DISCUSSION PAPER SERIES

No. 5793

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INTERNATIONAL MACROECONOMICS

Centre for Economic Policy Research

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ISSN 0265-8003

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Discussion Paper No. 5793
August 2006

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CEPR Discussion Paper No. 5793

August 2006

ABSTRACT

Fiscal Policy and the Term Structure: Evidence from the Case of Italy in the EMS and the EMU Periods*

We study the relationship between the term structure of interest rates and fiscal policy by considering the Italian case. Empirical analysis has been so far rather inconclusive on this important topic. We abscribe such evidence to three problems: identification, regime-switching and maturity effects. All these aspects are particularly relevant to the Italian case. We propose a parsimonious model with three factors to represent the whole yield curve, and we consider yield differentials between Italian and German Government bonds. To take into account the possibility of regime-switching, we explicitly include a hidden two-state Markov chain that represents market expectations. The model is estimated using Bayesian econometric techniques. We find that government debt and its evolution significantly influence the yield of government bonds, that such effects are maturity dependent and regime-dependent. Hence when investigating the effect of fiscal policy on the term-structure it is of crucial importance to allow for multiple regimes in the estimation.

JEL Classification: E0 and G0
Keywords: Bayesian estimation, fiscal policy, regime switching and term structure

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*We acknowledge the FIRB programme for financial support. We are indebted to Alessandro Missale, Gustavo Piga and participants in the FIRB group on "Public debt Management" for their comments.

Submitted 19 July 2006

1 Introduction

The relationship between the term structure of interest rates and fiscal policy is a topic of central relevance to policymakers and market participants given its implication for optimal public debt management and portfolio allocation.

Economic theory does not provide a definitive solution to the issue under study. As Elmendorf and Mankiw (1999) and Engen and Hubbard (2004) point out, different models predict very different effects of government debt and deficit on interest rates, depending on the assumptions and the macroeconomic framework of each model. Empirical analysis has been mostly inconclusive in solving the problem: in the last 20 years many studies have undertaken the task of testing various theoretical hypotheses, obtaining very different results depending on the data employed and the variables used[1].

The heterogeneity of the empirical evidence can be explained by a number of issues that we will explicitly address in this paper:

i) identification. As clearly pointed out by Laubach(2004) estimating the effects of government debt and deficits on the yield curve is complicated by the need to isolate the effects of fiscal policy from the effects of the business cycle and associated monetary policy actions.

ii) regime-switching. The effects of fiscal fundamentals on the term structure are not constant over time. The impact of current deficit and debt on the term structure could be different under different expectations for the full path of future fiscal fundamentals (see, for example, Gale and Orszag (2003) or Engen and Hubbard(2004)) or under different attitude towards risk in international bond markets (see, for example,Codogno et al (2004).

iii) Maturity effects. The response to fiscal policy of the yield curve could be a function of maturity and modelling empirically the response of entire yield curve to fiscal fundamentals is a complicated task (see Dai and Philippon(2005) for a recent attempt).

In this paper we use the evidence from the Italian case to address the first issue explicitly, we shall then deal with the second and third issues using respectively a regime switching econometric model and a factor model of the term structure.

Starting from the choice of Italian data note first that the evolution of Italian

[1] See Engen and Hubbard (2004) for a review of the main empirical findings to date.

fiscal fundamentals over time offers definitely some variability. As shown in Figure 1, from the beginning of the nineties to now (the period in which we have a well established market for government bonds and reliable data on the yield curve) the debt to GDP ratio and the deficit to GDP ratio went through different regimes with a strong expansion of the debt to GDP ratio in the first part of the sample followed by a contraction in the central part of the sample and a stabilization in the late observations.

Second, the existence of EMS in the first part of the sample and of EMU from 1999 onwards allows us to measure the impact of Italian fundamentals on the yield curve by considering spreads of Italian yields on German yields. Germany is the reference country for Europe on which the country risk premium is negligible and which was the clear leader in setting monetary policy during the EMS. As a consequence the identification of the effect of fiscal policy on the Italian yield curve should be made easier by analyzing the impact of fiscal variables on spreads on German rates. We believe that this choice helps us to isolate the effects of fiscal policy from the effects of the business cycle and associated monetary policy actions.

Third, we use the Nelson-Siegel decomposition of the yield curve in three factors, to extract at each sample point three factors to model the entire Italian and German term structure . We then follow Diebold and Li (2003) and apply time-series techniques on the NS factors to forecast bond-yield differentials between Italy and Germany and we explicitly model these factors as functions of macroeconomic and fiscal variables (*debt/GDP ratio* and its change over time).

Finally, we adopt an econometric regime switching technique in which we use some discrete-state hidden Markov variables to represent regimes of various sort in the economy. Since one of the main problems in studying fiscal policy is to understand the role of the expectations, we explicitly include nonobservable market expectations, modeling them as a hidden two-state Markov chain that influences the effects of the fiscal variables on the yield curve at any point in time. In particular, the hidden Markov variable is in state 1 when markets beliefs are pessimistic on fiscal fundamentals and the country is in a high-risk situation. When instead the hidden Markov chain is in state 0, markets beliefs are consistent with a low-risk scenario . This obviously introduces a strong nonlinearity in the model. The need for such nonlinearity can be seen once again from Figure 1, which plots together the spread between the Italian and German long-term component of the yield curve, the

Italian debt/GDP ratio and its change over time.

[**Insert Figure 1 here**]

Importantly, our modeling approach allows us to forecast the evolution of the yield curve in relation with different scenarios for the main economic variables, together with different regimes, and to compare the predicted curves under the alternative regimes. We estimate our model with data ranging from 1991 to 2006, a period of time that on the one hand is sufficiently long to allow a complete estimation of the model, and on the other hand presents evidence of the existence of different regimes. We employ Bayesian econometric techniques, which treat all the parameters in the model *and* the hidden Markov chain as random variables on which we assume a prior and obtain a posterior distribution at the end of the estimation process.

The rest of this paper is organized as follows: Section 2 presents our empirical model, together with the estimation techniques, Section 3 discusses the results, Section 4 presents some forecasting issues and Section 5 concludes. The Appendix reports extensive details on the Bayesian estimation methods employed.

2 The empirical model

In this section we present the empirical framework we use to study the effect of the fiscal policy variables on the yield curve. First we discuss how we model the German and the Italian yield curves then using three factors, then we illustrate our specification for the relation between factors macroeconomic and fiscal variables.

2.1 The yield curve and the Nelson-Siegel decomposition

Our basic data set consists of a set of zero-coupon equivalent German and Italian yields, measured at the following maturities[2]: 3-month, 6-month, 1-year, 2-year, 3-year, 5-year, 7-year, 10-year.

From this data set we construct financial factors by estimating at each point of our time series t, by non-linear least squares, on the cross-section of eight yields, the following Nelson-Siegel model:

[2]The data were extracted from DataStream, and they are posted on Favero's website at the following address: http:/www.igier.uni-bocconi.it/personal/favero
in the section working papers

$$y_{t,t+k} = L_t + SL_t \frac{1-\exp\left(-\frac{k}{\tau_1}\right)}{\frac{k}{\tau_1}} + C_t\left(\frac{1-\exp\left(-\frac{k}{\tau_1}\right)}{\frac{k}{\tau_1}} - \exp\left(-\frac{k}{\tau_1}\right)\right) \quad (1)$$

which is implicit in the instantaneous forward curve:

$$f_{tk} = L_t + SL_t \exp\left(-\frac{k}{\tau_1}\right) + C_t \frac{k}{\tau_1} \exp\left(-\frac{k}{\tau_1}\right) \quad (2)$$

The parameter τ_1 is kept constant over time[3], as this restriction decreases the volatility of the L_t, SL_t, C_t,making them more predictable in time. As discussed in Diebold and Li (2002) the above interpolant is very flexible and capable of accommodating several stylized facts on the term structure and its dynamics. In particular, L_t, SL_t, C_t, which are estimated as parameters in a cross-section of yields, can be interpreted as latent factors. L_t has a loading that does not decay to zero in the limit, while the loading on all the other parameters do so, therefore this parameter can be interpreted as the long-term factor, the level of the term-structure. The loading on SL_t is a function that starts at 1 and decays monotonically towards zero; it may be viewed a short-term factor, the slope of the term structure. In fact, $f_{1t} = L_t + SL_t$ is the limit when k goes to zero of the spot and the forward interpolant. We naturally interpret f_{1t} as the risk-free rate or the monetary policy instrument. Obviously SL_t, the slope of the yield curve, is nothing else than minus the spread in Campbell-Shiller. C_t is a medium term factor, in the sense that its loading starts at zero, increases and then decays to zero. Such factor captures the curvature of the yield curve. In fact, Diebold and Li show that it tracks very well the difference between the sum of the shortest and the longest yield and twice the yield at a mid range (2-year maturity). The three loadings are plotted in Figure 2.

[**Insert Figure 2 here**]

The repeated estimation of loadings using a cross-section of yields at different maturities allows to construct a time-series for our factors. We report in Figure 3 the three factors for Germany and Italy, while Figure 4 shows the goodness of fit of the Nelson and Siegel interpolation for all yields considered in our sample.

[3] We restrict τ_1 at the value of 0.87, which is the median, over the time series, of the estimated value of τ_1 in a four parameter version of the Nelson-Siegel interpolant.

[**Insert Figure 3 here**]

[**Insert Figure 4 here**]

In the remaining part of the work we will be referring to three "factors":

- $f_{1t} = L_t + SL_t$, the monetary policy instrument
- $f_{2t} = L_t$, the long term component
- $f_{3t} = C_t$, the medium term component

The aim of our work will be to assess the impact of fiscal policy variables and market expectations on each of these factors and to evaluate the use of these variables for predicting future values for the three factors.

