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ABSTRACT

A Lender-Based Theory of Collateral*

We consider an imperfectly competitive loan market in which a local (e.g., relationship) lender has valuable soft, albeit private, information, which gives her a competitive advantage vis-à-vis distant transaction lenders who provide arm's-length financing based on hard, publicly available information. The competitive pressure from transaction lenders forces the local lender to leave surplus to borrowers, which distorts the local lender's credit decision in the sense that she inefficiently rejects marginally profitable projects. Collateral mitigates this inefficiency by 'flattening' the local lender's payoff function, thus improving her payoff from precisely those projects that she inefficiently rejects. Our model predicts that technological innovations such as small business credit scoring that narrow the information advantage of local lenders vis-à-vis transaction lenders lead to higher collateral requirements, thus strengthening the role of collateral in local lending relationships.

JEL Classification: G21 and G32
Keywords: collateral, relationship lending vs transaction lending and soft information

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1 Introduction

About 80 percent of small business loans in the United States are secured by collateral. In dollar terms, the number is even close to 90 percent (Avery, Bostic, and Samolyk (1998)). Likewise, Berger and Udell (1998) report that 92 percent of financial institution loans to small businesses are backed by collateral. Understanding the role of collateral is important, not only because of its widespread use, but also because of its implications for monetary policy. Under the "financial accelerator" view of monetary policy transmission, a tightening of monetary policy and the associated increase in interest rates reduces collateral values, thus making it more difficult for borrowers to obtain funds, which reduces investment and economic growth.[1]

Over the past years, small business lending in the United States has witnessed an "information revolution" (Petersen and Rajan (2002)). Small business lending has historically been considered a local activity based on "soft" information culled from close contacts with borrowers and superior knowledge of local business conditions. In recent years, however, this image has changed. Advances in information technology, in particular the widespread adoption of small business credit scoring, have made it possible to underwrite "transaction loans" based solely on publicly available "hard" information without actually meeting the borrower.[2] As a consequence, incumbent local (e.g., relationship) lenders have been facing increasing competitive pressure from arm's-length "transaction lenders," especially large banks (e.g., Hannan (2003), Frame, Padhi, and Woosley (2004), Berger, Frame, and Miller (2005)).

These developments raise several important questions. As the competitive pressure from transaction lenders increases, what will happen to collateral requirements? Will local lenders reduce their collateral requirements, implying collateral may lose some of its current importance for small business loans? Or will collateral requirements increase? Finally, who will be affected the most by the changes in collateral requirements? Businesses for which local lenders have a relatively strong information advantage vis-à-vis transaction lenders, or businesses for which the information advantage of local lenders is relatively weak?

[1] See Bernanke, Gertler, and Gilchrist (1999) for further details and Aoki, Proudman, and Vlieghe (2004) for an application of the BGG model to housing collateral.

[2] Two pieces of hard information are especially important: the business owner's personal credit history obtained from consumer credit bureaus and information on the business itself obtained from mercantile credit information exchanges such as Dun & Bradstreet. While credit scoring has been used for a while in consumer lending, it has only recently been applied to small business lending after credit analysts found out that the business owner's personal credit history is highly predictive of the loan repayment prospects of the business. For an overview of small business credit scoring, see Mester (1997) and Berger and Frame (2005).

This paper proposes a novel theory of collateral that can address the above questions. We consider an imperfectly competitive loan market in which a local lender has an information advantage vis-à-vis distant transaction lenders. Precisely, the local lender has access to additional soft, albeit private, information allowing her to make a more precise estimate of the borrower's default likelihood. This gives the local lender a competitive advantage, which (generally) allows her to attract the borrower.[3] And yet, the fact that there is competition from transaction lenders is important as it provides the borrower with a positive outside option. In order to attract the borrower, the local lender must consequently offer him a share of the project's cash flows, which in turn distorts the local lender's credit decision: as she incurs the full project cost but receives only a fraction of the project's cash flows, the local lender only accepts projects whose expected cash flow is sufficiently larger than the project cost. In other words, the local lender rejects projects with a small but positive NPV.

Collateral can mitigate this inefficiency.[4] The fundamental role of collateral in our model is to "flatten" the local lender's payoff function. When collateral is added, the local lender's payoff exceeds the project's cash flow in low cash-flow states. Of course, her payoff in high cash-flow states must be reduced, or else the borrower's participation constraint is violated. As low cash flows are relatively more likely under low-NPV projects, however, the overall effect is that the local lender's payoff from low-NPV projects increases, and thus from precisely those projects that she inefficiently rejects. Hence, collateral improves the local lender's incentives to accept marginally positive projects, making her credit decision more efficient.

We consider two implications of the "information revolution" in small business lending, both

[3] This is consistent with Petersen and Rajan's (1994, 2002) observation that 95 percent of the smallest firms in their sample borrow from a single lender (1994), which is generally a local bank (2002). See also Petersen and Rajan (1995), who argue that "credit markets for small firms are local" and Guiso, Sapienza, and Zingales (2004), who refer to "direct evidence of the informational disadvantage of distant lenders in Italy." Like our model, Hauswald and Marquez (2003, 2005) and Almazan (2002) also assume that lenders who are located closer to a borrower have better information about the borrower. Our notion of imperfect loan market competition differs from Thakor (1996), who considers symmetric competition between multiple lenders.

[4] The fact that the local lender's credit decision is based on soft, private information is crucial for the inefficiency, and hence also for our argument for collateral. If the information was contractible, the local lender could contractually commit to the first-best credit decision even if this means committing to a decision rule that is ex post suboptimal. Likewise, if the information was observable but non-verifiable, the inefficiency could be eliminated through bargaining. Inderst and Mueller (2005) examine the implications of this inefficiency for the optimal security design. In their model, however, there is no collateral as borrowers are assumed to have no pledgeable wealth.

of which lead to an increase in competitive pressure from arm's-length transaction lenders. The first implication is that the information advantage of local lenders vis-à-vis distant transaction lenders has narrowed. Small business credit scoring models give a fairly accurate prediction of the likelihood that a loan applicant will default, thus considerably reducing the information uncertainty associated with small business loans made to borrowers located far away (Mester (1997); see also footnote 2). In our model, a narrowing of the local lender's information advantage vis-à-vis transaction lenders forces her to reduce the loan rate, implying the borrower receives a larger share of the project's cash flows. To mitigate the distortion in her credit decision, the local lender must raise the collateral requirement.

Our model therefore predicts that—following the widespread adoption of small business credit scoring models since the early 1990s and the associated increase in competition from arm's-length transaction lenders—the usage of collateral in local lending relationships where soft information is important should increase. We also obtain a cross-sectional prediction, namely, that borrowers who borrow locally and for whom the local lender has a relatively smaller information advantage should face higher collateral requirements. Consistent with this prediction, Petersen and Rajan (2002) find that small business borrowers who are located further away from their local lender are more likely to pledge collateral.

The second implication of the "information revolution" which we consider is that the direct costs of underwriting transaction loans have decreased in recent years. Similar to above, this leads to an increase in competitive pressure from transaction lenders, implying the local lender must reduce the loan rate and raise the collateral requirement. Accordingly, our model predicts that technological innovations reducing the costs of underwriting transaction loans should lead to a higher usage of collateral in local lending relationships where soft information is important. Moreover, this increase in collateral requirements should be relatively weaker for borrowers for whom the local lender has a greater information advantage.

As the sole role of collateral in our model is to minimize distortions in credit decisions based on soft information, collateral has no meaningful role to play in loans underwritten by transaction lenders. Indeed, while the vast majority of small business loans in the United States are collateralized (see first paragraph), small business loans made by transaction lenders on the basis of credit scoring are generally unsecured (Zuckerman (1996), Frame, Srinivasan, and Woosley (2001), Frame, Padhi, and Woosley (2004)).

While we are unaware of empirical studies examining how an increase in competitive pressure from arm's-length transaction lenders affects the usage of collateral in local lending relationships, Jiménez and Saurina (2004) and Jiménez, Salas, and Saurina (2005) provide some indirect support for our model. Using Spanish data, they find a positive relationship between collateral and bank competition measured by the Herfindahl index. Moreover, Jiménez, Salas, and Saurina (2006) find that this positive effect of competition on collateral is weaker the shorter is the duration of bank-borrower relationships, which is consistent with our model if the local lender's information advantage increases with the duration of borrower relationships.

To the best of our knowledge, related models of imperfect loan market competition—e.g., Boot and Thakor (2000) who consider competition between transaction lenders and relationship lenders, or Hauswald and Marquez (2003, 2005) who examine how information technology affects competition between differentially informed lenders—do not consider collateral. On the other hand, theoretical models of collateral—to the extent that they consider loan market competition—do not consider imperfect loan market competition between arm's-length transaction lenders and better informed local (or relationship) lenders, thus generating different empirical predictions than this paper. For instance, Besanko and Thakor (1987a) and Manove, Padilla, and Pagano (2001) both compare a monopolistic with a perfectly competitive loan market and find that collateral is only used in the latter. Somewhat closer in spirit to our model, Villas-Boas and Schmidt-Mohr (1999) consider an imperfectly competitive loan market with horizontally differentiated (albeit symmetric) lenders, showing that collateral requirements may either increase or decrease as loan market competition increases.

In addition to examining the role of imperfect loan market competition for collateral, our model also makes predictions for a *given* borrower-lender relationship—i.e., holding loan market competition constant. For instance, our model predicts that observably riskier borrowers should pledge more collateral and that—holding observable borrower risk constant—collateralized loans should be more likely to default. Both predictions are consistent with the empirical evidence: observably riskier borrowers indeed appear to pledge more collateral (e.g., Leeth and Scott (1989), Berger and Udell (1995), Dennis, Nandy, and Sharpe (2000)), while collateralized loans appear to be riskier ex post in the sense that they default more often (Jiménez and Saurina (2004), Jiménez, Salas, and Saurina (2005)) and have a worse performance in terms of payments past due and nonaccruals (Berger and Udell (1990)).

