocations could still be implemented. It follows that under sticky prices and fixed exchange rates it is always possible to achieve the same allocations as when prices are flexible and exchange rates are also flexible.

Under sticky prices there are allocations other than the ones achieved under flexible prices. It could still be the case that the allocation with the highest welfare would be amongst these. We show that this is not the case. Independently of the exchange rate regime, for each allocation that can be implemented under sticky prices there is one under flexible prices that can potentially improve welfare in both countries. The reason for this is that in order for sticky prices to be

relevant, because different firms face different price setting restrictions, firms that use the same technology and face the same demand conditions will charge different prices. This means that production will be inefficient and the inefficiency in production is not optimal in this second best environment. This result relates to the one in Diamond Mirrlees (1971) that shows that it is not optimal to tax intermediate goods when there are consumption taxes on the final goods.

When prices are sticky, one would think that flexible exchange rates would be useful in adjusting the relative price of goods to different shocks. Monetary policy would change with shocks so as to induce the response of the exchange rate that would affect directly the relative price of goods. In our model, because we allow for consumption taxes the relative price is the ratio of prices gross of consumption taxes, adjusted by the nominal exchange rate. When the exchange rate is fixed, consumption taxes have a direct effect on the relative price and change in response to shocks so that the necessary adjustments take place. Labor income taxes will also have to be adjusted so that other margins may not be affected. The change in fiscal policy is not automatic and would require a knowledge of the model and the shocks to be fully effective. But neither would the change in exchange rates that would be necessary to accomplish the same goal.

Exchange rate policy can play other roles such as completing the noncontingent international financial markets. Again, when the exchange rates are fixed, taxes, and interest rates common across countries, play that same role of allowing for the returns on assets traded internationally to vary with the shocks.

The first result that it is possible to implement the set of allocations under flexible prices, with constant prices and constant exchange rates is obtained in the model where labor is not mobile across countries. It does not hold in an alternative model where agents can work outside their country (Section 5). This implies that if prices are sticky there will be costs of restricting

monetary policy to be a single one for the different countries. As mentioned above this result goes against the standard intuition in the optimal currency area literature. The intuition for our result is simple. Labor mobility imposes additional equilibrium restrictions, in particular arbitrage conditions on the choice of where to work, that cannot be satisfied with the instruments that we consider. This of course does not imply that welfare with labor mobility is lower than without it.

Related literature reassesses Milton Friedman (1953)'s case for exchange rate flexibility, as a way of side-stepping the rigidity in relative price movements. Recent examples in the debate are, for instance, Devereux and Engel (2003) and Duarte and Obstfeld (2005).<sup>1</sup> Devereux and Engle (2003) provide an example with local currency pricing where exchange rate flexibility is of no use. Because the prices of goods are set in the currency of the consumers, the exchange rate cannot affect the relative price and the optimal allocation will be a third best, relative to the optimal allocation in our model where consumption taxes can be adjusted. Duarte and Obstfeld (2004) respond, showing that exchange rate flexibility can still be of use in a more complex environment with non tradeable goods. Even if exchange rate movements cannot affect the relative price of goods, they can still affect the allocations and improve welfare. Because the optimal exchange rate regime depends on the degree of exchange rate pass-through, it is important to allow for the decision on which currency prices are set in to be endogenous. Corsetti and Pesenti (2002) do this and show that there are two self validating regimes, one with fixed and another with flexible exchange rates. The flexible exchange rate regime provides higher welfare. We add to this debate by showing that the claims hinge on the focus on monetary policy only.<sup>2</sup> Once the choice of the exchange rate regime is considered in the context of the full choice of policy instruments including tax and debt policy, exchange rate flexibility can be replaced with a gain by fiscal instruments.

Cooper and Kempf (2004) make a similar point to ours in a very different context. They



explicitly model the Mundellian trade-off between the benefits of a monetary union in reducing transaction costs and the costs of the union in the ability to stabilize. Stabilization in their set up are risk sharing transfers between agents. If the government is able to stabilize using alternative fiscal instruments, then there are no costs of a monetary union.

In the same direction, but unable to establish the irrelevance results for the lack of fiscal instruments are papers such as Gali and Monacelli (2005) and Ferrero (2005). In Gali and Monacelli (2005) the government chooses the optimal level of public consumption in a monetary union with lump-sum taxes. Ferrero (2005) considers that lump-sum taxes are not available. The monetary policy instrument is the interest rate and the fiscal policy consists in choosing state-noncontingent public debt and state-contingent taxes on output by firms. In both papers it is established that the choice of fiscal policy helps attaining higher welfare. The major difference between our set up and the one of Ferrero (2005) is that we consider not only state-contingent labor income taxes but also consumption taxes.

Other related literature is on optimal fiscal and monetary policy in small open economies. Nicolini and Hevia (2004) consider a small open economy with flexible exchange rates and state-contingent assets. Prices are set in advance. In that set up the second best, flexible price equilibrium is implementable, but exchange rates must move across states. Benigno and Paoli (2004) is other related work. They also analyze optimal fiscal and policy in a small open economy, but in their set up the optimal flexible price allocation is not implementable.

Finally this paper also relates to Correia, Nicolini and Teles (2002) that analyzes the Ramsey problem in a closed economy with sticky prices. They show that environments with different sticky price restrictions are equivalent for stabilization purposes, when taxes can be used with the same flexibility as monetary policy.

One obvious criticism of our approach is that we do not observe flexible taxes as the model assumes. This could be relevant if our purpose was to assess the costs of a monetary union without further institutional reform, but that is not the purpose of this analysis. Our point is that we should not address the issue of the optimal monetary regime without allowing for alternative fiscal regimes. There could be fundamental reasons for taxes not to be state-contingent. But then, wouldn't those reasons also apply to monetary policy? And yet, we make no apologies for state-contingent monetary policies.

The paper proceeds as follows. In Section 2 we present a model with flexible prices and flexible exchange rates. In Section 3, we show that the set of implementable allocations in a flexible world where both prices and exchange rates are flexible can be implemented with constant producer price levels and fixed exchange rates. The result follows that neither sticky prices nor the exchange rate regime restrict the set of allocations. Furthermore, this set of allocations is optimal. Therefore, as argued in Section 4, there is no cost of fixed exchange rates, independently of the degree or type of price rigidity. Section 5 addresses the issue of labor mobility showing that with labor mobility the irrelevance results are not obtained.

