

# DISCUSSION PAPER SERIES

No. 5746

## A PORTFOLIO THEORY OF INTERNATIONAL CAPITAL FLOWS

Michael B Devereux and Makoto Saito

*INTERNATIONAL MACROECONOMICS*



**C**entre for **E**conomic **P**olicy **R**esearch

[www.cepr.org](http://www.cepr.org)

Available online at:

[www.cepr.org/pubs/dps/DP5746.asp](http://www.cepr.org/pubs/dps/DP5746.asp)

# A PORTFOLIO THEORY OF INTERNATIONAL CAPITAL FLOWS

**Michael B Devereux**, University of British Columbia and CEPR  
**Makoto Saito**, Hitotsubashi University

Discussion Paper No. 5746  
July 2006

Centre for Economic Policy Research  
90–98 Goswell Rd, London EC1V 7RR, UK  
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999  
Email: [cepr@cepr.org](mailto:cepr@cepr.org), Website: [www.cepr.org](http://www.cepr.org)

This Discussion Paper is issued under the auspices of the Centre's research programme in **INTERNATIONAL MACROECONOMICS**. Any opinions expressed here are those of the author(s) and not those of the Centre for Economic Policy Research. Research disseminated by CEPR may include views on policy, but the Centre itself takes no institutional policy positions.

The Centre for Economic Policy Research was established in 1983 as a private educational charity, to promote independent analysis and public discussion of open economies and the relations among them. It is pluralist and non-partisan, bringing economic research to bear on the analysis of medium- and long-run policy questions. Institutional (core) finance for the Centre has been provided through major grants from the Economic and Social Research Council, under which an ESRC Resource Centre operates within CEPR; the Esmée Fairbairn Charitable Trust; and the Bank of England. These organizations do not give prior review to the Centre's publications, nor do they necessarily endorse the views expressed therein.

These Discussion Papers often represent preliminary or incomplete work, circulated to encourage discussion and comment. Citation and use of such a paper should take account of its provisional character.

Copyright: Michael B Devereux and Makoto Saito

## ABSTRACT

### A Portfolio Theory of International Capital Flows\*

Recent global imbalances and large gross external financial movements have raised interest in modeling the relationship between international financial market structure and capital flows. This paper constructs a model in which the composition of national portfolios is an essential element in facilitating international capital flows. Each country chooses an optimal portfolio in face of real and nominal risk. Current account deficits are financed by net capital flows which reflect differential movements in the holdings of gross external assets and liabilities. A country experiencing a current account deficit will be accumulating both gross external liabilities and gross external assets. Net capital flows generate movements in risk premiums such that the rate of return on a debtor country's gross liabilities is lower than the return on its gross assets. This ensures stability of the world wealth distribution. An attractive feature of the model is that portfolio shares, returns, and the wealth distribution can be characterized analytically. A calibrated version of the model can match quite well the observed measures of gross and net external assets and liabilities for the US economy.

JEL Classification: F32, F34 and F41

Keywords: current account, international capital flows and portfolio

Michael B Devereux  
Department of Economics  
University of British Columbia  
997-1873 East Mall  
Vancouver BC, V6T 1Z1  
CANADA  
Tel: (1 604) 822 2542  
Email: [devm@interchange.ubc.ca](mailto:devm@interchange.ubc.ca)

Makoto Saito  
Faculty of Economics  
Hitotsubashi University  
2-1 Naka Kunitachi  
Tokyo 186-8601  
Japan  
Email: [makoto@econ.hit-u.ac.jp](mailto:makoto@econ.hit-u.ac.jp)

For further Discussion Papers by this author see:  
[www.cepr.org/pubs/new-dps/dplist.asp?authorid=136892](http://www.cepr.org/pubs/new-dps/dplist.asp?authorid=136892)

For further Discussion Papers by this author see:  
[www.cepr.org/pubs/new-dps/dplist.asp?authorid=164954](http://www.cepr.org/pubs/new-dps/dplist.asp?authorid=164954)

\* We would like to thank Toni Braun, Fumio Hayashi, Philip Lane, Anna Pavlova, Helene Rey and Cedric Tille for helpful comments. In addition, we thank seminar participants at UBC, Hitotsubashi University, Osaka University, the University of Tokyo, the Bank of Japan, the University of Dublin, the Bank of Canada, the Federal Reserve Bank of Dallas, Gerzensee, Aix-Marseille II, and Princeton. We are also grateful to Shiba Suzuki for research assistance. Devereux thanks the Bank of Canada, the Royal Bank of Canada, and SSHRC for financial support. Saito acknowledges the hospitality of the department of economics at UBC in which the current work was initiated. In addition, he is grateful for a grant-in-aid from the Ministry of Education and Science, Japan

Submitted 14 June 2006

# 1 Introduction

Recent global imbalances have revived interest in issues of current account dynamics and sustainability<sup>1</sup>. One of the key messages from this debate is that the evaluation of current account imbalances depends critically on the structure of international financial markets. In a series of influential papers, Lane and Milesi-Ferretti (2001, 2004, 2006) construct measures of gross assets and liabilities for a large sample of countries, and emphasize the rapid increase in the size of gross positions for many countries in recent years. Gourinchas and Rey (2004) and Tille (2004) note that the U.S. net international portfolio comprises substantial gross liabilities held in U.S. dollar denominations, but also substantial gross assets in non-U.S. currencies. As shown by Tille (2004), this significantly alters the link between the exchange rate and the current account. Much of the real adjustment to a large current account deficit could take place automatically through re-valuation effects on the U.S. net international portfolio.

Greenspan (2004) suggests that this process of ‘Financial Globalization’ may alter the interpretation of sustainability in the current account. Current account imbalances that are unsustainable in an environment with limited capital markets and small gross external positions may be consistent with long run sustainability if a country has large offsetting gross portfolios. But this perspective on the current account does not fit easily into the standard ‘intertemporal approach’ (see Obstfeld and Rogoff 1995), which typically abstracts away from issues of portfolio composition. Obstfeld (2004) and Lane and Milesi-Ferretti (2004) argue for the need to extend the standard open economy macro models to incorporate portfolio effects and gross external holdings.

This paper develops a simple model of international capital flows that are sustained by trade in nominal bonds with a time-varying mix of gross currency-denominated external assets and liabilities. Portfolio composition is a critical ingredient for international capital flows in the model. In the baseline version of the model if we allowed only for net capital flows, facilitated through trade in a real risk-free bond (as in the standard intertemporal model of the current account), no international flows of capital could be sustained at all. In this sense, the model highlights the message that an evaluation of current account sustainability must be tied intimately to the structure of financial markets. In a calibration to US data, we find that the model can match quite closely the size of gross and net external assets and liabilities of the US economy in recent years. In addition, an extended version of the model is consistent with the well known observation of *home bias* in equity holdings.

---

<sup>1</sup>See, for instance, Gourinchas and Rey (2004), Obstfeld (2004), Obstfeld and Rogoff (2004), Tille (2004), and Lane and Milesi Ferretti (2004).

The paper is built around a stochastic, continuous-time model in which countries trade in nominal bonds. In the baseline version of our model, all international asset trade is mediated through the use of these bonds. We derive a unique optimal portfolio structure for each country. The form of national portfolios depends critically on properties of nominal bond returns. When the price level is counter-cyclical (as we find in the data), countries hold short positions in their own currency bonds, and long positions in foreign currency bonds. Moreover, the structure of portfolios is an essential component of current account adjustment. Capital flows will take the form of ‘cross-hauling’, whereby a country in current account deficit will borrow by issuing liabilities in its own currency bonds, but simultaneously hedging this by accumulating assets in foreign currencies.

In the model, current account imbalances, sustained by time varying gross portfolios, allow for an exploitation of gains from intertemporal trade. Current accounts are sustainable in the following sense. When countries are holding an optimal bond portfolio, the share of world wealth held by any country follows a symmetric, stationary distribution. Current account imbalances are naturally self-correcting. The mechanism through which imbalances work themselves out in the model is through time-varying risk premiums on nominal bonds. An integral aspect of the stationarity of world wealth is that the return on debtor countries gross external liabilities tends to be lower than the return on their gross external assets. This process of stationarity could not operate in an environment with only net capital flows in a real risk-free asset.

A highly appealing aspect of the model is its tractability. We can obtain an exact analytical characterization of the portfolio composition, asset returns, and the distribution of country wealth shares. While in the baseline model, intertemporal gains from trade are solely tied to nominal bonds trading, we conduct a number of extensions. We enhance the asset menu to allow for simultaneous trade in nominal bonds and a real bond, as well as trade in equity. An interesting finding is that trade in nominal bonds may also support trade in a real risk-free bond, even though such an asset would not be traded in a symmetric environment in the absence of trade in nominal bonds. This is because nominal bonds trade renders the distribution of wealth stationary, thereby opening up gains to borrowing and lending among countries in a risk-free bond. The model also implies that trade in nominal bonds may substitute for trade in equity, leading countries to have a ‘home bias’ in equity holdings, even without exogenous restrictions on holding foreign equities<sup>2</sup>. We can also restrict the asset menu, to rule out trade in the nominal bonds of one currency (reflecting for instance the dominance of the US dollar in international capital markets). Finally, as

---

<sup>2</sup>The ‘home bias’ puzzle has been addressed by many authors. See Baxter and Jermann (1998), Heathcote and Perri (2004), Engel and Matsumoto (2006), Kollman (2006), and Hnatkowska (2005), among many others.

mentioned above, the model can be quantified. In a calibration we find that the model can account quite well for the gross and net external position of the US economy in recent years.

The paper is part of a growing literature on portfolio dynamics in open economy macroeconomics. One simplifying approach is to use a complete markets assumption. Engel and Matsumoto (2006) and Kollman (2006) develop models of equity and bond trade in two country models with complete markets. With complete markets however, the issue of current account sustainability and stationarity of the wealth distribution does not arise. Evans and Hnatkovska (2005), and Hnatkovska (2006) use a numerical method to solve an incomplete markets model. Their solution method is based on local approximations, and so the issue of stationarity is not explored. Pavlova and Rigobon (2003) construct a continuous-time stochastic model of exchange rates, and focus on aspects of asset pricing and the international transmission of stock prices.

There are limitations of our analysis. To focus exclusively on a portfolio approach to the current account, we have only a single world commodity, and full purchasing power parity (PPP). This means that the real exchange rate plays no role at all in current account adjustment. In this sense, our analysis is strictly a marriage of the intertemporal approach to the current account (Obstfeld and Rogoff 1996) with a Merton (1971) type consumption-portfolio model. In addition we restrict preferences to have an intertemporal elasticity of substitution of unity. This is the only tractable approach to portfolio dynamics in an economy with time-varying rates of return (see for instance, Devereux and Saito 1997).

Section 2 develops the basic model. Section 3 explores the equilibrium portfolio holdings in the model, and analyses the interaction among portfolio structure, capital flows, and the world wealth distribution. Section 4 sets out a number of extensions to the asset structure, and conducts an empirical calibration of the model. Section 5 concludes.

## 2 The Model

We take a one-good two-country model of a world economy<sup>3</sup>. In each country there is a risky linear technology which uses capital and generates expected instantaneous return  $\alpha_i$  with standard deviation  $\sigma_i$ , where  $i = h$  or  $f$ , signifying the ‘home’ or ‘foreign’ country. Capital can be turned into consumption without any cost. The return on technology  $i$  (in terms of

---

<sup>3</sup>It is possible to extend the model to many goods, and incorporate real exchange rate dynamics. However, the one-good framework is adequate for an analysis of the central issue we are concerned with here; that is, the interaction of portfolio choice and the current account.

the homogeneous good) is given by:

$$\frac{dQ_i}{Q_i} = \alpha_i dt + \sigma_i dB_i,$$

for  $i = h$  or  $f$ , where  $dB_i$  is the increment to a standard Weiner process. For simplicity, we assume that the returns on the two technologies are independent, so that

$$\lim_{\Delta t \rightarrow 0} \frac{Cov_t(\Delta B_h(t + \Delta t), \Delta B_f(t + \Delta t))}{\Delta t} = 0.$$

For the moment, assume that residents of one country cannot directly purchase shares in the technology of the other country (this is relaxed in a later section). Nominal bonds can be traded between the countries, however. Bonds may be denominated in home or foreign currency. Although the bonds are risk-free in nominal terms, their real returns are subject to inflation risk. Let inflation in country  $i$  be represented as<sup>4</sup>:

$$\frac{dP_i}{P_i} = \Pi_i dt + v_i dM_i.$$

Thus, inflation has mean  $\Pi_i$  and standard deviation  $v_i$ ,  $i = h$  and  $f$ .  $dM_i$  represents the increment to a standard Weiner process. The monetary policy followed by country  $i$  is represented by the parameters  $\Pi_i$  and  $v_i$ , and the covariance of  $dM_i$  with  $dB_i$ . We let

$$\lim_{\Delta t \rightarrow 0} \frac{Cov_t(\Delta M_i(t + \Delta t), \Delta B_i(t + \Delta t))}{\Delta t} = \lambda_i, \quad (1)$$

and

$$\lim_{\Delta t \rightarrow 0} \frac{Cov_t(\Delta M_i(t + \Delta t), \Delta M_j(t + \Delta t))}{\Delta t} = 0. \quad (2)$$

for  $i \neq j$ . Equation (2) here says that inflation shocks are independent across countries. This is not critical, but simplifies the algebra.