The above two predictions do not easily follow from existing models of collateral. Adverse selection models (e.g., Bester (1985), Chan and Kanatas (1987), Besanko and Thakor (1987a,b)) predict that safer borrowers within an observationally identical risk pool pledge more collateral. Likewise, moral hazard models (e.g., Chan and Thakor (1987), Boot and Thakor (1994)) are based on the premise that posting collateral improves borrowers' incentives to work hard, which reduces their default likelihood. A notable exception is Boot, Thakor, and Udell (1991), who combine observable borrower quality with moral hazard. Like this paper, they too find that observably riskier borrowers may pledge more collateral and that collateralized loans may be riskier ex post. Intuitively, if borrower quality and effort are substitutes, low-quality borrowers post collateral to commit to higher effort. While this reduces the default likelihood of low-quality borrowers, it remains still higher than that of high-quality borrowers due to the greater relative importance of borrower quality for default risk.

While most existing models of collateral assume agency problems on the part of borrowers, we are not the first to consider incentive problems on the part of the lender. Rajan and Winton (1995) examine the effect of collateral on the lender's ex post monitoring incentives. Monitoring is valuable because it allows the lender to claim additional collateral if the firm is in distress. In Manove, Padilla, and Pagano (2001), on the other hand, lenders protected by collateral screen too little ("lazy banks"). In our model, by contrast, collateral and screening are complements: without screening, there would be no role for collateral.

The rest of this paper is organized as follows. Section 2 lays out the basic model. Section 3 focuses on a given borrower-lender relationship. It shows why collateral is optimal in our model, derives comparative static results, and discusses related empirical literature. Section 4 considers robustness issues. Section 5 examines how technological innovations which increase the competitive pressure from transaction lenders affect the usage of collateral in local lending relationships. The related empirical literature is discussed along with our main predictions. Section 6 concludes. Appendix A shows that our basic argument for collateral extends to a setting with a continuum of cash flows. All proofs are in Appendix B.

2 The Model

Basic Setup

A firm ("the borrower") has an indivisible project with fixed investment cost $k > 0$.[5] The project's cash flow x is verifiable and can be either high ($x = x_h$) or low ($x = x_l$). The two cash-flow model is the simplest framework to illustrate our argument for collateral. In Appendix A we show that our argument straightforwardly extends to a setting with a continuum of cash

[5] Assuming a fixed project size limits the number of choice variables to two: collateral and loan rate. With few exceptions (e.g., Besanko and Thakor (1987b)), existing models of collateral all assume a fixed project size.

flows. The borrower has pledgeable assets w, e.g., business property, real estate, or receivables due in the future. We assume that $x_l + w < k$, implying the project cannot be financed by issuing a safe claim. The risk-free interest rate is normalized to zero.

Lender Types and Information Structure

There are two types of lenders: a local lender and distant "transaction lenders". Transaction lenders are perfectly competitive and provide arm's-length financing based solely on publicly available "hard" information.[6] Given this information, the project's success probability—i.e., the probability that $x = x_h$—is $p \in (0, 1)$. The corresponding expected project cash flow is $\mu := px_h + (1 - p)x_l$.

The fundamental difference between the local lender and transaction lenders is that the local lender has additionally access to valuable qualitative "soft" information allowing her to make a more precise estimate of the project's success probability.[7] For instance, the local lender may be already familiar with the borrower from previous lending relationships—i.e., she may be a typical "relationship lender" (see, e.g., Berger and Udell (1998), Boot (2000)). But even if the local lender has not had any prior lending relationship with the borrower, managing the borrower's accounts, familiarity with local business conditions, and experience with similar businesses in the region may provide the local lender with additional valuable information the transaction lenders do not have.[8]

We assume that the local lender's assessment of the borrower's project can be represented by a continuous variable $s \in [0, 1]$ with corresponding success probability p_s. In practice, s and p_s may be viewed as the local lender's internal rating of the borrower. The success probability p_s is increasing in s, implying the associated conditional expected project cash flow $\mu_s := p_s x_h + (1 - p_s)x_l$ is also increasing in s. Since the local lender's assessment is based on soft information which is difficult to verify vis-à-vis outsiders, we assume that s and p_s are private

[6] See the Introduction for details. The term "transaction lending" is due to Boot and Thakor (2000). In their model, like in our model, transaction lenders are "passive" in the sense that—other than providing arm's-length financing—they create no additional value.

[7] See, e.g., Brunner, Krahnen, and Weber (2000): "[P]rivate corporate ratings (internal ratings) reflect the core business of commercial banks, whose superior information as compared to an external assessment by the market allows a more precise estimate of the POD [probability of default]."

[8] See, e.g., Mester (1997): "The local presence gives the banker a good knowledge of the area, which is thought to be useful in the credit decision. Small businesses are likely to have deposit accounts at the small bank in town, and the information the bank can gain by observing the firm's cash flows can give the bank an information advantage in lending to these businesses."

information.[9] As for the borrower, we assume he lacks the skills and expertise to replicate the local lender's project assessment. After all, professional lenders have specialized expertise, which is why they are in the project-evaluation business.[10] In summary, neither the transaction lenders (for lack of access to soft information) nor the borrower (for lack of expertise) know s or p_s. Of course, the *expected* value of p_s is commonly known: consistency of beliefs requires that $p = \int_0^1 p_s f(s) ds$, where $f(s)$ is the density function associated with s.

To make the local lender's access to additional soft information valuable, we assume that $\mu_1 > k$ and $\mu_0 < k$. That is, the project's NPV is positive for high s and negative for low s. Consequently, having access to additional soft information allows the local lender to distinguish between positive- and negative-NPV projects. By contrast, transaction lenders can only observe the project's NPV based on hard information, which is $\mu - k$.

Financial Contracts

A financial contract specifies repayments $t_l \leq x_l$ and $t_h \leq x_h$ out of the project's cash flow, an amount of collateral $C \leq w$ to be pledged by the borrower, and repayments $c_l \leq C$ and $c_h \leq C$ made out of the pledged assets. The total repayment made by the borrower is thus $R_l := t_l + c_l$ in the bad state and $R_h := t_h + c_h$ in the good state.[11]

Given that the local lender has interim private information, a "standard" solution is to have the local lender offer an incentive compatible menu of contracts from which she chooses a contract after she has evaluated the borrower's project. As can be easily shown, introducing such a menu is (strictly) suboptimal in our model. Rather, it is uniquely optimal to have the local lender offer a single contract and then have her accept or reject the borrower on the basis of this contract. This is consistent with the notion that in many loan markets credit decisions

[9] Brunner, Krahnen, and Weber (2000) argue that "internal ratings should therefore be seen as private information. Typically, banks do not inform their customers of the internal ratings or the implied PODs [probability of default], nor do they publicize the criteria and methods used in deriving them." See also Boot (2000), who writes that "the information [collected by relationship lenders] remains confidential (proprietary)."

[10] See Manove, Padilla, and Pagano (2001). If the local lender holds additionally loans from other local businesses, she may also know more than any individual borrower because she additionally knows where the borrower's local competitors are headed (Boot and Thakor (2000)). Consistent with the notion that professional lenders are better than borrowers at estimating default risk, Reid (1991) finds that bank-financed firms are more likely to survive than firms funded by family investors.

[11] This excludes the possibility that the local lender "buys" the project *before* evaluating it. Using a standard argument, we assume that upfront payments made by the local lender would attract a potentially large pool of fraudulent borrowers, or "fly-by-night operators", with fake projects (see Rajan (1992)). This argument also rules out that the local lender pays a penalty to the borrower if the loan is not approved.

are plain accept or reject decisions: loan applicants are typically either accepted under the terms laid out in the initial contract offer or rejected (Saunders and Thomas (2001)).

Timeline and Competitive Structure of the Loan Market

There are three dates: $\tau = 0$, $\tau = 1$, and $\tau = 2$. In $\tau = 0$ the local lender and transaction lenders make competing offers. If the borrower goes to a transaction lender, he obtains financing under the terms laid out in the initial offer. (As transaction lenders have only access to publicly available information, making an offer to the borrower is de facto equivalent to accepting him.) If the borrower goes instead to the local lender, the local lender evaluates the borrower's project, which takes place in $\tau = 1$. If the local lender accepts the borrower, he obtains financing under the terms laid out in the initial offer.[12] If the borrower is rejected, he may still seek financing from transaction lenders. In $\tau = 2$ the project's cash flow is realized, and the borrower makes the contractually stipulated repayment.

To ensure the existence of a pure-strategy equilibrium, we assume the transaction lenders can observe if the borrower has previously sought credit from the local lender.[13] Given that transaction lenders are perfectly competitive, they can thus offer a "fresh" borrower—i.e., a borrower who has not previously sought credit from the local lender—the full project NPV based on hard information. In contrast, we assume that the local lender makes a take-it-or-leave-it offer that maximizes her own profits subject to matching the borrower's outside option from going to transaction lenders. Effectively, we thus give the local lender all the bargaining power. In Section 4 we show that our results hold for arbitrary distributions of bargaining powers. This includes, as a special case, the other polar case in which the initial contract maximizes the *borrower's* expected profit. In the same section, we also show that the local lender and the borrower will not renegotiate the initial contract after the project evaluation.

[12] We will revisit our assumption that the local lender makes her offer *before* the project evaluation in Section 4. At least in the case of small business lending, lenders indeed appear to make conditional ex ante offers specifying what loan terms borrowers will receive *if* the loan application is approved. At Chase Manhattan, for instance, applicants for small business loans are initially shown a pricing chart explaining in detail what interest rate they will get if their loan is approved. A copy is available from the authors.

[13] On the non-existence of pure-strategy equilibria in loan markets with differentially informed lenders, see Broecker (1990). When a borrower applies for a loan, the lender typically makes an inquiry into the borrower's credit history, which is subsequently documented in the borrower's credit report. Hence, potential future lenders can see precisely if, when, and from whom, the borrower has previously sought credit (Mester (1997), Jappelli and Pagano (2002)). See also footnote 14 below.

3 Optimal Credit Decision and Financial Contract

In our analysis of loan market competition in Section 5 we will show that the local lender may be sometimes unable to attract the borrower. Formally, there may be no solution to the local lender's maximization problem that would satisfy the borrower's participation constraint. In this section, we solve the local lender's maximization problem assuming that a solution exists. We first characterize general properties of the local lender's optimal credit decision (Section 3.1) and financial contract (Section 3.2). We then examine how the optimal contract depends on the borrower's pledgeable assets (Section 3.3). We conclude with some comparative static exercises and a discussion of the relevant empirical literature (Section 3.4).

