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ON THE VIRTUE OF BAD TIMES: AN ANALYSIS OF THE INTERACTION BETWEEN ECONOMIC FLUCTUATIONS AND PRODUCTIVITY GROWTH

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ABSTRACT

On the Virtue of Bad Times: An Analysis of the Interaction Between Economic Fluctuations and Productivity Growth*

This paper develops a simple model, which shows how economic fluctuations can stimulate growth. It is shown that firms tend to invest more in productivity growth during recessions, since the opportunity cost (in terms of forgone profits) of investing capital or labour resources in technological (or managerial) improvements is lower during recessions. It is then established that the average growth rate of the economy increases with the amplitude of the fluctuations and also with their frequency, provided that the initial average duration of recession phases is sufficiently low compared with that of the expansion phases. Finally, the main results of the paper are shown to be consistent with the empirical evidence recently produced by Davis-Haltiwanger (1990) or Blanchard-Diamond (1990) concerning the cyclical behaviour of job re-allocation.

JEL classification: C68, D21, D50, D90, D92
Keywords: productivity growth. recessions. expansions. productivity shocks. demand shocks. job destruction. job creation

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NON-TECHNICAL SUMMARY

This paper develops a simple model which formalizes the idea that economic fluctuations can stimulate productivity growth. We show that firms tend to invest more in productivity growth, either by reorganizing their production activities or by investing in technical progress, during recessions which are expected to be followed by an expansionary phase. In our framework, recessions tend to increase the rate of productivity growth because of an intertemporal substitution effect: the opportunity cost (in terms of forgone profits) of investing part of their capital or labour resources in technological improvements or in managerial reorganizations is lower during depression phases, and the more so when the discrepancy between booms and slumps is large.

In Section I of the paper we analyse the impact of economic fluctuations on productivity growth in a one-firm (or one-sector) model, where the sole producer must decide at each instant how to distribute its labour force between reorganization (or R&D) and production activities. Economic fluctuations occur randomly according to a stationary Markov process. We show that the average growth rate of the one-sector economy is an increasing function of the amplitude of the fluctuations, and that it increases with the frequency of recessions, provided that the initial frequency of recessions is lower than that of expansionary phases.

In Section II we develop a multifirm (multisector) extension of the basic model where firms are subject to uncorrelated idiosyncratic shocks of the same kind as in Section I. We find that the average growth rate (whose computation involves some additional technicalities) depends on the amplitude and frequency of fluctuations in a similar way as in the one-firm case. The difference is that in the multi-sector case the growth process introduces an additional discounting effect working through relative prices over time (the extent of which increases with the degree of substitutability across products).

In Section III we analyse the combined effects on productivity growth of idiosyncratic fluctuations (as formalized in Section II) and the occurrence of aggregate recessions. We find that these two components of economic fluctuations tend to reinforce each other, the more so when firms incur (large) fixed costs of production. More generally, the existence of large fixed costs of production will tend to strengthen the impact of economic fluctuations on growth.

In Section IV we show that under a natural interpretation of our model where firms continuously hire or fire workers, both in their 'reorganization' and in their 'production' divisions, the main results of the previous sections are consistent with the evidence in Davis and Haltiwanger (1990) or in Blanchard-Diamond (1990) concerning the cyclical behaviour of job creation and job destruction: job destructions are concentrated in recession periods, whereas job creations are subject to relatively smaller fluctuations over the cycle.

Introduction

For years the problem of the trade cycle and that of the dynamic process of economic growth have been recognized as being closely interrelated.

However, for several decades, the two phenomena had been investigated separately by the economic literature: in particular the accelerator-multiplier (or oscillator) model that followed and extended Keynes'General Theory had initially no trend component in it.

"As a pure cyclical model, it had therefore little resemblance to the cyclical fluctuations in the real world, where successive booms carried production to successively higher levels" (Kaldor, 1954).

More recent versions of the oscillator model (Hicks, 1950; Kalecki, 1938,...) looked to overcome this kind of criticism by superimposing a linear trend on the original model without altering its basic properties. The trend itself was not explained, but rather introduced from the outside either by assuming a linear percentage growth in population or by introducing an exogenous source of technical progress.

The same applies to Goodwin (1967), although his model was the first to provide a consistent explanation for the occurrence of endogenous economic fluctuations as the deterministic outcome of growth, thereby connecting the two phenomena in a less arbitrary way than the previous models. But, like in all other models of cycles, the process of economic growth was left fundamentally unexplained.

Analyzing the relationship between economic fluctuations and the determinants of growth, Kaldor (1954) concluded:

"... so far from the trend of growth determining the strength or durations of booms, it is the strength and duration of booms which shape the trend rate of growth. It is the economy in which business-men are reckless and speculative, (...) where high and growing profits are projected into the future and lead to the hasty adoption of "unsound" projects involving overexpansion, which is likely to show a higher rate of progress over longer periods. The same forces therefore which produce violent booms and slumps will also tend to produce a higher trend-rate of progress".

In this paper we develop a simple model which formalizes the idea that economic fluctuations can have a stimulating effect on productivity growth. More precisely, we show that recessions, when expected to be followed by a new expansionary phase, are times where firms tend to invest more into productivity growth, either by reorganizing their production activities or by investing in technical progress.[1] The main reason underlying the positive contribution of recessions to the rate of productivity growth is the following intertemporal substitution effect: the opportunity cost, in terms of foregone profits, of investing part of their capital or labor resources in technological improvements or in managerial reorganizations is lower during depression

[1] Our model is inspired by the work of Schumpeter (see 1939, 1950), who used to describe recessions as times where entrepreneurs invest in fundamental innovations rather than in imitation activities.

phases, and the more so when the discrepancy between booms and slumps is large.

More precisely we analyze the impact of economic fluctuations on productivity growth in a one-firm (or one-sector) model where the unique producer must decide at each instant how to distribute its labor force between reorganization (or R & D) and production activities. Economic fluctuations occur randomly (as in RBC models) according to a stationary Markov process. We show that the average growth rate of the one-sector economy is an increasing function of the <u>amplitude</u> of the fluctuations and that it also increases with the <u>frequency</u> of recessions provided that the initial frequency of recessions is lower than that of expansionary phases.

A particular stylized fact supporting our approach[2] is produced by Davis and Haltiwanger (1990) who show - on the basis of a US data set covering the period 1972-1985 and desegregated across manufacturing firms and sectors - that the amount of job reallocation across sectors and firms is larger in aggregate recessions than in aggregate booms. This suggests that a lot of activities are going on in recession, the value of which may not

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[2] Other pieces of evidence which do not invalidate our analysis include, first, the empirical finding (established by Saint-Paul (1991) on the basis on OECD data) that between 1974 and 1989 the developed countries that have experienced higher rates of productivity growth are also those that have experienced a higher of unemployment. The same data show that the variance of GDP in percentage terms and the average growth rate of the Solow residual appears to be positively correlated if one chooses Japan at the bottom of the correlation, the U.S. (or the U.K., France, Germany) in the middle of it, and Belgium at the top end. (However the comparison between the middle countries mentionned above, whose locations on the variance-growth curve are relatively very close to one another, shows no significant correlation positive or negative).

show up in GNP, like reorganization, implementation of new technologies, etc, which, in addition to being quite consistent with our model, may also explain the pro-cyclical behavior of productivity.

Two related recent papers include Hall(1991) and Gali and Hammour (1991). The latter, in particular, find, using time series data on the U.S., that the long-run effect of a negative demand shock on productivity is positive, which is quite consistent with our model.

The Model

We now present the main features of the model. We consider the case of a single representative firm. The firm is infinitely lived and at each point in time it can employ one unit of labor and thereby generate a current flow of profits equal to:

$$\Pi = y.e^{x} \tag{1}$$

where x is a technological parameter measuring the productivity of labor at that time, and y is a profitability parameter reflecting the current opportunities of the firm, e.g. the current demand for the firm's product or its current production costs. To simplify the analysis, we assume that the profitability parameter y can only take two values:

$$y \in \{y_R, y_E\},$$

where y_R measures the firm's profitability when the economy (i.e. the unique productive sector) is in <u>recession</u> and y_E measures the firm's profitability when the economy is in <u>expansion</u>; we naturally assume that $y_R < y_E$.

The state of the economy (i.e. the firm's profitability) is assumed to be random, and more precisely to follow a Markov process with parameters (ϵ,γ), where γ is the flow probability of a transition from expansion (state E) to recession (state R) while ϵ is the flow probability of the reverse transition.

At any time, the firm may decide to allocate a share (1-h) of its current profits to R&D activities so as to improve its technology x.