## II. The Model

The economy has two countries of equal size, the home country and the foreign country. In each country there is a representative household, a continuum of firms and a government. Each firm produces a distinct, perishable consumption good with labor only. In each period  $t = 0, 1, \dots, T$ , where  $T$  can be made arbitrarily large,<sup>3</sup> the economy experiences one of finitely many events  $s_t$ . The initial realization  $s_0$  is given. The set of all possible events in period  $t$  is denoted by  $S_t$ , the history of these events up to and including period  $t$ , which we call state at  $t$ ,  $(s_0, s_1, \dots, s_t)$ , is

denoted by  $s^t$ , and the set of all possible states in period  $t$  is denoted by  $S^t$ . The number of all possible states in period  $t$  is  $\#S^t$ . All the relevant variables for this world economy are a function of the state,  $s^t$ , but to simplify the notation we do not index formally the variables to the state.

There are markets for goods, labor, money, state-contingent debt and state-noncontingent debt. The labor market is segmented across countries. The state-contingent debt market is segmented across countries and across households and governments. The goods and the state-noncontingent debt are tradeable across countries and agents. In this section we assume that firms set prices every period with contemporaneous information. We also assume that exchange rates are flexible.

#### A. *The households*

The preferences of the home households are described by the expected utility function:

$$U = E_t \sum_{t=0}^T \beta^t u(C_{h,t}, C_{f,t}, L_t) \quad (1)$$

$C_{h,t}$  is the home composite consumption good that aggregates the goods produced by the home firms. There is a continuum of home firms in the unit interval  $[0, 1]$ , indexed by  $i$ .

$$C_{h,t} = \left[ \int_0^1 C_{h,t}(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 1, \quad (2)$$

where  $C_{h,t}(i)$  is the consumption of the good produced by firm  $i$ .  $C_{f,t}$  is the foreign composite consumption good aggregating the goods produced by the foreign firms,

$$C_{f,t} = \left[ \int_0^1 C_{f,t}(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \theta > 1. \quad (3)$$

There is also a continuum of these firms in the unit interval, indexed by  $j$ .  $L_t$  is leisure time and is equal to  $1 - N_t$ , where  $N_t$  is total time devoted to production.

The preferences of the foreign households are described by an identical expected utility function:

$$U = E_t \sum_{t=0}^T \beta^t u(C_{h,t}^*, C_{f,t}^*, L_t^*),$$

where  $C_{h,t}^*$  is the foreign households composite consumption of the goods produced in the home country and  $C_{f,t}^*$  is the foreign households composite consumption of the goods produced in the foreign country.

The households of either country minimize expenditure in the home and foreign goods to obtain a given aggregate consumption of either good. This implies

$$C_{h,t}(i) = \left( \frac{P_{h,t}(i)}{P_{h,t}} \right)^{-\theta} C_{h,t} \text{ and } C_{f,t}(j) = \left( \frac{P_{f,t}^*(j)}{P_{f,t}^*} \right)^{-\theta} C_{f,t},$$

where

$$P_{h,t} = \left[ \int_0^1 P_{h,t}(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}} \text{ and } P_{f,t}^* = \left[ \int_0^1 P_{f,t}^*(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$$

where  $P_{h,t}(i)$  is the price of the good produced by the home firm  $i$  in units of domestic currency, and  $P_{f,t}^*(j)$  is the price of the good produced by the foreign firm  $j$  in units of foreign currency.

Expenditure in either composite good purchased by the home households can then be written as

$$\int_0^1 P_{h,t}(i) C_{h,t}(i) di = P_{h,t} C_{h,t} \text{ and } \int_0^1 P_{f,t}^*(i) C_{f,t}(i) di = P_{f,t}^* C_{f,t}.$$

Similar expressions are obtained for the households of the foreign country.

The budget constraints can then be written in terms of the aggregate variables. The representative household of the home country at the beginning of each period  $t = 0, 1, \dots, T + 1$ ,<sup>4</sup> uses the nominal wealth  $\mathbb{W}_t$  to buy  $M_t$  (home money),  $B_{h,t}$  (home government noncontingent debt),  $B_{f,t}$  (foreign government noncontingent debt) and  $B_{t+1}$  (home private state-contingent debt). The home government noncontingent debt pays the gross return  $R_t$  in the domestic currency at the beginning of the following period, and the foreign government noncontingent debt pays gross return  $R_t^*$  in foreign currency. The price, normalized by the probability of occurrence of the state, at date  $t$  of one unit of domestic currency at a particular state at date  $t + 1$  is  $Q_{t,t+1}$ . There is no government state-contingent debt and the home household cannot buy foreign contingent debt. The price of one unit of foreign currency in units of home currency is  $\varepsilon_t$ . Thus, the following restrictions must be satisfied, respectively, for the home and the foreign households,

$$M_t + B_{h,t} + \varepsilon_t B_{f,t} + E_t B_{t+1} Q_{t,t+1} \leq \mathbb{W}_t. \quad (4)$$

$$M_t^* + \frac{B_{h,t}^*}{\varepsilon_t} + B_{f,t}^* + E_t B_{t+1}^* Q_{t,t+1}^* \leq \mathbb{W}_t^*.$$

In the home country there are taxes on the consumption of home produced goods,  $\tau_{h,t}$ , on the consumption of foreign produced goods,  $\tau_{f,t}$ , labor income  $\tau_{n,t}$  and profits. As the tax on profits is a lump-sum tax it is optimal that all profits be taxed away, so that the net profits are zero. There are corresponding taxes in the foreign country,  $\tau_{h,t}^*$ ,  $\tau_{f,t}^*$ ,  $\tau_{n,t}^*$ .

The wealth home households bring to date  $t + 1$  is

$$\begin{aligned} \mathbb{W}_{t+1} &= M_t + B_{h,t} R_t + \varepsilon_{t+1} B_{f,t} R_t^* + B_{t+1} + \\ &\quad (1 - \tau_{n,t}) W_t N_t - (1 + \tau_{h,t}) P_{h,t} C_{h,t} - (1 + \tau_{f,t}) \varepsilon_t P_{f,t}^* C_{f,t}. \end{aligned} \quad (5)$$

Money is used to purchase goods according to the following cash-in-advance constraints, for the home and foreign country, respectively,

$$(1 + \tau_{h,t})P_{h,t}C_{h,t} + (1 + \tau_{f,t})\varepsilon_t P_{f,t}^* C_{f,t} \leq M_t, \quad (6)$$

$$(1 + \tau_{h,t}^*)\frac{P_{h,t}}{\varepsilon_t}C_{h,t}^* + (1 + \tau_{f,t}^*)P_{f,t}^*C_{f,t}^* \leq M_t^*. \quad (7)$$

The households of the home country take prices, policies and initial wealth as given and choose allocations and asset positions that maximize expected utility (1) subject to the cash-in-advance constraints (6) and the budget constraints (4) and (5), together with  $\mathbb{W}_{T+1} \geq 0$ . The households of the foreign country solve a similar problem.

