DISCUSSION PAPER SERIES

No. 5711

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FINANCIAL ECONOMICS

Centre for Economic Policy Research

www.cepr.org

Available online at: www.cepr.org/pubs/dps/DP5711.asp

ISSN 0265-8003

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Discussion Paper No. 5711
June 2006

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CEPR Discussion Paper No. 5711

June 2006

ABSTRACT

Financing a Portfolio of Projects*

This paper shows that investors financing a portfolio of projects may use the depth of their financial pockets to overcome entrepreneurial incentive problems. While competition for scarce informed capital at the refinancing stage increases the investor's ex post bargaining position, it may nevertheless improve entrepreneurs' ex ante incentives. This is because projects funded by investors with 'shallow pockets' must have not only a positive NPV at the refinancing stage, but one that is higher than that of competing portfolio projects. We also show that, besides mitigating moral hazard, committing to shallow pockets may have benefits in dealing with adverse selection problems. Our paper may help to understand provisions used in venture capital finance that limit a fund's initial capital and make it difficult to add on more capital once the initial venture capital fund is raised. Our paper also provides a number of empirical implications, some of which have not yet been tested.

JEL Classification: G31 and G32
Keywords: deep versus shallow pockets and venture capital portfolio

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* We thank an anonymous referee, the editor (Bob McDonald), and seminar audiences at the London School of Economics and the First RICAFE Conference on Risk Capital and the Financing of European Innovative Firms for comments. Inderst and Münnich acknowledge financial support from the Financial Markets Group (FMG).

Submitted 08 May 2006

Venture capital finance takes place in an environment in which informational asymmetries and incentive problems are severe. Not only must venture capitalists assess the quality of investment proposals submitted to them for funding. Once the initial funding has taken place, entrepreneurs must be given the right incentives, and the performance of portfolio companies must be monitored on an ongoing basis. This paper departs from most of the existing literature by explicitly recognizing that venture capitalists manage a portfolio of projects. The need for portfolio management arises if the amount of capital—both financial and human—available to a given venture capital fund is limited, implying venture capitalists must carefully choose which portfolio projects to allocate their scarce financial and human resources to.[1] Indeed, by staging their investments, venture capitalists retain the right to deny capital infusions to particular projects in favor of other, more promising ones:

> "The most important mechanism for controlling the venture is staging the infusion of capital. ... Capital is a scarce and expensive resource for individual ventures. ... The credible threat to abandon a venture, even when the firm might be economically viable, is the key to the relationship between the entrepreneur and the venture capitalist. ... The seemingly irrational act of shutting down an economically viable entity is rational when viewed from the perspective of the venture capitalist confronted with allocating time and capital among various projects" (Sahlman (1990)).

The allocation of scarce resources to the most potent portfolio projects implies projects effectively compete with one another for limited "informed" capital at the refinancing stage.[2] As this naturally increases venture capitalists' ex post bargaining power, an intuitive conjecture might be that it reduces entrepreneurs' ex ante incentives. As we will show, however, the opposite may be true. While the entrepreneur's expected payoff from a *given* effort level is reduced ("bargaining power effect"), the difference in expected payoffs across effort levels may be increased ("competition effect"). Intuitively, competition for scarce informed capital introduces an additional incentive to have not only a positive NPV at the refinancing stage, but one that is higher than that of competing portfolio projects. If the competition effect outweighs the bargaining power effect, limiting the amount of informed capital can improve entrepreneurial incentives.

In this paper, we compare “constrained finance” (or “shallow pockets”)—i.e., committing to scarce informed capital in order to induce competition among entrepreneurs—with “unconstrained finance” (or “deep pockets”). While constrained finance may improve entrepreneurial incentives, it entails an allocational inefficiency: successful projects may not obtain capital at the refinancing stage. Accordingly, constrained finance should not be used for projects that have a high likelihood of being successful, but only for projects whose success likelihood is low. Indeed, venture capitalists acknowledge that they “go for the homerun” in order to offset the large number of failures in their portfolios (Sahlman (1990), Bygrave and Timmons (1992)).[3]

While the main focus of our model is on moral hazard, we show that constrained finance may have also advantages in dealing with adverse selection problems. Specifically, we show that separation between good and bad entrepreneurs may be impossible if investors have deep pockets, but possible if investors can choose between deep and shallow pockets. For certain parameter values, the unique equilibrium is a separating equilibrium in which good entrepreneurs choose constrained finance while bad ones choose unconstrained finance.

Evidence from venture capital funds and the partnership agreements governing them support the notion of competition for scarce financial and human capital among portfolio companies envisioned here. As is well known, “venture organizations will limit both how often they raise funds and the size of the funds that they raise” (Gompers and Lerner (1996)). Moreover, while venture capitalists raise a new fund every few years, partnership agreements include covenants preventing venture capitalists from co-investing in companies managed by other other funds of the same venture capitalist, implying once a fund is raised, it cannot be easily augmented by adding on more capital (Sahlman (1990), Fenn, Liang, and Prowse (1995), Gompers and Lerner (1996)).[4] Not only is a fund’s financial capital limited from the outset, but so is its human capital: partnership agreements often include covenants restricting the ability to add on more general partners—i.e., experienced venture capitalists—to an existing fund (Gompers and Lerner (1996)).[5] The consequence is that venture capitalists must carefully choose which portfolio companies to allocate their scarce financial and human capital to, which leads to precisely the sort of competition envisioned in this paper.

Most of the theoretical literature on venture capital finance considers the financing of a single project. Exceptions are Kanniainen and Keuschnigg (2003), Bernile, Cumming, and Lyandres (2005), and Fulghieri and Sevilir (2005), who all consider the optimal span of a venture capitalist's portfolio. In contrast, holding the span of the venture capitalist's portfolio fixed, we consider the benefits and costs of venture capitalists being capital constrained.

In a broader context, this paper shows that prominent arguments made in other strands of economics are also relevant for venture capital portfolio financing. Without trying to be exhaustive, let us point out three important parallels.

First, a potential disadvantage of constrained finance in our model is that it weakens entrepreneurs' bargaining position, thus reducing their incentives to exert effort. *If* entrepreneurs can be motivated to exert high effort, however—because the competition effect outweighs the bargaining power effect—then this disadvantage may turn into an advantage: due to the investor's stronger bargaining position, projects that would otherwise not be financially viable may now become viable. The general idea of strenghtening the (bargaining) position of the party whose contribution is relatively more important has been explored in several papers, notably in Grossman and Hart (1986), Hart and Moore (1990), Aghion and Tirole (1997), and—in a corporate financing context—Aghion and Bolton (1992) and Gertner, Scharfstein, and Stein (1994). Aghion and Bolton, in particular, make the point that strengthening the position of investors may render projects financially viable that might not be viable otherwise.

Second, the idea that competition for scarce capital may improve effort incentives ("competition effect") borrows from the labor tournament literature (Lazear and Rosen (1981), Nalebuff and Siglitz (1983)). There is one, albeit subtle, qualification: in many real-world tournaments, prizes are exogenously given, e.g., there is only one CEO position in a firm. In contrast, our model implies that in a portfolio financing context, investors can provide optimal incentives by carefully choosing the ratio of available capital to projects.

Third, there is an obvious parallel to the literature on soft-budget constraints, started by Kornai (1979, 1980) in the context of socialist economies and applied by Dewatripont and Maskin (1995) to financial commitment problems. There is, again, a subtle but noteworthy difference: in Dewatripont and Maskin's modelk, the role of hard budget constraints is to deter bad entrepreneurs from seeking financing ex ante. In our model, by contrast, the role of hard budget constraints—or shallow pockets—is to credibly commit to a tournament in order to elicit higher entrepreneurial effort.

Issues similar to those addressed in our paper are also addressed in the internal capital markets literature. On the positive side, internal capital markets may allow for an efficient ex post realloction of resources, known as "winner-picking" (Stein (1997), Matsusaka and Nanda (2002)).[6] On the negative side, the prospect of having resources reallocated away from them may weaken division managers' ex ante incentives (Brusco and Panunzi (2005)).[7] Interestingly, in our model the positive and negative sides are exactly the opposite: unlike in an internal capital market, the ex post resource allocation is *less* efficient under constrained finance, while entrepreneurs' ex ante incentives may be *improved*.[8]

Finally, our paper is related to the capital budgeting literature, notably Harris and Raviv (1996, 1998). The authors show that imposing a fixed spending limit—which can be relaxed at the cost of a subsequent audit—may be part of an optimal capital budgeting procedure. Like in our model, it may thus be optimal to rational capital even if doing so means forgoing positive NPV investments. The reason is different, though. In our model, capital rationing improves entrepreneurs' incentives. In Harris and Raviv's models, by contrast, capital rationing induces truthful revelation of division managers' private information while economizing on audit costs.

The rest of this paper is organized as follows. Section 1 describes the model. Section 2 examines the benefits and costs of constrained finance with respect to providing effort incentives. Section 3 considers the investors' optimal choice between constrained and unconstrained finance. Section 4 discusses the role of ex ante and interim asymmetric information. Section 5 concludes. All proofs are in the Appendix.

1 The Model

Agents and Technology

There are two types of agents: entrepreneurs, who have no wealth, and investors. Each entrepreneur has a project which requires an initial capital outlay of $I_1 > 0$ at $t = 0$. Projects can be refinanced at $t = 1$ at cost $I_2 > 0$. Refinancing is best understood as an expansion of the project. Projects that are not refinanced continue on a smaller scale in a sense made precise below.[9] At $t = 2$, each project generates a verifiable payoff of either $R > 0$ or zero.

At $t = 1$, when the refinancing decision is made, a project's "interim type" is $\psi \in \{n, l, h\}$. The interim type is only observed by the investor and entrepreneur. Projects with interim type $\psi = n$ are a failure and generate a zero payoff for sure. Projects with interim type $\psi = l$ or $\psi = h$ are a success, which implies it is efficient to refinance them. If a project with interim type $\psi \in \{l, h\}$ is refinanced, its probability of generating R is p_ψ, where $p_h > p_l$, implying its expected payoff is $R_\psi := p_\psi R$. By contrast, if a project with interim type $\psi \in \{l, h\}$ is not refinanced, its probability of generating R is p_0, which implies its expected payoff is $R^0 := p_0 R$.[10] Hence, the overall surplus from refinancing a project with interim type $\psi \in \{l, h\}$ is $r_\psi := R_\psi - R^0 - I_2$, which is positive, and where $r_h > r_l$ follows from our assumption that $p_h > p_l$.

With probability $1 - \tau$ the project's interim type is $\psi = n$, while with probability τ its interim type is either $\psi = l$ or $\psi = h$. Conditional on being a success, the probability of having interim type $\psi = h$ is q_θ, while the probability of having interim type $\psi = l$ is $1 - q_\theta$, where $\theta \in \{g, b\}$ represents the project's "ex ante type." Accordingly, the total probability that the project has interim type $\psi = h$ is τq_θ, while the total probability that it has interim type $\psi = l$ is $\tau(1 - q_\theta)$. We assume that $q_g > q_b$, i.e., good projects have a higher probability of becoming interim type $\psi = h$ than bad projects. Figure 1 summarizes the project technology.

[Figure 1 here]

We assume entrepreneurs can choose their ex ante type at $t = 0$. This choice is only observed by the entrepreneur ("moral hazard"). Choosing ex ante type θ yields private benefits B_θ at $t = 2$, where $B_b = B > B_g = 0$. These benefits are only obtained if the project is a success. As B constitutes the opportunity cost of choosing $\theta = g$ instead of $\theta = b$, we refer to B simply as "effort cost" and to the entrepreneur's choice of $\theta = g$ and $\theta = b$ as "high effort" and "low effort", respectively. Finally, we assume that $(q_g - q_b)(r_h - r_l) > B$, implying high effort is socially efficient.

Financing

Investors compete at $t = 0$ to provide financing to entrepreneurs. We specify that each investor optimally provides start-up finance to two entrepreneurs.[11] In principle, investors can raise sufficient capital initially so that at $t = 1$ they are able to refinance all projects that are worth refinancing. The central claim of this paper, however, is that investors may sometimes deliberately want to limit the amount of capital raised in order to create competition among entrepreneurs at the refinancing stage. As noted in the Introduction, evidence from venture capital funds and the partnership agreements governing them supports the notion of competition for scarce financial and human capital envisioned here.