Note that there are a number of alternatives to the approach that we have followed in extracting factors. In particular, Nelson and Siegel factors are not in general consistent with no-arbitrage and an alternative path to extracting factors is the estimation of a no-arbitrage term-structure model with fiscal indicators (see, Dai-Philippon(2005)). We favoured the NS approach because of its established superior forecasting performance (Diebold-Li(2005)) and because of the feasibility of the regime-switching analysis within this framework. We feel that having a model capable of accommodating regime switches is vital to identify the response of the yield curve to fiscal policy and such a step is practically impossible to implement within a no-arbitrage model of the term structure. Of course, as a consequence of this strategy care must be exercised in the interpretation of the extracted factors, as the no-arbitrage conditions have not been imposed to identify them.

2.2 The macroeconomic framework

The empirical model we present is based on the idea that by concentrating on the yield spreads between Italian and German bonds we shall be able to identify how market expectations and fiscal policy affect the Italian yield curve by changing the risk premium markets impose on bond yields. In particular, we concentrate on the difference between factors for the Italian and German yield curve relying upon the fact that Germany has been the reference country for monetary policy during the EMS with fundamentals always in line with the Maastricht Treaty; in the EMU period, with a common monetary policy, Germany has continued to be the reference country for Italy for the medium and the long end of the yield curve. We shall therefore concentrate on the difference between the monetary policy factor, the medium term factor and the level of the Italian and German yield curve.

In modelling the response of the yield curve to fiscal fundamentals we will allow explicitly for a regime switching model. In particular we focus on two regimes: a high-risk regime and a low-risk regime. This two regimes will be represented as an unobserved Markov chain, S_t, with only two states: 0 and 1. When the variable is in state 1, we are in the high risk regime. Notice that since we model S_t as a Markov chain, the information on future market expectations is all contained in S_t, and does not depend on what happened before. The probability of a regime switching depends on the current state and is completely determined once we know the transition probabilities of the Markov chain. The transition matrix will be:

$$\begin{array}{c|cc} & 0 & 1 \\ \hline 0 & q & 1-q \\ 1 & 1-p & p \end{array}$$

2.2.1 The first factor: the monetary policy instrument

Following most literature, the monetary policy instrument is modeled as a Taylor rule, where the monetary authorities set the short term rate based on inflation and the output gap. However, we modify the traditional Taylor rule in two directions. First of all, we model the *spread* between the Italian and the German instrument as the dependent variable, since we aim to explain the difference between the monetary policies of the two countries. Second, we make the rule depend on an exogenous monetary regime R_t, that coincides with the EMU regime (0 when the two countries have a unique monetary authority, 1 when they are separate; the reason for this choice is an easier comparison with the market expectations regime, S_t).

In our model, the spread between the Italian and the German instrument depends on a constant term, an autoregressive term, the spread between the Italian and the German inflation (supposing an implicit targeting of the German inflation) and the Italian output gap.

The first equation will be:

$$f_{1t} - f_{1t}^g = \alpha^R + \gamma_1^R(f_{1t-1} - f_{1t-1}^g) + \gamma_2^R(\pi_{t-1} - \pi_{t-1}^g) + \gamma_3^R(y_{t-1} - y_{t-1}^N) + e_{1t}$$

where f_1 represents the first factor in Italy, f_1^g the first factor in Germany, α^R is the monetary-regime-dependent constant of the equation, π is Italian inflation, π^g is German inflation, $(y - y^N)$ is the Italian output gap, and e_1 is a random error with mean 0 and variance σ_1^2. Notice that all the parameters depend on the exogenous monetary regime R_t. In the pre-EMU period the Italian reaction function is specified as that of a small open-economy Taylor-rule where the central bank implements the objective of exchange rate stability by defining the target values of the macroeconomic variables as those assumed by these variables in the reference country . Importantly the long-run equilibrium policy rates for Italy is equal to the German policy rates, but discrepancies between the two rates might occur in the short-run. The EMU period is instead characterized by interest rate equalization between Germany and Italy and the monetary policy instrument becomes a purely exogenous variable in the context of our adopted model.

2.2.2 The second factor (the long-term component) and the third factor (the medium-term component)

In order to concentrate on the effects of fiscal policy on the long term and medium term components of the yield curve, we use only two fiscal variables to explain the difference between the Italian and German factors: the *level* of the debt/GDP ratio over time and its *variation*. The *variation* of the debt/GDP ratio represents the direction of its evolution, and can be interpreted as the difference between the actual deficit/GDP ratio and the one that would stabilize the debt/GDP ratio. We model this relationship quite flexibly, allowing each of the variables to impact on the factors differently, depending on the regime of market expectations. This approach allows us to leave the burden of stating the relative importance of the variables to the data, while retaining a framework that allows for a direct economic interpretation of the results.

The second and third equations of our model will be:

$$\begin{aligned} f_{2t} - f_{2t}^{g} &= \delta_1^S (D_t - D_{t-1}) + \delta_2^S (D_t - 60) + e_{2t} \\ f_{3t} - f_{3t}^{g} &= \theta_1^S (D_t - D_{t-1}) + \theta_2^S (D_t - 60) + e_{3t} \end{aligned}$$

where f_2 and f_3 are the second and third factors for Italy, f_2^g and f_3^g are the ones for Germany, D_t is total Italian debt over GDP at time t, and e_2 and e_3 are errors with mean 0 and variances σ_2^2 and σ_3^2. The first regressor in the two models is defined the difference between current budget surplus and the debt stabilizing surplus. This term penalizes divergent debt dynamics but it is silent on the relative penalization of different level of the debt to GDP ratio. To this end we include a second regressor in which the Maastricht reference criterion is adopted to pin down 60 per cent as the optimal level for the debt-GDP ratio [4].

To estimate the model presented above we use a Bayesian approach. In particular, we approximate the joint posterior distribution of the parameters *and* the hidden Markov chain S_t using Gibbs sampling techniques. In this section we present the theory of Gibbs sampling and Bayesian estimation as they can be applied to estimating our model. Detailed technical details are provide in the Appendix.

2.3 The Bayesian approach

The estimation of our model requires computing marginal posterior distributions for the 16 parameters of the model (each parameter subject to switching regime actually consists of two parameters, one for each regime), the 3 variances, the transition matrix of the Markov chain and the T periods of the Markov chain. Obtaining these distributions analytically by integrating a large joint posterior distribution is a nearly impossible task.

To estimate our model we therefore use an approximation technique, the Gibbs sampler. This is a Markov Chain Monte Carlo simulation method for approximating joint and marginal distributions by sampling from conditional distributions. In particular, suppose that we are interested in obtaining the marginal posterior dis-

[4] Other empirical studies (see for example Ardagna, Caselli and Lane 2004) introduce quadratic terms and interactions among these variables to capture nonlinear effects. In our model, however, nonlinear effects are already captured by the hidden Markov chain, and adding those terms would strongly interefere with its estimation. Consequently, we leave all nonlinearities to S_t.

tribution

$$f(z_j \mid y) = \int \cdots \int f(z_1, ..., z_k \mid y) dz_1 ... dz_{j-1} dz_{j+1} ... dz_k$$

the marginal distribution of the variable $z_j \mid y$ from the joint distribution $f(z_1, ..., z_k \mid y)$. The idea of the Gibbs sampling is that we can obtain the marginal distribution as the limit of a procedure in which it is *not* required to know the joint distribution nor to integrate it. Instead, we just need to know the distribution of each variable conditional on the other variables. Note that since we use conditional *posterior* distributions, we need to obtain them from conditional prior distributions using standard Bayesian techniques before starting the sampler.

Given an arbitrary set of starting values for the remaining $k-1$ variables, $(z_2^0, ..., z_k^0)$, the procedure goes as follows:

1. Draw z_1^1 from the known conditional distribution $f(z_1 \mid z_2^0, ..., z_k^0, y)$
2. Then, draw z_2^1 from the known conditional distribution $f(z_2 \mid z_1^1, z_3^0 ..., z_k^0, y)$
3. Then, draw z_3^1 from the known conditional distribution $f(z_2 \mid z_1^1, z_2^1, z_4^0 ..., z_k^0, y)$

   $\vdots$

4. Finally, draw z_k^1 from the known conditional distribution $f(z_k \mid z_1^1, ..., z_{k-1}^1, y)$

At this point, we have drawn a whole set of values $(z_1^1, ..., z_k^1)$, which constitutes the first iteration of the procedure. Now, iterating steps 1 to 4 J times, always using the most recent draws when conditioning, gives us J sets $(z_1^j, ..., z_k^j)$. As Geman and Geman (1984) have shown, the joint and marginal distributions of $(z_1^j, ..., z_k^j)$ converge at an exponential rate to the joint and marginal posterior distributions of $z_1, ..., z_k$ as $J \to \infty$. Therefore, the empirical distributions obtained by iterating the Gibbs sampler enough times can be used as an approximations of the joint and marginal distributions of the parameters.

To insure the convergence of the sampling procedure, usually the first M results of the sampler are discarded, and only the remaining $J - M$ draws are used to approximate the joint and marginal distribution.