Among the first order conditions for the home and foreign households are the intertemporal conditions for the contingent assets,

$$Q_{t-1,t}\frac{u_{C_h}(t-1)}{P_{h,t-1}(1 + \tau_{h,t-1})} = \frac{\beta u_{C_h}(t)}{P_{h,t}(1 + \tau_{h,t})}, \text{ all } s^{t-1} \text{ and } s^t | s^{t-1}, 1 \leq t \leq T, \quad (8)$$

$$Q_{t-1,t}^*\frac{\varepsilon_{t-1}u_{C_h^*}(t-1)}{P_{h,t-1}(1 + \tau_{h,t-1}^*)} = \frac{\beta \varepsilon_t u_{C_h^*}(t)}{P_{h,t}(1 + \tau_{h,t}^*)}, \text{ all } s^{t-1} \text{ and } s^t | s^{t-1}, 1 \leq t \leq T, \quad (9)$$

the intertemporal conditions for the noncontingent assets,

$$\frac{u_{C_h}(t-1)}{P_{h,t-1}(1 + \tau_{h,t-1})} = \beta R_{t-1} E_{t-1} \left[ \frac{u_{C_h}(t)}{P_{h,t}(1 + \tau_{h,t})} \right], \text{ all } s^{t-1}, 1 \leq t \leq T, \quad (10)$$

$$\frac{\varepsilon_{t-1}u_{C_h}(t-1)}{P_{h,t-1}(1 + \tau_{h,t-1})} = \beta R_{t-1}^* E_{t-1} \left[ \frac{\varepsilon_t u_{C_h}(t)}{P_{h,t}(1 + \tau_{h,t})} \right], \text{ all } s^{t-1}, 1 \leq t \leq T, \quad (11)$$

$$\frac{u_{C_h^*}(t-1)}{P_{h,t-1}(1 + \tau_{h,t-1}^*)} = \beta R_{t-1} E_{t-1} \left[ \frac{u_{C_h^*}(t)}{P_{h,t}(1 + \tau_{h,t}^*)} \right], \text{ all } s^{t-1}, 1 \leq t \leq T, \quad (12)$$

$$\frac{\varepsilon_{t-1}u_{C_h^*}(t-1)}{P_{h,t-1}(1+\tau_{h,t-1}^*)} = \beta R_{t-1}^* E_{t-1} \left[ \frac{\varepsilon_t u_{C_h^*}(t)}{P_{h,t}(1+\tau_{h,t}^*)} \right], \text{ all } s^{t-1}, 1 \leq t \leq T, \quad (13)$$

and the intratemporal conditions,

$$\frac{u_L(t)}{u_{C_h}(t)} = \frac{W_t(1-\tau_{n,t})}{P_{h,t}R_t(1+\tau_{h,t})}, \text{ all } s^t, 0 \leq t \leq T \quad (14)$$

$$\frac{u_{C_h}(t)}{u_{C_f}(t)} = \frac{(1+\tau_{h,t})P_{h,t}}{(1+\tau_{f,t})\varepsilon_t P_{f,t}^*}, \text{ all } s^t, 0 \leq t \leq T \quad (15)$$

$$\frac{u_{L_t^*}(t)}{u_{C_f^*}(t)} = \frac{W_t^*(1-\tau_{n,t}^*)}{P_{f,t}^*R_t^*(1+\tau_{f,t}^*)}, \text{ all } s^t, 0 \leq t \leq T \quad (16)$$

$$\frac{u_{C_h^*}(t)}{u_{C_f^*}(t)} = \frac{(1+\tau_{h,t}^*)P_{h,t}}{(1+\tau_{f,t}^*)\varepsilon_t P_{f,t}^*}, \text{ all } s^t, 0 \leq t \leq T \quad (17)$$

The budget constraints of the households of each country, (4) and (5), together with the terminal conditions  $\mathbb{W}_{T+1} \geq 0$  and  $\mathbb{W}_{T+1}^* \geq 0$ , can be written as intertemporal budget constraints, that at the optimum will hold with equality. For the home country those constraints are the following

$$\begin{aligned} & \sum_{s=t}^T E_t Q_{t,s+1} [(1+\tau_{h,s})P_{h,s}C_{h,s} + (1+\tau_{f,s})\varepsilon_s P_{f,s}^* C_{f,s} - (1-\tau_{n,s})W_s N_s] \\ & + \sum_{s=t}^T E_t Q_{t,s+1} \left[ M_s \left( \frac{Q_{t,s}}{Q_{t,s+1}} - 1 \right) \right] = \mathbb{W}_t, \text{ all } s^t, 0 \leq t \leq T, \end{aligned}$$

where  $Q_{t,s} = Q_{t,t+1} \dots Q_{s-1,s}$ ,  $t \geq 0$ ,  $s \geq t+1$ , and  $Q_{t,t} = 1$ .

Using the marginal conditions, as well as the cash in advance constraints, in the intertem-

poral budget constraints, we can rewrite the budget constraints for the home country as

$$\sum_{s=t}^T \beta^{s-t} E_t \left[ (u_{C_h}(s) C_{h,s} + u_{C_f}(s) C_{f,s} - u_L(s) N_s) \right] = \mathbb{W}_t \frac{u_{C_h}(t)}{P_{h,t}(1 + \tau_{h,t})}, \text{ all } s^t, 0 \leq t \leq T, \quad (18)$$

and for the foreign country as

$$\sum_{s=t}^T \beta^{s-t} E_t \left[ (u_{C_h^*}(s) C_{h,s}^* + u_{C_f^*}(s) C_{f,s}^* - u_{L^*}(s) N_s^*) \right] = \mathbb{W}_t^* \frac{u_{C_f^*}(t)}{P_{f,t}^*(1 + \tau_{f,t}^*)}, \text{ all } s^t, 0 \leq t \leq T. \quad (19)$$

### B. The government

The government of each country includes both the fiscal authority and the monetary authority. We assume as is standard in this literature that aggregate public expenditures are exogenous. Each government only consumes goods produced by local firms, and chooses consumption of each good to minimize expenditure on the aggregate level of expenditures,  $G_t$  for the home country and  $G_t^*$ , for the foreign country

$$G_t = \left[ \int_0^1 G_{h,t}(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}, \theta > 1,$$

and

$$G_t^* = \left[ \int_0^1 G_{f,t}^*(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}, \theta > 1,$$

where  $G_{h,t}(i)$  is the home government consumption of the good produced by firm  $i$  and  $G_{f,t}^*(j)$  is the foreign government consumption of the good produced in that country by firm  $j$ .