A priori, it is not clear why the investor would not attempt to raise additional capital at $t = 1$ if both projects turn out to be successful. Indeed, we do not preclude the investor from trying to raise additional capital. However, as only the (inside) investor and the entrepreneur know the project's interim type, there exists a lemons problem vis-à-vis outside investors that may render outside financing infeasible, like in Rajan (1992). We relegate a formal analysis of this issue to Section 4.2. For the time being, we simply assume that the lemons problem at $t = 1$ is sufficiently strong to render outside financing infeasible.

The investor's choice is then between what we call *unconstrained finance* (or "deep pockets") and *constrained finance* (or "shallow pockets"). This choice is observable by entrepreneurs. Under unconstrained finance, investors raise sufficient capital so that they can potentially refinance both portfolio projects at $t = 1$, i.e., investors raise $2I_1 + 2I_2$. Under constrained finance, in contrast, investors only raise $2I_1 + I_2$ initially. Any capital currently not used is invested in the (short-term) "market technology", whose interest rate is normalized to zero.

Contracts and Renegotiations

Investors compete ex ante by offering contracts specifying for each entrepreneur E_i a share s_i of the project's final payoff. By restricting ourselves to sharing rules, we rule out transfer payments to entrepreneurs that are independent of the project's payoff. The usual motivation for this assumption is that guaranteed payoff-independent transfer payments would attract fraudulent entrepreneurs, or "fly-by-night operators" (Rajan (1992)), who would only apply to cash in the guaranteed transfer payment.[12]

Since the project's interim type is nonverifiable, the refinancing decision cannot be part of an initial contract. Hence, whether the project will be refinanced must be determined by negotiations between the investor and entrepreneur at $t = 1$. As part of these negotiations, the two parties may renegotiate the initial sharing rule s_i, which is why we henceforth use the term *renegotiations*. Despite the fact that the initial sharing rule is renegotiated, it is not meaningless, however: it defines the entrepreneur's and investor's payoffs if the project is not refinanced, and thus their outside options if the renegotiations break down. Where do the bargaining powers in the renegotiations emanate from? The entrepreneur's bargaining power simply stems from his ability to withdraw his inalienable and essential human capital. The investor's bargaining power, in contrast, stems from her decision rights with regard to the refinancing decision.[13]

The assumption that the project's interim type is nonverifiable is important. It implies the refinancing decision cannot be part of an initial contract, which in turn forces the investor and entrepreneur into a bargaining situation at the refinancing stage. Evidence from the venture capital literature supports this assumption. For instance, Gompers (1995) writes: "Each time capital is infused, contracts are written and negotiated ... Major review of progress, due diligence, and the decision to continue funding are generally done at the time of the refinancing." The fact that contracts are negotiated anew at the refinancing stage suggests it might be difficult to specify ex ante what precisely "progress" means. Indeed, Gompers (1995) rejects the alternative hypothesis of contingent follow-up financing based on observable "technology-driven milestones".[14] Similarly, Kaplan and Strömberg (2003) write: "We consider a financing round as a set of contracts agreed to on a particular date that determines the disbursements of funds from the VC to a company. A new financing round *differs from the contingent release of funds* in that the price and terms of the financing are not set in advance" (italics added).

2 Refinancing and Renegotiations

Solving the model backwards, we first consider the renegotiations at $t = 1$. Subsequently, we derive the entrepreneur's expected payoff at $t = 0$ taking into account the outcome of the renegotiations. We then compute the sensitivity of the entrepreneur's expected payoff with respect to his ex ante type. Comparing the sensitivities under unconstrained and constrained finance, we finally obtain what we call the "responsiveness condition".

2.1 Renegotiations under Unconstrained Finance

Under unconstrained finance, the investor has sufficient capital to refinance all projects that are worth refinancing. As a result, she cannot credibly threaten not to refinance a project with interim type $\psi \in \{l, h\}$, regardless of the interim type of the other portfolio project. Consequently, the refinancing decision for a particular project is independent of the other project, implying we can analyze the renegotiations with each entrepreneur separately.

Consider the renegotiations with entrepreneur E_i. Given that the investor knows E_i's interim type, renegotiations take place under symmetric information. We adopt the standard alternating offers bargaining procedure with open time horizon analyzed in Rubinstein (1982). While the bargaining procedure is open ended, bargaining frictions ensure that an agreeement is reached immediately. As for the specific type of bargaining friction employed here, we follow Binmore, Rubinstein, and Wolinsky (1986) and assume that after each round there is a probability δ that the renegotiations break down, in which case the project is not refinanced.[15]

Without loss of generality, we assume the investor makes the first offer, which E_i can either accept or reject.[16] The offer is to provide refinancing in return for a share of the project's payoff. If E_i rejects the investor's offer, he can—provided there has been no breakdown—make a counteroffer, and so on. It is crucial that the entrepreneur can make counteroffers. If all E_i could do is accept or reject the investor's offers, the investor could extract the entire surplus. E_i's continuation payoff at $t = 1$ would then always be $s_i R^0$ regardless of his interim type, which in turn implies there would be no difference between constrained and unconstrained finance in terms of incentive provision. However, a bargaining procedure in which only the investor can make offers would imply that she can credibly *commit* not to listen to any offers made by the entrepreneur, which seems to be difficult to implement in practice.[17]

The analysis of the bargaining game is straightforward. If a project with interim type $\psi_i \in \{l, h\}$ is not refinanced, it generates an expected payoff of R^0. Hence, if $\psi_i \in \{l, h\}$ the outside options in the renegotiations are $(1 - s_i)R^0$ and $s_i R^0$, respectively, while the surplus to be bargained over is r_{ψ_i}. Lemma 1 characterizes the equilibrium outcome of the bargaining game as $\delta \to 0$. The proof follows Binmore, Rubinstein, and Wolinsky (1986).

Lemma 1. *Under unconstrained finance, the investor's and entrepreneur E_i's continuation payoffs at $t = 1$ are as follows:*

i) If E_i has interim type $\psi_i = n$, both continuation payoffs are zero.

ii) If E_i has interim type $\psi_i \in \{l, h\}$, E_i's continuation payoff is $s_i R^0 + \frac{1}{2} r_{\psi_i}$ while the investor's *continuation payoff is* $(1 - s_i)R^0 + \frac{1}{2}r_{\psi_i}$.

Proof. See Appendix. ■

2.2 Renegotiations under Constrained Finance

Under constrained finance, the investor cannot refinance all projects that are worth refinancing, implying she can credibly threaten to use her scarce capital for the other portfolio project. The renegotiations with E_i thus depend on the interim type of the other entrepreneur, E_j, for two reasons. First, who the investor picks first to bargain with depends on who has a higher interim type. Second, the investor's outside option in the renegotiations with E_i depends on E_j's interim type, and vice versa.[18]

The extensive form of the bargaining game is as follows. The investor picks one of the two entrepreneurs, say E_i, and makes him an offer. If E_i accepts, the game ends. If E_i rejects the investor's offer, the negotiations with E_i break down with probability δ. If there has been no breakdown, E_i can make a counteroffer. If the investor accepts E_i's counteroffer, the game ends. If the investor rejects, the negotiations with E_i break down with probability δ. If the negotiations with E_i have not yet broken down, the investor picks again one of the two entrepreneurs, and so on. In contrast, if the negotiations with E_i have broken down, the investor must necessarily turn to E_j. Hence, the bargaining procedure is the same alternating offer procedure with open time horizon and risk of breakdown as in the case of unconstrained finance, except that after each round the investor can choose anew with whom to bargain next.

If at least one entrepreneur has interim type $\psi = n$, the outcome is trivially the same as under unconstrained finance. The interesting case is where neither entrepreneur has interim type $\psi = n$. As the following lemma shows, the investor can then extract a higher continuation payoff from her first pick, say E_i, relative to unconstrained finance. The downside is that she cannot realize any surplus with her second pick E_j as her capital has already been used up.

Lemma 2. *Under constrained finance, the investor's and the two entrepreneurs' continuation* *payoffs at $t = 1$ are as follows*:

i) If at least one entrepreneur has interim type $\psi = n$, all payoffs are as in Lemma 1.

ii) If neither entrepreneur has interim type $\psi = n$, and if the investor picks E_i first to bargain with, then

a) E_i's continuation payoff is $s_i R^0 + \frac{1}{2}(r_{\psi_i} - \frac{1}{2} r_{\psi_j})$,

b) E_j's continuation payoff is $s_j R^0$, and

c) the investor's continuation payoff is $(1 - s_i) R^0 + (1 - s_j) R^0 + \frac{1}{2}(r_{\psi_i} + \frac{1}{2} r_{\psi_j})$.

Proof. See Appendix. ■

Interestingly, if both entrepreneurs have the same interim type $\psi \in \{l, h\}$, the investor cannot extract the entire surplus from her first pick E_i even though the other entrepreneur is a perfect substitute. This may seem surprising. Why does the investor not deviate and go to the other entrepreneur E_j, who should be eager to obtain refinancing even under less favorable conditions given that he would otherwise only obtain $s_j R^0$? The reason is that E_j would not accept an offer that leaves him just a little more than his outside option payoff. He would instead reject the investor's offer and make a counteroffer that makes the investor indifferent between accepting and going back to her first pick E_i.

Consider finally the issue who the investor picks first to bargain with. First of all, note that the initial sharing rule s_i does not affect the investor's choice—this choice depends exclusively on the entrepreneurs' interim types. When the two interim types are not identical, the investor bargains first with the higher interim type. When the two interim types are identical, the investor is indifferent. In this case, we specify that she picks either of the two entrepreneurs with equal probability (see Proof of Lemma 2).

2.3 The Responsiveness Condition

Given Lemmas 1 and 2, we can compute the entrepreneur's expected payoff at $t = 0$. The derivation is in the Appendix. The entrepreneur's expected payoff under unconstrained finance is

$$\tau\left\{s_iR^0+\frac{1}{2}\left[r_l+q_{\theta_i}\left(r_h-r_l\right)\right]\right\}. \tag{1}$$

Below we consider the entrepreneur's effort choice problem. The more responsive is the entrepreneur's expected payoff to his ex ante type, the easier it will be to motivate him to choose $\theta = g$ rather than $\theta = b$. We obtain the responsiveness under unconstrained finance by subtracting the entrepreneur's expected payoff for $\theta_i = b$ from that for $\theta_i = g$:

$$\frac{1}{2}\tau\left(q_g-q_b\right)\left(r_h-r_l\right). \tag{2}$$

Importantly, the responsiveness does not correspond to the full difference in expected project values as the investor can extract part of this value in the renegotiations.

Likewise, the entrepreneur's expected payoff under constrained finance is

$$\tau\left\{s_iR^0+\frac{1}{2}\left[r_l+q_{\theta_i}\left(r_h-r_l\right)\right]\right\}-\frac{\tau^2}{8}\left\{r_l\left(3-q_{\theta_i}+q_{\theta_j}\right)+3q_{\theta_i}q_{\theta_j}\left(r_h-r_l\right)\right\}. \tag{3}$$

Under constrained finance, the two entrepreneurs compete for scarce informed capital. Consequently, if the other entrepreneur also has a profitable refinancing opportunity, the investor can extract more from a given entrepreneur than under unconstrained finance. Our key insight, however, is that offering constrained finance may nevertheless increase the responsiveness of the entrepreneur's expected payoff to his ex ante type: while the investor's stronger ex post bargaining position reduces the entrepreneur's expected payoff for a *given* ex ante type, the difference in expected payoffs across ex ante types can be increased. As will become clear shortly, we are interested in the case where both entrepreneurs choose $\theta = g$. Consequently, we obtain the responsiveness under constrained finance by setting $\theta_j = g$ and subtracting the entrepreneur's expected payoff for $\theta_i = b$ from that for $\theta_i = g$:

$$\frac{1}{2}\left(q_g-q_b\right)\tau\left\{\left(r_h-r_l\right)+\frac{\tau}{4}\left[r_l-3q_g\left(r_h-r_l\right)\right]\right\}. \tag{4}$$

Comparing the responsiveness under unconstrained finance, (2), with that under constrained finance, (4), establishes the following proposition.

Proposition 1. *The responsiveness of the entrepreneur's expected payoff to his ex ante type is higher under constrained finance than under unconstrained finance if and only if*

$$r_h - r_l < \frac{r_l}{3q_g}. \tag{5}$$

We will henceforth refer to (5) as the "responsiveness condition." It captures the tradeoff between two effects of competition for scarce informed capital under constrained finance:

Competition Effect: Under constrained finance, not being picked first to be bargained with implies that the entrepreneur will not receive refinancing in equilibrium, in contrast to unconstrained finance. As a result, competition for scarce informed capital introduces an additional incremental return to being picked first to be bargained with, thus making the entrepreneur's expected payoff more sensitive to his ex ante type.

