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ABSTRACT

Optimal Choice of Characteristics for a Non-Excludable Good*

I consider a model where a principal decides whether to produce one unit of an indivisible good (e.g. a private school) and which characteristics it will contain (emphasis on language or science). Agents (parents) are differentiated along two substitutable dimensions: a vertical parameter that captures their privately known valuation for the good (demand for private education), and an horizontal parameter that captures their observable differences in preferences for the characteristics. I analyze the optimal mechanism offered by the principal to allocate the good and show that the principal will produce a good with characteristics more on the lines of the preferences of the agent with the lowest valuation. Furthermore, if the principal has also a private valuation for the good, he will bias the choice of the characteristics against his own preferences.

JEL Classification: D44 and D62
Keywords: allocation mechanisms, externalities, mechanism design, non-excludable goods and vertical and horizontal differentiation

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1 Introduction

Consider the following problem. A principal (she) needs to decide whether to produce one unit of an indivisible good and, if she does, which characteristics it will contain. Production of the good affects positively the utility of two agents. These agents are differentiated along two dimensions. First, a vertical parameter, which captures their privately known valuations for the good. Second, a horizontal parameter, which captures their differences in preferences for the characteristics of the good.

The kind of examples we have in mind are the following. Suppose that a privately interested investor is deciding whether to construct a football stadium and where to locate it. Two neighboring cities are interested in the project. The vertical differentiation parameter is the cities' privately known demand for football. The horizontal differentiation is simply the physical location of the stadium: each city prefers to host it because this maximizes the identification of the residents with the team and minimizes their cost of attending a game. Last, the positive externality is captured by the fact that having the stadium between the two cities or even in the neighboring city is still better than not having the stadium at all. In this example, horizontal differentiation is literally interpreted as geographic distance. Naturally, it can also account for differences in tastes. For instance, suppose that an entrepreneur is deciding whether to open a new private school in the city. The school may put a special emphasis on languages or sciences, which becomes the horizontal differentiation parameter. Although residents find the initiative attractive, one group of parents cares specially about languages and the other about sciences. The vertical differentiation parameter is the overall desire to send their children to a private school.

It seems a priori natural to conclude that the stadium should be built in the city that values it most and the school should adapt its curriculum to the desire of the parents most willing to pay for private education. Interestingly, this is not always the case in practice. For instance, most French schools located in foreign countries adapt the curriculum to the preferences of local citizens even though French parents are most willing to enrol their children and prefer an emphasis on subjects traditionally taught in France.[1] One purpose of the present paper is to determine when this can occur.

More generally, we characterize the optimal contract offered by the principal to the two agents, given the asymmetry of information, the choice of characteristics, and the presence of positive externalities. Optimal contracting with asymmetric information and positive externalities has already been studied in the literature. Yet, previous studies always suppose that the

[1] A main objective of these institutions is to offer French education (and diploma) to French citizens located abroad. Parents who plan to come back to France or expect to travel from country to country in the future value highly the fact their children can get the same education at every location.

characteristics of the good are given prior to the contracting stage.[2] To the best of our knowledge, the endogenous choice of characteristics has been overlooked in the literature. This paper aims at filling that gap.

The literature studying contracting problems where positive externalities arise can be divided into two branches. The first branch analyzes contracting relationships when the principal contracts separately with several agents and when the contract between the principal and one agent generate positive externalities on other agents. This problem has been studied in a general setting in Segal (1999) and Segal and Whinston (2003).[3] It has also been investigated in specific settings by Cornelli (1996) and Lockwood (2000) among others.[4] In these articles, the item to be contracted upon is generally excludable and the Principal can a priori contract only with a subset of agents if she finds it optimal to. However, in our model the principal cannot produce one good for each agent (i.e. she cannot serve both agents separately, specify a different output level for each of them, or exclude some of them). Instead, her major decision is to determine the characteristics of the good. One could reinterpret "producing the good most preferred by one agent" as "selling the good to that agent". Thus, our setting resembles an auction, where the good can be allocated to one of the agents but the other one still enjoys a positive utility when this happens. However, unlike in the auction of an indivisible good, the principal is not forced to produce a good with the characteristics most liked by one agent. Instead, she chooses from a wide array of combinations, ranging from most preferred by one agent to equally appreciated by both of them. Also, it is formally not the same as the auction of a divisible good, either. In this type of auctions the owner may, in equilibrium, sell only a fraction of the good and keep the rest, a possibility not available in our setting.

The second branch studies the optimal contract between a principal and several agents when the item that is contracted upon affects the payoff of all agents. In particular, the literature on the provision of public goods in the tradition of Clarke (1971), Groves (1973) and D'Aspremont and Gérard-Varet (1979) offer mechanisms to implement the socially optimal level of public good with or without budget balance for the government.[5] A main ingredient in those articles,

[2]We mean by characteristic a property of the good on which agents disagree because their tastes differ. This rules out quality. Incentives to provide quality however have been studied for instance in Lewis and Sappington (1988) and (1991). See also Laffont and Tirole (1993) for a detailed analysis of the regulation of quality.

[3]Segal (1999) and Segal and Whinston (2003) analyze contracts in the presence of positive and negative externalities. Segal (1999) studies the nature of inefficiencies depending on whether contracts are observable or not. Segal and Whinston (2003) considers a larger family of games of contracting where contracts between the principal and one agent are not observed by other agents. The paper analyses general properties of equilibrium outcomes that must be satisfied by all equilibria of all games considered.

[4]Cornelli (1996) studies the optimal provision of a private good when the valuations of consumers are privately known and the firm has a high fixed cost of production. In that case, positive externalities arise between consumers, since purchasing the good affects positively the probability that the firm finds it profitable to produce it. Lockwood (2000) analyzes optimal contracts when the agents' marginal cost of effort is private information and the output of an agent is affected positively both by his effort and that of his co-workers.

[5]An important literature also discusses from a positive point of view how local public goods should be financed

as in the present paper, is that the good to be contracted upon is non-excludable and all agents benefit from its provision. However, our focus departs in two respects. First, the situations we have in mind are not necessarily decisions to produce public goods and we do not impose budget balance. Instead, we are concerned with participation constraints and want to ensure that all parties who enjoy the positive externality participate and contribute. Second and more importantly, in these analyses, the Principal has to decide over the quantity to be produced, and more quantity is always preferred by agents. By contrast, in our model, the Principal has to decide over an *attribute* on which agents disagree since the best characteristic for one agent is also the worst possible for the other. This generates a new trade-off for the contract designer.

The main features of the optimal contract are the following. We assume that the vertical and horizontal dimensions are substitutable, in the sense that the marginal importance attached by an agent to the characteristics of the good decreases as his valuation increases. We show that introducing a horizontal dimension generates a qualitative departure only when this assumption is satisfied. In that case, the principal always produces the good in the benchmark case of full information. Besides, she prefers to favor the agent with lowest valuation, that is to offer a good with characteristics more on the lines of his preferences than on the lines of the preferences of the other agent. Given the substitutability of the vertical and horizontal differentiation parameters, the loss in the revenue extracted from the high-valuation agent under this strategy is smaller than the gain in the revenue extracted from the low-valuation one.

Asymmetric information induces two distortions in the optimal contract, one for each agent. In fact, since production of the good affects the utility of the two agents, the optimal contract is such that the principal demands payments and grants informational rents to both of them. Interestingly, under incomplete information the principal favors *even more* the agent with lowest valuation than under full information. The idea is that the principal distorts the characteristics of the good offered in order to reduce the rents left to agents (the usual trade-off efficiency vs. rents). Due to substitutability of characteristics and valuation, marginal rents are greatest for the lowest valuation agent. Therefore, it is relatively more interesting to reduce the rents of this agent, which is achieved by selecting characteristics that are closer to his favorite ones. To sum up, positive externalities together with the capacity to extract payments from both agents induces the principal to select a convex combination of characteristics, with a slight tendency to favor the agent of lowest valuation. Asymmetric information exacerbates this bias, that is, it pushes the principal to make more extreme choices.

According to our optimal contract, when the reported valuations are sufficiently small, the principal commits not to produce the good, even if it always yields a net benefit to society.

by residents and land owners. It addresses the issue of which type of tax should be used taking into account how land prices affect location decisions as well as the size of the jurisdictions. See Scotchmer (2002) for a review.

This result is similar to the standard inefficiency in the auction literature, where the seller sometimes keeps the item even if her valuation is always smaller than that of every bidder. As in that literature, this ex-post inefficiency occurs because such commitment is an ex-ante optimal mechanism to reduce the expected informational rents. Overall, the presence of positive externalities alleviates that inefficiency but does not eliminate it.

Last, if one agent is also the producer of the good, he will bias the choice against his own preferences. This surprising result has a simple explanation. The principal trades-off two distortions when she selects the characteristics of the good. If one agent becomes the producer, one distortion disappears (an agent has no asymmetric information with himself) so that agent only needs to handle the distortion with the other agent. Thus, by increasing the bias in favor of the other agent, the producer reduces the informational rents and increases his overall utility.

The plan of the paper is the following. The model and the basic properties of the optimal mechanism are presented in section 2 and solved in section 3. Some extensions are discussed in section 4 and the concluding remarks are collected in section 5.

2 The model

2.1 Basic ingredients

We consider two agents A and B indexed by i and j. Each agent (from now on "he") is located at one extreme of a Hotelling line of measure N. Denoting by y_i the location of agent i, we have $y_A = 0$ and $y_B = N$. An indivisible good can be produced and then located somewhere on the line. We assume that agents have private information about their valuation $\theta_i \in [\underline{\theta}, \overline{\theta}]$ for this good (also referred to as "type"). Valuations are independently drawn from a common knowledge distribution $F(\theta_i)$ with continuous and strictly positive density $f(\theta_i)$. It also satisfies the standard monotone hazard rate property: $d\left[\frac{1-F(\theta)}{f(\theta)}\right]/d\theta < 0$. Agents are concerned about the location x of the good in the Hotelling line. We assume that x can take a finite but arbitrarily large number of locations, and we order these potential locations from closest to agent A to closest to agent B: $x \in \{0, 1, ..., N-1, N\}$. Denoting by γ_i $(= |x - y_i|)$ the distance between the location of the good and the location of agent i, the payoff of agent i takes the following form:

$$\pi(\theta_i - \gamma_i) \tag{1}$$

where $\pi' > 0$, $\pi'' < 0$ and, for technical convenience, $\pi''' \geqslant 0$. According to this formalization, the payoff is increasing in the valuation $(\partial\pi/\partial\theta_i > 0)$ and decreasing in the distance with the good $(\partial\pi/\partial\gamma_i < 0)$. Moreover, valuation is relatively more important the bigger the distance between the location of the agent and the location of the good $(\partial^2\pi/\partial\theta_i\partial\gamma_i > 0)$. In other words, high type agents are relatively less sensitive to distance. Overall, agents are differentiated along two substitutable dimensions captured by two parameters, a vertical differentiation parameter (the valuation for the good) and a horizontal differentiation parameter (the distance between the good and the agent).[6]

To be in the interesting case, we assume that the payoff of each agent when the good is produced is always greater than the payoff when it is not, which is normalized to zero. Formally, $\pi(\underline{\theta} - N) > 0$. As the reader can notice, our setting is characterized by *positive and type-dependent externalities*. Each agent prefers to have the good produced and the payoff of agents increases with their valuation, independently of the location of the good.