The home government issues state-noncontingent debt,  $B_{h,t} + B_{h,t}^*$ , and money,  $M_t^s$ , and taxes labor income and private consumption, as well as profits. The nominal financial liabilities of the home government at the start of period  $t$  are  $\mathbb{W}_t^g$ , which can be financed by issuing money and



public debt

$$M_t^s + B_{h,t} + B_{h,t}^* = \mathbb{W}_t^g.$$

The nominal financial liabilities the home government brings to the next period are

$$\begin{aligned} \mathbb{W}_{t+1}^g = & M_t^s + R_t B_{h,t} + R_t B_{h,t}^* + P_{h,t} G_t - \tau_{h,t} P_{h,t} C_{h,t} - \\ & \tau_{f,t} \varepsilon_t P_{f,t}^* C_{f,t} - \tau_{n,t} W_t N_t - \Pi_{h,t} \end{aligned}$$

where  $\Pi_{h,t}$  are the profits of the home firms that are fully taxed. We impose the terminal condition that government liabilities in the terminal period are zero,  $\mathbb{W}_{T+1}^g = 0$ . The home government period  $t$  intertemporal budget constraint can then be written as

$$\begin{aligned} \sum_{s=t}^T E_t Q_{t,s+1} [\tau_{h,s} P_{h,s} C_{h,s} + \tau_{f,s} \varepsilon_s P_{f,s}^* C_{f,s} + \tau_{n,s} W_s N_s + \Pi_{h,s} - P_{h,s} G_s] \\ + \sum_{s=t}^T E_t Q_{t,s+1} M_s^s \left( \frac{Q_{t,s}}{Q_{t,s+1}} - 1 \right) = \mathbb{W}_t^g, \text{ all } s^t, 0 \leq t \leq T. \end{aligned}$$

There is a similar condition for the government of the foreign country.

### C. Firms

In each country there is a continuum of firms in the unit interval. Each firm produces a distinct, perishable consumption good with a technology that uses labor only. Each home firm  $i$  has the production technology

$$Y_{h,t}(i) = A_t N_t(i), \text{ all } s^t, 0 \leq t \leq T, \quad (20)$$

where  $Y_{h,t}(i)$  is the production of good  $i$ ,  $N_t(i)$  is the labor used in the production of good  $i$ , and  $A_t$  is an aggregate technology shock in the home country. Good  $i$  can be used for private and public

consumption,  $Y_{h,t}(i) = C_{h,t}(i) + C_{h,t}^*(i) + G_t(i)$ . The technology in the foreign country is

$$Y_{f,t}(j) = A_t^* N_t^*(j), \text{ all } s^t, 0 \leq t \leq T, \quad (21)$$

where the technology parameter  $A_t^*$  is the same across firms but can be different from  $A_t$ . Each good  $j$  produced in the foreign country can be consumed by households or by the foreign government,

$$Y_{f,t}(j) = C_{f,t}(j) + C_{f,t}^*(j) + G_t^*(j).$$

Prices are flexible. The firms in the home country choose prices to maximize profits

$$\Pi_{h,t}(i) = P_{h,t}(i) Y_{h,t}(i) - W_t N_t(i), \text{ given the demand functions}$$

$$Y_{h,t}(i) = \left( \frac{P_{h,t}(i)}{P_{h,t}} \right)^{-\theta} Y_{h,t}, \text{ all } s^t, 0 \leq t \leq T$$

where  $Y_{h,t} = C_{h,t} + C_{h,t}^* + G_t$ , obtained using the demand functions of the home good at home and abroad, and given the production functions (20).

The home firms set a common price  $P_{h,t}(i) = P_{h,t}$  such that

$$\frac{W_t}{P_{h,t}} = \frac{\theta - 1}{\theta} A_t, \text{ all } s^t, 0 \leq t \leq T, \quad (22)$$

and the foreign firms set  $P_{f,t}^*(j) = P_{f,t}^*$  where

$$\frac{W_t^*}{P_{f,t}^*} = \frac{\theta - 1}{\theta} A_t^*, \text{ all } s^t, 0 \leq t \leq T. \quad (23)$$

#### D. Equilibrium

We now define a flexible price equilibrium as a vector of allocations  $\{C_{h,t}, C_{f,t}, N_t, C_{h,t}^*, C_{f,t}^*, N_t^*\}$  asset positions  $\{M_t, B_{h,t}, B_{f,t}, B_{t+1}, M_t^*, B_{h,t}^*, B_{f,t}^*, B_{t+1}^*\}$ , prices and policies  $\{P_{h,t}, W_t, R_t, Q_{t,t+1}, \tau_{h,t}, \tau_{f,t}, \tau_{n,t}, M_t^S, \varepsilon_t\}$  and  $\{P_{f,t}, W_t^*, R_t^*, Q_{t,t+1}^*, \tau_{h,t}^*, \tau_{f,t}^*, \tau_{n,t}^*, M_t^{*S}\}$  such that,

- (a) Given the initial wealth levels, prices and policy the households choose the relevant quantities that solve their problems;
- (b) Firms given prices and policy choose the relevant quantities that solve their problems;
- (c) For initial public liabilities the governments satisfy their budget constraints;
- (d) The markets are in equilibrium:

$$C_{h,t} + C_{h,t}^* + G_t = A_t N_t \quad (24)$$

$$C_{f,t} + C_{f,t}^* + G_t^* = A_t^* N_t^* \quad (25)$$

$$M_t^S = M_t \quad (26)$$

$$M_t^{*S} = M_t^* \quad (27)$$

$$B_{t+1} = 0 \quad (28)$$

$$B_{t+1}^* = 0 \quad (29)$$

The market clearing in the labor and noncontingent bond markets was already imposed.