Bargaining Power Effect: Under constrained finance, the investor can threaten to refinance the other entrepreneur when bargaining with her first pick, which creates additional bargaining power for the investor. As a result, the entrepreneur's return from being refinanced is reduced and the responsiveness is lowered.

If the responsiveness condition (5) holds, the entrepreneur's expected payoff under constrained finance is more sensitive to his ex ante type than under unconstrained finance. Put simply, constrained finance then provides stronger effort incentives than unconstrained finance. Intuitively, unconstrained finance provides effort incentives through the difference in final payoffs $r_h - r_l = R_h - R_l$ (see (2)). If this difference is large, the incentives provided under unconstrained finance are already quite substantial. Accordingly, the additional incentives provided under constrained finance through competition for scarce informed capital have then relatively little value, and the competition effect is dominated by the bargaining power effect. Conversely, if $r_h - r_l$ is small, the incentives provided under unconstrained finance are relatively small, and the additional incentives provided under constrained finance through competition for scarce informed capital offset the negative bargaining power effect. As we will show in the following section, (5) is a necessary, albeit not a sufficient, condition for constrained finance to be chosen.

3 Constrained versus Unconstrained Finance

3.1 Analysis

We will now analyze the investors' choice between constrained and unconstrained finance. There are exactly two cases in which investors will choose constrained finance. One is the case in which constrained finance is the only viable alternative, i.e., investors can only break even under constrained finance. The other is the case in which both alternatives are viable, but constrained finance provides entrepreneurs a higher expected payoff. As there is ex ante competition for entrepreneurs, investors choose constrained finance in this case.

As is easy to show, neither of these two cases is possible if constrained and unconstrained finance both implement the same level of effort. Hence, a *necessary* condition for constrained finance to be chosen is that it induces higher effort than unconstrained finance, i.e., constrained finance must implement $\theta = g$ while unconstrained finance must implement $\theta = b$. By (2) and (4), this in turn implies, firstly, that the responsiveness condition (5) must hold, and secondly, that the effort cost B must lie in the intermediate range

$$\frac{1}{2}(q_g - q_b)(r_h - r_l) \leq B < \frac{1}{2}(q_g - q_b)\left\{(r_h - r_l) + \frac{\tau}{4}\left[r_l - 3q_g(r_h - r_l)\right]\right\}. \tag{6}$$

Condition (6) has an intuitive interpretation.[19] If effort is not particularly costly so that even unconstrained finance can induce high effort, constrained finance cannot play out its advantage of providing *relatively* stronger effort incentives. Conversely, if effort is extremely costly so that even constrained finance cannot induce high effort, then again the fact that constrained finance provides relatively stronger effort incentives does not matter.

If these necessary conditions hold, the choice between constrained and unconstrained finance becomes straightforward. If only constrained finance is viable—i.e., investors can only break even under constrained finance—then constrained finance is chosen. Likewise, if only unconstrained finance is viable, then unconstrained finance is chosen. Finally, if constrained and unconstrained finance are both viable, competition for entrepreneurs implies that investors will choose the financing mode that provides entrepreneurs a higher expected payoff.

To see whether the project is financially viable, we must derive the investor's expected payoff at $t = 0$. The derivation is analogous to that of (1) and (3), with the addition that $\theta_i = b$ for unconstrained and $\theta_i = \theta_j = g$ for constrained finance (see Proof of Proposition 2). As the investor's expected payoff is decreasing in the entrepreneur's payoff share, the project is viable if and only if the investor's expected payoff is non-negative at $s_i = 0$. Accordingly, the project is viable under unconstrained finance if and only if

$$\pi_U^I := \tau \left\{ R^0 + \frac{1}{2} \left[r_l + q_b \left(r_h - r_l \right) \right] \right\} \geq I_1, \tag{7}$$

while it is viable under constrained finance if and only if

$$\pi_C^I := \tau \left\{ R^0 + \frac{1}{2} \left[r_l + q_g \left(r_h - r_l \right) \right] \right\} - \frac{\tau^2}{8} \left\{ r_l + q_g^2 \left(r_h - r_l \right) \right\} \geq I_1. \tag{8}$$

If constrained and unconstrained finance are both viable, ex ante competition among investors implies they will choose the financing mode that is better for entrepreneurs. The entrepreneur's expected payoff in this case can be easily derived from (1) and (3) and the investors' zero-profit condition (see Proof of Proposition 2). The following proposition summarizes the investors' optimal choice between unconstrained and constrained finance.

Proposition 2. *Suppose the responsiveness condition* (5) *holds and B satisfies* (6). *For any given investment cost I_1, projects whose success probability τ is sufficiently low will not be financed. For projects that are financially viable, the following holds*:

i) For projects whose investment cost is sufficiently high—provided the project is financed at all—only unconstrained finance will be chosen.

ii) For projects whose investment cost is low, other things equal, constrained finance will be *chosen if the project's success probability is low and unconstrained finance will be chosen if the project's success probability is high.*

Proof. See Appendix. ■

By Proposition 2, if either i) the responsiveness condition (5) is violated, implying unconstrained finance provides relatively stronger effort incentives than constrained finance, or if ii) the effort cost B is either too low or too high so that (6) is violated, implying constrained and unconstrained finance both implement the same effort, or if iii) the investment cost is too high, then constrained finance will *not* be chosen.[20] Conversely, if i)-iii) hold, then constrained finance will be chosen for relatively low success probabilities while unconstrained finance will be chosen for relatively high success probabilities.

Proposition 2 is illustrated in Figure 2. The success probability τ is depicted on the X-axis and the investment cost I_1 is depicted on the Y-axis. The vertically and horizontally shaded areas depict all (τ, I_1) combinations for which constrained and unconstrained finance are chosen, respectively. The unshaded area depicts all (τ, I_1) combinations for which the project is not financially viable.

Perhaps the simplest way to illustrate Proposition 2 is by fixing I_1 and drawing an imaginary horizontal line originating at I_1 that runs parallel to the X-axis. (In Proposition 2, "fixing I_1" is implied by "other things equal", which implies projects are only compared with respect to their success probabilities.) Holding I_1 fixed, the intersection of this horizontal line with the unshaded area shows all success probabilities for which the project is not viable, the intersection with the vertically shaded area shows all success probabilities for which constrained finance is chosen, while the intersection with the horizontally shaded area shows all success probabilities for which unconstrained finance is chosen.[21]

[Figure 2 here]

Part i) of Proposition 2 refers to values of I_1 that lie above the point where π_U^I and π_C^I intersect. For such high investment costs, the project is only viable if the success probability is also high, in which case unconstrained finance is chosen. Intuitively, for high success probabilities the allocational inefficiency induced by constrained finance—namely, that if both projects are successful, one of them will not be refinanced—weighs heavily in expected terms.

Part ii) of Proposition 2 refers to values of I_1 that lie below the intersection of π_U^I and π_C^I. Holding I_1 fixed, the horizontal line originating at I_1 intersects first with the unshaded area, then with vertically shaded area, and finally with the horizontally shaded area. Projects with relatively low success probabilities are thus financed under constrained finance, while projects with high success probabilities are financed under unconstrained finance.

In Figure 2, $\tau = \widehat{\tau}$ marks the critical success probability at which the entrepreneur's expected payoffs under constrained and unconstrained finance intersect.[22] If both financing modes are financially viable, constrained finance is chosen for success probabilites $\tau \leq \widehat{\tau}$, while unconstrained finance is chosen for success probabilites $\tau > \widehat{\tau}$. In the (vertically shaded) "lens-shaped" area, unconstrained finance is not viable, implying constrained finance is chosen also for success probabilities $\tau > \widehat{\tau}$.

Proposition 2 lends itself to two intuitive empirical implications. The first implication is that projects whose investment cost is very high should not be financed under constrained finance. This statement is independent of whether the two necessary conditions (5) and (6) hold. Unfortunately, a similarly strong statement cannot be made with respect to the question of when projects *should* be financed under constrained finance, for two reasons: i) the necessary conditions (5) and (6) may not hold, and ii) the investment cost may be too high so that Part i) of Proposition 2 applies. However, one can make the converse, and in some sense slightly weaker, statement that *if* projects are financed under constrained finance, then these projects must have, other things equal, lower success probabilities than comparable projects financed under unconstrained finance.

We conclude with a comparative statics exercise. The benefit of constrained finance in our model is that it may induce high entrepreneurial effort when unconstrained finance can only induce low effort. But if the efficiency loss from exerting low effort is relatively small, this benefit is also small. Intuitively, we might thus expect that constrained finance will be more likely to be chosen if the efficiency loss from exerting low effort is large, which is the case if q_b—the likelihood that exerting low effort generates a high interim type $\psi = h$—is small. The following corollary formalizes this intuition.

Corollary 1. *Other things equal, an increase in the efficiency loss from having low entrepreneurial effort makes it more likely that constrained finance will be chosen.*

Given the analysis in the Proof of Proposition 2, the Proof of Corollary 1 is immediate. In Figure 2, a decrease in q_b shifts both $\widehat{\tau}$ and π_U^I to the right, thus strictly expanding the range of success probabilities for which constrained finance is chosen.[23]

3.2 Empirical Implications

The first implication summarizes a key insight of our model:

Implication 1. *Other things equal, projects financed under constrained finance should have lower success probabilities than comparable projects financed under unconstrained finance.*

The intuition, which is at the heart of our model, is that for high success probabilities the allocational inefficiency induced by constrained finance—namely, that successful projects may not be refinanced—weighs heavily in expected terms, implying such projects will be financed under unconstrained finance.

Like Implication 1, the following implication has been already discussed in the previous section.

Implication 2. *Other things equal, projects whose investment cost is very high should not be financed under constrained finance.*

The intuition is closely related to that of Implication 1. Projects whose investment cost is very high require a high success probability to break even. But for high succcess probabilities, the benefits of constrained finance are outweighed by its costs.

The next empirical implication is a restatement of Corollary 1.

Implication 3. *Other things equal, projects should be more likely to be financed under constrained finance if the efficiency loss from having low entrepreneurial effort is large.*

There are two aspects to the entrepreneurs' effort problem in our model. The first, addressed in Implication 3, is how *important* is entrepreneurial effort? That is, what is the efficiency loss from having low (instead of high) entrepreneurial effort? Intuitively, if the efficiency loss from having low entrepreneurial effort is small, the benefits of constrained finance—namely, that it provides relatively stronger effort incentives—are also small and likely to be outweighed by the allocational inefficiency it induces.

The second aspect concerns the severity of the effort problem, i.e., how *costly* is effort? With regard to this issue, a necessary condition for constrained finance to be chosen is that effort is sufficiently costly. If effort is not particularly costly so that even unconstrained finance can induce high effort, constrained finance cannot play out its advantage of providing *relatively* stronger effort incentives. By the same token, entrepreneurial effort must not be too costly. If effort is extremely costly so that even constrained finance cannot induce high effort, constrained finance again loses its advantage. We thus have:

Implication 4. *Projects for which eliciting entrepreneurial effort is either not particularly costly or extremely costly should be financed under unconstrained finance.*

An immediate corollary to Implication 4 is that, other things equal, we should see that projects financed under constrained finance exhibit higher entrepreneurial effort. Importantly, our model does not predict that projects financed under constrained finance should have a higher ex post success likelihood. While in our model constrained finance is chosen only if it induces higher effort, Implication 1 states that projects financed under constrained finance should have a lower ex ante success probability. As the two effects go in opposite directions, the overall effect on the project's ex post success likelihood remains ambiguous.

Under unconstrained finance there is no allocational inefficiency: projects rejected at the refinancing stage are always negative NPV projects. By contrast, under constrained finance rejected projects may have either a negative or positive NPV.

Implication 5. *Projects rejected at the refinancing stage under constrained finance should on average have a higher NPV than projects rejected under unconstrained finance.*

It would seem that a natural corollary to Implication 5 is that projects rejected under constrained finance should find it easier to obtain outside finance. As we will show in Section 4.2, this may or may not be true. In particular, if the lemons problem faced by outside investors is sufficiently strong, then projects rejected under constrained and unconstrained finance may both find it impossible to attract outside finance.