Last, in order to better concentrate on the inefficiencies of the allocation due to the asymmetry of information, we assume that producing the good is costless for the principal and generates no delay.[7]

Given these ingredients, we are interested in determining how the good is optimally located on the Hotelling line. We assume that the location decision is in the hands of a third party (from now on "principal" or "she"). The efficient allocation mechanism will of course be affected by the objective function of the principal. We will concentrate first on the case in which the principal's objective is to maximize revenue. In Section 4, we will analyze the case in which the principal cares about welfare (section 4.1) and the case in which the principal derives a private benefit from the provision of the good (section 4.2).

2.2 Examples

The purpose of this subsection is to provide a few examples in which the ingredients of our theory are present and for which we believe our normative approach can be useful.[8]

Physical location of a non-excludable private or public good.

This corresponds to the example of the football stadium mentioned in the introduction. In that example, agents A and B are simply two neighboring cities. The vertical differentiation parameter θ_i is the demand for football of each city and the horizontal differentiation parameter is the distance between the city and the stadium. Our formalization captures the following features. The payoff of each city when the stadium is built increases with its demand for football ($\partial\pi/\partial\theta_i > 0$) and decreases with the distance between the city and the stadium ($\partial\pi/\partial\gamma_i < 0$), and inhabitants of a city supporting a football team are relatively more inclined to drive to

[6] We discuss the results obtained under the alternative assumption $\partial^2\pi/\partial\theta_i\partial\gamma_i < 0$ in Section 3.1.

[7] Naturally, our model easily generalizes to the case of a positive cost of production. This would not affect the results qualitatively.

[8] Of course, other forces not studied in the present paper might also be at work in some of the examples. For instance, the party we refer as the Principal might not have as much bargaining power in real life and parties might bargain instead of resorting to take-it-or-leave-it offers. Our theory provides an upper bound on the payoff the Principal can obtain in that situation.

attend an event ($\partial^2\pi/\partial\theta_i\partial\gamma_i > 0$). Also, each city prefers a stadium located far away rather than no stadium at all (positive externalities) and, the utility of cities increases with their valuation, independently of the location (type-dependent externalities). The principal represents for instance an investor willing to build and manage a new stadium, in which case her objective is to maximize revenue. Or, the principal is a local authority trying to make the two cities agree to finance a public stadium. Of course, the model can be applied to other decisions to locate a non-excludable good such as a shopping mall or a hospital.

Creation of a private school.

In that example, agents A and B are two types of parents. The vertical differentiation parameter θ_i is the valuation of a new private school by the parents and the characteristics of the good is the emphasis of the school on languages vs. sciences. Given our assumptions, the payoff of a group of parents increases with their valuation for private education and decreases with the distance between the actual emphasis of the school and their desired emphasis. Parents with a high valuation for private education are relatively more willing to compromise on emphasis. Also, each group of parents prefers to have a new school even if its main emphasis is not on their preferred subject and the utility of parents increases with their valuation, independently of the subject emphasized. Last, the principal represents an investor contemplating the possibility to open a new school and maximizes revenue, or a parent willing to offer a personalized education to his own children and deciding to offer this new concept to other parents as well.[9]

Services offered to club members.

The principal is the administrator of a private golf or tennis club and maximizes revenue or welfare of club members. The club accepts families (agent A) who enjoy other activities besides sports (e.g. socializing, using a restaurant) and individual (agent B) who come mainly to practice. Club members who have a high valuation for the club are relatively more willing to compromise on the services offered, and all members value the club independently of the services offered.

Development of a new product.

The principal represents a monopolist deciding to develop a new product and maximizes profit. Agents A and B are two groups of consumers. The parameter θ_i represents the unknown demand for the new good in each group and γ_i is the difference between the preferred and the actual characteristics of the good for group i. Our model captures the fact that consumers with a high valuation for the new good are relatively more willing to compromise on characteristics.

[9] Private schools are sometimes created at the initiative of parents who want a particular education for their children. This has been the case for instance of the Lycée International de Los Angeles combining a French education with an international component (http://www.lilaschool.com).

Also, each group prefers to have the possibility to buy the new good even if its main characteristic is not the preferred one.

2.3 First-best

From now on in this section, we assume the principal maximizes her expected revenue. In order to have a benchmark for comparison, we denote by x_F the first-best location. It maximizes the payoff of the principal under full information. For any possible location x, the Principal extracts all the surplus generated by the production of the good at that location. Formally, her total revenue is $\pi_A(\theta_A, x) + \pi_B(\theta_B, x)$. Therefore, the good is located at x_F such that

$$x_F = \arg\max_x \quad \pi_A(\theta_A, x) + \pi_B(\theta_B, x) \tag{2}$$

Lemma 1 *Under complete information,* $x_F \gtreqless N/2$ *when* $\theta_A \gtreqless \theta_B$.

Proof. See Appendix 3.

Under complete information, the principal always produces the good. Given the substitutability of the vertical and horizontal differentiation parameters, she prefers to favor the agent with lowest valuation, that is to offer a good with characteristics more on the lines of his preferences than on the lines of the preferences of the other agent. This is the case because, by doing so, the loss in the revenue extracted from the high-valuation agent is smaller than the gain in the revenue extracted from the low-valuation agent. In the next sections we study how asymmetric information modifies this allocation. In particular we want to determine whether it exacerbates the bias or not.

2.4 Properties of the mechanism under asymmetric information

Note that (i) the willingness to pay of each agent depends on his privately known valuation and (ii) every location affects the utility of both agents. Therefore, the principal must design a contract that provides the agents with the adequate incentives to reveal their information. Moreover, the payments of both agents in the optimal mechanism must be determined simultaneously.

The contract must specify an allocation rule and payments when both agents accept the contract but also when at least one refuses it. Indeed, given the presence of externalities, the outside option of each agent is *mechanism dependent*. Note that the objective of the principal is to extract as much payments as possible from both agents. Therefore, she benefits from designing a mechanism in which the outside option is the smallest possible. Following the standard contracting literature, we assume that the *principal can commit* to any mechanism offered to the agents and each agent accepts the contract when the utility of accepting is at least equal to the outside option.[10] Given the commitment ability, it is therefore immediate that, in the optimal mechanism, the principal will commit not to produce the good if at least one agent refuses the contract. The idea is simply that given the positive externalities, the worst possible scenario for any agent who refuses to participate is the one in which the good is never produced. Note that this threat is only credible if the principal can commit. On the other hand, it is costless for her, as it is only made off-the-equilibrium path.

Given that producing the good is costless for the principal and both agents always value the good then, in the absence of informational problems, the good will be produced with probability one and located somewhere in the Hotelling line. Under asymmetric information, the principal makes two choices: whether to produce the good and, if she produces the good, where to locate it. Denote by $e = \emptyset$ the event "the principal does not produce the good" (which, given our assumptions, is equivalent to producing the good but keeping it) and by $e = x \in \{0, ..., N\}$ the event "the good is produced and located at x".

From the revelation principle, we know that we can restrict the attention to a direct revelation mechanism. The principal offers a menu of contracts to each agent that depends on the pair of announced valuations $(\tilde{\theta}_A, \tilde{\theta}_B)$. The menu specifies a probability $p_x(\tilde{\theta}_A, \tilde{\theta}_B)$ of production at each possible location x, and a transfer $t_i(\tilde{\theta}_A, \tilde{\theta}_B)$ from each agent to the principal. We also denote by $p_\emptyset(\tilde{\theta}_A, \tilde{\theta}_B)$ the probability of not producing the good.

For notational convenience, let $\pi_i(\theta_i, x) \equiv \pi(\theta_i - |x - y_i|)$ be agent i's payoff when the good is located at x. Given that each agent is situated at one extreme of the line, we have:

$$\pi_A(\theta_A, x) = \pi(\theta_A - x) \qquad \text{and} \qquad \pi_B(\theta_B, x) = \pi(\theta_B - (N - x)) \tag{3}$$

Also, let $u_i(\theta_i, \tilde{\theta}_i)$ be the *expected utility* of agent i when his valuation is θ_i, he announces $\tilde{\theta}_i$ and the other agent discloses his true valuation θ_j. We denote by $u_i(\theta_i) \equiv u_i(\theta_i, \theta_i)$ his expected utility under truthful revelation. We have:

$$u_i(\theta_i, \tilde{\theta}_i) = \int_{\underline{\theta}}^{\overline{\theta}} \Big(\Big[\sum_{x=0}^{N} \pi_i(\theta_i, x) p_x(\tilde{\theta}_i, \theta_j) \Big] - t_i(\tilde{\theta}_i, \theta_j) \Big) dF(\theta_j)$$

A mechanism $\{p_x(\cdot), t_i(\cdot)\}$ is optimal if and only if it maximizes R, the expected revenue of the principal:

$$R = \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \Big[t_A(\theta_A, \theta_B) + t_B(\theta_A, \theta_B) \Big] dF(\theta_A) dF(\theta_B)$$

[10]This last (standard) assumption is made without loss of generality. If we assume alternatively that the agent refuses the contract or accepts it only with some positive probability in case of indifference, then the principal needs to break the indifference by giving an arbitrarily small compensation.

and satisfies three kinds of constraints. First, *incentive-compatibility*, which states that each agent must prefer to state his true valuation rather than any other one:

$$u_i(\theta_i) \geqslant u_i(\theta_i, \tilde{\theta}_i) \qquad \forall\, i, \theta_i, \tilde{\theta}_i$$