2.4 The Gibbs sampler applied to the model

As explained earlier, the model can be summarized by the following equations (where the superscript S indicates the dependence of the parameter on the *unobserved*

Markov chain S_t whose states can be 0 or 1, and the superscript R indicates the dependence on the *known* variable R_t representing the monetary regime).

$$\begin{aligned}
f_{1t} - f_{1t}^g &= \alpha^R + \gamma_1^R(f_{1t-1} - f_{1t-1}^g) + \gamma_2^R(\pi_{t-1} - \pi_{t-1}^g) + \gamma_3^R(y_{t-1} - y_{t-1}^N) + e_{1t} \\
f_{2t} - f_{2t}^g &= \delta_1^S(D_t - D_{t-1}) + \delta_2^S(D_t - 60) + e_{2t} \\
f_{3t} - f_{3t}^g &= \theta_1^S(D_t - D_{t-1}) + \theta_2^S(D_t - 60) + e_{3t}
\end{aligned}$$

Every time a coefficient depends on the hidden Markov variable S_t, or on the observable EMU regime variable R_t, this dependence can be expressed as the sum of two components. For example, γ_1^S can be written, expliciting the hidden Markov variable, as $\gamma_1^0 + \gamma_1^1 S_t$. We can therefore rewrite the model as:

$$\begin{aligned}
f_{1t} - f_{1t}^g &= \alpha^0 + \alpha^1 R_t + \gamma_1^0(f_{1t-1} - f_{1t-1}^g) + \gamma_1^1(f_{1t-1} - f_{1t-1}^g)R_t + \gamma_2^0(\pi_{t-1} - \pi_{t-1}^g) \\
&\quad + \gamma_2^1(\pi_{t-1} - \pi_{t-1}^g)R_t + \gamma_3^0(y_{t-1} - y_{t-1}^N) + \gamma_3^1(y_{t-1} - y_{t-1}^N)R_t + e_{1t} \\
f_{2t} - f_{2t}^g &= \delta_1^0(D_t - D_{t-1}) + \delta_1^1(D_t - D_{t-1})S_t + \delta_2^0(D_t - 60) + \delta_2^1(D_t - 60)S_t + e_{2t} \\
f_{3t} - f_{3t}^g &= \theta_1^0(D_t - D_{t-1}) + \theta_1^1(D_t - D_{t-1})S_t + \theta_2^0(D_t - 60) + \theta_2^1(D_t - 60)S_t + e_{3t}
\end{aligned}$$

where the variance-covariance matrix of the error is:

$$Q = \begin{bmatrix} \sigma_1^2 & 0 & 0 \\ 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_3^2 \end{bmatrix}$$

since the errors of the various equations are assumed to be independent.

Notice that while R_t is known and therefore enters the model simply as a *dummy* variable, the chain S_t is not observed and must therefore be estimated along with the other parameters: it is the presence of S_t to introduce nonlinearities in the model.

Using the Bayesian approach to the estimation problem, and considering that we observe all the exogenous variables except for S_t, the random variables on which we have a prior distribution and for which we want to obtain a posterior distribution, are:

- The time series $S_1..., S_T$, that we will indicate as $\tilde{S}_T$
- The transition values of the Markov chain (in particular, we need only two of them to represent the whole transition matrix: p and q)

- $C_1 = \{\alpha^0, \alpha^1, \gamma_1^0, \gamma_1^1, \gamma_2^0, \gamma_2^1, \gamma_3^0, \gamma_3^1\}$: the coefficients of the first equation
- $C_2 = \{\delta_1^0, \delta_1^1, \delta_2^0, \delta_2^1\}$: the coefficients of the second equation
- $C_3 = \{\theta_1^0, \theta_1^1, \theta_2^0, \theta_2^1\}$: the coefficients of the third equation
- $\sigma_1^2, \sigma_2^2, \sigma_3^2$: the three variances of the equation errors

To estimate the model, we use the Gibbs sampling technique by generating draws from the conditional posterior distribution of each of the variables (or groups of variables) above conditional on the values of the other variables. To insure the convergence of the Gibbs sampler, we run it for $15,000$ iterations, discarding the first $5,000$.

The details of the procedure to obtain the conditional posterior distribution for the parameters above are left for the Appendix. While some of them (such as the variances or the coefficients) employ standard Bayesian techniques, others, such as drawing a value for the entire Markov chain S_t, require special calculations.

2.5 The data

To estimate our model, we use quarterly data ranging from 1991:2 to 2006:1. The source for the Nelson-Siegel factor series are the yields of Italian and German bonds as reported by DataStream (for the short term rate, series ECITLxM and ECWG-MxM where x=3,6,12 ; for the longer term, series BMITxxY and BMBDxxY where xx=02, 03, 05, 07, 10). After the bootstrap procedure to extract zero-coupon yields, we estimate the Nelson-Siegel factors for each of the 60 time periods of our sample, thus obtaining the time series for the three factors from 1991:2 to 2006:1.

The output gap and the debt/GDP ratio series for Italy were constructed by interpolating the annual OECD data, while for Italian and German inflation (CPI) quarterly data were directly available from the OECD dataset.

3 Results

In this section we discuss the main estimation results, stressing in particular the importance of market expectations and fiscal policy.

For each coefficient of the three equations that we modeled as regime-dependent we estimate two different quantities. The first one, with superscript 0, is the estimate of the effect of the variable on the factor when the regime (R_t for the monetary instrument equation and S_t for the long and medium term equations) is 0. The second one, with superscript 1, is the estimate of the variation of the effect of the variable due to the switch of regime to 1. For example, for the coefficient δ_2, which measures the effect of the debt/GDP ratio on the long-term spread, we estimate two separate parameters. δ_2^0 measures the effect of a change of the the debt/GDP ratio on the spread when markets expect a low-risk situation ($S_t = 0$). δ_2^1 measures the variation of this effect due *exclusively* to the regime change, i.e. if markets begin not to trust the future of the Italian economic situation (switch from $S_t = 0$ to $S_t = 1$). Thus, the total effect of the debt/GDP ratio on the long-term spread when $S_t = 1$ can be calculated as $\delta_2^0 + \delta_2^1$.

We also generate estimates for the historical realization of the Markov chain S_t. This allows us to assess market expectations about the economy for each point in time from 1991 to 2006.

3.1 The monetary policy instrument

The following Table shows the Bayesian estimates for the parameters of the first equation. For each parameter we report the mean and standard deviation of the prior distribution used and the mean (estimate) and standard deviation of the marginal posterior distribution. Notice that when the mean or SD of a prior distribution is not reported, a noninformative prior was used. All estimates in bold are significant at the 5% level.

	Prior		Posterior	
	Mean	SD	Mean	SD
α^0	0	0.01	-0.0003	0.0017
α^1	0.01	0.01	0.0083	0.0045
γ_1^0	0	0.3	-0.04	0.22
γ_1^1	0.8	0.3	**0.76**	0.23
γ_2^0	0	0.03	0.007	0.029
γ_2^1	0.1	0.03	**0.11**	0.03
γ_3^0	0	0.03	-0.0005	0.0018
γ_3^1	0.1	0.03	0.0005	0.0024
σ_1^2	-	-	**0.00008**	0.00001

Remember that the first equation was:

$$f_{1t} - f_{1t}^{g} = \alpha^{R} + \gamma_1^{R}(f_{1t-1} - f_{1t-1}^{g}) + \gamma_2^{R}(\pi_{t-1} - \pi_{t-1}^{g}) + \gamma_3^{R}(y_{t-1} - y_{t-1}^{N}) + e_{1t}$$

and that the regime it refers to is the (observable) EMU regime.

As we can see from the table, we have a clear differentiation of the equation depending on the monetary regime. When $R_t = 0$ (EMU regime), there is only one monetary authority, policy rates for Germany and Italy coincide and, as we expect, all the parameters with superscript 0 are not significantly different from 0. Outside the EMU, when $R_t = 1$, we find a strong coefficient for the persistence of the spread between the monetary instruments of Italy and Germany ($\gamma_1^1 + \gamma_1^0 = 0.72$). Besides, we find evident signs of a pure inflation targeting, since the coefficient for the difference between the Italian and German inflation is positive and significant while the coefficient for the output gap is not. Remember, however, that what is explained here is not the level of the Italian monetary instrument, but the spread between the Italian and the German one. The results indicate that, before the EMU, an increase of the Italian inflation over the level of the German inflation would prompt the Italian authorities to react increasing the overnight interest rate relative to the level of the German one. The fact that the long-run response of the monetary policy differential with respect to the inflation differential is less than one does not imply that the Italian central bank has not been following the Taylor principle of tightening monetary policy so as to cause an increase in real policy rates in presence of an inflationary shocks, it just tell us that the Bundesbank has been more aggressive toward inflation than the bank of Italy.

3.2 The second factor: the long-term component or level of the yield curve

The following Table shows the estimates of the parameters of the second equation:

	Prior		Posterior	
	Mean	SD	Mean	SD
δ_1^0	0	0.1	0.004	0.003
δ_1^1	0.1	0.1	0.0042	0.0024
δ_2^0	0	0.1	**0.015**	0.004
δ_2^1	0.1	0.1	**0.043**	0.005
σ_2^2	-	-	**0.00006**	0.00002

Remember that the equation was:

$$f_{2t} - f_{2t}^g = \delta_1^S(D_t - D_{t-1}) + \delta_2^S(D_t - 60) + e_{2t}$$

Fiscal policy seems to be a very significant factor in explaining long term yields. The level of the debt/GDP ratio is significant in explaining the long-term spread of the two yield curves, with a positive effect under both states of market expectations. Whenever the debt/GDP ratio increases, markets react increasing the risk premium on the long-term component of the curve.

Notice however the large and significant effect that market expectations have on determining the size of the effect of government debt on bond yields. While the coefficient in the situation of low perceived risk is only $\delta_2^0 = 0.015$, the coefficient when markets are worried is $\delta_2^0 + \delta_2^1 = 0.058$, four time as much as the case $S_t = 0$. When markets are worried and perceive the situation as high-risk, the weight given to fiscal policy in determining the risk premia increases noticeably.