A related empirical implication concerns the *likelihood* that projects will be rejected at the refinancing stage. Under unconstrained finance this likelihood is simply $1 - \tau$. By contrast, under constrained finance the likelihood of rejection is strictly higher.[24] Moreover, we know from Implication 1 that projects for which constrained finance is chosen should have lower ex ante success probabilities to begin with. As both effects go in the same direction, we have:

Implication 6. *Projects financed under constrained finance should have a higher likelihood of being rejected at the refinancing stage than projects financed under unconstrained finance.*

4 Adverse Selection

In this section we consider the role of asymmetric information both at the ex ante and the refinancing stage. In our base model, we have assumed that entrepreneurs can choose their ex ante type. In Section 4.1 we now assume instead that the ex ante type is chosen by nature, while only the entrepreneur can observe nature's choice. Hence, we consider an adverse selection problem instead of a moral hazard problem.

In Section 4.2 we consider the role of asymmetric information at the refinancing stage. While the (inside) investor and the entrepreneur know the project's interim type, outside investors do not. In our base model, we have assumed that the resulting lemons problem is sufficiently strong to render outside financing at the refinancing stage infeasible. We now formally show under what conditions this will be the case. Moreover, in case outside financing at the refinancing stage *is* feasible, we will show that our results still hold qualitatively.

4.1 Ex Ante Asymmetric Information

Contrary to our base model, we now assume that the entrepreneur's ex ante type is chosen by nature prior to $t = 0$. With probability α nature chooses $\theta = g$, while with probability $1 - \alpha$ nature chooses $\theta = b$. Entrepreneurs know their ex ante type, but investors do not. Hence, at $t = 0$ when investors compete for entrepreneurs, the former face an adverse selection problem. To simplify the exposition, we assume projects are financially viable. From our previous analysis, we know that this is the case if the initial investment I_1 is not too large.

Suppose for the moment that unconstrained finance is the only financing mode available to investors. We consider competitive equilibria à la Rothschild-Stiglitz (1976). As explained previously, the initial sharing rule s_i does not affect the investor's choice as to which project she will refinance. Consequently, separation between ex ante types $\theta = g$ and $\theta = b$ cannot be achieved by offering a menu of initial sharing rules: both types of entrepreneurs would strictly prefer the highest sharing rule offered. The following result is then immediate.

Lemma 3. *Suppose unconstrained finance is the only financing mode available to investors. Then the unique competitive equilibrium is a pooling equilibrium in which all entrepreneurs receive the same sharing rule regardless of their ex ante type.*

We now argue that allowing investors to choose between constrained and unconstrained finance may enable them to separate between type $\theta = g$ and type $\theta = b$ entrepreneurs. Recall from Proposition 1 that if the responsiveness condition (5) holds, the payoff differential across ex ante types is larger under constrained finance. This implies that (5) is a necessary, albeit not a sufficient, condition for separation across types to occur. To achieve separation, the difference in the responsiveness between constrained and unconstrained finance must additionally be sufficiently large so that separation can be achieved at sufficiently favorable terms for type $\theta = g$ entrepreneurs. Moreover, the allocational inefficiency induced by constrained finance must not be too large. Otherwise, investors offering constrained finance will be unable to offer mutually profitable contracts that can achieve separation.

In addition to these conditions, we obtain the usual condition arising in competitive screening models that the probability α of type $\theta = g$ entrepreneurs must not be too large. The following proposition establishes conditions under which the above requirements are met and separation between type $\theta = g$ and type $\theta = b$ entrepreneurs can be achieved. As in Rothschild and Stiglitz (1976), we restrict consideration to pure-strategy equilibria.

Proposition 3. *Consider the following separating equilibrium: entrepreneurs with ex ante type $\theta = b$ receive unconstrained finance, while entrepreneurs with ex ante type $\theta = g$ receive constrained finance. Suppose the responsiveness condition* (5) *holds. Then this separating equilibrium exists and is the unique competitive equilibrium if*

$$\tau \leq \frac{(q_g - q_b)(r_h - r_l)}{r_l + q_g^2 (r_h - r_l)}$$

and

$$\alpha \leq \min \left\{ \frac{\tau}{8} \frac{r_l - 3q_g (r_h - r_l)}{r_h - r_l}, \frac{1}{2} \left[1 - \tau \frac{r_l + q_g^2 (r_h - r_l)}{(q_g - q_b)(r_h - r_l)} \right] \right\}.$$

Proof. See Appendix. ■

4.2 Interim Asymmetric Information and Outside Finance

While there is perfect competition for entrepreneurs at $t = 0$, we have assumed that the (inside) investor is the only source of funding at the refinancing stage. That is, we have assumed that projects which are not refinanced by the inside investor also cannot obtain refinancing from outside investors. Intuitively, the market for outside finance may shut down at the refinancing stage due to a "lemons problem": while the insiders—i.e., the entrepreneur and the inside investor—know the project's interim type, outside investors do not. If successful projects are pooled with "lemons"—i.e., projects with interim type $\psi = n$—then outside investors may be unable to make an offer that can both attract successful projects and allow them to break even.

We will now establish the foundations for this assumption. Precisely, we will show two things. Firstly, we will show that there is always an equilibrium in which the market for outside finance shuts down at the refinancing stage, thus validating the assumption in our base model. Second, to the extent that there is also an equilibrium in which outside finance is feasible, we will show that our results still hold qualitatively. Intuitively, while the inside investor is then no longer the only potential provider of capital at the refinancing stage, she is still the only provider of *informed* capital as only she, but not outside investors, knows the project's interim type. Accordingly, outside finance will command a lemons premium which provides the inside investor (again) with a strong bargaining position: while projects do not compete for *scarce* capital at the refinancing stage, they now compete for *cheaper* (informed) capital.

For a lemons problem to exist at the refinancing stage, type $\psi = n$ projects must have an incentive to seek outside finance. Otherwise, the pool of projects seeking outside finance would consist only of positive NPV projects. In our model thus far, insiders do not *strictly* benefit from luring outside investors into the refinancing of a type $\psi = n$ project. But they do if we change our model as follows: suppose type $\psi = n$ projects—instead of having a zero success probability—have a small but positive probability p_n of generating $R > 0$. If p_n is small, refinancing a type $\psi = n$ project remains a negative NPV investment.[25] Most importantly, however, this modification has no effect on our previous results. In particular, the renegotiatons between the entrepreneur and the inside investor remain exactly the same: there is still no refinancing of type $\psi = n$ projects by the inside investor, and type $\psi = n$ projects still generate a zero payoff if not refinanced. However, the insiders now strictly benefit from luring outside investors into the refinancing of a type $\psi = n$ project: while they have nothing to lose, they may gain $R - D$ with probability p_n.

The market for outside finance at $t = 1$ operates as follows. Projects, represented by the insiders, express their willingness to seek outside finance. Outside investors then compete to provide funds I_2 in return for a share $D \leq R$ of the project's payoff.[26] Given the modification introduced above, the insiders now strictly prefer to seek outside finance for unsuccessful projects. By contrast, the insiders may have something to lose from seeking costly outside finance for *successful* projects. As successful projects are pooled with lemons, outside finance may only be available at unfavorable terms ("lemons premium"). If these terms are sufficiently unfavorable, the insiders may prefer not to refinance a successful project—thus realizing R^0 instead—to seeking costly outside finance. Formally, the insiders will seek outside finance for a type $\psi \in \{l, h\}$ project if and only if

$$\lambda_\psi := p_\psi(R - D) - R^0 \geq 0. \tag{9}$$

The difference

$$r_\psi - \lambda_\psi = p_\psi R - I_2 - p_\psi(R - D) = p_\psi D - I_2$$

represents the lemons premium associated with costly outside finance. If there was no asymmetric information vis-à-vis outsiders, the insiders could always obtain funds I_2 in return for a repayment $F = I_2/p_\psi$, thus realizing an expected payoff of $p_\psi(R - F) = p_\psi R - I_2$. If there is asymmetric information, however, outside investors will demand a higher repayment $D > F$ due to the possibility of financing a lemon.

Our equilibrium concept is that of perfect Bayesian Nash equilibrium in which outside investors rationally anticipate which projects seek outside finance. Given these rational beliefs, outside investors compete themselves down to zero profits. The following result characterizes all (pure-strategy) equilibria under constrained and unconstrained finance.

Proposition 4. *Under unconstrained finance, the market for outside finance at the refinancing stage shuts down completely. Likewise, under constrained finance, there exists always an equilibrium in which the market for outside finance shuts down. Depending on* τ, *there may exist two additional equilibria under constrained finance*: *if* τ *is sufficiently large, there exists an equilibrium in which all three interim types have access to costly outside finance at the refinancing stage, while if* τ *lies in some intermediate range, there exists an equilibrium in which only interim types* $\psi \in \{n, h\}$ *have access to costly outside finance.*

Proof. See Appendix. ■

The intuition underlying Proposition 4 is straightforward. Given that any offer made by outside investors also attracts all lemons, outside investors must set D relatively high to break even. Outside finance thus involves a lemons premium, which makes it costly. Under unconstrained finance, the inside investor has sufficient funds to refinance all successful projects. There is thus no need to draw on costly outside finance, which implies the only projects seeking outside finance will be lemons, which in turn implies the market for outside finance will shut down. Likewise, under constrained finance there is always an equilibrium in which the market for outside finance shuts down: irrespective of τ or other parameter values, if the outside investors believe that only lemons will seek outside finance, then outside finance becomes infeasible. This validates the assumption made in our base model that the only source of funding at the interim stage is the inside investor.

But Proposition 4 also shows that there may—at least for certain parameter values—exist additional equilibria under constrained finance in which outside finance is feasible at the refinancing stage.[27] Arguably, since outside finance commands a lemons premium, the inside investor will always find it optimal to use up her capital of I_2 to refinance *one* of the two projects (unless both are failures, of course). But if outside finance is feasible, this means the other project may now *also* be refinanced—depending on the project's interim type, of course—implying inside and outside finance may coexist at the refinancing stage.

Given that there may be an equilibrium in which projects that are not refinanced by the inside investor have access to outside finance, it is important to check if our previous results hold qualitatively if outside finance at $t = 1$ is costly but feasible. For the sake of brevity, we only consider the equilibrium in Proposition 4 in which all three interim types have access to costly outside finance. It is easy to verify that qualitatively similar results obtain with regard to the other equilibrium in which only type $\psi = n$ and type $\psi = h$ projects have access to costly outside finance. The following proposition establishes the analogue of the responsiveness condition (5) for the case in which outside finance is costly but feasible.

Proposition 5. *Consider the equilibrium in Proposition 4 in which all three interim types have access to costly outside finance. Given this equilibrium, the responsiveness of the entrepreneur's expected payoff to his ex ante type is higher under constrained finance than under unconstrained finance if and only if*

$$(r_h - \lambda_h) - (r_l - \lambda_l) < \frac{r_l - \lambda_l}{3q_g}. \tag{10}$$

Proof. See Appendix. ■

The responsiveness condition is now expressed in terms of the lemon premium $r_\psi - \lambda_\psi$ as the insiders now bargain over the cost savings from using cheaper informed capital. Importantly, however, the responsiveness condition retains its basic qualitative structure from Proposition 1. This points to the crucial driver behind the responsiveness condition: there must be a benefit to being refinanced by the *inside* investor, which implies there will be a benefit to being a high interim type, which in turn implies there will be a benefit to exerting high effort. Whether this benefit arises because not being refinanced by the inside investor means not being refinanced at all, as in our base model, or whether it arises because not being refinanced by the inside investor means a lower surplus due to the use of costly outside finance, as above, is irrelevant for the central argument of our model.

5 Conclusion

This paper shows that investors financing a portfolio of investment projects may use the depth of their financial pockets to overcome entrepreneurial agency problems. Limiting the amount of capital allows investors to credibly commit to a tournament among portfolio projects for (cheaper) informed capital at the refinancing stage. While this improves the investor's ex post bargaining position, thus reducing the entrepreneur's expected payoff, it may nevertheless improve the entrepreneur's incentives. This is because projects funded by investors with scarce capital must have not only a positive NPV at the refinancing stage, but one that is higher than that of competing portfolio projects. As a consequence, committing to "shallow" pockets may be optimal despite the induced allocational inefficiency that positive NPV projects may not be refinanced.

Committing to shallow pockets (or "constrained finance") may have also benefits in dealing with adverse selection problems. If all investors have deep pockets ("unconstrained finance"), separation between good and bad entrepreneurs may be impossible. If investors can choose between constrained and unconstrained finance, however, such separation may be possible. In the separating equilibrium in question, bad entrepreneurs are financed under unconstrained finance while good ones are financed under constrained finance.