Second, *individual-rationality*, which implies that each agent must be willing to accept the contract offered by the principal (recall that in case of non-acceptance of the contract the good is never allocated, so the agent's reservation utility is zero):[11]

$$u_i(\theta_i) \geqslant 0 \qquad \forall\, i, \theta_i$$

Last, the allocation rule must be *feasible*:[12]

$$p_x(\theta_A, \theta_B) \geqslant 0 \quad \forall\, x, \theta_i, \theta_j \qquad \text{and} \qquad \sum_{x=0}^{N} p_x(\theta_A, \theta_B) \leqslant 1 \quad \forall\, \theta_i, \theta_j$$

Lemma 2 *The optimal mechanism solves the following program $\mathcal{P}$:*

$$\mathcal{P}: \max_{p_x(\theta_A,\theta_B)} \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \sum_{x=0}^{N} p_x(\theta_A, \theta_B) \Bigg[\pi_A(\theta_A, x) - \frac{\partial \pi_A(\theta_A, x)}{\partial \theta_A} \frac{1 - F(\theta_A)}{f(\theta_A)}$$
$$+ \pi_B(\theta_B, x) - \frac{\partial \pi_B(\theta_B, x)}{\partial \theta_B} \frac{1 - F(\theta_B)}{f(\theta_B)} \Bigg] dF(\theta_A) dF(\theta_B)$$

$$\text{s. t.} \quad \sum_{x=0}^{N} \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial \pi_A}{\partial \theta_A} \times \frac{\partial p_x}{\partial \theta_A}\, dF(\theta_B) \geqslant 0 \quad \text{and} \quad \sum_{x=0}^{N} \int_{\underline{\theta}}^{\overline{\theta}} \frac{\partial \pi_B}{\partial \theta_B} \times \frac{\partial p_x}{\partial \theta_B}\, dF(\theta_A) \geqslant 0 \tag{M}$$

$$p_x(\theta_A, \theta_B) \geqslant 0 \quad \forall\, x \quad \text{and} \quad \sum_{x=0}^{N} p_x(\theta_A, \theta_B) \leqslant 1 \tag{F}$$

Proof. See Appendix 1.

The *net surplus* of agents A and B when the good is located at x are $\pi_A(\theta_A, x)$ and $\pi_B(\theta_B, x)$, respectively. Under complete information, this also corresponds to their willingness to pay and therefore to the maximum revenue that the principal can extract. Naturally, these net surplus are increasing in the agents' valuations and decreasing in the distance between the location of

[11] In particular a given agent cannot refuse to participate in the contract and produce the good on his own. Under this alternative assumption, the outside option would be type-dependent and countervailing incentives would arise (see Maggi and Rodriguez (1995) and Jullien (2000)). Besides, given the presence of externalities, the decision of each agent to produce the good would also affect the outside option of the other agent. This alternative analysis is out of the scope of the present paper. However, we analyze in Section 4.2. the case in which one agent designs the contract (that is becomes principal) and produces the good.

[12] Another way to rewrite the constraint is $p_{\emptyset}(\theta_i, \theta_j) + \sum_{x=0}^{N} p_x(\theta_i, \theta_j) = 1$.

the agent and the location of the good. Asymmetric information introduces a distortion in the agents' willingness to pay. Denote by:

$$\Phi_A(\theta_A, x) = \pi_A(\theta_A, x) - \frac{\partial \pi_A(\theta_A, x)}{\partial \theta_A} \frac{1 - F(\theta_A)}{f(\theta_A)} \tag{4}$$

$$\Phi_B(\theta_B, x) = \pi_B(\theta_B, x) - \frac{\partial \pi_B(\theta_B, x)}{\partial \theta_B} \frac{1 - F(\theta_B)}{f(\theta_B)} \tag{5}$$

the *virtual surplus* of agents A and B when the good is located at x. It represents the surplus that the principal can extract from both agents when she locates the good at x (first term in equations (4) and (5)) adjusted for the informational rents that she is obliged to grant to both agents due to the asymmetry of information (second term in equations (4) and (5)). Lemma 2 thus states that the principal will choose the location that maximizes these virtual surplus under the standard monotonicity (M) and feasibility (F) constraints.[13] Given the concavity of π and the monotone hazard rate property of the distribution $F(\cdot)$, then for all x the virtual surplus increase with the valuations of agents: $\partial\Phi_A/\partial\theta_A > 0$ and $\partial\Phi_B/\partial\theta_B > 0$. Note that the analysis of the allocation mechanism considered here is an adaptation of the procedure introduced by Myerson (1981) in the context of an auction.

3 The optimal location

We can now proceed to the analysis of the optimal contract offered by the principal to the two agents given the existing asymmetry of information. We first study the case in which the good can only be located at the two extremes of the Hotelling line ($x \in \{0, N\}$, section 3.1). This restricted model has some interesting properties and several analogies with the auction literature. We then analyze the more general case in which the good can be situated in a finite but arbitrarily large number of locations ($x \in \{0, 1, ..., N-1, N\}$, section 3.2).

3.1 Optimal contract with two possible locations

The principal's choice when the good can only be located at $x = 0$ or $x = N$ is quite interesting. In fact, this problem is formally identical to the optimal auction of an indivisible good with two bidders (A and B), private valuations and positive type-dependent externalities. To see the analogy, note that the principal has three alternatives. First, she may decide not to produce the good, in which case both agents get utility 0. Second, she may produce the good and locate it at $x = 0$, in which case agent A gets utility $\pi(\theta_A)$ and agent B gets utility $\pi(\theta_B - N)$. Third, she may produce the good and locate it at $x = N$, in which case agent A gets utility $\pi(\theta_A - N)$ and agent B gets utility $\pi(\theta_B)$. Now, call $v_i(\theta_i) \equiv \pi(\theta_i)$ and $\alpha_i(\theta_i) \equiv \pi(\theta_i - N)$ ($< v_i$ for all θ_i).

[13] Recall that monotonicity is the second-order condition which states that revealing the true valuation $\tilde{\theta}_i = \theta_i$ must be globally optimal. Feasibility just ensures that the functions $p_x(\cdot)$ are well-defined probabilities.

Locating the good at $x = 0$ and at $x = N$ in our model is thus formally equivalent to selling the good to agent A and to agent B respectively: the agent who purchases it gets utility v_i (increasing in his type θ_i) and the other one enjoys a positive externality α_j (also increasing in his type θ_j).[14]

Using Lemma 2, equations (4)-(5) and ignoring for the moment constraint (M), it is immediate that in the optimal mechanism:

$$\begin{array}{lllll} \text{If} & \Phi_A(\theta_A, 0) + \Phi_B(\theta_B, 0) & > & \max\{\, 0, \Phi_A(\theta_A, N) + \Phi_B(\theta_B, N)\,\}, & \text{then} \quad p_0(\theta_A, \theta_B) = 1 \\ \text{If} & \Phi_A(\theta_A, N) + \Phi_B(\theta_B, N) & > & \max\{\, 0, \Phi_A(\theta_A, 0) + \Phi_B(\theta_B, 0)\,\}, & \text{then} \quad p_N(\theta_A, \theta_B) = 1 \end{array}$$

Also, denote by $r_i(\theta_j, x)$ the value of θ_i such that $\Phi_i(r_i(\theta_j, x), x) + \Phi_j(\theta_j, x) = 0$.[15] At this point, we can state our first result.

Proposition 1 *With two possible locations $x \in \{0, N\}$, the optimal contract is such that:*

$$\begin{cases} p_0(\theta_A, \theta_B) = 1 & \textit{if} \;\; \theta_A < \theta_B \;\; \textit{and} \;\; \theta_A > r_A(\theta_B, 0) \\ p_N(\theta_A, \theta_B) = 1 & \textit{if} \;\; \theta_B < \theta_A \;\; \textit{and} \;\; \theta_B > r_B(\theta_A, N) \\ p_\emptyset(\theta_A, \theta_B) = 1 & \textit{otherwise} \end{cases}$$

In equilibrium, the expected utility of agent $i = \{A, B\}$ is

$$u_i(\theta_i) = \int_{\underline{\theta}}^{\theta_i} \int_{\underline{\theta}}^{\overline{\theta}} \left[p_0(s, \theta_j) \frac{\partial \pi_i}{\partial s}(s, 0) + p_N(s, \theta_j) \frac{\partial \pi_i}{\partial s}(s, N) \right] dF(\theta_j) ds.$$

Proof. See Appendix 2.

When the principal decides where to locate the good, she compares the virtual surplus at each location. Since externalities are positive and type-dependent, the surplus depends on the valuations of both agents. The two distortions capture the idea that, in order to induce truthful revelation of types, the principal must grant informational rents to both agents independently of the final location of the good.

The optimal mechanism described in Proposition 1 has some interesting properties. First, the good will never be produced where the agent with highest valuation is located (formally, $\theta_A > \theta_B \Rightarrow x \neq 0$ and $\theta_B > \theta_A \Rightarrow x \neq N$). This is due to the type-dependency of the externality and the substitutability between the vertical and horizontal dimensions (or complementarity between valuation and distance $\partial^2 \pi / \partial \theta_i \partial \gamma_i > 0$). The key issue is that, at any location, the principal extracts payments from *both agents*. So, suppose that $\theta_A > \theta_B$. By definition, A is

[14] Obviously, what matters for the analogy is the existence of only two possible locations, one closer to A and one closer to B (that is, the same reinterpretation holds if for example $x \in \{1, N-1\}$).

[15] If, for some pairs (θ_j, x), $\Phi_i(\theta_i, x) + \Phi_B(\theta_j, x) > 0$ for all θ_i then $r_i(\theta_j, x) \equiv \underline{\theta}$ and if, also for some pairs (θ_j, x), $\Phi_i(\theta_i, x) + \Phi_j(\theta_j, x) < 0$ for all θ_i then $r_i(\theta_j, x) \equiv \overline{\theta}$.

willing to pay more than B to have the good at his own location. However, by locating the good at $x = N$ rather than at $x = 0$, the loss in the revenue extracted from agent A is smaller than the gain in the revenue extracted from agent B.