3.1 General Properties of the Optimal Credit Decision

To obtain a benchmark, let us first derive the first-best optimal credit decision. Given that $\mu_s < k$ for low s and $\mu_s > k$ for high s, and given that μ_s is increasing and continuous in s, there exists a unique first-best cutoff value $s_{FB} \in (0,1)$ given by $\mu_{s_{FB}} = k$ such that the project's NPV is positive if $s > s_{FB}$, zero if $s = s_{FB}$, and negative if $s < s_{FB}$. The first-best credit decision is thus to accept the project if and only if $s \geq s_{FB}$ or, equivalently, if and only if

$$p_s \geq p_{s_{FB}} := \frac{k - x_l}{x_h - x_l}. \tag{1}$$

Let us next derive the local lender's privately optimal credit decision. The local lender accepts the project if and only if her conditional expected payoff

$$U_s(R_l, R_h) := p_s R_h + (1 - p_s) R_l$$

equals or exceeds k. It is immediate that we can exclude contracts under which the project is either accepted or rejected for *all* $s \in [0,1]$. As $R_l = t_l + c_l \leq x_l + w < k$, this implies that $R_h > k$. Given that p_s is increasing in s, this in turn implies that $U_s(R_l, R_h)$ is strictly increasing in s, which finally implies that the local lender accepts the project if and only if $s \geq s^*(R_l, R_h)$, where $s^*(R_l, R_h) \in (0,1)$ is unique and given by $U_{s^*}(R_l, R_h) = k$. Like the first-best optimal credit decision, the local lender's privately optimal credit decision thus follows a cutoff rule: accept the project if and only if the project assessment is sufficiently positive. Like previously, we can alternatively express the optimal credit decision in terms of a critical success probability. Accordingly, the local lender accepts the project if and only if

$$p_s \geq p_{s^*} := \frac{k - R_l}{R_h - R_l}. \tag{2}$$

The above results are summarized in the following lemma.

Lemma 1. *The first-best optimal credit decision is to accept the borrower if and only if* $p_s \geq p_{s_{FB}}$, *where* $p_{s_{FB}}$ *is given by* (1). *By contrast, the local lender's privately optimal credit decision is to accept the borrower if and only if* $p_s \geq p_{s^*}$, *where* p_{s^*} *is given by* (2).

3.2 General Properties of the Optimal Financial Contract

The following lemma simplifies the analysis further.

Lemma 2. *Borrowers who are initially attracted by the local lender but rejected after the project evaluation cannot obtain financing elsewhere.*

Proof. See Appendix.

The Proof of Lemma 2 shows that the project's expected NPV conditional on being rejected by the local lender is non-positive, implying transaction lenders will refrain from making an offer.[14] To understand the intuition, note first that the local lender makes positive expected profits: if $s < s^*$ she rejects the borrower, if $s = s^*$ she makes zero profit ($U_s = k$), and if $s > s^*$ she makes a strictly positive profit ($U_s > k$), which constitutes her *informational rent* from making her credit decision under private information. But if the local lender can attract the borrower while making positive expected profits, this implies she *must* create additional surplus, which in turn implies rejected projects *must* disappear from the market: if rejected projects could still obtain financing, implying all projects would eventually be financed (by someone), no additional surplus would be created.

Equipped with Lemmas 1 and 2, we can now set up the local lender's maximization problem. The local lender chooses R_l and R_h to maximize her expected payoff

$$U(R_l,R_h) := \int_{s^*}^{1} \left[U_s\left(R_l,R_h\right) - k\right] f(s)ds,$$

subject to the constraint $U_{s^*}(R_l,R_h) = k$ characterizing her privately optimal credit decision (see Lemma 1), and the borrower's participation constraint

$$V(R_l,R_h) := \int_{s^*}^{1} V_s(R_l,R_h)f(s)ds \geq \overline{V}, \tag{3}$$

[14] Recall that transaction lenders know from the borrower's credit report whether the borrower has previously sought credit from the local lender. There is a famous anecdote, albeit in the context of consumer credit scoring, where Lawrence Lindsay, then Governor of the Federal Reserve System, was denied a Toys 'R' Us credit card by a fully automated credit scoring system based on the fact that he had too many inquiries in his credit report stemming from previous credit card and loan applications (Mester (1997)).

where

$$V_s(R_l, R_h) := \mu_s - U_s(R_l, R_h) = p_s(x_h - R_h) + (1 - p_s)(x_l - R_l)$$

denotes the borrower' expected payoff conditional on s.

Two brief comments are in order. First, in (3) the borrower's payoff is zero with probability $F(s^*)$, which reflects the insight from Lemma 2 that rejected borrowers cannot obtain financing elsewhere. Second, given that the maximum which the transaction lenders can offer the borrower is the project's full NPV based on hard information, we have that $\overline{V} = \max\{0, \mu - k\}$.

By standard arguments, the borrower's participation constraint must bind, implying the local lender receives any surplus in excess of $\overline{V}$. As the residual claimant, the local lender thus designs a contract inducing herself to make as efficient as possible a credit decision. As the following proposition shows, the optimal contract stipulates a positive amount of collateral.

Proposition 1. *There exists a uniquely optimal financial contract. If $\overline{V} > 0$ the borrower pledges a positive amount of collateral $C \in (0, w]$ such that the local lender receives $R_l = x_l + C$ in the bad state and $R_h \in (R_l, x_h)$ in the good state. If $\overline{V} = 0$ the local lender receives the full project cash flow, i.e., $R_l = x_l$ and $R_h = x_h$.*

Proof. See Appendix.[15]

The case where $\overline{V} = 0$ is a special case that arises only because we assumed the local lender has all the bargaining power. If the borrower had positive bargaining power, we would have $\overline{V} > 0$ even if the borrower's outside option was zero, i.e., even if $\mu - k \leq 0$ (see Section 4). Evidently, if $\overline{V} = 0$ there is no role for collateral: the local lender can extract the full project cash flow, which implies her credit decision is first-best optimal.

The interesting case is where $\overline{V} > 0$. In this case, the local lender cannot extract the full project cash flow, implying her expected payoff $U_s(R_l, R_h)$ is less than the expected project cash flow μ_s for all $s \in [0, 1]$. In particular, it holds that $U_{s_{FB}}(R_l, R_h) < \mu_{s_{FB}} = k$, i.e., the local lender does not break even at $s = s_{FB}$. As $U_s(R_l, R_h)$ is strictly increasing in s, this in turn implies that $s^* > s_{FB}$, i.e., the local lender's privately optimal cutoff exceeds the first-best cutoff. Put simply, the local lender is "too conservative": she rejects projects with a low but positive NPV.

[15] The optimal repayment R_h in the good state if $\overline{V} > 0$ is uniquely determined by the borrower's binding participation constraint (3) after inserting $R_l = x_l + C$. In case of indifference, we stipulate that repayments are first made out of the project's cash flow.

Collateral can mitigate this inefficiency. Firstly, collateral should optimally be added in the bad state, implying that $R_l > x_l$. This improves the local lender's payoff primarily from low-NPV projects and thus from precisely those projects that are inefficiently rejected. By contrast, adding collateral in the good state (i.e., $R_h > x_h$) would improve the local lender's payoff primarily from high-NPV projects that are accepted anyway. The optimal solution is thus to "flatten" the local lender's payoff function by adding collateral in the bad state—thus increasing R_l—and simultaneously decreasing R_h to satisfy the borrower's participation constraint. Arguably, these two payoff adjustments have opposite effects on the local lender's cutoff s^* : increasing R_l pushes s^* down and thus closer towards s_{FB} while decreasing R_h drags s^* away from s_{FB}. And yet, the overall effect is that s^* will be pushed down.

To see why s^* will be pushed down, suppose the local lender's optimal cutoff is currently $s^* = \hat{s}$, and suppose we increase R_l and simultaneously decrease R_h such that conditional on $s \geq \hat{s}$ the borrower's expected payoff $\int_{\hat{s}}^1 V_s(R_l, R_h) f(s) ds$ remains unchanged. While "on average"—i.e., over the interval $[\hat{s}, 1]$—the borrower remains equally well off, his conditional expected payoff $V_s(R_l, R_h)$ is now higher at high values of $s \in [\hat{s}, 1]$ and lower at low values of $s \in [\hat{s}, 1]$. The opposite holds for the local lender. Her conditional expected payoff $U_s(R_l, R_h)$ is now higher at low values of $s \in [\hat{s}, 1]$ and lower at high values of $s \in [\hat{s}, 1]$. Consequently, the local lender's payoff function has become "flatter" over the interval $[\hat{s}, 1]$. Most importantly, her conditional expected payoff $U_s(R_l, R_h)$ at $s = \hat{s}$ is now greater than k, which implies $\hat{s}$ is no longer her optimal cutoff. Indeed, as $U_s(R_l, R_h)$ is strictly increasing in s, the (new) optimal cutoff must be lower than $\hat{s}$, i.e., s^* has been pushed down.[16]

Similar to the effect on the local lender's optimal cutoff s^*, the increase in R_l and simultaneous decrease in R_h—when viewed in isolation—have opposite effects on the local lender's profits. The overall effect, however, is that the local lender's profits must increase. Intuitively, the fact that s^* is pushed down towards s_{FB} implies that additional surplus has been created. As the borrower's participation constraint holds with equality, this additional surplus must accrue to the local lender.

For expositional convenience, we shall write the optimal repayment R_h in the good state in terms of an optimal loan rate r, where $R_h := (1 + r)k$. As the risk-free interest rate has been

[16]The Proof of Proposition 1 in the Appendix shows that the increase in R_l and simultaneous decrease in R_h analyzed here is actually feasible, i.e., it does not violate the borrower's participation constraint. In fact, both the local lender and the borrower are strictly better off as a result of the optimal cutoff being pushed down. The local lender can therefore, in a final step, increase R_h again—thus pushing the optimal cutoff even further down—until the borrower's participation constraint binds.

normalized to zero, r also represents the required risk premium. By Proposition 1, the optimal contract is then fully characterized by two variables, r and C.