The covariance term  $\lambda_i$  in equation (1) is a critical parameter. It describes the cyclical characteristics of the price level, and hence of the real return on nominal bonds. Although we allow for any value of  $\lambda_i$ , such that  $-1 < \lambda_i < 1$ , most of the discussion will focus on the case where  $\lambda_i < 0$ , so that the price level is countercyclical.

What is the evidence on  $\lambda$ ? Using Hodrick-Prescott (HP) filtered outputs and CPI's based on U.S. quarterly data, Kydland and Prescott (1990) report a coefficient of  $-0.57$  for the period between 1954 and 1989, and Cooley and Hansen (1995) report  $-0.52$  for the period

---

<sup>4</sup>We do not explicitly model a source of demand for money. As in Woodford (2003), we can think of the model as representing a 'cashless economy'. What matters is that there is an asset whose payoff depends on the price level, and monetary policy can generate a particular distribution for the price level.

Table 1: Evidence on correlation coefficients between GDP and CPI ( $\lambda$ )

	sample period	HP-filtered	log-differenced
US	1980-2004	-0.417	-0.214
	1955-2000	-0.565	
Japan	1980-2004	-0.041	-0.085
	1955-2000	-0.132	
Canada	1981-2004	-0.706	-0.392
France	1980-2004	-0.050	-0.085
UK	1980-2004	-0.573	-0.350

between 1954 and 1991. Table 1 presents our own estimates using more recent data for a wider sample of countries. We note that the estimates are substantially lower when we use first difference in logarithm (which is more appropriate for our model) rather than applying the HP filter, but the estimates are all negative nonetheless.

Let the instantaneous nominal return on currency  $i$  bonds be  $\widehat{R}_i$ . Then the real return on bond  $i$  is

$$(R_i - \Pi_i)dt - v_i dM_i,$$

where  $R_i = \widehat{R}_i + v_i^2$  is an adjusted nominal interest rate<sup>5</sup>. This will be determined endogenously as part of the world bond market equilibrium<sup>6</sup>.

The budget constraint for the home country may then be written as:

$$dW_h = W_h [\omega_T^h(\alpha_h - r_h) + \omega_h^h(R_h - \Pi_h - r_h) + \omega_f^h(R_f - \Pi_f - r_h) + r_h] dt \quad (3) \\ - C_h dt + W_h (\omega_T^h \sigma_h dB_h - \omega_h^h v_h dM_h - \omega_f^h v_f dM_f),$$

where  $\omega_T^h$ ,  $\omega_h^h$ , and  $\omega_f^h$  are the portfolio shares, respectively, of the domestic technology, home currency nominal bonds, and foreign currency nominal bonds. It facilitates the presentation of the model to allow for a non-traded domestic real risk-free bond, with return  $r_h$ . The equilibrium value of  $r_h$  can be used as a measure the risk-free real interest rate in the home economy<sup>7</sup>. Since all home agents are alike, this bond is in zero supply in equilibrium.  $C_h$

<sup>5</sup>The adjustment factor comes from a Jensen's inequality term in evaluating the real return on nominal bonds.

<sup>6</sup>In this nominal bond equilibrium, long-term bonds are redundant assets and derivatives of instantaneous nominal bonds. Therefore, given the equilibrium path of instantaneous nominal interest rates, determined by equations (9) and (10) together with equation (16), longer-term nominal interest rates are derived completely by arbitrage pricing.

<sup>7</sup>Devereux and Saito (1997) investigate a case where instantaneous real bonds are traded between two countries in the absence of cross-border equity trading. A later section of the paper extends the model to

denotes consumption of the representative home household<sup>8</sup>.

Assume that each country is populated by a continuum of identical agents, that preferences are identical across countries, and given by logarithmic utility,

$$E_0 \int_0^\infty \exp(-\rho t) \ln C_i(t) dt,$$

where  $\rho$  is the rate of time preference.

## 2.1 Optimal Consumption and Portfolio Rules

This section derives optimal consumption and portfolio rules. At present, we simplify the analysis by assuming identical drift and diffusion parameters, so that,  $\alpha_h = \alpha_f = \alpha$ ,  $\sigma_h = \sigma_f = \sigma$ ,  $\Pi_h = \Pi_f = \Pi$ ,  $\nu_h = \nu_f = \nu$ , and  $\lambda_h = \lambda_f = \lambda$ . Section 4 will explore cases with differences in parameters across countries.

With logarithmic utility, home country expected utility maximizing agents follow the myopic consumption rule:

$$C = \rho W.$$

The optimal portfolio rules may be obtained as the solution to:

$$\begin{bmatrix} \omega_T^h \\ \omega_h^h \\ \omega_f^h \end{bmatrix} = \begin{bmatrix} \sigma^2 & -\lambda\sigma\nu & 0 \\ -\lambda\sigma\nu & \nu^2 & 0 \\ 0 & 0 & \nu^2 \end{bmatrix}^{-1} \begin{bmatrix} \alpha - r_h \\ R_h - \Pi - r_h \\ R_f - \Pi - r_h \end{bmatrix} \quad (4)$$

A similar set of conditions hold for the foreign country. In some cases, we make the following parameter assumptions governing the behavior of optimal portfolios:  $\sigma^2 + \lambda\nu\sigma > 0$  and  $\nu^2 + \lambda\nu\sigma > 0$ . These conditions ensure that the behavior of portfolio demands satisfy regularity properties. In particular, the first condition ensures that a rise in the risk-free rate reduces the demand for home currency nominal bonds, while the second ensures that a rise in the risk-free rate reduces the demand for shares in the domestic technology. These

---

allow for trade in a real bond.

<sup>8</sup>Note that despite the fact that all capital flows are facilitated with nominal bonds, the exchange rate plays no independent role in our analysis. Since we have the single-good world and PPP holds, then the rate of the change in the exchange rate  $S (= \frac{P_h}{P_f})$  is just determined residually by:

$$d \ln S = \left( \Pi_h - \frac{1}{2}v_h^2 - \Pi_f + \frac{1}{2}v_f^2 \right) dt + v_h dM_h - v_f dM_f.$$

The model could be extended to a multi-good environment with the real exchange rate playing a critical role. At present, however, we focus on the single-good case so as to emphasize the role of portfolio structure in current account adjustment.

conditions are not necessary for the key stability results developed below, but they make the exposition substantially easier.

## 2.2 Autarky versus Complete Markets

To provide a reference point, we describe the outcome in the two polar cases of autarky (no asset trade of any kind), and complete markets (implemented by full trade in shares of each country's technology).

Under autarky, the risk-free rate in each country is given by  $r = \alpha - \sigma^2$ . Equilibrium nominal bond holdings (in either currency) are zero, so that  $\omega_T^i = 1$ . The equilibrium nominal interest rate on home currency bonds is  $R_h = \Pi + \alpha - \sigma^2 - \lambda v \sigma$ . This includes a risk premium term  $\lambda v \sigma$ . When  $\lambda < 0$ , the home nominal bond is a bad hedge against technology risk, and must have a return higher than the risk-free rate, adjusted for inflation. The zero-trade equilibrium interest rate on foreign currency bonds is  $R_f = \Pi + \alpha - \sigma^2$ . Since the foreign price level is independent of home outputs, it is a better hedge against consumption risk when  $\lambda < 0$ , and therefore carries a lower autarky return than the home currency bond.

From a welfare perspective, with preferences given by (2), the relevant measure of expected consumption (or wealth) growth in any equilibrium is the *risk-adjusted growth rate*, given by:

$$\lim_{\Delta t \rightarrow 0} E_t \left[ \frac{\Delta \ln C_i(t + \Delta t)}{\Delta t} \right] = \lim_{\Delta t \rightarrow 0} \frac{E_t \left( \frac{\Delta C_i(t + \Delta t)}{C_i(t)} \right) - \frac{1}{2} \text{Var}_t \left( \frac{\Delta C_i(t + \Delta t)}{C_i(t)} \right)}{\Delta t}.$$

In autarky, the risk-adjusted growth rate is given by  $\alpha - \rho - \frac{1}{2}\sigma^2$ .

With unrestricted trade in shares of technology, there are effective complete markets. Trade in nominal assets is then redundant. The equilibrium share of each technology will be  $\omega_T^i = \frac{1}{2}$ , and the risk-free rate will be  $r = \alpha - \frac{1}{2}\sigma^2$ . Risk-pooling under complete markets implies a higher risk-free interest rate than in autarky. Finally, the risk-adjusted growth rate with complete markets is  $\alpha - \rho - \frac{1}{4}\sigma^2$ . Complete markets improve welfare by raising the risk-adjusted consumption growth rate.

## 2.3 Nominal Bond Trading Equilibrium

Since autarky nominal returns on home and foreign currency bonds differ, there are gains from trade in nominal bonds. At any moment in time, an equilibrium in the market for home and foreign currency bonds determines the nominal rates of return  $R_h$  and  $R_f$ . Nominal bond

market clearing conditions are given as:

$$\omega_h^h W_h + \omega_h^f W_f = 0, \quad (5)$$

$$\omega_f^h W_h + \omega_f^f W_f = 0. \quad (6)$$

In addition, since equilibrium holdings of the non-traded risk-free bond must be zero in each country, the portfolio shares of the domestic technology plus the two nominal bonds must add to one, in each country:

$$\omega_T^h + \omega_h^h + \omega_f^h = 1, \quad (7)$$

$$\omega_T^f + \omega_h^f + \omega_f^f = 1. \quad (8)$$

These four conditions may be solved for  $R_h$ ,  $R_f$ ,  $r_h$ , and  $r_f$ . Define  $\theta = \frac{W_f}{W_h + W_f}$  as the ratio of foreign wealth to world wealth. From equations (5) through (8), we may write the solution for nominal interest rates and domestic risk-free rates as  $R_h(\theta)$ ,  $R_f(\theta)$ ,  $r_h(\theta)$ , and  $r_f(\theta)$ . The solutions are quite tedious to derive, but can be written as:

$$R_h(\theta) = \left[ \alpha + \Pi - \sigma^2 + \frac{1}{2} (\lambda^2 \sigma^2 - \lambda \sigma v) \right] + \Phi_h(\theta), \quad (9)$$

$$R_f(\theta) = \left[ \alpha + \Pi - \sigma^2 + \frac{1}{2} (\lambda^2 \sigma^2 - \lambda \sigma v) \right] + \Phi_f(\theta), \quad (10)$$

$$r_h(\theta) = \left[ \alpha - \sigma^2 + \frac{1}{2} \lambda^2 \sigma^2 \right] + \Gamma_h(\theta), \quad (11)$$

and

$$r_f(\theta) = \left[ \alpha - \sigma^2 + \frac{1}{2} \lambda^2 \sigma^2 \right] + \Gamma_f(\theta), \quad (12)$$

where we define the expressions  $\Phi_i$  and  $\Gamma_i$  as:

$$\Phi_h(\theta) = \frac{\lambda \sigma (2\theta - 1) (v + \lambda \sigma (1 - 2\theta)) (v + 2\lambda v \sigma - \sigma^2 (\lambda^2 - 2))}{2 (v^2 + 2\lambda v \sigma + \sigma^2 (2 - (1 - 2\theta)^2 \lambda^2))},$$

$$\Phi_f(\theta) = \frac{v - \lambda \sigma (1 - 2\theta)}{v + \lambda \sigma (1 - 2\theta)} \Phi_h(\theta),$$

$$\Gamma_h(\theta) = \frac{\lambda^2 \sigma^2 (2\theta - 1) (v^2 + 2\lambda v \sigma \theta + \sigma^2 (2\theta - 1) (\lambda^2 - 2))}{2 (v^2 + 2\lambda v \sigma + \sigma^2 (2 - (1 - 2\theta)^2 \lambda^2))},$$

and

$$\Gamma_f(\theta) = \frac{\lambda^2 \sigma^2 (2\theta - 1) (v^2 + 2\lambda v \sigma (1 - \theta) + \sigma^2 (2\theta - 1) (\lambda^2 - 2))}{2 (v^2 + 2\lambda v \sigma + \sigma^2 (2 - (1 - 2\theta)^2 \lambda^2))}.$$