Second, a standard result in the auction literature is the existence of an ex-post inefficiency. Even if the auctioneer's utility of keeping the good is smaller than the bidders' lowest valuation, in equilibrium the good may not be sold.[16] Under positive externalities, this inefficiency is diminished but still persists: for some pairs of valuations (θ_A, θ_B) the principal does not produce the good ($e = \emptyset$) even though each agent derives a positive utility under all locations.[17] The reason for such inefficiency is the usual one. Under asymmetric information, the principal must grant some rents to the agents to induce truthful revelation of their type. In order to reduce these rents, the principal produces the good with lower probability than in the first-best case (the standard trade-off efficiency vs. rents). Note that r_i is the analogue of a reserve price for bidder i in an auction mechanism. However, instead of being fixed, it depends negatively on the valuation of the other agent ($\partial r_i / \partial \theta_j < 0$). This is again due to the type-dependency of the externality. As the valuation of one agent increases, his willingness to pay at any given location also increases. Therefore, the minimum valuation of the other agent above which the principal finds it optimal to produce the good decreases.

Third, we can perform some comparative statics about the effect of the externality on the optimal contract. Note that $\partial \pi(\theta_i - N)/\partial N < 0$. This means that, in this model with two possible locations, the size of the externality is inversely related to the length of the Hotelling line. We show that $\partial r_i(\theta_j, x)/\partial N > 0$. As the externality increases (i.e. as N decreases) the regulator can extract more payoff from the agents. Therefore, the event $e = x \in \{0, N\}$ becomes relatively more profitable than the event $e = \emptyset$ (i.e. r_i decreases). These results are depicted in Figure 1.

[INSERT FIGURE 1 HERE]

The reader might be interested in the results obtained when the payoff function satisfies the assumption $\partial^2 \pi / \partial \theta_i \partial \gamma_i < 0$ (which is the case in our model when $\pi''(\cdot) > 0$), that is when high type agents are also relatively more concerned about distance. In that case, the good is located in 0 (resp. N) when $\theta_A > \theta_B$ (resp. $\theta_A < \theta_B$).[18] Given the agent with the highest valuation is also the least willing to compromise on location, the decision is biased towards that agent. This leads to the two following immediate conclusions. First, when $\partial^2 \pi / \partial \theta_i \partial \gamma_i < 0$, horizontal differentiation does not introduce any qualitative departure compared to the model with vertical differentiation only. In other words, the interesting case arise when $\partial^2 \pi / \partial \theta_i \partial \gamma_i > 0$. Second,

[16] See Myerson (1981).

[17] More precisely and from Proposition 1, the pairs of valuations (θ_A, θ_B) such that $p_\emptyset(\theta_A, \theta_B) = 1$ are those that satisfy $\theta_A < r_A(\theta_B, 0)$ and $\theta_B < r_B(\theta_A, N)$.

[18] In that alternative model it is also necessary to assume $\pi'''(\cdot) \leqslant 0$ to avoid bunching.

observing that a good is located close to the interest of the party who enjoys it the least (as in the case of the French school) cannot be reconciled with the assumption $\partial^2\pi/\partial\theta_i\partial\gamma_i < 0$.

3.2 Optimal contract with several possible locations

We now turn to analyze the more general setting in which the number of potential locations for the good is finite but arbitrarily large ($x \in \{0, 1, ..., N\}$). This case cannot be reinterpreted as an auction of an indivisible good with externalities. Formally, it shares some features with the auction of a divisible good:[19] for example, locating the good at $x = N/2$ is similar to selling half of the good to one agent and half to the other one. However, there is a crucial difference between the two interpretations. In fact, not producing the good in our model ($e = \emptyset$) corresponds to not selling it in the auction case, and locating the good somewhere in the line ($e = x$) corresponds to splitting it entirely between the two bidders. Yet, in auctions of divisible goods there is a third possibility implicitly ruled out in our setting, which is to sell a fraction of the good and keep the rest.[20]

We denote by x_S the optimal second-best location. It maximizes the sum of the virtual surplus, that is the payoff of the principal given the asymmetry of information:

$$x_S \;=\; \arg\max_{x} \quad \Phi_A(\theta_A, x) + \Phi_B(\theta_B, x) \tag{6}$$

We can now state our second result.

Proposition 2 *When the set of locations is arbitrarily large, the optimal contract is such that:*[21]

$$\begin{cases} p_{x_S}(\theta_A, \theta_B) = 1 & \textit{if} \;\; \Phi_A(\theta_A, x_S) + \Phi_B(\theta_B, x_S) > 0 \\ p_{\emptyset}(\theta_A, \theta_B) = 1 & \textit{otherwise} \end{cases}$$

In equilibrium, the expected utility of agent $i = \{A, B\}$ is

$$u_i(\theta_i) = \int_{\underline{\theta}}^{\theta_i} \int_{\underline{\theta}}^{\overline{\theta}} \sum_{x=0}^{N} p_x(s, \theta_j) \frac{\partial \pi_i}{\partial s}(s, x) dF(\theta_j) ds.$$

The location x_S is such that: $\frac{\partial x_S}{\partial \theta_A} > 0$, $\frac{\partial x_S}{\partial \theta_B} < 0$ and $x_S \gtreqless x_F \gtreqless N/2$ for all $\theta_A \gtreqless \theta_B$. Furthermore, $\frac{\partial r_i(\theta_j, x_S)}{\partial \theta_j} < 0$.

<u>Proof</u>. See Appendix 3.[22]

[19] See Maskin and Riley (1989).

[20] In other words, our setting can be reinterpreted as the auction of a divisible good under the restriction that the auctioneer must either keep the good or allocate it entirely between the bidders.

[21] The optimal contract can also be written as $p_{x_S}(\theta_A, \theta_B) = 1$ if $\theta_i > r_i(\theta_j, x_S)$ and $p_{\emptyset}(\theta_A, \theta_B) = 1$ otherwise.

[22] Note that the problem is formally different from the allocation of a good to one person. Therefore, the proof does not follow Myerson (1981) and needs to be adapted to our specific contracting problem.

The first important conclusion of Proposition 2 is that the basic location principle highlighted in Proposition 1 extends to the case of a large number of possible locations. Basically, the principal first determines which location x_S maximizes the virtual surplus, and then compares this total payoff with the payoff under no production of the good. If both agents have the same valuation, then the good will be located halfway between the two. As before, when types are different, the good is located closer to the agent with lowest valuation, although it will not necessarily be at the exact location of the agent ($\theta_A \gtrless \theta_B \Leftrightarrow x_S \gtrless N/2$). Given the cost of rents due to asymmetric information, the principal may again decide not to produce the good ($e = \emptyset$). However, the ability to choose from a wider range of locations makes this event relatively less likely to occur than in Proposition 1. Moreover, the good is located more efficiently than in Proposition 1.

It is interesting to notice that asymmetric information induces the principal to increase the distance between the location of the good and that of the agent who values it most, relative to the socially optimal level (formally, $\theta_A \gtrless \theta_B \Leftrightarrow x_S \gtrless x_F$). In fact, the principal has to manage simultaneously two distortions (one for each agent), $\frac{\partial \pi_A}{\partial \theta_A} \frac{1-F(\theta_A)}{f(\theta_A)}$ and $\frac{\partial \pi_B}{\partial \theta_B} \frac{1-F(\theta_B)}{f(\theta_B)}$, and both increase with the distance between the agent and the good. As the valuation θ_i of an agent increases, the distortion becomes less sensitive to the distance γ_i. Therefore, in order to decrease the rents, it becomes relatively more interesting to bring the location of the good closer to the agent with lowest valuation.

Now, suppose that we allow the principal to locate the good outside the imaginary line that connects the two agents. Naturally, any choice outside $[0, N]$ is inefficient: it is always possible to increase the utility of both agents by situating the good within that segment. Yet, since the principal sometimes takes the (also inefficient) decision of not producing the good, one can think that the principal will make use of this extra possibility. This intuition is incorrect. In fact, the reason for no production is a simple cost-benefit trade-off. Recall that informational rents are increasing in the agent's valuation. By not producing the good if the valuation is sufficiently low, the principal gives no rents to an agent with that valuation and, most importantly, decreases the rents proportionally if his valuation is above that value. This gain is compared to the loss of no production. By contrast, the alternative of producing the good and locating it outside the Hotelling line, has costs but no benefits: the choice is inefficient and still forces the principal to grant informational rents for truthful revelation of the agents' valuation. Hence, in equilibrium, the good will never be located outside $[0, N]$. These properties are graphically represented in Figure 2.

[INSERT FIGURE 2 HERE]

The mechanism described in Proposition 2 immediately extends to the case in which more than two agents are affected by the location of the good. Formally, suppose that there are

M agents, indexed by $k \in \{1, 2, ..., M\}$. Agent k is located at $y_k \in [0, N]$, and we denote by $\pi_k(\theta_k, x) \equiv \pi(\theta_k - |x - y_k|)$ the payoff of agent k when the good is located at x. Also, we call $\theta = (\theta_1, \theta_2, ..., \theta_M)$ the vector of valuations. The optimal location is such that:[23]

$$\begin{cases} p_{x_M}(\theta) = 1 & \text{if } \sum_{k=1}^{M} \Phi_k(\theta_k, x_M) > 0 \\ p_{\emptyset}(\theta) = 1 & \text{otherwise} \end{cases}$$

where $x_M = \arg\max_x \sum_{k=1}^{M} \Phi_k(\theta_k, x)$. Not surprisingly, production takes place only if the vector of valuations θ is above a certain level. Also, if agents are evenly distributed over the Hotelling line then, other things being equal, the good will be located in the sector where agents have lowest willingness to pay for the good.

3.3 A numerical example

In order to provide a quantitative idea of the differences between the optimal locations under full and asymmetric information (x_F and x_S), we consider the following numerical example:

$$\pi(\theta_i - \gamma_i) = 4(\theta_i - \gamma_i) - (\theta_i - \gamma_i)^2 \qquad \text{and} \qquad \theta_i \sim \text{U}[1, 2]$$

We also let $N = 1$ and we assume that x can take any value in $[0, 1]$, so that $\theta_i - \gamma_i \in [0, 2]$. Using (3)-(4)-(5)-(6)-(2), we immediately obtain the following expressions for the optimal locations:[24]

$$x_F = \frac{1}{2} + \frac{1}{2}(\theta_A - \theta_B) \qquad \text{and} \qquad x_S = \begin{cases} 0 & \text{if } \theta_A - \theta_B < -1/2 \\ \frac{1}{2} + (\theta_A - \theta_B) & \text{if } \theta_A - \theta_B \in [-1/2, 1/2] \\ 1 & \text{if } \theta_A - \theta_B > 1/2 \end{cases}$$

Note that, under asymmetric information, the good will always be located closer to the agent with lowest valuation than under full information. Moreover, as long as x_F and x_S are interior, the distortion increases as the difference in the valuations of the agents $|\theta_A - \theta_B|$ increases. Also, if the difference between valuations is sufficiently important ($|\theta_A - \theta_B| > 1/2$), then the agent with lowest valuation enjoys the good at his favorite location.

