

# DISCUSSION PAPER SERIES

No. 5756

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*INTERNATIONAL MACROECONOMICS*



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Discussion Paper No. 5756  
July 2006

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CEPR Discussion Paper No. 5756

July 2006

## ABSTRACT

### The Brevity and Violence of Contractions and Expansions\*

Early studies of business cycles argued that contractions in economic activity were briefer (shorter) and more violent (rapid) than expansions. This paper systematically investigates this claim and in the process discovers a robust new business cycle fact: expansions and contractions in output are equally brief and violent but contractions in employment are briefer and more violent than expansions. The difference arises because employment typically lags output around peaks but both series roughly coincide in their troughs. We discuss the performance of existing business cycle models in accounting for this fact, and conclude that none can fully account for it. We then show that a simple model that combines three familiar ingredients - labor hoarding, a choice of when to scrap old technologies, and job training or job search - can account for the business cycle fact.

JEL Classification: E23, E24, E32 and J60

Keywords: asymmetric cycles, business cycles, economic expansions and contractions and unemployment

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\* We are grateful to Bill Dupor, Jonathan Parker, David Romer, Mark Watson, and several seminar participants for useful comments.

Submitted 25 June 2006

# 1 Introduction

In a series of studies, Wesley Mitchell (1913, 1927, 1946 with Burns) collected a set of facts on the U.S. business cycle. Most of them have been thoroughly scrutinized since then and have survived the test of time. Today, it is well-established that: fluctuations occur in aggregate activity and not in particular sectors; cycles are recurrent but not periodic; cycles have at least two different stages, expansions and contractions; once the economy enters one of the stages, it stays there for some time, so detecting turning points is important for forecasting; and there are regular and predictable co-movements between variables over the cycle that can be expressed as relative variances and lead-and-lag correlations.<sup>1</sup>

There is another fact, however, that while emphasized by Mitchell in all of his works, has not received as much attention. In his words: “Business contractions appear to be briefer and more violent than business expansions.” (Mitchell, 1927: 333). The main aim of this paper is to empirically investigate whether this belongs among the set of business cycle stylized facts.

To answer this question, we consider different measures of business activity both in the product market (industrial production, GDP, personal income, sales, etc.) and in the labor market (the unemployment rate, total employment, etc.). We use different detrending procedures and several algorithms to detect peaks and troughs in the data and split each time-series into expansions and contractions. We measure the brevity of an expansion (contraction) as the number of periods from trough (peak) to peak (trough), and we measure violence by considering different estimators of the rate of change in business activity. Finally, we compare brevity and violence during expansions and contractions by looking at their averages and their distributions, both visually and statistically.

After going over hundreds of different combinations of these methods, we reach one robust conclusion: contrary to Mitchell’s claim, expansions and contractions in output are equally brief and violent but, just as Mitchell wrote, contractions in employment are briefer and more violent than expansions. The difference between output and employment comes from a difference in the timing of turning points: peaks in employment typically lag peaks in output, whereas the troughs in both series are roughly coincident. Because we find that these patterns are so robust, we propose them as a new business cycle fact.

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<sup>1</sup>Zarnowitz (1992) and Stock and Watson (1999) summarize the established business cycle facts.

The second contribution of this paper is to ask whether existing business cycle models can account for this fact. We conclude that there is no available theory that can simultaneously account for all of its parts. While some theories can explain why output and unemployment can move in opposite directions during parts of the business cycle, or why contractions in employment are briefer than expansions, or why contractions in employment are more violent than expansions, there is no existing single theory that can account for all three parts of the fact.

We then present a simple theory that can qualitatively account for the fact by combining three ingredients. The first ingredient is labor hoarding: if firms can vary production by changing the number of employees (an extensive margin) or the number of hours per worker (an intensive margin), then output and employment can move differently during the business cycle. The second ingredient is technology adoption or the optimal timing of creative destruction: if firms can sustain ageing technologies for a while, expansions in employment can persist even when output has started declining. The third ingredient is that job separations can occur abruptly while job creations take time, accounting for the asymmetry in the violence of employment changes. This is the case in theories of training that emphasize that employers can fire quickly but need time to train new workers, and in theories of job search in which employees can quit quickly but need time to find a new job.

Section 2 of the paper presents a baseline case that illustrates the new business cycle fact. Section 3 discusses our methods for measuring the brevity and violence of contractions and expansions more systematically and exhaustively. Section 4 contains our main results, while section 5 considers their robustness. Section 6 uses our findings to evaluate existing theories of asymmetric business cycles and section 7 presents a new simple model to match the facts. Section 8 concludes and discusses the implications of our findings for European unemployment hysteresis and jobless recoveries.

### *The related literature*

There is a large empirical literature on asymmetric business cycles that we cannot do full justice to here. Relative to this paper, the literature fits broadly into three branches. The first branch, starting with Neftci (1984) and DeLong and Summers (1986) looks at skewness in either the level or in the changes in economic activity.<sup>2</sup> Skewness in levels

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<sup>2</sup>See also Falk (1986), Sichel (1989, 1993), Rothman (1991), Verbrugge (1997), Belaire-Franch and Peiro

would imply that the economy spends more time above or below trend. Our emphasis is instead on the behavior of the economy when it is expanding or contracting. Skewness in changes evaluates whether economic activity is more likely to increase or to fall. Yet, while economic activity is generally rising during an expansion, there are some periods where it actually falls. Using our data, we find that in a typical U.S. expansion, output actually falls about one-fifth of the times, and in a typical contraction, output actually rises during one-fourth of the periods. An asymmetry between the dates when economic activity falls and rises does not imply or is implied by an asymmetry between the business cycle phases of expansion and contraction. While skewness is interesting in its own right, it does not address the brevity and violence of expansions and contractions.

A second branch in the literature, starting with the seminal contribution of Hamilton (1989), estimates regime-switching models and examines whether there are differences between the two regimes.<sup>3</sup> The typical finding in these studies is that the dynamics of recessions are significantly different from the dynamics of booms. Our paper distinguishes itself from this literature because we are not looking at whether contractions are generally different from expansions. Rather, we focus on a more specific difference: whether they are briefer and more violent. This focus allows us to be more precise and to have more powerful tests of this particular type of asymmetry. It implies, of course, that even if we fail to find differences in brevity and violence, there may still be other forms of asymmetry.

A third branch of the empirical literature has focused on specific types of asymmetries. McQueen and Thorley (1993) found that peaks tend to be round, while troughs are sharp. Diebold and Rudebusch (1990) found no evidence that expansions and contractions are duration dependent—whether they are more likely to end as they last longer.<sup>4</sup> Finally, Diebold and Rudebusch (1992) and Watson (1994) compared the duration of expansions and contractions in the post-war not to each other but to the equivalent moments in pre World War II data. Relative to these articles, this paper focusses on a different type of asymmetry and compares post-war expansions and contractions.

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(2003), and Bai and Ng (2005).

<sup>3</sup>See Acemoglu and Scott (1994), Ramsey and Rothman (1995), and Hamilton (2005) who use close variants of the Hamilton model. Beaudry and Koop (1993), Hussey (1992), Hess and Iwata (1997), Montgomery et al (1998), Rothman (1998), and Koop and Potter (1999) use other non-linear models to look for business cycle asymmetries. Clements and Krolzig (2003) bridge the two first branches of the literature, using a regime-switching model to look for skewness.

<sup>4</sup>See also McCulloch (1975), Sichel (1991), Durland and McCurdy (1994), and Lam (2004).

Relative to previous empirical work, this paper therefore contributes the investigation of one new type of asymmetry (brevity and violence) between the two states of the business cycle (contractions and expansions) and systematically comparing output to employment.

Regarding theory, there are several existing models of business cycle asymmetries. Some have argued that some constraints bind during booms but not recessions or vice-versa, whether they are credit constraints (Kocherlakota, 2000) or capacity constraints (Gilchrist and Williams, 2000, and Hansen and Prescott, 2005). Others have emphasized learning about productivity (Chalkley and Lee, 1998, and Van Nieuwerburgh and Veldkamp, 2006) or mismatches between skills and technologies (Jovanovic, 2006). An alternative is that during periods of lower activity, it is cheaper to adopt new technologies (Caballero and Hammour, 1996) or to fill a job vacancy (Burgess, 1992). Mortensen and Pissarides (1994) note that jobs are destroyed as soon as their value is negative, but it takes time for potential matches with positive value to form. Millard et al. (1997) investigate the performance of some of these models above at fitting the persistence of unemployment in response to shocks during recessions and booms. We provide two main contributions to this theoretical literature. First, our new business cycle fact puts forward a new challenge to these models. Second, we propose a simple new model that can account for the fact.

## 2 A benchmark case

We start our empirical investigation by looking at one specific case. The most common measure of business activity is quarterly real GDP. We de-trend the log of this series using a modified HP filter due to Rotemberg (1999) that avoids some of the problems that the conventional HP filter has at the edges of the sample. Then, we identify peaks and troughs using the standard algorithm of Bry and Boschan (1971).<sup>5</sup> The top panel of figure 1 shows the periods of expansion and contraction in output.

To measure brevity, we simply compute the average number of quarters during expansions and contractions. The average expansion in GDP lasted 11.5 quarters, whereas the average contraction lasted 9.5 quarters. A simple t-test of equal duration versus the alternative of longer expansions has a p-value of 0.22. Looking at the whole distribution of

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<sup>5</sup>Both the Rotemberg modified HP filter and the Bry and Boschan algorithm will be described in more detail in section 3.

durations, we find that only 60% of expansions were longer than the median duration of a contraction. Moving next to violence, we compute the average change in the series during expansions and contractions. The average growth of GDP during an expansion is 0.49%, while average growth during a contraction is -0.51%. A t-test that these numbers add up to zero has a p-value of 0.44 so, at typical significance levels, we accept the null hypothesis of equal violence. Moreover, again only 60% of expansions were less violent than the median contraction, and the distribution of violence in expansions is very similar to the distribution in contractions. This baseline case therefore contradicts Mitchell's assertion.

**Result 1:** *Expansions and contractions in output are equally brief and violent.*

[...FIGURE 1...]

To investigate further, we look at another series: the log of 1 minus the unemployment rate. The state of the labor market is sometimes seen as being as important as the level of GDP to assess the state of the business cycle, and the unemployment rate is its most used measure. The bottom panel of figure 1 shows the expansions and contractions in this series, again after using the modified-HP filter and the Bry and Boschan algorithm.

The results for employment are strikingly different from those for GDP. The average expansion in employment lasted 18 quarters, 10 quarters more than the average contraction. The test that these are equal has a t-statistic of 2.63, and a p-value of 0.00 against the one-sided alternative that contractions are briefer. In fact, in the entire sample, there isn't a single expansion that lasted shorter than the median duration of a contraction. The data overwhelmingly points to shorter contractions than expansions in employment. Moving to violence, the average growth rate of employment during an average expansion was 0.18% whereas the average growth during contractions was more than double: -0.39%. The t-statistic is 3.41, which overwhelmingly rejects equality in the absolute value of growth rates in favor of the alternative that contractions are more violent. Looking at the whole distribution, every single expansion in employment in the post-war had an average growth rate lower than the median absolute growth rate during a contraction. Therefore, focussing on employment leads to the opposite conclusion from looking at output.

**Result 2:** *Contractions in employment are briefer and more violent than expansions.*

To understand why output and employment are so different, figure 2 plots the average



business cycle dynamics of both series. Starting from each employment trough, we recorded output and employment before and after 20 quarters, and averaged these over cycles. We also recorded the date of the previous and next peak in both employment and GDP, as well as the date of the nearer trough in GDP. The figure shows that employment and output both trough at around the same time. On average, troughs in employment lag output by only 0.13 quarters or about 12 days. The difference in duration between the two series therefore comes almost exclusively from peaks in employment lagging those in output by about 2.25 quarters or 203 days. Employment therefore is a lagging indicator of output cycles only when coming down but not when going up.

**Result 3:** *Employment lags output at peaks but coincides with it at troughs.*

[...FIGURE 2...]

Part of the difference between output and employment is due to the two brief cycles in the late 1960s and mid 1990s that appear in GDP but not in the employment rate (or the NBER). Excluding those two cycles, the case for briefer and more violent contractions in output slightly strengthens but remains very weak. In this case, the average expansion now lasts 15.9 quarters and the average contraction 10.2 quarters, with a p-value of 0.06 in a test of equality between the two. Growth during an average expansion is now 0.51% while that during an average contraction is -0.59%, and the p-value is now 0.25. Result 1 is not just due to more output cycles.

Why are the NBER dates so different from our output cycles? Because the NBER eclectically looks at many series to reach its decisions, we cannot definitely answer this question. Still, we can offer some clues. Using the Bry and Boschan algorithm on GDP that is not de-trended, we can reproduce almost exactly the NBER dates. Since output trends up, only in rare instances does it actually decline leading the NBER to call a contraction. The positive trend automatically leads to longer expansions and shorter contractions, so the question of whether contractions are brief and violent stops being interesting with trending data. Moreover, an increase in trend growth automatically leads to even rarer contractions, so brevity and violence in trending data depend on the trend, not just on the cycle. For both reasons, we work with de-trended data.<sup>6</sup>

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<sup>6</sup>The NBER itself has not always been consistent about de-trending. While post 1927, it has focussed

To conclude, figure 3 summarizes the peak-to-peak dynamics of output and employment suggested by the three results. Starting from a trough, employment in a recovery rises at a slower pace than output. Output eventually reaches its peak and starts falling, while employment keeps rising at a tame pace. Only almost 7 months after the peak in output does employment finally reach its peak, after which it falls sharply catching up with output at the next trough.

[...FIGURE 3...]

### 3 Measurement algorithms and methods for testing

Our empirical investigation requires a measure of business activity, an algorithm that de-trends it, picks turning points and measures brevity and violence, and a method to systematically compare them.

#### 3.1 The series and de-trending

For our baseline results, we measure output using the log of industrial production (IP) and employment using the log of one minus the unemployment rate. Our data are quarterly and cover the period from 1948:1 to 2005:1.

As explained earlier, we remove the upward trend in output. The unemployment rate does not trend up or down, but it has a significant low frequency component driven by demographic changes. Using the raw series can lead to misleadingly observing very short or very long business cycle phases around the time of changes in this component. Therefore, we also de-trend unemployment.<sup>7</sup>

There is no consensus on what is the best way to de-trend economic series. We use four algorithms that broadly capture four different views of the source of trends. The first view sees trends as deterministic but subject to occasional abrupt changes in growth rates. To represent this view, we compute the trend by fitting a linear regression of time, allowing for breaks in the slope in 1973:4 and 1995:4 to capture the productivity slowdown.<sup>8</sup> The

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on trending data, Romer (1994) convincingly shows that the business cycle dates for the 1884-1927 period came from looking at de-trended data. This is consistent with Mitchell's own view, which seems to have hesitated between de-trending or not, as discussed by Romer.

<sup>7</sup>Cycles on de-trended data are sometimes called "growth cycles" (Zarnowitz, 1989).

<sup>8</sup>We experimented with close alternatives dates for the breaks and found no substantial differences.

second view agrees that the trend is deterministic, but models changes that occur smoothly. We fit a polynomial function of time to the series, using measures of goodness of fit to pick the order of the polynomial. The third view associates trends with possibly stochastic movements affecting the low frequency of a series. We use the Baxter and King (1999) band-pass filter to extract cycles of duration between 6 and 32 quarters. For output, we found that other choices than the conventional 6-32 led to very similar results. For the unemployment rate, it is more common to extract only very low frequency trends, so we also pursue the alternative 2-80 quarters specification for the band-pass filter. This leads to somewhat different implications for the measured violence of unemployment, which we will discuss later. A fourth view of the trend insists that it should be smooth and uncorrelated with the cycle. We calculate it using the minimization algorithm of Rotemberg (1999), which builds on the Hodrick-Prescott procedure but performs better at the edge of the sample.<sup>9</sup>

### 3.2 Detecting expansions and contractions

Expansions and contractions are defined by peaks and troughs. The peak of the cycle marks the end of an expansion and the beginning of a contraction, while the trough marks the end of a contraction and the beginning of an expansion.<sup>10</sup> We consider four different algorithms to detect peaks and troughs; each has virtues and flaws, so by considering several we ensure the robustness of the results. There are certainly alternatives, but we pick these four as broadly representative of the available menu.<sup>11</sup> The appendix describes each method formally.

The first method, which we label the window method, searches for local extremes. It starts by smoothing the series using a 5-quarter centered moving average to remove high-frequency noise. Then, at each date, it forms a symmetric window with  $N$  quarters around each side of the date. If the date is a maximum (minimum) in the window, then it becomes a candidate peak (trough). Finally, to ensure that peaks and troughs alternate, we take the

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<sup>9</sup>We obtained very similar results using the Hodrick-Prescott filter. Missing from our list of de-trending algorithms are unobserved-components models. We avoided these because they impose tight statistical restrictions on the series that affect their symmetry.

<sup>10</sup>Note that expansions and contractions are not booms and recessions. The latter refer to the economy being above or below trend, whereas expansions and contractions refer to it rising or falling. A boom includes both the final part of an expansion and the initial part of a contraction.

<sup>11</sup>For a discussion of alternative methods to pick turning points, see Canova (1999), Harding and Pagan (2002), and Zarnowitz and Ozyildirim (2002).

later of two consecutive peaks (troughs). We chose  $N$  so that the number of turning points was not too different from the number found by the NBER. We set  $N=5$ , but the results are similar if  $N$  is 3 or 7.

Second is the reversal method, which looks for reversals in the successive changes in the series. This method finds peaks at dates which are preceded by  $N$  periods of successive increases and  $N-1$  quarters of successive decreases. Troughs are dates preceded by  $N$  decreases and followed by  $N-1$  increases. This method captures the often-held view that a contraction is a period of some quarters of negative growth. We chose  $N=3$ , for the same reasons as in the window method.

Third is the Bry and Boschan (1971) approach. Bry and Boschan found that their algorithm reproduces the set of turning points picked by Burns and Mitchell (1946) and the NBER. King and Plosser (1994) and Watson (1994) also used this method to detect turning points. While the exact algorithm contains several steps, it can be broadly described as follows. First, the algorithm smooths the series using a 1-year centered moving average and looks for peaks and troughs in a manner akin to the window method. Second, it smooths the series using an alternative moving average (a Spencer filter) that allows it to have sharper changes, and again looks for turning points. Third, it looks for turning points in a shorter (3-month) centered moving average. Finally, the algorithm looks for peaks and troughs in the unsmoothed series using a series of criteria to eliminate mistakes that may be caused by erratic movements.

Our fourth algorithm is in the Markov regime-switching tradition and is due to Chauvet and Hamilton (2005). It assumes that a series  $x(t)$  alternates between two states, so either  $x(t) = x_1(t)$  or  $x(t) = x_2(t)$ . The state is a latent variable that follows a first-order Markov chain where the probability of staying in state 1 is  $p_1$  and the probability of staying in state 2 is  $p_2$ . In each state,  $\Delta x_1(t) \sim N(\mu_1, \sigma_1^2)$  and  $\Delta x_2(t) \sim N(\mu_2, \sigma_2^2)$ . Associated with this statistical model is a likelihood function that we can maximize to find estimates of the six parameters  $(p_1, p_2, \mu_1, \mu_2, \sigma_1^2, \sigma_2^2)$ . Note that we use this approach, not as a model of the stochastic process, but solely as an algorithm to provide a statistic of the sample path. It provides estimates at each date of the probability of being in either of the two states, that we use to define expansions as the dates when the probability of being in the high-mean state is higher than 50%, and contractions when it falls below 50%. In practice the estimated probability is above 80% and below 20% most of the time, so the results are not sensitive

to the 50%-cutoff rule. An important caveat to this approach is that it does not impose that the two states correspond to expansions and contractions. Indeed, when we use this algorithm, we find that the two states corresponded to pre and post 1984, marking the fall in output volatility that has been called “The Great Moderation.” We extend the model to allow for  $\sigma_1^2$  and  $\sigma_2^2$  to differ pre and post 1984:3, raising the number of parameters to eight.<sup>12</sup> While the states identified by the algorithm then more closely resemble expansions and contractions, one should still keep this caveat in mind.

### 3.3 Measuring brevity and violence

We call the *duration* of a contraction the number of periods from peak to trough. The duration of an expansion is the number of periods from trough to peak. *Brevity* is understood as smaller duration.

*Violence* refers to the rate of change of  $x(t)$  while in an expansion or in a contraction. We use three related measures of violence. The first and simplest is *steepness*: the (absolute value of) the average change in the series. It captures well the idea that a contraction is violent if activity is falling quickly. Note that, for a de-trended series, the numerator in steepness (the total change in the series from one turning point to the next) must on average be the same for expansions and contractions. Since the denominator in steepness equals duration, then a brief series will tend to be a violent series as well, although not necessarily so. Our second measure of violence is *sharpness*, and it equals the square root of the average squared change in  $x(t)$ . It is easy to show that  $sharpness^2 = steepness^2 + VAR(\Delta x(t))$ . A contraction is therefore sharper if it is steep but also if the series jerks around by more during contractions than during expansions. Our third measure, *slope*, is the least-squares coefficient on a linear trend from a regression of  $x(t)$  on the trend and an intercept. The first two measures are sensitive to the exact location of peaks and troughs, whereas slope is more robust to measurement error in spotting these dates.

If during a contraction (or expansion) a series falls exactly linearly, then steepness = sharpness = slope. Otherwise, sharpness adds to steepness a measure of how volatile the series is, while slope makes the measure of violence less dependent on the exact location of

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<sup>12</sup>We have looked at a few quarters before and after this exact date and obtained similar results. A less appealing alternative is to impose  $\sigma_1^2 = \sigma_2^2$ , since it constrains differences in violence solely to differences in  $\mu_i$ . When we tried this alternative, the resulting turning points were quite similar.

the turning points.

### 3.4 Statistical inference

The algorithms discussed so far produce a set of measures of duration in expansions and contractions  $\{D^E(i, p), D^C(i, p)\}$  and of violence  $\{V^E(i, p), V^C(i, p)\}$ . The index  $i$  refers to the cycle within a series, while  $p$  indexes the procedure used to transform the data, to identify turning points, and to measure violence. We employ three approaches to infer whether contractions are different from expansions.