3.3 The Role of the Borrower's Pledgeable Assets for the Optimal Credit Decision and Financial Contract

While Proposition 1 provides a qualitative characterization of the optimal financial contract, it remains to derive the *specific* solution to the local lender's maximization problem, i.e., the specific optimal loan rate and collateral as a function of the borrower's pledgeable assets w. As discussed previously, if $\overline{V} = 0$ the first best can be attained trivially without the help of collateral. In what follows, we focus on the nontrivial case where $\overline{V} > 0$.

There are two subcases. If the borrower has insufficient pledgeable assets to attain the first best, then the uniquely optimal contract stipulates that he pledges all his assets as collateral. In contrast, if the borrower has sufficient pledgeable assets, then there exist unique values C_{FB} and r_{FB}, which are jointly determined by the borrower's binding participation constraint (3) with $\overline{V} = \mu - k$ and the equation

$$p_{s_{FB}}(1 + r_{FB})k + (1 - p_{s_{FB}})(x_l + C_{FB}) = k, \tag{4}$$

where $p_{s_{FB}}$ has been defined in (1). Solving these two equations yields unique values

$$C_{FB} = \frac{(k - x_l)(\mu - k)}{\int_{s_{FB}}^{1}(\mu_s - k)f(s)ds} \tag{5}$$

and

$$r_{FB} = \frac{1}{k}\left[x_h - C_{FB}\frac{x_h - k}{k - x_l}\right] - 1. \tag{6}$$

We thus have the following proposition.

Proposition 2. *If the borrower has sufficient pledgeable assets $w \geq C_{FB}$, the first best can be implemented with the uniquely optimal optimal financial contract (r_{FB}, C_{FB}) defined in* (5)-(6). *In contrast, if $w < C_{FB}$ the local lender's credit decision is inefficient: she rejects projects with a low but positive NPV. The uniquely optimal financial contract then stipulates that the borrower pledges all his assets as collateral, i.e., $C = w$.*[17]

Proof. See Appendix.

[17] The corresponding optimal loan rate $r := R_h/k - 1$ in case $w < C_{FB}$ is uniquely determined by the borrower's binding participation constraint (3) after inserting $R_l = x_l + w$.

Proposition 2 shows that there is a natural limit to how "flat" the local lender's payoff function should be. Even in the ideal case where the borrower has sufficient pledgeable assets to attain the first best, the local lender's payoff function will not be completely flat: her payoff in the bad state is then $R_l = x_l + C_{FB}$, which is strictly less than her corresponding payoff in the good state, $R_h = (1 + r_{FB})k$.[18]

3.4 Comparative Static Analysis

In Section 5 we will derive empirical implications regarding the role of imperfect loan market competition for collateral. In this section, by contrast, we focus on a *given* borrower-lender relationship—i.e., holding loan market competition constant.

Collateral and Credit Likelihood

The first implication follows directly from Propositions 1 and 2. Borrowers who can pledge the first-best collateral C_{FB} have the highest acceptance likelihood, namely, $1 - F(s_{FB})$. In contrast, borrowers who—because of binding wealth constraints—can only pledge $C = w < C_{FB}$ have a lower acceptance likelihood. Moreover, within the group of borrowers facing binding wealth constraints, those who have more pledgeable assets have a higher acceptance likelihood, i.e., $1 - F(s^*)$ is increasing in C for all $C < C_{FB}$.[19]

Corollary 1. *Borrowers who can pledge more collateral have a higher likelihood of obtaining credit.*

To test this prediction, one would ideally like to have bank-level data, including data on rejected loan applications. A possible, albeit imperfect, substitute is to use firm-level data. Cole, Goldberg, and White (2004) employ firm-level data from the 1993 National Survey of Small Business Finances (NSSBF), which asks small businesses in the United States about their borrowing experiences, including whether they have been granted or denied credit, and if so, under what terms. Consistent with Corollary 1, the authors find that collateral has a positive effect on the likelihood of obtaining credit.

[18] The difference between these two payoffs is

$$(1 + r_{FB})k - (x_l + C_{FB}) = k - x_l - (x_h - k)\frac{\int_0^{s_{FB}}(\mu_s - k)f(s)ds}{\int_{s_{FB}}^1(\mu_s - k)f(s)ds},$$

which is strictly positive as $x_h > k > x_l$ and $\mu_s - k > 0$ for all $s > s_{FB}$ while $\mu_s - k < 0$ for all $s < s_{FB}$.

[19] This is shown in the Proof of Proposition 1.

Theoretical models of collateral typically assume that borrowers have unlimited wealth. A notable exception is Besanko and Thakor (1987a). In their model, sufficiently wealthy borrowers obtain credit with probability one, while wealth-constrained borrowers face a positive probability of being denied credit. By contrast, in our model all borrowers—i.e., even those with sufficient pledgeable assets $w \geq C_{FB}$—face a positive probability of being denied credit.

Collateral and Observable Borrower Risk

While borrowers do not have private information in our model, they may differ with regard to observable characteristics. In what follows, we consider a mean-preserving spread in the project's cash-flow distribution to examine variations in observable borrower risk.

Corollary 2. *Observably riskier borrowers face higher collateral requirements. If they are unable to pledge more collateral, they face a higher likelihood of being denied credit.*

Proof. See Appendix.

While the local lender receives the full project cash flow x_l (plus collateral) in the bad state, her payoff in the good state is capped at $R_h = (1+r)k$. All else equal—i.e., holding the loan rate and collateral requirement constant—the local lender's expected payoff thus decreases after a mean-preserving spread. Most importantly, she no longer breaks even at her previously optimal cutoff, implying that absent any adjustment of the loan terms the optimal cutoff would increase. By the same logic as in Propositions 1 and 2, the local lender therefore raises the collateral requirement. The last sentence in Corollary 2 is based on the same argument as Corollary 1.

Given the difficulty of finding a good proxy for observable borrower risk, empirical studies have employed a variety of different proxies. And yet, all of these studies find a positive relationship between observable borrower risk and loan collateralization (e.g., Leeth and Scott (1989), Berger and Udell (1995), Dennis, Nandy, and Sharpe (2000), Jiménez, Salas, and Saurina (2005)). To our knowledge, Boot, Thakor, and Udell (1991) are the only theoretical model of collateral that considers variations in observable borrower risk. As discussed in the Introduction, they too find that observably riskier borrowers may pledge more collateral and, moreover, that collateralized loans may be riskier ex post, which is the issue we shall turn to next.

Collateral and Ex Post Default Likelihood

The fact that observably riskier borrowers pledge more collateral already implies that collateralized loans will have a higher ex post default likelihood. Interestingly, this prediction follows from our model *even if we control for observable borrower risk.* In our model, the average default likelihood within the pool of accepted borrowers under a lenient credit policy (low s^*) is higher than under a conservative credit policy (high s^*). Formally, the average default likelihood conditional on the borrower being accepted is

$$D := \int_{s^*}^{1} (1 - p_s) \frac{f(s)}{1 - F(s^*)} ds, \tag{7}$$

where $f(s)/[1 - F(s^*)]$ is the density of s conditional on $s \geq s^*$. Given that $1 - p_s$ is decreasing in s, and given that s^* is decreasing in the amount of collateral, an increase in collateral thus implies a higher average default likelihood of accepted borrowers.

Corollary 3. *Collateralized loans have a higher ex post default likelihood.*

Corollary 3 is consistent with evidence by Jiménez and Saurina (2004) and Jiménez, Salas, and Saurina (2005), who find that collateralized loans have a higher probability of defaulting in the year after the loan was granted. Similarly, Berger and Udell (1990), using past dues and nonaccruals as proxies for default risk, find that collateralized loans are riskier ex post.

As discussed in the Introduction, existing models of collateral—with the exception of Boot, Thakor, and Udell (1991)—generally make the opposite prediction that collateralized loans are safer, not riskier. In adverse selection models (e.g., Bester (1985), Chan and Kanatas (1987), Besanko and Thakor (1987a,b)), this is because safe borrowers can reveal their type by posting collateral. In moral hazard models (e.g., Chan and Thakor (1987), Boot and Thakor (1994)), it is because collateral improves the incentives of borrowers to work hard, which reduces their default likelihood.

4 Robustness

Thus far we have assumed that the local lender has all the bargaining power ex ante. Moreover, it was assumed that the local lender's decision to reject the borrower is not subject to renegotiations. In this section, we show that our results are robust to allowing for bargaining both at the ex ante and interim stage.

4.1 Ex Ante Bargaining

Suppose the local lender and the borrower can bargain over the loan terms ex ante. Given that there is symmetric information, it is reasonable to assume that the two choose a contract on the Pareto frontier. Contracts on the Pareto frontier are derived by maximizing the utility of one side subject to leaving the other side a given utility. This is precisely what we did when we maximized the local lender's expected payoff subject to leaving the borrower a utility of $V = \overline{V}$. By varying the borrower's utility, we can thus trace out the Pareto frontier $U = u(V)$.[20] By Proposition 1, each point (U, V) on the Pareto frontier is associated with a uniquely optimal financial contract $(r(V), C(V))$. Note that we could alternatively solve the dual problem in which the *borrower's* expected payoff is maximized subject to leaving the local lender a given reservation utility. The Pareto frontier would be the same.

As the borrower's utility under ex ante bargaining may exceed his outside option, we must introduce some additional notation. Accordingly, let $\hat{V} = \max\{0, \mu - k\}$ denote the borrower's outside option from going to transaction lenders. The local lender's outside option is zero. Provided there exists a mutually acceptable contract, we assume that the solution is determined by Nash bargaining, where b and $1-b$ denote the borrower's and local lender's bargaining power, respectively. The bargaining solution (U, V) thus maximizes the Nash product $(V - \hat{V})^b U^{1-b} = (V - \hat{V})^b [u(V)]^{1-b}$, implying the borrower's expected utility V is the solution to

$$\frac{b}{1-b} = -u'(V)\frac{V - \hat{V}}{u(V)}. \tag{8}$$

Hence, the optimal financial contract is obtained in precisely the same way as in Section 3, except that now we have $\overline{V} = V$, where V is given by (8).

Proposition 3. *Suppose the local lender and the borrower can bargain over the loan terms ex ante. Irrespective of the distribution of bargaining powers, the optimal financial contract is the same as in Proposition 1, except that now* $\overline{V} = V$, *where* V *is given by* (8).