4 Extensions

The benchmark model developed in section 2 can be extended in a number of directions. In this section we study two that we find particularly relevant. In the first one, the production and location of the good is decided by a benevolent regulator who maximizes the welfare of

[23] Given the close analogy with Proposition 2, the proof is omitted. Of course, it is available upon request.

[24] We have assumed previously that the number of locations is finite. However, our results continue to hold when this number is arbitrarily large, and in particular if x is a continuous variable.

society (section 4.1). Indeed, it is difficult to reconcile the revenue-maximizing assumption with some applications of our theory, such as the decision to locate a public good between two communities.[25] In the second one, this decision is taken by one of the agents (section 4.2) to capture the fact that the planner (e.g. a parent/entrepreuner) can also have a private interest in the project (e.g. the private school).

4.1 Optimal location selected by a social planner

Suppose that, instead of a privately interested party, the principal is a benevolent utilitarian regulator. Given asymmetry of information and the conflict of interests between the two agents, she must design an incentive contract, much in the lines of the revelation scheme developed in section 2. More precisely, the regulator offers to each agent a menu that specifies, for every pair of announced valuations $(\tilde{\theta}_A, \tilde{\theta}_B)$, a probability $p_x(\tilde{\theta}_A, \tilde{\theta}_B)$ of locating the good at x together with a subsidy $s_i(\tilde{\theta}_A, \tilde{\theta}_B)$ to agent i. The key assumption in the whole regulation literature is that subsidies are socially costly: \$1 transferred to an agent is raised through distortionary taxation and costs $\$(1+\lambda)$ to taxpayers, with $\lambda > 0$.[26]

We denote by $\hat{u}(\theta_i, \tilde{\theta}_i)$ the expected utility of agent i when he has a valuation θ_i, he announces $\tilde{\theta}_i$, and agent j reports his true valuation θ_j, then:

$$\hat{u}(\theta_i, \tilde{\theta}_i) = \int_{\underline{\theta}}^{\overline{\theta}} \sum_{x=0}^{N} \pi_i(\theta_i, x) p_x(\tilde{\theta}_i, \theta_j) + s_i(\tilde{\theta}_i, \theta_j) dF(\theta_j)$$

The objective function of the utilitarian regulator, denoted by W, is to maximize social welfare. Given the shadow cost λ of public funds, the social welfare is simply the payoff of the agents when the good is produced at x (π_A and π_B) minus the social costs of transferring an amount of funds s_A and s_B from the consumers to the agents. Formally:

$$W = \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \sum_{x=0}^{N} p_x(\theta_A, \theta_B) \Big[\pi_A(\theta_A, x) + \pi_B(\theta_B, x)\Big] - \lambda s_A(\theta_A, \theta_B) - \lambda s_B(\theta_A, \theta_B) dF(\theta_A) dF(\theta_B)$$

The regulator's optimization program is thus to maximize W under the usual incentive-compatibility ($\hat{u}(\theta_i, \theta_i) \geqslant \hat{u}(\theta_i, \tilde{\theta}_i)$ for all $i, \theta_i, \tilde{\theta}_i$), individual-rationality[27] ($\hat{u}(\theta_i, \theta_i) \geqslant 0$ for all i, θ_i) and fea-

[25] We take a neutral approach and assume the Principal is benevolent. In the case of local public goods however, it is conceivable that the manager of the jurisdiction acts more on behalf of a certain type of residents. The objective function of local authorities as well as the subsequent effect on optimal provision of local public goods are addressed in Urban Economics. See for instance Hamilton (1975), Wildasin (1979) and Scotchmer (1994) among others. See also Scotchmer (2002) for a review.

[26] For the seminal analyses of optimal regulation under asymmetric information, see Baron and Myerson (1982) and Laffont and Tirole (1986). In the first paper transfers are not costly but society attaches a higher weight to consumers than to firms. In the second one, each party has equal weight but transfers are costly. Both models yield similar insights in terms of the optimal mechanism.

[27] Here again, the principal uses out-of-equilibrium threats. Indeed, the highest the rent left to the agent the smallest the welfare. Therefore, the principal pushes the agents towards their worst outside option (not allocating the good if at least one agent refuses to participate).

sibility ($p_x(\theta_A, \theta_B) \geqslant 0$ for all x and $\sum_{x=0}^{N} p_x(\theta_A, \theta_B) \leqslant 1$) constraints for agents A and B. Following the same steps as in Lemma 2, the program can be rewritten as:[28]

$$\mathcal{P_W} : \max_{p_x(\theta_A,\theta_B)} \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \sum_{x=0}^{N} p_x(\theta_A, \theta_B) \Big[\pi_A(\theta_A, x) - \frac{\lambda}{1+\lambda} \frac{\partial \pi_A(\theta_A, x)}{\partial \theta_A} \frac{1 - F(\theta_A)}{f(\theta_A)}$$

$$+ \pi_B(\theta_B, x) - \frac{\lambda}{1+\lambda} \frac{\partial \pi_B(\theta_B, x)}{\partial \theta_B} \frac{1 - F(\theta_B)}{f(\theta_B)} \Big] dF(\theta_A) dF(\theta_B)$$

$$\text{s. t.} \quad \text{(M)-(F)}$$

We now define the functions $\Lambda_i(\theta_i, x)$, $\hat{r}_i(\theta_j, x; \lambda)$ and $x_W(\lambda)$ which are the analogue of $\Phi_i(\theta_i, x)$, $r_i(\theta_j, x)$ and x_S to the regulation case:

$$\Lambda_i(\theta_i, x) = \pi_i(\theta_i, x) - \frac{\lambda}{1+\lambda} \frac{\partial \pi_i(\theta_i, x)}{\partial \theta_i} \frac{1 - F(\theta_i)}{f(\theta_i)}$$

$$\Lambda_i(\hat{r}_i(\theta_j, x; \lambda), x) + \Lambda_j(\theta_j, x) = 0$$

$$x_W(\lambda) = \arg\max_x \quad \Lambda_A(\theta_A, x) + \Lambda_B(\theta_B, x) \tag{7}$$

and we get the following result.

Proposition 3 *When a regulator chooses the location, the optimal contract is such that:*

$$\begin{cases} p_{x_W}(\theta_A, \theta_B) = 1 & \text{if } \Lambda_A(\theta_A, x_W) + \Lambda_B(\theta_B, x_W) > 0 \\ p_{\emptyset}(\theta_A, \theta_B) = 1 & \text{otherwise} \end{cases}$$

The location x_W is such that: $\frac{\partial x_W}{\partial \theta_A} > 0$, $\frac{\partial x_W}{\partial \theta_B} < 0$ and $x_S \gtrless x_W \gtrless x_F \gtrless N/2$ for all $\theta_A \gtrless \theta_B$. Furthermore, $\frac{\partial x_W(\lambda)}{\partial \lambda} \gtrless 0$ for all $\theta_A \gtrless \theta_B$, $x_W(0) = x_F$ and $x_W(\infty) = x_S$.

Proof. Immediate given (6), (2), (7), $\mathcal{P_W}$ and Proposition 2.

The characteristics of the optimal contract offered by a benevolent regulator and a privately interested party are very similar: location of the good closer to the agent with lowest valuation, distortion due to asymmetric information, possibility of not producing the good, etc. The main difference is that, in the regulation case, the relative weights of efficiency vs. rent extraction in the objective function of the principal are entirely determined by λ, the shadow costs of transferring public funds.

When transferring funds from taxpayers to agents is costless ($\lambda = 0$), there is no social loss of taking \$1 from taxpayers and giving it to an agent, which means that the regulator will be interested exclusively in the efficiency of her action. She will therefore take the same decisions as under full information ($x_W(0) = x_F$ and $\hat{r}_i(\theta_j, x_W(0); 0) = \underline{\theta}$), even if it comes at the

[28] Note that we assume that the gross consumer's surplus is 0. This is without loss of generality.

expense of a substantial subsidy. On the other extreme, if subsidies from taxpayers to agents are prohibitively costly ($\lambda = \infty$), then the regulator's objective is formally equivalent to maximize welfare under the constraint that agents can be taxed but not subsidized ($s_i \leqslant 0$). This case is identical to the case of a privately interested principal, who trades-off efficiency and rents but will never choose to subsidize agents. The optimal decision therefore coincides with that of Proposition 2: $x_W(\infty) = x_S$ and $\hat{r}_i(\theta_j, x_W(\infty); \infty) = r_i(\theta_j, x_S)$. In the general case where the cost of public funds is positive but finite $\lambda \in (0, \infty)$, efficiency and rents are again traded-off. The regulator is more concerned with increasing efficiency and less concerned with decreasing rents than a privately interested party, simply because the utility of agents is now part of her objective function. Naturally, this is reflected in her choices: $x_S \gtreqless x_W \gtreqless x_F$ for all $\theta_A \gtreqless \theta_B$ and $\hat{r}_i(\theta_j, x_W(\lambda); \lambda) \in (\underline{\theta}, r_i(\theta_j, x_S))$.

The problem analyzed in this section can be interpreted as the optimal allocation of a public good. In that respect, the analysis provides an alternative and complementary perspective of the problem already studied in the literature. The framework analyzed so far is such that each agent's valuation is a function $v(\theta, q)$ where θ is his type and q the quantity of public good. Given that all agents prefer more quantity to less, the main issue is to design a mechanism to prevent them from understating their type, getting away with a low payment while enjoying the public good (positive externality). In our setting the valuation functions of agent A and B depend on the *location* x of the public good instead of the quantity provided. The fact that agents do not have the same preferences over locations introduces a new dimension to the standard problem of the social planner. Given the positive externality, the incentives to underreport are still present but the principal can use the location choice to mitigate them.