We can also interpret the results considering the effect of a regime switch keeping everything else constant (*including* the debt/GDP ratio). If markets become worried about the potential negative consequences of the government's fiscal policy, and thus their expectations change to $S_t = 1$, the long term risk premium increases by 0.43 percentage points *for every* 10 percentage points in the Italian debt/GDP ratio. Since yields at longer maturities depend almost completely on this factor, this means that these coefficients capture most of the effect of fiscal policy at long maturities. Besides, since the second factor is the level of the yield curve, it means that the *whole* yield curve is shifted above by 0.43 percentage points. For short maturities, instead, one must take into account that fiscal policy has an indirect additional effect through the other factors, that may increase or decrease the size of this relation.

A similar argument holds for the change in the debt/GDP ratio. In this case, however, even if the estimates are positive (meaning that if the government sets a deficit higher than the stabilizing one, markets react increasing the risk premia), they are not very significant, being the coefficient for the case $S_t = 1$ significant only at the 10% level.

3.3 The third factor: the medium-term component or curvature of the yield curve

The Table below presents the estimates for the parameters of the third equation:

$$f_{3t} - f_{3t}^{g} = \theta_1^S (D_t - D_{t-1}) + \theta_2^S (D_t - 60) + e_{3t}$$

	Prior		Posterior	
	Mean	SD	Mean	SD
θ_1^0	0	0.1	0.014	0.008
θ_1^1	0.1	0.1	-0.008	0.012
θ_2^0	0	0.1	0.007	0.010
θ_2^1	0.1	0.1	**0.076**	0.014
σ_3^2	-	-	**0.00040**	0.00009

The estimates clearly indicate that in periods when markets fear an unsustainable behavior of the Italian policymakers, the weight given to fiscal policy is significant, although not as large as the one for the level (second factor). In particular, notice the estimate of 0.076 for the additional effect of the debt/GDP ratio on the medium term rates in periods of worried markets. Translating it to the effect on the yield curve, it means that, for every 10 percentage points in the Italian debt/GDP ratio, markets increase the risk premium, in the high-risk scenario, by around 0.25 percentage points in the medium term range (3 - 4 years maturities).

Notice that medium term yields bear the summed effects of the second and third factor: combining them, we can see that an increase of the debt/GDP ratio by 10 percentage points results in a total increase of about 0.7 percentage points for the medium-term yields. Summing these effects up, it appears that fiscal policy is extremely relevant in explaining the medium-term risk premium, and market expectations are a very significant factor in determining this effect.

3.4 Market expectations

Figure 5 below shows (solid line) the estimate of the states of the Markov chain S_t in each period. In each iteration of the Gibbs sampler, a whole realization of the chain was generated randomly from its posterior distribution, and the estimate corresponds to the mean of the empirical distribution.

[**Insert Figure 5 here**]

Remember that $S_t = 1$ when markets put a higher risk premium on long term and medium-term rates in response to fiscal policy. Notice how Italy was perceived to be in a high-risk situation until 1997 (with a decrease between 1993 and 1994), while since then it has converged to a low-risk situation, that is still going on today, in which markets do not penalize the fiscal policy of Italian governments with higher risk premia.

The same Figure plots the exogenous monetary regime (EMU) R_t with a dotted line. What emerges comparing the two regimes is the extent to which markets anticipated in 1997 the positive effects of the convergence that brought Italy to the EMU in 1999. The expectations regime switches to the low-risk situation two years before the monetary regime switch.

Finally, the following table reports the estimated transition matrix for the Markov chain S_t.

	0	1
0	0.94	0.06
1	0.16	0.84

It is interesting to note that, while both states are heavily persistent, the probability of switching from the bad-case situation to the good one is more than twice as much as the opposite. That is, persistence is lower in the high-risk situation.

As we can see from the Figures above, nowadays both the monetary regimes and the expectations regime are in a favorable state (zero). However, in the future we may see changes of one, or both, regimes. The effects of these switches on the Italian yield curve, even if all the other variables remained equal, would be very large. The next section explores this issue.

4 A forecast exercise

In this section we use the estimated model to forecast the evolution of the Italian yield curve up until 2007:4 under various scenarios for the regimes.

In order to forecast future Italian yields, we must make projections for the macroeconomic variables and the German factors into the future, since all these variables are exogenous in the model. To do that, we use OECD forecasts for the variables for which they are available (Italian inflation, italian output gap, and italian debt) while we project the values as of 2006:1 to the future for the other variables. We then use these predicted future paths to generate the future italian yield curve under different scenarios for S_t and R_t.

Our forecast exercise aims more to underline how market expectations affect the yield curve, than to predict its actual future evolution, since we do not have reliable forecasts for all the exogenous variables. However, it is an important exercise, because it shows how, *all the exogenous variables being equal across the different scenarios,* the underlying monetary and expectations regimes affect the evolution of the yield curve.

We build our scenarios combining different values for the two regimes. In the good-case scenario ($R_t = 0, S_t = 0$), Italy remains in the EMU and markets have low-risk expectations about the Italian economic situation. Since the Italian debt is projected to rise, we expect only a slight increase of the long-term component of the yield curve, while we expect the short-run component to remain stable.

In the second scenario, Italy is in the EMU but markets become worried and expect a high-risk situation ($R_t = 0, S_t = 1$), so that the short-term component is still tied to the German one but the long-term component will reflect the change of market perceptions.

The third scenario is the bad-case scenario in which Italy exits the EMU and markets switch to a high-risk situation. We ignore the case where Italy exits the EMU but markets remain in a low-risk situation as very little realistic

Figure 6 below plots together the current Italian curve and the predicted curves for Germany and Italy under the three scenarios.

[**Insert Figure 6 here**]

As we can see, in the cases in which Italy remains in the EMU, market expectations play a major role in determining the long-term component of the yield curve (more than 2 percentage points). In the bad-case scenario, the monetary instrument jumps at 6%, and the rest of the curve is shifted upwards due to the negative expectations, reaching yields of 8% for the medium-term maturities.

In the following graph (Figure 7) we plot the evolution of the yield curve from 2006:1 to 2007:4 in the bad-case scenario (i.e. imagining that since 2006:2 Italy exits the EMU and markets switch to the high-risk regime). After an initial jump upwards due to the regime change, the short-term rate increases steadily responding to the projected path for inflation, and so does the long-term component in response to the evolution of debt.

[**Insert Figure 7 here**]

5 Conclusions

To study the impact of fiscal policy and the yield curve we have considered NS factors and modelled the difference between the Italian and German yield curves as a function of macroeconomic and fiscal policy variables. We modelled the nonlinearity in the relation between the yield curve and fiscal fundamentals with a regime switching model, where different regimes are captured by a nonobservable Markov chain with two possible states. When the state is 1 (according to our estimates, before 1996-97) markets put a higher risk premium on government bonds, especially at longer maturities. When the state is 0 (after 1996-97) markets perceived to be in a low-risk regime.

Our results clearly show that fiscal fundamentals help in forecasting the yield curve. However, the use of hidden Markov chains as a way of capturing changes in regime is crucial to pin down these effects. Within this framework, Bayesian econometric methods provide us with a very efficient estimation procedure (an application of the Gibbs sampling), to tackle the problem of the underlying nonlinearity.

6 Appendix A: Econometric Details

In this appendix we show how to obtain the conditional posterior distribution of each parameter (or group of parameters as defined in Section 3), explicitly stating the corresponding prior we used and how we obtained the posterior used in the Gibbs sampler to draw a value in each iteration.

6.0.1 Generating $\tilde{S}_T$ conditional on $p, q, C_1, C_2, C_3, \sigma_1^2, \sigma_2^2, \sigma_3^2$ and the data.

Generating $\tilde{S}_T$ from the conditional distribution is the most complicated part of the process, because the chain is completely unobserved and is therefore considered as a series of T random variables. In particular, we want to extract a value from the conditional distribution

$$g(\tilde{S}_T \mid p, q, C_1, C_2, C_3, \sigma_1^2, \sigma_2^2, \sigma_3^2, \tilde{y}_T)$$

where $\tilde{y}_T$ represents the data up to time T (in our case, the vector of the three factors).

Suppressing for ease of notation the dependence on the various parameters, we want to obtain:

$$g(\tilde{S}_T \mid \tilde{y}_T)$$

In order to do that, we follow the procedure presented in Kim and Nelson (1998):

1. First of all, obtain the series $\Pr\{S_t \mid \tilde{y}_t\}$ for every t. This can be achieved using the Hamilton filter.

2. Then, starting from the value of S_T generated from $\Pr\{S_T \mid \tilde{y}_T\}$, obtain the whole series $\tilde{S}_T$ generated recursively from $\Pr\{\tilde{S}_T \mid \tilde{y}_T\}$.

The two steps are described below.

The Hamilton filter Let ψ_t be the information set at time t. The Hamilton filter is a procedure that allows to recursively obtain the series $\Pr\{S_t \mid \psi_t\}$ given the set of parameters and the series $\eta_{t|t-1}^{i,j}$ and $v_{t|t-1}^{i,j}$, respectively the forecast error and variance, which are used to calculate the likelihood function of the data.

The filter is initialized using the steady-state probability of the Markov chain, and proceeds as follows, for each t, and for each i and j:

1. Given $\Pr\{S_{t-1} = i \mid \psi_{t-1}\}$ calculate:

$$\Pr\{S_{t-1} = i, S_t = j \mid \psi_{t-1}\} = \Pr\{S_{t-1} = i \mid \psi_{t-1}\} \cdot \Pr\{S_t = j \mid S_{t-1} = i\}$$

where the last term is the known transition probability (on which we are conditioning when generating $\tilde{S}_t$).