Let $v > 0$ denote the speed of the firm's technical progress. This speed will be endogenously determined, as a choice variable of the firm, by the trade-off between current profits (net of R&D costs) and the future NPV of the firm which increases with the technological parameter x. More formally, to induce a technological improvement v.dt during the time-interval dt, the firm will have to sacrifice a share $(1-h(v))$ of its current profits[1] during the same time interval. We assume:

$$H1 \; : \; h(0) = 1; \; h' < 0; \; h'' < 0. \qquad (\textit{Figure } 4)$$

Figure 1

Let $V_R(x)$ and $V_E(x)$ denote the firm's NPV when its current productivity parameter is equal to x and the economy is respectively in a current state of recession or in current state

[1] Assuming constant returns to scale, this is implied by the firm having to devote a share $(1-h(v))$ of its labor force to R&D activities.

of expansion.[2] If $r > 0$ is the rate at which the firm discounts its future profits, the value functions V_E and V_E will satisfy the following Bellman equations:

$$
\left.
\begin{array}{c}
V_R(x) = \underset{v}{Max}[y_R h(v) e^x . dt \\
+ (1-rdt).\{V_R(x+vdt)(1-\varepsilon dt) + \varepsilon dt V_E(x+vdt)\}] \\
\& \\
V_E(x) = \underset{v}{Max}[y_E h(v) e^x . dt \\
+ (1-rdt).\{V_E(x+vdt)(1-\gamma dt) + \gamma dt V_R(x+vdt)\}]
\end{array}
\right\}
\qquad \text{(B)}
$$

In words, if the firm is currently passing through a recession, and devotes the fraction $(1-h(v))$ of its current profits $y_R e^x$ during the time interval dt, then at the end of this time interval (which the firm discounts at rate r) the firm will have reached the technological level x+vdt; furthermore, by then, the economy will have switched to an expansionary phase with probability εdt and remained in recession with the complementary probability; hence the first Bellman equation in (B). The second Bellman equation is identically interpreted except that the two states R and E must be permuted and the transition probability ε must be replaced by the symmetric flow probability γ.

The optimal rate of productivity growth v_R in recession phases is given by the first-order condition:

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2 To the extent that the firm's optimization problem is stationary, V_R and V_E do not directly depend upon time t.

$$e^{x}.y_R h'(v_R) + V_R'(x) = 0 \qquad (2)$$

where $V' = dV/dx$.

[(2) is simply obtained from the first Bellman equation in (B) by letting $dt \to 0$ and by differentiating w.r.t. v.]

Similarly, the optimal rate of productivity growth v_E in expansion phases is given by:

$$e^{x}.y_E h'(v_E) + V_E'(x) = 0 \qquad (3)$$

Assuming, which is indeed the case in equilibrium, that the firm's values V_R and V_E evolve like current profits according to:

$$V_i(x) = V_i^o.e^{\alpha x},$$

the above conditions can be simply reexpressed as:

$$h'(v_i) = -\frac{V_i^o}{y_i}, \quad i \in \{E,R\}. \qquad (4)$$

Then, if we assume that the cost of R&D is convex, i.e. $h''() < 0$ for $h(v) \in (0,1)$, we can sense from (4) that the firm will invest a <u>higher share</u> of its labor force into research where the economy is <u>recession</u> than when it is in expansion. Indeed, the ratio between the NPV $V_i(x)$ and the current profits $\Pi_i(x) = e^x.y_i$, i.e. $V_i(x)/\Pi_i(x) = V_i^o/y_i$, will be higher in recession periods since both V_R^o and V_E^o are discounted sums of profits over the firm's life time where both recessions and expansion phases occur with probability one; this in turn implies that the ratio of NPVs in expansion and recession should be smaller than the corresponding ratio between current profit flows in the two states of the economy.

More formally:

Proposition 1: (a) Assuming that $h''(v) < 0$ for $h(v) \in (0,1)$, there exists a unique [stable] equilibrium (v_E, v_R).

(b) Furthermore, if both v_E and v_R are interior solutions, we have: $v_E < v_R$.

In other words, the economy will grow at a slower rate in expansions than in recessions due to the fact that the opportunity cost of research is lower in the latter case. Our result supports the empirical observation that firms tend to reorganize their production activities more during recession periods. It also supports the observation that sectoral reorganization (e.g. labor reallocation) is positively correlated with the occurrence of recessions. (See Davis-Haltwanger (1990), Blanchard-Diamond (1989) and Section 4 below.)

Proof of Proposition 1: (a) Letting $dt \to 0$ in the Bellman equations (B), we obtain the following differential system in $V_R(x)$ and $V_E(x)$:

$$0 = -(r+e)\,V_R(x) + v_R V_R'(x) + eV_E(x) + h(v_R)\,y_R e^x \qquad (5)$$

$$0 = -(r+\gamma)\,V_E(x) + v_E V_E'(x) + \gamma V_R(x) + h(v_E)\,y_E e^x \qquad (6)$$

This yields: $V_i(x) = V_i^o.e^x$, $i \in (R,E)$, where:

$$(r+e-v_R)\,V_R^o = eV_E^o + h(v_R)\,y_R \qquad (7)$$

and

$$(r+\gamma-v_E)\,V_E^o = \gamma V_R^o + h(v_E)\,y_E. \tag{8}$$

Using the first-order conditions (4) above, we end up with a non-linear system in (v_E, v_R):

$$\left.\begin{array}{ll} h'(v_E) = \dfrac{1}{\theta e}\left[(r+e-v_R)\,h'(v_R)+h(v_R)\right] = \dfrac{1}{\theta}F(v_R) & (\#_1) \\ h'(v_R) = \dfrac{\theta}{\gamma}\left[(r+\gamma-v_E)\,h'(v_E)+h(v_E)\right] = \theta G(v_E) & (\#_2) \end{array}\right\} \tag{\#}$$

where $\theta = y_E/y_R$.

The uniqueness of v_E, v_R results from their being the outcome of the firm's <u>concave</u>[3] maximization problem. Furthermore this equilibrium is "stable" in the sense that:

$$\left(\frac{dv_R}{dv_E}\right)_{/\#_2} \cdot \left(\frac{dv_E}{dv_R}\right)_{/\#_1} > 1 \tag{9}$$

[This last inequality follows from the fact that

$$\left(\frac{dv_R}{dv_E}\right)\left(\frac{dv_E}{dv_R}\right) = \left(\frac{r+e-v_R}{e}\right)\left(\frac{r+\gamma-v_E}{\gamma}\right) > 1$$

by (7) and (8) above.]

This establishes part (a).

(b) We can rewrite the above equations (7) and (8) as:

Given that the firm's initial value is necessarily lower in recession than in expansion (i.e. $V_R^o < V_E^o$), we have, from (10):

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3 As long as we restrict the analysis to the domain where $h'' < 0$.

$$\left.\begin{aligned} r + \epsilon \frac{V_R^o - V_E^o}{V_R^o} &= v_R - \frac{h(v_R)}{h'(v_R)} \\ &\& \\ r + \gamma \frac{V_E^o - V_R^o}{V_E^o} &= v_E - \frac{h(v_E)}{h'(v_E)} \end{aligned}\right\} \qquad (10)$$

$$f(v_R) < f(v_E)$$

where $f(v) = -h(v)/h'(v)$; Part (b) then follows from the fact that $f'(v) = h.h''(v)/(h'(v))^2 < 0$ whenever $h(v) \in (0,1)$. □

Using this basic result, we shall now analyze <u>how the average growth rate of the economy varies with both the amplitude and the frequency of random fluctuations</u>.

Given that the random process is Markovian with flow probabilities of transition ϵ and γ respectively from expansion to recession and vice-versa, the firm will spend on average a fraction $(\gamma/\gamma+\epsilon)$ of its life-time in recession and the remaining fraction in an expansion phase. Hence the average growth rate of the firm's productivity will simply be:

$$g = \frac{\gamma}{\gamma+\epsilon} v_R + \frac{\epsilon}{\gamma+\epsilon} v_E. \qquad (11)$$

A first straightforward Corollary of Proposition 1 is that $\partial g/\partial r < 0$. Indeed, it follows from equations (#) above[4] that both v_E and v_R are decreasing functions of the discount rate r; this is not surprising: the higher the interest rate, the less firms will invest in R&D whatever the state of the economy.

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[4] And from the fact that $(dv_E/dv_R)(dv_R/dv_E) > 1$.