4.2 Optimal location when one agent is also the producer

Suppose now that agent A is in charge of deciding if he produces the good and where he locates it. Naturally, he will use his decision power to extract payments from agent B. In order to keep the simplest possible structure of the game and also to better isolate the changes in the incentives of the new decision-maker to select a given location, we assume that B observes A's valuation θ_A for the good. Given this assumption, B does not have anything to infer from the mechanism proposed by A, and therefore A has no incentives to use the contract design to signal any information.[29]

Agent A will again design an optimal revelation mechanism, just like the principal and the regulator did in the previous settings. More precisely, he will offer to B a menu of contracts $\{p_x(\tilde{\theta}_B), t_B(\tilde{\theta}_B)\}$ such that, for each announced valuation $\tilde{\theta}_B$ (and given the publicly observed valuation θ_A), agent B pays a transfer $t_B(\tilde{\theta}_B)$ to agent A and the good is located at x with

[29] For a thoughtful analysis of contracting with an informed principal, see Maskin and Tirole (1990, 1992).

probability $p_x(\tilde{\theta}_B)$. If we denote by R_A the expected revenue of A (that is the sum of his own valuation and the expected transfer raised from agent B) and by $u_B^*(\theta_B, \tilde{\theta}_B)$ the utility of agent B with valuation θ_B who announces $\tilde{\theta}_B$, we get:

$$R_A = \int_{\underline{\theta}}^{\overline{\theta}} t_B(\theta_B) dF(\theta_B) + \pi_A(\theta_A, x)$$

$$u_B^*(\theta_B, \tilde{\theta}_B) = \sum_{x=0}^{N} \pi_B(\theta_B, x) p_x(\tilde{\theta}_B) - t_B(\tilde{\theta}_B)$$

The objective of agent A is then to maximize his utility R_A under the following constraints on agent B. First, incentive-compatibility ($u^*(\theta_B, \theta_B) \geqslant u^*(\theta_B, \tilde{\theta}_B)$ for all $\theta_B, \tilde{\theta}_B$). Second, individual-rationality ($u^*(\theta_B, \theta_B) \geqslant 0$ for all θ_B). And third, feasibility ($p_x(\theta_B) \geqslant 0$ for all x and $\sum_{x=0}^{N} p_x(\theta_B) \leqslant 1$). Following a similar procedure as in Lemma 2, we can rewrite this optimization program in the following form:

$$\mathcal{P}_\mathcal{A}: \quad \max_{p_x(\theta_B)} \int_{\underline{\theta}}^{\overline{\theta}} \sum_{x=0}^{N} p_x(\theta_B) \left[\pi_A(\theta_A, x) + \pi_B(\theta_B, x) - \frac{\partial \pi_B(\theta_B, x)}{\partial \theta_B} \frac{1 - F(\theta_B)}{f(\theta_B)} \right] dF(\theta_B)$$

$$\text{s. t.} \quad \sum_{x=0}^{N} \frac{\partial \pi_B}{\partial \theta_B} \times \frac{\partial p_x}{\partial \theta_B} \geqslant 0 \tag{$\mathrm{M_B}$}$$

$$p_x(\theta_B) \geqslant 0 \quad \forall x \quad \text{and} \quad \sum_{x=0}^{N} p_x(\theta_B) \leqslant 1 \tag{$\mathrm{F_B}$}$$

The interpretation is straightforward. In the new optimization program $\mathcal{P}_\mathcal{A}$, the only parameter of asymmetric information is the valuation of agent B. Since it is only required to grant informational rents to that agent, the objective function is the sum of the *net surplus of agent A and the virtual surplus of agent B* ($\pi_A(\theta_A, x)$ and $\Phi_B(\theta_B, x)$ respectively). The monotonicity ($\mathrm{M_B}$) and feasibility ($\mathrm{F_B}$) constraints of agent B are the same as in Lemma 2, except that now the valuation θ_A is known. We denote by x_A the location that maximizes the surplus from agent A's perspective:

$$x_A \;=\; \arg\max_x \;\; \pi_A(\theta_A, x) + \Phi_B(\theta_B, x) \tag{8}$$

and we can state our last result.

Proposition 4 *When agent A chooses the location, the optimal contract is such that:*

$$\begin{cases} p_{x_A}(\theta_B) = 1 & \textit{if} \;\; \pi_A(\theta_A, x_A) + \Phi_B(\theta_B, x_A) > 0 \\ p_\emptyset(\theta_B) = 1 & \textit{otherwise} \end{cases}$$

The location x_A is such that: $\frac{\partial x_A}{\partial \theta_A} > 0$, $\frac{\partial x_A}{\partial \theta_B} < 0$ and $x_A > \max\{x_S, x_F\}$ for all θ_A and θ_B.

Proof. Immediate given (6), (2), (8), $\mathcal{P}_A$ and Proposition 2.

The properties of the optimal mechanism are very similar to those in Propositions 1 and 2: depending on agent B's reported valuation, either the good is not produced ($e = \emptyset$) or it is situated at the location where the surplus is maximized ($e = x_A$). The main novelty of this case is that, independently of the valuations (θ_A, θ_B), agent A will locate the good farther away from his own preferred location than the principal would do ($x_A > x_S$), and also farther away than under full information ($x_A > x_F$). Although it may at first seem striking, the idea is quite simple. When agent A chooses the location, there is only one asymmetry of information, with respect to agent B's valuation. In order to reduce B's informational rents, it is then unambiguously better to bring the good closer to that agent. That same logic applies when we compare agent A's optimal choice with the full information case.[30]

5 Concluding remarks

In this paper, we have analyzed the optimal choice of a principal who decides whether to produce one unit of an indivisible good and which characteristics it will contain. We have shown that if the utility of agents is differentiated along two substitutable dimensions (an intrinsic willingness to pay for the good and a preference for characteristics), the principal offers a good with characteristics more on the lines of the preferences of the agent with the lowest valuation. Moreover, asymmetric information exacerbates this bias, i.e. pushes the principal to make more extreme choices.

The analysis suggests that it is profitable to bias decisions against the preferences of the most interested parties. For instance, even though parents with a strong preference for private education might prefer an emphasis on science, a private school will be more successful if emphasis is put on some other subjects. Coming back to our anecdote, according to our analysis, the reason why the French schools adapt the program to the tastes of local citizens is simply that in order to attract them, the school must offer something of value to them.[31] Given French parents are ready to give up some features of French education as long as the main philosophy is preserved, the school maximizes its revenue by adopting that strategy. Examples of such biases can be found in other economic situations. For example, Operas generally schedule an important number of well known performances and only a few rare productions. This suggests that it is relatively easier to attract people who truly enjoy Opera rather than people who attend it only on occasion.

[30] Needless to say that, given the symmetry between A and B, the insights would be the same if the contract were proposed by agent B rather than by agent A.

[31] For instance, local citizens might face specific constraints set by local Colleges, or might have a different educational culture.

The results rest on the assumption that individuals are differentiated along two dimensions. They assign an intrinsic valuation to the good but they have different preferences for its characteristics. Absent the second dimension, the principal would take a decision on the lines of the agent who values it most because it is the only way to generate a social value. However, in our setting, the principal generates a social value also by choosing characteristics: on the one hand, the investor can locate the stadium in the city where there is already a high number of football supporters and on the other hand, he can locate it in a city in which residents go to football events if and only if they host them. In this setting, taking a decision on the lines of the agent who values the good most is not optimal when the inhabitants of the first city are more willing to compromise provided that the stadium is built. In other words, we have shown that the optimal allocation of a non-excludable good is affected crucially by the characteristics it contains and how they are perceived by economic agents.

Appendix

Appendix 1. Note that for all $i = \{A, B\}$

$$u_i(\theta_i, \tilde{\theta}_i) = u_i(\tilde{\theta}_i, \tilde{\theta}_i) - \sum_{x=0}^{N} \int_{\underline{\theta}}^{\overline{\theta}} p_x(\tilde{\theta}_i, \theta_j)[\pi_i(\tilde{\theta}_i, x) - \pi_i(\theta_i, x)]dF(\theta_j).$$

The incentive compatibility constraint is equivalent to

$$u_i(\theta_i, \theta_i) \geqslant u_i(\tilde{\theta}_i, \tilde{\theta}_i) + \sum_{x=0}^{N} \int_{\underline{\theta}}^{\overline{\theta}} p_x(\tilde{\theta}_i, \theta_j)[\pi_i(\theta_i, x) - \pi_i(\tilde{\theta}_i, x)]dF(\theta_j). \tag{9}$$

Using this inequality twice, the incentive compatibility constraint is equivalent to

$$\begin{aligned} \sum_{x=0}^{N} \int_{\underline{\theta}}^{\overline{\theta}} p_x(\tilde{\theta}_i, \theta_j)[\pi_i(\theta_i, x) - \pi_i(\tilde{\theta}_i, x)]dF(\theta_j) &\leqslant u_i(\theta_i, \theta_i) - u_i(\tilde{\theta}_i, \tilde{\theta}_i) \\ &\leqslant \sum_{x=0}^{N} \int_{\underline{\theta}}^{\overline{\theta}} p_x(\theta_i, \theta_j)[\pi_i(\theta_i, x) - \pi_i(\tilde{\theta}_i, x)]dF(\theta_j). \end{aligned} \tag{10}$$

Given that $\pi_i(\theta, x)$ is increasing in θ for all $i = \{A, B\}$, the agent reveals truthfully if the monotonicity condition (M) is satisfied:

$$\sum_{x=0}^{N} \int_{\underline{\theta}}^{\overline{\theta}} p_x(\tilde{\theta}_i, \theta_j)[\pi_i(\theta_i, x) - \pi_i(\tilde{\theta}_i, x)]dF(\theta_j) \leqslant \sum_{x=0}^{N} \int_{\underline{\theta}}^{\overline{\theta}} p_x(\theta_i, \theta_j)[\pi_i(\theta_i, x) - \pi_i(\tilde{\theta}_i, x)]dF(\theta_j) \quad \forall\, \tilde{\theta}_i \leqslant \theta_i.$$

(10) must hold for all $\tilde{\theta}_i$ and all $\theta_i = \tilde{\theta}_i + \delta$ with $\delta > 0$. Taking the Riemann integral, the agent reveals truthfully if the following condition (LO) is also satisfied:

$$u_i(\theta_i) - u_i(\tilde{\theta}_i) = \int_{\tilde{\theta}_i}^{\theta_i} \sum_{x=0}^{N} \int_{\underline{\theta}}^{\overline{\theta}} p_x(s, \theta_j) \frac{\partial \pi_i}{\partial s}(s, x) dF(\theta_j) ds$$