2. When y_t becomes available at the end of time t, we can update the probability in the following way:

$$\begin{aligned}&\Pr\{S_{t-1}=i,S_t=j\mid\psi_t\}=\Pr\{S_{t-1}=i,S_t=j\mid\psi_{t-1},y_t\}\\ =\;&\frac{g(y_t\mid S_{t-1}=i,S_t=j,\psi_{t-1})\cdot\Pr\{S_{t-1}=i,S_t=j\mid\psi_{t-1}\}}{g(y_t\mid\psi_{t-1})}\end{aligned}$$

where the likelihood $g(y_t \mid S_{t-1}=i, S_t=j, \psi_{t-1})$ can be calculated obtaining the forecast error and variance $\eta^{i,j}_{t|t-1}$ and $v^{i,j}_{t|t-1}$ and applying the decomposition of the likelihood function as:

$$\begin{aligned}g(y_t \quad | \quad & S_{t-1}=i,S_t=j,\psi_{t-1})=\\ = \quad & (2\pi)^{-\frac{1}{2}}\mid v^{i,j}_{t|t-1}\mid^{\frac{1}{2}}\exp\{-\frac{1}{2}\left(\eta^{i,j}_{t|t-1}\right)'\left(v^{i,j}_{t|t-1}\right)^{-1}\left(\eta^{i,j}_{t|t-1}\right)\}\end{aligned}$$

and where

$$g(y_t\mid\psi_{t-1})=\sum_{i=1}^{2}\sum_{j=1}^{2}\left[g(y_t\mid S_{t-1}=i,S_t=j,\psi_{t-1})\cdot\Pr\{S_{t-1}=i,S_t=j,\psi_{t-1}\}\right]$$

3. Finally, obtain $\Pr\{S_t=j\mid\psi_t\}$ as $\sum_i\Pr\{S_{t-1}=i,S_t=j\mid\psi_t\}$ and repeat from step 1.

Generating $\tilde{S}_t$ - the Kim filter With the Hamilton filter we have obtained the whole series of $g\{S_t\mid\psi_t\}$, or $g\{S_t\mid\tilde{y}_t\}$. We can now recursively generate the whole series $S_1,...,S_T\mid\tilde{y}_T$ using the following result:

$$\begin{aligned}g(\tilde{S}_T\mid\tilde{y}_T) &= g(S_1,...,S_T\mid\tilde{y}_T)\\ &= g(S_T\mid\tilde{y}_T)\cdot g(\tilde{S}_{T-1}\mid S_T,\,\tilde{y}_T)\\ &= g(S_T\mid\tilde{y}_T)\cdot g(S_{T-1}\mid S_T,\,\tilde{y}_T)\cdot g(S_{T-2}\mid S_{T-1},\;S_T,\;\tilde{y}_T)\cdot...\cdot\\ &\quad\; g(S_1\mid S_2,...,S_{T-1},\;S_T,\;\tilde{y}_T)\\ &= g(S_T\mid\tilde{y}_T)\cdot g(S_{T-1}\mid S_T,\,\tilde{y}_{T-1})\cdot g(S_{T-2}\mid S_{T-1},\;\tilde{y}_{T-2})\cdot...\cdot\\ &\quad\; g(S_1\mid S_2,\;y_1)\\ &= g(S_T\mid\tilde{y}_T)\cdot\prod_{t=1}^{T-1}g(S_t\mid S_{t+1},\;\tilde{y}_t)\end{aligned}$$

where the derivation depends on the Markov property of S_t: conditional on S_t, all values of S_t from $t+1$ to T, and all values of y_t from t to T , contain no information beyond that in S_t.

The Kim filter starts generating S_T from the distribution obtained as the last step of the Hamilton filter. We then proceed backwards to generate each S_t from $g(S_t \mid S_{t+1},\ \tilde{y}_t)$, using the S_{t+1} generated in the preceding iteration. In particular, we generate it using:

$$\begin{aligned} g(S_t \mid S_{t+1},\ \tilde{y}_t) &= \frac{g(S_t,\ S_{t+1} \mid\ \tilde{y}_t)}{g(S_{t+1} \mid\ \tilde{y}_t)} \\ &= \frac{g(S_{t+1} \mid S_t, \tilde{y}_t) \cdot g(S_t \mid \tilde{y}_t)}{g(S_{t+1} \mid\ \tilde{y}_t)} \\ &= \frac{g(S_{t+1} \mid S_t) \cdot g(S_t \mid \tilde{y}_t)}{g(S_{t+1} \mid\ \tilde{y}_t)} \\ &\propto g(S_{t+1} \mid S_t) \cdot g(S_t \mid \tilde{y}_t) \end{aligned}$$

where the first term is the (known) transition probability and the second term was derived with the Hamilton filter.

6.0.2 Generating the regression coefficients (C_1, C_2 and C_3) conditional on $\tilde{S}_T,\ p, q, \sigma_1^2, \sigma_2^2, \sigma_3^2$ and the data

Conditional on $\tilde{S}_T$, $p, q, \sigma_1^2, \sigma_2^2$ and σ_3^2 the three equations are independent of one other. Each equation can be written as a standard linear model

$$Y = X\beta + \varepsilon$$

where the variance of the error is known. In particular, the three representations will be:

$$\begin{bmatrix} f_{12}-f_{12}^g \\ f_{13}-f_{13}^g \\ \vdots \\ f_{1T}-f_{1T}^g \end{bmatrix} = \begin{bmatrix} 1 & R_2 & f_{11}-f_{11}^g & \left(f_{11}-f_{11}^g\right)R_2 & \pi_1-\pi_1^g \\ 1 & R_3 & f_{12}-f_{12}^g & \left(f_{12}-f_{12}^g\right)R_3 & \pi_2-\pi_2^g \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & R_T & f_{1T-1}-f_{1T-1}^g & \left(f_{1T-1}-f_{1T-1}^g\right)R_T & \pi_{T-1}-\pi_{T-1}^g \end{bmatrix}$$

$$\begin{bmatrix} \left(\pi_1-\pi_1^g\right)R_2 & y_1-y_1^N & \left(y_1-y_1^N\right)R_2 \\ \left(\pi_2-\pi_2^g\right)R_3 & y_2-y_2^N & \left(y_2-y_2^N\right)R_3 \\ \vdots & \vdots & \vdots \\ \left(\pi_{T-1}-\pi_{T-1}^g\right)R_T & y_{T-1}-y_{T-1}^N & \left(y_{T-1}-y_{T-1}^N\right)R_T \end{bmatrix} \begin{bmatrix} \alpha^0 \\ \alpha^1 \\ \gamma_1^0 \\ \gamma_1^1 \\ \gamma_2^0 \\ \gamma_2^1 \\ \gamma_3^0 \\ \gamma_3^1 \end{bmatrix} + \begin{bmatrix} e_{12} \\ e_{13} \\ \vdots \\ e_{1T} \end{bmatrix}$$

$$\begin{bmatrix} f_{22}-f_{22}^g \\ f_{23}-f_{23}^g \\ \vdots \\ f_{2T}-f_{2T}^g \end{bmatrix} = \begin{bmatrix} D_2-D_1 & (D_2-D_1)S_2 & D_2-60 & (D_2-60)S_2 \\ D_3-D_2 & (D_3-D_2)S_3 & D_3-60 & (D_3-60)S_3 \\ \vdots & \vdots & \vdots & \vdots \\ D_T-D_{T-1} & (D_T-D_{T-1})S_T & D_T-60 & (D_T-60)S_T \end{bmatrix} \begin{bmatrix} \delta_1^0 \\ \delta_1^1 \\ \delta_2^0 \\ \delta_2^1 \end{bmatrix} + \begin{bmatrix} e_{22} \\ e_{23} \\ \vdots \\ e_{2T} \end{bmatrix}$$

$$\begin{bmatrix} f_{32}-f_{32}^g \\ f_{33}-f_{33}^g \\ \vdots \\ f_{3T}-f_{3T}^g \end{bmatrix} = \begin{bmatrix} D_2-D_1 & (D_2-D_1)S_2 & D_2-60 & (D_2-60)S_2 \\ D_3-D_2 & (D_3-D_2)S_3 & D_3-60 & (D_3-60)S_3 \\ \vdots & \vdots & \vdots & \vdots \\ D_T-D_{T-1} & (D_T-D_{T-1})S_T & D_T-60 & (D_T-60)S_T \end{bmatrix} \begin{bmatrix} \theta_1^0 \\ \theta_1^1 \\ \theta_2^0 \\ \theta_2^1 \end{bmatrix} + \begin{bmatrix} e_{32} \\ e_{33} \\ \vdots \\ e_{3T} \end{bmatrix}$$

Therefore, while the three sets of coefficients will be generated separately (each from its own matrix representation), they can be treated using the same family of priors and posteriors, since they can all be reduced to the case of a linear regression model with known variance. Notice that S_t (contrary to R_t) is in general *not known*. In this step of the Gibbs sampler, however, we condition on the whole series $\tilde{S}_T$: therefore, it behaves here as an *observed* dummy variable.

Let the prior distribution we use for the coefficient vector β (that, in the three cases, is made of the parameter sets C_1, C_2 or C_3, depending on which coefficient set we are generating) be a Multivariate Normal distribution with mean b_0 and covariance matrix B_0:

$$\beta \mid \sigma^2 \sim N(b_0, B_0)$$

with b_0 and B_0 known and set by the researcher.