The equilibrium conditions that determine the allocations  $\{C_{h,t}, C_{f,t}, N_t, C_{h,t}^*, C_{f,t}^*, N_t^*\}$ , asset positions  $\{M_t, B_{h,t}, B_{f,t}, B_{t+1}, M_t^*, B_{h,t}^*, B_{f,t}^*, B_{t+1}^*\}$ , prices and policies  $\{P_{h,t}, W_t, R_t, Q_{t,t+1}, \tau_{h,t}, \tau_{f,t}, \tau_{n,t}, M_t^S, \varepsilon_t\}$  and  $\{P_{f,t}, W_t^*, R_t^*, Q_{t,t+1}^*, \tau_{h,t}^*, \tau_{f,t}^*, \tau_{n,t}^*, M_t^{*S}\}$  are (6) - (19), (22) - (29) and the

intertemporal budget constraints of the home country that can be obtained by adding up the home government budget constraints and the home household budget constraints, as follows

$$\sum_{s=t}^T E_t Q_{t,s+1} [P_{h,s} (C_{h,s} + G_s) + \varepsilon_s P_{f,s}^* C_{f,s} - W_s N_s - \Pi_{h,s}] = \mathbb{W}_t^e, \text{ all } s^t, 0 \leq t \leq T, \quad (30)$$

where  $\mathbb{W}_t^e = \mathbb{W}_t - \mathbb{W}_t^g$ , are the foreign assets owned by the home country. Using the market clearing conditions and the expression for profits, the constraints can be written as

$$\sum_{s=t}^T E_t Q_{t,s+1} [\varepsilon_s P_{f,s}^* C_{f,s} - P_{h,s} C_{h,s}^*] = \mathbb{W}_t^e, \text{ all } s^t, 0 \leq t \leq T, \quad (31)$$

### III. Equilibria under flexible prices

Our purpose in this section is to assert a major result of the paper that has implications for equilibrium allocations with sticky prices and fixed exchange rates. We show that the set of equilibrium allocations under flexible prices can be implemented with policies such that the price level in either country will be constant over time, and such that the nominal exchange rate will also be constant over time. In order to do this we show that for a given equilibrium allocation  $\{C_{h,t}, C_{f,t}, N_t, C_{h,t}^*, C_{f,t}^*, N_t^*\}$  the equilibrium conditions are all satisfied with constant producer price levels in each country equal to arbitrary numbers, and a constant nominal exchange rate, also equal to an arbitrary number.

The proposition follows:

Proposition 1. *Any flexible equilibrium allocation can be implemented with  $P_{h,t} = P_{h,0}$ ,*

*$P_{f,t}^* = P_{f,0}^*$ ,  $\varepsilon_t = \varepsilon_0$  (and  $R_t = R_t^*$ ).*

**Proof:** Without loss of generality we take  $T = 1$ . In the beginning of period  $t = 2$  the assets market opens to liquidate debts. This means that the wealth of the households in period  $t = 2$ , in either country, is zero,  $\mathbb{W}_2 = 0$  and  $\mathbb{W}_2^* = 0$ .

We take as given an arbitrary equilibrium allocation  $\{C_{h,t}, C_{f,t}, N_t, C_{h,t}^*, C_{f,t}^*, N_t^*\}$  for  $t = 0, 1$ , in the set defined above. We show that there are constant prices with  $P_{h,t} = P_{h,0}$ ,  $P_{f,t}^* = P_{f,0}^*$ , and fixed exchange rates,  $\varepsilon_t = \varepsilon_0$ , which implies that  $R_t = R_t^*$ , that satisfy the equilibrium equations for that allocation which are (6) - (19), (22) - (29) and (31).

First, this allocation satisfies trivially the two feasibility constraints, (24) and (25), as it is an equilibrium allocation. For given  $P_{h,0}$ ,  $P_{f,0}^*$ ,  $\varepsilon_0$  we use the remaining equilibrium conditions to determine the values for the policy variables and remaining prices.

The firms' conditions determine  $W_t$  and  $W_t^*$

$$\frac{W_t}{P_{h,0}} = \frac{\theta - 1}{\theta} A_t, \quad t = 0, 1,$$

$$\frac{W_t^*}{P_{f,0}^*} = \frac{\theta - 1}{\theta} A_t^*, \quad t = 0, 1.$$

The period 0 intertemporal budget constraints for the two representative households are

$$\sum_{t=0}^1 \beta^t E_0 \left[ (u_{C_h}(t) C_{h,t} + u_{C_f}(t) C_{f,t} - u_L(t) N_t) \right] = \mathbb{W}_0 \frac{u_{C_h}(0)}{P_{h,0}(1 + \tau_{h,0})}$$

and

$$\sum_{t=0}^1 \beta^t E_0 \left[ (u_{C_h^*}(t) C_{h,t}^* + u_{C_f^*}(t) C_{f,t}^* - u_{L^*}(t) N_t^*) \right] = \mathbb{W}_0^* \frac{u_{C_h^*}(0)}{\frac{P_{h,0}}{\varepsilon_0} (1 + \tau_{h,0}^*)}$$

which are satisfied by appropriately choosing  $\tau_{h,0}$  and  $\tau_{h,0}^*$ .

Given a common process for the nominal interest rate

$$R_t = R_t^*$$

to be determined later, and  $\tau_{h,0}$  and  $\tau_{h,0}^*$ , can use the following equations

$$\frac{u_{C_h}(0)}{(1 + \tau_{h,0})} = \beta R_0 E_0 \frac{u_{C_h}(1)}{(1 + \tau_{h,1})}$$

$$\frac{u_{C_h^*}(0)}{(1 + \tau_{h,0}^*)} = \beta R_0 E_0 \frac{u_{C_h^*}(1)}{(1 + \tau_{h,1}^*)}$$

$$u_{C_h}(1) C_{h,1} + u_{C_f}(1) C_{f,1} - u_L(1) N_1 = \mathbb{W}_1 \frac{u_{C_h}(1)}{P_{h,0}(1 + \tau_{h,1})}, \quad s^1 \in S^1$$

$$u_{C_h^*}(1) C_{h,1}^* + u_{C_f^*}(1) C_{f,1}^* - u_{L^*}(1) N_1^* = \mathbb{W}_1^* \frac{u_{C_h^*}(1)}{\frac{P_{h,0}}{\varepsilon_0}(1 + \tau_{h,1}^*)}, \quad s^1 \in S^1$$

to determine  $\tau_{h,1}$ ,  $\tau_{h,1}^*$ ,  $\mathbb{W}_1$  and  $\mathbb{W}_1^*$ .

Can use the intratemporal conditions

$$\frac{u_L(t)}{u_{C_h}(t)} = \frac{W_t(1 - \tau_{n,t})}{P_{h,0}R_t(1 + \tau_{h,t})}, \quad t = 0, 1$$

$$\frac{u_{C_h}(t)}{u_{C_f}(t)} = \frac{(1 + \tau_{h,t})P_{h,0}}{(1 + \tau_{f,t})\varepsilon_0 P_{f,0}^*}, \quad t = 0, 1$$

$$\frac{u_{L_t^*}(t)}{u_{C_f^*}(t)} = \frac{W_t^*(1 - \tau_{n,t}^*)}{P_{f,0}^*R_t(1 + \tau_{f,t}^*)}, \quad t = 0, 1$$

$$\frac{u_{C_h^*}(t)}{u_{C_f^*}(t)} = \frac{(1 + \tau_{h,t}^*)P_{h,0}}{(1 + \tau_{f,t}^*)\varepsilon_0 P_{f,0}^*}, \quad t = 0, 1$$

to determine  $\tau_{n,t}$ ,  $\tau_{f,t}$ ,  $\tau_{n,t}^*$ ,  $\tau_{f,t}^*$ , for  $t = 0, 1$ .