Proof. See Appendix.

While ex ante bargaining does not affect the qualitative properties of the optimal financial contract, it does affect the *specific* solution, i.e., the specific optimal loan rate and collateral requirement, implying Proposition 2 must be modified accordingly. Evidently, if the borrower's bargaining power is zero ($b \to 0$), we are back to the specific solution in Proposition 2. As the borrower's bargaining power increases, $\overline{V}$ increases correspondingly, implying the borrower's utility exceeds his outside option. Generalizing (5) to arbitrary values of $\overline{V}$, we obtain

$$C_{FB} := \frac{(k - x_l)\overline{V}}{\int_{s_{FB}}^{1} (\mu_s - k) f(s) ds}, \tag{9}$$

[20]While the Pareto frontier is decreasing by construction, it is convenient to assume that it is smooth and concave. A standard way to ensure concavity of the Pareto frontier is to allow lotteries over contracts.

which implies the first-best amount of collateral is increasing in $\overline{V}$. Finally, if $b \to 1$ we obtain the other polar case in which the borrower has all the bargaining power. The optimal financial contract is now the solution to the specific dual problem in which the borrower makes a take-it-or-leave-it offer that maximizes his expected payoff subject to leaving the local lender a reservation utility of zero. Interestingly, the local lender's participation constraint in this case is slack: as she makes her credit decision under private information, she can always extract an informational rent (see Section 3.2). This is arguably different from models in which the agency problem lies with the borrower. In such models, if the borrower has all the bargaining power or the loan market is perfectly competitive, lenders generally make zero profits.

4.2 Interim Bargaining

We will now reconsider our assumption that the local lender's decision to reject the borrower is not subject to renegotiations. Clearly, if the borrower could observe the local lender's project assessment, any inefficiency would be renegotiated away. Precisely, if $s \in [s_{FB}, s^*)$ the local lender and the borrower would change the loan terms to allow the local lender to break even. Given that the borrower cannot observe the local lender's project assessment, however, such a mutually beneficial outcome may not arise. In fact, despite the inefficiency in case $s^* > s_{FB}$, the original loan terms will not be renegotiated in equilibrium

We consider the following simple renegotiation game. After the local lender has evaluated the borrower's project, either she or the borrower can make a take-it-or-leave-it offer to replace the original loans terms with new ones.[21] If the local lender makes the offer the borrower must agree, while if the borrower makes the offer the local lender must agree. If the two cannot agree, the original loan terms remains in place.

Proposition 4. *Suppose the local lender and the borrower can renegotiate the original loan terms after the local lender has evaluated the borrower's project. Regardless of who can make the contract offer at the interim stage, the original loan terms remain in place.*

Proof. See Appendix.

The intuition is straightforward. As only the local lender can observe s, the borrower does

[21] To the best of our knowledge, there exists no suitable axiomatic bargaining concept à la Nash bargaining to analyze surplus sharing under private information; hence the restriction to the two polar cases in which either the borrower or the local lender makes a take-it-or-leave-it offer. Our results would remain unchanged if the local lender and the borrower could make alternating offers as long as there is no additional sorting variable.

not know whether $s < s^*$ or $s \geq s^*$. In the first case, adjusting the loan terms to the local lender's benefit would allow her to break even, thus avoiding an inefficient rejection. In the second case, however, the local lender would have accepted the project anyway. Adjusting the loan terms would then merely constitute a wealth transfer to the local lender. By Proposition 4 , the *expected* value to the borrower from adjusting the loan terms—given that he does not know whether $s < s^*$ or $s \geq s^*$—is negative.

Finally, we might ask whether it would ever be *suboptimal* to set the loan terms ex ante. That is, would the local lender ever prefer to wait until after the project evaluation?[22] The answer is no. Suppose the local lender waits until after the project evaluation. In this case, any equilibrium of the signaling game in which the borrower is attracted must provide him an expected utility of at least $\overline{V}$. Moreover, while waiting allows the local lender to "fine-tune" her offer to the outcome of her project assessment, she can accomplish the same by offering an incentive-compatible menu of contracts ex ante from which she chooses at the interim stage. As is easy to show, offering such a menu is strictly suboptimal in our model.[23] Consequently, there is no benefit to the local lender from waiting with her offer until after the project evaluation.

5 Imperfect Loan Market Competition and Collateral

Thus far we have focused on a *given* borrower-lender relationship—i.e., holding loan market competition constant. We will now examine changes in loan market competition. Specifically, we will examine how advances in information technology that increase the competitive pressure from transaction lenders affect loan rates and collateral requirements.

5.1 Changes in the Local Lender's Information Advantage

As discussed in the Introduction, one implication of the "information revolution" in small business lending is that the information advantage of local lenders has narrowed. This is especially true since the early 1990s when small business credit scoring has been adopted on a broad scale

[22] As the borrower has the same information ex ante and at the interim stage, it is immediate that he would make the same offer in $\tau = 0$ and $\tau = 1$.

[23] Intuitively, allowing the local lender to choose from a menu after the project evaluation creates a "self-dealing problem" as the local lender would always pick the contract that is ex post optimal for her. This makes it harder to satisfy the borrower's participation constraint, with the result that the local lender's privately optimal cutoff s^* will be strictly higher (and thus less efficient) than under the single optimal contract from Proposition 2.

in the United States.[24] Small business credit scoring models give a fairly accurate prediction of the likelihood that a borrower will default based solely on hard information, especially credit reports, thus considerably reducing the information uncertainty associated with small business loans made to borrowers located far away.[25]

To obtain a continuous yet simple measure of the local lender's information advantage vis-à-vis transaction lenders, we assume it is now only with probability $0 < q \leq 1$ that the local lender has a more precise estimate of the project's success probability. Our base model corresponds to the case where $q = 1$. Like in our base model, we assume that the local lender's actual success probability estimate is her private information. We obtain the following result.

Proposition 5. *There exists a threshold $\widehat{q}$ such that borrowers for whom the local lender's information advantage is large ($q \geq \widehat{q}$) go to the local lender while borrowers for whom the local lender's information advantage is small ($q < \widehat{q}$) borrow from transaction lenders.*

Conditional on going to the local lender ($q \geq \widehat{q}$), borrowers for whom the local lender's information advantage is smaller (lower q) face lower loan rates but higher collateral requirements.

Proof. See Appendix.

Why is a small (but positive) information advantage not always sufficient to attract the borrower?[26] Like in our base model, borrowers who are rejected by the local lender are unable to obtain financing elsewhere. Hence, from the borrower's perspective, going to the local lender and being rejected is worse than borrowing directly from transaction lenders. To attract the borrower, the local lender must therefore offer him a loan rate that is below the rate offered by transaction lenders, which implies the local lender must create additional surplus. But merely creating *some* additional surplus is not enough: as the local lender extracts an informational rent (see Section 3.2), she can only promise a fraction of the created surplus to the borrower,

[24] The first bank in the United States to adopt small business credit scoring was Wells Fargo in 1993 using a proprietary credit scoring model. Already in 1997—only two years after Fair, Isaac & Co. introduced the first commercially available small business credit scoring model—70 percent of the (mainly large) banks surveyed in the Federal Reserve's Senior Loan Officer Opinion Survey responded that they use credit scoring in their small business lending (Mester (1997)).

[25] Frame, Srinivasan, and Woosley (2001) conclude: "[C]redit scoring lowers information costs between borrowers and lenders, thereby reducing the value of traditional, local bank lending relationships."

[26] The threshold $\widehat{q}$ in Proposition 5 may not always be interior. For instance, if $\mu - k \leq 0$ the borrower's outside option is zero, implying the local lender can attract the borrower for all $q > 0$.

implying that—in order to attract the borrower—the additional surplus created by the local lender must be sufficiently large, i.e., q must be sufficiently high.

Before we link Proposition 5 to advances in information technology narrowing the local lender's information advantage vis-à-vis transaction lenders, it is worth pointing out that Proposition 5 has also cross-sectional implications. Precisely, borrowers who choose to borrow locally ($q \geq \widehat{q}$) and for whom the local lender's information advantage is smaller (lower q) face lower loan rates but higher collateral requirements. Intuitively, a decrease in q implies that the local lender creates less surplus in terms of screening out negative-NPV projects. Holding the loan rate constant, a decrease in q thus reduces the borrower's expected payoff, violating his (previously binding) participation constraint. In order to attract the borrower, the local lender must consequently lower the loan rate. But a lower loan rate implies the borrower receives a larger share of the project's cash flows, which in turn implies the local lender must raise the collateral requirement to minimize the distortion in her credit decision.

As the sole role of collateral in our model is to minimize distortions in credit decisions based on soft information, collateral has no meaningful role to play in loans underwritten by transaction lenders. Indeed, while the vast majority of small business loans in the United States are collateralized (Avery, Bostic, and Samolyk (1998), Berger and Udell (1998)), small business loans made by transaction lenders on the basis of credit scoring are generally unsecured (Zuckerman (1996), Frame, Srinivasan, and Woosley (2001), Frame, Padhi, and Woosley (2004)). In addition, our model predicts that—with respect to those borrowers who choose to borrow locally—loans will be more collateralized if the local lender's information advantage is smaller. Consistent with this prediction, Petersen and Rajan (2002) find that small business borrowers who are located further away from their local lender are more likely to pledge collateral.[27] To the extent that the local lender's information advantage increases with the duration of borrower relationships, Proposition 5 is also consistent with evidence by Berger and Udell (1995) and Degryse and van Cayseele (2000), who both find that longer borrower relationships are associated with less collateral.[28]

We can alternatively interpret Proposition 5 in terms of a change in the local lender's infor-

[27] The vast majority of the small firms in Petersen and Rajan's (2002) sample borrow from a single local lender—the median bank-borrower distance is only four miles.