Then,

$$\begin{aligned} g(\beta \mid \sigma^2) &= (2\pi)^{-\frac{k}{2}} |B_0|^{-\frac{1}{2}} \exp\{-\frac{1}{2}\,(\beta - b_0)'\, B_0^{-1}\,(\beta - b_0)\} \\ &\propto \exp\{-\frac{1}{2}\,(\beta - b_0)'\, B_0^{-1}\,(\beta - b_0)\} \end{aligned}$$

Given that by hypothesis

$$\varepsilon \sim N(0, \sigma^2 I_T)$$

where σ^2 will be equal to σ_1^2 for the first equation, σ_2^2 for the second and σ_3^2 for the last one, the likelihood of the model will be:

$$\begin{aligned} L(\beta \mid \sigma^2, Y) &= (2\pi\sigma^2)^{-\frac{T}{2}} \exp\{-\frac{1}{2\sigma^2}(Y - X\beta)'(Y - X\beta)\} \\ &\alpha \exp\{-\frac{1}{2\sigma^2}(Y - X\beta)'(Y - X\beta)\} \end{aligned}$$

The posterior distribution can therefore be obtained as:

$$\begin{aligned} g(\beta \mid \sigma^2, Y) &\propto \exp\{-\frac{1}{2}\,(\beta - b_0)'\, B_0^{-1}\,(\beta - b_0)\} \cdot \exp\{-\frac{1}{2\sigma^2}(Y - X\beta)'(Y - X\beta)\} \\ &= \exp\{-\frac{1}{2}\,(\beta - b_0)'\, B_0^{-1}\,(\beta - b_0) - \frac{1}{2\sigma^2}(Y - X\beta)'(Y - X\beta)\} \end{aligned}$$

Rearranging terms (see Judge et al. 1982) it can be shown that the posterior distribution will be a multivariate Normal distribution as well, so that the Normal distribution is the prior conjugate to the model:

$$\beta \mid \sigma^2, y \sim N(b_1, B_1)$$

where

$$\begin{aligned} b_1 &= (B_0^{-1} + \sigma^{-2} X'X)^{-1} (B_0^{-1} b_0 + \sigma^{-2} X'Y) \\ B_1 &= (B_0^{-1} + \sigma^{-2} X'X)^{-1} \end{aligned}$$

We can therefore sample the values for the coefficient sets from the three Normal posterior distributions obtained above.

6.0.3 Generating the variances σ_1^2, σ_2^2, σ_3^2 conditional on $\tilde{S}_T, p, q, C_1, C_2, C_3$ and the data

Conditional on $\tilde{S}_T$, the regression coefficients, the transition probabilities and the data, each equation can be treated independently from the others. Since we assume that the errors are independent and homoskedastic, in each case we can sample the variance σ^2 of each of the three equations using the known results for the Bayesian linear regression model. Again, the same results can be obtained to generate values for σ_1^2, σ_2^2 and σ_3^2 in each equation independently, using the matrix representation described above.

Let the prior for σ^2, conditional on β, be the noninformative prior:

$$g\left(\sigma^2 \mid \beta\right) \propto \frac{1}{\sigma^2}$$

The distribution of the inverse, $\frac{1}{\sigma^2}$, will be:

$$g\left(\frac{1}{\sigma^2} \mid \beta\right) \propto \sigma^2$$

The likelihood function for $\frac{1}{\sigma^2}$ is (from the normality assumption about ε):

$$\begin{aligned} L(\frac{1}{\sigma^2} \mid \beta, Y) &= \left(2\pi\sigma^2\right)^{-\frac{T}{2}} \exp\{-\frac{1}{2\sigma^2}(Y - X\beta)'(Y - X\beta)\} \\ &\propto \left(\frac{1}{\sigma^2}\right)^{\frac{T}{2}} \exp\{-\frac{1}{\sigma^2}\frac{(Y - X\beta)'(Y - X\beta)}{2}\} \end{aligned}$$

Combining that with the likelihood function obtained above, we get the posterior:

$$\begin{aligned} g(\frac{1}{\sigma^2} \mid \beta, y) &\propto \sigma^2 \cdot \left(\frac{1}{\sigma^2}\right)^{\frac{T}{2}} \exp\{-\frac{1}{\sigma^2}\frac{(Y - X\beta)'(Y - X\beta)}{2}\} \\ &= \left(\frac{1}{\sigma^2}\right)^{\frac{T}{2}-1} \exp\{-\frac{1}{\sigma^2}\frac{(Y - X\beta)'(Y - X\beta)}{2}\} \\ &= \left(\frac{1}{\sigma^2}\right)^{\frac{v_1}{2}-1} \exp\{-\frac{1}{\sigma^2}\frac{\delta_1}{2}\} \end{aligned}$$

where

$$\begin{aligned} v_1 &= T \\ \delta_1 &= (Y - X\beta)'(Y - X\beta) \end{aligned}$$

The posterior distribution of $\frac{1}{\sigma^2}$ is a Gamma with parameters $\frac{v_1}{2}$ and $\frac{2}{\delta_1}$. Therefore, the posterior distribution of σ^2, from which we sample, will be an *inverted* gamma, with parameters $\frac{v_1}{2}$ and $\frac{\delta_1}{2}$:

$$\sigma^2 \mid \beta, Y \sim IG(\frac{v1}{2}, \frac{\delta 1}{2})$$

6.0.4 Generating the transition probabilities conditional on $\tilde{S}_T, \sigma_1^2, \sigma_2^2, \sigma_3^2$, C_1, C_2, C_3 and the data

Conditional on $\tilde{S}_T$, the transition probabilities p and q are independent on the other parameters and the data. We use again a Bayesian approach to generate p and q. We will have two priors for p and q and consider the information in $\tilde{S}_T$ as the data needed to obtain the posterior distribution.

Let the priors for the two probabilities be from the Beta family:

$$\begin{aligned} p &\sim beta(u_{11}, u_{10}) \\ q &\sim beta(u_{00}, u_{01}) \end{aligned}$$

with $u_{11}, u_{10}, u_{00}, u_{01}$ known.

Assuming that p and q are independent, we have

$$g(p, q) \propto p^{u_{11}-1}(1-p)^{u_{10}-1}q^{u_{00}-1}(1-q)^{u_{01}-1}$$

The likelihood function for p and q is given by:

$$L(p, q \mid \tilde{S}_T) = p^{n_{11}}(1-p)^{n_{10}}q^{n_{00}}(1-q)^{n_{01}}$$

where n_{ij} represents the number of transitions from i to j in $\tilde{S}_T$, since it is the likelihood of obtaining exactly that sequence of states.

The posterior distribution will be:

$$
\begin{aligned}
g(p, q \mid \tilde{S}_T) &\propto g(p, q) L(p, q \mid \tilde{S}_T) \\
&= p^{u_{11}-1}(1-p)^{u_{10}-1} q^{u_{00}-1}(1-q)^{u_{01}-1} p^{n_{11}}(1-p)^{n_{10}} q^{n_{00}}(1-q)^{n_{01}} \\
&= p^{u_{11}+n_{11}-1}(1-p)^{u_{10}+n_{10}-1} q^{u_{00}+n_{00}-1}(1-q)^{u_{01}+n_{01}-1}
\end{aligned}
$$

The two independent posterior distributions will therefore be Beta again (it is the conjugate prior):

$$
\begin{aligned}
p \mid \tilde{S}_T &\sim beta(u_{11} + n_{11}, u_{10} + n_{10}) \\
q \mid \tilde{S}_T &\sim beta(u_{00} + n_{00}, u_{01} + n_{01})
\end{aligned}
$$

From these two distributions the two parameters will be generated for the Gibbs sampler.

References

[1] Adamo, M. et al. Optimal stategies for the issuance of public debts securities, *International Journal of Theorethical and Applied Finance, forthcoming*

[2] Albert, J. H. and Chib, S. (1993). Bayes Inference via Gibbs Sampling of Autoregressive Time Series Subject to Markov Mean and Variance Shifts, *Journal of Business & Economic Statistics, 11, No. 1*

[3] Ang, A. and Piazzesi, M. (2000). A No-Arbitrage Vector Autoregression of Term Structure Dynamics, with Macroeconomic and Latent Variables, *Working Paper, Columbia University.*

[4] Ardagna, S., Caselli, F. and Lane, T. (2004). Fiscal discipline and the cost of public debt service: some estimates for OECD countries, *NBER Working Papers 10788*

[5] Bernadell, C. , Coche, J. and Nyholm, K. (2005). Yield curve prediction for the strategic investor, *ECB Working Papers, N*o 472

[6] Blanchard O. and Summers, L. (1984). Perspective on High World Real Interest Rates, *Brookings Papers on Economic Activity, vol. 1984, n. 2*, 273-324

[7] Campbell, J.Y., Shiller, R.J. (1991). Yield spreads and interest rate movements: a bird's eye view. *Review of Economic Studies, 57*, 495–514

[8] Carriero, A., Favero, C.A. and Kaminska, I. (2004). Financial factors, macroeconomic information and the expectations theory of the term structure of interest rates, *CEPR discussion papers, 4301*

[9] Codogno, L., Favero, C. and Missale, A. (2003). Yield spreads on EMU government bonds, *Economic Policy, 37,* 527-529

[10] Dai, Q., and Philippon, T. (2005). Fiscal policy and the term structure of interest rates, *NBER Working Papers 11574*

[11] Dai, Q. and Singleton, K. (2000). Specification analysis of affine term structure models, *Journal of Finance, 55,* 1943-1978

[12] Dahlquist, M., Svensson, L.E.O. (1996). Estimating the term structure of interest rates for monetary policy analysis, *Scandinavian Journal of Economics 98*, 163–183.