Using these solutions with the optimal rules from the portfolio problem allows us to write the equilibrium home country portfolio shares as:

$$\omega_h^h = \frac{\theta \lambda \sigma (v^2 + 2\sigma^2 + 2\lambda \sigma v + \lambda \sigma v (1 - 2\theta))}{v (v^2 + 2\sigma^2 + 2\lambda \sigma v - \sigma^2 \lambda^2 (1 - 4\theta(1 - \theta)))}, \quad (13)$$

and

$$\omega_f^h = -\frac{\theta \lambda \sigma (v^2 + 2\sigma^2 + 2\lambda \sigma v - \lambda \sigma v (1 - 2\theta))}{v (v^2 + 2\sigma^2 + 2\lambda \sigma v - \sigma^2 \lambda^2 (1 - 4\theta(1 - \theta)))}. \quad (14)$$

The model has an appealing recursive structure. Given the myopic consumption rule, portfolio equilibrium has the property that returns depend only on the current world distribution of wealth, captured by the term  $\theta$ . The dynamics of  $\theta$  itself may be constructed from the wealth dynamics (3) and the equivalent process for the foreign country, which in turn depend on the portfolio rules (13) and (14), as well as the market returns (9)-(12). Before examining the properties of the distribution of  $\theta$  however, it is helpful to explain the effects of  $\theta$  on returns and portfolio shares. We begin with a special case of equal wealth levels in each country.

### 3 Portfolios, Returns, and Capital Flows

#### 3.1 A Special Case: $\theta = 0.5$

Take first the case where  $\theta = 0.5$ , so that wealth levels are equal in the home and foreign countries. From inspection of the above solutions for interest rates, we see that  $\Phi_i(0.5) = \Gamma_i(0.5) = 0$ . Hence,

$$R_i = \alpha + \Pi - \sigma^2 + \frac{1}{2} (\sigma^2 \lambda^2 - \lambda \sigma v),$$

and

$$r_i = \alpha - \sigma^2 + \frac{1}{2} \sigma^2 \lambda^2.$$

The real risk-free interest rate falls between that of autarky and complete markets. Since  $-1 < \lambda < 1$ , the real risk-free rate is higher than under autarky, but lower than under complete markets. If  $\lambda = 0$ , the nominal bond would play no role at all as a hedge against technology (and therefore consumption) risk, and the presence of nominal bonds leaves the equilibrium real risk-free rate unchanged at the autarky rate. But as  $|\lambda|$  moves closer to one, bonds can act as a real hedge against technology risk, and the fall in consumption risk

raises the real risk-free rate in each country. In the limit, as  $|\lambda|$  goes to one, we approach the risk-free rate under complete markets.

The movement of the nominal interest rate, relative to autarky, depends on the sign of  $\lambda$ . When  $\lambda < 0$ , the home currency bond is a poor hedge against consumption risk for the home country, and has a high autarky return. By the same token, however, it is a relatively good hedge against foreign consumption risk, and has a low autarky return in the foreign economy. With nominal bond trade, the home country will sell home currency bonds to foreign residents, and the new equilibrium nominal return will be in between the home and foreign country autarky returns. The equivalent mechanism works for the foreign currency bond.

When  $\theta = 0.5$ , the risk-adjusted growth rate may be written as:

$$\alpha - \rho - \frac{1}{2}\sigma^2 \left(1 - \frac{\lambda^2}{2}\right).$$

When  $\lambda = 0$ , this is identical to that under autarky, while when  $|\lambda| = 1$ , the bond trading regime attains the risk-adjusted growth rate under complete markets.

Each country's portfolio of nominal bonds hedges its country-specific risk on the real technology. With  $\theta = 0.5$ , net foreign assets are zero in each country, so that  $\omega_h^h + \omega_f^h = \omega_h^f + \omega_f^f = 0$ . But in order to hedge technology risk, countries find it advantageous to hold different *gross* positions in each currency. The optimal portfolio in this case is:

$$\omega_h^h = \frac{1}{2} \frac{\lambda\sigma}{v}, \quad \omega_f^h = -\frac{1}{2} \frac{\lambda\sigma}{v}.$$

When  $\lambda < 0$ , the home country takes a short position in home currency bonds and a long position in foreign currency bonds. Since in this case, the return on home nominal bonds is pro-cyclical, a negative gross holding of home currency bonds acts as an effective hedge against consumption risk. Similarly, a positive gross holding of foreign currency bonds allows it to share in the foreign technology process. This portfolio structure exploits the different returns processes on home and foreign currency bonds to allow for the risk from technology shocks to be partly pooled across countries.

When  $\lambda > 0$ , the process works in reverse. Then, the home currency bond is a good hedge against home consumption risk, and home residents will hold positive quantities of home currency bonds, and negative amounts of foreign currency bonds.

### 3.2 Effects of Marginal Variation in $\theta$

The results can be easily extended to general values of  $\theta$ . From (13) and (14), it is clear that, given assumptions made so far and  $\lambda < 0$ ,  $\omega_h^h < 0$  and  $\omega_h^f > 0$  for all  $\theta$  such that  $0 < \theta < 1$ . Hence, for all levels of  $\theta$ , the home country issues its own currency bonds short, and holds a long position in foreign currency bonds. In addition, summing (13) and (14), we get

$$\omega_h^h + \omega_f^h = -\frac{2\theta(2\theta - 1)\lambda^2\sigma^2}{v^2 + 2\sigma^2 + 2\lambda\sigma v - \sigma^2\lambda^2 + 4\sigma^2\lambda^2\theta(1 - \theta)}. \quad (15)$$

These solutions also confirm that when  $\theta = 0.5$ , the net foreign asset position is zero. More generally however, we see that  $\omega_h^h + \omega_f^h < 0$  ( $> 0$ ) as  $\theta > 0.5$  ( $< 0.5$ ). Hence, while the home country is a gross creditor in foreign currency bonds, and a gross debtor in home country bonds, these positions only balance exactly when the two countries have equal wealth. As the foreign share of world wealth rises above 0.5, the home country becomes a net debtor.

Trade in nominal bonds allows for *net* flows of capital between countries. Net capital flows would not be possible if there were trade only in a real risk-free bond, since (as shown above) the autarky return on a risk-free bond is equal in the two countries with symmetric technologies.

How is it that nominal bond trade facilitates capital flows (or intertemporal trade) when real bonds do not? The key feature is the interaction between the *gross* and *net* asset positions held by countries. Differentiating equations (13) and (14) at  $\theta = 0.5$ , we see that a rise in  $\theta$  has the following effect on the home country's portfolio:

$$\left. \frac{d\omega_h^h}{d\theta} \right|_{\theta=0.5} = \frac{\lambda\sigma(2\sigma^2 + \lambda\sigma v + v^2)}{v(2\sigma^2 + 2\lambda\sigma v + v^2)},$$

and

$$\left. \frac{d\omega_f^h}{d\theta} \right|_{\theta=0.5} = -\frac{\lambda\sigma(2\sigma^2 + 3\lambda\sigma v + v^2)}{v(2\sigma^2 + 2\lambda\sigma v + v^2)}.$$

When  $\lambda < 0$ , the first expression is negative, and the second is positive. Hence, beginning at  $\theta = 0.5$ , a rise in foreign relative wealth will be followed by a rise in home gross borrowing in home currency bonds, matched however by a rise in gross lending in foreign currency bonds. But these effects do not cancel out. From (15) it is clear that  $\left. \frac{d\omega_h^h}{d\theta} + \frac{d\omega_f^h}{d\theta} \right|_{\theta=0.5} < 0$ , for  $\lambda < 0$ . The home country engages in net foreign borrowing as  $\theta$  rises. It uses this borrowing to invest in the home technology. Conversely, the foreign country becomes a net foreign creditor, and reduces the share of its wealth invested in its own production technology.

Thus, in the regions of a symmetric equilibrium, capital will flow to the less wealthy

country. But this capital flow will take place through a ‘cross-hauling’ effect. When  $\lambda < 0$ , the home country engages in net foreign borrowing by issuing more home currency bonds, but at the same time hedging the risk on these bonds by acquiring more foreign currency assets. In this sense, large gross portfolio positions are not redundant - rather they are an essential leverage mechanism in facilitating net capital flows between countries.

The net capital flow is also reflected in the behavior of interest rates. From the solutions for  $R_h$  and  $R_f$ , we find that:

$$\left. \frac{dR_h}{d\theta} \right|_{\theta=0.5} = \frac{\lambda\sigma v(v^2 + 2\lambda\sigma v + 2\sigma^2 - \sigma^2\lambda^2)}{2\sigma^2 + 2\lambda\sigma v + v^2},$$

and

$$\left. \frac{dR_f}{d\theta} \right|_{\theta=0.5} = -\frac{\lambda\sigma v(v^2 + 2\lambda\sigma v + 2\sigma^2 - \sigma^2\lambda^2)}{2\sigma^2 + 2\lambda\sigma v + v^2}.$$

The first expression is negative, while the second is positive, for  $\lambda < 0$ <sup>9</sup>. Thus, a rise in the share of world wealth for the foreign country drives down the return on home currency bonds, while pushing up the return on foreign currency bonds. Intuitively, as the foreign country increases its wealth, its portfolio preferences dominate the global bond markets. Since its levered portfolio requires it to be long in home currency bonds and short in foreign currency bonds, it increases its demand for home currency bonds, while increasing its supply of foreign currency bonds. This is reflected in the movements in the returns on the bonds. It may also be shown in a similar manner that a rise in  $\theta$ , beginning at  $\theta = 0.5$ , raises the home risk-free interest rate, and reduces the foreign risk-free rate.

The gross portfolio position, when combined with the evolution of returns that are driven by relative wealth dynamics, allows for gains from intertemporal trade in an economy with nominal bonds, even though there are no gains when only a real bond is traded in a symmetric case. To see the intuition, take the position  $\theta = 0.5$ , where the two countries have exactly equal net wealth, and given the symmetry in the model, the current account of each country is zero. Say that there is a rise in  $W_f$ , driven for instance by a positive technology shock in the foreign country. This will raise  $\theta$ . If there were trade only in a real risk-free bond, this would simply permanently increase the foreign country’s expected consumption, and have no impact at all on the home country. But with trade in nominal bonds, the rise in  $\theta$  increases the foreign country’s demand for home currency bonds (supply of foreign currency bonds), in the case  $\lambda < 0$ . The fall in  $R_h$  and rise in  $R_f$  reduces the effective cost of borrowing for the home country, leading it to a higher net foreign debt, higher investment in the domestic

---

<sup>9</sup>Note that  $v^2 + 2\lambda\sigma v + 2\sigma^2 - \sigma^2\lambda^2 > (v - \sigma)^2 + (1 - \lambda^2)\sigma^2 > 0$  as long as  $\lambda > -1$ . At this point, our simplifying assumption  $\sigma^2 + \lambda v\sigma > 0$  or  $v^2 + \lambda v\sigma > 0$  plays no role.

technology, and a higher level of wealth and consumption. In this manner, the original positive technology shock in the foreign economy is shared by the home economy.