[28] While these findings are consistent with our model only to the extent that the local lender's information advantage increases with the length of borrower relationships, they are directly consistent with Boot and Thakor (1994), who model relationship lending as a repeated game showing that collateral decreases with the duration of borrower relationships.

mation advantage for any *given* borrower. As discussed above, with the widespread adoption of small business credit scoring since the early 1990s this information advantage appears to have narrowed. According to Proposition 5, a narrowing of the local lender's information advantage vis-à-vis transaction lenders has two effects. First, marginal borrowers for whom the local lender has only a relatively small information advantage will switch to transaction lenders. Indeed, various studies document that transaction lenders using small business credit scoring have successfully expanded their small business lending to borrowers located outside of their own markets (Hannan (2003), Frame, Padhi, and Woosley (2004), Berger, Frame, and Miller (2005)).[29] Second, borrowers who continue to borrow from their local lender will face lower loan rates but higher collateral requirements. We are unaware of empirical studies investigating how the widespread adoption of small business credit scoring over the past years has affected the loan terms in local lending relationships.

5.2 Changes in the Costs of Transaction Lending

A second—and in a sense more immediate—implication of the "information revolution" in small business lending is that the costs of underwriting transaction loans have decreased over time. Processing costs for small business loans based on credit scoring have decreased considerably (Mester (1997)), input databases for credit scoring models have become larger, and credit reports can now be sent instantly and at relatively low costs over the internet (DeYoung, Hunter, and Udell (2004), Berger and Frame (2005)).[30]

To examine the implications of a decrease in the costs of transaction lending, we assume that underwriting a transaction loan involves a cost of κ. Arguably, this cost will be initially incurred by the respective lender. As the market for transaction loans is perfectly competitive, however, it will be ultimately borne by the borrower. Accordingly, the borrower's outside option from going to transction lenders is now $\overline{V} = \max\{0, \mu - k - \kappa\}$. Evidently, if $\overline{V} = 0$ a change in κ has no effect in our model. In the following, we thus focus on the interesting case where $\overline{V} = \mu - k - \kappa > 0$. Our base model corresponds to the case where $\kappa = 0$.

Like in the case of a decrease in q, a decrease in the costs of transaction lending implies

[29] See also Berger and Frame (2005): "[T]echnological change—including the introduction of SBCS [small business credit scoring]—may have increased the competition for small business customers and potentially widened the geographic area over which these firms may search for credit. Presumably, a small business with an acceptable credit score could now shop nationwide through the Internet among lenders using SBCS."

[30] At the same time, there appears to be little evidence that advances in information technology have had a significant direct impact on relationship lending (DeYoung, Hunter, and Udell (2004)).

that the local lender will lose marginal borrowers to transaction lenders. This is precisely what Boot and Thakor (2000) show in their analysis of competition between transaction lenders and relationship lenders. What is less obvious, however, is to what extent this decrease in the costs of transaction lending might affect collateral requirements. We obtain the following result.

Proposition 6. *A decrease in the costs of transaction lending (lower κ) forces the local lender to lower the loan rate and increase the collateral requirement. The increase in collateral requirement for a given decrease in κ is stronger for borrowers for whom the local lender has a relatively smaller information advantage (lower q).*

Proof. See Appendix.

A decrease in the direct costs of transaction lending increases the value of the borrower's outside option, thus increasing the competitive pressure from transaction lenders. To attract the borrower, the local lender must consequently lower the loan rate. Like in the case of a narrowing of the local lender's information advantage, this implies the local lender must raise the collateral requirement to minimize the distortion in her credit decision. Interestingly, the increase in collateral requirement is stronger for borrowers for whom the local lender has a smaller information advantage; the intuition is the same as why these borrowers face higher collateral requirements in the first place (see Proposition 5).

We are unaware of empirical studies investigating how changes in the costs of transaction lending over the past years have affected the usage of collateral in small business loans. There is, however, evidence that the usage of collateral increases with loan market competition, which is broadly consistent with our argument. Using Spanish data, Jiménez and Saurina (2004) and Jiménez, Salas, and Saurina (2005) document a positive relation between collateral and bank competition measured by the Herfindahl index. Moreover, Jiménez, Salas, and Saurina (2006) find that this positive effect of bank competition on collateral is decreasing with the length of bank-borrower relationships, which is consistent with the last statement in Proposition 6 if the local lender's information advantage increases with the duration of borrower relationships.

To our knowledge, related models of imperfect loan market competition (e.g., Boot and Thakor (2000), Hauswald and Marquez (2003, 2005)) do not consider collateral. On the other hand, theoretical models of collateral do not consider imperfect loan market competition between arm's-length transaction lenders and local (relationship) lenders, thus generating different empirical predictions than this paper. For instance, Besanko and Thakor (1987a) and Manove, Padilla, and Pagano (2001) both find that collateral is used in a perfectly competitive but not in a monopolistic loan market. Closer in spirit to our model, Villas-Boas and Schmidt-Mohr (1999) consider an imperfectly competitive loan market with horizontally differentiated lenders, showing that collateral may either increase or decrease as loan market competition increases.

6 Conclusion

This paper offers a novel argument for collateral based on the notion that collateral mitigates distortions in credit decisions based on soft information. Our argument is entirely lender-based; there is no borrower moral hazard or adverse selection.

In our model, there is a local (e.g., relationship) lender who has access to additional soft, albeit private, information allowing her to make a more precise estimate of the borrower's default likelihood than distant "transaction lenders" who provide arm's-length financing based on publicly available, hard information. While the local lender has a competitive advantage, the competition from transaction lenders is nonetheless important as it provides the borrower with a positive outside option that the local lender must match. In order to attract the borrower, the local lender must consequently leave him some of the surplus from the project, which in turn distorts her credit decision in the sense that she rejects marginally profitable projects. Collateral "flattens" the local lender's payoff function, i.e., it improves the local lender's payoff from projects with a relatively high likelihood of low cash flows, and thus from precisely those marginally profitable projects that she inefficiently rejects.

The fact that the local lender's credit decision is based on soft and private information is crucial for the inefficiency studied here, and hence for the optimality of collateral. If the information was observable and contractible, the local lender could contractually commit to the first-best credit decision even if this means committing to a decision rule that is ex post suboptimal for her. Likewise, if the information was observable but non-verifiable, the inefficiency could be eliminated through bargaining at the interim stage.

Given that our model is cast as an imperfectly competitive loan market in which the local lender has an information advantage vis-à-vis transaction lenders, we can draw implications regarding the effects of technological improvements that strengthen the competitive position of transaction lenders. For instance, we find that technological innovations such as small business credit scoring that narrow the information advantage of incumbent local lenders lead to lower loan rates but higher collateral requirements, thus strengthening the usage of collateral in local lending relationships. Similarly, innovations that lower the costs of underwriting transaction loans lead to greater competitive pressure from transaction lenders, lower loan rates, and higher collateral requirements. Interestingly, the increase in collateral requirements is stronger for borrowers for whom the local lender has only a relatively weak information advantage, e.g., borrowers who are located further away from the local lender or borrowers with whom the local lender has no existing lending relationship.

7 Appendix A: Continuum of Cash Flows

This section shows that our argument for why collateral is optimal extends to a continuum of cash flows. Unlike the two cash-flow model in the main text, it not only shows that collateral is used only in low cash-flow states, but it also shows how precisely repayments are made out of the pledged assets as a function of the project's cash flow when cash flows are continuous.

We assume that the project cash flow x is distributed with atomless CDF $G_s(x)$ over the support $X := [0, \overline{x}]$, where $\overline{x} > 0$ may be either finite or infinite. The density $g_s(x)$ is everywhere continuous and positive. In case $\overline{x}$ is infinite, we assume that $\mu_s := \int_X x g_s(x) dx$ exists for all $s \in [0,1]$. We moreover assume that $G_s(x)$ satisfies the Monotone Likelihood Property (MLRP), which states that for any pair $(s, s') \in S$ with $s' > s$, the ratio $g_{s'}(x)/g_s(x)$ is strictly increasing in x for all $x \in X$.

When there is a continuum of cash flows, a financial contract specifies a repayment schedule $t(x) \leq x$ out of the project's cash flow, an amount $C \leq w$ of collateral, and a repayment schedule $c(x) \leq C$ out of the pledged assets. It is convenient to write $R(x) := t(x) + c(x)$. We make the standard assumption that $R(x)$ is nondecreasing for all $x \in X$ (e.g., Innes (1990)). The local lender's and the borrower's expected payoffs are then $U_s(R) := \int_X R(x) g_s(x) dx$, $V_s(R) := \mu_s - U_s(R)$, $U(R) := \int_{s^*}^1 [U_s(R) - k] f(s) ds$ and $V(R) := \int_{s^*}^1 V_s(R) f(s) ds$, respectively. Analogous to the analysis in the main text, the local lender's privately optimal cutoff s^* is given by the optimality condition $U_{s^*(R)}(R) = k$. The local lender's program is to maximize $U(R)$ subject to the borrower's participation constraint $V(R) \geq \overline{V}$.

The following result extends Proposition 1 to the case with a continuum of cash flows.

Proposition. *The optimal financial contract when there is a continuum of cash flows stipulates a repayment* $R \in (0, \overline{x})$ *and an amount of collateral* $C \in (0, w]$ *such that the local lender receives* $R(x) = x + C$ *if* $x \leq R$ *and* $R(x) = R$ *if* $x > R$.

As far as the repayments out of the project's cash flow are concerned, we have $t(x) = x$ for $x \leq R$ and $t(x) = R$ for $x > R$. Collateral is used as follows: if $x \leq R - C$ the local lender receives the entire collateral, i.e., $c(x) = C$, if $R - C < x \leq R$ the local lender receives a fraction

$c(x) = R - x$ of the pledged assets (after liquidation), and if $x > R$, the local lender receives no repayment out of the pledged assets because the project's cash flow is sufficient to make the contractually stipulated repayment.

To prove the proposition, suppose to the contrary that the optimal contract stipulated a repayment schedule $R(x)$ that is different from the one in the proposition. We can then construct a new repayment schedule $\widetilde{R}(x) = \min\{x + \widetilde{C}, \widetilde{R}\}$ where $\widetilde{C} = w$, and where $\widetilde{R}$ satisfies

$$\int_{s^*(R)}^{1} \left[\int_X z(x) g_s(x) dx \right] f(s) ds = 0, \tag{10}$$

with $z(x) := \widetilde{R}(x) - R(x)$. (Holding the local lender's cutoff fixed at $s^*(R)$, all expected payoffs remain unchanged.)[31] By construction of $\widetilde{R}(x)$, there exists a value $0 < \widetilde{x} < \overline{x}$ such that $z(x) \geq 0$ for all $x < \widetilde{x}$ and $z(x) \leq 0$ for all $x > \widetilde{x}$, where the inequalities are strict on a set of positive measure.