To complete the proof we need to show that both (M) and (LO) imply (9). Consider $\tilde{\theta}_i \leqslant \theta_i$, the necessary conditions imply:

$$\begin{aligned} u_i(\theta_i, \theta_i) &= u_i(\tilde{\theta}_i, \tilde{\theta}_i) + \int_{\tilde{\theta}_i}^{\theta_i} \sum_{x=0}^{N} \int_{\underline{\theta}}^{\overline{\theta}} p_x(s, \theta_j) \frac{\partial \pi_i}{\partial s}(s, x) dF(\theta_j) ds \\ &\geq u_i(\tilde{\theta}_i, \tilde{\theta}_i) + \int_{\tilde{\theta}_i}^{\theta_i} \sum_{x=0}^{N} \int_{\underline{\theta}}^{\overline{\theta}} p_x(\tilde{\theta}_i, \theta_j) \frac{\partial \pi_i}{\partial s}(s, x) dF(\theta_j) ds \\ &= u_i(\tilde{\theta}_i, \tilde{\theta}_i) + \sum_{x=0}^{N} \int_{\underline{\theta}}^{\overline{\theta}} p_x(\tilde{\theta}_i, \theta_j)[\pi_i(\theta_i, x) - \pi_i(\tilde{\theta}_i, x)]dF(\theta_j). \end{aligned}$$

The seller maximizes her expected revenue (the sum of transfers) under constraints (M) and (LO) (to induce truthtelling), the individual rationality constraint and the feasibility constraints (F). The expected transfer paid by agent i is:

$$\int_{\underline{\theta}}^{\overline{\theta}} t_i(\theta_i,\theta_j)dF(\theta_j) = \int_{\underline{\theta}}^{\overline{\theta}} \sum_{x=0}^{N} \pi_i(\theta_i,x)p_x(\theta_i,\theta_j)dF(\theta_j) - u_i(\theta_i)$$

Given (LO), the utility can be rewritten as

$$u_i(\theta_i) = \int_{\underline{\theta}}^{\theta_i} \sum_{x=0}^{N} \int_{\underline{\theta}}^{\overline{\theta}} p_x(s,\theta_j)\frac{\partial \pi_i}{\partial s}(s,x)dF(\theta_j)ds + u_i(\underline{\theta}).$$

To minimize the rent left to agents and satisfy the incentive compatibility constraint, the seller sets $u_i(\underline{\theta}) = 0$. Replacing the expression of the expected utility and integrating by parts, the expected utility of the seller is:

$$R = \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \sum_{x=0}^{N} p_x(\theta_A,\theta_B) \sum_{i=A,B} \left[\pi_i(\theta_i,x) - \frac{\partial \pi_i}{\partial \theta_i}(\theta_i,x)\frac{1-F(\theta_i)}{f(\theta_i)}\right] dF(\theta_A)dF(\theta_B)$$

The problem of the seller is then to maximize the previous expression under the remaining constraints (M) and (F).[32] □

Appendix 2. The expected revenue of the seller can be rewritten as

$$R = \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \sum_{x=0,N} p_x(\theta_i,\theta_j)\left[\Phi_A(\theta_A,x) + \Phi_B(\theta_B,x)\right] dF(\theta_A)dF(\theta_B)$$

To simplify notations $\Phi(\theta_A,\theta_B,x) = \Phi_A(\theta_A,x) + \Phi_B(\theta_B,x)$.

• Given the monotone hazard rate property and $\pi'' < 0$, $\Phi(\theta_A,\theta_B,x)$ is increasing in both θ_A and θ_B. Then, for all θ_B, there exists $r_A(\theta_B,0)$ such that $\Phi(\theta_A,\theta_B,0) \geqslant 0$ if $\theta_A \geqslant r_A(\theta_B,0)$. Similarly, there exists $r_B(\theta_A,N)$ such that $\Phi(\theta_A,\theta_B,N) \geqslant 0$ if $\theta_B \geqslant r_B(\theta_B,N)$.

• Given the monotone hazard rate property, $\pi'' < 0$ and $\pi''' \geqslant 0$, we have

$$\Phi_1(\theta_A,\theta_B,0) - \Phi_1(\theta_A,\theta_B,N) = \left[\pi'(\theta_A) - \pi'(\theta_A - N)\right]\left[1 - \frac{d}{d\,\theta_A}\left[\frac{1-F(\theta_A)}{f(\theta_A)}\right]\right]$$

$$-\frac{1-F(\theta_A)}{f(\theta_A)}[\pi''(\theta_A) - \pi''(\theta_A - N)] < 0$$

and $\Phi(\theta_A,\theta_B,N) = \Phi(\theta_A,\theta_B,0)$ when $\theta_A = \theta_B$. Then, for all $\theta_A < \theta_B$, $\Phi(\theta_A,\theta_B,0) > \Phi(\theta_A,\theta_B,N)$ and for all $\theta_A > \theta_B$, $\Phi(\theta_A,\theta_B,0) < \Phi(\theta_A,\theta_B,N)$.

[32] Note that the proof is similar to Myerson (1981).

Combining the two previous points, the allocation rule in proposition 1 maximizes the revenue of the seller. It is the optimal contract if it satisfies also (F) and (M). It is immediate that (F) holds. Differentiating $\Phi(r_A(\theta_B, 0), \theta_B, 0) = 0$ with respect to θ_B yields $\frac{d}{d\theta_B} r_A(\theta_B, 0) < 0$. Similarly $\frac{d}{d\theta_A} r_B(\theta_A, N) < 0$. Moreover, $r_A^{-1}(\theta_A, 0) = r_B(\theta_A, N)$. There exist possibly many values θ^* such that $r_B(\theta^*, N) = r_A^{-1}(\theta^*, 0)$. Note that $\frac{d}{d\theta_A} r_B(\theta_A, N)|_{\theta^*} = -\frac{\Phi_1(\theta^*,\theta^*,N)}{\Phi_2(\theta^*,\theta^*,N)}$ where

$$\Phi_1(\theta^*, \theta^*, N) = \pi'(\theta^* - N) \left[1 - \frac{d}{d\theta}\left[\frac{1 - F(\theta^*)}{f(\theta^*)}\right]\right] - \pi''(\theta^* - N)$$

$$\Phi_2(\theta^*, \theta^*, N) = \pi'(\theta^*) \left[1 - \frac{d}{d\theta}\left[\frac{1 - F(\theta^*)}{f(\theta^*)}\right]\right] - \pi''(\theta^*).$$

Given $\pi'' < 0$ and $\pi''' \geqslant 0$, $\frac{d}{d\theta_A} r_B(\theta_A, N)|_{\theta^*} \leqslant -1$. This ensures that θ^* is unique. Moreover for all $\theta_A < \theta^*$, $r_B(\theta_A, N) > r_A^{-1}(\theta_A, 0)$ and for all $\theta_A > \theta^*$, $r_B(\theta_A, N) < r_A^{-1}(\theta_A, 0)$. Let us denote the probability of allocating the good in location x for an agent with valuation θ_A by $P_x(\theta_A) = \int_{\underline{\theta}}^{\overline{\theta}} p_x(\theta_A, \theta_B) dF(\theta_B)$, then

$$\text{If } \theta_A < \theta^* : \quad P_0(\theta_A) = 1 - F(r_A^{-1}(\theta_A, 0)), \qquad P_N(\theta_A) = 0;$$

$$\text{If } \theta_A > \theta^* : \quad P_0(\theta_A) = 1 - F(\theta_A), \quad P_N(\theta_A) = F(\theta_A) - F(r_B(\theta_A, N)).$$

We need to check that $\pi'(\theta_A)\frac{d}{d\theta_A}P_0(\theta_A) + \pi'(\theta_A - N)\frac{d}{d\theta_A}P_N(\theta_A) \geqslant 0$. It comes immediately that (M) is satisfied for all $\theta_A < \theta^*$. Given $\pi'' < 0$, (M) is also satisfied when $\theta_A > \theta^*$. Overall, the allocation rule in Proposition 1 satisfies (M), and it is the optimal contract.

Last note that the virtual surplus is a function of N. Let $\phi(\theta_A, \theta_B, 0; N) \equiv \Phi(\theta_A, \theta_B, 0)$ and $\phi(\theta_A, \theta_B, N; N) \equiv \Phi(\theta_A, \theta_B, N)$. Both functions decrease in N. As a consequence $r_A(\theta_B, 0)$ and $r_B(\theta_A, N)$ increase in N. □

Appendix 3. The expected revenue of the seller is

$$R = \int_{\underline{\theta}}^{\overline{\theta}} \int_{\underline{\theta}}^{\overline{\theta}} \sum_{x=0}^{N} p_x(\theta_i, \theta_j)\Phi(\theta_A, \theta_B, x) dF(\theta_A) dF(\theta_B)$$

In the remainder of the proof, we assume that K locations are available between 0 and N and we denote each location by x_k with $k = \{0, ..., K\}$. Moreover, $x_0 = 0$ and $x_K = N$ and there exists k^* such that $x_{k^*} = N/2$.

• Consider x_k and θ_A such that $x_k = \arg\max \Phi(\theta_A, \theta_B, x)$. Assume also that the optimal location when agent A's type is $\theta'_A > \theta_A$ is $y < x_k$. Overall we have,

$$\Phi(\theta_A, \theta_B, x_k) > \Phi(\theta_A, \theta_B, y)$$

$$\Phi(\theta'_A, \theta_B, y) > \Phi(\theta'_A, \theta_B, x_k)$$

Then, adding the two inequalities,

$$\Phi(\theta_A, \theta_B, x_k) + \Phi(\theta'_A, \theta_B, y) > \Phi(\theta_A, \theta_B, y) + \Phi(\theta'_A, \theta_B, x_k).$$

This implies that:

$$\Phi(\theta_A, \theta_B, x_k) - \Phi(\theta_A, \theta_B, y) > \Phi(\theta'_A, \theta_B, x_k) - \Phi(\theta'_A, \theta_B, y).$$

Noting that $\Phi_{31}(\theta_A, \theta_B, x) > 0$, the last inequality yields to a contradiction. As a consequence, if $x_k = \arg\max \Phi(\theta_A, \theta_B, x)$, then $\arg\max \Phi(\theta'_A, \theta_B, x) \geqslant x_k$. Overall the optimal location $x_S(\theta_A, \theta_B)$ is non-decreasing in θ_A. By a similar argument and using the fact that $\Phi_{32}(\theta_A, \theta_B, x) < 0$, we get that $x_S(\theta_A, \theta_B)$ is non-increasing in θ_B. As a consequence, for all θ_A there exists a subset of locations $\mathcal{X}(\theta_A)$ such that if k and $k+2$ are in $\mathcal{X}(\theta_A)$, then $k+1 \in \mathcal{X}(\theta_A)$ (by continuity) and,

(i) for all $k \in \mathcal{X}(\theta_A)$ there exist $h_{x_k}(\theta_A)$ and $h_{x_{k-1}}(\theta_A) > h_{x_k}(\theta_A)$ both increasing in θ_A such that $x_k = \arg\max \Phi(\theta_A, \theta_B, x)$ if $\theta_B \in [h_{x_k}(\theta_A), h_{x_{k-1}}(\theta_A)]$;

(ii) for all $k \notin \mathcal{X}(\theta_A)$, the good is not located in x_k.