[13] Diebold, F. X. and Li, C. (2003). Forecasting the Term Structure of Government Bond Yields, *NBER Working Papers 10048*

[14] Diebold, F.X. , Rudebush, G.D. and Aruoba, S.B. (2006). The macroeconomy and the yield curve: a dynamic latent factor approach, *Journal of econometrics, 131*, 309–338

[15] Duffee, G. (2002). Term premia and interest rate forecasts in affine models, *Journal of Finance 57*, 405–443

[16] Elmendorf, D. W. and Mankiw, N. G. (1999). *Handbook of Macroeconomics,* Chapter 25, Vol 1, Part C, 1615-1669

[17] Engen, E. and Hubbard, R. G. (2004). Federal government debts and interest rates, *NBER Working Papers 10068*

[18] Evans, P. (1985), Do large deficits produce high interest rates?, *American Economic Review, vol. 75, n.1*, 68-87

[19] Evans, C.L., Marshall, D. (1998). Monetary policy and the term structure of nominal interest rates: evidence and theory. *Carnegie-Rochester Conference Series on Public Policy 49*, 53–111

[20] Favero, C.A., Giavazzi, F. and Spaventa, L. (1997). High Yields: the spread on German interest rates, *The Economic Journal, 107, 443*

[21] Gale,W. G. and Orszag, P. R. (2003). The Economic Effects of Long-Term Fiscal Discipline, *Urban-Brookings Tax Policy Center Discussion Paper 8*

[22] Geman, S. and Geman, D. (1984). Stochastic Relaxation, Gibbs Distributions, and the Bayesian Restoration of Images, *IEEE Transactions on Pattern Analysis and Machine Intelligence, 6,* 721-741

[23] Hoerdahl, P., Tristani, O., Vestin, D. (2004). A joint econometric model of macroeconomic and term structure dynamics. *Manuscript, European Central Bank*

[24] Judge, G. G., Hill, R. C., Griffiths, W. E., Lutkepohl, H. and Lee, T., *Introduction to the theory and practice of econometrics*, John Wiley & Sons, New York, 1998

[25] Kim, C. and Nelson, C., *State-space models with regime switching,* MIT Press, Cambridge, MA, 1998

[26] Laubach, T. (2004). The Effects of Budget Deficits on Interest Rates: A Review of Empirical Results, *Public Debt (Rome: Bank of Italy),* pp. 427-45.

[27] Miller, S. and Russek, F. (1996). Do federal deficits affect interest rates? Evidence from three econometric methods, *Journal of Macroeconomics, vol. 18, n.3*, 403-428

[28] Nelson, C.R. and Siegel, A.F. (1987). Parsimonious modeling of the yield curves, *Journal of Business, 60, 4*

[29] OECD Economic Outlook n.78, December 2005.

[30] Plosser, C. (1987). Fiscal Policy and the Term Structure, *Journal of Monetary Economics, 20*, 343-367

[31] Rudebusch, G.D. (1995). Federal reserve interest rate targeting, rational expectations, and the term structure. *Journal of Monetary Economics 35*, 245–274.

[32] Taylor, J.B (1993), Discretion versus Policy Rules in Practice, *Carnegie-Rochester Conference Series on Public Policy, 39, 195-214*