Figure 2

Hence $\partial g/dr < 0$ from the above (11). □

More interesting is the relationship between growth and both the magnitude and the frequency of economic fluctuations. First, we can analyze the effect on growth of an increase in the ratio $\theta = \gamma_E/\gamma_R$ which measures the amplitude of the random fluctuations. Using the two equations (#) above, we can indeed establish:

<u>Proposition 2</u>: Under the same assumption as in Proposition 1, we have: $dv_R/d\theta > 0$; $dv_E/d\theta < 0$; and $dg/d\theta > 0$ provided $h'''(.)$ is not too negative.

In particular, if the R&D cost is close to be quadratic, the average growth rate of the economy will be positively correlated with the amplitude of the fluctuations. In any case, the relative incentive for the firm to invest in technological progress (or in reorganization) during recession phases does increase with the amplitude parameter θ. This result formalizes one aspect of the relationship between cycles and growth that had been envisaged, although not explicitly, by various authors and especially Schumpeter. On the other hand, as far as we know, only the reverse causality from growth to cycles has been formally analyzed in the existing literature. (See Matthews

(1967) and Lordon (1991) for good surveys of the "growth and cycles" literature.)

Proof of Proposition 2: Differentiating the system of equations (#) with respect to v_E, v_R and θ, we get:

$$h''(v_R)\,dv_R = \frac{(r+\gamma-v_E)\,h'(v_E)+h(v_E)}{\gamma}\,d\theta + \frac{\theta}{\gamma}\,(r+\gamma-v_E)\,h''(v_E)\,dv_E \tag{12}$$

$$h''(v_E)\,dv_E = -\frac{1}{\theta^2}\left[\frac{(r+e-v_R)\,h'(v_R)+h(v_R)}{e}\right]d\theta + \frac{1}{\theta e}\,(r+e-v_R)\,h''(v_R)\,dv_R \tag{13}$$

Substituting (13) into (12) and again using (#), we obtain:

$$\left.\begin{aligned} dv_R.\,h''(v_R).\left(1-\frac{(r+\gamma-v_E)\,(r+e-v_R)}{\gamma e}\right) &= \frac{h(v_E)}{\gamma}\,d\theta \\ &\& \\ dv_E.\,h''(v_E).\left(1-\frac{(r+\gamma-v_E)\,(r+e-v_R)}{\gamma e}\right) &= -\frac{1}{\theta^2}\,\frac{h(v_R)}{e}\,d\theta \end{aligned}\right\} \tag{14}$$

Given that $h'' < 0$ and that $(r+\gamma-v_E)(r+e-v_R) > \gamma e$ by Proposition 1, we have shown that $\partial v_R/\partial\theta > 0$ and $\partial v_E/\partial\theta < 0$.

Finally, the overall effect on growth of increasing the size of fluctuations is determined by:

$$\frac{dg}{d\theta} = \frac{\gamma}{\gamma+e} \cdot \frac{dv_R}{d\theta} + \frac{e}{\gamma+\epsilon} \cdot \frac{dv_E}{d\theta}$$

$$= \frac{\gamma}{(\gamma+\epsilon)\left(1 - \frac{(r+\gamma-v_E)(r+e-v_R)}{\gamma e}\right)} \cdot \left[\frac{h(v_E)}{h''(v_R)} - \frac{1}{\theta^2} \cdot \frac{h(v_R)}{h''(v_E)}\right]$$

< 0 sign ?

Given that $\theta > 1$, the expression in brackets will be negative when h is [almost] quadratic, since:

$v_E < v_R$ implies $h(v_E) > h(v_R)$, i.e. $+ 1/\theta^2 h(v_R) < h(v_E)$.

More generally, it suffices that $h(v_R)h''(v_R) > h(v_E)h''(v_E)$ for the expression in brackets to be negative (since $h'' < 0$); as $v_R > v_E$, it suffices that $h.h''$ be increasing in v, or equivalently that h''' not be too negative. □

To complete our comparative static analysis of the one-firm case, we still have to analyze how the average growth rate g is affected by an increase in the <u>frequency</u> of upward or downward economic fluctuations. Consider first an increase in the frequency of recessions, i.e. in the flow probability γ. Given that the rate of technological progress is higher during recessions (Proposition 1 above), an increase in γ will have the direct effect of <u>increasing</u> the average growth rate g, i.e.: $\partial g/\partial\gamma > 0$. In other words, to the extent that recessions induce firms to invest a higher share of inputs into research activities, an increase in their frequency has a direct positive effect on growth. The indirect effect through the endogenous speeds of technological progress v_R and v_E goes, not surprisingly, in the opposite direction; i.e.:

Proposition 3:

$$\frac{dv_E}{d\gamma} < 0; \quad \frac{dv_R}{d\gamma} < 0; \quad \frac{dv_E}{de} > 0; \quad \frac{dv_R}{de} > 0.$$

Intuitively, the more frequent the recessions, the less the firm is likely to take advantage of having reached a higher productivity level; this, in turn, will tend to discourage research activities whatever the current state of the economy (R or E). The opposite will hold if one increases the frequency of expansion phases.

Proof of Proposition 3: Differentiating the above equations (#) with respect to e, v_E, v_R, we have:

$$\left.\begin{aligned}&(y_R h'(v_R) - y_E h'(v_E))\,de\\ &+ (r+e-v_R)\,y_R h''(v_R)\,dv_R - e y_E h''(v_E)\,dv_E = 0\\ &\&\\ &(r+\gamma-v_E)\,y_E h''(v_E)\,dv_E\\ &= \gamma y_R h''(v_R)\,dv_R.\end{aligned}\right\} \tag{15}$$

Substituting the latter equation into the former, we get:

$$\begin{aligned}&(y_R h'(v_R) - y_E h'(v_E))\,de\\ &= y_E h''(v_E)\left[e - \frac{(r+e-v_R)\,(r+\gamma-v_E)}{\gamma}\right].dv_E\end{aligned} \tag{16}$$

From Proposition 1 we know that the expression in brackets is negative and therefore the RHS of (16) is positive whenever $h'' < 0$. Moreover, from the first-order conditions (4) we have:

$$y_R h'(v_R) - y_E h'(v_E) = (V_E^o - V_R^o) > 0.$$

This, in turn, implies that $dv_E/de > 0$, which by the second equation (15) implies also that $dv_R/de > 0$. By symmetry, if we differentiate (#) with respect to γ, v_E and v_R we end up with the equation:

$$\begin{aligned}&\left(y_E h'(v_E) - y_R h'(v_R)\right)d\gamma \\ &= y_R h''(v_R)\left(\gamma - \frac{(r+e-v_R)(r+\gamma-v_E)}{e}\right).dv_R.\end{aligned} \qquad (16)'$$

The RHS of (16)' is again positive, but now its LHS will be negative equal to $(V_R^o - V_E^o)d\gamma$; hence $dv_R/d\gamma < 0$, which by the second equation (15) will imply that $dv_E/d\gamma < 0$. □

This proposition implies an ambiguous effect on the average growth rate g of an increase in either the frequency of recessions or that of expansion phases. For example, the direct effect of an increased frequency of recessions is to increase growth (since $v_R > v_E$) whereas the indirect effect working through $dv_R/d\gamma$ and $dv_E/d\gamma$ goes in the opposite direction.

The overall effect on growth will generally be indeterminate, except when the amplitude parameter θ is close to 1, in which case the following result can be established:

Proposition 4: When θ is sufficiently close to 1, we have:

$$\gamma > e \Rightarrow \frac{dg}{d\gamma} < 0$$

and

$$\gamma < e \Rightarrow \frac{dg}{d\gamma} > 0.$$

In words, when initially the frequency of recessions is higher than that of expansion phases, an additional increase in γ will essentially discourage the firm (or sector) from pursuing research activities since the expansion phases, where the benefits of these activities could best be captured, are expected to become even more rare or unlikely.

At the opposite, if the expansion phases are expected to be relatively more frequent, the direct positive effect on growth of an increased frequency of recessions will eventually dominate. (Figure 3).