### 3.3 Endogenous $\theta$ : A Stationarity Result

An implication of the model is that the country with a higher level of wealth is a net creditor. Is the wealth distribution stable? For this to be the case, it must be that home wealth grows faster than foreign wealth, when  $\theta > 0.5$ . To answer this question, we must explicitly derive the dynamics of  $\theta$ . Using Ito's lemma and equation (3), we may write the diffusion process governing  $\theta$  as:

$$d\theta = \theta(1 - \theta)F(\theta)dt + \theta(1 - \theta)G(\theta)dB, \quad (16)$$

where the functional forms of  $F(\theta)$ ,  $G(\theta)$ , and  $dB$  are described in the Appendix. The asymptotic distribution of  $\theta$  must satisfy either; (a)  $\theta \rightarrow 1$ , (b)  $\theta \rightarrow 0$ , or (c)  $\theta$  follows a stable distribution in  $(0, 1)$ . Given the form of (16), clearly  $\theta = 1$  and  $\theta = 0$  are absorbing states. But the following proposition establishes the conditions under which (c) will apply.

**Proposition 1** *For  $\lambda \neq 0$ ,  $\theta$  has a symmetric ergodic distribution in  $(0, 1)$  centered at  $\theta = \frac{1}{2}$ .*

*Proof.* See Appendix. ■

The content of this proposition is illustrated through the effect of  $\theta$  on risk-adjusted growth rates of wealth. As before, we define the risk-adjusted growth rate for country  $i$  as:

$$g_i(\theta) = \lim_{\Delta t \rightarrow 0} E_t \left[ \frac{\Delta \ln W_i(t + \Delta t)}{\Delta t} \right] = \lim_{\Delta t \rightarrow 0} \frac{E_t \left( \frac{\Delta W_i(t + \Delta t)}{W_i(t)} \right) - \frac{1}{2} Var_t \left( \frac{\Delta W_i(t + \Delta t)}{W_i(t)} \right)}{\Delta t}.$$

Then,  $\theta$  has an ergodic distribution if it cannot access the boundaries 0 or 1. Defining the difference between the foreign and home risk-adjusted growth rate as  $\delta(\theta) = g_f(\theta) - g_h(\theta)$ , this property holds if the probability of reaching either is zero. For the lower bound, this is the case if  $\delta(0) > 0$ . Likewise, the probability of reaching the upper bound is zero if  $\delta(1) < 0$ . This just says that as the home country gets arbitrarily wealthy, relative to the foreign country, the foreign country's risk-adjusted growth rate exceeds that of the home country. Likewise, if the foreign country's wealth increases arbitrarily relative to that of the home country, then the home risk-adjusted growth rate will exceed that of the foreign country. The Proposition establishes that, for  $\lambda \neq 0$ , this property always holds.

We may show this directly by computing  $\delta(\theta)$ . The Appendix shows that  $\delta(\theta)$  may be written as:

$$\delta(\theta) = \frac{\lambda^2 \sigma^2 (1 - 2\theta)(v^2 + 2\lambda\sigma v + 2\sigma^2 - \sigma^2 \lambda^2)(v^2 + 2\lambda\sigma v + 2\sigma^2)}{(4\sigma^2 \theta \lambda^2 (\theta - 1) + \sigma^2 \lambda^2 - 2\lambda\sigma v - 2\sigma^2 - v^2)^2}.$$

The denominator is always positive, and the numerator is positive (negative) for  $\theta < 0.5$  ( $> 0.5$ ), as long as  $\lambda \neq 0$ . Moreover, this satisfies the conditions

$$\delta(0) = \frac{\lambda^2 \sigma^2 (v^2 + 2\lambda\sigma v + 2\sigma^2)}{v^2 + 2\lambda\sigma v + 2\sigma^2 - \sigma^2 \lambda^2} > 0,$$

$$\delta(1) = -\frac{\lambda^2 \sigma^2 (v^2 + 2\lambda\sigma v + 2\sigma^2)}{v^2 + 2\lambda\sigma v + 2\sigma^2 - \sigma^2 \lambda^2} < 0,$$

and  $\delta(0.5) = 0$ .

Hence, for  $\theta > 0.5$ , when the foreign country is relatively wealthy, the home risk-adjusted growth rate exceeds that of the foreign country, and  $\theta$  falls. The same dynamics occur in reverse when  $\theta < 0.5$ . These expressions also make clear that the distribution of  $\theta$  is symmetric.

Stationarity is ensured whether  $\lambda$  is positive or negative. In either case, agents can make use of nominal bonds to hedge internationally against consumption risk, holding short the home (foreign) currency bond if  $\lambda < 0$ , and conversely if  $\lambda > 0$ . But if  $\lambda = 0$ , then nominal bond returns are independent of consumption risk in either country, and they will not be held in equilibrium (i.e.  $\omega_j^i = 0$ , for all  $i$  and  $j$ ). In this case, the stationarity result fails.

The stationarity of the world wealth distribution is directly tied to the composition of gross portfolio holdings and the dynamics of nominal returns in a very intuitive way. Take the case  $\lambda < 0$ . Then when  $\theta > 0.5$ , the foreign country is wealthier than the home country, and also a net creditor. But as we saw in the last section, the foreign country's demand for home currency bonds and supply of foreign currency bonds pushes down  $R_h$  and pushes up  $R_f$ , leading to a lower cost of borrowing for the home country, encouraging it to invest more in its domestic technology. Home currency bond returns will approach  $\alpha + \Pi - \sigma^2$ , and foreign currency bond returns approach  $\alpha + \Pi - \sigma^2 - v\lambda\sigma$ , as  $\theta \rightarrow 1$ . Since the expected return on the domestic technology exceeds that on its nominal asset portfolio, this increases the risk-adjusted expected growth rate for the home country, relative to the foreign country. As a result,  $\theta$  is driven back towards 0.5 again. In effect, it is the levered portfolio composition and its implication for the net borrowing costs for the debtor country as the wealth distribution evolves that ensures the stability of the wealth distribution itself. Current account imbalances are naturally self-correcting when agents hold an optimal currency portfolio of international

debt.

While this interpretation is based on a negative value of  $\lambda$ , this is not necessary for the stability result. If  $\lambda > 0$ , then the equivalent stabilizing force takes place, but now with the foreign country holding positive (negative) amounts of foreign (home) currency bonds. Stability is ensured because it is always the case that countries hold a gross portfolio such that their cost of borrowing falls as the rest of the world gets wealthier<sup>10</sup>.

If  $\lambda = 0$ , the portfolio composition is indeterminate, since bonds can then play no role as a hedge against technology risk. In fact, agents will hold no bonds at all. Since technologies are identical, there can be no gains from trade in international bonds at all. Any innovations to wealth are permanent. Clearly then the wealth distribution will not be stationary. In fact,  $\theta$  will be characterized by hysteresis in technology shocks will give rise to an expected permanent increase in wealth without international asset trade at all.

As demonstrated in the Appendix, the model allows for an explicit solution for the distribution of wealth. Figure 1 illustrates the distribution of  $\ln(\frac{W_f}{W_h}) = \ln(\frac{\theta}{1-\theta})$  for different values of  $\lambda$  with  $\sigma = v = 0.02$ <sup>11</sup>. In each case, the distribution has zero mean. But the  $\lambda$  parameter matters substantially for the shape of the distribution. For high absolute values of  $\lambda$ , the distribution is tightly centered around zero. But as  $\lambda$  falls in absolute value, the distribution becomes substantially spread out. This means that the speed of convergence in the wealth distribution depends critically on the size of  $\lambda$ . For high absolute values of  $\lambda$ , convergence is much faster. More concretely, in the above example represented by Figure 1, the unconditional probability that  $\theta$  (the foreign wealth share) is between 40% and 60% is 34.8% for  $\lambda = -0.9$ , 27.0% for  $\lambda = -0.8$ , but it reduces to 11.8% for  $\lambda = 0.5$  and 7.6% for  $\lambda = -0.3$ .

### 3.4 Characteristics of Equilibrium

Figures 2 through 5 describe some features of the equilibrium under a set of structural parameters,  $\alpha = 0.03$ ,  $\sigma = 0.02$ ,  $\nu = 0.02$ ,  $\Pi = 0.02$ , and  $\lambda = -0.5$ . These figures illustrate

---

<sup>10</sup>It is interesting that the effective risk premium on net foreign debt displays dynamics that are the opposite of those that are assumed in some international macro models that incorporate ad-hoc debt related risk premiums designed to ensure a stationary distribution of wealth (see the discussion in Schmitt-Grohe and Uribe 2003). In these models, there is typically a target level of net foreign debt such that, if a country's debt exceeds the target, the cost of borrowing rises, leading it to curtail spending, and reduce net foreign debt so that the country's wealth approaches a steady state. In our model, there is no target level of net foreign debt, and national wealth is non-stationary. Rather the adjustment takes place through *relative* national wealth levels moving away from their unconditional mean. Stability is achieved by a debtor country which increases its investment so that the growth of its wealth exceeds that of the creditor country.

<sup>11</sup>A major reason for adopting  $\ln \frac{W_f}{W_h}$  instead of  $\frac{W_f}{W_h + W_f}$  is that the former definition can illuminate the behavior at tails of wealth distribution with a support of  $(-\infty, +\infty)$  rather than  $(0, 1)$ .

the relationship between the relative wealth ratio  $\theta$ , country portfolio weights, real and nominal interest rates, and risk-adjusted growth rates.

Figure 2 shows how the home country's portfolio weights depend on  $\theta$ . As  $\theta$  falls towards zero, the home country dominates the world capital market, and its bond portfolio shares in both currencies are very low. But as  $\theta$  rises, it increases its holdings of foreign currency bonds, and balances this by issuing home currency bonds. As we saw above, for  $\theta < 0.5$ , ( $\theta > 0.5$ ) the former (latter) effect dominates, and it is a net creditor (debtor).

Figure 3 illustrates the pattern of real risk-free rates as a function of  $\theta$ . From the analysis at  $\theta = 0.5$ , we know that the home country risk-free rate is increasing in  $\theta$  at  $\theta = 0.5$ , and the foreign risk-free rate is decreasing in  $\theta$  at that point. But the behavior of risk-free rates away from  $\theta = 0.5$  may be quite different. In the figure, we see that risk-free rates may be non-monotonic in  $\theta$ . As  $\theta$  rises above 0.5, the home country risk-free rate first rises, as income growth rises when capital flows go into home technology. But, for relatively low  $v$ , this also involves increasing the riskiness of the home portfolio, which tends to push down the risk-free rate. As  $\theta$  continues to increase, this second effect can dominate, and  $r_h$  may fall in  $\theta$ . But if monetary uncertainty is high enough (that is,  $v$  high enough), then the home risk-free rate will be monotonically increasing in  $\theta$ . The logic is as follows. As  $\theta$  rises towards unity,  $R_h$  approaches  $\alpha + \Pi - \sigma^2$ , while  $R_f$  approaches  $\alpha + \Pi - \sigma^2 - \lambda\sigma v$ , which is high for a high value of  $v$ . Since the home country holds positive amounts of the foreign currency bond, this pushes up its growth rate, and hence the home risk-free rate.

Figure 4 shows a similar pattern but in terms of nominal returns  $R_h$  and  $R_f$ . As the home (foreign) country increases its share of world wealth, it pushes down the return on the foreign (home) currency bond. Similarly, because the home (foreign) country issues its own currency bond in these circumstances, the return on this bond must increase when it dominates world wealth.

Finally, Figure 5 illustrates the relationship between  $\theta$  and risk-adjusted growth rates. For the reasons discussed above, risk-adjusted growth rates are monotonic in  $\theta$ . As  $\theta$  approaches either boundary, the risk-adjusted growth rate of the smaller country will always exceed that of the larger country.

## 4 Applications of the Basic Model

### 4.1 Extension to Trade in a Real Risk-free Bond

So far all capital flows were facilitated only by trade in nominal bonds. In many traditional open economy macroeconomic models however, it is assumed that there is a real bond traded

across countries, where the return is denominated in terms of a world consumption basket. We now extend the model to allow for simultaneous trade in a world risk-free bond *as well as* nominal bonds. We show that the existence of nominal bond trade actually generates gains from trade in *real* bonds because of the stationarity induced by nominal bonds. In addition, as we see, this substantially simplifies some of the main features of the model.