First, we plot the cumulative distribution functions (cdf's) for each measure across  $i$ , and see whether the cdf for the duration of contractions tends to lie to the left of the cdf for expansions, while the reverse describes the cdf's for violence. This allows us to graphically infer whether contractions tend to be briefer and more violent than expansions. At the extreme, if  $F(D^C(., p)) \geq F(D^E(., p))$  and  $F(V^C(., p)) \leq F(V^E(., p))$ , then the duration of expansions first-order stochastically dominates the duration of contractions, while the opposite is true of violence.

Second, we test the null hypotheses of equal average duration  $E[D^E(., p)] = E[D^C(., p)]$  against the one-sided alternative of shorter contractions  $E[D^C(., p)] < E[D^E(., p)]$ , and equal violence  $E[V^E(., p)] = E[V^C(., p)]$  against more violence in contractions  $E[V^C(., p)] > E[V^E(., p)]$ . If duration and violence are independent over  $i$ , then a standard t-test of equality of means is efficient in a finite sample under normality, and asymptotically efficient otherwise. The assumption of independence may be an issue. Section 4.4 investigates whether it is so, by using a bootstrap to produce distributions for the t-statistic when the duration and violence are correlated across successive cycles.

Third, we test the null hypothesis that the distributions from which duration and violence are drawn are the same for expansions and contractions. To test this null, we use a Wilcoxon rank-sum test, computing the exact p-values for each sample size. Diebold and Rudebusch (1992) note that this test can be quite efficient even in small samples. It also requires the assumption of independent draws, so we again employ the bootstrap to calculate its distribution if there is serial correlation.

One possible criticism of these tests is that they treat the  $D(\cdot)$  and the  $V(\cdot)$  as observations, even though these are the product of the algorithms that we described so far. We do not think that this is a matter of too much concern. Most macroeconomic series, like output

or consumption, are also the result of algorithms with many steps that add, subtract, average, interpolate and smooth. Since our algorithms are symmetric, they do not create any asymmetry between expansions and contractions beyond the one already in the data. Still, we address this concern in section 4.4. We use estimated symmetric models for output and employment to generate artificial times-series for these of the same length as our sample on which we apply our algorithms and tests. We repeat this for many artificial series and investigate whether we could erroneously draw the wrong conclusions regarding brevity and violence of contractions and expansions in these symmetric-by-construction observations.

## 4 Main results

Figure 4 plots the cdf's for output for brevity across all 16 methods (4 for de-trending and 4 for detecting turning points). The distributions typically lie on top of each other without a discernible difference between expansions and contractions. In contrast, figure 5 plots the cdf's for the duration of unemployment. In almost all cases, the distribution during expansions either strictly stochastically dominates that for contractions or almost always lies to the right of it.

[...FIGURES 4 AND 5...]

Tables 1a to 1d present the average duration of expansions and contractions, as well as the t and W statistics and the respective p-values for the tests of equal means and equal distributions. While the results vary across different methods, the average length of an expansion in output is about 11 quarters long, whereas the average contraction is about 8 quarters long. In most cases, the difference is not statistically significant at the 5% level. However, the average length of an expansion in unemployment is about 16 quarters, whereas the average length of a contraction is only about 7 quarters long. The difference is significant at the 5% level for most cases, and at the 1% level for many of them. With regards to duration, the data strongly suggests that contractions in employment are briefer than expansions in employment, but contractions and expansions in output are equally long.

[...TABLES 1a, 1b, 1c, AND 1d...]

To understand what lies behind the difference in brevity, within each method, we compared the dates at which peaks and troughs occur in output and in employment. The typical finding is that peaks in employment lag peaks in output by between 1 and 3 quarters, whereas troughs in employment are typically within one quarter of troughs in output. The brevity in the contractions in employment is due to employment starting to decline only after output has already been declining for some time. The contractions in both output and employment end around the same time.

Turning to violence, figures 6a to 6c show the cdf's for the three measures of violence in output. As with duration, the cdf's for violence in expansions and contractions are typically very close, except when the Chauvet-Hamilton algorithm is used. In figures 7a to 7c are the cdf's for the violence of unemployment. The typical finding is that contractions are substantially more violent than expansions.

[...FIGURES 6a, 6b, 6c, 7a, 7b, AND 7c...]

Tables 2a to 2d show the results from the  $t$  and  $W$  tests. There are now some rejections of symmetry for output, although for the large majority of the cases, we accept the null hypotheses of equal mean or equal distribution of violence in expansions and contractions in output. Out of the 96 tests, we reject symmetry at the 5% significance level only in 12 cases, and only once at the 1% level, and all of these rejections occur when the Chauvet-Hamilton algorithm for picking turning points is used. The robust conclusion is that contractions and expansions in output are equally violent.

In employment, on the other hand, most tests (73 out of 96) reject the null at the 5% level. Curiously, almost all of the acceptances (21 out of 23) arise in the case when the band-pass filter is used to extract de-trend unemployment as corresponding to fluctuations between 6 and 32 quarters. Table 2e uses instead the band-pass filter to extract the fluctuations between 2 and 80 quarters, thus only removing the very low frequency movements in unemployment that are associated with demographic changes. Note that in this case, the inferences for output are almost unchanged. For employment however, instead of only 3 rejections at the 5% level, there are now 16 rejections. There is no answer as to what is the right choice of parameters for the band-pass filter, but we can offer two observations. First, that the typical choices in the literature on unemployment are closer to those in table 2e. Second, that in both tables 2d and 2e, employment is more likely to be asymmetric than



output, in the sense of lower p-values. We therefore conclude that, while the results with regards to violence are not as overwhelming as with regards to brevity, the evidence strongly supports the view that contractions in employment are more violent than expansions.

[...TABLES 2a, 2b, 2c, 2d, AND 2e...]

## 5 Inspecting the robustness of the results

### 5.1 The frequency of the observations

One may fear that quarterly data might not be fine enough to accurately detect turning points. It is unclear that this would bias our measures of brevity and violence in any particular direction, or that it would do so differentially for output and employment. Still, it is conceivable that if, for instance, employment typically peaks in January, April, July, and October then quarterly data will induce a systematic delay in identifying these peaks.

Table 3 reports the results from our algorithms and tests using monthly seasonally-adjusted observations for industrial production and the unemployment rate. To conserve space, we report only the average brevity and violence across all methods and the number of times that the null hypotheses are rejected at the 5% and 1% levels. As before, the evidence for no asymmetry between expansions and contractions in output is strong. There are very few rejections of the nulls and they are almost all due to using the Chauvet-Hamilton algorithm. For employment, the results are not as strong as those with quarterly data, but one stills rejects symmetry for the majority of cases.

[...TABLE 3...]

### 5.2 The series used

In this section we investigate whether there is something special about the series for industrial production and the unemployment rate that is inducing our results.

Turning first to output, we extend our analysis to consider GDP and non-farm business output to ensure that our results are not driven by some specific features of the industrial sector. As a second check, we see whether inventories or indirect taxes and depreciation may enhance or abate asymmetries by considering series for real sales and real personal income.

As a third check, we break output into consumption, investment and government spending and look for asymmetries in these series. Table 4 displays the results. The basic statistical inference, that contractions and expansions are equally brief and violent is unchanged.

We have also compared the timing of peaks and troughs across the different output series. Excluding government expenditures, the dates are typically similar, within 2 quarters of each other in most cases. This gives us some confidence that our dating of turning points correctly identifies the business cycle in output.

[...TABLE 4...]

Looking next at employment, so far we have focussed on the log of one minus the unemployment rate. To investigate whether it is the labor force or total employment that is creating the asymmetry, we look for brevity and violence in the total number of employed according to the household survey. Table 5 shows that contractions in total employment are still briefer and more violent than expansions, though there are not as many rejections of symmetry as before.

An alternative is to use the payroll employment numbers from the payroll survey. This survey is often judged to be more reliable because it covers more workers. However, it is probably less accurate for the purpose of detecting turning points, because of the difficulties in accounting for firm births and deaths. It takes at least nine months for a new firm to get into the sample for the payroll survey. The BLS adjusts the raw data for firm births and deaths by first imputing the growth rate of firms that cannot be sampled to equal the growth of those sampled and then fitting an ARIMA model to the actual net birth/death data from unemployment insurance records from the past 5 years to adjust the imputation. Both the imputation and the short sample of the ARIMA can bias the survey towards failing to detect sharp reversals. Table 5 performs our analysis using total employment from the payroll survey. There is stronger evidence of asymmetry now than when the household survey was used. We should also note that for both measures of employment there were many more rejections at significance levels between 5% and 10%. The presentation of the results in Table 5 underemphasizes the pervasiveness of asymmetries.

[...TABLE 5...]

Next, we look separately at the employment rate for younger and older workers. The evidence of briefer and more violent contractions is stronger among workers over 24 than it is for workers between 16 and 24, but it is present for both.

Another labor market variable that attracts considerable attention is the participation rate. It is well-established that the labor force expands and shrinks during the business cycle with the participation rate typically lagging the business cycle. When we applied our algorithms to participation, we found that we could typically not reject the null of symmetry. However, we also could typically not find many turning points, since the participation rate does not fluctuate very much, so we should not put too much weight on these results.

Finally, we looked at hours per worker. Table 5 shows the results, which indicate that, typically, contractions and expansions are equally brief and violent, although there are some rejections. The results are less clear-cut for this variable, and its turning points can sometimes be quite different than those found for employment or output. We could not draw any conclusive inference on whether hours per worker resemble output or employment the most when it comes to asymmetries on duration and violence.

Table 6 looks at a series from another time period: monthly pig-iron production between 1877 and 1929. If one looks solely at the t-test for same average duration, then there is considerable evidence for shorter contractions than expansions in this output series that Mitchell and others focussed on. However, looking at either the Wilcoxon test or at any of the measures of violence, the evidence for asymmetry is much weaker.

[...TABLE 6...]

### **5.3 Inspecting the turning-point algorithms**

Another potential worry is that our algorithms for finding turning points may be unduly affecting the results. Partly, we have addressed this fear by using four different algorithms and finding that they give similar results. One may still have issues with the specifics of each algorithm and we try to address these here.

A first concern arises with the window algorithm. When it identifies two successive candidate peaks (or troughs), we took the latter. Our reasoning was that, during expansions, the series may have very short-lived blips downward that lead to incorrectly detecting a peak there. The reverse reasoning applies to contractions. We also tried an alternative selection

rule, that takes the higher of the two candidate peaks. We found that the dates of turning points were almost entirely unchanged.

While the algorithms treat expansions and contractions symmetrically, one may still wonder whether they have some hidden feature that leads to asymmetries. We implement a simple and effective test of this possibility. Taking each series, we reverse its time-ordering and run our algorithms. Looking from the perspective of the present in the direction of the past, expansions now become contractions and contractions become expansions. We found that the algorithms pick out the same turning point dates in 87% of the cases, with the failures evenly distributed between peaks and troughs.

Yet a third strategy to check whether the algorithms are doing the right job is to simulate artificial data and see whether the right turning points are detected. As a data-generating process we use a Chauvet-Hamilton model with parameters that imply symmetric expansions and contractions.<sup>13</sup> We simulated 1000 samples of the same length as our data, and ran our turning-point algorithms on each, recording whether they detected turning point at the right dates. We found that all four of our methods to detect turning points have a close to 100% success rate, as long as the preceding expansion (or contraction) lasts for more than 2 quarters. Only in the unlikely event that an expansion (or contraction) lasts for less than 6 months do our algorithms fail to detect the turning point.

A fourth concern might be that the difference that we find between output and employment is driven by finding many short and symmetric cycles for output and only a few and very asymmetric cycles for employment. While our results highlight the need to consider cycles for output and employment separately, if the two series had a very different total number of cycles, one would grow suspicious. We checked if this was the case by computing the difference between the number of cycles in output and in employment. The average difference across the 16 classifications was 1.2, so the algorithms are detecting approximately the same number of cycles in output and employment, and excluding the extra cycles in output, as we did in section 2, does not alter the results.

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<sup>13</sup>The choices of parameters values will be described in section 4.4.

## 5.4 Robustness of the statistical tests

The  $t$  and  $W$  tests have two potentially worrisome features. First, they are based on small samples of expansions and contractions, typically around 23. Second, they use the assumption of independent draws of duration and violence. We address both concerns using one Monte Carlo experiment that allows for serial correlation.

It is possible that if an expansion lasts longer than average, then perhaps the next contraction is shorter (or longer) than average. For instance, the last two long expansions in the U.S. were followed by short contractions. We started by estimating an AR(1) on the sequence of durations for contractions and expansions demeaned by their group averages. Using the series that results from a polynomial trend and the window-method on output, the autoregressive coefficient is 0.2. Notwithstanding the recent experience, typically in the post-war, a longer-than-usual expansion (contraction) is followed by a longer-than usual contraction (expansion).<sup>14</sup> The autoregressive parameter for unemployment is 0.6.

The next step was to simulate artificial samples of data. We use the estimated AR(1) but set the mean duration of both recessions and contractions to 7.9 quarters. Using this symmetric data generating process, we draw innovations from a normal distribution to generate 23 observations. We then run our algorithms and construct the  $t$  and  $W$  statistics. Repeating this 1000 times generates an empirical distribution for these statistics, under the assumption of symmetry but now allowing for serial correlation. Table 7 compares the  $t$  and  $W$  statistics that we obtained in the actual data with these distributions. The bootstrap p-values for the test of symmetry in duration are quite close to those using the asymptotic distributions, although (as expected) they tend to be more conservative. Still, for the majority of cases, as before we reject symmetry for employment but do not reject it for output.

[...TABLE 7...]

We use a similar bootstrap for the three measures of violence. The results are in tables 8a to 8d, and the conclusions are similar to those in table 7. The bootstrap p-values are

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<sup>14</sup>The coefficient is around 0.2 across all methods of de-trending and all methods for detecting turning points, with one exception. The durations identified by the Chauvet-Hamilton algorithm lead to an auto-correlation parameter that is negative but close to zero.

quite close to the asymptotic p-values and the statistical significance tests at the 5% or 1% level lead to almost always the same conclusion.

[...TABLES 8a, 8b, 8c, 8d...]

A second concern with our tests is that we treat the  $D(\cdot)$  and the  $V(\cdot)$  as observations. Insofar as these are measured with error, the standard errors used for our tests may underestimate the sampling error. We conduct a second Monte Carlo exercise to investigate this issue. We use a Chauvet-Hamilton model as a data-generating process, with the Markov transition own-probabilities both set to 0.90, so that the average duration of contractions and expansions is 10 quarters for both output and unemployment. We allow the variances to change pre and post 1984, but impose that the changes in the series in the two states have the same mean and variance. These are then estimated for output and unemployment separately. Using this symmetric data-generating process, we simulate 1000 samples for output and unemployment. Treating these as data, for each sample we run our algorithm to detect turning points and construct the  $t$  and  $W$  statistics. Tables 9 and 10a to 10d display the p-values for our statistics in the real data, using the simulated distributions.

[...TABLES 9, 10a, 10b, 10c, AND 10d...]

The results are a little surprising. It turns out that the p-values are typically lower than before. The rejections of symmetry for employment are stronger than before, whereas for output, one can still typically not reject symmetry at least at the 5% level. The exception is when the Chauvet-Hamilton algorithm to detect turning points is used. For these cases the p-values increase considerably. The combination of the results in tables 7 to 10 indicates that the window, reversal and Bry and Boschan methods coupled with  $t$  and  $W$  tests, do surprisingly better than expected at assessing the symmetry of expansions and contractions.

## 5.5 An alternative approach to statistical inference

An alternative approach to statistical inference to the one we have taken so far is to commit to a statistical model that completely characterizes the observations of output and employment and allows for, but does not require, asymmetries between expansions and

contractions. With this model in hand, one can then test for symmetry between expansions and contractions. We pursue this alternative in this section.

Our model of the data is the version of the Chauvet-Hamilton model described in section 3. Whereas in that section, the model was treated as an algorithm to detect turning points, here it is treated as a full statistical representation of the data on output and employment. The parameters  $(p_1, p_2, \mu_1, \mu_2, \sigma_{1,pre}^2, \sigma_{2,pre}^2, \sigma_{1,post}^2, \sigma_{2,post}^2)$  are estimated by maximum likelihood. We then use likelihood-ratio tests for the null hypotheses of equal brevity:  $p_1 = p_2$  and equal violence, either measured as equal steepness  $\mu_1 = \mu_2$  or equal sharpness:  $\mu_1 = \mu_2$ ,  $\sigma_{1,pre}^2 = \sigma_{2,pre}^2$ , and  $\sigma_{1,post}^2 = \sigma_{2,post}^2$ .

The results are in table 11. The estimates of the probability of remaining in a contraction or an expansion are quite high, reflecting the persistence of business cycle phases. Moreover, they tend to be close to each other. Therefore, we cannot reject the hypothesis that contractions are as long as expansions for both output and employment. As for violence, for output, at the 5% significance level, we cannot reject the hypothesis of equal steepness but reject equal sharpness. For employment, we reject equal violence for both measures of violence at the 5% level.

[...TABLE 11...]

## 6 Implications of our findings for existing theories

There are a few models that generate asymmetric business cycles. In this section, we ask whether they can account for our finding that there is one precise form of asymmetry between expansions and contractions in employment that is not present in output cycles.

Credit constraints are a source of asymmetry in Kocherlakota (2000). Large negative shocks can lead to large cuts in production since agents cannot borrow, whilst positive shocks are attenuated using savings. While credit constraints explain the different reaction to positive and negative shocks, they do not account for the difference between expansions and contractions. Moreover, credit constraints should affect both output and employment equally. The same problem arises with theories that emphasize capacity constraints. Gilchrist and Williams (2000) and Hansen and Prescott (2005) argue that during booms expanding production requires expending resources to set up more plants. In recessions

instead, some plants are not used and can be re-activated or de-activated at no cost. This model generates asymmetries in both output and employment.

Jovanovic (2006) proposes a model in which firms must adopt technologies without knowing whether they are a good fit for their production process. As bad fits lower output by more than good fits raises it, output is negatively skewed. While skewness is an important feature of the data, it is conceptually distinct from brevity and violence of expansions and contractions. Caballero and Hammour (1996) analyze an economy in which firms at each date face the option of paying a cost to scrap their old technology and adopt a new one. With technological progress, they show that this creative destruction should be bunched around recessions, when the marginal profitability of production is lower. If new technologies are embodied in jobs, then there will be a sharp increase in unemployment around recessions. This model generates violent and short-lived contractions in employment. However, in their model, output follows the same dynamics as employment.

Increasing returns to scale can be another source of asymmetry. Acemoglu and Scott (1997) argue that investment in maintenance today not only raises productivity today but also lowers the cost of adopting new technologies tomorrow. Past shocks therefore affect the profitability of current investments and thus the economy's response to shocks. While their model is flexible enough to account for different types of asymmetries between prolonged expansions and prolonged contractions, it emphasizes investment as the source of asymmetries and output as its reflection. Our findings emphasize that the key is employment.

Chalkley and Lee (1998) and Veldkamp and van Nieuwerburgh (2006) argue that when output is high, investors face less uncertainty. Around peaks, they therefore respond to bad shocks quickly, leading to violent contractions, at least initially. Around troughs, there is less precision of information so the response to positive shocks is slow. These theories can account for differences in violence. However, they lead to a difference between contractions and expansions in output and investment, but not necessarily in employment. Moreover, they do not generate asymmetries in brevity.

From the perspective of the labor market, Burgess (1992) argues that the cost of adjusting employment for a firm depend on the tightness of the labor market. In booms, the labor market is tight and employment moves slowly, whereas in slumps, the market is slack, and employment moves quickly. Our finding however was that expansions were different from contractions, rather than booms different than slumps. In the model of Burgess, the



initial stage of contractions would be more violent than its later stages (and the reverse for expansions), but on average, expansions and contractions would be equally violent.

The model of job creation and job destruction of Mortensen and Pissarides (1994) can generate different violence during expansions and contractions. In their model, job destruction occurs immediately once the value to the firm and the worker of being matched is negative. Job creation on the other hand takes place only with some probability. Thus, employment can fall quickly and violently, but it must expand slowly. However, output equals employment so it is asymmetric as well. Moreover, expansions are as brief as contractions.

Overall, we conclude that none of the existing theories can account for all of: (1) symmetric expansions and contractions in output, (2) briefer and more violent contractions than expansions in employment and (3) employment lagging output at peaks but not at troughs. Next, we ask whether some of the features in these models can be combined to match all three facts.

## 7 Three ingredients to fit the facts

In this section, we build a simple stylized model of the business cycle. Using the economic mechanisms in the previous section, our aim is to find what combination of ingredients is required to generate the three findings describing the business cycle dynamics in figure 3.

### 7.1 Setup of the model

The structure of the model is standard. Time is continuous and there are two types of agents: households and firms. There is a unit mass of infinitely-lived households that discount the future at rate  $r$  and obtain a flow of utility that is linear in consumption ( $c_{i,t}$ ) and hours worked ( $h_{i,t}$ ). The assumption of linear utility implies that the interest rates is always  $r$  and also that agents are risk neutral, both of which simplify the model considerably. Household  $i$  maximizes:

$$U_i = \int_0^{\infty} e^{-rt} (c_{i,t} - mh_{i,t}) dt, \quad (1)$$

$$s.t. \quad : \quad db_{i,t}/dt = \omega_{i,t} + \pi_t k_{i,t} - \kappa k_{i,t} - c_{i,t} + rb_{i,t}. \quad (2)$$

In the budget constraint, assets ( $b_{i,t}$ ) increase by the wage received by this household ( $\omega_{i,t}$ ) and by the returns from renting capital ( $k_{i,t}$ ) at price  $\pi_t$  in the market. Every instant, the agent incurs a cost  $\kappa$  associated with maintaining and repairing the capital stock when it is being employed by firms in the use of a specific technology. Agents can trade bonds with each other that on aggregate are in zero net supply ( $\int_0^1 b_{i,t} di = 0$  at all  $t$ ), and they own capital that on aggregate is in fixed one unit supply ( $\int_0^1 k_{i,t} di = 1$  at all  $t$ ).