Proof of Proposition 4: Let $\theta = 1+\alpha$, where α is a small positive number. We can then rewrite the above equations (#) in v_R and v_E as:

$$h'(v_E) = \frac{1}{e}(1-\alpha)[(r+e-v_R)h'(v_R)+h(v_R)]$$
$$h'(v_R) = \frac{1}{\gamma}(1+\alpha)[(r+\gamma-v_E)h'(v_E)+h(v_E)], \quad (\#)'$$

Let us denote: $v_R = v+w_R$ and $v_E = v+w_E$, where v would be the stationary rate of productivity growth if $\theta = 1$ (i.e. if $y_E = y_R$), and let $\rho = h'(v)/h''(v) > 0$. When θ is close to 1, we can use the first-order linear approximations:

$$h'(v_i) = h'(v) + w_i.h''(v), \quad i = E,R.$$

Substituting in (#)', we end up with the linear system:

$$w_E = -\alpha\rho + \frac{1}{e}[(r+e-v)w_R - \rho w_R]$$

$$(\#)"$$

$$w_R = \alpha\rho + \frac{1}{\gamma}[(r+\gamma-v)w_E - \rho w_E].$$

This system yields:

$$w_E = \frac{\gamma(r-(v+\rho))\alpha\rho}{e\gamma-(r+e-(v+\rho))(r+\gamma-(v+\rho))} = \frac{N}{D},$$

$$\text{and} \quad w_R = -\frac{e}{\gamma}\, w_E.$$

[D > 0 by stability of the solution to (#).]

We can then compute the total derivative:

$$\frac{dg}{d\gamma} = \underbrace{\frac{\gamma}{\gamma+e} \cdot \frac{dw_R}{d\gamma}}_{<\,0} + \underbrace{\frac{e}{\gamma+e} \cdot \frac{dw_E}{d\gamma}}_{<\,0} + \underbrace{(w_R - w_E)\,\frac{e}{(\gamma+e)^2}}_{>\,0}$$

[indirect effects] *[direct effect]*

Using the fact that:

$$\frac{dw_R}{d\gamma} = \frac{-\alpha\rho(r-(v+\rho))^2 e}{D^2} \quad (<0), \quad \text{and}$$

$$\frac{dw_E}{d\gamma} = \frac{-\alpha\rho(r-(v+\rho))^2(r+e-(v+\rho))}{D^2} \quad (<0).$$

we end up with:

$$\frac{dg}{d\gamma} = -\frac{\alpha\rho}{D^2}\,(r-(v+\rho))^2 . (\gamma-e).$$

Hence:

$$\frac{dg}{d\gamma} > 0 \iff \gamma < \varepsilon. \qquad \square$$

Two remarks to conclude this section:

(a) The overall effect of an increased frequency of fluctuations involving both $d\gamma > 0$ and $d\varepsilon > 0$ on growth will be ambiguous, and again depend on the relative values of ε and γ. Suppose for example that both γ and ε were to be multiplied by the same factor $1+\pi > 1$, π small. Then, in the case where θ is close to 1, we know from Proposition 4 that if γ is initially larger than ε, we have $dg/d\gamma < 0$ and $dg/d\varepsilon > 0$. Then, $dg = \partial g/\partial\gamma . d\gamma + \partial g/\partial\varepsilon . d\varepsilon$ will be negative if γ was initially sufficiently larger than ε. By symmetry, dg will be positive if initially the expansion phases were significantly more frequent than the recessions.

(b) One can show that the introduction of <u>fix costs</u> of production reinforces the <u>indirect</u> effects of ε or γ on g: The above analysis, together with the first-order conditions (4), imply that $dV^o_R/d\gamma < 0$ and $dV^o_E/d\gamma < 0$.

Now when γ becomes sufficiently large, we may end up with:

$$V^o_E > f > V^o_R,$$

where f is the fix cost of operating. But as soon as the value V^o_R falls below the fix cost f, the firm will only operate (or innovate) during the expansionary phases, so that the average growth rate will fall down to $v_E < \gamma/\gamma+\varepsilon \; v_R + \varepsilon/\gamma+\varepsilon \; v_E$. This establishes our claim. $\square$

(c) It is possible to extend the model to take into account heterogeneity across firms. This is done in appendix A. The intuitions are basically unaltered compared to the single-firm case we consider in this section. The analysis is however useful to analyse how, in a general equilibrium framework, the results are affected by the degree of complementarity across differentiated firms and, more generally, firm's interactions with each other.

(d) The main effect at work here is the intertemporal substitution effect : it is optimal for the firm to reorganize activities and implement new technologies during a temporary recession, to benefit more from the following boom. A permanent recession, i.e. a once-and-for-all drop in y_E and y_R has no impact on productivity growth, since it only depends on relative levels. If fixed production costs are present, it may be the case that even a permanent recession has a positive impact on productivity growth. This is demonstrated in appendix B in the context of the multi-firm version of the model. The intuition is quite simple: if the fixed costs is associated with production activities and not with development activities, and if it is not proportional to the firm's level of technological development, then a permanent recession is marked by a drop in profits that the firm can overcome by investing in development activities, which would contribute to cover the fixed cost in the long run, thus offsetting the direct, negative impact of the recession. This is, in some sense, a "lame duck effect": low productivity firms are in danger in recession because it becomes increasingly difficult for them to cover their fixed costs. This creates an incentive for reorganization.

CONCLUSION

Our model has highlighted the interaction between aggregate fluctuations and economic growth. In particular, we have emphasized the idea that aggregate fluctuations may well be valuable as a stimulant for growth.

This hypothesis is related to a long standing debate in the literature, and there are some indications that it might be validated by the data. On the other hand, more empirical research is clearly needed in order to assess the importance of the various effects we have pointed out in this paper. For example, is our analysis supported by existing cross-sectoral dynamic data on R&D or by evidence on industry turnover along the business cycle ?

On the theory side, a natural research direction consists in analysing the feedback from growth to fluctuations: in particular, as we have argued above, it is natural to suppose that the productivity improvements generated by a recession, are themselves the engine of a further recovery. Such analysis would be the first step towards a theory where both growth and fluctuations would be endogenous.

Other extensions would involve introducing unemployment in the analysis sketched in Section IV, e.g. following Pissarides (1990), so as to account for the observation that the rate of unemployment is positively correlated with the rate of labor reallocation across firms and sectors. Finally one might try to close the model further, e.g. by introducing intersectorial correlations between idiosyncratic fluctuations or by explicitly modeling the demand side of the economy.

Appendix A: The Multisector Case

In this section we consider a more complete version of the previous model where we can analyze the role of aggregate and idiosyncratic shocks separately. We assume that there are a continuum of firms (or sectors) with a total mass of 1. Let any particular firm be indexed by $i \in [0,1]$. Each firm is located on the technological axis. Let x_{it} denote its position at time t, as in the previous section. We assume that each firm is characterized by a productivity parameter ρ_{it}. The way this productivity parameter affects costs will be specified further below. In addition to that, firms are subject to an idiosyncratic shock which follows the same process as in the previous section: when the firm is in an expansionary state, its productivity parameter is:

$$\rho_{it} = y_E e^{x_{it}} h(v_{it})$$

where v_{it} is, as in the previous section, its speed in technological adoption. When the firm is in the recessionary state, its productivity parameter is:

$$\rho_{it} = y_R e^{x_{it}} h(v_{it})$$

where $y_R < y_E$. Again, ϵ is the flow probability of a transition from state E to state R, while γ is the flow probability of the reverse transition.

We assume that the firms all produce differentiated goods. Let p_{it} be the price of firm i's good at time t, and let p_t be the aggregate price level. Following recently popular specifications (Blanchard and Kiyotaki (1987), Dixit and Stiglitz (1979)), we assume that the demand for good i is, at time t:

$$z_{it} = A_t \frac{\left(\frac{p_{it}}{p_t}\right)^{-\eta}}{p_t} \quad ; \quad \eta > 1. \tag{2.1}$$

where A_t is aggregate demand at time t, and p_t the aggregate price level, defined as follows:

$$p_t = \left(\int p_{it}^{1-\eta} di\right)^{1/(1-\eta)}$$

One can then define aggregate production:

$$Z_t = \frac{A_t}{p_t} = \left(\int z_{it}^{(\eta-1)/\eta} \, di\right)^{\eta/(\eta-1)}$$

The simplest way to interpret this equation is to assume an open economy selling its goods on the world market: p_t is an index of the relative price of home goods; A_t/p_t is the demand for home goods, where A_t is world expenditure on home goods; the demand for good i will then be isoelastic in p_{it} provided home goods enter in the utility function of world consumers through a CES aggregate consumption index. Aggregate fluctuations then come from fluctuations in the relative demand from home goods.[5]

We assume that the cost of production of firm i when it produces z_{it} is:

$$c_{it}(z_{it}) = \left(\frac{z_{it}}{\rho_{it}}\right)^{\beta} \qquad (2.2)$$

where $\beta \geq 1$. When $\beta = 1$, each firm has a constant returns production function with productivity ρ_{it}, whereas when $\beta = +\infty$, each firm has a fixed capacity equal to ρ_{it}.