Assume then that the set-up is exactly the same as before except for that there is free international trade in a real risk-free bond. The optimal portfolio rules remain as before, as do the equations (5) and (6). Instead of (7) and (8) however, we have a world market clearing equation in the real risk-free bond, given as:

$$(\omega_T^h + \omega_h^h + \omega_f^h - 1)W_h + (\omega_T^f + \omega_h^f + \omega_f^f - 1)W_f = 0. \quad (17)$$

The two nominal bond market clearing conditions and (17) may be solved for  $R_h$ ,  $R_f$ , and  $r$ . The solutions can be written as:

$$R_h(\theta) = \left[ \alpha + \Pi - \sigma^2 + \frac{1}{2} (\lambda^2 \sigma^2 - \lambda \sigma v) \right] - \frac{\sigma \lambda (1 - \lambda^2) [v(1 - 2\theta) + \sigma \lambda (1 - 4\theta(1 - \theta))]}{2(1 - \lambda^2(1 - 2\theta(1 - \theta)))}, \quad (18)$$

$$R_f(\theta) = \left[ \alpha + \Pi - \sigma^2 + \frac{1}{2} (\lambda^2 \sigma^2 - \lambda \sigma v) \right] - \frac{\sigma \lambda (1 - \lambda^2) [v(1 - 2(1 - \theta)) + \sigma \lambda (1 - 4\theta(1 - \theta))]}{2(1 - \lambda^2(1 - 2\theta(1 - \theta)))}, \quad (19)$$

and

$$r(\theta) = \left[ \alpha - \sigma^2 + \frac{1}{2} \lambda^2 \sigma^2 \right] + \frac{\sigma^2 \lambda^2 (1 - \lambda^2) [1 - 4\theta(1 - \theta)]}{2(1 - \lambda^2(1 - 2\theta(1 - \theta)))}. \quad (20)$$

Qualitatively, (18) and (19) are similar to the previous results. For  $\lambda < 0$  ( $\lambda > 0$ ),  $R_h$  is declining (increasing) in  $\theta$ , since the home currency (foreign currency) denominated bond is in higher demand when the foreign country increases its share of world wealth. A similar property holds for  $R_f$ . The unified world risk-free rate  $r_h$  is now increasing (decreasing) in  $\theta$  for  $\theta < 0.5$  ( $\theta > 0.5$ ), and is maximized at  $\theta = 0.5$ .

The particular interest of this case lies in the portfolio mix of real and nominal bond holdings that agents will choose. Using these expressions for equilibrium returns in combination with the solutions from the portfolio problem allows us to write the equilibrium home country portfolio shares as:

1. Net share of wealth in bonds

$$1 - \omega_T^h = \frac{\theta \lambda^2 (1 - 2\theta)}{(1 - \lambda^2(1 - 2\theta(1 - \theta)))} \quad (21)$$

2. Share of wealth in risk-free bonds

$$1 - \omega_T^h - \omega_h^h - \omega_f^h = \frac{\theta\lambda^2(1 - 2\theta)(v + \sigma\lambda)}{v(1 - \lambda^2(1 - 2\theta(1 - \theta)))} \quad (22)$$

3. Share of wealth in nominal bonds

$$\omega_h^h + \omega_f^h = -\frac{\sigma\theta\lambda^3(1 - 2\theta)}{v(1 - \lambda^2(1 - 2\theta(1 - \theta)))} \quad (23)$$

4. Share of wealth in home currency denominated bonds

$$\omega_h^h = \frac{\sigma\theta\lambda(1 - \lambda^2(1 - \theta))}{v(1 - \lambda^2(1 - 2\theta(1 - \theta)))} \quad (24)$$

5. Share of wealth in foreign currency denominated bonds

$$\omega_f^h = -\frac{\sigma\theta\lambda(1 - \lambda^2\theta)}{v(1 - \lambda^2(1 - 2\theta(1 - \theta)))} \quad (25)$$

As before, the net share of wealth in bonds is zero when  $\theta = 0.5$ , and this share falls as  $\theta$  increases. The home country is always short (long) in risk-free bonds when  $\theta > 0.5$  ( $\theta < 0.5$ ), for  $\lambda \neq 0$ . Also, for  $\lambda < 0$  ( $\lambda > 0$ ), the country is always short (long) in home currency bonds, and long (short) in foreign currency bonds, for all values of  $\theta$ . But now, in contrast to the previous results, the *net* position in nominal bonds depends upon both  $\theta$  and the sign of  $\lambda$ .

When  $\lambda < 0$ , the home country's net nominal bond position mirrors both its overall bond position and its risk-free bond position. That is, it is a net nominal debtor whenever  $\theta > 0.5$ . But when  $\lambda > 0$ , the opposite applies; its net nominal bond position is the opposite of its overall bond position, and its risk-free bond position. That is, it is long (short) on nominal bonds whenever  $\theta > 0.5$  ( $\theta < 0.5$ ). The intuition behind this is that the net nominal bond position is dominated by the country's holdings of home currency bonds, whether short if  $\lambda < 0$  or long if  $\lambda > 0$ . The existence of the real risk-free bond reduces the diversification potential of the foreign currency bond, given that its returns are orthogonal to the domestic production technology. Thus, when  $\lambda > 0$ , the home country wishes to have a long position in home currency bonds, but unlike the previous case, this is counterbalanced by the real risk-free bond. As a result, its net position in nominal bonds are positive when  $\lambda > 0$  and  $\theta > 0.5$ , even though the country's overall bond holdings are negative.

Using the same set of parameters as before ( $\alpha = 0.03$ ,  $\sigma = 0.02$ ,  $\nu = 0.02$ , and  $\Pi = 0.02$ ), Figure 6 illustrates this process with both positive and negative values of  $\lambda$ . We see that,

when  $\lambda > 0$ , net nominal bond holdings are opposite in sign to total bond holdings, or total risk-free bond holdings. In particular, the key aspect of the previous section, that net capital flows are facilitated by taking a larger short position than a long position in nominal bonds, is not true of this case.

What determines the extent to which the investor will rely on nominal bonds relative to real risk-free bonds? Again, the key parameter is  $\lambda$ . Based on the same parameters as above, Figures 7 and 8 show, for the case of  $\lambda < 0$ , that for low absolute values of  $\lambda$ , home residents will hold a bond portfolio dominated by risk-free bonds. But as  $\lambda$  increases in absolute value, the composition of the portfolio switches to net nominal bond holdings, and risk-free bond holdings play a much more minor role.

Stationarity of the wealth distribution continues to hold in this extended model. Using the same procedure as before, we compute the function  $\delta(\theta)$  (defined as  $g_f(\theta) - g_h(\theta)$ ), describing the difference between the foreign and home countries' risk-adjusted consumption growth rates, as:

$$\delta(\theta) = \frac{\sigma^2 \lambda^2 (1 - 2\theta) \left[1 - \frac{3}{2} \lambda^2 \left(1 - \frac{2}{3} \lambda^2\right)\right]}{\left[1 - \lambda^2 (1 - 2\theta(1 - \theta))\right]^2}. \quad (26)$$

It is easy to see that  $\delta(0) > 0$ ,  $\delta(1) < 0$ , and  $\delta(0.5) = 0$ , as before.

These results also reveal an interesting feature of the coexistence of real and nominal bonds. Without nominal bonds, there are no gains from trade in risk-free bonds at all, when technologies are symmetric (recall that the autarky risk-free rates are the same). So why is it that risk-free bonds are traded *simultaneously* with nominal bonds in a stationary environment? The answer can be seen from (11) and (12) above. When there are only nominal bonds traded, consumption growth is not equalized across countries, and the implicit equilibrium risk-free rates will differ across countries, except when  $\theta = 0.5$ . This gives rise to gains from intertemporal trade, as in conventional models of the current account. That is, when  $\theta > 0.5$ , the foreign country is temporarily wealthier than the home country. Without trade in a risk-free bond, it would have a lower risk-free rate than the home country, indicating that it would wish to save some of the temporarily higher wealth if an international risk-free bond market were opened<sup>12</sup>. Likewise, in the same circumstance, the home country would wish to borrow as its wealth is temporarily below its long run mean. But it is the trade in nominal bonds that induces stationarity of the wealth distribution in the first place. If there were no nominal bonds traded, then the distribution of  $\theta$  is no longer stationary (as discussed for the case with  $\lambda = 0$ ), and thus, there are no gains at all from trade in a

---

<sup>12</sup>It is important to emphasize that the underlying technology shocks in our model are permanent, but when Proposition 1 applies, their effect on relative wealth positions are temporary.

risk-free bond in a symmetric environment<sup>13</sup>.

## 4.2 Introduction of a Tradable Technology

This subsection relaxes the assumption that shares in the national production technologies are non-tradable across countries. That is, we extend the model to allow for trade in both (nominal) bond and shares in technologies (or equities). In order to maintain the characteristic of incomplete markets while allowing for both bond and equity trade, we introduce another linear technology in each country, and allow for trade in shares in this technology. The tradable technology of country  $i$  is characterized as

$$\frac{dQ_i^E}{Q_i^E} = \beta dt + \epsilon dB_i^E$$

for  $i = h$  or  $f$ , where  $dB_i^E$  is an increment to the standard Weiner process uncorrelated with  $dB_i$ , and correlated with  $dM_i$  with the coefficient  $\lambda$ <sup>14</sup>.

Now, in addition to investments in its own non-tradable technology  $\omega_T^h$  ( $\omega_T^f$ ), the home (foreign) country can invest in its own tradable technology with a portfolio weight  $\omega_{Th}^h$  ( $\omega_{Tf}^f$ ), and the tradable technology of the foreign (home) country with a portfolio weight  $\omega_{Th}^h$  ( $\omega_{Tf}^f$ ). Holding of a share in technology in this model is equivalent to making a direct investment in the production technology. Thus, we impose a restriction that investors are not allowed to take a short position on the tradable technologies. Then, the following portfolio restrictions must be satisfied for both countries:  $\omega_T^h + \omega_{Th}^h + \omega_{Tf}^h + \omega_h^h + \omega_f^h = 1$  with  $\omega_T^h \geq 0$ ,  $\omega_{Th}^h \geq 0$ , and  $\omega_{Tf}^h \geq 0$ , and  $\omega_T^f + \omega_{Tf}^f + \omega_{Th}^f + \omega_f^f + \omega_h^f = 1$  with  $\omega_T^f \geq 0$ ,  $\omega_{Tf}^f \geq 0$ , and  $\omega_{Th}^f \geq 0$ .

To illustrate the properties of this extended model, we make the additional assumptions;  $\alpha = \beta$ , and  $\sigma = \epsilon = \nu$ . These assumptions are not essential, but help to simplify the exposition.

At  $\theta = 0.5$ , we find that as long as  $\lambda^2 \leq \frac{1}{2}$ , then  $\omega_T^h = \frac{1-\lambda^2}{3-4\lambda^2} > 0$ ,  $\omega_{Th}^h = \frac{1-\lambda^2}{3-4\lambda^2} > 0$ , and  $\omega_{Tf}^h = \frac{1-2\lambda^2}{3-4\lambda^2} > 0$  in equilibrium; that is,  $\omega_T^h + \omega_{Th}^h + \omega_{Tf}^h = 1$  holds, with a long position on all of the three linear technologies. In the nominal bond markets,  $\omega_h^h = -\omega_h^f = \frac{\lambda}{2(3-4\lambda^2)}$ . As in the previous case without direct trade in shares of the production technologies, the home (foreign) country still takes a short position in the home currency bond, and a long position in the foreign currency bond, when  $\lambda$  is negative, and  $\lambda^2 \leq \frac{1}{2}$ .

<sup>13</sup>Devereux and Saito (1997) demonstrate that for asymmetric cases, one country with a high risk/high return technology and the other with a low risk/low return technology would constitute the stationary wealth distribution.

<sup>14</sup>An indirect correlation between  $dB_i^E$  and  $dB_i$  does not show up at the variance-covariance matrix because it converges to zero as  $\Delta t \rightarrow 0$  by a higher-order effect.