Our model lends itself to several testable implications. For instance, a key implication of our model is that, other things equal, projects financed under constrained finance should have lower ex ante success probabilities than comparable projects financed under unconstrained finance. The intuition, which lies at the heart of our model, is that for high success probabilities the allocational inefficiency induced by constrained finance weighs heavily in expected terms, implying such projects are better financed under unconstrained finance. The same intuition holds for projects with high investment costs as such projects require a high success probability in order to be financially viable. On the other hand, the main benefit of constrained finance in our model is that it may provide stronger effort incentives to entrepreneurs. Hence, another empirical implication is that constrained finance should be more likely to be used if the efficiency loss from having low entrepreneurial effort is large.

6 Appendix

Proof of Lemma 1. Claim i) is obvious. As for claim ii), denote by $y_i := (1 - s_i)R^0$ and $z_i := s_i R^0$ the investor's and E_i's respective continuation payoffs if the project is not refinanced, and by $v_i := R_{\psi_i} - I_2$ and $w_i := v_i - (y_i + z_i) = r_{\psi_i}$ their combined continuation payoffs and net surplus, respectively, from refinancing a project with interim type $\psi_i \in \{l, h\}$.

Given that the proof is fairly standard, we shall be brief. We characterize offers by the continuation payoff X which the offer leaves to E_i. The investor always offers X^I, while E_i always offers X^E. If the investor must respond to E_i's offer, she accepts any X^E satisfying

$$v_i - X^E \geq \delta y_i + (1 - \delta)(v_i - X^I). \tag{11}$$

The right-hand side in (11) represents the investor's payoff from rejecting E_i's offer: with probability δ the negotiations with E_i break down, in which case the investor receives y_i. If the negotiations do not break down, the investor makes her counteroffer X^I. Similarly, if E_i must respond to the investor's offer, he accepts any X^I satisfying

$$X^I \geq \delta z_i + (1 - \delta)X^E. \tag{12}$$

As usual, offers along the equilibrium path must make the respective counterparty indifferent between accepting and rejecting, implying (11)-(12) must hold with equality. Solving (11) for X^E and inserting the result in (12), we have

$$X^I = \frac{\delta z_i + (1 - \delta)\delta(v_i - y_i)}{\delta(2 - \delta)}, \tag{13}$$

which E_i accepts immediately.

By L'Hôpital's rule, E_i's equilibrium continuation payoff as $\delta \to 0$ is

$$\lim_{\delta \to 0} X^I = \frac{v_i - y_i + z_i}{2} = z_i + \frac{w_i}{2} = s_i R^0 + \frac{r_{\psi_i}}{2}, \tag{14}$$

implying the investor's equilibrium continuation payoff as $\delta \to 0$ is

$$\lim_{\delta \to 0} v_i - X^I = v_i - z_i - \frac{w_i}{2} = y_i + \frac{w_i}{2} = (1 - s_i) R^0 + \frac{r_{\psi_i}}{2}.$$

Note that the same equilibrium continuation payoffs would obtain if instead of solving for X^I we solves for X^E and took the limit as $\delta \to 0$, i.e., $\lim_{\delta \to 0} X^I = \lim_{\delta \to 0} X^E$. Consequently, instead of letting the investor make the first offer, we could equally assume that E_i makes the first offer; the equilibrium continuation payoffs would be identical. ■

Proof of Lemma 2. Claim i) is obvious. As for claim ii), we use the same notation as in the Proof of Lemma 1, except that we use subscripts i and j to distinguish between E_i and E_j. If $\psi_i \in \{l, h\}$, $\psi_j \in \{l, h\}$, and $\psi_i \neq \psi_j$, we specify that the investor picks the entrepreneur with the higher interim type. Without loss of generality, we assume this is E_i. We will confirm below that this strategy on the part of the investor is optimal. If $\psi_i = \psi_j$ the investor is indifferent. In this case, we specify that the investor randomly picks an entrepreneur (with equal probability), with whom she then bargains until there is either a breakdown or an agreement.[28] Again without loss of generality, we assume that this is E_i.

Analogous to the Proof of Lemma 1, the investor always offers x_i^I and accepts any counteroffer x_i^E satisfying

$$v_i - x_i^E + y_j \geq \delta(y_i + v_j - X_j^I) + (1 - \delta)(y_j + v_i - x_i^I). \tag{15}$$

In (15), X_j^I denotes the investor's offer to E_j if he is the only entrepreneur still around, i.e., if the negotiations with E_i have already broken down. We already know from Lemma 1 what this offer will be. In contrast, x_i^E and x_i^I denote E_i's and the investor's offers, respectively, if both entrepreneurs are still around. Note the difference to (11): if the investor accepts E_i's offer, she realizes—in addition to $(v_i - x_i^E)$—also her outside option payoff y_j with E_j, whose project is not refinanced. By contrast, if the investor rejects E_i's offer, the negotiations with E_i break down with probability δ, in which case she continues with E_j. Finally, if the negotiations with E_i have not broken down, the investor makes her counteroffer x_i^I. As for E_i, he always offers x_i^E and accepts any counteroffer x_i^I satisfying

$$x_i^I \geq \delta z_i + (1-\delta) x_i^E. \tag{16}$$

Analogous to the Proof of Lemma 1, (15)-(16) must hold with equality. Solving (15) for x_i^E and inserting the result in (16), we obtain

$$x_i^I = \frac{\delta z_i + (1-\delta)\delta(v_i - y_i + y_j - v_j + X_j^I)}{\delta(2-\delta)}, \tag{17}$$

which E_i accepts immediately.

Analogous to (14), we obtain $\lim_{\delta \to 0} X_j^I = z_j + w_j/2$. Using L'Hôpital's rule, we thus have that E_i's equilibrium continuation payoff as $\delta \to 0$ is

$$\begin{aligned}
\lim_{\delta \to 0} x_i^I &= \frac{v_i - y_i - \frac{w_j}{2} + z_i}{2} = z_i + \frac{1}{2}(w_i - \frac{w_j}{2}) \\
&= s_i R^0 + \frac{1}{2}(r_{\psi_i} - \frac{r_{\psi_j}}{2}),
\end{aligned}$$

which implies the investor's *total* equilibrium continuation payoff (i.e., including her outside option payoff y_j realized with E_j) as $\delta \to 0$ is

$$\begin{aligned}
v_i - z_i - \frac{1}{2}(w_i - \frac{w_j}{2}) + y_j &= y_i + \frac{1}{2}(w_i + \frac{w_j}{2}) + y_j \\
&= (1-s_i)R^0 + (1-s_j)R^0 + \frac{1}{2}(r_{\psi_i} + \frac{r_{\psi_j}}{2}).
\end{aligned}$$

As in the Proof of Lemma 1, we could have equally solved for x_i^E and taken the limit as $\delta \to 0$; the equilibrium continuation payoffs are identical.

It remains to show that if both entrepreneurs are still around and $\psi_i \neq \psi_j$, the investor does not find it profitable to deviate and make an offer to the entrepreneur with the lower interim type, E_j. Suppose the investor deviates and offers x_j^I to E_j while accepting any x_j^E satisfying

$$v_j - x_j^E + y_i \geq \delta(y_j + v_i - X_i^I) + (1-\delta)(y_j + v_i - x_i^I). \tag{18}$$

In (18), if the investor rejects E_j's offer and the negotiations with E_j break down, the investor must necessarily switch back to E_i. However, the investor also switches back to E_i if the negotiations with E_j have not broken down.[29] As for E_j, he offers x_j^E and accepts any x_j^I satisfying

$$x_j^I \geq \delta z_j + (1-\delta)x_j^E. \tag{19}$$

As previously, (18)-(19) must hold with equality. Solving (18) for x_j^E and inserting the result in (19) yields

$$x_j^I = \delta z_j + (1-\delta)(v_j + y_i - y_j - v_i + \delta X_i^I + (1-\delta)x_i^I). \tag{20}$$

To confirm that the investor does not find it profitable to deviate, we must show that

$$v_i - x_i^I + y_j \geq v_j - x_j^I + y_i. \tag{21}$$

Inserting x_j^I from (20) into (21) and rearraning, (21) transforms to

$$\delta z_j - \delta(v_j + y_i - y_j - v_i) + (1-\delta)\delta X_i^I \geq x_i^I \delta(2-\delta). \tag{22}$$

Next, inserting (17) into (22), dividing through by δ, and rearranging, (22) transforms to

$$(1-\delta)(X_i^I - X_j^I) \geq (z_i - z_j) - \delta(v_i - y_i + y_j - v_j). \tag{23}$$

Note that from (13) we have that

$$X_i^I = \frac{\delta z_i + (1-\delta)\delta(v_i - y_i)}{\delta(2-\delta)}$$

and

$$X_j^I = \frac{\delta z_j + (1-\delta)\delta(v_j - y_j)}{\delta(2-\delta)}.$$

Finally, inserting X_i^I and X_j^I into (23), multiplying through by $\delta(2-\delta)$, and rearranging, (23) transforms to

$$\delta[(v_i - y_i - z_i) - (v_j - y_j - z_j)] = \delta(w_i - w_j) = \delta(r_{\psi_i} - r_{\psi_j}) \geq 0,$$

which holds by assumption. ■

Proof of Proposition 1. It remains to derive (1) and (3). Consider first the derivation of (1). Under unconstrained finance, the probabilities of having interim type $\psi = n$, $\psi = l$, and $\psi = h$ are $1 - \tau$, $\tau(1 - q_{\theta_i})$, and τq_{θ_i}, respectively. Multiplying these probabilities with the respective continuation payoffs from Lemma 1 and rearranging yields (1).

Consider next the derivation of (3). Given that the investor picks the entrepreneur with the higher interim type first, and if indifferent she picks each of the two entrepreneurs with equal probability (see Proof of Lemma 2), Lemma 2 implies the following expected continuation payoffs at $t = 1$ for E_k, an arbitrary entrepreneur: zero if $\psi_k = n$, $s_k R^0$ if $\psi_k = l$ and $\psi_{j \neq k} = h$, $s_k R^0 + \frac{1}{2}(r_h - \frac{1}{2} r_l)$ if $\psi_k = h$ and $\psi_{_{j \neq k}} = l$, $s_k R^0 + \frac{1}{2} r_{\psi_k}$ if $\psi_k \in \{l, h\}$ and $\psi_{_{j \neq k}} = n$, and $s_k R^0 + \frac{1}{8} r_\psi$ if $\psi_k = \psi_{_{j \neq k}} = \psi \in \{l, h\}$. Multiplying these expected continuation payoffs with the respective joint probabilities for interim types (ψ_i, ψ_j) and rearranging yields (3). The respective joint probabilities are $\tau^2 q_{\theta_i} q_{\theta_j}$ for (h, h), $\tau^2 (1 - q_{\theta_i})(1 - q_{\theta_j})$ for (l, l), $(1 - \tau)^2$ for (n, n), $\tau(1 - q_{\theta_i})(1 - \tau)$ for (l, n), $\tau(1 - q_{\theta_j})(1 - \tau)$ for (n, l), $\tau q_{\theta_i}(1 - \tau)$ for (h, n), $\tau q_{\theta j}(1 - \tau)$ for (n, h), $\tau^2 q_{\theta_i}(1 - q_{\theta_j})$ for (h, l), and $\tau^2 q_{\theta j}(1 - q_{\theta_i})$ for (l, h). ■

Proof of Proposition 2. Analogous to the derivation of (1) and (3) in the Proof of Proposition 1, we can derive the investor's expected payoff at $t = 0$. Under unconstrained finance, the investor's expected payoff at $t = 0$ is

$$\tau \left\{ (1 - s_i) R^0 + \frac{1}{2} [r_l + q_{\theta_i} (r_h - r_l)] \right\} - I_1, \tag{24}$$

while under constrained finance, it is

$$\tau \left\{ (1 - s_i) R^0 + \frac{1}{2} [r_l + q_{\theta_i} (r_h - r_l)] \right\} - \frac{\tau^2}{8} \left\{ \left(1 + 3q_{\theta_j} - 3q_{\theta_i}\right) r_l + q_{\theta_i} q_{\theta_j} (r_h - r_l) \right\} - I_1. \tag{25}$$

If the two conditions (5) and (6) hold, we have $\theta_i = b$ in the case of unconstrained finance and $\theta_i = \theta_j = g$ in the case of constrained finance. Accordingly, (24) and (25) transform to

$$\tau \left\{ (1 - s_i) R^0 + \frac{1}{2} [r_l + q_b (r_h - r_l)] \right\} - I_1 \tag{26}$$

and

$$\tau\left\{(1-s_i)\,R^0+\frac{1}{2}\left[r_l+q_g\,(r_h-r_l)\right]\right\}-\frac{\tau^2}{8}\left\{r_l+q_g^2\,(r_h-r_l)\right\}-I_1, \tag{27}$$

respectively. Setting $s_i = 0$ in (26) and (27), respectively, we obtain $\pi_U^I - I_1$ and $\pi_C^I - I_1$ as defined in (7) and (8) in the main text.