Idiosyncratic fluctuations are thus fluctuations in the firm's state, E or R, while aggregate fluctuations are fluctuations in the aggregate demand index A_t.

Each firm's production and price policy at time t can then be derived from profit maximization using (2.1) and (2.2). One has:

$$p_{it} = K_p A_t^{\frac{\beta-1}{1+\eta(\beta-1)}} \; p_t^{\frac{(\beta-1)(\eta-1)}{1+\eta(\beta-1)}} \; \rho_{it}^{\frac{-\beta}{1+\eta(\beta-1)}}$$

$$z_{it} = K_z A_t^{\frac{1}{1+\eta(\beta-1)}} \; p_t^{\frac{\eta-1}{1+\eta(\beta-1)}} \; \rho_{it}^{\frac{\eta\beta}{1+\eta(\beta-1)}} = z_t(x_{it}, v_{it})$$

$$\pi_{it} = K_\pi A_t^{\frac{\beta}{1+\eta(\beta-1)}} \; p_t^{\frac{\beta(\eta-1)}{1+\eta(\beta-1)}} \; \rho_{it}^{\frac{\beta(\eta-1)}{1+\eta(\beta-1)}} \qquad (2.3)$$

where the K's are positive constants. This last equation can be rewritten:

where:

[5] Alternatively, one could interpret aggregate demand A_t as nominal money, p_t as nominal prices, and the cost function as nominal costs under rigid nominal wages.

$$\pi_{it} = M_t^j \, e^{\mu x_{it}} G(v_{it}) \tag{2.4}$$

$$M_t^j = K_* A_t^{\frac{\beta}{1+\eta(\beta-1)}} \; P_t^{\frac{\beta(\eta-1)}{1+\eta(\beta-1)}} \; y_j^{\frac{\beta(\eta-1)}{1+\eta(\beta-1)}}$$

$$\mu = \frac{\beta(\eta-1)}{1+\eta(\beta-1)}$$

$$G(v) = h(v)^{\mu}$$

and j ∈ {E,R} is the firm's state. Therefore, a unit move on the technological axis yields a relative increase of μ in terms of profits instead of 1 as in the previous section.

This implies that the firm's position on the technological axis will enter in its value function as a multiplicative term in $e^{\mu x}$.

Let $V_t^E e^{\mu x_{it}}$ (*resp.* $V_t^R e^{\mu x_{it}}$) be firm i's net present value

when it is located at x_{it} at time t and when it is in expansion (resp. recession). Then one has:

$$rV_t^E = \operatorname*{Argmax}_v M_t^E G(v) + \frac{dV_t^E}{dt} + \mu v V_t^E + \gamma\left(V_t^R - V_t^E\right) \qquad (2.5)$$

$$rV_t^R = \operatorname*{Argmax}_v M_t^R G(v) + \frac{dV_t^R}{dt} + \mu v V_t^R + e\left(V_t^E - V_t^R\right). \qquad (2.6)$$

This yields the following first-order condition for v:

$$G'(v_{jt}) = -\mu V_t^j / M_t^j, \qquad (2.7)$$

where j ∈ {R,E} is the firm's state. This equation is quite similar to equation (4) in the previous section.

The above results can be summarized in the following:

<u>PROPOSITION 2.1</u>: At each time t, the speed of firm i in state j, v_{it}^j, is determined by equation (2.7); the firm's value functions V_t^j evolve according to:

$$0 = M_t^E G(v_{it}^E) + \frac{dV_t^E}{dt} - \left(r+\gamma-\mu v_{it}^E\right)V_t^E + \gamma V_t^R$$

$$0 = M_t^R G(v_{it}^R) + \frac{dV_t^R}{dt} - \left(r+e-\mu v_{it}^E\right)V_t^R + eV_t^E$$

Before we study the impact of aggregate fluctuations in this model, we first characterize the aggregate steady-state of this economy when $A_t = A$ is a constant. If aggregate output Z grows at rate g, the price level p_t must decline at rate g. This implies that M_t can be written:

$$M_t^j = M_0 e^{-\mu g t} y_j^\mu .$$

This implies that the solution to (2.5), (2.6) and (2.7) must be such that $V_t^j = V_0^j e^{-\mu g t}$. Hence, as in the previous section, the firm moves at a constant speed v_E in expansion and v_R in recession, with:

$$G'(v_E) = \frac{-\mu V_0^E}{M_0 y_E^\mu}$$

$$G'(v_R) = \frac{-\mu V_0^R}{M_0 y_R^\mu}$$

and, from (2.5) and (2.6):

$$0 = M_0 y_E^\mu h(v_E)^\mu - (r+\mu g+\gamma-\mu v_E)\, V_0^E + \gamma V_0^R \qquad (2.8)$$

$$0 = M_0 y_R^{\mu} h(v_R)^{\mu} - (r+\mu g+e-\mu v_R)\, V_0^E + eV_0^E. \qquad (2.9)$$

Equations (2.8) and (2.9) are identical to those derived in the previous section, except that r is replaced by $r' = r+\mu g$, v_i by μv_i, y_i by $M_0 y_j^{\mu}$, and h by h^{μ}. The average growth rate g enters in the equations as an additional <u>discounting effect</u> because the more the economy grows, the faster the price level declines. Hence the future is more heavily discounted because output will be sold at a lower price. This discounting effect is greater when μ is greater, i.e. when there is more substitutability across products or when decreasing returns are strong.

The determination of the growth rate is illustrated in figure 2.1. The GG schedule defines the growth rate as a function of the effective discount rate r'. The way it is determined is identical to what was obtained in the previous section, once the adequate transformations have been made. The DD schedule defines the effective discount rate as $r' = r+\mu g$. The equilibrium growth rate is determined by the intersection of the two schedules. Comparative statics are easy to do: whenever a parameter has a positive impact on the growth rate in the previous section, it will shift the GG locus upwards here. Hence it will have the same effect as in the previous section. Thus, one gets

<u>PROPOSITION 2.2</u>: $dg/d\theta > 0$ provided G''' is not too negative;

$$\frac{dv_E}{d\gamma} < 0; \quad \frac{dv_R}{d\gamma} < 0; \quad \frac{dv_E}{de} > 0; \quad \frac{dv_R}{de} > 0.$$

Proof: Notice that h is replaced by G and apply propositions 1.2 and 1.3.

□

To complete our characterization of the steady state, we need to derive an explicit expression for the aggregate growth rate g.

As will be seen below, there does not exist a stationary distribution of firms across states and relative locations: the set of locations on the technological axis keeps spreading as time elapses. However, aggregate production asymptotically grows at a constant rate g, which is what matters for our present purposes.

To compute this aggregate growth rate, let us denote by $f_j(t,x)dx$ the number of firms in state j at locations between x and x + dx at time t. f_R and f_E evolve according to:

$$\frac{\partial f_R}{\partial t} = - v_R \frac{\partial f_R}{\partial x} - e f_R + \gamma f_E \qquad (2.10)$$

$$\frac{\partial f_E}{\partial t} = - v_E \frac{\partial f_E}{\partial x} - \gamma f_E + e f_R \qquad (2.11)$$

Inspection of (2.9) and (2.10) reveals that there is no stationary (i.e. invariant by translation) distribution of firms. This is due to the fact that the distribution of firms spans all the locations between the slowest-moving firm (which is always in expansion and moves at speed v_E), and the fastest-moving firm (which is always in recession and moves at speed v_R). But the gap between these two firms does not stop increasing and at an exponential rate.

However, this is no real problem for our purpose, since we are only interested in deriving an expression for the aggregate growth rate. Equations (2.10) and (2.11) can in fact readily be aggregated, leading to:

PROPOSITION 2.3: The steady-state growth rate of the economy is given by:

$$g = \frac{\left(\mu(v_E+v_R) - (e+\gamma) + \sqrt{(\mu(v_E-v_R)+e-\gamma)^2+4e\gamma}\right)}{2\mu}$$

Proof: Let us define:

$$Z_t^j = \left(\int_0^{+\infty} z_t(x, v_j)^{(\eta-1)/\eta} f_j(t, x)\, dx\right)^{\eta/(\eta-1)}, \; j \in \{E, R\}$$

i.e. output aggregated over all firms in state j. Let us also define $\tilde{Z}_t^j = Z_t^{j(\eta-1)/\eta}$. Then (2.10) and (2.11) yield, after using the definition of $z_t(.,.)$ and after integrating by parts:

$$\dot{\tilde{Z}}_t^R = \left(\mu v_R - e - g\frac{(\eta-1)^2}{\eta(1+\eta(\beta-1))}\right)\tilde{Z}_t^R + \gamma\tilde{Z}_t^E \qquad (2.12)$$

$$\dot{\tilde{Z}}_t^E = \left(\mu v_E - \gamma - g\frac{(\eta-1)^2}{\eta(1+\eta(\beta-1))}\right)\tilde{Z}_t^E + \gamma\tilde{Z}_t^R. \qquad (2.13)$$

These two equations imply that in steady state the $(\tilde{Z}_t^R, \tilde{Z}_t^E)$ vector grows at a rate equal to the highest eigen value of this system and becomes proportional to the corresponding eigen vector. Let λ be this eigen value. Then, since $Z_t = (\tilde{Z}_t^R + \tilde{Z}_t^E)^{\eta/(\eta-1)}$, one has $g = \lambda\eta/(\eta-1)$. Substituting this into the characteristic equation of the (2.12)-(2.13) system yields the following second degree equation:

$$(\mu v_R - e - \mu g)(\mu v_E - \gamma - \mu g) - eg = 0$$

thus completing the proof.