Firms use a Cobb-Douglas production function combining capital ( $K$ ) and labor ( $L$ ):

$$Y_t = L_t^\alpha K_t^{1-\alpha}. \quad (3)$$

There is free access to this technology, which combined with constant returns to scale implies perfect competition. Capital is rented from households every period in a competitive market. When a firm hires each worker, they set the hours of work and the wage. To preserve zero profits by firms in equilibrium, we assume that the worker captures the entire surplus from the relationship. This assumption is not essential—the combination of perfect competition by firms and linear utility function of workers implies that any other rule to divide the surplus leads to the same aggregate equilibrium.

The first non-standard feature of the model is that total labor equals the integral of the product of different tasks that each occupy one worker. The first unit of labor spent at a task has a marginal product  $AA_i$ , while any extra time  $l_i$  has a lower marginal product  $A$ :

$$L_t = \int_1^T A_t(A_i q_{i,t} + l_{i,t}) dF(A_i, T), \quad (4)$$

with  $0 \leq q_{i,t} \leq 1$ ,  $l_{i,t} \geq 0$ , and  $h_{i,t} = q_{i,t} + l_{i,t}$  for all  $i$ . These assumptions capture the presence of diminishing returns to work effort as well as the benefits of specialization, since they imply that the first hours of a skilled worker at her specific task are more productive than her overtime. Specialization also entails costs in that operating each task requires using a fixed  $z$  units of labor to cover the accounting and administrative work of managing each job.<sup>15</sup> This cost is positive but not so large that in equilibrium no worker is hired. The firm's problem is then to choose every instant how many workers to have ( $N$ ) and how many hours of work in each task ( $q_i$  and  $l_i$ ) so as to maximize profits  $Y_t - \int \omega_{i,t} di - \pi_t K_t$ .

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<sup>15</sup>While our model does not have a government, these could also be interpreted as including the pecuniary costs of buying worker's insurance or minimum social security contributions.

The second key feature of our model is the distinction between the general productivity in the economy ( $A$ ) and the technology that is available ( $T$ ). The productivity of each task  $A_i$  is distributed according to  $F(A_i, T)$  with support  $[1, T]$  and positive mass everywhere. Our only assumption on this distribution is that the higher is the top technology available, the more productive is the technology on average if its top tasks are being used. (Formally, if  $T > T'$  then  $\int_x^T A_i dF(A_i, T) > \int_x^{T'} A_i dF(A_i, T')$  for all  $x$ .) A higher  $T$  therefore unambiguously implies a better technology. Both productivity and technology are stochastic and vary over the business cycle. We make the simplest possible assumption on their dynamics: productivity growth (de-trended) can take on two values,  $g$  during expansions and  $-g$  during contractions, while technology alternates between  $T^E$  during expansions and  $T^C < T^E$  during contractions. The economy switches between expansions and contractions according to a Poisson process with arrival rate  $\lambda$ .

The competitive equilibrium in this economy is an allocation of output, consumption, jobs, and hours worked, such that households maximize utility and firms maximize profits, the market where firms sell their goods to households clears, the labor market where firms hire workers clears, and the bonds and capital markets clear. The equilibrium is efficient and solves a social planner problem.

## 7.2 First ingredient: hours and workers

To fit our findings, it must be possible that sometimes output contracts and employment expands, while at other times output and employment move in the same direction but at different speed. If firms can vary the number of hours and output by each worker, as in theories of labor hoarding, this is possible.<sup>16</sup> This is possible in the model above since firms can either hire more workers or increase the overtime hours of existing workers.

The equilibrium level of employment is determined by firms trading off the specialization benefits of a task with its administrative costs. Firms optimally follow a threshold rule: if a task has a productivity  $A_i$  above or equal to a threshold  $x$ , it is operated; otherwise, it is not. The optimal threshold  $x^*$  is equal to  $1 + z$ . Because of the administrative costs, not all possible jobs are filled—rather than fill the worst available job at cost  $z$  with productivity

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<sup>16</sup>However, note that the standard model of labor hoarding and capacity utilization cannot produce these results. If firms choose the intensity at which to use their workers and if increasing intensity raises the marginal benefit of hiring the extra worker, then employment, effort, and output all move in the same direction.

$A$ , a firm would prefer to increase the overtime in the other jobs at no fixed cost but with the same marginal productivity. The number of jobs is then  $N = 1 - F(x, T)$ .

As for the equilibrium level of hours, note that since overtime is equally productive across all tasks,  $l_i$  is the same for all  $i$ . In equilibrium the marginal product of an extra hour of work equals its marginal disutility  $b$ , so total overtime hours are:  $Nl = [\alpha A^\alpha / b]^{1/(1-\alpha)} - \int_x^T A_i dF(A_i, T)$ . Since in equilibrium  $K = 1$ , total output is  $Y = [\alpha A^\alpha / b]^{\alpha/(1-\alpha)}$ .

It is now clear how, by allowing for labor hoarding, our simple model is able to separate fluctuations in output and employment. If productivity  $A$  changes, output will change but employment will not. Changes in output are made possible by changes in overtime hours, without the need for any change in the number of jobs. Changes in employment are instead driven by changes in the distribution of skills, which we modelled through changes in the top technology  $T$ . When  $T$  rises, a higher number of tasks have productivity above the threshold  $x$  so employment rises.

Figure 8 illustrates the peak-to-peak dynamics of this economy. With just this ingredient, both output and employment are still symmetric with respect to brevity and violence.

[...FIGURE 8...]

### 7.3 Second ingredient: choosing technology adoption

Inspired by the theories of creative destruction, we give firms the freedom to choose when to shift technologies. To this end, we now assume that when a turning point arrives, the old technology remains available, but the cost of operating an obsolete technology is  $\kappa e^{\bar{\kappa}\tau}$ , growing exponentially at the rate  $\bar{\kappa}$  with the time elapsed since the turning point  $\tau$ . Firms can therefore choose whether to stay with the old technology and pay a higher rent on capital to cover the higher maintenance costs, or switch to the new technology for which maintenance costs are a constant  $\kappa$ .<sup>17</sup>

Social welfare depends on productivity  $A$ , the number of workers  $N$ , the technology used  $S \in \{E, C\}$ , and the time  $\tau$  that an old technology has been in use since a new technology

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<sup>17</sup>While inspired by theories of technology adoption and creative destruction, our setup is different from the typical models in this literature. In those models, technological changes also lead to changes in aggregate productivity ( $A$ ) and labor costs ( $z$ ). In our setup, these would lead to asymmetries in output and equal brevity in employment, thus failing to match the facts.

arrived:

$$W(A, N, S, \tau) = [\alpha A^\alpha / b]^{\alpha / (1 - \alpha)} + b \left( \int_x^{T^S} A_i dF(A_i, T^S) - N(1 + z) \right) - \kappa e^{\bar{r}\tau}. \quad (5)$$

The first term gives the utility from consuming output, while the second term is the surplus that is generated from the ability to work with a marginal product  $A_i$  at an opportunity cost  $b$ . The third term is the cost of maintaining the capital being used with the current technology. Note that if the  $T^E$  technology is being used during an expansion or the  $T^C$  technology during a contraction, then  $\tau = 0$ , whereas otherwise  $\tau$  is the time elapsed since the last turning point.

Consider now the problem facing the economy when a trough arrives. It can either stay with the old technology with benefit  $W(A, N^C, C, \tau)$  from then on, or switch to the new technology, adjust its workforce, and earn  $W(A, N^E, E, 0)$ . From equation (5),  $W(A, N^E, E, 0) > W(A, N^C, C, \tau)$  for all  $\tau$  so the economy switches to the new technology immediately in troughs. Consider next what happens when a peak arrives. In the first instant after the peak,  $W(A, N^E, E, 0) > W(A, N^C, C, 0)$ , so the economy stays with the old technology. As time elapses though,  $W(A, N^E, E, \tau) - W(A, N^C, C, 0)$  falls monotonically with  $\tau$  until the time  $\tau^*$  arrives when it equals zero. At this point, firms adopt the new technology.

The business cycle dynamics are in figure 9. When a peak arrives, now firms choose to stay with the old technology. In the instant after the shock, the old technology produces on average more than the new technology and the maintenance costs are still the same. As time progresses though, the old technology becomes increasingly more costly to operate. At some point, firms switch to the new technology and employment falls. When the trough arrives, firms switch to the new technology immediately: it is more productive and has initially the same cost of operation. The model now generates brevity in employment.

[...FIGURE 9...]

#### 7.4 Third ingredient: Job destructions are immediate but job creation takes time

There are two alternative ways to formalize this ingredient, one taking the perspective of the firm, and another the perspective of the worker. We present each separately.

*The firm's perspective: costs of training.* The new assumption is that firms can no longer costlessly and instantaneously adjust the number of workers  $N$ . Instead the total number of jobs evolves over time according to

$$dN/dt = H - F - \delta N, \quad (6)$$

where  $H \geq 0$  are hires,  $F \geq 0$  are fires, and  $\delta$  is a rate of exogenous separations. The key assumption is that there are asymmetric adjustment costs of hiring and firing workers. Firing is costly, but the marginal cost per worker is constant at  $\beta$  labor hours. Hiring new workers instead involves training them, and training is subject to decreasing returns to scale.<sup>18</sup> To hire  $H$  workers requires using labor according to the training function  $l_T^\gamma$ , with  $\gamma < 1$ . The adjustment costs therefore are:

$$C(F, H) = b\beta F + bH^{1/\gamma} \quad (7)$$

To solve for the new equilibrium, we use a recursive representation of social welfare. Let  $V(\cdot)$  denote the value function when productivity is  $A$ , employment is  $N$ , the state of technology is  $S$ , and an old technology has been used for  $\tau$  periods. Then:

$$rV(A, N, S, \tau) = \max_{F, H} [W(A, N, S, \tau) - C(F, H) + E(dV)/dt], \quad (8)$$

subject to (5), (6), (7) and  $dA/dt = \pm g$ . Using Ito's lemma:

$$E(dV)/dt = \pm gV_A(N, S, \tau) + V_N(N, S, \tau)(H - F - \delta N) + \lambda(V(N, \bar{S}, 0) - V(N, S, 0)) + V_\tau d\tau/dt, \quad (9)$$

where  $\bar{S}$  is the element of  $\{E, C\}$  that is not  $S$ . The second term captures the change in value from the switch of technology following the arrival of a turning point.

The characterization of the equilibrium is lengthy so we relegate it to the appendix, and describe its main features here. At any state of the business cycle, the economy can be in three regions. If employment is above  $\bar{N}^S$ , then there is an immediate burst of firing driving

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<sup>18</sup>This asymmetry between the cost of increasing and reducing employment is supported by the evidence surveyed in Hamermesh and Pfann (1996). There is also strong empirical evidence for fixed adjustment costs, which we do not include in the model. The empirical studies so far have not looked for any asymmetry in fixed costs between increasing and reducing employment.

employment down to  $\bar{N}^S$ . Since there are constant returns to firing, all of it takes place at once. If employment is in the range between a lower threshold  $\underline{N}^S$  and  $\bar{N}^S$  it is optimal to have neither firing nor hiring. Employment in this range falls at rate  $\delta$  as separations occur voluntarily at no cost for firms. In the third and final range (below  $\underline{N}^S$ ), there is positive hiring. If employment is above  $\hat{N}^S$ , then hiring is lower than voluntary separations; if it is below, hiring exceeds voluntary separations. At  $\hat{N}^S$ , hiring equals separations and employment is in a steady state.

Figure 10 plots the business cycle dynamics starting from a peak at which the economy is in a steady state, and assuming that  $T^E$  is sufficiently higher than  $T^C$ , so that  $\bar{N}^C < \hat{N}^E$ . When the peak arrives, the economy maintains the old technology until it is too costly to do so. When that happens, there is a switch to the contraction technology and firms want to lay off workers. Because the marginal cost of layoffs is constant, they do all the firing at once. They do not fire all the way to the new steady state, however, as the firm can use the exogenous separations (which are costless) to deplete the stock of remaining workers. As a result, employment first falls sharply, as firing is bunched, and then contracts at the rate  $\delta$  for a while. After that, firms start hiring to ensure that employment does not fall so much that repleting the stock of workers during an expansion is very costly. Employment keeps on falling but now at a declining rate towards its new steady state. When a trough arrives, the new technology is immediately adopted. Firms want to hire more workers but face increasing marginal costs of doing so. They therefore choose to hire gradually so employment slowly rises. Contractions in employment are more violent than expansions.

[...FIGURE 10...]

*The worker's perspective: job search.* An alternative setup that leads to similar dynamics assumes that workers can quit their jobs instantly, but only find a new job with some probability, as in Mortensen and Pissarides (1994). In this model, firms face only one decision: whether to open a vacancy or not. It is costless to open a vacancy and all firms are identical so, as long as the marginal product of an extra worker is positive, then all firms post vacancies. Otherwise, no firm posts a vacancy. In our model, the marginal product of having an extra worker is  $b(x - 1 - z)$ , which falls with employment and is higher if a superior technology is in use.

Workers currently employed can remain in their job or quit, while those unemployed can search for a job or not. Letting  $J(S)$  denote the value of having a job in state  $S$ , and  $U(S)$  the value of being unemployed, the Bellman equation for employed workers is:

$$rJ(S) = \max \{ b(x - 1 - z) + \delta(U(S) - J(S)) + \lambda(J(\bar{S}) - J(S)); rU(S) \}, \quad (10)$$

where the second option is to quit. The value of being unemployed is:

$$rU(S) = \max \{ -\eta + \pi(S)(J(S) - U(S)) + \lambda(U(\bar{S}) - U(S)); 0 \}, \quad (11)$$

where  $\eta$  is a search cost and  $\pi(S)$  is the probability of finding a job.<sup>19</sup> This probability comes from a Cobb-Douglas matching function so that  $\pi(S) = (\text{vacancies/searchers})^{1-\theta}$ , increasing at a declining rate on the ratio of vacancies to job-searchers. Employment evolves according:

$$dN/dt = -\delta N + \pi(S)\text{searchers} - \text{quits}. \quad (12)$$

The detailed solution of this model is in the appendix. As in the case of adjustment costs to the firm, there are three relevant regions. If employment is above  $\bar{M}^S$ , the marginal product of a job is negative, and workers quit their jobs. Employment falls abruptly up to the point where the marginal product of the extra job is zero. If employment is between  $\underline{M}^S$  and  $\bar{M}^S$ , workers stay in their job, but the unemployed are indifferent as to whether to search for a job or not. Only a few search (or all follow a mixed strategy searching with some probability), since the probability of finding a job is high but the return to having a job is small. Employment falls as long as the rate  $\delta$  at which voluntary separations take place is sufficiently high.<sup>20</sup> Finally, below  $\underline{M}^S$ , all of the unemployed search for a job. If employment is above  $\hat{M}^S$ , there are few vacancies so the job-finding rate is smaller than the job separation rate and employment falls; below  $\hat{M}^S$  there is more job-hiring and employment rises; and at  $\hat{M}^S$  employment is in a steady state. As before, we assume

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<sup>19</sup>Note that, unlike before, the assumption that workers collect the entire surplus from a match plays an important role here. Since only workers pay costs of searching, it ensures that the equilibrium is still Pareto efficient. This allows us to abstract from the details of the bargaining process (Mortensen and Pissarides, 1994) or appropriability problems (Caballero and Hammour, 1996).

<sup>20</sup>We focus on this case, but if  $\delta$  is low, the qualitative dynamics of employment during the cycle are the same. In this case, the steady state level of employment  $\hat{M}^S > \underline{M}^S$ , and the economy converges to it slowly just as in the case that we focus on.



that technology during expansions is sufficiently better than technology during contractions which ensures  $\hat{M}^E > \bar{M}^C$ .

The qualitative dynamics of this economy are just like in figure 10. Now, when the switch to the contraction technology occurs, a set of jobs have negative surplus. Workers quit these immediately, and employment falls abruptly. Afterwards, only a few of the unemployed search for a job. This is not enough to compensate for the exogenous separations, so employment falls rapidly up to the point where all unemployed start searching for jobs. Employment from then on declines at a decreasing speed towards a steady state. When the trough arrives, there is an immediate switch to the new technology, and employment starts rising at a declining speed. Now, contractions in employment are more violent than expansions because quitting occurs instantly, whereas finding a job takes time.

## 7.5 Alternative ingredients

The end result of combining the three ingredients are the business cycle dynamics in figure 10. It fits the three results that we found robustly characterize the U.S. post-war data. Our model is simple and has some special assumptions, but we see these as virtues. They allow us to point precisely to the ingredients that are needed to qualitatively fit the facts, and can therefore serve to guide future theories.

There are certainly alternatives to the ingredients that we have used. For instance, instead of being able to vary hours instead of the number of workers to affect production, it is possible that firms can vary capital utilization (Greenwood et al, 1998), organizational capital (van Rens, 2004), or organizational restructuring (Koenders and Rogerson, 2005). As an alternative to the delayed technology adoption in our second ingredient, perhaps firms instead switch between modes of governance (Philippon, 2005) or face uncertainty on future productivity that combined with fixed costs of switching creates an option value of waiting. Finally, as an alternative to our third ingredient, maybe newly formed firm-worker matches face uncertainty on their joint productivity and learn about it gradually, in which case, a sudden contraction in employment creates a steady flow of short-term jobs and recurring job losses while good matches are found leading to tame expansions in employment (Pries, 2004).

None of the papers described in the previous paragraph is able to explain our empirical findings. But the simple model in this section points to the directions in which the economic

mechanisms that they suggest would have to be modified to explain the business cycle fact on brevity and violence. Once alternative theories are formulated, we can then use micro-level data to distinguish between them. We are currently undertaking this research.

## 8 Conclusion

This paper investigated the claim that business contractions are briefer and more violent than business expansions. We examined different series for business activity, different measures of expansions and contractions, different definitions of brevity and violence, and different approaches to statistically infer whether there is a difference. The robust conclusion was that contractions in output are as brief and violent as expansions, but contractions in employment are briefer and more violent than expansions. Typically employment peaks a few quarters before output, while the troughs in both series are roughly coincident.

While some existing theories on asymmetric business cycles offer clues on how to explain this fact, none can fully account for it. To fit our empirical findings, one needs a theory that allows for output and employment to follow different dynamics, for employment to remain high as output is falling, and for employment to fall abruptly but rise steadily. We built a model that can qualitatively account for the findings by combining labor hoarding, the ability to choose when to scrap old technologies, and the difficulty of creating jobs relative to destroying them. Future work can take our theoretical results in one of two directions. Either it can refine the assumptions in the model to the point where it can be used to quantitatively match the facts as well, or it can explore how to use alternative ingredients to generate the same set of facts.

The results in this paper inform two current debates. The first is on the persistence of high unemployment in Europe. After rising abruptly in a few years in the 1980s, unemployment in many European countries has remained stubbornly high. This paper has found that brief and sharp increases in unemployment followed by protracted reductions are a robust feature of the U.S. economy as well. The difference between Europe and the U.S. is of the magnitude in the pace of decrease in unemployment.

The second debate is on jobless recoveries in the U.S. We have found that on average, in the post-war, troughs in employment and output have coincided so jobless recoveries are not the norm. However, we have also found that starting from a trough, employment

expands at a slow pace in the beginning of a recovery. This may lead to an impression of joblessness at the start of a recovery. The decline in volatility in the last 20 years may have made recoveries even tamer. Whether the properties of the business cycle have changed can only be resolved with the accumulation of time and data.

## 9 Appendix

### 9.1 Turning point algorithms

*Window method:* For a given series,  $\{x_t\}_{t=1}^T$ , the window method with window size  $w$  begins by identifying dates in the range  $[w + 1, T - w]$  where  $x_t \leq \min(\{x_s\}_{s=t-w}^{t+w})$ . Such dates are tentatively labeled as troughs. A similar operation yields a set of tentative peaks. The method then imposes the requirement that peaks and troughs alternate. This is achieved by retaining the latest of a series of successive turning points of the same type. We found that the window method was sensitive to noise and therefore pre-smoothed the data using a five-quarter, centered moving average.

*Reversal method:* The reversal method requires two parameters representing the “reversal pattern” that identifies a turning point. A (3, 2) reversal (the one we use) identifies a peak as an episode in which the series rises for three successive quarters and then immediately falls for two successive quarters. Once a tentative set of turning points has been identified, the requirement that turning points alternate is imposed in the same manner as in the window method.

*Bry-Boschan:* The Bry-Boschan procedure is described by Bry and Boschan (1971) and King and Plosser (1994). It was originally developed for monthly data and we adapt it to quarterly data in the same manner as King and Plosser: the quarterly value is repeated for each month of the quarter. Our procedure differs from that described by King and Plosser in that we use a 10 month moving average in the first step and a 6 month moving average in the third. We use the programs made available by Monch and Uhlig (2004).

*Chauvet-Hamilton:* Chauvet and Hamilton (2005) fit a two-state Markov-switching model to the first differences of GDP in which each observation is drawn from normal distribution with a common variance and a mean that depends on the state. We expand their model to allow the variance of the first differences to change between states. As explained in the text, we also allow the variance to change before and after 1984Q3. Contractions are then defined as periods in which the smoothed regime probability is greater than 0.5 for the state with the smaller mean first difference. The remaining dates are classified as expansions. The model is estimated by numerical maximum likelihood.

## 9.2 Solution of the training model

Recall that the problem is to find:

$$rV(A, N, S, \tau) = \max_{F, H} \left[ \begin{array}{l} W(A, N, S, \tau) - C(F, H) + V_N(N, S, \tau) (H - F - \delta N) \\ + \lambda (V(N, \bar{S}, 0) - V(N, S, 0)) \pm gV_A(N, S, \tau) + V_\tau d\tau/dt \end{array} \right], \quad (13)$$

subject to the constraints  $H \geq 0, F \geq 0$ , and  $dN/dt = H - F - \delta N$ . Note that  $W_{AN}(\cdot)$  and  $W_{N\tau}(\cdot)$  are always zero and so  $V_{AN}(\cdot) = V_{N\tau}(\cdot) = 0$ . The necessary conditions for optimality are:

$$-b\beta - V_N(S) \leq 0, \quad F \geq 0, \quad F(b\beta + V_N(S)) = 0 \quad (14)$$

$$-(b/\gamma)H^{1/\gamma-1} + V_N(S) \leq 0, \quad H \geq 0, \quad H \left[ V_N(S) - (b/\gamma)H^{1/\gamma-1} \right] = 0, \quad (15)$$

$$(r + \delta)V_N(S) = b(x - 1 - z) + V_{NN}(S) (H - F - \delta N) + \lambda (V_N(\bar{S}) - V_N(S)). \quad (16)$$

To save on length, we report only the  $S$  argument of the value functions. The three conditions in (14) are the first-order condition for  $F$ , the constraint on fires, and a complementary slackness condition. The equivalent conditions for hires are in (15). The envelope theorem condition with respect to employment is in (16), which used the fact that  $N = 1 - f(x)/T$ .