We next derive the entrepreneur's expected payoff at $t = 0$ if the project is viable and investors compete themselves down to zero profits. Setting (26) and (27) equal to zero, solving for s_i, and inserting the result in (1) (with $\theta_i = b$) and (3) (with $\theta_i = \theta_j = g$), respectively, we have that E_i's equilibrium expected payoff under unconstrained finance is

$$\pi_U^E - I_1 := \tau\left\{R^0 + r_l + q_b\,(r_h - r_l) + B\right\} - I_1, \tag{28}$$

while his equilibrium expected payoff under constrained finance is

$$\pi_C^E - I_1 := \tau\left\{R^0 + r_l + q_g\,(r_h - r_l)\right\} - \frac{\tau^2}{2}\left\{r_l + q_g^2\,(r_h - r_l)\right\} - I_1. \tag{29}$$

We finally establish the functional properties of π_U^I, π_C^I, π_U^E, and π_C^E. Once these properties are established, the remainder of the proof is trivial. By inspection, π_U^I and π_U^E are both linear and strictly increasing in τ. Moreover, both are zero at $\tau = 0$, and π_U^E lies strictly above π_U^I for all $\tau > 0$.[30] Likewise, it is easily shown that π_C^I and π_C^E are both strictly concave, increasing in τ, and zero at $\tau = 0$. Note that

$$\lim_{\tau\to 0}\frac{d\pi_C^E}{d\tau} - \lim_{\tau\to 0}\frac{d\pi_U^E}{d\tau} = (q_g - q_b)\,(r_h - r_l) - B > 0,$$

where the inequality follows from our assumption that $\theta = g$ is socially optimal. Hence, π_C^E lies strictly above π_U^E for small τ, implying it crosses π_U^E exactly once from the left. In Figure 2, this intersection point was denoted by $\widehat{\tau}$. Straightforward calculations show that

$$\widehat{\tau} := 2\frac{(q_g - q_b)\,(r_h - r_l) - B}{r_l + q_g^2\,(r_h - r_l)} < 1,$$

where the inequality follows from $2(q_g - q_b)\,(r_h - r_l) < r_l + q_g^2\,(r_h - r_l)$.[31] Likewise, note that

$$\lim_{\tau\to 0}\frac{d\pi_C^I}{d\tau} - \lim_{\tau\to 0}\frac{d\pi_U^I}{d\tau} = \frac{1}{2}(q_g - q_b)\,(r_h - r_l) > 0,$$

which establishes that π_C^I lies strictly above π_U^I for small τ, implying it crosses π_U^I exactly once from the left as depicted in Figure 2. Denote the intersection of π_C^I and π_U^I by $\widetilde{\tau}$. Straightforward calculations show that

$$\widetilde{\tau} := 4\frac{(r_h - r_l)(q_g - q_b)}{r_l + q_g^2 (r_h - r_l)} > \widehat{\tau}.$$

Associated with $\widetilde{\tau}$ is a critical value of I_1, which is equal to the value of π_U^I (or, equivalently, the value of π_C^I) at $\tau = \widetilde{\tau}$. Denote this critical value by $\widetilde{I}_1$. From (7) we have that

$$\widetilde{I}_1 := \widetilde{\tau}\left\{R^0 + \frac{1}{2}\left[r_l + q_b (r_h - r_l)\right]\right\}. \tag{30}$$

Case i) of Proposition 2 then holds for $I_1 > \widetilde{I}_1$ while Case ii) holds for $I_1 \leq \widetilde{I}_1$. ■

Proof of Proposition 3. Denote by s_C and s_U the equilibrium sharing rules offered by constrained and unconstrained investors, respectively. A separating equilibrium in which type $\theta = g$ entrepreneurs prefer constrained finance and type $\theta = b$ entrepreneurs prefer unconstrained finance exists if i) s_C and s_U are incentive compatible, ii) investors' and entrepreneurs' participation constraints hold, and iii) there exists no other offer that can break the proposed separating equilibrium. We now address each of these three conditions in turn.

Consider first incentive compatibility. In the proposed equilibrium, unconstrained investors attract only type $\theta = b$ entrepreneurs and make zero profits. Setting (24) with $\theta_i = b$ and $s_i = s_U$ equal to zero and solving for s_U, we obtain

$$s_U = 1 + \frac{1}{2}\frac{r_l + q_b (r_h - r_l)}{R^0} - \frac{I_1}{\tau R^0}. \tag{31}$$

Consider next s_C. Incentive compatibility for type $\theta = b$ entrepreneurs requires that constrained investors offer s_C such that type $\theta = b$ entrepreneurs weakly prefer unconstrained finance. Consequently, s_C must satisfy[32]

$$\begin{aligned} &\tau\left\{s_U R^0 + \frac{1}{2}\left[r_l + q_b (r_h - r_l)\right]\right\} \\ \geq\ &\tau\left\{s_C R^0 + \frac{1}{2}\left[r_l + q_b (r_h - r_l)\right]\right\} - \frac{\tau^2}{8}\left\{r_l (3 - q_b + q_g) + 3 q_b q_g (r_h - r_l)\right\}, \end{aligned}$$

which transforms to

$$s_C \leq s_U + \frac{\tau}{8}\frac{r_l\left(3-q_b+q_g\right)+3q_bq_g\left(r_h-r_l\right)}{R^0}, \tag{32}$$

where s_U was defined in (31).

Incentive compatibility for type $\theta = g$ entrepreneurs, in turn, requires that these entrepreneurs weakly prefer constrained finance:[33]

$$\tau\left\{s_C R^0+\frac{1}{2}\left[r_l+q_g\left(r_h-r_l\right)\right]\right\}-\frac{3\tau^2}{8}\left\{r_l+q_g^2\left(r_h-r_l\right)\right\} \geq \tau\left\{s_U R^0+\frac{1}{2}\left[r_l+q_g\left(r_h-r_l\right)\right]\right\},$$

which transforms to

$$s_C \geq s_U + \frac{3\tau}{8}\frac{r_l+q_g^2\left(r_h-r_l\right)}{R^0}. \tag{33}$$

By inspection, (32) and (33) can be jointly satisfied if and only if $\frac{r_l}{3q_g} > r_h - r_l$, i.e., if and only if the responsiveness condition (5) holds.

Consider next the participation constraints. The entrepreneurs' expected payoff is always non-negative, while s_U was constructed such that unconstrained investors break even. From (25) with $\theta_i = \theta_j = g$ and $s_i = s_C$ we have that constrained investors' expected payoff is non-negative if

$$\tau\left\{\left(1-s_C\right)R^0+\frac{1}{2}\left[r_l+q_g\left(r_h-r_l\right)\right]\right\}-\frac{\tau^2}{8}\left\{r_l+q_g^2\left(r_h-r_l\right)\right\}-I_1 \geq 0,$$

which transforms to

$$s_C \leq s_U + \frac{1}{2}\frac{\left(q_g-q_b\right)\left(r_h-r_l\right)}{R^0} - \frac{\tau}{8}\frac{r_l+q_g^2\left(r_h-r_l\right)}{R_0}. \tag{34}$$

This condition is compatible with (33) if

$$s_U + \frac{1}{2}\frac{\left(q_g-q_b\right)\left(r_h-r_l\right)}{R^0} - \frac{\tau}{8}\frac{r_l+q_g^2\left(r_h-r_l\right)}{R_0} \geq s_U + \frac{3\tau}{8}\frac{r_l+q_g^2\left(r_h-r_l\right)}{R^0},$$

which transforms to

$$\tau \leq \frac{\left(q_g-q_b\right)\left(r_h-r_l\right)}{r_l+q_g^2\left(r_h-r_l\right)}.$$

Finally, existence of the proposed separating equilibrium requires that there exists no other—in this case: pooling—offer that can break the separating equilibrium and allows investors to break even. Analogous to (31), the zero-profit pooling offer is given by

$$s_P = 1 + \frac{1}{2}\frac{r_l + (\alpha q_g + (1-\alpha)\, q_b)\,(r_h - r_l)}{R^0} - \frac{I_1}{\tau R^0}.$$

For type $\theta = g$ entrepreneurs to prefer s_C to s_P, it must hold that

$$\tau\left\{s_C R^0 + \frac{1}{2}\left[r_l + q_g\,(r_h - r_l)\right]\right\} - \frac{3\tau^2}{8}\left\{r_l + q_g^2\,(r_h - r_l)\right\} \geq \tau\left\{s_P R^0 + \frac{1}{2}\left[r_l + q_g\,(r_h - r_l)\right]\right\},$$

which transforms to

$$s_C \geq s_P + \frac{3\tau}{8}\frac{r_l + q_g^2\,(r_h - r_l)}{R^0}. \tag{35}$$

Condition (35) is compatible with (32) if

$$s_U + \frac{\tau}{8}\frac{r_l\,(3 - q_b + q_g) + 3q_g q_b\,(r_h - r_l)}{R^0} \geq s_P + \frac{3\tau}{8}\frac{r_l + q_g^2\,(r_h - r_l)}{R^0},$$

which transforms to

$$\alpha \leq \frac{\tau}{8}\frac{r_l - 3q_g\,(r_h - r_l)}{r_h - r_l}.$$

Likewise, (35) is compatible with the zero-profit constraint (34) if

$$s_U + \frac{1}{2}\frac{(q_g - q_b)\,(r_h - r_l)}{R^0} - \frac{\tau}{8}\frac{r_l + q_g^2\,(r_h - r_l)}{R_0} \geq s_P + \frac{3\tau}{8}\frac{r_l + q_g^2\,(r_h - r_l)}{R^0},$$

which transforms to

$$\alpha \leq \frac{1}{2}\left[1 - \tau\frac{r_l + q_g^2\,(r_h - r_l)}{(q_g - q_b)\,(r_h - r_l)}\right].$$

Finally, if the above conditions hold, any candidate pooling equilibrium can be broken by the separating offers s_U and s_C, which establishes uniqueness. ■

Proof of Proposition 4. The argument for why under unconstrained finance the market for outside finance must shut down has been stated in the main text. Consider next constrained finance. By (9), if interim type $\psi = l$ weakly prefers to seek outside finance, then interim type $\psi = h$ strictly prefers to seek outside finance. This immediately implies that we have three equilibrium candidates under constrained finance: (i) no project has access to outside finance, (ii) all three interim types have access to outside finance, and (iii) only interim types $\psi = n$ and

$\psi = h$ have access to outside finance. Importantly, there cannot exist an equilibrium in which only interim types $\psi = n$ and $\psi = l$ have access to outside finance, and there obviously cannot exist an equilibrium in which only successful projects have access to outside finance: any offer that attracts sucecssful projects will also attract all lemons. We now consider all three candidate equilibria in turn.

Equilibrium in which no project has access to outside finance at the refinancing stage

This is trivially always an equilibrium. If outside investors believe that only lemons will seek outside finance, the market for outside finance shuts down completely.

Equilibrium in which all three interim types have access to outside finance

We first characterize outside investors' rational beliefs, which we denote by $\pi(\psi)$. In the proposed equilibrium, there is exactly one project seeking outside finance in every state of nature.[34] With probability $\tau^2 q_{\theta_i} q_{\theta_j}$ both projects have interim type $\psi = h$. Hence, the conditional probability that the project seeking outside finance has interim type $\psi = h$ is $\pi(h) = \tau^2 q_{\theta_i} q_{\theta_j}$. Likewise, with probability $1 - \tau^2$ at least one project has interim type $\psi = n$. The conditional probability that the project seeking outside finance has interim type $\psi = n$ is thus $\pi(n) = 1 - \tau^2$. Finally, with probability $\tau^2(1 - q_{\theta_i} q_{\theta_j})$ at least one project has interim type $\psi = l$ and no project has interim type $\psi = n$. The conditional probability that the project seeking outside finance has interim type $\psi = l$ is thus $\pi(l) = \tau^2(1 - q_{\theta_i} q_{\theta_j})$.[35]

Given these beliefs, the zero-profit repayment D required by outside investors is

$$D = \frac{I_2}{\tau^2 q_{\theta_i} q_{\theta_j} p_h + \tau^2(1 - q_{\theta_i} q_{\theta_j}) p_l + (1 - \tau^2) p_n}. \tag{36}$$

The proposed equilibrium exists if i) interim types $\psi = l$ and $\psi = h$ weakly prefer outside finance, and ii) there exists a repayment $D \leq R$ satisfying (36). By our previous arguments, if interim type $\psi = l$ weakly prefers outside finance, then interim type $\psi = h$ strictly prefers outside finance. Hence, the proposed equilibrium exists if and only if (36) and

$$p_l(R - D) \geq R^0 \tag{37}$$

hold. Note that (37) implies $D < R$. Inserting (36) into (37) and rearranging, we obtain the requirement that

$$\tau^2 \geq \left(\frac{p_l I_2}{p_l R - R^0} - p_n\right)\left(\frac{1}{q_{\theta_i} q_{\theta_j}(p_h - p_l) + p_l - p_n}\right), \tag{38}$$

which implies for an equilibrium to exist in which all three interim types have access to costly outside finance, τ must be sufficiently large.[36].