In spite of the more complicated expression just derived for the aggregate growth rate g, straightforward computations can show that the analysis of the comparative static effects of v_E, v_R, e and γ on g produces the same results as in the previous section:

PROPOSITION 2.4: $\partial g/\partial e < 0$, $\partial g/\partial \gamma > 0$, $\partial g/\partial v_E > 0$, $\partial g/\partial v_R > 0$.

Proof: obvious.

PROPOSITION 2.5: $dg/d\gamma > 0$ if θ is close to one and $\gamma < e$.
$dg/de > 0$ if θ is close to one and $\gamma > e$.

Proof: When θ is close to one, v_E is close to v_R. A first-order Taylor expansion of the expression in proposition 2.3 yields:

$$g = \frac{(\gamma v_R + e v_E)}{(e+\gamma)}.$$

Since the expression for g is locally the same as in the previous section, using proposition 1.4 with the appropriate substitutions as in proposition 2.2 establishes the result.

The analysis of the multisector case is thus essentially the same as that of the one-firm case developed in the previous section; but we have highlighted how growth can now be dampened by its own discounting effect, and solved the non-trivial problem of computing the aggregate growth rate of an economy where the rate of technological adoption differs across firms or sectors.

This characterization of the aggregate steady state will now serve as a benchmark to study the impact on productivity growth of fluctuations in aggregate demand A_t.

We now analyze the effect of aggregate demand shocks on the firms' decisions to invest in technological progress. Aggregate demand is given by:

$$D_t = \frac{A_t}{p_t},$$

where p_t is the aggregate price in the economy at date t, which can be interpreted as the relative [aggregate] price on the world market if the economy is open. An aggregate demand shock is a shock on the relative value of foreign demand A_t.

We now study the effect of aggregate recessions, respectively when such recession is meant to last for a long period of time and when it is meant to be temporary.

To save on cumbersome notations we assume $\eta = +\infty = \beta$. As it should become clear as we go ahead, this latter restriction involves no major loss of generality.

Firms' investments in technological growth at each point in time are governed by the equations:

$$\left.\begin{aligned} h'(v_{tE}) &= \frac{V_{tE}}{p_t \cdot y_E} \text{ in state } E \text{ (idiosyncratic expansion)} \\ h'(v_{tR}) &= -\frac{V_{tR}}{p_t \cdot y_R} \text{ in state } R \text{ (idiosyncratic recession)} \end{aligned}\right\} \quad (3.1)$$

where $V_{tj}e^x$ is the firms' PDV of future profits at time t in state j and technological location x.

The value functions V_{tj} now evolve according to:

$$\left.\begin{aligned} 0 &= p_t y_{tE} h(v_{tE}) - (r+\gamma-v_{tE}) V_{tE} + dV_{tE}/dt + \gamma V_{tR} \\ 0 &= p_t y_{tR} h(v_{tR}) - (r+e-v_{tR}) V_{tR} + dV_{tR}/dt + e V_{tE} \end{aligned}\right\} \quad (3.2)$$

Note that equations (3.1) and (3.2) are valid even out of steady-state. In the steady-state situation analyzed in the previous section, we find:

$$A_t = A = \text{constant}$$

and

$$\frac{A}{P_t} \;(= D_t) \;= \int_0^1 y_{tj(i)} \cdot h(v_{tj(i)})\, e^{x(i)} di \;(= S_t)$$

where S_t is the aggregate supply at time t with individual firms being indexed by $i \in [0,1]$.

This implies:

and

with an average growth rate for the whole economy equal to:

$$p_t = p_o . e^{-gt} \text{ with } p_o = \frac{A}{S_o}; \quad V_{tj} = V_{oj} . e^{-gt};$$

$$v_{tE} \equiv v_E; \quad v_{tR} \equiv v_R;$$

$$g = \frac{(v_E+v_R)-(\epsilon+\gamma)+\sqrt{(v_E-v_R+\epsilon-\gamma)^2+4\epsilon\gamma}}{2} \tag{3.3}$$

Now let us perturb this aggregate steady-state and consider first the effect of a permanent (unanticipated) recession, i.e. of a permanent drop in A_t (say at time t_o). Clearly, the aggregate demand parameter A_t does not intervene in the above equations, except through the assumption that A_t is constant in steady-state. Therefore, a permanent change in A has no effect on the speed of technological adoption, nor on the growth rate. The effect of a permanent recession is illustrated in Figure A1: there is a once-and-for-all drop in the output level, and the economy proceeds as before, at the same growth rate. In other words, the economy jumps instantaneously to the new steady state, and the growth rate is unaffected since it does not depend on levels.

Figure A1

The impact of a temporary recession is quite different, however. To see this, consider for example the impact of an infinitesimal recession at time t where A drops by ΔA during an

Figure A2

infinitesimal time interval dt. Clearly, present discounted values V_{tE} and V_{tR} are not affected by such an infinitesimal change. If velocities v_E and v_R were unchanged, aggregate supply S_t would be unchanged, so that the price level would drop by $\Delta A/S_t$. But by equation (3.1), this would imply higher velocities. Hence v_E and v_R have to change. In what direction? Clearly, they both have to go up. For this, one just has to show that the price level has to drop, implying an increase in v_E and v_R by virtue of equation (3.1). Suppose by contradiction that the price level increases. Then by equation (3.1), velocities go down, which increases aggregate supply. But then, since A_t drops, p_t also has to drop, which yields the desired contradiction. Hence p_t has to go down, implying an increased pace of technological adoption. Therefore, temporary recessions have a positive impact on growth. When any such recession is over, firms enjoy the pre-recession growth rate, but will be on a higher output path if the recession had not happened. This is illustrated in Figure A3.

Figure A3

The main effect underlying this positive contribution of recessions to the long-run productivity of the economy is an intertemporal substitution effect: it is profitable to use a period of temporarily low aggregate demand to implement technological improvements because the profit loss from doing so is low relative to expansionary periods. Intuitively, the effect is stronger, the deeper the recession. A deeper recession will imply a larger drop in p_t, and hence, according to equation (3.1), a faster pace of technological improvement. Similarly, the shorter the recession, the larger the effect on v_{Et} and v_{Rt}. This is because V_{Et} and V_{Rt} will drop by less if the recession is shorter, thus increasing the absolute value of the right-hand side of (3.1) for a given price drop, hence inducing higher adoption speeds. Consequently, the net effect on productive capacity in the long-run is ambiguous in general, since shorter recessions imply faster productivity growth, but during shorter periods.

To obtain more precise comparative static results one needs to linearize the system of equations (3.1) and (3.2) which describe the dynamics of the economy. One natural case where such linearization can be performed is when the recession is of small magnitude, i.e. $A_t = A(1+a_t)$ with $|a_t|$ small. Since the intuition developed above is independent of idiosyncratic shocks, we shall conduct the formal analysis in the case where there is a single individual state $E = R$, i.e. $\theta = 1$. The analysis could then be extended to the general case where $\theta > 1$ (possibly keeping θ close to 1 in order to preserve the linearization). We will choose instead to analyze the interaction between aggregate recessions and idiosyncratic fluctuations in a simplified version of our model where firms' decisions to invest in technological progress are bang-bang with $v_t \in \{0,v\}$, $v > 0$. (Subsection (b) below.)