Note that it is never optimal to have positive hires and fires. Any policy that involves this is dominated by a policy that leads to the same net change in employment at a lower cost by lowering either hires or fires to zero. There are therefore three optimal regions: when there is firing, when there is hiring, and when there is neither. If firing is positive, then (14) implies that  $V_N(S) = -b\beta$ . If hiring is positive, then (15) implies that  $V_N(S) = (b/\gamma)H^{1/\gamma-1}$ . When there is neither firing nor hiring, (16) implies that  $V_N(S)$  falls monotonically with  $N$  from 0 to  $-b\beta$ . Because  $V(S)$  is concave,  $V_N(S)$  must weakly fall with  $N$  so: for  $N$  above  $\bar{N}^S$ , there is firing and  $V_N(S) = -b\beta$ ; for  $N$  between  $\underline{N}^S$  and  $\bar{N}^S$ ,  $V_N(S)$  rises as  $N$  falls through voluntary separations equalling 0 at  $\underline{N}^S$ ; and below  $\underline{N}^S$   $V_N(S) = (b/\gamma)H^{1/\gamma-1}$  and there are positive hires.

Next we characterize the equilibrium values of these thresholds, and the dynamics of employment. We focus on the equilibrium in which  $\bar{N}^C < \hat{N}^E$ —we will later find the condition on parameters for this to hold. If  $N > \bar{N}^E$ , then there is firing in both states.

Condition (16) during expansions implies that

$$\bar{N}^E = 1 - f(1 + z - \beta(r + \delta))/T^E, \quad (17)$$

the number of fires  $F = N - \bar{N}^E$ , as the economy jumps to the threshold immediately. When  $\underline{N}^E < N < \bar{N}^E$ ,  $F = H = 0$ , and  $dN/dt = -\delta N$ . To find  $\underline{N}^E$ , since at this point  $V_N(S) = 0$ , (16) implies that:

$$\underline{N}^E = 1 - f(1 + z + \beta\lambda)/T^E. \quad (18)$$

Note that employment is lower than the value for which  $x = 1 + z$  as long as  $\lambda \neq 0$  because the possibility of a technological change leads firms to hold back on the number of workers to lower future costs of firing. Finally, below  $\underline{N}^E$ , taking time derivatives of (15) to substitute for  $V_{NN}$  in (16), one finds the dynamic system:

$$\frac{dH}{dt} (1/\gamma - 1) H^{1/\gamma-2} = (r + \delta + \lambda) H^{1/\gamma-1} - \gamma(x - 1 - z) + \beta\lambda\gamma, \quad (19)$$

$$(f'(x)/T^E) \frac{dx}{dt} = -H + \delta (1 - f(x)/T^E), \quad (20)$$

in the variables  $x$  and  $H$ . The unique steady state of this system is  $\hat{H}^E = \delta \hat{N}^E$ ,  $\hat{N}^E = 1 - f(\hat{x}^E)/T^E$ , and  $\hat{x}^E$  solves the non-linear equation:

$$\hat{x}^E = 1 + z + \beta\lambda + (r + \delta + \lambda) [\delta (1 - f(\hat{x}^E)/T^E)]^{1/\gamma-1} / \gamma \quad (21)$$

Very similar steps give the contraction thresholds:  $\bar{N}^C$ ,  $\underline{N}^C$ , and  $\hat{N}^C$ . The last thing to check is that  $\bar{N}^C < \hat{N}^E$ . Condition (16) for  $S = C$  becomes, after rearranging:

$$\bar{x}^C = 1 + z - \beta(r + \delta) - (\lambda/b) [V_N(E) + b\beta]. \quad (22)$$

Since  $V_N(E) > -b\beta$ , comparing (21) and (22), we see that  $\bar{x}^C < \hat{x}^E$ , and so  $f(\bar{x}^C) < f(\hat{x}^E)$ . Now,  $\bar{N}^C < \hat{N}^E$  requires that  $f(\bar{x}^C)/f(\hat{x}^E) > T^C/T^E$ . This will hold for sure as long as  $T^C/T^E$  is small enough. Finally, note that our assumption that  $\bar{x}^S > 1$  requires that  $T^C$  and  $T^E$  are sufficiently large. We can see this in (21) and (22).

### 9.3 Solution of the job-search model

Since, for any fixed  $N$ , the marginal product of a job is higher in expansions than contractions, it must be that  $J(E) \geq J(C)$  and  $U(E) \geq U(C)$ . The choice of workers is whether to quit or stay in their job, while the choice of the unemployed is whether to search for jobs or not. Therefore, there are in principle 4 possible regions. However, since if there are quits,  $J(S) = U(S)$  which implies  $U(S) = 0$ , no one searches. Therefore, there are only three regions: when workers quits and no one searches for a job so  $J(S) = U(S) = 0$  and employment falls; when no one quits and the unemployed are indifferent between looking for a job or not, so  $J(S) > 0 = U(S)$  and employment may fall or rise depending on  $\delta$ ; and when no one quits and everyone searches for a job so  $J(S) > 0$  and  $U(S) > 0$ . Since the value functions are concave in  $N$ , these correspond to three regions in employment:  $(\bar{M}^S, 1]$ ,  $(\underline{M}^S, \bar{M}^S]$  and  $[0, \underline{M}^S]$ , and if there is a steady state  $\hat{M}^S$  it must be in the last region.

As before, we focus on the case where  $\hat{M}^E > \bar{M}^C$  to lower the number of possible regions for employment across the two states. Consider first the region where  $N > \bar{M}^E$  and we are in an expansion. Here, workers quit their job and employment falls abruptly to  $\bar{M}^E$ . The threshold is at the point where the worker is indifferent between quitting or staying. This occurs when  $b(x - 1 - z) = 0$  so:

$$\bar{M}^E = 1 - f(1 + z)/T^E \quad (23)$$

Second, we look at the region where  $N \in (\underline{M}^E, \bar{M}^E]$ . Here,  $U(E) = 0$  and  $J(E) > 0$ , so combining the two Bellman equations:

$$\pi(E) = \frac{\eta(r + \delta + \lambda)}{b(x - 1 - z)}. \quad (24)$$

Now, firms post vacancies as long as their marginal product is positive, so  $\bar{M}^E - N$  vacancies are posted. The equation above can then be solved for how many searchers look for a job:

$$\text{searchers} = (\bar{M}^E - N) \left[ \frac{b(x - 1 - z)}{\eta(r + \delta + \lambda)} \right]^{1/(1-\theta)}, \quad (25)$$

while the dynamics of employment are  $dN/dt = -\delta N + \pi(E)\text{searchers}$ . As stated in the text, we assume that  $\delta$  is high enough that  $dN/dt < 0$  as long as  $\text{searchers} < 1 - \underline{M}^E$ . This

second threshold is given by the solution to the non-linear equation:

$$\underline{M}^E = 1 - f \left[ 1 + z + \left( \frac{1 - \underline{M}^E}{\bar{M}^E - \underline{M}^E} \right)^{1-\theta} \frac{\eta(r + \delta + \lambda)}{b} \right] / T^E \quad (26)$$

For  $N \in (\bar{M}^C, \underline{M}^E)$ ,  $J(E) > U(E) > 0$  so all workers stay in their jobs and all the unemployed search for a job. The dynamics of employment are then  $dN/dt = -\delta N + (\bar{M} - N)^{1-\theta}(1 - N)^\theta$ . Employment is clearly falling at a declining rate as long as  $N$  is above the steady state  $\hat{M}^E$ , which is the solution of:

$$\delta \hat{M}^E = \left( \bar{M}^E - \hat{M}^E \right)^{1-\theta} (1 - \hat{M}^E)^\theta \quad (27)$$

The solution for the thresholds during a contraction follows along the same lines.

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Table 1a. Duration of output and employment, linearly de-trended with breaks

|                          |                      |              | Average | t-statistic<br>(p-value) | W-statistic<br>(p-value) |
|--------------------------|----------------------|--------------|---------|--------------------------|--------------------------|
| Industrial<br>Production | Window               | Expansions   | 11.273  | 1.072                    | 1.18                     |
|                          |                      | Contractions | 8.167   | (0.142)                  | (0.130)                  |
|                          | Reversal             | Expansions   | 10.417  | 0.706                    | 0.226                    |
|                          |                      | Contractions | 8.250   | (0.240)                  | (0.421)                  |
|                          | Bry-Boschan          | Expansions   | 11.364  | 1.220                    | 0.681                    |
|                          |                      | Contractions | 7.546   | (0.111)                  | (0.260)                  |
|                          | Regime-<br>switching | Expansions   | 24.800  | 0.973                    | 2.121*                   |
|                          |                      | Contractions | 16.000  | (0.165)                  | (0.041)                  |
| Employment<br>Rate       | Window               | Expansions   | 17.875  | 2.401**                  | 1.99*                    |
|                          |                      | Contractions | 8.778   | (0.008)                  | (0.037)                  |
|                          | Reversal             | Expansions   | 15.900  | 2.698**                  | 2.357*                   |
|                          |                      | Contractions | 6.600   | (0.003)                  | (0.018)                  |
|                          | Bry-Boschan          | Expansions   | 18.375  | 2.754**                  | 3.105**                  |
|                          |                      | Contractions | 8.222   | (0.003)                  | (0.006)                  |
|                          | Regime-<br>Switching | Expansions   | 15.222  | 1.505                    | 1.208                    |
|                          |                      | Contractions | 9.222   | (0.066)                  | (0.129)                  |

Notes: The time unit is one quarter. t-statistics are for a test of means with p-values from the Normal distribution. W-statistics are for a Wilcoxon test of distributions with p-values from the exact finite sample distribution. \* and \*\* denote significance at the 5% and 1% levels respectively.

Table 1b. Duration of output and employment, polynomially de-trended

|                          |                      |              | Average | t-statistic<br>(p-value) | W-statistic<br>(p-value) |
|--------------------------|----------------------|--------------|---------|--------------------------|--------------------------|
| Industrial<br>Production | Window               | Expansions   | 10.818  | 0.760                    | 0.423                    |
|                          |                      | Contractions | 8.583   | (0.224)                  | (0.347)                  |
|                          | Reversal             | Expansions   | 10.583  | 0.774                    | 0.511                    |
|                          |                      | Contractions | 8.167   | (0.219)                  | (0.315)                  |
|                          | Bry-Boschan          | Expansions   | 11.273  | 1.062                    | 0.362                    |
|                          |                      | Contractions | 8.000   | (0.144)                  | (0.370)                  |
|                          | Regime-<br>switching | Expansions   | 24.400  | 0.900                    | 2.121*                   |
|                          |                      | Contractions | 16.333  | (0.184)                  | (0.041)                  |
| Employment<br>Rate       | Window               | Expansions   | 17.625  | 2.318*                   | 1.99*                    |
|                          |                      | Contractions | 9.000   | (0.010)                  | (0.037)                  |
|                          | Reversal             | Expansions   | 15.300  | 2.39**                   | 2.041*                   |
|                          |                      | Contractions | 7.200   | (0.008)                  | (0.032)                  |
|                          | Bry-Boschan          | Expansions   | 18.000  | 2.633**                  | 3.45**                   |
|                          |                      | Contractions | 8.000   | (0.004)                  | (0.003)                  |
|                          | Regime-<br>switching | Expansions   | 15.111  | 1.435                    | 1.016                    |
|                          |                      | Contractions | 9.444   | (0.076)                  | (0.170)                  |

Notes: The time unit is one quarter. t-statistics are for a test of means with p-values from the Normal distribution. W-statistics are for a Wilcoxon test of distributions with p-values from the exact finite sample distribution. \* and \*\* denote significance at the 5% and 1% levels respectively.

Table 1c. Duration of output and employment, band-pass filter de-trended

|                          |                      |              | Average | t-statistic<br>(p-value) | W-statistic<br>(p-value) |
|--------------------------|----------------------|--------------|---------|--------------------------|--------------------------|
| Industrial<br>Production | Window               | Expansions   | 8.615   | 0.687                    | 0.82                     |
|                          |                      | Contractions | 7.714   | (0.246)                  | (0.215)                  |
|                          | Reversal             | Expansions   | 6.579   | 1.308                    | 1.531                    |
|                          |                      | Contractions | 5.263   | (0.096)                  | (0.069)                  |
|                          | Bry-Boschan          | Expansions   | 9.000   | 1.145                    | 1.71                     |
|                          |                      | Contractions | 7.357   | (0.126)                  | (0.052)                  |
|                          | Regime-<br>switching | Expansions   | 11.600  | 0.531                    | 0.767                    |
|                          |                      | Contractions | 9.455   | (0.298)                  | (0.234)                  |
| Employment<br>Rate       | Window               | Expansions   | 9.917   | 1.93*                    | 2.023*                   |
|                          |                      | Contractions | 7.615   | (0.027)                  | (0.030)                  |
|                          | Reversal             | Expansions   | 7.177   | 1.016                    | 1.416                    |
|                          |                      | Contractions | 6.059   | (0.155)                  | (0.085)                  |
|                          | Bry-Boschan          | Expansions   | 10.417  | 2.616**                  | 3.391**                  |
|                          |                      | Contractions | 7.000   | (0.004)                  | (0.002)                  |
|                          | Regime-<br>switching | Expansions   | 13.111  | 1.025                    | 1.243                    |
|                          |                      | Contractions | 9.800   | (0.153)                  | (0.121)                  |

Notes: The time unit is one quarter. t-statistics are for a test of means with p-values from the Normal distribution. W-statistics are for a Wilcoxon test of distributions with p-values from the exact finite sample distribution. \* and \*\* denote significance at the 5% and 1% levels respectively.

Table 1d. Duration of output and employment, modified-HP filter de-trended

|                          |                      |              | Average | t-statistic<br>(p-value) | W-statistic<br>(p-value) |
|--------------------------|----------------------|--------------|---------|--------------------------|--------------------------|
| Industrial<br>Production | Window               | Expansions   | 11.273  | 1.072                    | 1.18                     |
|                          |                      | Contractions | 8.167   | (0.142)                  | (0.130)                  |
|                          | Reversal             | Expansions   | 9.615   | 0.724                    | 0.685                    |
|                          |                      | Contractions | 7.615   | (0.235)                  | (0.256)                  |
|                          | Bry-Boschan          | Expansions   | 10.000  | 1.083                    | 0.627                    |
|                          |                      | Contractions | 7.333   | (0.139)                  | (0.276)                  |
|                          | Regime-<br>switching | Expansions   | 25.000  | 1.023                    | 2.121*                   |
|                          |                      | Contractions | 15.833  | (0.153)                  | (0.041)                  |
| Employment<br>Rate       | Window               | Expansions   | 17.250  | 2.17*                    | 1.736                    |
|                          |                      | Contractions | 9.333   | (0.015)                  | (0.057)                  |
|                          | Reversal             | Expansions   | 15.300  | 2.39**                   | 2.041*                   |
|                          |                      | Contractions | 7.200   | (0.008)                  | (0.032)                  |
|                          | Bry-Boschan          | Expansions   | 18.000  | 2.633**                  | 3.45**                   |
|                          |                      | Contractions | 8.000   | (0.004)                  | (0.003)                  |
|                          | Regime-<br>switching | Expansions   | 15.222  | 1.491                    | 1.016                    |
|                          |                      | Contractions | 9.333   | (0.068)                  | (0.170)                  |

Notes: The time unit is one quarter. t-statistics are for a test of means with p-values from the Normal distribution. W-statistics are for a Wilcoxon test of distributions with p-values from the exact finite sample distribution. \* and \*\* denote significance at the 5% and 1% levels respectively.

Table 2a. Violence of output and employment, linearly de-trended with breaks

|                              |       | <u>Steepness</u> |                          |                          | <u>Sharpness</u> |                          |                          | <u>Slope</u> |                          |                          |
|------------------------------|-------|------------------|--------------------------|--------------------------|------------------|--------------------------|--------------------------|--------------|--------------------------|--------------------------|
|                              |       | Average          | t-statistic<br>(p-value) | W-statistic<br>(p-value) | Average          | t-statistic<br>(p-value) | W-statistic<br>(p-value) | Average      | t-statistic<br>(p-value) | W-statistic<br>(p-value) |
| <u>Industrial Production</u> |       |                  |                          |                          |                  |                          |                          |              |                          |                          |
| Window                       | Exp.  | 0.013            | 0.434                    | 0.362                    | 0.021            | 0.554                    | 0.668                    | 0.015        | 0.067                    | 0                        |
|                              | Cont. | -0.015           | (0.332)                  | (0.370)                  | 0.024            | (0.290)                  | (0.263)                  | -0.015       | (0.473)                  | (0.500)                  |
| Reversal                     | Exp.  | 0.010            | 0.88                     | 0.802                    | 0.020            | 0.695                    | 0.802                    | 0.012        | 0.018                    | 0.113                    |
|                              | Cont. | -0.013           | (0.189)                  | (0.221)                  | 0.023            | (0.244)                  | (0.221)                  | -0.012       | (0.493)                  | (0.466)                  |
| Bry-<br>Boschan              | Exp.  | 0.013            | 0.191                    | 0.289                    | 0.022            | 0.479                    | 0.681                    | 0.016        | 0.401                    | 0.16                     |
|                              | Cont. | -0.014           | (0.424)                  | (0.398)                  | 0.025            | (0.316)                  | (0.260)                  | -0.014       | (0.344)                  | (0.449)                  |
| Regime-<br>switching         | Exp.  | 0.007            | 1.624                    | 1.326                    | 0.014            | 2.105*                   | 2.121*                   | 0.007        | 1.381                    | 1.326                    |
|                              | Cont. | -0.0155          | (0.052)                  | (0.123)                  | 0.027            | (0.018)                  | (0.041)                  | -0.016       | (0.084)                  | (0.123)                  |
| <u>Employment Rate</u>       |       |                  |                          |                          |                  |                          |                          |              |                          |                          |
| Window                       | Exp.  | 0.002            | 2.258*                   | 2.739*                   | 0.004            | 2.244*                   | 2.267*                   | 0.002        | 1.977*                   | 2.126*                   |
|                              | Cont. | -0.004           | (0.012)                  | (0.010)                  | 0.006            | (0.012)                  | (0.023)                  | -0.004       | (0.024)                  | (0.030)                  |
| Reversal                     | Exp.  | 0.001            | 4.028**                  | 4.086**                  | 0.004            | 2.671**                  | 2.961**                  | 0.002        | 3.562**                  | 3.896**                  |
|                              | Cont. | -0.004           | (0.000)                  | (0.001)                  | 0.006            | (0.004)                  | (0.006)                  | -0.004       | (0.000)                  | (0.001)                  |
| Bry-<br>Boschan              | Exp.  | 0.002            | 3.208**                  | 4.335**                  | 0.004            | 2.103*                   | 1.99*                    | 0.002        | 2.901**                  | 3.309**                  |
|                              | Cont. | -0.004           | (0.001)                  | (0.001)                  | 0.006            | (0.018)                  | (0.037)                  | -0.004       | (0.002)                  | (0.004)                  |
| Regime-<br>switching         | Exp.  | 0.001            | 2.638**                  | 1.726                    | 0.002            | 5.439**                  | 6.971**                  | 0.001        | 2.798**                  | 2.200*                   |
|                              | Cont. | -0.003           | (0.004)                  | (0.057)                  | 0.006            | (0.000)                  | (0.000)                  | -0.003       | (0.003)                  | (0.025)                  |

Notes: The time unit is one quarter. t-statistics are for a test of means with p-values from the Normal distribution. W-statistics are for a Wilcoxon test of distributions with p-values from the exact finite sample distribution. \* and \*\* denote significance at the 5% and 1% levels respectively.



Table 2b. Violence of output and employment, polynomial de-trended

|                              |       | <u>Steepness</u> |                          |                          | <u>Sharpness</u> |                          |                          | <u>Slope</u> |                          |                          |
|------------------------------|-------|------------------|--------------------------|--------------------------|------------------|--------------------------|--------------------------|--------------|--------------------------|--------------------------|
|                              |       | Average          | t-statistic<br>(p-value) | W-statistic<br>(p-value) | Average          | t-statistic<br>(p-value) | W-statistic<br>(p-value) | Average      | t-statistic<br>(p-value) | W-statistic<br>(p-value) |
| <u>Industrial Production</u> |       |                  |                          |                          |                  |                          |                          |              |                          |                          |
| Window                       | Exp.  | 0.013            | 0.431                    | 0.423                    | 0.022            | 0.526                    | 0.545                    | 0.015        | 0.06                     | 0.241                    |
|                              | Cont. | -0.015           | (0.333)                  | (0.347)                  | 0.024            | (0.299)                  | (0.304)                  | -0.015       | (0.476)                  | (0.416)                  |
| Reversal                     | Exp.  | 0.010            | 1.318                    | 1.226                    | 0.021            | 0.735                    | 0.743                    | 0.011        | 0.674                    | 0.454                    |
|                              | Cont. | -0.014           | (0.094)                  | (0.121)                  | 0.024            | (0.231)                  | (0.239)                  | -0.013       | (0.250)                  | (0.335)                  |
| Bry-<br>Boschan              | Exp.  | 0.013            | 0.404                    | 0.731                    | 0.022            | 0.372                    | 0.362                    | 0.015        | 0.167                    | 0.12                     |
|                              | Cont. | -0.014           | (0.343)                  | (0.243)                  | 0.024            | (0.355)                  | (0.370)                  | -0.015       | (0.434)                  | (0.464)                  |
| Regime-<br>switching         | Exp.  | 0.007            | 1.707*                   | 2.121*                   | 0.014            | 2.279*                   | 2.882*                   | 0.008        | 1.431                    | 1.562                    |
|                              | Cont. | -0.016           | (0.044)                  | (0.041)                  | 0.026            | (0.011)                  | (0.015)                  | -0.016       | (0.076)                  | (0.089)                  |
| <u>Employment Rate</u>       |       |                  |                          |                          |                  |                          |                          |              |                          |                          |
| Window                       | Exp.  | 0.002            | 2.358**                  | 2.739*                   | 0.004            | 2.289*                   | 2.267*                   | 0.002        | 2.081*                   | 2.739*                   |
|                              | Cont. | -0.004           | (0.009)                  | (0.010)                  | 0.006            | (0.011)                  | (0.023)                  | -0.004       | (0.019)                  | (0.010)                  |
| Reversal                     | Exp.  | 0.001            | 3.765**                  | 3.24**                   | 0.004            | 2.603**                  | 2.961**                  | 0.002        | 3.325**                  | 3.391**                  |
|                              | Cont. | -0.004           | (0.000)                  | (0.003)                  | 0.006            | (0.005)                  | (0.006)                  | -0.004       | (0.000)                  | (0.003)                  |
| Bry-<br>Boschan              | Exp.  | 0.002            | 3.44**                   | 3.72**                   | 0.004            | 2.401**                  | 2.592*                   | 0.002        | 3.055**                  | 3.45**                   |
|                              | Cont. | -0.004           | (0.000)                  | (0.002)                  | 0.006            | (0.008)                  | (0.014)                  | -0.004       | (0.001)                  | (0.003)                  |
| Regime-<br>switching         | Exp.  | 0.001            | 2.508**                  | 1.511                    | 0.002            | 5.42**                   | 6.971**                  | 0.001        | 2.616**                  | 1.838*                   |
|                              | Cont. | -0.003           | (0.006)                  | (0.081)                  | 0.006            | (0.000)                  | (0.000)                  | -0.003       | (0.004)                  | (0.047)                  |

Notes: The time unit is one quarter. t-statistics are for a test of means with p-values from the Normal distribution. W-statistics are for a Wilcoxon test of distributions with p-values from the exact finite sample distribution. \* and \*\* denote significance at the 5% and 1% levels respectively.