We can use the cash in advance constraints

$$(1 + \tau_{h,t})P_{h,0}C_{h,t} + (1 + \tau_{f,t})\varepsilon_0 P_{f,0}^* C_{f,t} = M_t, t = 0, 1$$

$$(1 + \tau_{h,t}^*)\frac{P_{h,0}}{\varepsilon_0} C_{h,t}^* + (1 + \tau_{f,t}^*)P_{f,0}^* C_{f,t}^* = M_t^*, t = 0, 1$$

to determine  $M_t$  and  $M_t^*$ , for  $t = 0, 1$ .

The home country intertemporal budget constraints can be used to determine the nominal interest rates  $R_t$ . The budget constraints for period  $t = 1$  are

$$\mathbb{W}_1^e = \frac{1}{R_1} [\varepsilon_0 P_{f,0}^* C_{f,1} - P_{h,0} C_{h,1}], s^1 \in S^1$$

which give  $\#S^1$  interest rates  $R_1$  as a function of the value for  $\mathbb{W}_1^e$ . Given these values for  $R_1$  the budget constraint for period  $t = 0$ ,

$$\begin{aligned} \mathbb{W}_0^e &= \frac{1}{R_0} [\varepsilon_0 P_{f,0}^* C_{f,0} - P_{h,0} C_{h,0}] + \\ &E_0 \frac{Q_{0,1}}{R_1} [\varepsilon_0 P_{f,0}^* C_{f,1} - P_{h,0} C_{h,1}]. \end{aligned}$$

determines uniquely  $R_0$ .

The flexible price allocation can be supported with constant producer prices for both goods and a constant exchange rate, for arbitrary initial levels  $P_{h,0}$ ,  $P_{f,0}^*$  and  $\varepsilon_0$ . The common interest rate is a function of the level of the net foreign assets of the home country in period  $t = 1$ . This, for the case of  $t = 0, 1$ , gives one degree of freedom for  $\mathbb{W}_1^e$ .

The proof extends to any finite horizon economy,  $t = 0, \dots, T$ , with  $T$  arbitrarily large. There are  $\#S^{t-1}$  degrees of freedom for  $\mathbb{W}_t^e$  for every  $t = 1, \dots, T$ . ■

We have shown that for any equilibrium allocation,  $\{C_{h,t}, C_{f,t}, N_t, C_{h,t}^*, C_{f,t}^*, N_t^*\}$ , the equilibrium conditions can be satisfied by asset positions, prices and policies such that producer prices and exchange rates are arbitrary constants,  $P_{h,t} = P_{h,0}$ ,  $P_{f,t}^* = P_{f,0}^*$ ,  $\varepsilon_t = \varepsilon_0$ . This means that the full set of equilibrium allocations can be implemented under fixed exchange rates with producer prices in both countries that are constant over time.

Taxes play a particular role when equilibria have constant producer prices and exchange rates. Since prices are constant and so is the exchange rate, consumption taxes in one good relative to the other will have to move if relative prices are to move, which will happen in general if there are different shocks in different countries. Consumption taxes play another role, when public debt is noncontingent, that of replicating state-contingent real debt. We have assumed, as is standard in this literature, that internationally traded assets are state-noncontingent. Nominal interest rates, that in a fixed exchange rate regime are common across countries, can play the role of replicating state-contingent international debt. Consumption taxes also affect the households margin between consumption and labor. Labor income taxes will have to adjust for those effects. Since prices are constant and technologies in the two countries are varying, the nominal wages will have to move in response to shocks and move differently in different countries. Money supply will also have to move to respond to shocks to satisfy the cash in advance constraints.

One first implication of the result in the proposition is that fixed exchange rates do not restrict the set of allocations under flexible prices. This is an interesting result in itself, in particular, as in our model, when asset markets are incomplete. However, the issue of whether there are costs of a fixed exchange rate regime is typically associated with the presence of some type of price rigidity, as argued by Friedman (1953). If there are restrictions on how producer prices are set, and exchange rates are fixed, it may be the case that there will be restrictions on the relative prices of



the goods produced in the different countries.

It is particularly surprising that fixed exchange rates do not restrict the set of allocations also when producer prices are constant over time. Can both producer prices and exchange rates be constant over time? Yes, as long as taxes can change so that the terms of trade, real wages, and debt levels can move with the shocks.

In the following section we assume that firms are restricted in the setting of prices.

#### **IV. Sticky prices**

The model considered above has full flexibility of prices and exchange rates. However, the discussion of the costs of the exchange rate regime is particularly interesting insofar as there is some type of nominal rigidity. We now assume that prices are sticky in some or in all goods produced. We start by showing that there are fiscal and monetary policies in a fixed exchange rate regime that can achieve any flexible price equilibrium allocation even when there are nominal rigidities that can be different across countries. Proposition 1 will be instrumental to show this result.

Under sticky prices there are allocations other than the ones that can be implemented under flexible prices. However, as we show here, those allocations are dominated in terms of welfare, so that indeed we can conclude that the fixed exchange rate regime is of no cost whether prices are flexible or sticky.

We assume that firms set prices as in Calvo (1983) staggered price setting, which is a commonly used assumption in the sticky price literature. We assume that the firms set prices in the currency of their country. In each country, starting from an historical common price, at every date, each firm can optimally set its price with some probability less than one, that can differ across countries. Because there is a continuum of firms, the probability is also the share of firms that

optimally revise the price in each period.

In general, staggered price setting leads to inefficient differences in prices across firms. Although in a given country firms are otherwise identical, have the same linear technology and face identical demand functions, they may charge different prices. Thus, the relative price of the goods they produce may be different from one. The only case in which this will not occur is when firms that in each period have the opportunity of choosing a new price decide to maintain the same price. The price setting restrictions in this case will not be binding and the producer price level in each country will be constant. The equilibrium conditions will be identical to the equilibrium conditions of the flexible price economy when producer prices are constant across periods.

Since, as stated in Proposition 1, under flexible prices it is possible to implement the full set of equilibrium allocations with constant prices and fixed exchange rates, it follows that under sticky prices it is also possible to implement that same set, also with fixed exchange rates.

It is clear that under sticky prices there are allocations that are not implementable under flexible prices. That is the case whenever otherwise identical firms set different prices. It turns out, as we show in the Appendix, that the set of flexible price allocations dominates in terms of welfare the set of allocations under sticky prices. Since agents are heterogeneous across countries, the meaning of welfare dominance is the usual one, of a potential Pareto movement where lump sum transfers between agents are implicitly assumed.