As in section 2, let $X_t = \int_0^\infty e^x.f(t,x)dx$, where $f(t,x)dx$ is the number of firms at technological locations between x and x+dx at time t. We have: $S_t = h(v_t)/X_t = D_t = A_t/p_t$. Substituting p_t for $h(v_t)X_t/A_t$ in equations (3.1) and (3.2), one gets the following dynamic system:

$$\left.\begin{aligned} h'(v_t) &= -V_t h(v_t)\,\frac{X_t}{A_t} \quad (\leftarrow (3.1)) \\ 0 &= \frac{A_t}{X_t} - (r-v_t)V_t + \frac{dV_t}{dt} \quad (\leftarrow (3.2)) \\ \dot{X}_t &= v_t X_t \end{aligned}\right\} \qquad (3.4)$$

The steady-state of this system with $A_t = A$ is determined by:

$$v_t = \bar{v};\ h'(\bar{v})/h(\bar{v}) = -1/r$$

$$V_t = \bar{V}e^{-\bar{v}t};\ \bar{V} = A/rX_0$$

$$X_t = X_0 e^{-\bar{v}t}$$

To linearize the system (3.4) around this steady state, we rewrite:

$$A_t = A(1+a_t)$$

$$X_t = X_0 e^{\bar{v}t}(1+x_t)$$

$$V_t = (\bar{V}+u_t)\, e^{-\bar{v}t}$$

and:

$$v_t = \bar{v}+\omega_t$$

Note that there are two dynamic variables, V_t and X_t. X_t is predetermined, while V_t can jump only in response to unanticipated shocks.

The linearized system can be reduced to the following two dynamic equations:

$$\left.\begin{aligned} \dot{u}_t &= rq_1 u_t + Aq_1 x_t/X_0 - Aq_1 a_t/X_0 \\ \dot{x}_t &= (X_0/A)\, q_0 u_t + (1/r)\, q_0 x_t - (1/r)\, q_0 a_t \end{aligned}\right\} \qquad (3.5)$$

where:

$q_0 = \left(h'(\bar{v})^2 - h(\bar{v})h''(\bar{v})\right)/h(\bar{v})^2 > 0$ and

$q_1 = -h''(\tilde{v})h(\tilde{v})/(h'(\tilde{v})^2 - h(\tilde{v})h''(\tilde{v})) > 0.$

The velocity deviation is then determined residually by:

$$\omega_t = (X_0/A)q_0 u_t + (1/r)q_0 x_t - (1/r)q_0 a_t \qquad (3.6)$$

The system (3.5) has a unit root (meaning a zero root here), which captures the fact that transitory shocks to a_t have permanent effects on x_t. The other root λ is explosive, implying uniqueness of the equilibrium path. It is easy to use equations (3.5) to compute the impact of an aggregate recession on the path of x_t. For this assume $a_t = 0$ for $t < 0$, $a_t = -\Delta a < 0$ for $0 \le t \le T$ and $a_t = 0$ again for $t > T$. Straightforward computations yield:

$$x_t = \bar{x}(1 - e^{-\lambda t}), \quad \text{for } 0 \le t \le T$$

$$x_t = \bar{x}, \quad \text{for } t > T.$$

And

$$\bar{x} = \frac{\Delta a(e^{\lambda T}-1)}{r^2 q_0 q_1 + e^{\lambda T}} \qquad (3.7)$$

The path for x_t is depicted in Figure 2.4. The temporary recession implies a permanent increase in productivity x. Equation (3.7) shows that the cumulated increase is directly proportional to the depth of the recession and increases in its length. $\bar{x}$ reaches however an upper bound as the length of the recession increases. Therefore $\bar{x}/T$, which is the average increase in growth due to the recession, vanishes as T goes to infinity. Figure 2.5 illustrates the behaviour of velocity ω_t: it jumps at the start of the recession, then steadily declines, and jumps back to its steady-state value at the end of the recession. Figure 2.6 illustrates the behaviour of output.

The above analysis suggests a positive impact on productivity growth of having a succession of temporary recessions. One natural extension would consist in introducing such sequence of temporary recessions explicitly into our model and then try to derive some comparative statics results on the average growth rate as a function of the duration, depth and frequency of aggregate recessions. This, however, goes beyond the scope of the present paper.

Other extensions involve, first, analyzing the cross-effects between aggregate recessions and idiosyncratic fluctuations (case where $\theta >> 1$);[6] and second, investigating the extent to which

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[6] Remark: In general the relative impact of idiosyncratic versus aggregate recession depends on η and β. When η tends to one, μ tends to zero, and M_t^j tends to A_t (see Section 2 above). This implies that profits do not depend on the firm's idiosyncratic state, i.e. $V_t^E = V_t^R$. This in turn implies $v_t^E = v_t^R$. Hence idiosyncratic recessions do not induce faster technological adoption. When η tends to 1, complementarities become so strong that the productivity of a firm does not affect its profits. This intuition should however be taken with caution, because it does not imply that aggregate recessions do not have any impact on v. When μ tends to 0, equation (2.7) becomes $h'(v_{it})/h(v_{it}) = -V_t^j/A_t^j$, so that a temporary aggregate recession will have a large positive impact on v_{it}, as argued above. When μ goes to zero, the benefits of a higher speed are reduced, but so are its costs. Hence a parameter which shifts current profits relative to net present value will still have an impact on velocity. The effect of y_j on profits will tend to disappear, so that firms will choose the same speed no matter what their idiosyncratic state is. On the other hand, the effect of A_t on profits will not vanish.

It follows from this discussion that when complementarities are stronger, idiosyncratic recessions will be less effective than aggregate recessions in increasing the pace of technological adoption.

the above results can be affected by the introduction of positive fix costs of production for firms. These two things are done in appendix B within a simple version of our model where firms' investment in technological progress take the form of a zero-one decision.

Appendix B: The cross-effects of aggregate recessions and idiosyncratic fluctuations when firms face non-zero fix costs of production

In this section we extend the above analysis, first by formally reintroducing idiosyncratic fluctuations ($\theta > 1$) and second by introducing fix flow costs of production for firms, which we denote by $f \geq 0$. To simplify the analysis we assume that the only technological choice available to firms at any instant in time is either to invest all their human resources in productivity growth

$$(i.e.\ v_i = v > 0 \text{ with } h(v) = 0)$$

or not to invest them at all

$$(i.e.\ v_i = 0 \text{ with } h(0) = 1).$$

Presumably, at any instant t, firms in individual state j will only invest in technological progress if $p_t.e^x \leq S_i$, where from Section 1 (Proposition 1) we expect that $S_E < S_R$, given that firms invest more in technological progress during [individual] recessions. In any case we shall start assuming the latter inequality and then show that it is indeed satisfied in equilibrium.

Now, suppose that $p_t.e^x > S_R$. Then, neither firms experiencing an idiosyncratic boom nor those experiencing an idiosyncratic recession will invest in technological progress. The value functions $V_j(t,x)$, $j = E,R$ will then evolve according to the following differential equations, similar to the above (3.2) but with $v_j = 0$:

$$\left.\begin{aligned} 0 &= p_o e^{-gt} e^x y_R - f - (r+e) V_R(t,x) + e V_E(t,x) + \frac{\partial V_R}{\partial t}(t,x) \\ 0 &= p_o e^{-gt} e^x y_E - f - (r+\gamma) V_E(t,x) + \gamma . V_R(t,x) + \frac{\partial V_E}{\partial t}(t,x) \end{aligned}\right\} \quad (3.8$$

[Here x remains constant since no firm invests in technological progress.]

This differential system yields:

$$V^R(t,x) = -\frac{f}{r} + p_0 e^{-gt}.e^x\left[\frac{\overline{y}}{r+g} - \frac{\Delta y}{(\gamma+e)(r+g+\gamma+e)}\right]$$

$$\text{where } \overline{y} = \frac{\gamma y_R + e y_E}{\gamma+e}\ ;\ \Delta y = y_E - y_R.$$

Now firms facing individual recessions will decide not to invest in technical progress if and only if:

$$p_0 e^{-gt} e^x y_R - f > v.\frac{\partial V_R}{\partial x}(t,x) \qquad (3.9)$$

increase in the firm's value if they invest into the speed v of technical progress.

i.e.:

$$p_t.e^x > S_R = \frac{f}{y_R - v\left[\frac{\bar{y}}{r+g} - \frac{\Delta y}{(\gamma+e)(r+g+\gamma+e)}\right]} \qquad (3.10)$$

One easily sees from (3.10) that an aggregate recession, even if permanent, will tend to make (3.10) become violated (if p_0 decreases sufficiently and/or if f is sufficiently large) and therefore will encourage firms experiencing idiosyncratic recessions to invest more in technological progress. This is in contrast with the above subsection where we saw that in the absence of fix costs of production, only temporary recessions could have a stimulating effect on growth.