Claim 1. $s^*(\widetilde{R}) < s^*(R)$.

Proof. By (10) and continuity of $g_s(x)$ in s, there exists a value $\widetilde{s}$ satisfying $s^*(R) < \widetilde{s} < 1$, where $\int_X z(x) g_{\widetilde{s}}(x) dx = 0$. From $\widetilde{s} > s^*(R)$ and MLRP it follows that $g_{s^*(R)}(x)/g_{\widetilde{s}}(x)$ is strictly decreasing in x such that

$$\begin{aligned} \int_X z(x) g_{s^*(R)}(x) dx &= \int_{x \leq \widetilde{x}} z(x) g_{\widetilde{s}}(x) \frac{g_{s^*(R)}(x)}{g_{\widetilde{s}}(x)} dx + \int_{x > \widetilde{x}} z(x) g_{\widetilde{s}}(x) \frac{g_{s^*(R)}(x)}{g_{\widetilde{s}}(x)} dx \\ &> \frac{g_{s^*(R)}(\widetilde{x})}{g_{\widetilde{s}}(\widetilde{x})} \int_X z(x) g_{\widetilde{s}}(x) dx = 0. \end{aligned}$$

Given that $\int_X z(x) g_{s^*(R)}(x) dx > 0$ and $\int_X R(x) g_{s^*(R)}(x) dx = k$ from the definition of $s^*(R)$, we have that $\int_X \widetilde{R}(x) g_{s^*(R)}(x) dx > k$. As $U_s(\widetilde{R})$ is strictly increasing in s, we have that $s^*(\widetilde{R}) < s^*(R)$. Q.E.D.

Observe that the new cutoff $s^*(\widetilde{R})$ may lie *below* s_{FB}. In this case, we can make the following adjustment:

Claim 2. *In case* $s^*(\widetilde{R}) < s_{FB}$ *for* $\widetilde{C} = w$, *we can adjust the new contract by decreasing* $\widetilde{C}$ *and increasing* $\widetilde{R}$ *such that* (10) *continues to hold while* $s^*(\widetilde{R}) = s_{FB}$.

Proof. Take first some contract $(\widehat{R}, \widehat{C})$ such that $\widehat{R} > \widetilde{R}$ and $\widehat{C} < \widetilde{C}$ and (10) holds with $z(x) := \widehat{R}(x) - \widetilde{R}(x)$. From (10)—together with $\widehat{R} > \widetilde{R}$ and $\widehat{C} < \widetilde{C}$—it follows that there exists

[31] Existence and uniqueness of a value $\widetilde{R}$ solving (10) follows from the fact that i) the local lender's payoff is continuous and strictly increasing in $\widetilde{R}$ for a given cutoff, and ii) the left-hand side of (10) is strictly positive at $\widetilde{R} = \overline{x}$ and strictly negative at $\widetilde{R} = 0$.

a value $0 < \widetilde{x} < \overline{x}$ such that $z(x) \geq 0$ for all $x > \widetilde{x}$ and $z(x) \leq 0$ for all $x < \widetilde{x}$, where the inequalities are strict on a set of positive measure. By the argument in Claim 1, this implies that $s^*(\widehat{R}) > s^*(\widetilde{R})$. As we decrease $\widehat{C}$ and adjust $\widehat{R}$ accordingly to satisfy (10), we have from the definition of s^* and continuity of $g_s(x)$ that $s^*(\widehat{R})$ increases continuously. Given that $s^*(\widehat{R}) > s_{FB}$ at $\widehat{C} = 0$, the claim follows immediately. Q.E.D.

We show next that the borrower is not worse off under the new contract $(\widetilde{R}, \widetilde{C})$.

Claim 3. $V(\widetilde{R}) \geq V(R)$.

Proof. We can distinguish between three cases.

Case 1: $s^*(R) = s_{FB}$. The claim follows immediately from (10) and $s^*(R) = s^*(\widetilde{R})$.

Case 2: $s^*(R) > s_{FB}$. In this case, it follows from the construction of $\widetilde{R}(x)$ that $s_{FB} \leq s^*(\widetilde{R}) < s^*(R)$ and that the borrower's expected payoff remains unchanged if he is accepted if and only if $s \geq s^*(R)$. Hence, $V(\widetilde{R}) \geq V(R)$ follows if $V_s(\widetilde{R}) \geq 0$ for all $s \in [s^*(\widetilde{R}), s^*(R)]$. To see that this is the case, note first that $V_{s^*(\widetilde{R})}(\widetilde{R}) \geq 0$ since $U_{s^*(\widetilde{R})}(\widetilde{R}) = k$ and $s_{FB} \leq s^*(\widetilde{R})$. It thus remains to show that $V_s(\widetilde{R})$ is nondecreasing in s. Partial integration yields

$$V_s(\widetilde{R}) = \int_{\widetilde{R}-\widetilde{C}}^{\overline{x}} [1 - G_s(x)]\, dx - \widetilde{C}. \tag{11}$$

MLRP implies that $G_s(x)$ is strictly decreasing in s for all $0 < x < \overline{x}$. By (11) this implies that $V_s(\widetilde{R})$ is strictly increasing in s.

Case 3: $s^*(R) < s_{FB}$. In this case, it follows from the construction of $\widetilde{R}(x)$ that $s^*(\widetilde{R}) = s_{FB}$. It remains to show that $V_s(\widetilde{R}) \leq 0$ for all $s \in [s^*(\widetilde{R}), s_{FB}]$. From $s^*(\widetilde{R}) = s_{FB}$—implying that $U_{s_{FB}}(\widetilde{R}) = 0$—it follows that $V_{s_{FB}}(\widetilde{R}) = 0$, while the argument in Case 2 implies that $V_s(\widetilde{R})$ is nondecreasing in s. Together, this implies that $V_s(\widetilde{R}) \leq 0$ for all $s \in [s^*(\widetilde{R}), s_{FB}]$. Q.E.D.

Summing up, we have constructed a new contract $(\widetilde{R}, \widetilde{C})$ with the following characteristics: i) $\widetilde{R}(x) = \min\{x+\widetilde{C}, \widetilde{R}\}$; ii) (10) is satisfied; iii) if $s^*(R) \geq s_{FB}$ it holds that $s_{FB} \leq s^*(\widetilde{R}) \leq s^*(R)$, where $s^*(\widetilde{R}) < s^*(R)$ if $s^*(R) > s_{FB}$; iv) if $s^*(R) < s_{FB}$ it holds that $s^*(R) < s^*(\widetilde{R}) = s_{FB}$; v) $V(\widetilde{R}) \geq V(R)$. The new contract thus satisfies the borrower's participation constraint while the local lender is not worse off. In fact, she is strictly better off if $s^*(\widetilde{R}) \neq s^*(R)$, which follows immediately from (10) and optimality of s^*. Finally, if the original contract implements the first best, i.e., if $s^*(\widetilde{R}) = s^*(R) = s_{FB}$, then the repayment made out of the pledged assets is strictly lower under the new contract, i.e., $\int_{s_{FB}}^1 \left[\int_X c(x) g_s(x) dx\right] f(s) ds > \int_{s_{FB}}^1 \left[\int_X \widetilde{c}(x) g_s(x) dx\right] f(s) ds$. Q.E.D.

8 Appendix B: Proofs

Proof of Lemma 2. Suppose to the contrary that the project's NPV conditional upon rejection was positive, i.e., that

$$\int_0^{s^*} (\mu_s - k)\frac{f(s)}{F(s^*)}ds > 0. \tag{12}$$

Note that this immediately implies that $\mu - k > 0$: if the project's unconditional NPV was non-positive, its NPV conditional upon rejection would have to be negative. Given that transaction lenders are perfectly competitive, (12) is what a rejected borrower would obtain in $\tau = 1$ when seeking funding from transaction lenders. In $\tau = 0$, the borrower's expected payoff from going to the local lender is consequently

$$\int_{s^*}^1 [\mu_s - U_s(R_l, R_h)]f(s)ds + \int_0^{s^*} (\mu_s - k)f(s)ds, \tag{13}$$

while his payoff from going to transaction lenders is $\mu - k > 0$. Requiring that (13) is equal to or greater than $\mu - k$ and using the fact that $\mu = \int_0^1 \mu_s f(s)ds$ yields the requirement that

$$\int_{s^*}^1 [U_s(R_l, R_h) - k]\, f(s)ds \leq 0,$$

which contradicts the fact that $U_s(R_l, R_h) > k$ for all $s > s^*$. Q.E.D.

Proof of Propositions 1 and 2. It is convenient to prove the two propositions jointly. Since the case where $\overline{V} = 0$ is obvious, we focus on the nontrivial case where $\overline{V} > 0$. To make the dependency of s^* on R_l and R_h explicit, we shall write $s^* = s^*(R_l, R_h)$ for convenience. The following observations are all obvious. First, if we increase R_l while holding R_h constant, $U(R_l,R_h)$ increases while $s^*(R_l, R_h)$ decreases. Second, if we increase R_h while holding R_l constant, $U(R_l, R_h)$ increases while $s^*(R_l, R_h)$ decreases. Third, $s^*(R_l, R_h)$ is continuous in both R_l and R_h, implying that $V(R_l,R_h)$ and $U(R_l, R_h)$ are also both continuous.

The following two auxiliary results simplify the analysis.