The proposition follows.

*Proposition 2. In a world economy with noncontingent bond markets and Calvo (1983) staggered price setting there is no cost of a fixed exchange rate regime, independently of the degree of price rigidity.*

**Proof** (sketch): In Proposition 1 we show that the set of allocations under flexible prices is implemented with policies that generate constant prices and exchange rates, equal to arbitrary numbers. For the policies that induce prices to be equal to the historical initial prices of the Calvo firms,  $P_{h,0}$  and  $P_{f,0}^*$ , and exchange rates equal to any constant<sup>5</sup>, the equilibrium conditions under Calvo (1983) will be exactly the ones under flexible prices. This establishes that the flexible price set of allocations is feasible with Calvo price setting and fixed exchange rates. It remains to show that the set is optimal, in the sense that for every allocation in the set under sticky prices, there is one in the set under flexible prices that is a potential Pareto improvement. This is done in the Appendix. ■

The result in this proposition can be extended to any other form of price stickiness, such as prices set in advance, Taylor (1980) staggered prices, or Rotemberg (1982) adjustment costs of changing prices. For the case where prices are set in advance, let the initial prices  $P_{h,0}$  and  $P_{f,0}^*$  be exogenously given and the other period prices  $P_{h,t}$  and  $P_{f,t}^*$  be set in advance for  $k$  periods, for a finite  $k$ . Proposition 1 implies that adding those restrictions to the flexible price economy still allows to implement the set of allocations under flexible prices, in a fixed exchange rate regime. The argument of welfare dominance of the flexible price set also applies here.

We have analyzed flexible versus fixed exchange rate regimes. The analysis clearly follows through in a monetary union. The interest rate will be common as under fixed exchange rates. The money supply in each country obviously does not have to be the same.

We have assumed that prices are set in the currency of the producer. We could alternatively have assumed local currency pricing. The results would follow through. For the policies that support constant producer prices and constant exchange rates, local currency price setting restrictions would not have any impact. Contrary to what is argued extensively in the literature that does not allow

for fiscal policy instruments, it does not make a difference whether prices are set in the currency of the producer or the consumer.

## V. Labor mobility

In the literature of optimal currency areas the lack of labor mobility is one of the justifications for the costs of a monetary union with asymmetric member countries. A result of this paper is that the opposite is true. Labor immobility is a necessary condition for the irrelevance of the exchange regime.

Proposition 1 was stated for the case where labor cannot move across countries. It does not apply when labor is mobile. To see this we assume that workers can choose to work in foreign firms being taxed at home. They consume at home. This is one way of modelling labor mobility. There are alternative ways but the same arguments go through.

For the home households, total labor  $N_t$  is split between work at home  $N_{h,t}$  and work abroad  $N_{f,t}$ ,

$$N_t = N_{h,t} + N_{f,t}. \quad (32)$$

Similarly for the foreign country,  $N_t^*$  is split between  $N_{h,t}^*$ , which is labor in the home country, and  $N_{f,t}^*$ , which is labor in the foreign country,

$$N_t^* = N_{h,t}^* + N_{f,t}^*. \quad (33)$$

The market clearing conditions in the goods market, (24), become

$$C_{h,t} + C_{h,t}^* + G_t = A_t [N_{h,t} + N_{h,t}^*] \quad (34)$$

$$C_{f,t} + C_{f,t}^* + G_t^* = A_t^* [N_{f,t} + N_{f,t}^*] \quad (35)$$

The conditions of the households problem are the same except for an additional arbitrage condition on where to work, that equates the two wages

$$W_t = \varepsilon_t W_t^* \quad (36)$$

The price setting conditions for the firms are also unchanged. The country budget constraints (31) become

$$\begin{aligned} & \sum_{s=t}^T E_t Q_{t,s+1} [\varepsilon_s P_{f,s}^* C_{f,s} - P_{h,s} C_{h,s}^* + W_s N_{h,s}^* - \varepsilon_s W_s^* N_{f,s}] \\ &= \mathbb{W}_t - \mathbb{W}_t^g, \text{ all } s^t, 0 \leq t \leq T, \end{aligned}$$

Notice that full labor mobility implies one additional constraint per state to the equilibrium conditions. The wage in the same currency must be equal across countries. As shown in the proof of Proposition 1, there are multiple policies that support each allocation under flexible prices with constant prices and exchange rates. Under sticky prices and fixed exchange rates the degrees of freedom are the number of states at  $t - 1$ ,  $\#S^{t-1}$ , for each period  $t$ . These degrees of freedom are not enough to satisfy the additional equilibrium restrictions, which are as many as the number of states in  $t$ ,  $\#S^t$ . This is stated in the proposition below.

*Proposition 3. When prices are sticky, in a fixed exchange rate regime (or monetary union), labor immobility is a necessary condition to implement the set of flexible price equilibrium allocations.*

**Proof:** The nominal wages cannot satisfy simultaneously the price setting conditions (22) and (23), with  $P_{h,t} = P_{h,0}$  and  $P_{f,t}^* = P_{f,0}^*$ ,

$$\frac{W_t}{P_{h,0}} = \frac{\theta - 1}{\theta} A_t, \text{ all } s^t, 0 \leq t \leq T, \quad (37)$$

and

$$\frac{W_t^*}{P_{f,0}^*} = \frac{\theta - 1}{\theta} A_t^*, \text{ all } s^t, 0 \leq t \leq T, \quad (38)$$

and the arbitrage condition

$$W_t = \varepsilon_0 W_t^*. \quad (39)$$

Therefore the equilibrium allocations under flexible prices and exchange rates with mobile labor cannot be decentralized with constant exchange rates and producer prices. ■

When labor is mobile, and prices are sticky, the exchange rate regime matters. In particular, while with flexible exchange rates it is possible to implement the set of allocations under flexible prices, that is not the case in a fixed exchange rate regime.

The fact that with labor mobility there are costs of a fixed exchange rate regime, while there are no such costs when labor is immobile, does not mean that labor mobility is undesirable. We are not comparing environments with and without labor mobility, but rather environments with and without fixed exchange rates, when labor is immobile or when it is mobile.

## VI. Concluding remarks

Under a flexible exchange rate regime, monetary policy in each country can freely respond to shocks; may respond to country specific shocks or may respond differently from other countries to common

shocks. Instead, in a monetary union there is a unique monetary policy for the members of the union. This implies restrictions in the use of policy; the exchange rate must be constant over time and the nominal interest rate must be equal across countries. Are these restrictions relevant to achieve the optimal equilibrium allocations? Does the answer to this question change with the introduction of nominal rigidities, like staggered price setting?