Now suppose that $p_t.e^x < S_E$, so that firms would always invest in technological progress at date t whichever idiosyncratic state they are presently experiencing. In that case, the value functions $V_j(t,x)$ evolve according to the following differential system, also derived from (3.2) with h(v) = 0 and $v_j = v$ for j = E,R.

$$\left.\begin{aligned} 0 &= -(r+e)V_R(t,x) + eV_E(t,x) + \frac{\partial V_R}{\partial t}(t,x) + v.\frac{\partial V_R}{\partial x}(t,x) \\ 0 &= -(r+\gamma)V_E(t,x) + \gamma V_R(t,x) + \frac{\partial V_E}{\partial t}(t,x) + v.\frac{\partial V_E}{\partial x}(t,x) \end{aligned}\right\} \quad (3.11)$$

Let $W_j(t,x-vt) = V_j(t,x)$, where we denote $x-vt = z$. The modified value functions W_j satisfy the differential system:

This homogeneous system involves no derivatives w.r.t. z and therefore the solutions W_E and W_R to this system must be independent of z, i.e. of x. So will the value functions V_E and

$$0 = -(r+\epsilon)W_R(z,t) + e.W_E(z,t) + \frac{\partial W_R}{\partial t}(z,t)$$

$$0 = -(r+\gamma)W_E(z,t) + \gamma.W_R(z,t) + \frac{\partial W_E}{\partial t}(z,t) \qquad (3.12)$$

V_R.

The necessary and sufficient conditions for firms to invest in technical progress when experiencing an idiosyncratic boom is then simply:

$$p_t.e^x.y_E - f < v.\frac{\partial V_E}{\partial x}(t,x),$$

where the RHS of this inequality is equal to zero when $p_t.e^x < S_E$ as we have just established.

We can then rewrite the above inequality as:

$$p_t.e^x < \frac{f}{y_E} = S_E. \qquad (3.12)$$

and easily verify from (3.10) and (3.12) that indeed $S_R > S_E$. Now looking at (3.10) and (3.12) one can easily get to the following conclusions:

(a) If f = 0, i.e. in the absence of fix costs of production, firms will never invest in technical progress during an idiosyncratic boom whereas they may invest during an idiosyncratic recession provided the denominator in the RHS of (3.10) becomes negative. This in turn will necessarily occur if $y_E - y_R = \Delta y$ is sufficiently large, i.e. if idiosyncratic fluctuations are of sufficiently large magnitude. Condition (3.10) also points to a mutually reinforcing effect of aggregate recessions and idiosyncratic fluctuations, which, in the particular case where firms investments in technology are of the zero-one type, appears explicitly only when the fix cost of production is strictly positive.

(b) If $f > 0$, firms will invest in technical progress more often in recessions than during an idiosyncratic boom, given that $S_E < S_R$. In fact firms experiencing a boom will only invest in technology when they make losses by producing at too high a fix cost. Furthermore, the larger the magnitude of idiosyncratic fluctuations $(y_E - y_R)$, the more stimulating aggregate recessions will be for growth. [Aggregate recessions in the form of a drop in the aggregate price p_0 will induce firms to invest more in productivity growth, the larger the difference $y_E - y_R$ and/or the larger the fix cost f', as it appears from 3.10 above.]

(c) The increased growth rate induced by an increase either in fix cost f or in the magnitude of fluctuations Δy will retroact on the decision of firms experiencing an idiosyncratic recession. On the one hand, a higher growth rate reduces the NPV of technical progress: this is the discounting effect pointed out in Section 2. On the other hand, a higher growth rate makes the revenues of firms that do not invest in technical progress fall faster below the current fix cost value (which also grows at rate g); this in turn should encourage them to invest more in productivity growth. The overall effect of an increased fix cost on growth will be positive (for the same reasons as in Section 2 above), but all the more so when initially the fix costs of production incurred by firms are high.

A final remark to conclude this section: we have just shown that in case firms face non-zero fix costs of production, even a durable ("permanent") aggregate recession has a stimulating effect on technical progress. From then on, one could "easily" generate endogenous cycles by making investments in technical progress become a major component of aggregate demand: an aggregate recession, through its stimulating effects on R&D investments would induce aggregate demand to boom up again which in turn would discourage firms from investing further in technical progress, thereby triggering a new aggregate recession etc ... Formalizing this latter idea would provide us with an endogenous theory of both growth and cycles. The formal development of such theory goes beyond the scope of this paper, and will rather be the subject of further research.

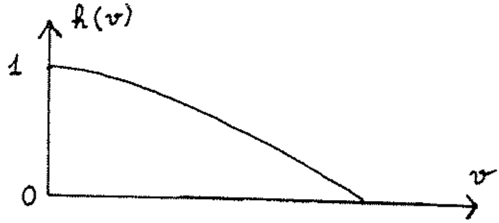

Figure 1

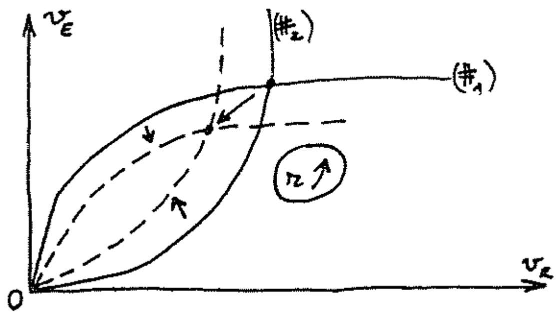

Figure 2

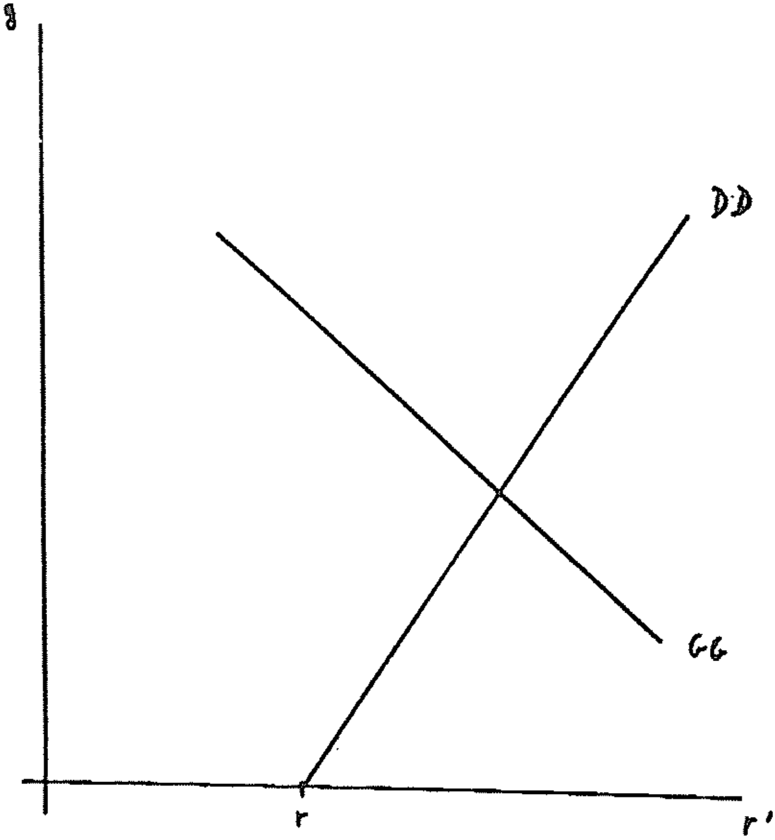

Figure 2.1

(A1)

A2

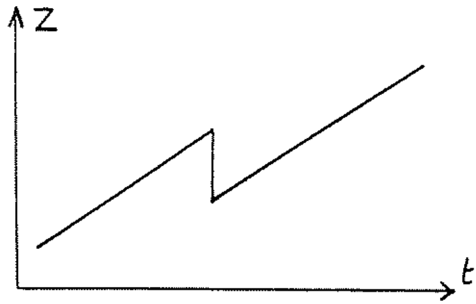

Figure 1

A3

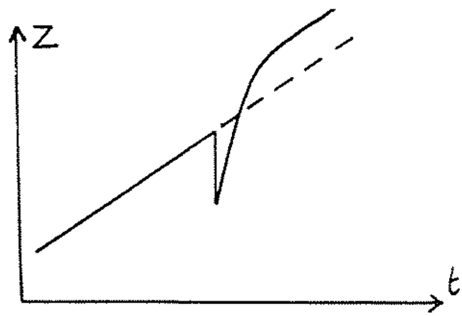

Figure 2