Equilibrium in which only interim types $\psi = n$ and $\psi = h$ have access to outside finance

In this equilibrium, there is exactly one project seeking outside finance if either both projects have interim type $\psi = h$ or if at least one project has interim type $\psi = n$. Hence, the conditional probability that the project seeking outside finance has interim type $\psi = l$ is $\pi(l) = 0$, the conditional probability that it has interim type $\psi = h$ is $\pi(h) = \tau^2 q_{\theta_i} q_{\theta_j} / [(1 - \tau^2) + \tau^2 q_{\theta_i} q_{\theta_j}]$, and the conditional probability that it has interim type $\psi = n$ is $\pi(n) = (1 - \tau^2)/[(1 - \tau^2) + \tau^2 q_{\theta_i} q_{\theta_j}]$.

Given these beliefs, the zero-profit repayment D required by outside investors is

$$D = \frac{I_2}{\xi(\tau)}, \tag{39}$$

where

$$\xi(\tau) := \frac{\tau^2 q_{\theta_i} q_{\theta_j} p_h + (1 - \tau^2) p_n}{\tau^2 q_{\theta_i} q_{\theta_j} + 1 - \tau^2}$$

is strictly increasing in τ with $\lim_{\tau \to 0} \xi(\tau) = p_n$ and $\lim_{\tau \to 1} \xi(\tau) = p_h$.

The proposed equilibrium exists if i) interim type $\psi = h$ weakly prefers outside finance, ii) interim type $\psi = l$ prefers no refinancing to outside finance, and iii) there exists a repayment $D \leq R$ satisfying (39). Hence, the proposed equilibrium exists if and only if (39) and

$$p_h (R - D) \geq R^0 > p_l (R - D) \tag{40}$$

hold. Note that the first inequality implies $D < R$. Inserting (39) with equality into (40) we obtain

$$p_h \left(R - \frac{I_2}{\xi(\tau)}\right) \geq R^0 > p_l \left(R - \frac{I_2}{\xi(\tau)}\right).$$

Since $r_\psi := p_\psi R - R^0 - I_2 > 0$ for $\psi \in \{l, h\}$, the second inequality is violated if $\xi(\tau) \geq p_l$. Given that $\xi(\tau)$ is increasing in τ, this implies τ must not be too large. On the other hand, given that $\lim_{\tau \to 0} \xi(\tau) = p_n$ and our assumption that p_n is small, the first inequality is violated if τ is sufficiently small. ■

Proof of Proposition 5. As in Section 2.2, we first derive the entrepreneurs' continuation payoffs at $t = 1$ under constrained finance. As the basic structure of the bargaining game is the same as in Section 2.2, we confine ourselves to reporting the equilibrium continuation payoffs as $\delta \to 0$. The main difference to our base model concerns the insiders' total payoff if a project is not refinanced by the inside investor. In our base model, this payoff was zero for projects with interim type $\psi = n$ and R^0 for projects with interim type $\psi \in \{l, h\}$. Now, given that projects have access to costly outside finance, the insiders' total payoff if the project is not refinanced by the inside investor is $\lambda_\psi := p_\psi(R - D)$ for all three interim types, where, unlike our base model, it now holds that $p_n > 0$. The only exception is when both projects have interim type $\psi = n$: as only one project can be presented to outside investors, the insiders' total payoff in this case is λ_n from the project presented to outside investors and zero from the other project.

Consider first the case where $\psi_i = \psi_j = n$. If E_j is the last entrepreneur to be bargained with, E_j and the investor each realize $\frac{1}{2}\lambda_n$. Consider next the negotiations with E_i, who the investor picks first. As the investor can credibly threaten to present instead E_j's project to outside investors, equilibrium continuation payoffs are—analogous to Lemma 2—$\frac{1}{2}(\lambda_n - \frac{1}{2}\lambda_n) = \frac{1}{4}\lambda_n$ for E_i and zero for E_j.

Consider next the case where $\psi_i \in \{l, h\}$ and $\psi_j = n$. By optimality (see Proof of Lemma 2), the investor bargains first with E_i. Moreover, if the negotiations with E_i break down, it is optimal to present E_i's project to outside investors, not E_j's.[37] Hence, the investor and E_i bargain over the cost savings from using inside funds, $r_{\psi_i} - \lambda_{\psi_i}$, implying E_i's equilibrium continuation payoff is the sum of $s_i R^0 + \frac{1}{2}\lambda_{\psi_i}$ and $\frac{1}{2}(r_{\psi i} - \lambda_{\psi_i})$, which equals $s_i R^0 + \frac{1}{2} r_{\psi_i}$. Naturally, E_j's equilibrium continuation payoff is then $\frac{1}{2}\lambda_{\psi_j}$.

Consider finally the case where $\psi_i \in \{l,h\}$ and $\psi_j \in \{l,h\}$. Suppose E_j is the last entrepreneur to be bargained with. Payoffs now depend on whether the investor has already used up her funds for E_i. If the investor's funds have already been used up, E_j realizes $s_jR^0 + \frac{1}{2}\lambda_{\psi_j}$, while the investor realizes $(1-s_j)\,R^0 + \frac{1}{2}\lambda_{\psi_j}$ from bargaining with E_j. If the investor's funds are still available, E_j and the investor bargain over the cost savings from using inside funds, $r_{\psi_j} - \lambda_{\psi_j}$. Consequently, E_j realizes the sum of $s_jR^0 + \frac{1}{2}\lambda_{\psi_j}$ and $\frac{1}{2}(r_{\psi_j} - \lambda_{\psi_j})$, which equals $s_jR^0 + \frac{1}{2}r_{\psi_j}$, while the investor realizes the sum of $(1-s_j)\,R^0 + \frac{1}{2}\lambda_{\psi_j}$ and $\frac{1}{2}(r_{\psi_j} - \lambda_{\psi_j})$ from bargaining with E_j, which equals $(1-s_j)\,R^0 + \frac{1}{2}r_{\psi_j}$. Consider next the negotiations between the investor and her first pick, E_i. If the negotiations break down, E_i realizes $s_iR^0 + \frac{1}{2}\lambda_{\psi_i}$, while the investor realizes the sum of $(1-s_i)\,R^0 + \frac{1}{2}\lambda_{\psi_i}$ and $(1-s_j)\,R^0 + \frac{1}{2}r_{\psi_j}$. On the other side, the surplus over which E_i and the investor bargain is $r_{\psi_i} - \lambda_{\psi_i} - \frac{1}{2}(r_{\psi_j} - \lambda_{\psi_j})$. Hence, E_i's equilibrium continuation payoff is the sum of $s_iR^0 + \frac{1}{2}\lambda_{\psi_i}$ and $\frac{1}{2}[r_{\psi_i} - \lambda_{\psi_i} - \frac{1}{2}(r_{\psi_j} - \lambda_{\psi_j})]$, which equals $s_iR^0 + \frac{1}{2}[r_{\psi_i} - \frac{1}{2}(r_{\psi_j} - \lambda_{\psi_j})]$. Naturally, E_j's equilibrium continuation payoff is then $s_jR^0 + \frac{1}{2}r_{\psi_j}$.

Consider next the issue who is picked first to be bargained with. If $\psi_i \neq \psi_j$, we know from the Proof of Lemma 2 that the investor picks the entrepreneur with the higher interim type first. In contrast, if $\psi_i = \psi_j$, the investor picks both entrepreneurs with equal probability. We thus have the following *expected* continuation payoffs for E_k, an arbitrary entrepreneur: $\frac{1}{8}\lambda_n$ if $\psi_k = \psi_{j\neq k} = n$, $s_kR^0 + \frac{1}{2}r_{\psi_k}$ if $\psi_k \in \{l,h\}$ and $\psi_{j\neq k} = n$, $\frac{1}{2}\lambda_{\psi_n}$ if $\psi_k = n$ and $\psi_{j\neq k} \in \{l,h\}$, $s_kR^0 + \frac{1}{8}(r_\psi + 3\lambda_\psi)$ if $\psi_k = \psi_{j\neq k} = \psi \in \{l,h\}$, $s_kR^0 + \frac{1}{2}[r_h - \frac{1}{2}(r_l - \lambda_l)]$ if $\psi_k = h$ and $\psi_{j\neq k} = l$, and $s_kR^0 + \frac{1}{2}\lambda_l$ if $\psi_k = l$ and $\psi_{j\neq k} = h$.

Given these expected continuation payoffs, we can now analogous to (3) compute E_i's expected payoff at $t = 0$. We obtain[38]

$$\tau^2 q_{\theta_i} q_{\theta_j} \left\{ s_iR^0 + \frac{1}{8}\left[r_h + 3\lambda_h\right] \right\} + \tau^2 q_{\theta_i}\left(1 - q_{\theta_j}\right) \left\{ s_iR^0 + \frac{1}{2}\left[r_h - \frac{1}{2}\left[r_l - \lambda_l\right]\right] \right\}$$
$$+\tau^2\left(1 - q_{\theta_i}\right) q_{\theta_j} \left\{ s_iR^0 + \frac{1}{2}\lambda_l \right\} + \tau^2\left(1 - q_{\theta_i}\right)\left(1 - q_{\theta_j}\right) \left\{ s_iR^0 + \frac{1}{8}\left[r_l + 3\lambda_l\right] \right\}$$
$$+\tau\left(1 - \tau\right) \left\{ s_iR^0 + \frac{1}{2}\left[r_l + q_{\theta_i}\left(r_h - r_l\right)\right] \right\} + \left(1 + 2\tau - 3\tau^2\right)\frac{1}{8}\lambda_n,$$

which simplifies to

$$\tau\left\{s_i R^0+\frac{1}{2}\left[r_l+q_{\theta_i}\left(r_h-r_l\right)\right]\right\}+\left(1+2\tau-3\tau^2\right)\frac{1}{8}\lambda_n \qquad (41)$$
$$-\frac{\tau^2}{8}\left\{\left(r_l-\lambda_l\right)\left(3-q_{\theta_i}+q_{\theta_j}\right)+3q_{\theta_i}q_{\theta_j}\left[r_h-\lambda_h-\left(r_l-\lambda_l\right)\right]\right\}.$$

Having derived E_i's expected payoff at $t=0$, we next compute the responsiveness under constrained finance for the case in which all three interim types have access to costly outside finance. Analogous to (4), we obtain the responsiveness from (41) by setting $\theta_j = g$ and subtracting E_i's expected payoff for $\theta_i = b$ from that for $\theta_i = g$. We have

$$\frac{1}{2}\tau\left(q_g-q_b\right)\left[\left(r_h-r_l\right)-\frac{\tau}{4}\left[3q_g\left[r_h-\lambda_h-\left(r_l-\lambda_l\right)\right]-\left(r_l-\lambda_l\right)\right]\right]. \qquad (42)$$

Comparing (2) with (42), we finally obtain the responsiveness condition (10). ■

7 References

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Notes

[1]Silver (1985) writes: "The need for greater amounts of venture capital, frequently not cited in the business plan, occurs sooner than expected. Because the Murphy's law affliction attacks most venture capital portfolios, there arises a serious need for portfolio management."

[2]Refinancing by uninformed outside investors is at best more costly and at worst unavailable: "if the original partnership is unwilling to arrange for additional financing, it is unlikely that any other partnership will choose to do so; the reluctance of the original partnership is a strong signal that the company is a poor investment" (Fenn, Liang, and Prowse (1995)). Consistent with this notion, Bruno and Tyebjee (1983) find that being denied follow-up financing by a previous-round venture capitalist reduces a portfolio company's chances of obtaining financing from outside investors by 74 percent. See Section 4.2 for a formal analysis of this issue.

[3]Sahlman reports the results of one survey of venture capital investments showing that 34.5 percent of invested capital resulted in a loss and another 30 percent resulted in returns in the low to middle single digits. Conversely, less than 7 percent of invested capital resulted in payoffs of more than ten times the original amount invested.