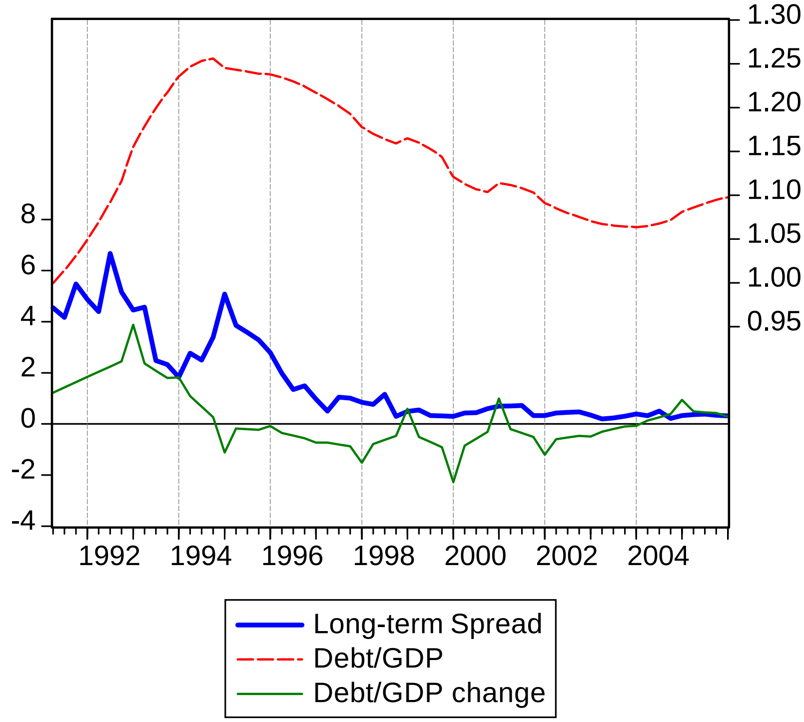

Figure 1: *ITA-GER long term spread, Italian Debt/GDP ratio and its variation*

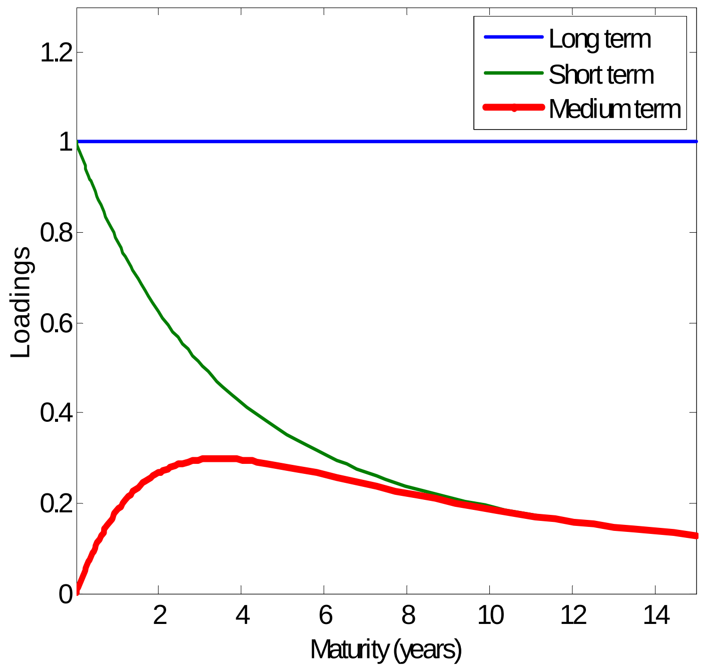

Figure 2: *The three functions of the Nelson-Siegel decomposition*

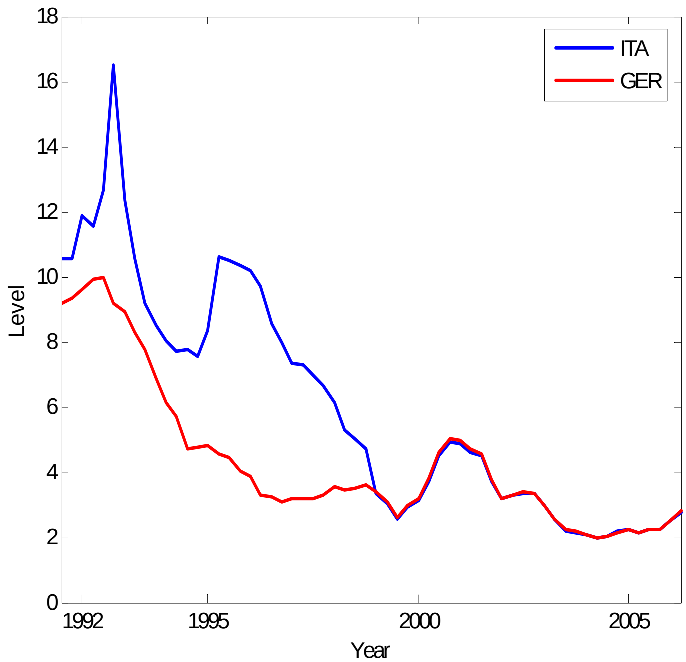

Figure 3a: *monetary policy instrument*

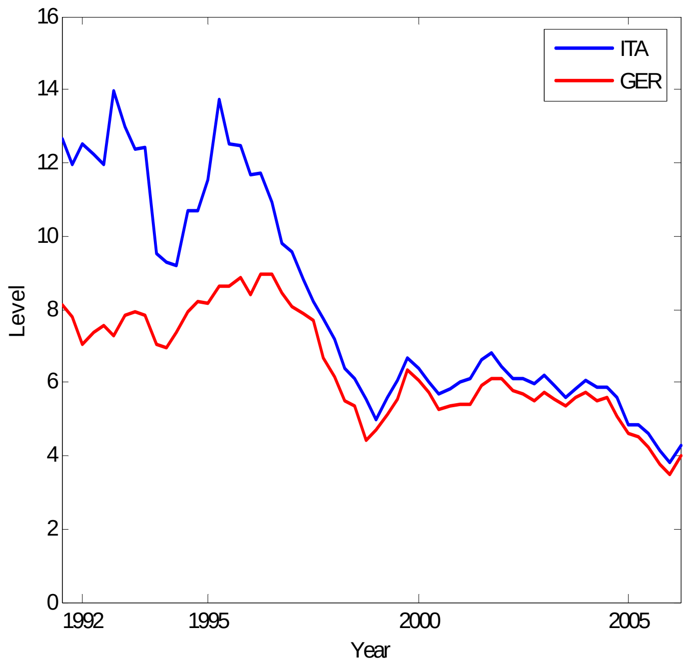

Figure 3b: *long term factor*

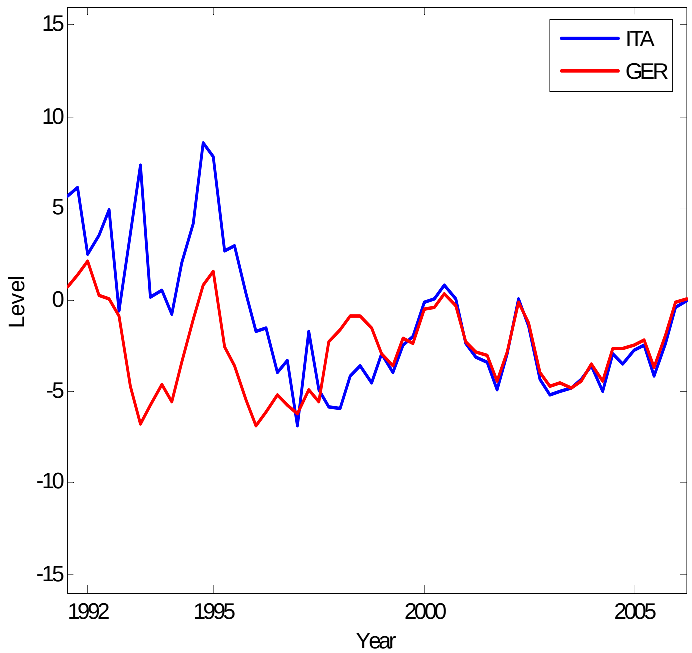

Figure 3c: *medium term factor*

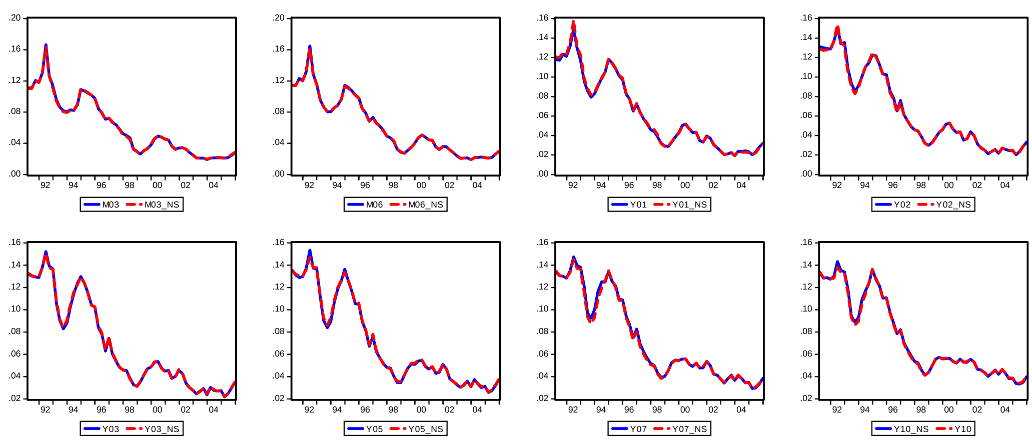

Figure 4a: *Italian plots of actual vs. estimated (Nelson-Siegel) bond yields*

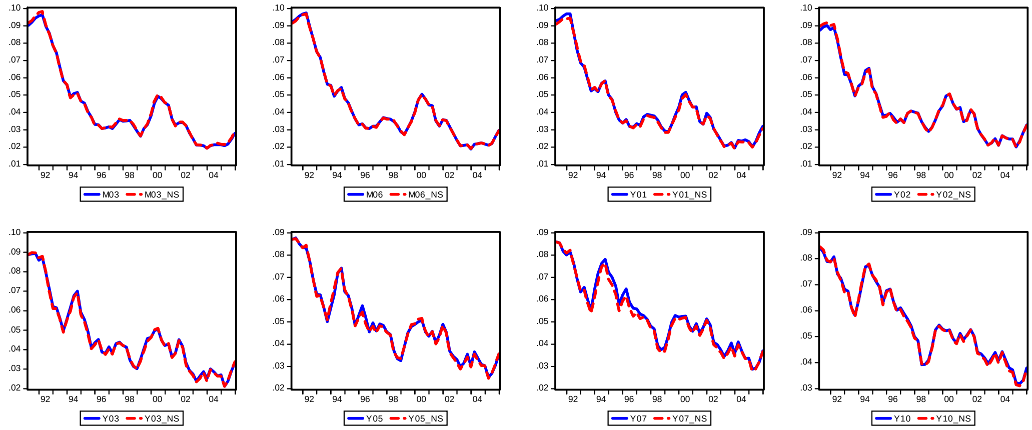

Figure 4b: *German plots of actual vs. estimated (Nelson-Siegel) bond yields*

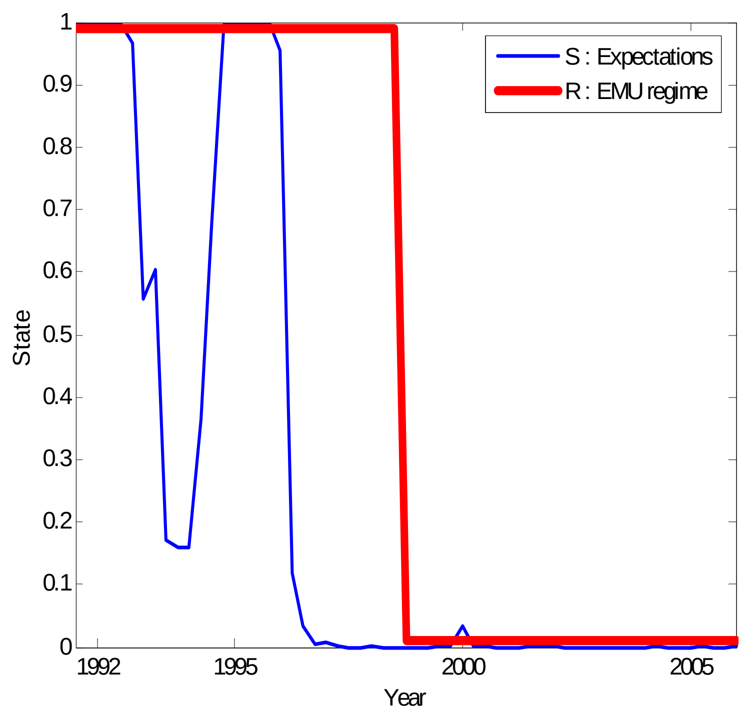

Figure 5: *the expectations regime and the monetary regime*

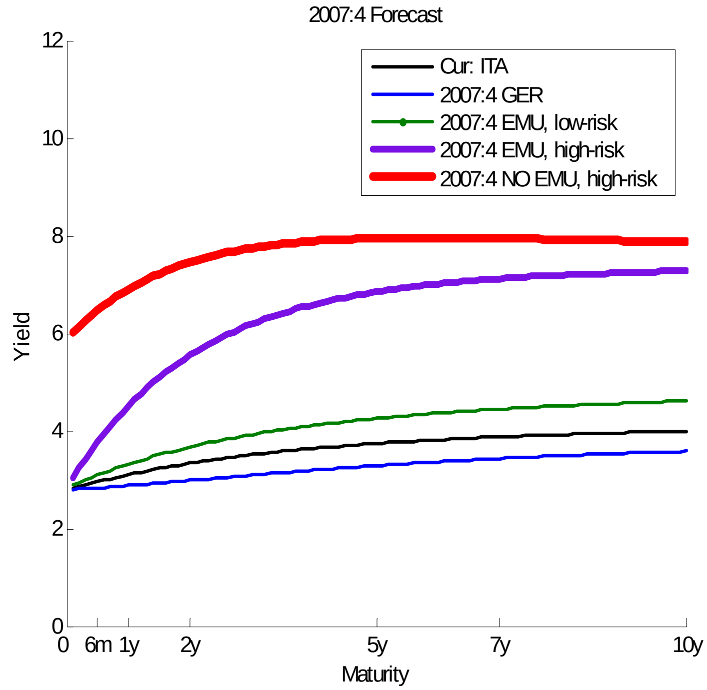

Figure 6: *Forecast of the yield curve in 2007:4*

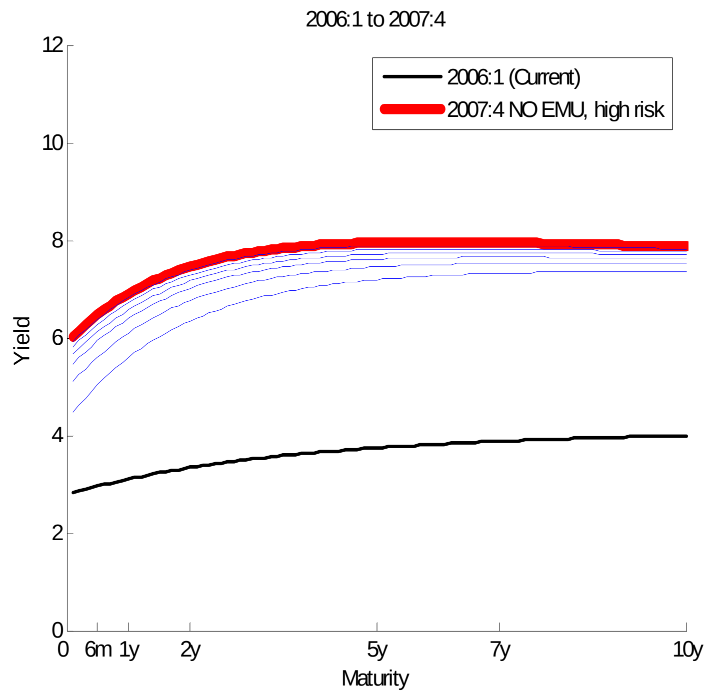

Figure 7: *Evolution of the yield curve under the bad-case scenario*