Under this parameterization and  $\theta = 0.5$ , the home and foreign bond market clearing condition determine equilibrium interest rates equal to  $R_h = R_f = \alpha + \Pi - \frac{[2+\lambda(3-3\lambda-4\lambda^2)]\sigma^2}{6-8\lambda^2}$ , while the corresponding risk-free rates ( $r_h$  and  $r_f$ ) are equal to  $\alpha - \frac{(2-3\lambda^2)\sigma^2}{6-8\lambda^2}$ . At the lower limit of  $\lambda^2 = 0$ , nominal bonds play no role in hedging consumption risk, and an equilibrium is characterized by each country dividing its wealth equally over the three technologies (the domestic non-tradable and two tradable technologies). The equilibrium risk-free rate is then equal to  $\alpha - \frac{1}{3}\sigma^2$ . At the upper limit of  $\lambda^2 = \frac{1}{2}$ , on the other hand, the equilibrium risk-free rate reaches  $\alpha - \frac{1}{4}\sigma^2$ , which is equivalent to the risk-free rate in the complete markets case where each country divides its wealth equally over the two domestic and two foreign technologies. Those results imply that as long as  $\lambda$  is non-zero, nominal bond trading can still play an effective role in sharing country-specific shocks.

Again,  $\theta$  has a stationary distribution in  $(0, 1)$  centered at  $\theta = 0.5$ . Define the difference between the foreign and home risk-adjusted growth rate defined as  $\delta(\theta) = g_f(\theta) - g_h(\theta)$ . It is possible to show that  $\delta(0.5) = 0$ , and

$$\delta(1) = -\delta(0) = -\frac{\lambda^2(5 + 6\lambda)\sigma^2}{9(1 + \lambda) [(1 + \lambda) + 4(1 - 2\lambda^2)]} < 0,$$

as long as  $\lambda^2 \leq \frac{1}{2}$ . Therefore, as before, when a country's share of world wealth falls, its relative growth rate increases. Despite the ability to trade equity, the underlying force behind the stability property for  $\theta$  is the presence of nominal bond trading, just as in the previous case.

Note an interesting implication of the portfolio share holdings in this case. As  $\lambda^2 \rightarrow \frac{1}{2}$ , we find that  $\omega_{Tf}^h \rightarrow 0$ ,  $\omega_T^h \rightarrow \frac{1}{2}$ , and  $\omega_{Th}^h \rightarrow \frac{1}{2}$ . Thus, as the nominal bond markets converge more towards supporting a complete market allocation, the direct holding of foreign equity goes to zero, and home agents hold 100 percent of the home technologies (non-tradable and tradable). Thus, although direct trade in equity is possible, an equilibrium is characterized by complete *home bias* in equity holdings. The intuitive reason for this is that the bond portfolio held by residents of each country represents a perfect claim on the foreign technology in the case when  $\lambda^2 \rightarrow \frac{1}{2}$ .

### 4.3 One-way Capital Flows

Another special case of the model emerges when trade can take place only in bonds denominated in one currency. This might be seen as reflecting the dominance of the US dollar in international financial markets, represented by a restriction that only US dollar assets are acceptable in international exchange. This places a restriction on the ability to trade, since

even if  $\lambda < 0$ , the foreign country cannot issue its own currency debt.

To sketch out this case, we make the simplifying assumption that  $\nu = \sigma$ . Optimal portfolio rules ( $\omega_T^h$ ,  $\omega_h^h$ ,  $\omega_T^f$ , and  $\omega_h^f$ ) are still determined by a version of equation (4) with  $\lambda_h = \lambda < 0$  for the home country choice and  $\lambda_f = 0$  for the foreign country. Then, two portfolio restrictions ( $\omega_T^h + \omega_h^h = 1$ ,  $\omega_T^f + \omega_h^f = 1$ ) and a bond market clearing ( $(1-\theta)\omega_h^h + \theta\omega_h^f = 0$ ) may be used to determine the equilibrium nominal interest rates on home currency bonds ( $R_h$ ), and the risk-free real interest rate on implicit (non-traded) real bonds ( $r_h$  and  $r_f$ ). The home country's holding of home bonds is given by

$$\omega_h^h = \frac{1}{2} \frac{\lambda\theta}{\lambda\theta + 1}. \quad (27)$$

As before, the home country has a negative position in home currency bonds, when  $\lambda < 0$ . The difference now however is that (27) represents both the *gross* and *net* bond position of the home country. When  $\lambda < 0$ , the home country always has a negative net foreign asset position. Capital flows from the rest of the world to the home country, since the tradable nominal bond is a better hedge against foreign consumption risk than against home consumption risk.

Even in this case however,  $\theta$  has a stationary distribution in  $(0, 1)$ . The unconditional mode of  $\theta$  is  $\theta = \frac{-1+\sqrt{1+\lambda}}{\lambda}$  as long as  $\lambda$  satisfies  $0 < \frac{-1+\sqrt{1+\lambda}}{\lambda} < 1$ . This can be shown as follows. Given the difference between the foreign and home risk-adjusted growth rate defined as  $\delta(\theta) = g_f(\theta) - g_h(\theta)$ , we obtain  $\delta(\frac{-1+\sqrt{1+\lambda}}{\lambda}) = 0$ ,  $\delta(0) = \frac{\lambda^2\sigma^2}{4} > 0$ , and  $\delta(1) = -\frac{\lambda^2\sigma^2}{4+4\lambda} < 0$ . The stationarity of  $\theta$  is driven by the effects of changes in interest rates on net foreign assets. When  $\lambda < 0$ , a rise in  $\theta$  reduces the nominal interest rate on home currency debts, thereby raising the risk-adjusted growth of the home country.

In addition, the mode of the wealth distribution in this case of one-way capital flows is now governed by  $\lambda$ . For  $\lambda < 0$ , the foreign country has a higher expected share of world wealth in the long run, and this share is increasing, the greater is  $\lambda$  in absolute value. Given  $\alpha = 0.03$ ,  $\sigma = \nu = 0.02$ , and  $\lambda_h = -0.5$ , Figure 9 depicts the foreign and home risk-adjusted growth; in this case, the risk-adjusted growth rates are equal to each other at  $\theta \approx 0.586$ .

When the two countries grow equally at  $\theta = \frac{-1+\sqrt{1+\lambda}}{\lambda}$ , the equilibrium nominal interest rate on home currency bonds ( $R_h$ ) is equal to  $\Pi + \alpha - \sigma^2\sqrt{1+\lambda}$ . At that time, the home country finances its domestic production by  $\frac{1}{2}(\frac{1}{\sqrt{1+\lambda}} - 1)W_h$  from the foreign country, while the foreign country holds home currency bonds equal to  $\frac{1}{2}(1 - \sqrt{1-\lambda})W_f$ . Under the same assumption as above, Figure 10 shows positions on home currency bonds ( $\omega_h^h$  and  $\omega_h^f$ ) as a function of  $\theta$ .

## 4.4 Asymmetry in Structural Parameters

Here we explore the impact of asymmetries in structural parameters. To see the intuition, we first evaluate the marginal effect of a change in a particular parameter on net foreign asset positions of the home country, starting from  $\omega_h^h + \omega_f^h = 0$  at  $\theta = 0.5$  in the symmetric case ( $\alpha_h = \alpha_f = \alpha$ ,  $\sigma_h = \sigma_f = \sigma$ ,  $\nu_h = \nu_f = \nu$ , and  $\lambda_h = \lambda_f = \lambda < 0$ ). This approach ensures that stationarity still applies in the neighborhood of the symmetric case that we have investigated in detail. We maintain our simplifying assumption ( $\sigma^2 + \lambda\nu\sigma > 0$  and  $\nu^2 + \lambda\nu\sigma > 0$ ).

With regard to production parameters, we find that:

$$\left. \frac{\partial(\omega_h^h + \omega_f^h)}{\partial\alpha_h} \right|_{\alpha_h=\alpha_f, \sigma_h=\sigma_f, \lambda_h=\lambda_f < 0, \nu_h=\nu_f} = -\frac{1}{\nu^2 + 2\sigma(\sigma + \lambda\nu)} < 0,$$

and

$$\left. \frac{\partial(\omega_h^h + \omega_f^h)}{\partial\sigma_h} \right|_{\alpha_h=\alpha_f, \sigma_h=\sigma_f, \lambda_h=\lambda_f < 0, \nu_h=\nu_f} = \frac{1}{2} \frac{(\sigma + \lambda\nu) + (3 - \lambda^2)\sigma}{\nu^2 + 2\sigma(\sigma + \lambda\nu)} > 0.$$

A marginal increase in  $\alpha_h$  tips the home country towards a net foreign debtor status, as the more profitable opportunities in the home economy attract capital from the foreign country. A marginal increase in  $\sigma_h$ , has the opposite effect, as a result of hedging production risk through a long position in foreign currency bonds.

A marginal increase in home inflation risk captured by  $\nu_h$  makes net foreign assets positive for the same reason as in an increase in  $\sigma_h$ .

$$\left. \frac{\partial(\omega_h^h + \omega_f^h)}{\partial\nu_h} \right|_{\alpha_h=\alpha_f, \sigma_h=\sigma_f, \lambda_h=\lambda_f < 0, \nu_h=\nu_f} = -\frac{1}{2\nu} \frac{\lambda\sigma(\nu + \lambda\sigma)}{\nu^2 + 2\sigma(\sigma + \lambda\nu)} > 0,$$

Finally, as production shocks become more negatively correlated with inflation shocks in the home country, it becomes a net debtor, as intuitively, short positions in home currency bonds become a more effective instrument to hedge production risk.

$$\left. \frac{\partial(\omega_h^h + \omega_f^h)}{\partial\lambda_h} \right|_{\alpha_h=\alpha_f, \sigma_h=\sigma_f, \lambda_h=\lambda_f < 0, \nu_h=\nu_f} = \frac{1}{2} \frac{\sigma(\nu - \lambda\sigma)}{\nu^2 + 2\sigma(\sigma + \lambda\nu)} > 0.$$

## 4.5 A Simple Calibration

Can the model match empirical observations on gross and net international investment positions? Here we sketch a simple quantitative investigation for the US economy. Think of

the US as the home country, and assume that the home and foreign countries are identical, except for the correlation coefficient  $\lambda$ . We choose a set of parameters to roughly match predicted moments with the observed moments of the U.S. economy with regard to consumption growth, inflation rates, and external portfolio positions. For the sample period between 1980 and 2004, U.S. consumption grew at 3.4% per year on the average with 1.2% standard deviation, while inflation rates were 3.6% on average, with 1.3% standard deviation. As reported in Lane and Milesi-Ferretti (2006), US net external assets in 2003 were approximately  $-24\%$  of GDP. This reflected gross external liabilities of approximately 96 percent of GDP, with gross external assets of 71 percent of GDP.

We set parameters as follows:  $\alpha = 0.084$ ,  $\sigma = 0.012$ ,  $\Pi = 0.036$ ,  $\nu = 0.013$ , and  $\rho = 0.054$ . These ensure that the expected growth rate and volatility of consumption and inflation match those of the US economy. From our estimates of  $\lambda$  reported above, we set  $\lambda = -0.2$  for the US economy. For the rest of the world, the estimates vary, but in order to match the US net foreign asset position within the confines of the model (since all other parameters are set to be equal across countries), it is necessary that  $\lambda$  for the rest of the world to be less negative than that of the US. Accordingly, we set  $\lambda = -0.1$ , following the estimates for Japan.

Given this parameterization, the conditions for long-run stationarity hold with a modal value of  $\theta$  equal to 0.511. Table 2 illustrates the implications of the model for gross and net external positions at  $\theta = 0.511$ . The mean output of the home country in the model is  $\alpha W_h$  (as in an ‘AK’ growth model). This, implies that relative gross external assets and liabilities are respectively  $\frac{\omega_f^h}{\alpha}$  and  $\frac{\omega_h^h}{\alpha}$ , and the relative net external position is  $\frac{\omega_h^h + \omega_f^h}{\alpha}$ . In the baseline case of Table 2, gross and net external positions are surprisingly close to those of the US economy in 2003; gross liabilities in the model are 98.2% of GDP, gross assets are 71.6% of GDP, and net assets are  $-26.6\%$  of GDP. Table 2 also reports some sensitivity analysis allowing for different values of  $\lambda$  for both the US and the rest of the world. In all cases, we report the asset positions around the modal value of  $\theta$  implied by the assumed values for  $\lambda_h$  and  $\lambda_f$ .

Table 3 looks at deviations from the unconditional mode of  $\theta$  for the baseline case of  $\lambda_h = -0.2$  and  $\lambda_f = -0.1$ . As implied by our theoretical discussion, an increase in the foreign wealth share increases the net external liabilities of the US economy, while also increasing both gross external liabilities and (somewhat less) gross external assets.