Table 2c. Violence of output and employment, band-pass filter de-trended

|                              |       | <u>Steepness</u> |                          |                          | <u>Sharpness</u> |                          |                          | <u>Slope</u> |                          |                          |
|------------------------------|-------|------------------|--------------------------|--------------------------|------------------|--------------------------|--------------------------|--------------|--------------------------|--------------------------|
|                              |       | Average          | t-statistic<br>(p-value) | W-statistic<br>(p-value) | Average          | t-statistic<br>(p-value) | W-statistic<br>(p-value) | Average      | t-statistic<br>(p-value) | W-statistic<br>(p-value) |
| <u>Industrial Production</u> |       |                  |                          |                          |                  |                          |                          |              |                          |                          |
| Window                       | Exp.  | 0.011            | 0.483                    | 0.623                    | 0.015            | 0.593                    | 0.575                    | 0.013        | 0.295                    | 0.334                    |
|                              | Cont. | -0.013           | (0.315)                  | (0.275)                  | 0.018            | (0.277)                  | (0.291)                  | -0.014       | (0.384)                  | (0.378)                  |
| Reversal                     | Exp.  | 0.009            | 0.899                    | 0.71                     | 0.013            | 0.81                     | 0.858                    | 0.010        | 0.788                    | 0.74                     |
|                              | Cont. | -0.011           | (0.184)                  | (0.244)                  | 0.016            | (0.209)                  | (0.201)                  | -0.012       | (0.215)                  | (0.235)                  |
| Bry-<br>Boschan              | Exp.  | 0.011            | 0.89                     | 0.721                    | 0.015            | 0.704                    | 0.721                    | 0.012        | 0.698                    | 0.672                    |
|                              | Cont. | -0.014           | (0.187)                  | (0.244)                  | 0.018            | (0.241)                  | (0.244)                  | -0.015       | (0.243)                  | (0.259)                  |
| Regime-<br>switching         | Exp.  | 0.004            | 1.631                    | 1.527                    | 0.008            | 2.266*                   | 2.047*                   | 0.004        | 1.479                    | 1.367                    |
|                              | Cont. | -0.009           | (0.051)                  | (0.076)                  | 0.017            | (0.012)                  | (0.031)                  | -0.009       | (0.070)                  | (0.099)                  |
| <u>Employment Rate</u>       |       |                  |                          |                          |                  |                          |                          |              |                          |                          |
| Window                       | Exp.  | 0.002            | 0.77                     | 0.866                    | 0.003            | 0.847                    | 0.978                    | 0.002        | 0.672                    | 0.755                    |
|                              | Cont. | -0.003           | (0.221)                  | (0.203)                  | 0.004            | (0.198)                  | (0.174)                  | -0.003       | (0.251)                  | (0.235)                  |
| Reversal                     | Exp.  | 0.002            | 0.773                    | 0.528                    | 0.003            | 0.841                    | 0.946                    | 0.002        | 0.736                    | 0.528                    |
|                              | Cont. | -0.002           | (0.220)                  | (0.305)                  | 0.003            | (0.200)                  | (0.179)                  | -0.003       | (0.231)                  | (0.305)                  |
| Bry-<br>Boschan              | Exp.  | 0.002            | 1.36                     | 1.092                    | 0.003            | 1.054                    | 1.092                    | 0.002        | 1.239                    | 1.035                    |
|                              | Cont. | -0.003           | (0.087)                  | (0.147)                  | 0.004            | (0.146)                  | (0.147)                  | -0.003       | (0.108)                  | (0.160)                  |
| Regime-<br>switching         | Exp.  | 0.001            | 1.263                    | .979                     | 0.002            | 2.846**                  | 2.773**                  | 0.001        | 1.832*                   | 1.719                    |
|                              | Cont. | -0.002           | (0.103)                  | (0.178)                  | 0.004            | (0.002)                  | (0.009)                  | -0.002       | (0.033)                  | (0.056)                  |

Notes: The time unit is one quarter. t-statistics are for a test of means with p-values from the Normal distribution. W-statistics are for a Wilcoxon test of distributions with p-values from the exact finite sample distribution. \* and \*\* denote significance at the 5% and 1% levels respectively.

Table 2d. Violence of output and employment, modified-HP filter de-trended

|                              |       | <u>Steepness</u> |                          |                          | <u>Sharpness</u> |                          |                          | <u>Slope</u> |                          |                          |
|------------------------------|-------|------------------|--------------------------|--------------------------|------------------|--------------------------|--------------------------|--------------|--------------------------|--------------------------|
|                              |       | Average          | t-statistic<br>(p-value) | W-statistic<br>(p-value) | Average          | t-statistic<br>(p-value) | W-statistic<br>(p-value) | Average      | t-statistic<br>(p-value) | W-statistic<br>(p-value) |
| <u>Industrial Production</u> |       |                  |                          |                          |                  |                          |                          |              |                          |                          |
| Window                       | Exp.  | 0.014            | 0.268                    | 0.241                    | 0.022            | 0.484                    | 0.668                    | 0.016        | 0.201                    | 0.181                    |
|                              | Cont. | -0.015           | (0.394)                  | (0.416)                  | 0.024            | (0.314)                  | (0.263)                  | -0.015       | (0.420)                  | (0.440)                  |
| Reversal                     | Exp.  | 0.011            | 0.616                    | 0.378                    | 0.020            | 0.39                     | 0.429                    | 0.013        | 0.238                    | 0.378                    |
|                              | Cont. | -0.012           | (0.269)                  | (0.362)                  | 0.022            | (0.348)                  | (0.343)                  | -0.012       | (0.406)                  | (0.362)                  |
| Bry-<br>Boschan              | Exp.  | 0.012            | 0.04                     | 0.056                    | 0.021            | 0.309                    | 0.397                    | 0.015        | 0.522                    | 0.17                     |
|                              | Cont. | -0.012           | (0.484)                  | (0.489)                  | 0.023            | (0.379)                  | (0.356)                  | -0.013       | (0.301)                  | (0.444)                  |
| Regime-<br>switching         | Exp.  | 0.006            | 1.767*                   | 2.121*                   | 0.014            | 2.362**                  | 2.882*                   | 0.007        | 1.394                    | 1.326                    |
|                              | Cont. | -0.016           | (0.039)                  | (0.041)                  | 0.027            | (0.009)                  | (0.015)                  | -0.016       | (0.082)                  | (0.123)                  |
| <u>Employment Rate</u>       |       |                  |                          |                          |                  |                          |                          |              |                          |                          |
| Window                       | Exp.  | 0.002            | 2.079*                   | 2.416*                   | 0.004            | 2.158*                   | 2.126*                   | 0.003        | 1.847*                   | 2.126*                   |
|                              | Cont. | -0.004           | (0.019)                  | (0.018)                  | 0.006            | (0.015)                  | (0.030)                  | -0.004       | (0.032)                  | (0.030)                  |
| Reversal                     | Exp.  | 0.001            | 3.751**                  | 3.549**                  | 0.004            | 2.543**                  | 2.961**                  | 0.002        | 3.266**                  | 3.24**                   |
|                              | Cont. | -0.004           | (0.000)                  | (0.002)                  | 0.006            | (0.006)                  | (0.006)                  | -0.004       | (0.001)                  | (0.003)                  |
| Bry-<br>Boschan              | Exp.  | 0.002            | 3.406**                  | 4.365**                  | 0.004            | 2.343**                  | 2.093*                   | 0.002        | 3.001**                  | 3.45**                   |
|                              | Cont. | -0.004           | (0.000)                  | (0.001)                  | 0.006            | (0.010)                  | (0.032)                  | -0.004       | (0.001)                  | (0.003)                  |
| Regime-<br>switching         | Exp.  | 0.001            | 2.860**                  | 2.200*                   | 0.002            | 5.375**                  | 6.971**                  | 0.001        | 2.914**                  | 2.200*                   |
|                              | Cont. | -0.003           | (0.002)                  | (0.025)                  | 0.006            | (0.000)                  | (0.000)                  | -0.003       | (0.002)                  | (0.025)                  |

Notes: The time unit is one quarter. t-statistics are for a test of means with p-values from the Normal distribution. W-statistics are for a Wilcoxon test of distributions with p-values from the exact finite sample distribution. \* and \*\* denote significance at the 5% and 1% levels respectively.

Table 2e. Violence of output and employment, band-pass (2,80) filter de-trended

|                              |       | <u>Steepness</u> |                          |                          | <u>Sharpness</u> |                          |                          | <u>Slope</u> |                          |                          |
|------------------------------|-------|------------------|--------------------------|--------------------------|------------------|--------------------------|--------------------------|--------------|--------------------------|--------------------------|
|                              |       | Average          | t-statistic<br>(p-value) | W-statistic<br>(p-value) | Average          | t-statistic<br>(p-value) | W-statistic<br>(p-value) | Average      | t-statistic<br>(p-value) | W-statistic<br>(p-value) |
| <u>Industrial Production</u> |       |                  |                          |                          |                  |                          |                          |              |                          |                          |
| Window                       | Exp.  | 0.014            | 0.024                    | 0.06                     | 0.022            | 0.358                    | 0.484                    | 0.016        | -0.364                   | -0.301                   |
|                              | Cont. | -0.014           | (0.491)                  | (0.488)                  | 0.023            | (0.360)                  | (0.325)                  | -0.014       | (0.358)                  | (0.393)                  |
| Reversal                     | Exp.  | 0.011            | 0.553                    | 0.226                    | 0.021            | 0.265                    | 0.327                    | 0.013        | -0.313                   | -0.327                   |
|                              | Cont. | -0.012           | (0.290)                  | (0.420)                  | 0.022            | (0.396)                  | (0.381)                  | -0.012       | (0.377)                  | (0.381)                  |
| Bry-<br>Boschan              | Exp.  | 0.014            | -0.147                   | 0.000                    | 0.023            | 0.202                    | 0.423                    | 0.017        | -0.634                   | -0.241                   |
|                              | Cont. | -0.013           | (0.441)                  | (0.500)                  | 0.024            | (0.420)                  | (0.347)                  | -0.014       | (0.263)                  | (0.416)                  |
| Regime-<br>switching         | Exp.  | 0.009            | 0.050                    | -0.564                   | 0.014            | 1.102                    | 0.681                    | 0.010        | 0.044                    | -0.223                   |
|                              | Cont. | -0.010           | (0.480)                  | (0.306)                  | 0.020            | (0.135)                  | (0.268)                  | -0.011       | (0.483)                  | (0.433)                  |
| <u>Employment Rate</u>       |       |                  |                          |                          |                  |                          |                          |              |                          |                          |
| Window                       | Exp.  | 0.002            | 1.508                    | 1.427                    | 0.004            | 1.838*                   | 1.62                     | 0.003        | 1.356                    | 1.243                    |
|                              | Cont. | -0.003           | (0.066)                  | (0.091)                  | 0.005            | (0.033)                  | (0.067)                  | -0.004       | (0.088)                  | (0.121)                  |
| Reversal                     | Exp.  | 0.001            | 3.602**                  | 3.717**                  | 0.004            | 2.453**                  | 2.705**                  | 0.002        | 3.116**                  | 2.83**                   |
|                              | Cont. | -0.004           | (0.000)                  | (0.001)                  | 0.006            | (0.007)                  | (0.009)                  | -0.004       | (0.001)                  | (0.007)                  |
| Bry-<br>Boschan              | Exp.  | 0.002            | 2.237*                   | 2.263*                   | 0.004            | 1.529                    | 1.523                    | 0.002        | 1.795*                   | 1.719                    |
|                              | Cont. | -0.003           | (0.013)                  | (0.022)                  | 0.005            | (0.063)                  | (0.078)                  | -0.003       | (0.036)                  | (0.056)                  |
| Regime-<br>switching         | Exp.  | 0.001            | 2.686**                  | 2.33*                    | 0.002            | 5.066**                  | 5.829**                  | 0.001        | 2.762**                  | 2.200*                   |
|                              | Cont. | -0.002           | (0.004)                  | (0.020)                  | 0.006            | (0.000)                  | (0.000)                  | -0.003       | (0.003)                  | (0.025)                  |

Notes: The time unit is one quarter. t-statistics are for a test of means with p-values from the Normal distribution. W-statistics are for a Wilcoxon test of distributions with p-values from the exact finite sample distribution. \* and \*\* denote significance at the 5% and 1% levels respectively.

Table 3. Duration and violence of output and employment, with monthly observations

|                          |                                       | Duration | Violence<br>(Steepness) | Violence<br>(Sharpness) | Violence<br>(Slope) |
|--------------------------|---------------------------------------|----------|-------------------------|-------------------------|---------------------|
| Industrial<br>Production | Average difference                    | -10.60   | -0.009                  | 0.003                   | -0.011              |
|                          | Fraction of rejections<br>at 5% level | 3/16     | 3/16                    | 4/16                    | 3/16                |
|                          | Fraction of rejections<br>at 1% level | 3/16     | 3/16                    | 4/16                    | 3/16                |
|                          |                                       | 3/16     | 4/16                    | 4/16                    | 4/16                |
| Employment<br>Rate       | Average difference                    | -22.71   | -0.002                  | 0.001                   | -0.002              |
|                          | Fraction of rejections<br>at 5% level | 11/16    | 10/16                   | 4/16                    | 10/16               |
|                          | Fraction of rejections<br>at 1% level | 12/16    | 11/16                   | 6/16                    | 10/16               |
|                          |                                       | 3/16     | 4/16                    | 1/16                    | 8/16                |
|                          |                                       | 8/16     | 7/16                    | 4/16                    | 10/16               |

Notes: In each cell, the first row is based on the Wilcoxon test and the second on the test of means. The time unit is one month. The averages and fractions are across the 16 combinations of methods of de-trending and detecting turning points. Differences are contractions less expansions.

Table 4. Duration and violence of output, using different series

|                                |                                       | Duration                   | Violence<br>(Steepness) | Violence<br>(Sharpness) | Violence<br>(Slope) |
|--------------------------------|---------------------------------------|----------------------------|-------------------------|-------------------------|---------------------|
| GDP                            | Average difference                    | -5.30 (-1.64)              | -0.010 (-0.010)         | 0.002 (0.001)           | -0.011 (-0.010)     |
|                                | Fraction of rejections<br>at 5% level | 5/16 (1/12)<br>4/16 (0/12) | 4/16 (0/12)             | 3/16 (0/12)             | 4/16 (0/12)         |
|                                | Fraction of rejections<br>at 1% level | 1/16 (0/12)<br>4/16 (0/12) | 4/16 (0/12)             | 0/16 (0/12)             | 4/16 (0/12)         |
|                                |                                       |                            | 4/16 (0/12)             | 0/16 (0/12)             | 4/16 (0/12)         |
| Non-farm<br>Business<br>Output | Average difference                    | -6.02 (-1.17)              | -0.013 (-0.014)         | 0.000 (0.000)           | -0.014 (-0.015)     |
|                                | Fraction of rejections<br>at 5% level | 4/16 (0/12)<br>4/16 (0/12) | 4/16 (0/12)             | 0/16 (0/12)             | 3/16 (0/12)         |
|                                | Fraction of rejections<br>at 1% level | 3/16 (0/12)<br>3/16 (0/12) | 3/16 (0/12)             | 0/16 (0/12)             | 3/16 (0/12)         |
|                                |                                       |                            | 3/16 (0/12)             | 0/16 (0/12)             | 3/16 (0/12)         |
| Real Sales                     | Average difference                    | -6.12 (-0.90)              | -0.010 (-0.013)         | 0.003 (0.002)           | -0.011 (-0.014)     |
|                                | Fraction of rejections<br>at 5% level | 2/16 (0/12)<br>2/16 (0/12) | 0/16 (0/12)             | 0/16 (0/12)             | 0/16 (0/12)         |
|                                | Fraction of rejections<br>at 1% level | 1/16 (0/12)<br>2/16 (0/12) | 0/16 (0/12)             | 0/16 (0/12)             | 0/16 (0/12)         |
|                                |                                       |                            | 0/16 (0/12)             | 0/16 (0/12)             | 0/16 (0/12)         |
| Real Personal<br>Income        | Average difference                    | -3.53 (-1.98)              | -0.007 (-0.008)         | 0.002 (0.001)           | -0.008 (-0.008)     |
|                                | Fraction of rejections<br>at 5% level | 3/16 (0/12)<br>3/16 (0/12) | 3/16 (0/12)             | 3/16 (0/12)             | 3/16 (0/12)         |
|                                | Fraction of rejections<br>at 1% level | 0/16 (0/12)<br>0/16 (0/12) | 2/16 (0/12)             | 3/16 (0/12)             | 3/16 (0/12)         |
|                                |                                       |                            | 3/16 (0/12)             | 3/16 (0/12)             | 3/16 (0/12)         |
| Consumption                    | Average difference                    | -16.20 (-1.45)             | -0.006 (-0.007)         | 0.003 (0.001)           | -0.007 (-0.008)     |
|                                | Fraction of rejections<br>at 5% level | 1/16 (0/12)<br>0/16 (0/12) | 0/16 (0/12)             | 0/16 (0/12)             | 0/16 (0/12)         |
|                                | Fraction of rejections<br>at 1% level | 0/16 (0/12)<br>0/16 (0/12) | 0/16 (0/12)             | 0/16 (0/12)             | 0/16 (0/12)         |
|                                |                                       |                            | 3/16 (0/12)             | 0/16 (0/12)             | 1/16 (0/12)         |
| Investment                     | Average difference                    | -1.19 (-0.65)              | -0.039 (-0.047)         | 0.003 (-0.001)          | -0.043 (-0.051)     |
|                                | Fraction of rejections<br>at 5% level | 0/16 (0/12)<br>1/16 (1/12) | 0/16 (0/12)             | 4/16 (0/12)             | 0/16 (0/12)         |
|                                | Fraction of rejections<br>at 1% level | 0/16 (0/12)<br>0/16 (0/12) | 0/16 (0/12)             | 1/16 (0/12)             | 0/16 (0/12)         |
|                                |                                       |                            | 0/16 (0/12)             | 4/16 (0/12)             | 0/16 (0/12)         |
| Government<br>Spending         | Average difference                    | 21.03 (1.64)               | -0.011 (-0.011)         | -0.005 (-0.002)         | -0.013 (-0.013)     |
|                                | Fraction of rejections<br>at 5% level | 0/16 (0/12)<br>3/16 (0/12) | 2/16 (1/12)             | 0/16 (0/12)             | 3/16 (2/12)         |
|                                | Fraction of rejections<br>at 1% level | 0/16 (0/12)<br>3/16 (0/12) | 0/16 (0/12)             | 0/16 (0/12)             | 1/16 (0/12)         |
|                                |                                       |                            | 2/16 (0/12)             | 0/16 (0/12)             | 1/16 (0/12)         |

Notes: In each cell, the first row is based on the Wilcoxon test and the second on the test of means. The time unit is one month. The averages and fractions are across the 16 combinations of methods of de-trending and detecting turning points. Differences are contractions less expansions. In parentheses are the results excluding the use of the Chauvet-Hamilton algorithm.