[4]Bartlett (1995) and Brooks (1999) provide in-depth discussions of venture partnership agreements.

[5]While at first glance peculiar, the motive for this covenant stems from concerns by limited partners that "by adding less experienced general partners, venture capitalists may reduce the burden on themselves" (Gompers and Lerner (1996)). Besides, it is not easy to find skilled venture capitalists that can be added to a given fund: "[T]he skills needed for successful venture capital investing are difficult and time-consuming to acquire. During periods when the ... demand for venture capital has shifted, adjustments in the number of venture capitalists ... take place very slowly" (Gompers (1995)).

[6]A distinct, albeit somewhat related, point is made by Gertner, Scharfstein, and Stein (1994), who argue that assets from defaulting projects can be redeployed more efficiently in an internal capital market.

[7]For a similar argument, see Rotemberg and Saloner (1994) and Inderst and Laux (2005). In contrast, in Stein's (2002) model, managerial incentives to produce information may be weaker or stronger in a hierarchy.

[8]In winner-picking models à la Stein (1997), the amount of resources that can be allocated across projects in an internal capital market is the same as under stand-alone finance. However, headquarters has the authority to redistribute assets from "losers" to "winners" while stand-alone financiers lack this authority. Hence, headquarters has advantages but no disadvantages. In contrast, in our model constrained and unconstrained investors have the same authority to reallocate resources, but constrained investors have fewer resources available. Hence, as far as the resource allocation goes, constrained investors have disadvantages but no advantages.

[9]While it is natural to think of I_2 as financial capital, it may alternatively represent human capital on the part of the investor, who must expend time and resources to coach the project.

[10]The fact that R^0 does not depend on the project's interim type simplifies the analysis, but is not crucial.

[11]By managing more than two projects, which represents the optimal span of the investor's portfolio in our model, the investor would spread herself too thin in the projects' critical start-up phase.

[12]Suppose there is a potentially large pool of fly-by-night operators—ex ante indistinguishable from genuine entrepreneurs—who have projects generating a zero payoff for sure. Knowing that they will receive a guaranteed payment, all these operators would apply for financing, in which case the investor's expected profit would quickly become negative. In contrast, under a sharing

rule the fly-by-night operators have nothing to gain from applying. Indeed, if there is an epsilon cost, they will strictly prefer not to apply.

[13]Leaving the decision rights with regard to the refinancing decision with the investor is optimal given our fly-by-night operator assumption. If the entrepreneur had the decision rights, a fraudulent entrepreneur could extract a bribe at $t = 1$: by virtue of his decision rights, he could force the investor to invest I_2 at the refinancing stage, which is a negative NPV undertaking given that projects by fly-by-night operators generate a zero payoff for sure. Hence, the two sides will strike a deal whereby the operator cedes his decision rights to the investor in return for a bribe. Anticipating this bribe, all operators would apply for financing.

[14]"Tangible assets may be easy to monitor without formal evaluation. A venture capitalist can tell if a machine is still bolted to the floor. ... Conversations with practicioners, however, indicate that they normally make continuation decisions when a new financing round occurs. Venture capitalists evaluate a firm based on performance progress, not whether a machine is still bolted down" (Gompers (1995)).

[15]Modelling bargaining frictions by a risk of breakdown is standard. In contrast to the case where bargaining frictions take the form of delay, it ensures that the two parties' outside options are always of relevance. That bilateral bargaining with a risk of breakdown, but not bargaining with delay, can support the axiomatic Nash bargaining solution with threatpoints has been shown by Binmore, Rubinstein, and Wolinsky (1986).

[16]As is standard in the literature, we consider the limit as bargaining frictions go to zero, i.e., $\delta \rightarrow 0$. In the limit, it is irrelevant who makes the first offer. See the Proof of Lemma 1 for details.

[17]Besides, the notion that the investor can extract the entire surplus at $t = 1$ does not square easily with our assumption that the entrepreneur is essential for the project's continuation.

[18]This is provided both entrepreneurs are still around, i.e., there has been no breakdown.

[19]If entrepreneurs are indifferent between $\theta = b$ and $\theta = g$, we assume without loss of generality that they choose $\theta = b$. Note that if the responsiveness condition (5) holds, there exists always a nonempty set of B values satisfying (6).

[20]To be precise, Proposition 2 does actually not require that (5) and (6) hold for all $\tau > 0$—the two conditions only need to hold for (sufficiently large) success probabilities for which constrained finance is viable.

[21]It is easy to construct a numerical example. For instance, if $q_b = 1/4$, $q_g = 1/2$, $r_l = 7$, $r_h = 11$, $R^0 = 8$, and $B = 1/2$, (5) and (6) hold for all $\tau > 0$. Given the expressions for the investor's and entrepreneur's expected payoffs derived in the Appendix, it can be easily verified that $\widehat{\tau} = 1/8$, while π_U^I and π_C^I intersect at $\tau = 1/2$, implying Case i) of Proposition 2 holds if $I_1 \geq 6$ while Case ii) holds if $I_1 < 6$. For example, if $I_1 = 1$ the project is not viable if $\tau < 0.0805$, constrained finance is chosen if $0.0805 \leq \tau \leq 1/8$, while unconstrained finance is chosen if $\tau > 1/8$.

[22]The derivation of $\widehat{\tau}$ and the entrepreneur's payoffs under constrained and unconstrained finance are found in the Proof of Proposition 2. There, it is also shown that $\hat{\tau}$ lies to the left of the intersection of π_U^I and π_C^I as depicted in Figure 2.

[23]Moreover, a decrease in q_b makes it more likely that Case ii) in Proposition 2 applies, for two reasons: the set of admissible B values satisfying (6) becomes larger, and the fact that π_U^I shifts to the right implies that the critical investment cost above which Case i) applies is shifted upwards.

[24]Straightforward calculations show that the likelihood that a project is rejected at the refinancing stage under constrained finance is $1 - \tau + \frac{1}{2}\tau^2$.

[25]Precisely, it must hold that $p_n < I_2/R$.

[26]In a two-payoff model with one payoff being $R > 0$ and the other payoff being zero, any feasible financial contract must necessarily involve a positive repayment if the payoff is R.

[27]The conditions for the equilibrium in which all three interim types have access to costly outside finance and the one in which only interim types $\psi \in \{n, h\}$ have access to costly outside finance are not mutually exclusive. It is easy to find values of τ for which both equilibria exist (in addition to the equilibrium in which the market for outside finance shuts down, which always exists).

[28]One can show that in the limit as $\delta \to 0$ the same outcome would obtain if instead of the investor sticking to her first-round pick she randomizes in every round. The analysis involves somewhat longer equations, though.

[29]To prove optimality of the investor's strategy, it suffices to consider one-stage deviations. See Fudenberg and Tirole (1992), Theorem 4.2.

[30]Strictly speaking, (28) and (29) are only meaningful for values of τ for which the project is viable, i.e., values for which (26) and (27) are non-negative. This rules out $\tau = 0$. However, given that all functions in question are strictly increasing and either linear or strictly concave, considering the functions' behavior at $\tau = 0$ provides us with information as to their behavior relative to each other for larger, admissible values of τ.

[31]Dividing through by $(r_h - r_l)$ and rearranging, we obtain $2(q_g - q_b) - q_g^2 < \frac{r_l}{r_h - r_l}$, which holds by (5).

[32]The left-hand side corresponds to (1) with $\theta_i = b$ and $s_i = s_U$, while the right-hand side corresponds to (3) with $\theta_i = b$, $\theta_j = g$, and $s_i = s_C$.

[33]The left-hand side corresponds to (3) with $\theta_i = \theta_j = g$ and $s_i = s_C$, while the right-hand side corresponds to (1) with $\theta_i = g$ and $s_i = s_U$.

[34]If both projects have interim type $\psi = n$, it is optimal for the insiders to present only one project to outside investors as the latter would otherwise rationally conclude that both projects are unsuccessful.

[35]If one project has interim type $\psi = h$ and the other has interim type $\psi = l$, it is optimal for the insiders to finance the former internally and to present the latter to outside investors.

[36]Recall that p_n is assumed to be small. If, e.g., p_n was close to p_l, (38) would trivially hold for all $\tau \geq 0$.

[37]We assume if the negotiations with E_i over the use of inside funds break down, the investor and E_i can still negotiate over the surplus realized from using costly outside funds. An alternative assumption would be that the breakdown is "complete" in the sense that *any* negotiations with E_i are made impossible. While the definition of what precisely a breakdown of negotiations means affects the form of the responsiveness condition derived below, our qualitative results do not hinge on it.

[38]Recall the joint probabilities for interim types (ψ_i, ψ_j) stated in the Proof of Lemma 2.

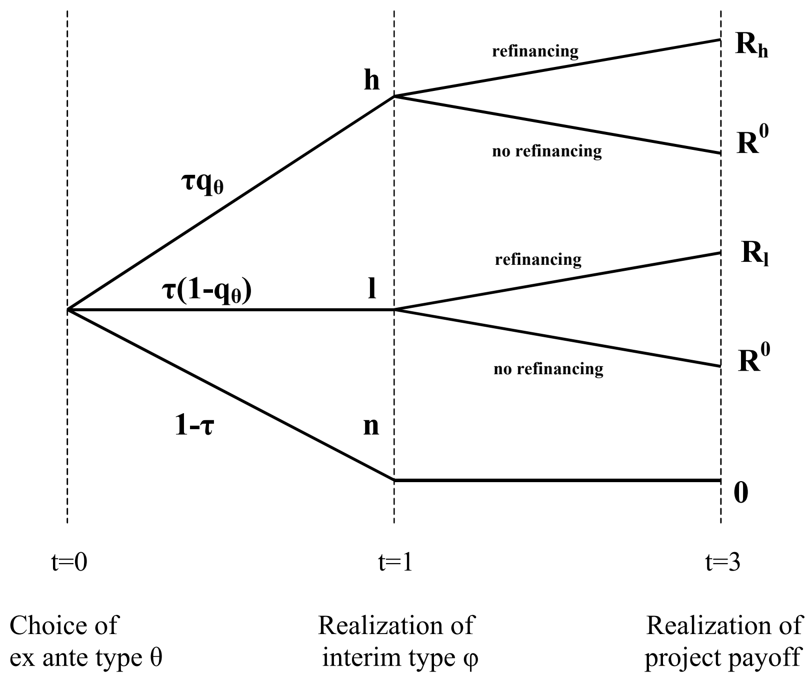

Figure 1: Summary of Project Technology

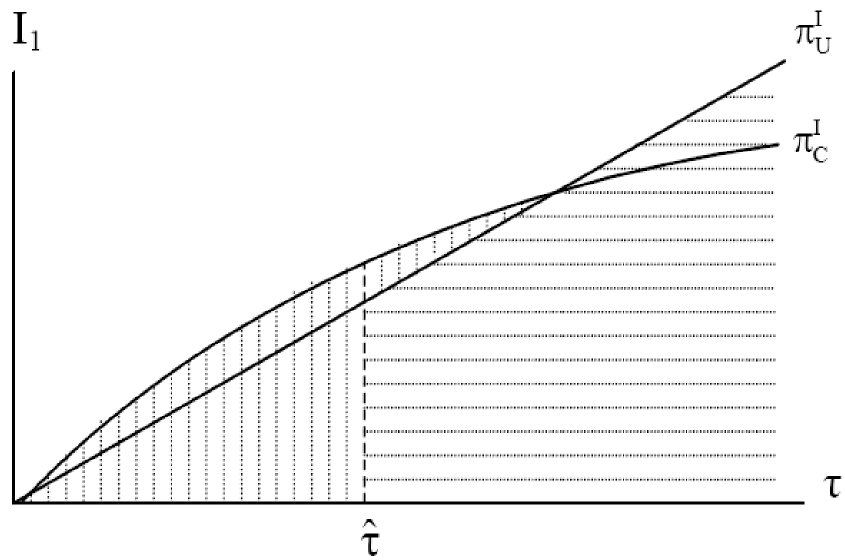

Figure 2: Illustration of Proposition 2. π_U^I represents the investor's expected gross payoff under unconstrained financed defined in (7), while π_C^I represents her expected gross payoff under constrained financed defined in (8). The entrepreneur's expected payoff is larger (smaller) under constrained finance if $\tau < \widehat{\tau}$ (if $\tau > \widehat{\tau}$). The shaded area depicts all combinations of investment costs I_1 and success probabilities τ for which the project is financially viable, implying it will be financed. In the vertically shaded area the investor chooses constrained finance, while in the horizontally shaded area she chooses unconstrained finance.