## 5 Conclusions

This paper develops a tractable model of international capital flows in which the existence of nominal bonds and the portfolio composition of net foreign assets is an essential element

Table 2: Gross and net positions at centered  $\theta$ 

$\lambda_h$	$\lambda_f$	modal $\theta$	$\omega_h^h/\alpha$	$\omega_f^h/\alpha$	$(\omega_h^h + \omega_f^h)/\alpha$
-0.20	-0.20	0.500	-1.099	+1.099	0.000
-0.20	-0.15	0.506	-1.043	+0.906	-0.137
-0.20	-0.10	0.511	-0.982	+0.716	-0.266
-0.20	-0.13	0.508	-1.019	+0.829	-0.190
-0.15	-0.08	0.507	-0.731	+0.557	-0.174
-0.10	-0.03	0.508	-0.429	+0.247	-0.182

Table 3: Gross and net positions at various values of  $\theta$  at  $\lambda_h = -0.20$  and  $\lambda_f = -0.10$ 

$\theta$	$\omega_h^h/\alpha$	$\omega_f^h/\alpha$	$(\omega_h^h + \omega_f^h)/\alpha$
0.300	-0.559	+0.429	-0.130
0.400	-0.756	+0.566	-0.190
0.511 (mode)	-0.982	+0.716	-0.266
0.600	-1.169	+0.834	-0.335
0.700	-1.386	+0.964	-0.422

in facilitating capital flows between countries. Nominal bonds issued in different currencies differ in the degree to which they can hedge country specific consumption risk. This leads countries to have distinct gross positions of currency-denominated bonds in their national portfolios. By adjusting their gross positions in each currency's bonds, countries can achieve an optimally hedged change in their net foreign assets (or their current account), thus facilitating international capital flows. Moreover, the risk characteristics of optimal portfolios ensures that current account movements are sustainable; net debtor countries pay lower rates of return on their gross liabilities than they receive on their gross assets. This ensures that the distribution of wealth across countries is stationary.

The modeling approach can be extended in a number of dimensions. First, we could do a more explicit welfare evaluation, comparing welfare across different bond trading regimes, as well as computing the welfare implications of alternative monetary policy rules. Secondly, we could introduce a multi-commodity structure, and explore the interaction between the evolution of growth and net debt positions and real exchange rates. We leave these issues for future research.

# Appendix

## Process of Wealth Distribution $\theta$

To obtain the process of wealth distribution  $\theta (= \frac{W_f}{W_h + W_f})$ , we define  $m_h(\theta) = \lim_{\Delta t \rightarrow 0} \frac{E_t \left[ \frac{\Delta W_h(t+\Delta t)}{W_h(t)} \right]}{\Delta t}$ ,  
 $m_f(\theta) = \lim_{\Delta t \rightarrow 0} \frac{E_t \left[ \frac{\Delta W_f(t+\Delta t)}{W_f(t)} \right]}{\Delta t}$ ,  $n_h(\theta) = \lim_{\Delta t \rightarrow 0} \frac{Var_t \left[ \frac{\Delta W_h(t+\Delta t)}{W_h(t)} \right]}{\Delta t}$ ,  $n_f(\theta) = \lim_{\Delta t \rightarrow 0} \frac{Var_t \left[ \frac{\Delta W_f(t+\Delta t)}{W_f(t)} \right]}{\Delta t}$ ,  
and  $n_{hf}(\theta) = \lim_{\Delta t \rightarrow 0} \frac{Cov_t \left[ \frac{\Delta W_h(t+\Delta t)}{W_h(t)}, \frac{\Delta W_f(t+\Delta t)}{W_f(t)} \right]}{\Delta t}$ . Then, using Ito's lemma, we can derive the process of wealth distribution  $\theta (= \frac{W_f}{W_h + W_f})$  as

$$d\theta = \theta(1 - \theta) [F(\theta)dt + G(\theta)dB], \quad (28)$$

where

$$F(\theta) = m_f(\theta) - m_h(\theta) - \theta n_f(\theta) + (1 - \theta)n_h(\theta) + (2\theta - 1)n_{hf}(\theta),$$

$$G(\theta) = \sqrt{n_h(\theta) + n_f(\theta) - 2n_{hf}(\theta)},$$

and

$$dB = \frac{1}{G(\theta)} \left[ \omega_T^f(\theta)\sigma dB_f - \omega_f^f(\theta)v dM_f - \omega_h^f(\theta)v dM_h \right] \\ - \frac{1}{G(\theta)} \left[ \omega_T^h(\theta)\sigma dB_h - \omega_h^h(\theta)v dM_h - \omega_f^h(\theta)v dM_f \right].$$

$dB(t)$  is newly defined as the increment to a standard Brownian motion. Note here that

$$\lim_{\Delta t \rightarrow 0} \frac{E_t [\Delta B(t + \Delta t)]}{\Delta t} = 0, \quad \lim_{\Delta t \rightarrow 0} \frac{Var_t [\Delta B(t + \Delta t)]}{\Delta t} = 1.$$

## Stationarity of Wealth Distribution $\theta$

To make theorems 16 and 18 of Skorohod (1989) applicable, we consider the process of  $\kappa$  or  $\ln \frac{\theta}{1-\theta}$  ( $= \ln \frac{W_f}{W_h}$ ) instead of  $\theta$ . The process of  $\kappa$  is derived as

$$d\kappa = \delta(\theta)dt + G(\theta)dB, \quad (29)$$

where  $\theta = \frac{\exp(\kappa)}{1+\exp(\kappa)}$ , and  $\delta(\theta) = g_f(\theta) - g_h(\theta)$ . As defined in the main text,  $\delta(\theta)$  represents the difference in risk-adjusted wealth growth between the two countries. Given equilibrium

asset pricing characterized by equations (9) through (12),  $\delta(\theta)$  is computed as

$$\delta(\theta) = \frac{\lambda^2 \sigma^2 (1 - 2\theta)(v^2 + 2\lambda\sigma v + 2\sigma^2 - \sigma^2 \lambda^2)(v^2 + 2\lambda\sigma v + 2\sigma^2)}{(4\sigma^2 \theta^2 \lambda^2 - 4\theta \lambda^2 \sigma^2 + \sigma^2 \lambda^2 - 2\lambda\sigma v - 2\sigma^2 - v^2)^2}. \quad (30)$$

We then introduce the following integrals:

$$\begin{aligned} I_1 &= \int_{-\infty}^0 \exp \left[ - \int_0^w c(u(v)) dv \right] dw, \\ I_2 &= \int_0^{\infty} \exp \left[ - \int_0^w c(u(v)) dv \right] dw, \end{aligned}$$

and

$$M = \int_0^{\infty} \left[ \frac{2}{G(u(w))^2} \exp \left[ \int_0^w c(u(v)) dv \right] \right] dw,$$

where

$$c(u(v)) = \frac{2\delta(u(v))}{G(u(v))^2}, \quad (31)$$

and

$$u(v) = \frac{\exp(v)}{1 + \exp(v)}.$$

According to the above theorems of Skorohod (1989), if  $I_1 = \infty$ ,  $I_2 = \infty$ , and  $M < \infty$ , then  $\kappa$  has a unique ergodic distribution in  $(-\infty, +\infty)$ ; accordingly,  $\theta$  has a unique ergodic distribution in  $(0, 1)$ .

A function  $c(\cdot)$  characterized by equation (31) plays a key role in determining stationarity of  $\kappa$ . Saito (1997) demonstrates that if  $c(0) > 0$  and  $c(1) < 0$ , then  $\kappa(\theta)$  has a unique ergodic distribution under some regulatory conditions. The process of  $\kappa$  or equation (29) always satisfies  $c(0) > 0$  and  $c(1) < 0$ , because from equation (30),

$$\delta(0) = \frac{\lambda^2 \sigma^2 (v^2 + 2\lambda\sigma v + 2\sigma^2)}{v^2 + 2\lambda\sigma v + 2\sigma^2 - \sigma^2 \lambda^2} > 0,$$

and

$$\delta(1) = -\frac{\lambda^2 \sigma^2 (v^2 + 2\lambda\sigma v + 2\sigma^2)}{v^2 + 2\lambda\sigma v + 2\sigma^2 - \sigma^2 \lambda^2} < 0,$$

given finite  $G(0)$  and  $G(1)$ . Note that  $v^2 + 2\lambda\sigma v + 2\sigma^2 > (v - \sigma)^2 > 0$  and  $v^2 + 2\lambda\sigma v + 2\sigma^2 - \sigma^2 \lambda^2 > (v - \sigma)^2 + (1 - \lambda^2)\sigma^2$  as long as  $\lambda > -1$ .

## Density Function of Wealth Distribution $\ln \frac{W_f}{W_h}$

According to Gihman and Skorohod (1972), given the process of  $\kappa$  ( $= \ln \frac{W_f}{W_h}$ ) or equation (29), a density function of  $\kappa$  is derived as

$$\frac{2\mu}{G(u(\kappa))^2} \exp \left[ \int_0^\kappa c(u(v)) dv \right],$$

where  $\mu$  is chosen such that  $\mu \int_0^\infty \left[ \frac{2}{G(u(w))^2} \exp \left[ \int_0^w c(u(v)) dv \right] \right] dw = 1$ . Figure 1 depicts density functions of  $\kappa$  or  $\ln \frac{W_f}{W_h}$  for  $\lambda = -0.9, -0.8, -0.5,$  and  $-0.3$  when  $\sigma = v = 0.02$ .

## References

- [1] Cooley, T. F. and G. D. Hansen (1995), "Money and the Business Cycle," in T. H. Cooley (editor), *Frontiers of Business Cycle Research*, Princeton: Princeton University Press.
- [2] Baxter, M. and U. Jermann (1997), "The International Diversification Puzzle is Worse than You Think," *American Economic Review* 87, 170-180.
- [3] Devereux, M. and M. Saito (1997), "Growth and Risk Sharing in a Two Country Model," *Journal of International Economics* 42, 453-81.
- [4] Dooley, M. P. and P. Isard (1979), "The Portfolio-balance Model of Exchange Rates," *International Finance Discussion Paper 141*, Board of Governors of the Federal Reserve System, U.S.
- [5] Engel, C. and A. Matsumoto (2006), "Portfolio Choice in a Monetary Open Economy DSGE Model," mimeo., University of Wisconsin.
- [6] Evans, M. and V. Hnatkovska (2005), "International Capital Flows, Returns, and World Financial Integration," mimeo., Georgetown University.
- [7] Gihman, I. and A. V. Skorohod (1972), *Stochastic Differential Equations*, New York: Springer-Verlag.
- [8] Gourinchas P. O. and H. Rey (2004), "International Financial Adjustment," mimeo., U. C. Berkeley.
- [9] Greenspan, A. (2004), "Globalization and Innovation," a speech made at Federal Reserve Bank of Chicago.
- [10] Heathcote, J., and F. Perri (2004), "The International Diversification Puzzle is Not as Bad as you Think," mimeo., NYU.
- [11] Hnatkovska, V. (2005), "Home Bias and High Turnover: Dynamic Portfolio Choice with Incomplete Markets," mimeo., Georgetown University.