The conventional wisdom is that there are costs of a fixed exchange rate regime, or a monetary union, resulting from the loss in ability to use policy for stabilization purposes. The costs are taken to be higher the stronger are the asymmetries across countries in shocks and their transmission, and the stronger are the nominal rigidities. Instead, we show that in an environment with nominal rigidities, whatever the type of price setting PCP (producer currency pricing) or LCP (local currency pricing), the exchange rate regime, whether flexible or fixed exchange rates, is irrelevant once fiscal policy instruments are taken into account. This is the main result of the paper. We also show that in order for the costs of the monetary union to be zero labor cannot be mobile.

One possible objection to our analysis, as well as to the related literature that uses both fiscal and monetary policy instruments, is that we do not incorporate informational restrictions in the policy choice and also do not take into account lack of ability to commit. The assumptions of private information on the part of the government and inability to commit in the presence of a time inconsistency problem may justify policy that does not respond to contingencies, such as the inflation cap in the analysis in Athey, Atkeson and Kehoe (2005).

## APPENDIX

In this appendix we show that for each allocation under sticky prices, with flexible exchange rates, there is an allocation under flexible prices that gives at least as high welfare to one country without reducing the welfare of the other country.

Assuming that lump sum transfers are feasible between countries, the set of implementable allocations under flexible prices  $\{C_{h,t}, C_{f,t}, N_t, C_{h,t}^*, C_{f,t}^*, N_t^*\}$  as well as initial taxes and exchange rate  $\{\tau_{h,0}, \tau_{h,0}^*, \varepsilon_0\}$  will be characterized by the following conditions:

$$\sum_{t=0}^1 \beta^t E_0 [(u_{C_h}(t) C_{h,t} + u_{C_f}(t) C_{f,t} - u_L(t) N_t)] = \mathbb{W}_0 \frac{u_{C_h}(0)}{P_{h,0}(1 + \tau_{h,0})} \quad (\text{A.1})$$

$$\sum_{t=0}^1 \beta^t E_0 [(u_{C_h^*}(t) C_{h,t}^* + u_{C_f^*}(t) C_{f,t}^* - u_{L^*}(t) N_t^*)] = \mathbb{W}_0^* \frac{u_{C_h^*}(0)}{\frac{P_{h,0}}{\varepsilon_0}(1 + \tau_{h,0}^*)} \quad (\text{A.2})$$

$$C_{h,t} + C_{h,t}^* + G_t = A_t N_t, \quad (\text{A.3})$$

$$C_{f,t} + C_{f,t}^* + G_t^* = A_t^* N_t^*. \quad (\text{A.4})$$

We do not impose as a restriction the budget constraint between countries, because we allow for transfers between these. The remaining equilibrium conditions determine the policy and prices. Denote the set of allocations that satisfy these conditions by  $E^f$ .

Given Pareto weights there will be an optimal allocation that can be decentralized with a choice of initial conditions  $\mathbb{W}_0^e$ . The Pareto weights can be chosen to be such that the optimal allocation is implemented with the actual initial  $\mathbb{W}_0^e$ .

Under sticky prices the set of equilibrium conditions cannot be summarized by a small set of implementability conditions as under flexible prices. The allocations  $\{C_{h,t}, C_{f,t}, N_t, C_{h,t}^*, C_{f,t}^*, N_t^*\}$



are restricted by the same intertemporal budget constraints as in the flexible price case above, (A.1) and (A.2). For given prices  $\left\{ \frac{P_{h,t}(i)}{P_{h,t}}, \frac{P_{f,t}(j)}{P_{f,t}} \right\}$ , they are restricted by the feasibility conditions

$$(C_{h,t} + C_{h,t}^* + G_t) \int_0^1 \left( \frac{P_{h,t}(i)}{P_{h,t}} \right)^{-\theta} di = A_t N_t \quad (\text{A.5})$$

$$(C_{f,t} + C_{f,t}^* + G_t) \int_0^1 \left( \frac{P_{f,t}(j)}{P_{f,t}} \right)^{-\theta} dj = A_t^* N_t^* \quad (\text{A.6})$$

the conditions that define the aggregate price levels,  $P_{h,t} = \left[ \int_0^1 P_{h,t}(i)^{1-\theta} di \right]^{\frac{1}{1-\theta}}$  and  $P_{f,t}^* = \left[ \int_0^1 P_{f,t}^*(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}}$ , as well as all the remaining equilibrium conditions. Let the set of allocations that satisfy these restrictions be denoted by  $E^s$ .

It is straightforward to show that  $D \equiv \int_0^1 \left( \frac{P_{h,t}(i)}{P_{h,t}} \right)^{-\theta} di \geq 1$  and  $D^* = \int_0^1 \left( \frac{P_{f,t}(j)}{P_{f,t}} \right)^{-\theta} dj \geq 1$ .  $D = 1$  when  $\frac{P_{h,t}(i)}{P_{h,t}} = 1$  and  $D^* = 1$  when  $\frac{P_{f,t}(j)}{P_{f,t}} = 1$ .

The set of allocations under flexible prices dominates the set under sticky prices, meaning that for each allocation in  $E^s$  there is at least one allocation in  $E^f$  with at least one of the goods in larger or equal quantity and none smaller. The intertemporal budget constraints are the same but the feasibility conditions are different, being A.5 and A.6 more restrictive than A.3 and A.4, and there are additional equilibrium restrictions over  $E^s$  that are absent from  $E^f$ . Moreover, the restrictions over the allocations under sticky prices are exactly the same only when  $P_{h,t}(i) = P_{h,0}$  and  $P_{f,t}(j) = P_{f,0}$ . ■

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## NOTES

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<sup>1</sup>See also Obstfeld (2004) and Duarte (2004).

<sup>2</sup>On recent work on optimal monetary policy in a currency area see Benigno (2004).

<sup>3</sup>The assumption of a finite, even if arbitrarily large, time horizon considerable simplifies the analysis and is as reasonable an assumption as the more standard one of an infinite horizon.

<sup>4</sup>Notice that the decision on assets is made also in period  $T + 1$ , while the last period in which agents produce and consume is  $T$ . The assumption that there is an additional subperiod with an assets market for the clearing of debts guarantees that money has value in a finite horizon economy. Agents will want to take money to period  $T + 1$  to settle debts, that in the aggregate must be with the government. If the finite horizon economy ended with a goods market at  $T$ , then sellers would not accept money in period  $T$ , and therefore money would not have value, not only in that period but in every period.

<sup>5</sup>The exchange rate could be equal to one for the case of a monetary union,