Table 5. Duration and violence of employment, using different series

|                                   |                                       | Duration      | Violence<br>(Steepness) | Violence<br>(Sharpness) | Violence<br>(Slope) |
|-----------------------------------|---------------------------------------|---------------|-------------------------|-------------------------|---------------------|
| Total<br>Employment               | Average difference                    | -5.35 (-6.88) | -0.006 (-0.006)         | 0.002 (0.002)           | -0.006 (-0.007)     |
|                                   | Fraction of rejections<br>at 5% level | 5/16 (6/16)   | 3/16 (4/16)             | 7/16 (8/16)             | 5/16 (5/16)         |
|                                   |                                       | 4/16 (6/16)   | 3/16 (4/16)             | 9/16 (11/16)            | 4/16 (4/16)         |
|                                   | Fraction of rejections<br>at 1% level | 3/16 (4/16)   | 3/16 (4/16)             | 4/16 (5/16)             | 3/16 (4/16)         |
| 3/16 (4/16)                       |                                       | 3/16 (4/16)   | 5/16 (6/16)             | 3/16 (4/16)             |                     |
| Total<br>Employment<br>(Payroll)  | Average difference                    | -4.66 (-5.73) | -0.008 (-0.008)         | 0.002 (0.003)           | -0.008 (-0.009)     |
|                                   | Fraction of rejections<br>at 5% level | 5/16 (6/16)   | 8/16 (9/16)             | 5/16 (5/16)             | 4/16 (5/16)         |
|                                   |                                       | 6/16 (8/16)   | 9/16 (11/16)            | 7/16 (8/16)             | 5/16 (7/16)         |
|                                   | Fraction of rejections<br>at 1% level | 0/16 (0/16)   | 1/16 (1/16)             | 1/16 (2/16)             | 0/16 (0/16)         |
| 0/16 (0/16)                       |                                       | 2/16 (2/16)   | 2/16 (3/16)             | 0/16 (0/16)             |                     |
| Employment<br>Rate<br>16 – 24 Yrs | Average difference                    | -3.74 (-3.75) | -0.008 (-0.008)         | 0.002 (0.002)           | -0.009 (-0.009)     |
|                                   | Fraction of rejections<br>at 5% level | 6/16 (6/16)   | 1/16 (0/16)             | 2/16 (2/16)             | 1/16 (0/16)         |
|                                   |                                       | 6/16 (6/16)   | 1/16 (1/16)             | 3/16 (3/16)             | 0/16 (0/16)         |
|                                   | Fraction of rejections<br>at 1% level | 3/16 (2/16)   | 0/16 (0/16)             | 0/16 (0/16)             | 0/16 (0/16)         |
| 3/16 (2/16)                       |                                       | 0/16 (0/16)   | 0/16 (0/16)             | 0/16 (0/16)             |                     |
| Employment<br>Rate<br>Over 25 Yrs | Average difference                    | -5.24 (-5.75) | -0.004 (-0.005)         | 0.001 (0.002)           | -0.005 (-0.005)     |
|                                   | Fraction of rejections<br>at 5% level | 8/16 (7/16)   | 7/16 (8/16)             | 6/16 (7/16)             | 5/16 (5/16)         |
|                                   |                                       | 8/16 (7/16)   | 7/16 (8/16)             | 6/16 (7/16)             | 6/16 (7/16)         |
|                                   | Fraction of rejections<br>at 1% level | 2/16 (1/16)   | 4/16 (4/16)             | 0/16 (0/16)             | 2/16 (2/16)         |
| 5/16 (5/16)                       |                                       | 4/16 (5/16)   | 1/16 (1/16)             | 3/16 (3/16)             |                     |
| Participation<br>Rate             | Average difference                    | 2.51 (2.28)   | -0.001 (-0.001)         | 0.000 (0.000)           | -0.002 (-0.002)     |
|                                   | Fraction of rejections<br>at 5% level | 3/16 (2/16)   | 3/16 (3/16)             | 1/16 (1/16)             | 2/16 (1/16)         |
|                                   |                                       | 4/16 (3/16)   | 3/16 (3/16)             | 2/16 (2/16)             | 2/16 (2/16)         |
|                                   | Fraction of rejections<br>at 1% level | 1/16 (0/16)   | 3/16 (3/16)             | 0/16 (0/16)             | 1/16 (0/16)         |
| 1/16 (0/16)                       |                                       | 2/16 (2/16)   | 1/16 (0/16)             | 1/16 (0/16)             |                     |
| Hours per<br>Worker               | Average difference                    | -2.24 (-3.18) | -0.003 (-0.003)         | 0.000 (0.000)           | -0.004 (-0.003)     |
|                                   | Fraction of rejections<br>at 5% level | 3/16 (4/16)   | 3/16 (4/16)             | 0/16 (0/16)             | 3/16 (4/16)         |
|                                   |                                       | 2/16 (3/16)   | 3/16 (4/16)             | 0/16 (0/16)             | 4/16 (6/16)         |
|                                   | Fraction of rejections<br>at 1% level | 2/16 (3/16)   | 2/16 (3/16)             | 0/16 (0/16)             | 2/16 (3/16)         |
| 2/16 (3/16)                       |                                       | 3/16 (4/16)   | 0/16 (0/16)             | 3/16 (4/16)             |                     |

Notes: In each cell, the first row is based on the Wilcoxon test and the second on the test of means. The time unit is one month. The averages and fractions are across the 16 combinations of methods of de-trending and detecting turning points. Differences are contractions less expansions. In parentheses are the results using the parameters (2,80) for the band-pass filter.

Table 6. Duration and violence of pre-war pig iron production

|                                     |                                       | Duration | Violence<br>(Steepness) | Violence<br>(Sharpness) | Violence<br>(Slope) |
|-------------------------------------|---------------------------------------|----------|-------------------------|-------------------------|---------------------|
| Pig Iron<br>Production<br>1877-1929 | Average difference                    | -0.20    | -0.087                  | 0.019                   | -0.094              |
|                                     | Fraction of rejections<br>at 5% level | 10/16    | 1/16                    | 3/16                    | 1/16                |
|                                     |                                       | 1/16     | 4/16                    | 3/16                    | 2/16                |
|                                     | Fraction of rejections<br>at 1% level | 7/16     | 1/16                    | 3/16                    | 1/16                |
| 1/16                                |                                       | 1/16     | 3/16                    | 0/16                    |                     |

Notes: In each cell, the first row is based on the Wilcoxon test and the second on the test of means. The time unit is one month. The averages and fractions are across the 16 combinations of methods of de-trending and detecting turning points. Differences are contractions less expansions.

Table 7. Bootstrap p-values for tests of same duration of expansions and contractions

|                          |                      | Linear detrended |             | Polynomial detrended |             | Band-pass filtered |             | Modified HP-filtered |             |
|--------------------------|----------------------|------------------|-------------|----------------------|-------------|--------------------|-------------|----------------------|-------------|
|                          |                      | t-statistic      | W-statistic | t-statistic          | W-statistic | t-statistic        | W-statistic | t-statistic          | W-statistic |
| Industrial<br>Production | Window               | 0.111            | 0.135       | 0.210                | 0.434       | 0.235              | 0.249       | 0.111                | 0.135       |
|                          | Reversal             | 0.228            | 0.465       | 0.207                | 0.395       | 0.067              | 0.073       | 0.222                | 0.294       |
|                          | Bry- Boschan         | 0.082            | 0.294       | 0.112                | 0.461       | 0.099              | 0.057       | 0.110                | 0.334       |
|                          | Regime-<br>switching | 0.146            | 0.022*      | 0.167                | 0.022*      | 0.287              | 0.274       | 0.127                | 0.022*      |
| Employment<br>Rate       | Window               | 0.004**          | 0.034*      | 0.005**              | 0.034*      | 0.009**            | 0.029*      | 0.006**              | 0.057       |
|                          | Reversal             | 0.001**          | 0.009**     | 0.004**              | 0.029*      | 0.129              | 0.096       | 0.004**              | 0.029*      |
|                          | Bry- Boschan         | 0.001**          | 0.005**     | 0.002**              | 0.001**     | 0.002**            | 0.002**     | 0.002**              | 0.001**     |
|                          | Regime-<br>switching | 0.039*           | 0.135       | 0.049*               | 0.200       | 0.125              | 0.117       | 0.040*               | 0.200       |

Notes: The p-values refer to the tests in tables 1a to 1d. \* and \*\* denote significance at the 5% and 1% levels respectively.



Table 8a. Bootstrap p-values for tests of same violence of expansions and contractions, linear detrended

|                       |                  | Steepness   |             | Sharpness   |             | Slope       |             |
|-----------------------|------------------|-------------|-------------|-------------|-------------|-------------|-------------|
|                       |                  | t-statistic | W-statistic | t-statistic | W-statistic | t-statistic | W-statistic |
| Industrial Production | Window           | 0.359       | 0.388       | 0.224       | 0.190       | 0.464       | 0.471       |
|                       | Reversal         | 0.257       | 0.262       | 0.173       | 0.136       | 0.475       | 0.491       |
|                       | Bry-Boschan      | 0.446       | 0.409       | 0.259       | 0.179       | 0.381       | 0.440       |
|                       | Regime-switching | 0.116       | 0.143       | 0.004**     | 0.002**     | 0.175       | 0.186       |
| Employment Rate       | Window           | 0.049*      | 0.020*      | 0.002**     | 0.002**     | 0.090       | 0.068       |
|                       | Reversal         | 0.002**     | 0.004**     | 0.001**     | 0.001**     | 0.014*      | 0.006**     |
|                       | Bry- Boschan     | 0.012*      | 0.004**     | 0.004**     | 0.004**     | 0.027*      | 0.016*      |
|                       | Regime-switching | 0.032*      | 0.094       | 0.000**     | 0.000**     | 0.030*      | 0.068       |

Notes: The p-values refer to the tests in table 2a. \* and \*\* denote significance at the 5% and 1% levels respectively.

Table 8b. Bootstrap p-values for tests of same violence of expansions and contractions, polynomial detrended

|                       |                  | Steepness   |             | Sharpness   |             | Slope       |             |
|-----------------------|------------------|-------------|-------------|-------------|-------------|-------------|-------------|
|                       |                  | t-statistic | W-statistic | t-statistic | W-statistic | t-statistic | W-statistic |
| Industrial Production | Window           | 0.360       | 0.364       | 0.232       | 0.239       | 0.498       | 0.443       |
|                       | Reversal         | 0.155       | 0.170       | 0.156       | 0.158       | 0.325       | 0.381       |
|                       | Bry-Boschan      | 0.369       | 0.289       | 0.307       | 0.336       | 0.437       | 0.487       |
|                       | Regime-switching | 0.106       | 0.050       | 0.001**     | 0.001**     | 0.170       | 0.146       |
| Employment Rate       | Window           | 0.044*      | 0.020*      | 0.001**     | 0.002**     | 0.078       | 0.035*      |
|                       | Reversal         | 0.003**     | 0.011*      | 0.001**     | 0.001**     | 0.016*      | 0.013*      |
|                       | Bry- Boschan     | 0.006**     | 0.006**     | 0.001**     | 0.001**     | 0.023*      | 0.013*      |
|                       | Regime-switching | 0.036*      | 0.124       | 0.000**     | 0.000**     | 0.038*      | 0.102       |

Notes: The p-values refer to the tests in table 2b. \* and \*\* denote significance at the 5% and 1% levels respectively.

Table 8c. Bootstrap p-values for tests of same violence of expansions and contractions, band-pass detrended

|                          |                  | Steepness   |             | Sharpness   |             | Slope       |             |
|--------------------------|------------------|-------------|-------------|-------------|-------------|-------------|-------------|
|                          |                  | t-statistic | W-statistic | t-statistic | W-statistic | t-statistic | W-statistic |
| Industrial<br>Production | Window           | 0.343       | 0.299       | 0.208       | 0.210       | 0.430       | 0.421       |
|                          | Reversal         | 0.249       | 0.289       | 0.141       | 0.113       | 0.298       | 0.305       |
|                          | Bry-Boschan      | 0.253       | 0.289       | 0.170       | 0.179       | 0.315       | 0.317       |
|                          | Regime-switching | 0.115       | 0.115       | 0.001**     | 0.003**     | 0.157       | 0.186       |
| Employment<br>Rate       | Window           | 0.279       | 0.245       | 0.126       | 0.092       | 0.325       | 0.305       |
|                          | Reversal         | 0.279       | 0.330       | 0.127       | 0.092       | 0.309       | 0.359       |
|                          | Bry- Boschan     | 0.149       | 0.198       | 0.077       | 0.068       | 0.202       | 0.245       |
|                          | Regime-switching | 0.166       | 0.224       | 0.001**     | 0.001**     | 0.107       | 0.126       |

Notes: The p-values refer to the tests in table 2c. \* and \*\* denote significance at the 5% and 1% levels respectively.

Table 8d. Bootstrap p-values for tests of same violence of expansions and contractions, modified-HP detrended

|                          |                  | Steepness   |             | Sharpness   |             | Slope       |             |
|--------------------------|------------------|-------------|-------------|-------------|-------------|-------------|-------------|
|                          |                  | t-statistic | W-statistic | t-statistic | W-statistic | t-statistic | W-statistic |
| Industrial<br>Production | Window           | 0.410       | 0.428       | 0.258       | 0.190       | 0.427       | 0.428       |
|                          | Reversal         | 0.309       | 0.364       | 0.302       | 0.275       | 0.415       | 0.373       |
|                          | Bry-Boschan      | 0.474       | 0.491       | 0.335       | 0.308       | 0.350       | 0.440       |
|                          | Regime-switching | 0.091       | 0.050       | 0.001**     | 0.001**     | 0.173       | 0.186       |
| Employment<br>Rate       | Window           | 0.060       | 0.038*      | 0.003**     | 0.002**     | 0.104       | 0.068       |
|                          | Reversal         | 0.003**     | 0.008**     | 0.001**     | 0.001**     | 0.016*      | 0.016*      |
|                          | Bry- Boschan     | 0.008**     | 0.004**     | 0.001**     | 0.003**     | 0.025*      | 0.013*      |
|                          | Regime-switching | 0.017*      | 0.050       | 0.000**     | 0.000**     | 0.027*      | 0.068       |

Notes: The p-values refer to the tests in table 2d. \* and \*\* denote significance at the 5% and 1% levels respectively.

Table 9. Bootstrap p-values for tests of same duration of expansions and contractions

|                          |                      | Linear detrended |             | Polynomial detrended |             | Band-pass filter |             | Modified-HP filtered |             |
|--------------------------|----------------------|------------------|-------------|----------------------|-------------|------------------|-------------|----------------------|-------------|
|                          |                      | t-statistic      | W-statistic | t-statistic          | W-statistic | t-statistic      | W-statistic | t-statistic          | W-statistic |
| Industrial<br>Production | Window               | 0.05             | 0.07        | 0.11                 | 0.33        | 0.23             | 0.28        | 0.06                 | 0.06        |
|                          | Reversal             | 0.17             | 0.43        | 0.14                 | 0.30        | 0.04*            | 0.07        | 0.19                 | 0.24        |
|                          | Bry-<br>Boschan      | 0.04*            | 0.23        | 0.05                 | 0.37        | 0.12             | 0.07        | 0.07                 | 0.27        |
|                          | Regime-<br>switching | 0.27             | 0.09        | 0.30                 | 0.11        | 0.43             | 0.43        | 0.31                 | 0.15        |
| Employment<br>Rate       | Window               | 0.00**           | 0.01**      | 0.00**               | 0.01**      | 0.01*            | 0.03*       | 0.00**               | 0.02*       |
|                          | Reversal             | 0.00**           | 0.01**      | 0.00**               | 0.01*       | 0.09             | 0.10        | 0.00**               | 0.02*       |
|                          | Bry-<br>Boschan      | 0.00**           | 0.00**      | 0.00**               | 0.00**      | 0.00**           | 0.00**      | 0.00**               | 0.00**      |
|                          | Regime-<br>switching | 0.24             | 0.28        | 0.24                 | 0.31        | 0.39             | 0.40        | 0.29                 | 0.36        |

Notes: The p-values refer to the tests in tables 1a to 1d. \* and \*\* denote significance at the 5% and 1% levels respectively.

Table 10a. Bootstrap p-values for tests of same violence of expansions and contractions, linear detrended

|                       |                  | Steepness   |             | Sharpness   |             | Slope       |             |
|-----------------------|------------------|-------------|-------------|-------------|-------------|-------------|-------------|
|                       |                  | t-statistic | W-statistic | t-statistic | W-statistic | t-statistic | W-statistic |
| Industrial Production | Window           | 0.31        | 0.33        | 0.22        | 0.17        | 0.49        | 0.48        |
|                       | Reversal         | 0.18        | 0.20        | 0.18        | 0.12        | 0.48        | 0.43        |
|                       | Bry-Boschan      | 0.41        | 0.36        | 0.25        | 0.19        | 0.35        | 0.45        |
|                       | Regime-switching | 0.26        | 0.29        | 0.05*       | 0.05        | 0.28        | 0.29        |
| Employment Rate       | Window           | 0.01**      | 0.00**      | 0.01**      | 0.01**      | 0.02*       | 0.01*       |
|                       | Reversal         | 0.00**      | 0.00**      | 0.00**      | 0.00**      | 0.00**      | 0.00**      |
|                       | Bry- Boschan     | 0.00**      | 0.00**      | 0.01*       | 0.02*       | 0.00**      | 0.00**      |
|                       | Regime-switching | 0.20        | 0.30        | 0.00**      | 0.00**      | 0.17        | 0.23        |

Notes: The p-values refer to the tests in table 2a. \* and \*\* denote significance at the 5% and 1% levels respectively.

Table 10b. Bootstrap p-values for tests of same violence of expansions and contractions, polynomial detrended

|                       |                  | Steepness   |             | Sharpness   |             | Slope       |             |
|-----------------------|------------------|-------------|-------------|-------------|-------------|-------------|-------------|
|                       |                  | t-statistic | W-statistic | t-statistic | W-statistic | t-statistic | W-statistic |
| Industrial Production | Window           | 0.30        | 0.29        | 0.25        | 0.21        | 0.46        | 0.39        |
|                       | Reversal         | 0.11        | 0.12        | 0.18        | 0.18        | 0.26        | 0.32        |
|                       | Bry-Boschan      | 0.32        | 0.21        | 0.30        | 0.33        | 0.44        | 0.44        |
|                       | Regime-switching | 0.27        | 0.23        | 0.03*       | 0.02*       | 0.30        | 0.29        |
| Employment Rate       | Window           | 0.00**      | 0.00**      | 0.00**      | 0.00**      | 0.02*       | 0.01**      |
|                       | Reversal         | 0.00**      | 0.00**      | 0.00**      | 0.00**      | 0.00**      | 0.00**      |
|                       | Bry- Boschan     | 0.00**      | 0.00**      | 0.01**      | 0.00**      | 0.00**      | 0.00**      |
|                       | Regime-switching | 0.21        | 0.33        | 0.00**      | 0.00**      | 0.18        | 0.27        |

Notes: The p-values refer to the tests in table 2b. \* and \*\* denote significance at the 5% and 1% levels respectively.

Table 10c. Bootstrap p-values for tests of same violence of expansions and contractions, band-pass detrended

|                       |                  | Steepness   |             | Sharpness   |             | Slope       |             |
|-----------------------|------------------|-------------|-------------|-------------|-------------|-------------|-------------|
|                       |                  | t-statistic | W-statistic | t-statistic | W-statistic | t-statistic | W-statistic |
| Industrial Production | Window           | 0.24        | 0.19        | 0.17        | 0.19        | 0.35        | 0.34        |
|                       | Reversal         | 0.10        | 0.13        | 0.08        | 0.08        | 0.13        | 0.13        |
|                       | Bry-Boschan      | 0.14        | 0.18        | 0.14        | 0.15        | 0.21        | 0.21        |
|                       | Regime-switching | 0.24        | 0.27        | 0.04*       | 0.06        | 0.24        | 0.27        |
| Employment Rate       | Window           | 0.12        | 0.10        | 0.09        | 0.08        | 0.19        | 0.15        |
|                       | Reversal         | 0.12        | 0.21        | 0.06        | 0.05        | 0.14        | 0.20        |
|                       | Bry- Boschan     | 0.04*       | 0.08        | 0.07        | 0.07        | 0.08        | 0.10        |
|                       | Regime-switching | 0.31        | 0.36        | 0.02*       | 0.02*       | 0.20        | 0.23        |

Notes: The p-values refer to the tests in table 2c. \* and \*\* denote significance at the 5% and 1% levels respectively.

Table 10d. Bootstrap p-values for tests of same violence of expansions and contractions, modified-HP detrended

|                       |                  | Steepness   |             | Sharpness   |             | Slope       |             |
|-----------------------|------------------|-------------|-------------|-------------|-------------|-------------|-------------|
|                       |                  | t-statistic | W-statistic | t-statistic | W-statistic | t-statistic | W-statistic |
| Industrial Production | Window           | 0.37        | 0.38        | 0.27        | 0.21        | 0.42        | 0.43        |
|                       | Reversal         | 0.26        | 0.33        | 0.29        | 0.26        | 0.41        | 0.37        |
|                       | Bry-Boschan      | 0.48        | 0.47        | 0.35        | 0.30        | 0.30        | 0.43        |
|                       | Regime-switching | 0.27        | 0.23        | 0.03*       | 0.02*       | 0.30        | 0.31        |
| Employment Rate       | Window           | 0.01**      | 0.00**      | 0.00**      | 0.01**      | 0.02*       | 0.01*       |
|                       | Reversal         | 0.00**      | 0.00**      | 0.00**      | 0.00**      | 0.00**      | 0.00**      |
|                       | Bry- Boschan     | 0.00**      | 0.00**      | 0.00**      | 0.01**      | 0.00**      | 0.00**      |
|                       | Regime-switching | 0.20        | 0.27        | 0.01**      | 0.01**      | 0.17        | 0.26        |

Notes: The p-values refer to the tests in table 2d. \* and \*\* denote significance at the 5% and 1% levels respectively.

Table 11. Maximum likelihood estimates and tests on a statistical model

| <u>Panel A. Industrial Production</u>  |            |                 |
|--|------------|-----------------|
| Maximum-likelihood estimates   | Estimates  | Standard Errors |
| $p_1$  | 0.9441     | 0.1147          |
| $p_2$  | 0.9198     | 0.1202          |
| $\mu_1$  | 0.0053     | 0.0010          |
| $\mu_2$  | -0.0102    | 0.0024          |
| $\sigma_{1,pre}$   | 0.0124     | 0.0015          |
| $\sigma_{2,pre}$   | 0.0327     | 0.0029          |
| $\sigma_{1,post}$  | 0.0066     | 0.0007          |
| $\sigma_{2,post}$  | 0.0100     | 0.0014          |
| Likelihood ratio tests   | Statistics | p-values        |
| $p_1=p_2$  | 0.46       | 0.50            |
| $\mu_1 = -\mu_2$   | 2.71       | 0.10            |
| $\mu_1 = -\mu_2, \sigma_{1,pre} = \sigma_{2,pre}, \sigma_{1,post} = \sigma_{2,post}$ | 46.40**    | 0.00            |
| <u>Panel B. Employment Rate</u>  |            |                 |
| Maximum-likelihood estimates   | Estimates  | Standard Errors |
| $p_1$  | 0.9237     | 0.1056          |
| $p_2$  | 0.8869     | 0.1234          |
| $\mu_1$  | 0.0009     | 0.0002          |
| $\mu_2$  | -0.0022    | 0.0005          |
| $\sigma_{1,pre}$   | 0.0019     | 0.0002          |
| $\sigma_{2,pre}$   | 0.0070     | 0.0007          |
| $\sigma_{1,post}$  | 0.0014     | 0.0001          |
| $\sigma_{2,post}$  | 0.0022     | 0.0003          |
| Likelihood ratio tests   | Statistics | p-values        |
| $p_1=p_2$  | 0.77       | 0.38            |
| $\mu_1 = -\mu_2$   | 5.66*      | 0.02            |
| $\mu_1 = \mu_2, \sigma_{1,pre} = \sigma_{2,pre}, \sigma_{1,post} = \sigma_{2,post}$  | 71.36**    | 0.00            |

Notes: The likelihood function was maximised using a quasi-Newton method. \* and \*\* denote significance at the 5% and 1% levels respectively.