- [12] Kollman, R. (2006), "International Portfolio Equilibrium and the Current Account," mimeo., University of Paris.
- [13] Kydland, F. and E. Prescott (1990), "Business Cycles: Real Facts and a Monetary Myth," Federal Reserve Bank of Minneapolis Quarterly Review 14, 3-18.
- [14] Lane, P. R. and G. M. Milesi-Ferretti (2001), "The External Wealth of Nations: Measures of Foreign Assets and Liabilities for Industrial and Developing Countries," Journal of International Economics 55, 263-94.
- [15] — (2004), "Financial Globalization and Exchange Rates," CEP Discussion Paper DP0662, Centre for Economic Performance, LSE.
- [16] — (2006), "The External Wealth of Nations, Mark II: Revised and Extended Estimates of Foreign Assets and Liabilities, 1970-2004," IMF Discussion Paper 06/69.
- [17] Merton, R. (1971), "Optimal Consumption and Portfolio Rules in a Continuous Time Model," Journal of Economic Theory 3, 373-413.
- [18] Obstfeld, M. (2004), "External Adjustment," Review of World Economics 140, 541-568.
- [19] Obstfeld, M. and K. Rogoff (1996), Foundations of International Macroeconomics, MIT Press.
- [20] Obstfeld, M. and K. Rogoff (1995), "The Intertemporal Approach to the Current Account", in G. Grossman and K. Rogoff, eds Handbook of International Economics, Volume III, Elsevier Science, Amsterdam.
- [21] Obstfeld, M. and K. Rogoff (2002), "Global Implications of Self-Oriented Monetary Policies," Quarterly Journal of Economics 117, 503-536.
- [22] Obstfeld, M. and K. Rogoff (2004), "The Unsustainable US Current Account: Revisited," National Bureau of Economic Research Working Paper 10869.
- [23] Pavlova, A. and R. Rigobon (2003), "Asset Prices and Exchange Rates," MIT Sloan School Working Paper 4322-03.
- [24] Saito, M. (1997), "A Note on Ergodic Distributions in Two-Agent Economies," Journal of Mathematical Economics 27, 133-41.
- [25] Schmitt-Grohe, S. and M. Uribe (2003) "Closing Small Open Economy Models," Journal of International Economics 61, 163-185.
- [26] Skorohod, A. V. (1989), Asymptotic Methods in the Theory of Stochastic Differential Equations, Providence: American Mathematical Society.
- [27] Tille, C. (2005), "Financial Integration and the Wealth Effect of Exchange Rate Fluctuations," mimeo., Federal Reserve Bank of New York.
- [28] Woodford, M. (2003), Interest and Prices, Cambridge: MIT Press.

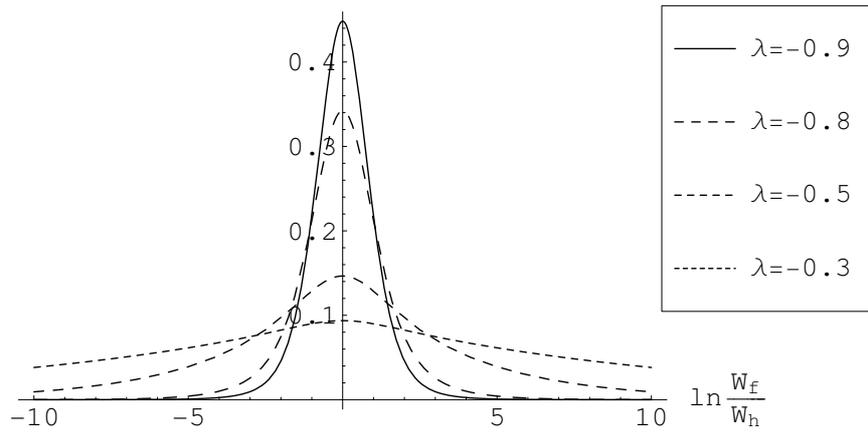


Figure 1: Density Functions of  $\kappa$  ( $= \ln \frac{W_f}{W_h}$ ) for Various Correlation Coefficients  $\lambda$

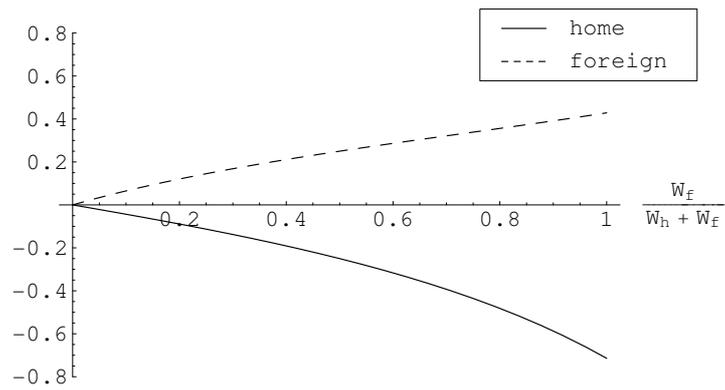


Figure 2: Portfolio Weights of Nominal Bonds for the Home Country ( $\omega_h^h$  and  $\omega_f^h$ )

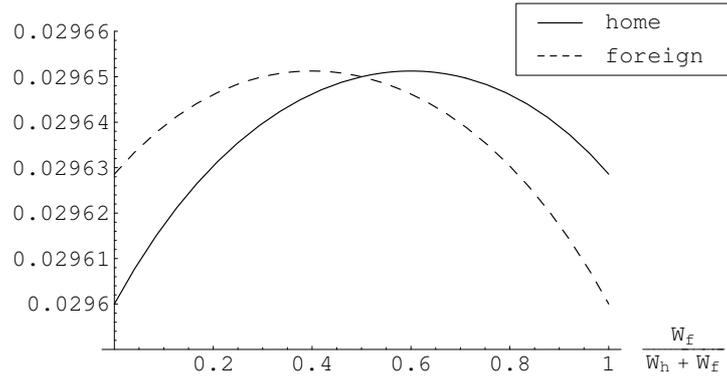


Figure 3: Real Risk-free Rates ( $r_h$  and  $r_f$ )

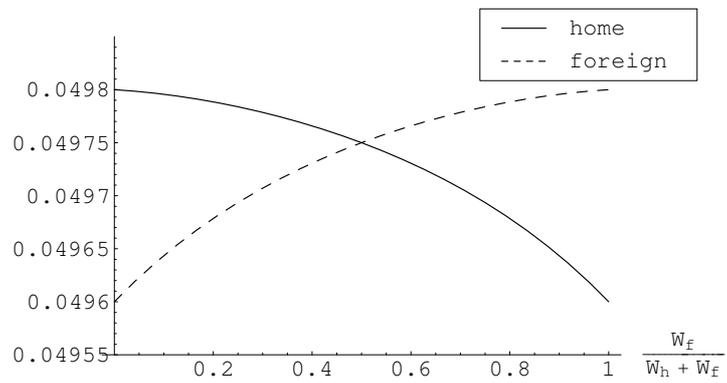


Figure 4: Nominal Interest Rates ( $R_h$  and  $R_f$ )

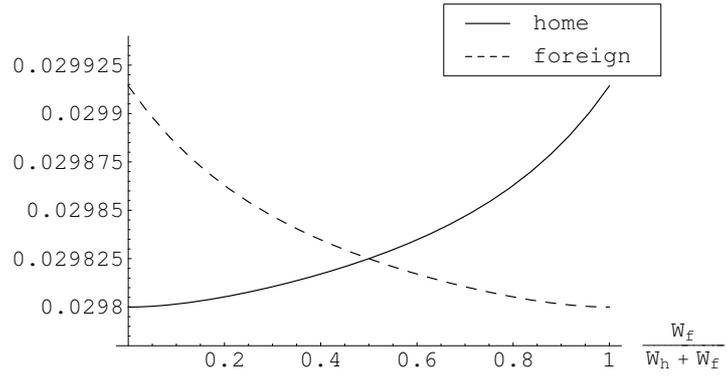


Figure 5: Risk-adjusted Wealth Growth ( $g_h$  and  $g_f$ )

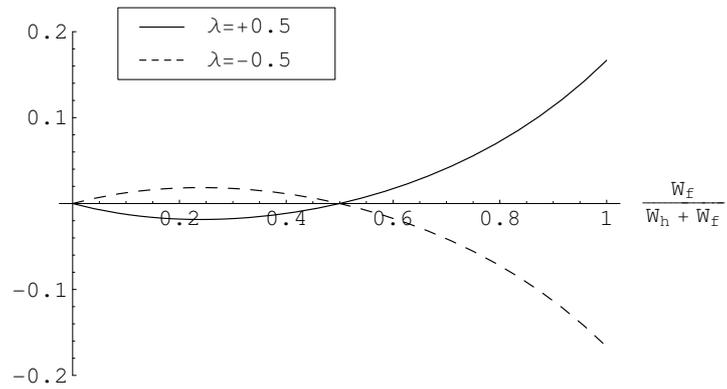


Figure 6: Nominal Bond Positions with Positive and Negative  $\lambda$

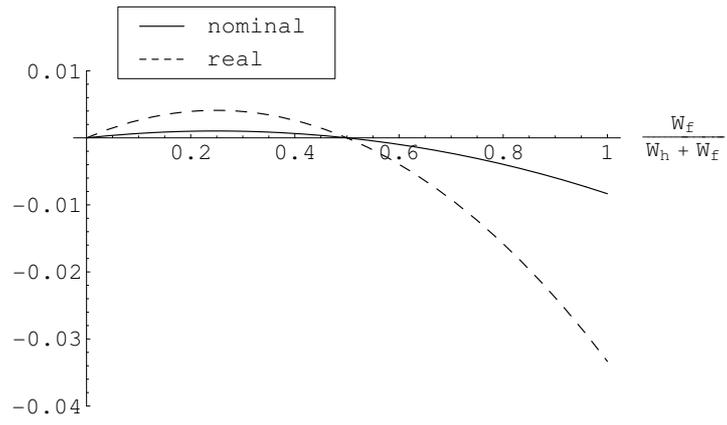


Figure 7: Nominal and Real Bond Positions with  $\lambda = -0.2$

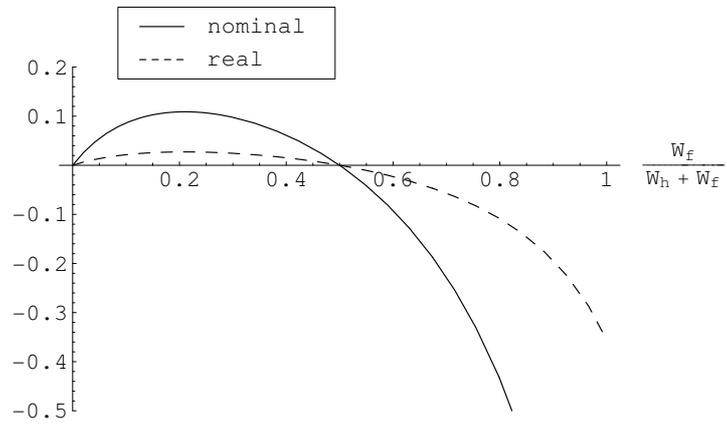


Figure 8: Nominal and Real Bond Positions with  $\lambda = -0.8$

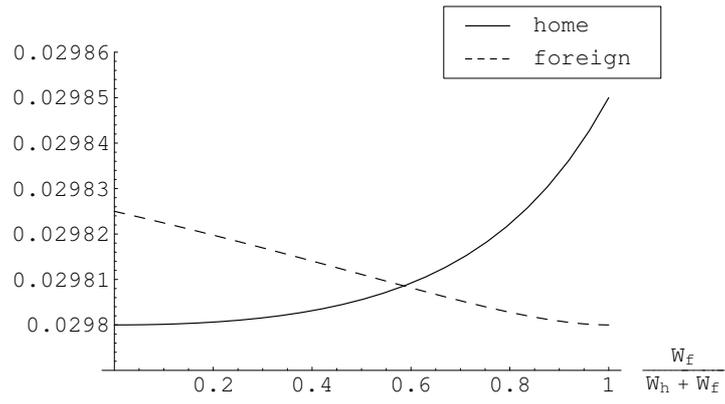


Figure 9: Risk-adjusted Wealth Growth with Short-sales Constraints ( $g_h$  and  $g_f$ )

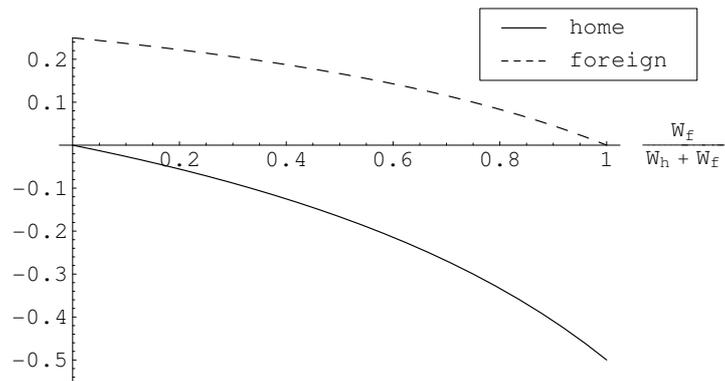


Figure 10: Positions on Home Currency Bonds with Short-sales Constraints ( $\omega_h^h$  and  $\omega_h^f$ )