• For all x, $\Phi(\theta_A, \theta_B, x)$ is increasing in θ_B. Consider $k \in \mathcal{X}(\theta_A)$, there exists $r_B(\theta_A, x_k)$ such that for all $\theta_B \geqslant r_B(\theta_A, x_k)$, $\Phi(\theta_A, \theta_B, x_k) \geqslant 0$. By the same argument as in Appendix 2, $r_B(\theta_A, x_k)$ is decreasing in θ_A.

• Note that $h_{x_{k-1}}(\theta_A)$ is such that $\Phi(\theta_A, h_{x_{k-1}}(\theta_A), x_{k-1}) > \Phi(\theta_A, h_{x_{k-1}}(\theta_A), x_k)$. Then, if $h_{x_{k-1}}(\theta_A) > r_B(\theta_A, x_k)$, we have also $h_{x_{k-1}}(\theta_A) > r_B(\theta_A, x_k)$.

Combining the previous results, for all θ_A, there exists a subset $\tilde{\mathcal{X}}(\theta_A) = \{\underline{k}(\theta_A), ..., \overline{k}(\theta_A)\} \subset \mathcal{X}(\theta_A)$ such that k is the optimal location when $\theta_B \in [g_{x_k}(\theta_A), g_{x_{k-1}}(\theta_A)]$ where $g_{x_{\overline{k}(\theta_A)}}(\theta_A) = r_B(\theta_A, x_{\overline{k}(\theta_A)})$ and $g_{x_k}(\theta_A) = h_{x_k}(\theta_A)$ for all $k < \overline{k}(\theta_A)$. The mechanism satisfies (F). We need to check it satisfies also (M).

• Pose $P_k(\theta_A) \equiv \int_{\underline{\theta}}^{\overline{\theta}} p_{x_k}(\theta_A, \theta_B) dF(\theta_B)$,

$$\begin{aligned} P_k(\theta_A) &= F(h_{x_{k-1}}(\theta_A)) - F(h_{x_k}(\theta_A)) && \text{if } k \in \tilde{\mathcal{X}}(\theta_A) - \overline{k}(\theta_A) \\ &= F(h_{x_{\overline{k}(\theta_A)-1}}(\theta_A)) - F(r_B(\theta_A, x_{\overline{k}(\theta_A)}) && \text{if } k = x_{\overline{k}(\theta_A)} \\ &= 0 && \text{otherwise} \end{aligned}$$

We can rewrite (M) as

$$\sum_{k=0}^{N} \frac{d\pi_A}{d\theta_A}(\theta_A, x_k) \times \frac{dP_k}{d\theta_A}(\theta_A) \geqslant 0.$$

Note that

$$\sum_{k=0}^{N} \frac{d\pi_A}{d\theta_A}(\theta_A, x_k) \times \frac{dP_k}{d\theta_A}(\theta_A) = \sum_{k \in \tilde{\mathcal{X}}(\theta_A)} \frac{d\pi_A}{d\theta_A}(\theta_A, x_k) \left[f(g_{x_{k-1}}(\theta_A)) \frac{dg_{x_{k-1}}}{d\theta_A}(\theta_A) - f(g_{x_k}(\theta_A)) \frac{dg_{x_k}}{d\theta_A}(\theta_A) \right]$$

$$= \sum_{k \in \tilde{\mathcal{X}}(\theta_A) - \overline{k}(\theta_A)} f(h_{x_k}(\theta_A)) \frac{dh_{x_k}}{d\theta_A} \left[\frac{d\pi_A}{d\theta_A}(\theta_A, x_{k+1}) - \frac{d\pi_A}{d\theta_A}(\theta_A, x_k) \right]$$

$$- \; f(r_B(\theta_A, x_{\overline{k}(\theta_A)})) \frac{dr_B}{d\theta_A} \frac{d\pi_A}{d\theta_A}(\theta_A, x_{\overline{k}(\theta_A)})$$

Given $\pi'' < 0$, $h_{x_k}(\theta_A)$ increasing in θ_A and $r_B(\theta_A, x_k)$ decreasing in θ_A, then (M) is satisfied.

• Note that when $\theta_A = \theta_B$, the optimal location is $x_S = N/2$. Since the optimal location is increasing in θ_A and decreasing in θ_B, $x_S \leqslant N/2$ when $\theta_A < \theta_B$, and $x_S \geqslant N/2$ when $\theta_A > \theta_B$.

• Under full information, the surplus is $\pi_A(\theta_A, x) + \pi_B(\theta_B, x) = \Pi(\theta_A, \theta_B, x)$. This surplus has the same properties as $\Phi(\theta_A, \theta_B, x)$. By the same reasoning as under incomplete information, the optimal location $x_F(\theta_A, \theta_B)$ is increasing in θ_A and decreasing in θ_B. Moreover, when $\theta_A = \theta_B$, the optimal location is $x_F = N/2$. Then, $x_F < N/2$ when $\theta_A < \theta_B$, and $x_F > N/2$ when $\theta_A > \theta_B$. Last, given that $x_S = \arg\max \Phi(\theta_A, \theta_B, x)$ and $x_F = \arg\max \Pi(\theta_A, \theta_B, x)$, we have:

$$\Phi(\theta_A, \theta_B, x_S) \geqslant \Phi(\theta_A, \theta_B, x_F)$$

$$\Pi(\theta_A, \theta_B, x_F) \geqslant \Pi(\theta_A, \theta_B, x_S)$$

Adding the two inequalities,

$$\Phi(\theta_A, \theta_B, x_S) - \Pi(\theta_A, \theta_B, x_S) \geqslant \Phi(\theta_A, \theta_B, x_F) - \Pi(\theta_A, \theta_B, x_F), \quad \text{or}$$

$$\left[\frac{d\pi_A(\theta_A, x_F)}{d\theta_A} - \frac{d\pi_A(\theta_A, x_S)}{d\theta_A} \right] \frac{1 - F(\theta_A)}{f(\theta_A)} \geqslant \left[\frac{d\pi_B(\theta_B, x_S)}{d\theta_B} - \frac{d\pi_B(\theta_B, x_F)}{d\theta_B} \right] \frac{1 - F(\theta_B)}{f(\theta_B)}$$

Assume $\theta_A > \theta_B$ and $x_S < x_F$, the previous inequality yields a contradiction. Then, when $\theta_A > \theta_B$ we have $x_S > x_F$. Similarly, when $\theta_A < \theta_B$ the optimal locations are such that $x_S < x_F$.

• Last, the number of locations K can be arbitrarily large and the result continues to hold. □

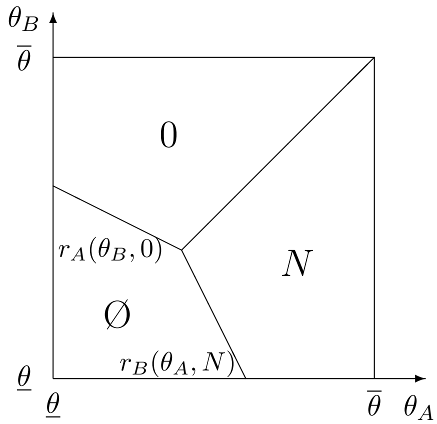

FIGURE 1: optimal location when $x \in \{0, N\}$

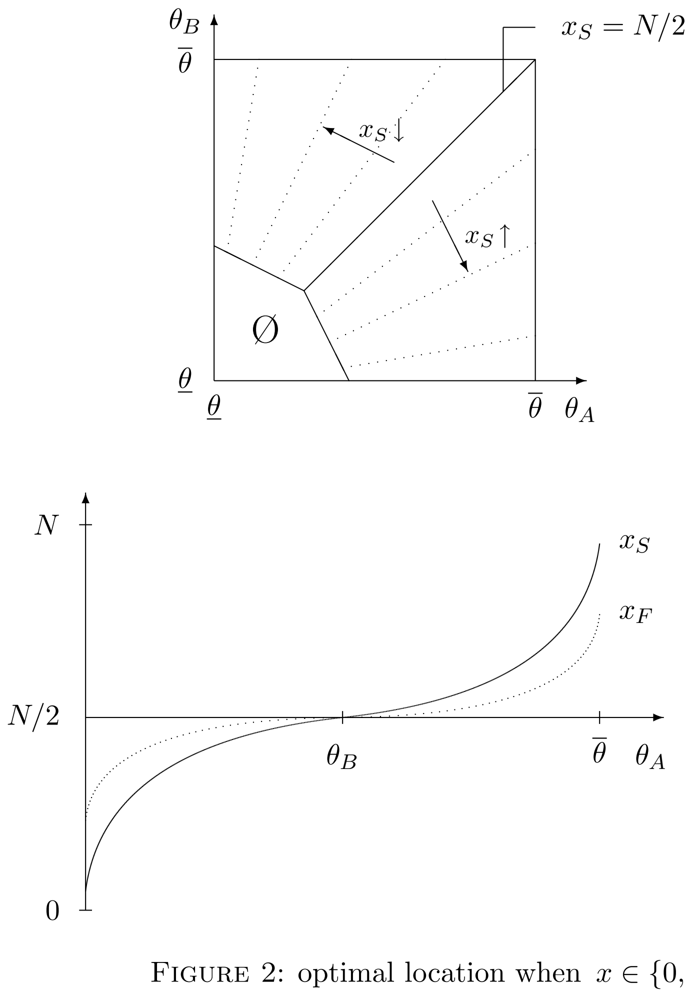

FIGURE 2: optimal location when $x \in \{0, 1, ..., N-1, N\}$

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