Figure 1: Contractions and expansions in the baseline case for output, employment and the NBER

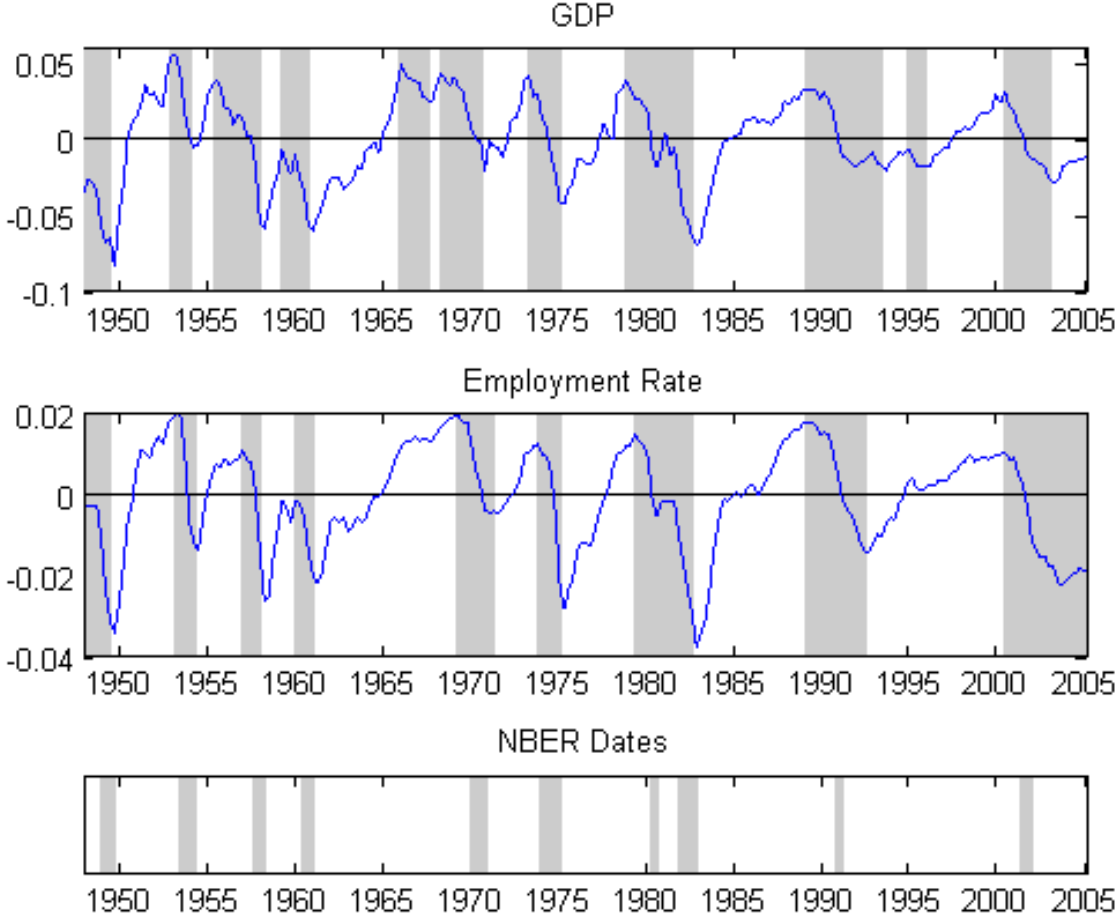


Figure 2: Average business cycle dynamics for output and employment in the baseline case

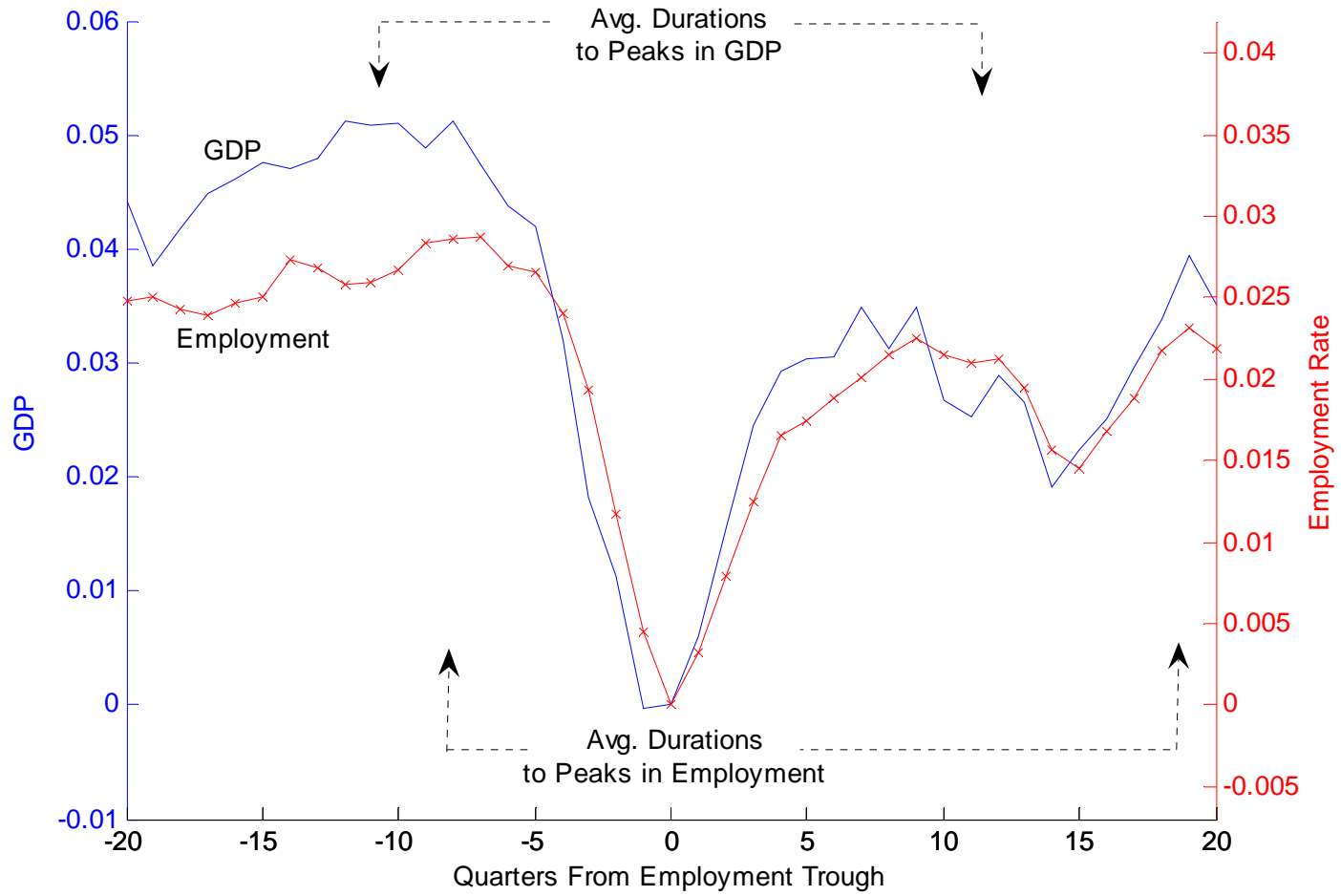




Figure 3: Representative peak-to-peak business cycle dynamics

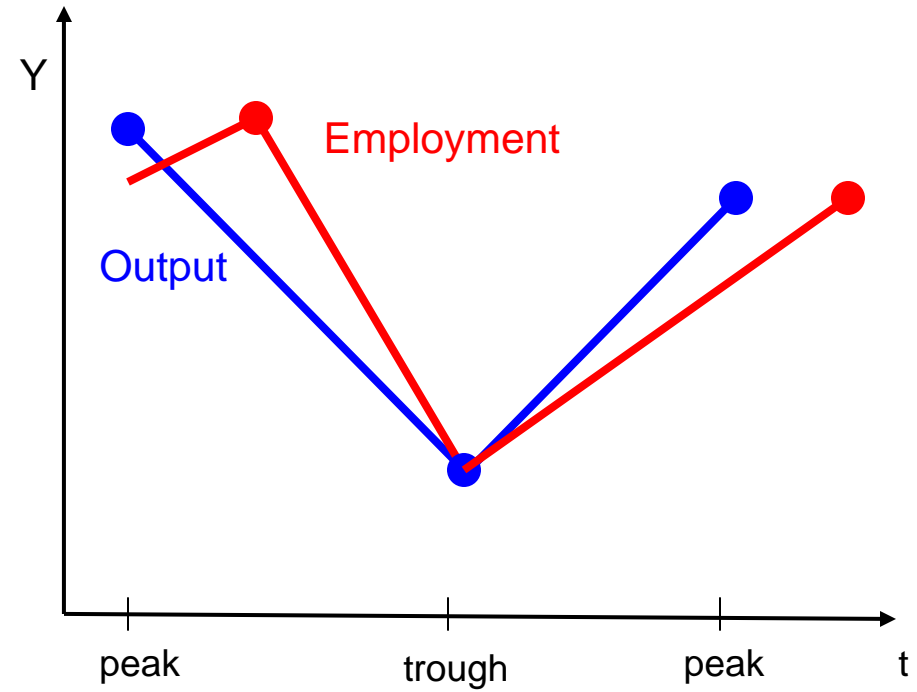


Figure 4: CDF's for the duration of expansions and contractions in industrial production

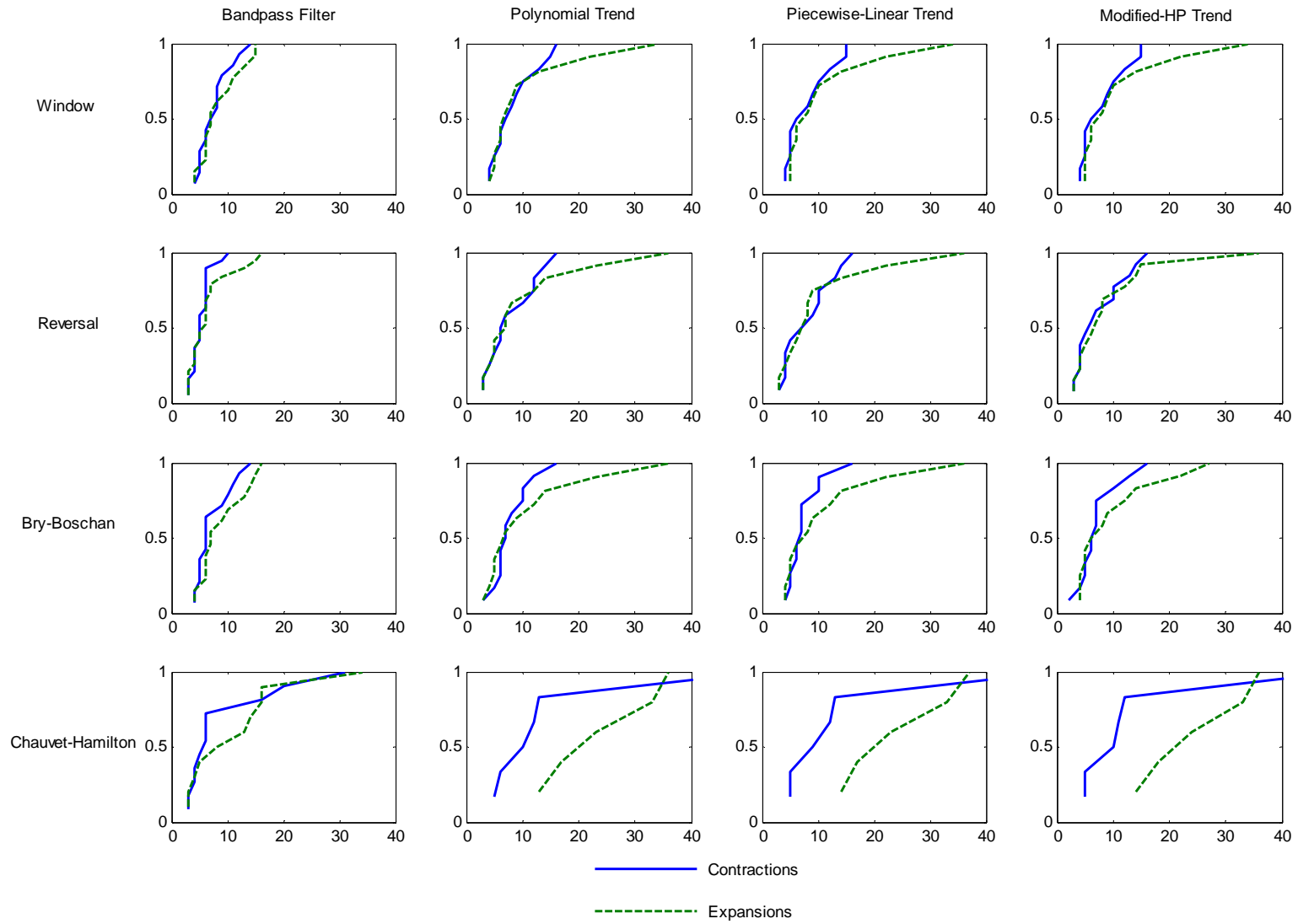


Figure 5: CDF's for the duration of expansions and contractions in the employment rate

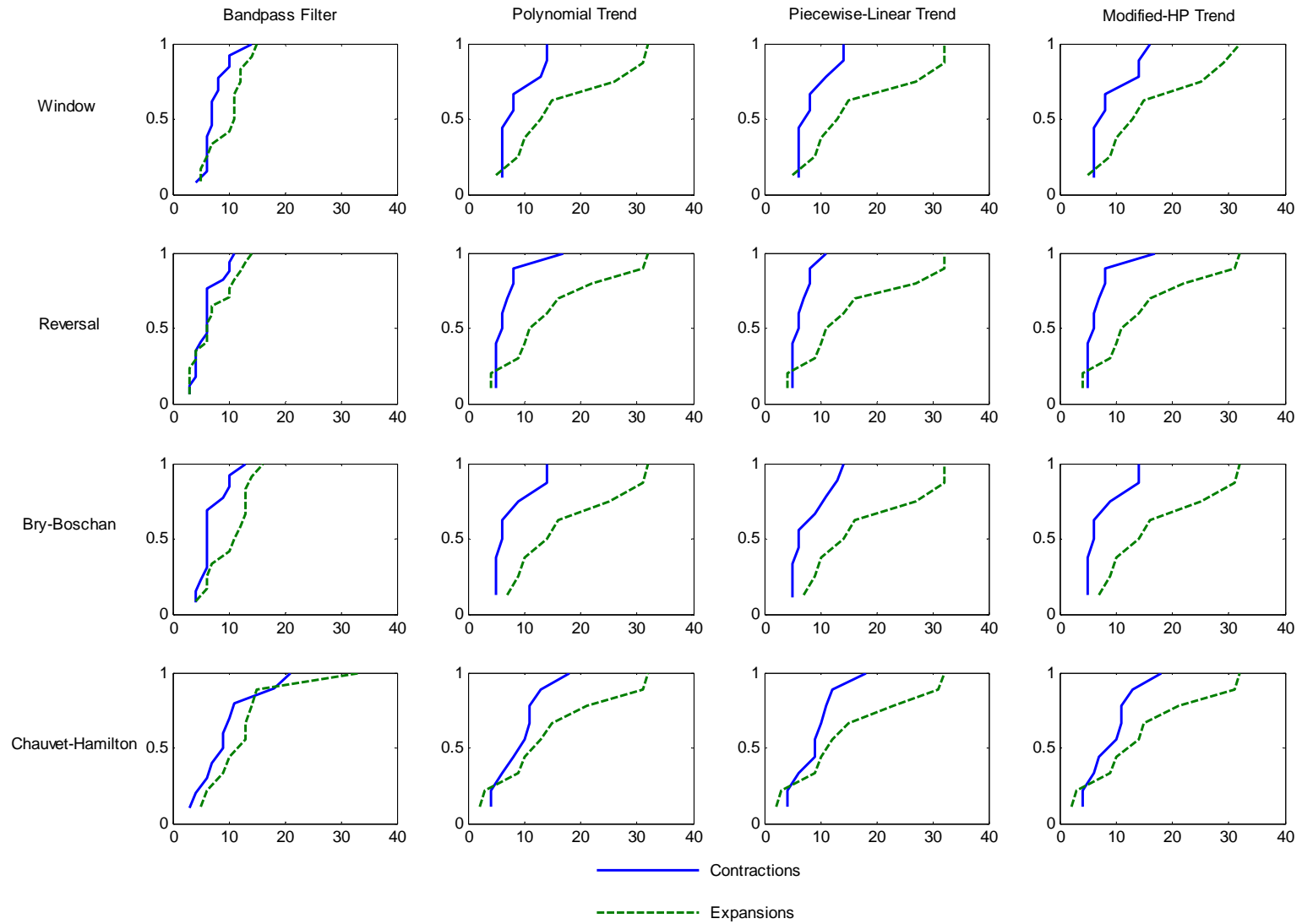


Figure 6a: CDF's for the steepness of expansions and contractions in industrial production

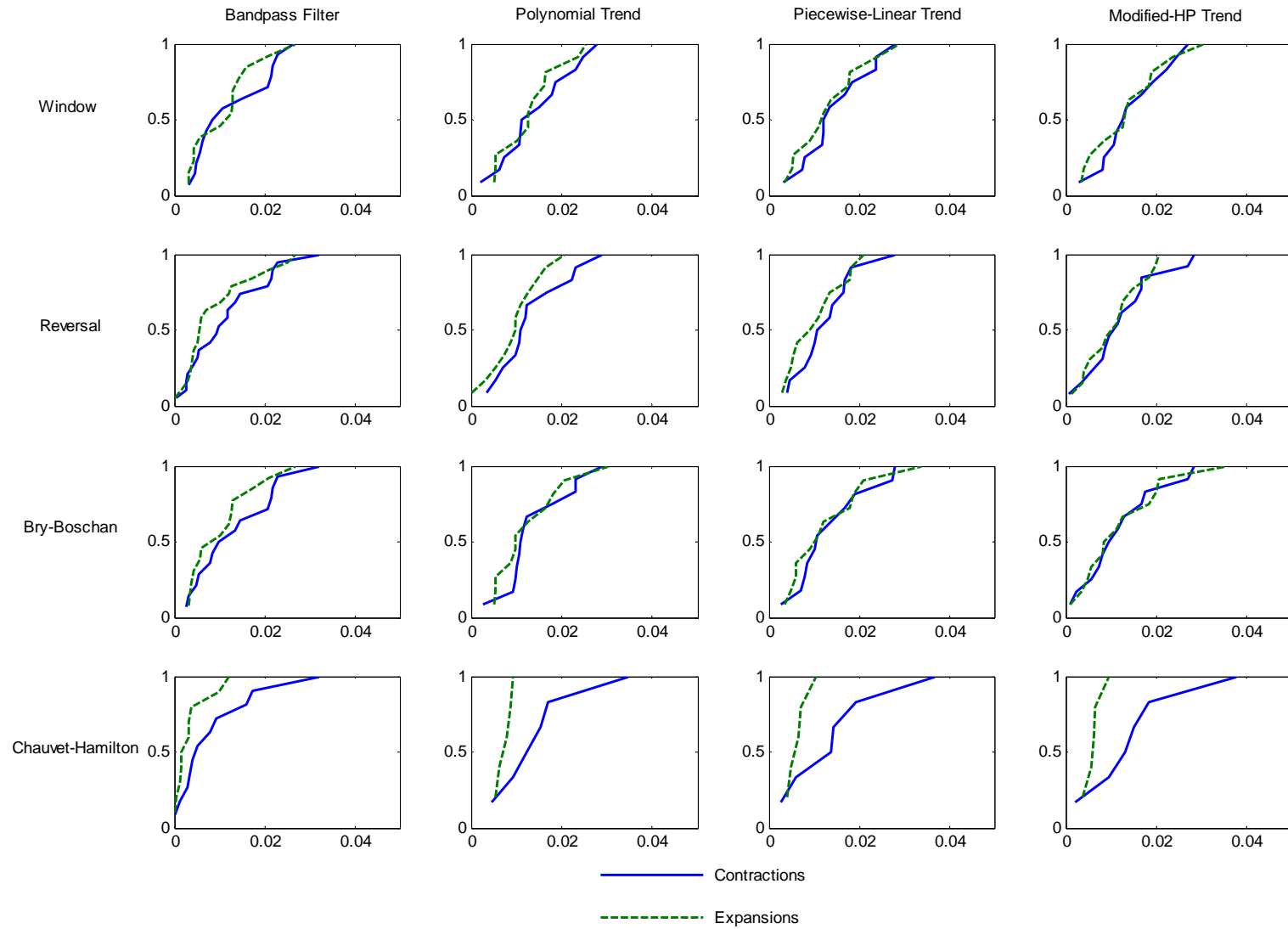


Figure 6b: CDF's for the sharpness of expansions and contractions in industrial production

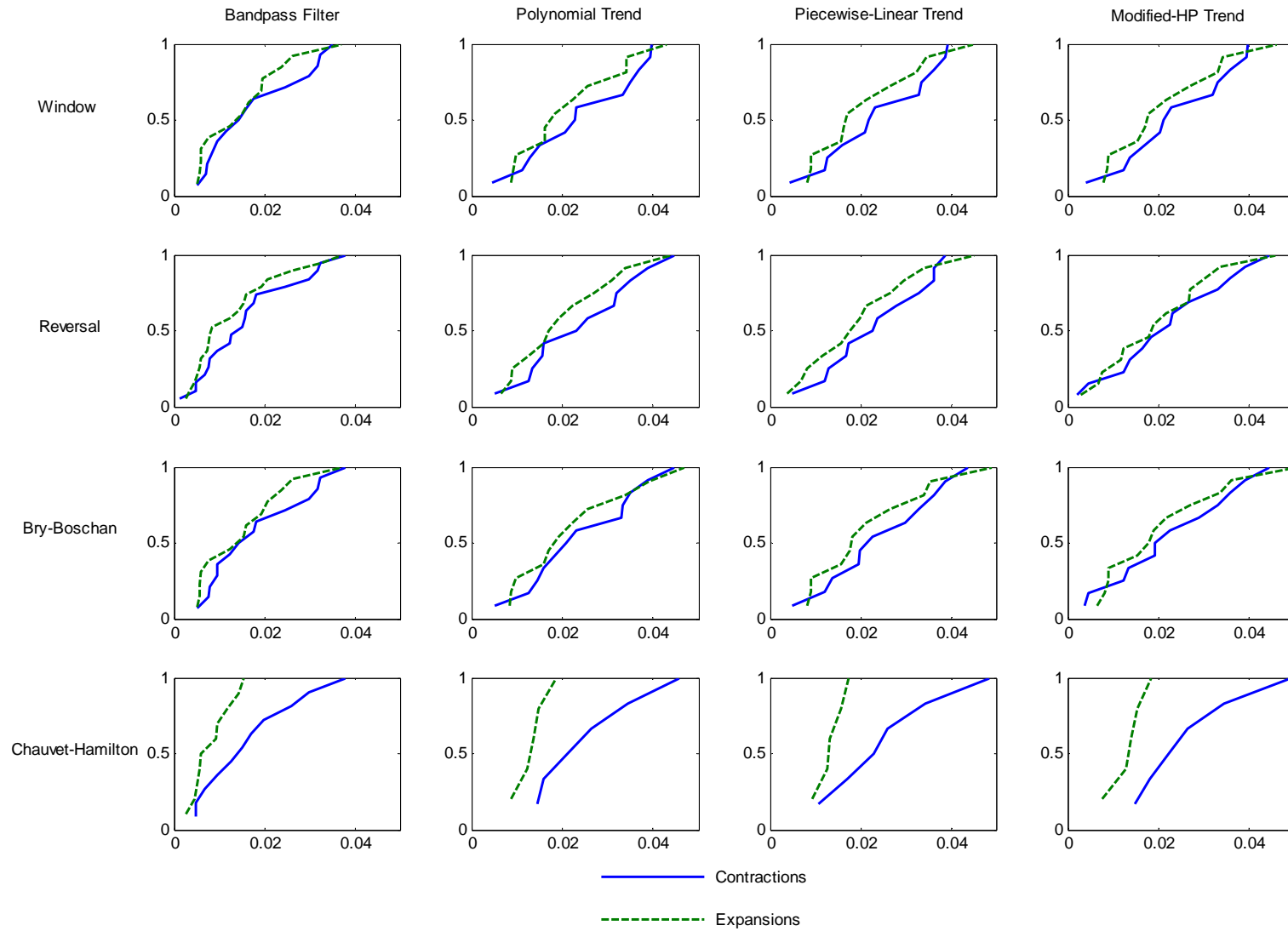


Figure 6c: CDF's for the slope of expansions and contractions in industrial production

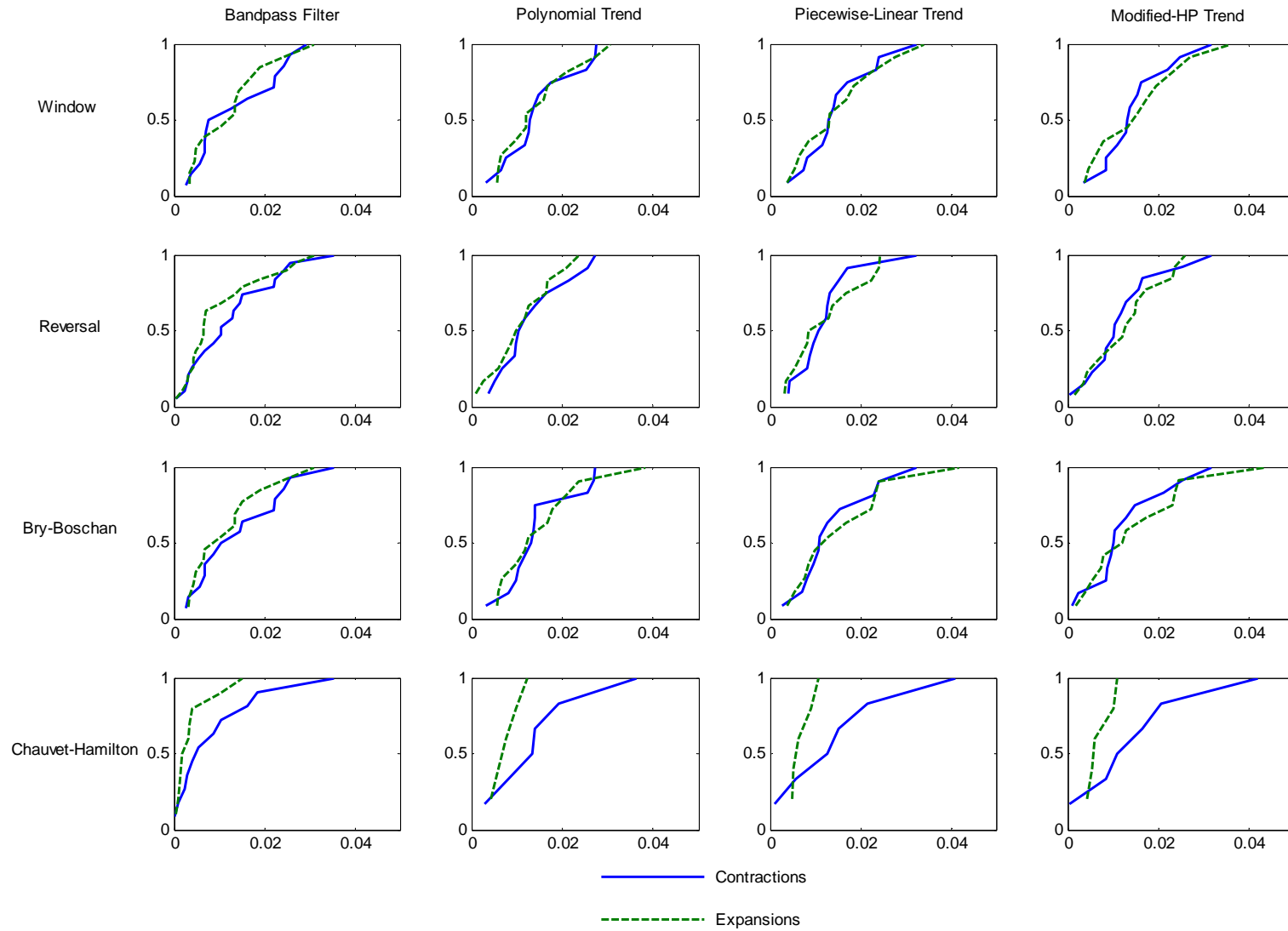


Figure 7a: CDF's for the steepness of expansions and contractions in the employment rate

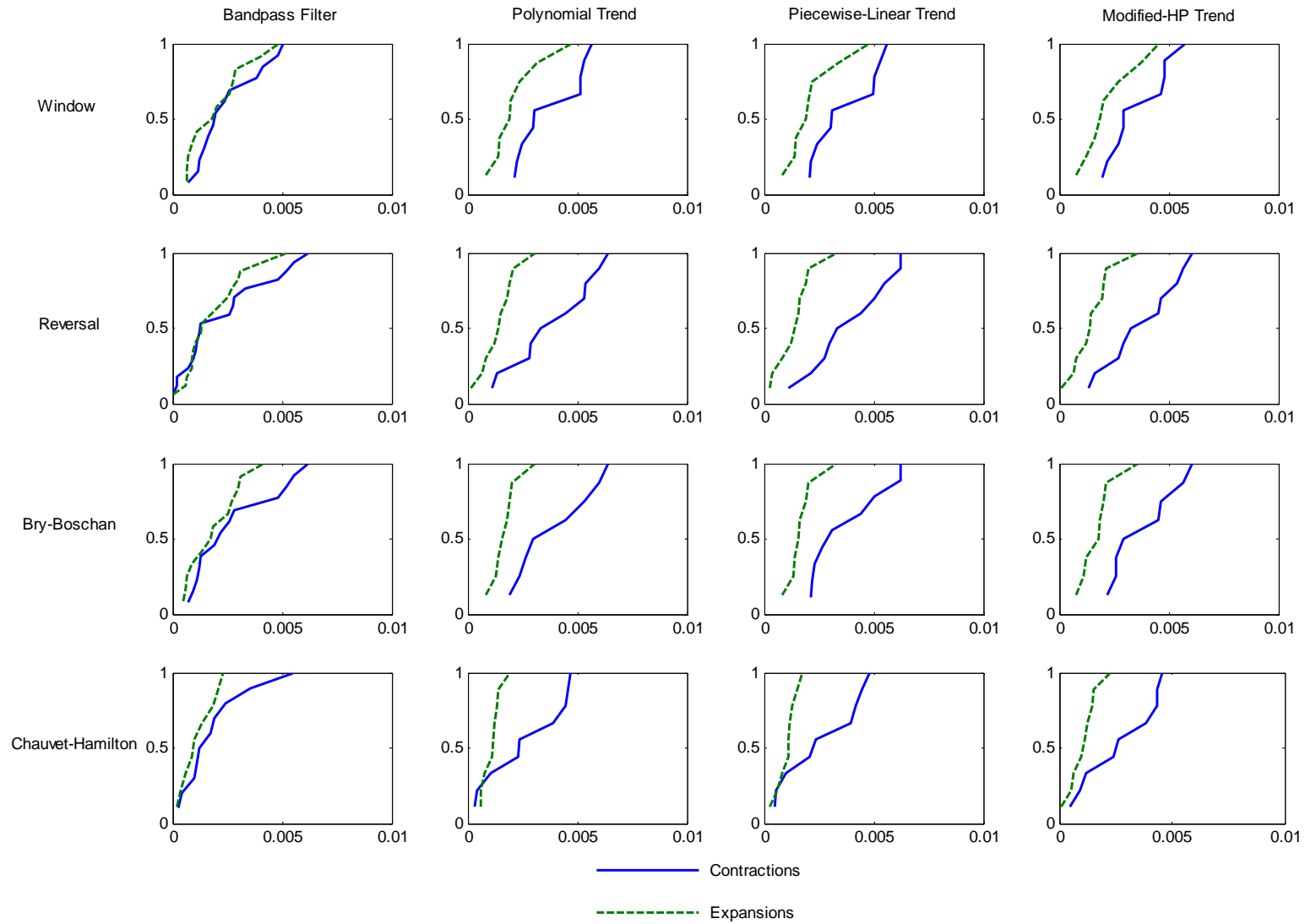


Figure 7b: CDF's for the sharpness of expansions and contractions in the employment rate

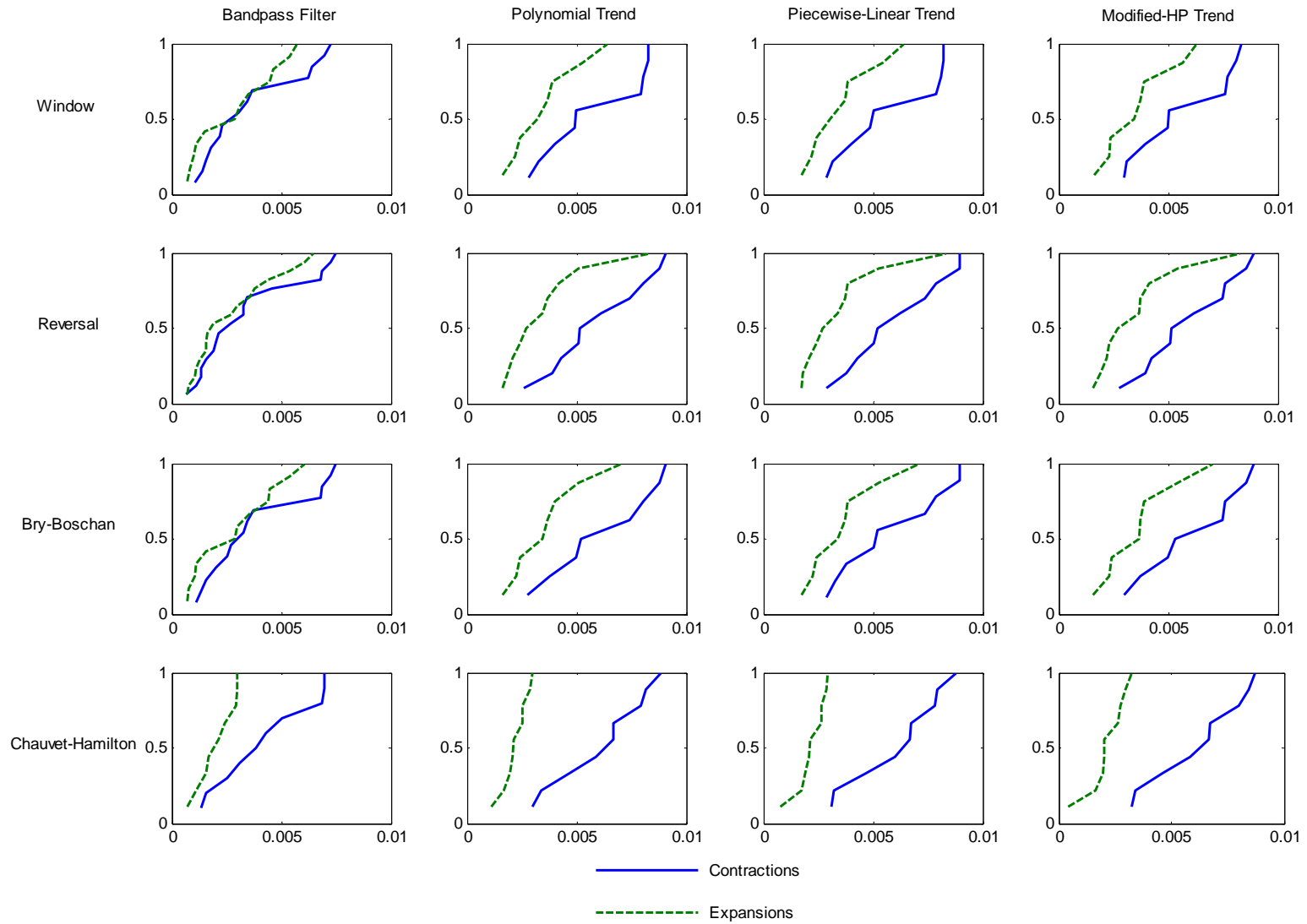




Figure 7c: CDF's for the slope of expansions and contractions in the employment rate

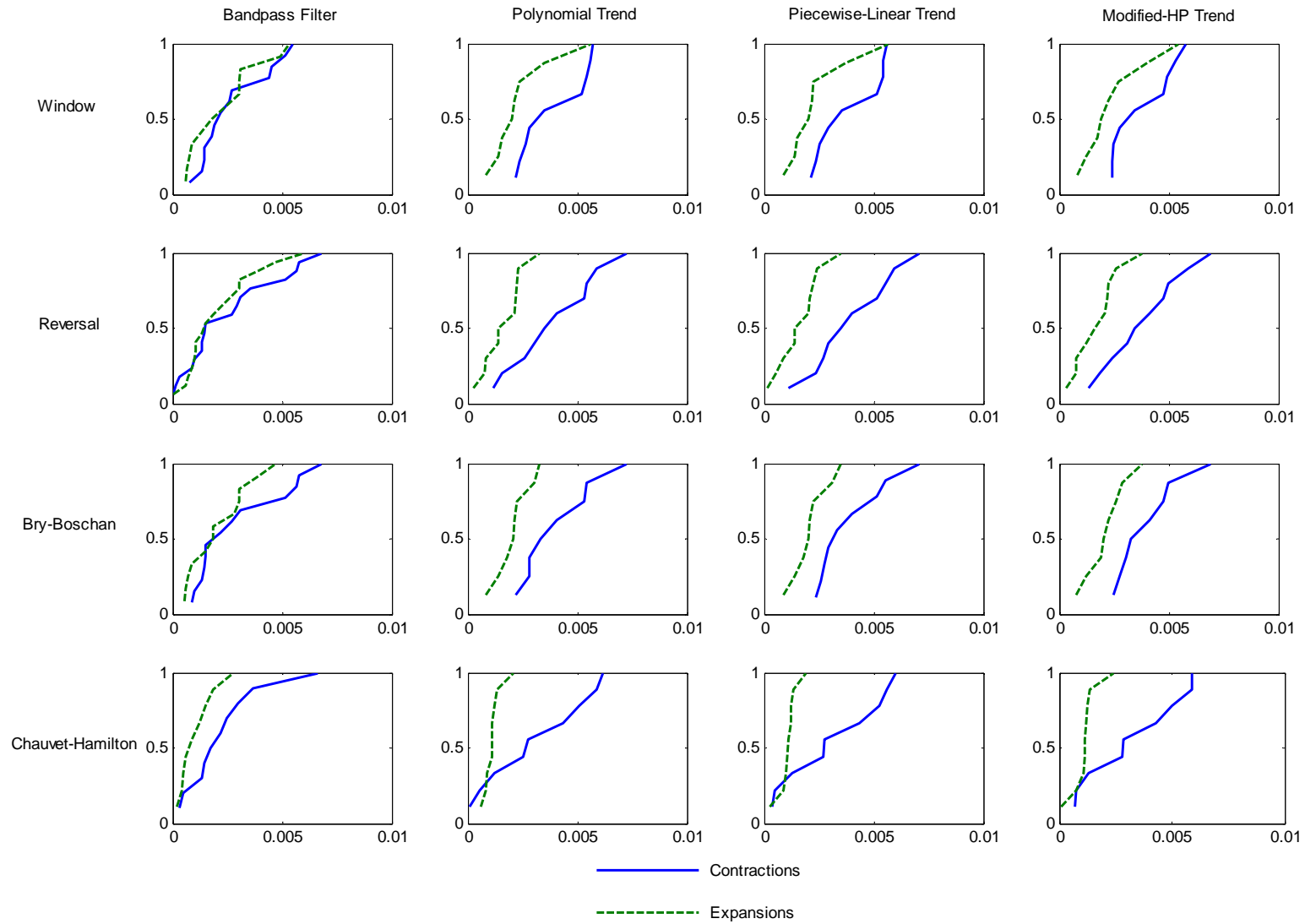


Figure 8: Output and employment dynamics with labor hoarding

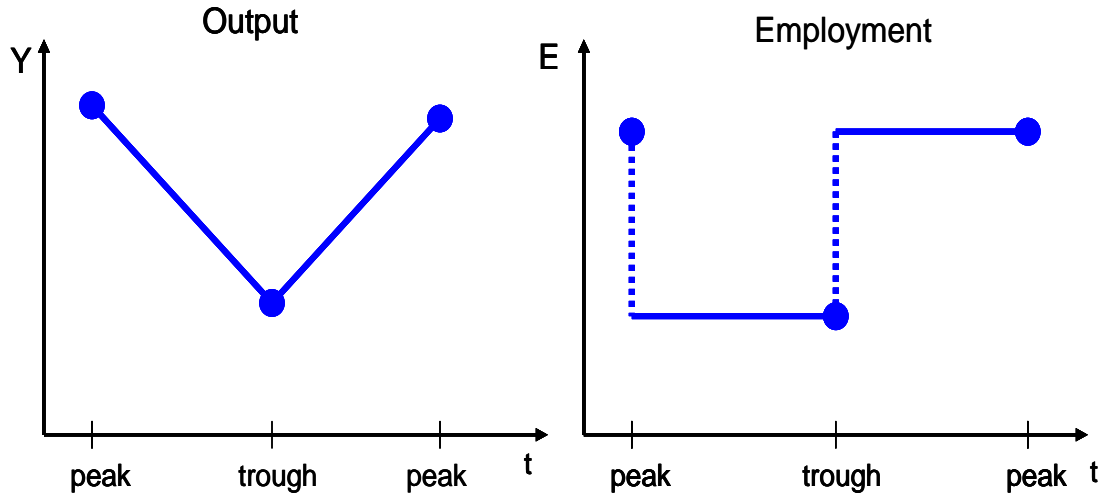


Figure 9: Output and employment dynamics with choice of timing of technology changes

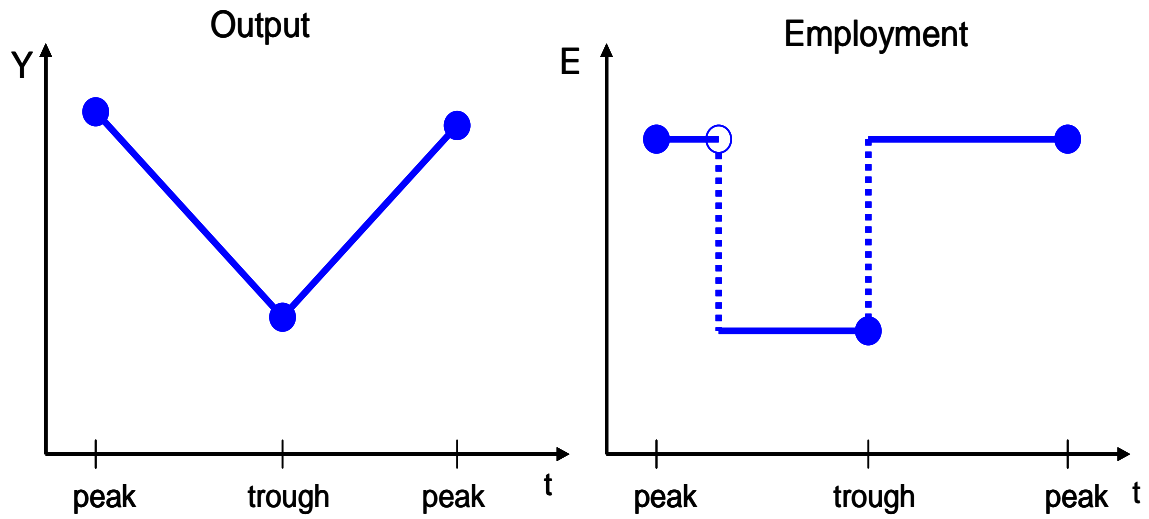


Figure 10: Output and employment dynamics with training or job search