Claim 1. *Take two contracts (R_l, R_h) and $(\widetilde{R}_l, \widetilde{R}_h)$ with $\widetilde{R}_l > R_l$ and $\widetilde{R}_h < R_h$. If the local lender's optimal cutoff is the same under both contracts, i.e., if $s^*(R_l, R_h) = s^*(\widetilde{R}_l, \widetilde{R}_h)$, then $V_s(\widetilde{R}_l, \widetilde{R}_h) > V_s(R_l, R_h)$ for all $s > s^*$.*

Proof. Since $s^*(R_l, R_h) = s^*(\widetilde{R}_l, \widetilde{R}_h) = s^*$, we have that $U_{s^*}(\widetilde{R}_l, \widetilde{R}_h) = U_{s^*}(R_l, R_h)$ and therefore that $V_{s^*}(\widetilde{R}_l, \widetilde{R}_h) = V_{s^*}(R_l, R_h)$. Given that $\widetilde{R}_h - \widetilde{R}_l < R_h - R_l$ and

$$V_s(\widetilde{R}_l, \widetilde{R}_h) - V_s(R_l, R_h) = (R_l - \widetilde{R}_l) + p_s[(R_h - R_l) - (\widetilde{R}_h - \widetilde{R}_l)],$$

the fact that p_s is strictly increasing in s implies that $V_s(\widetilde{R}_l, \widetilde{R}_h) - V_s(R_l, R_h)$ must be strictly increasing in s. In conjunction with $V_{s^*}(\widetilde{R}_l, \widetilde{R}_h) = V_{s^*}(R_l, R_h)$, this implies that $V_s(\widetilde{R}_l, \widetilde{R}_h) > V_s(R_l, R_h)$ for all $s > s^*$. Q.E.D.

Claim 2. *Take two contracts (R_l, R_h) and $(\widetilde{R}_l, \widetilde{R}_h)$, where $\widetilde{R}_l > R_l$ and $\widetilde{R}_l < \widetilde{R}_h < R_h$ satisfy*

$$\int_{s^*(R_l,R_h)}^{1} [V_s(\widetilde{R}_l, \widetilde{R}_h) - V_s(R_l, R_h)] f(s) ds = 0. \tag{14}$$

That is, holding the cutoff fixed at $s^*(R_l, R_h)$, the borrower's (and thus also the local lender's) expected payoffs under the two contracts are the same. It then holds that $s^*(\widetilde{R}_l, \widetilde{R}_h) < s^*(R_l, R_h)$.

Proof. We can transform (14) to

$$\int_{s^*(R_l,R_h)}^{1} [p_s[(R_h - R_l) - (\widetilde{R}_h - \widetilde{R}_l)] - (\widetilde{R}_l - R_l)] \frac{f(s)}{1 - F(s^*)} ds = 0. \tag{15}$$

As p_s is strictly increasing in s and $R_h - R_l > \widetilde{R}_h - \widetilde{R}_l$ by construction, (15) implies that

$$p_{s^*(R_l,R_h)}[(R_h - R_l) - (\widetilde{R}_h - \widetilde{R}_l)] - (\widetilde{R}_l - R_l) < 0,$$

and therefore that $V_{s^*(R_l,R_h)}(\widetilde{R}_l, \widetilde{R}_h) < V_{s^*(R_l,R_h)}(R_l, R_h)$. Since $U_{s^*(R_l,R_h)}(R_l, R_h) = k$ from the definition of $s^*(R_l, R_h)$, this implies that $U_{s^*(R_l,R_h)}(\widetilde{R}_l, \widetilde{R}_h) > k$. As $U_s(\widetilde{R}_l, \widetilde{R}_h)$ is strictly increasing in s and $U_{s^*(\widetilde{R}_l,\widetilde{R}_h)}(\widetilde{R}_l, \widetilde{R}_h) = k$ from the definition of $s^*(\widetilde{R}_l, \widetilde{R}_h)$, this implies that $s^*(\widetilde{R}_l, \widetilde{R}_h) < s^*(R_l, R_h)$. Q.E.D.

We now prove the assertion in Proposition 1 that the optimal contract is unique and that it has a positive amount of collateral in the low cash-flow state, i.e., $R_l > x_l$, where $R_l - x_l \in (0, w]$. The fact that $R_h < x_h$ follows trivially from the borrower's participation constraint (3): if $R_l > x_l$ but $R_h \geq x_h$, the borrower would not break even. We prove the assertion separately for the case where (R_l, R_h) is first-best optimal, i.e., $s^*(R_l, R_h) = s_{FB}$ (Case 1) and second-best optimal, i.e., $s^*(R_l, R_h) > s_{FB}$ (Case 2). In Case 2, we specifically prove that $R_l = x_l + w$ as asserted in Proposition 2. We finally show that it cannot be true that $s^*(R_l, R_h) < s_{FB}$.

Case 1. Suppose under the optimal contract (R_l, R_h) it holds that $s^*(R_l, R_h) = s_{FB}$. We then have from (1) and (2) that

$$\frac{k - R_l}{R_h - R_l} = \frac{k - x_l}{x_h - x_l}, \tag{16}$$

which uniquely pins down R_h for a given value of R_l. As we increase R_l while decreasing R_h to satisfy (16), we know from Claim 1 that $V_s(R_l, R_h)$ increases strictly for all $s > s^*(R_l, R_h) = s_{FB}$.

Consequently, $V(R_l, R_h)$ also increases strictly. The requirement that $s^*(R_l, R_h) = s_{FB}$ in conjunction with the fact that (3) holds with equality thus pins down a unique pair (R_l, R_h). It remains to show that $R_l > x_l$. If $R_l = x_l$, (16) would imply that $R_h = x_h$ and thus that $V(R_l, R_h) = 0$, violating (3). By Claim 1, any lower value $R_l < x_l$ (together with $R_h > x_h$ to satisfy (16)) would imply an even lower value of $V(R_l, R_h)$ and therefore also violate (3).

Case 2. Suppose under the optimal contract (R_l, R_h) it holds that $s^*(R_l, R_h) > s_{FB}$. We first show that in this case it must hold that $R_l = x_l + w$. We argue to a contradiction and assume that $R_l < x_l + w$. We can then construct a new contract $(\widetilde{R}_l, \widetilde{R}_h)$ with $\widetilde{R}_l > R_l$ and $\widetilde{R}_l < \widetilde{R}_h < R_h$ that satisfies (3) and is strictly preferred by the local lender, contradicting the optimality of (R_l, R_h). We construct $(\widetilde{R}_l, \widetilde{R}_h)$ as follows. Starting from $\widetilde{R}_l = R_l$ and $\widetilde{R}_h = R_h$, we continuously increase $\widetilde{R}_l$ and decrease $\widetilde{R}_h$ so that condition (14) in Claim 2 holds. From Claim 2, we then know that $s^*(\widetilde{R}_l, \widetilde{R}_h) < s^*(R_l, R_h)$, while $s^*(\widetilde{R}_l, \widetilde{R}_h)$ decreases continuously as we increase $\widetilde{R}_l$ and decrease $\widetilde{R}_h$. We continue to increase $\widetilde{R}_l$ and decrease $\widetilde{R}_h$ until one of the following two conditions is satisfied. Either the borrower's wealth constraint binds, i.e., $\widetilde{R}_l = x_l + w$ (Case 2i), or it holds that $s^*(\widetilde{R}_l, \widetilde{R}_h) = s_{FB}$ (Case 2ii).[32]

We now show that the local lender strictly prefers $(\widetilde{R}_l, \widetilde{R}_h)$ to (R_l, R_h) and that $(\widetilde{R}_l, \widetilde{R}_h)$ satisfies the borrower's participation constraint. The first claim is obvious. The local lender's expected payoff under $(\widetilde{R}_l, \widetilde{R}_h)$ is

$$\begin{aligned}
& \int_{s^*(\widetilde{R}_l, \widetilde{R}_h)}^{1} [U_s(\widetilde{R}_l, \widetilde{R}_h) - k] f(s) ds \\
= & \int_{s^*(\widetilde{R}_l, \widetilde{R}_h)}^{s^*(R_l, R_h)} [U_s(\widetilde{R}_l, \widetilde{R}_h) - k] f(s) ds + \int_{s^*(R_l, R_h)}^{1} [U_s(\widetilde{R}_l, \widetilde{R}_h) - k] f(s) ds \\
> & \int_{s^*(R_l, R_h)}^{1} [U_s(R_l, R_h) - k] f(s) ds,
\end{aligned}$$

which follows from $s^*(\widetilde{R}_l, \widetilde{R}_h) < s^*(R_l, R_h)$, the fact that $(\widetilde{R}_l, \widetilde{R}_h)$ and (R_l, R_h) satisfy (14), implying that $\int_{s^*(R_l, R_h)}^{1} [U_s(R_l, R_h) - k] f(s) ds = \int_{s^*(R_l, R_h)}^{1} [U_s(\widetilde{R}_l, \widetilde{R}_h) - k] f(s) ds$, and the fact that $U_s(\widetilde{R}_l, \widetilde{R}_h) > k$ for all $s > s^*(\widetilde{R}_l, \widetilde{R}_h)$ from the definition of $s^*(\widetilde{R}_l, \widetilde{R}_h)$.

It remains to show that $(\widetilde{R}_l, \widetilde{R}_h)$ satisfies the borrower's participation constraint (3). Since (R_l, R_h) satisfies (3) by construction—it is assumed to be the optimal contract—and $(\widetilde{R}_l, \widetilde{R}_h)$ satisfies (14), this is true if $V_s(\widetilde{R}_l, \widetilde{R}_h) \geq 0$ for all $s \in [s^*(\widetilde{R}_l, \widetilde{R}_h), s^*(R_l, R_h))$. To see that this condition holds, note first that $V_{s^*(\widetilde{R}_l, \widetilde{R}_h)}(\widetilde{R}_l, \widetilde{R}_h) \geq 0$ since $U_{s^*(\widetilde{R}_l, \widetilde{R}_h)}(\widetilde{R}_l, \widetilde{R}_h) = k$ from the

[32] In Case 2i it holds that $s^*(\widetilde{R}_l, \widetilde{R}_h) \geq s_{FB}$. It does not matter if we subsume the case where $\widetilde{R}_l = x_l + w$ and $s^*(\widetilde{R}_l, \widetilde{R}_h) = s_{FB}$ hold jointly under Case 2i or Case 2ii.

definition of $s^*(\widetilde{R}_l, \widetilde{R}_h)$ and $\mu_{s^*(\widetilde{R}_l,\widetilde{R}_h)} \geq k$ due to $s^*(\widetilde{R}_l, \widetilde{R}_h) \geq s_{FB}$. It therefore suffices to show that $V_s(\widetilde{R}_l, \widetilde{R}_h)$ is nondecreasing in s. Given that p_s is increasing in s, this is true if

$$x_h - \widetilde{R}_h \geq x_l - \widetilde{R}_l. \tag{17}$$

To see that (17) holds, consider first Case 2i where $\widetilde{R}_l = x_l + w$. Since $(\widetilde{R}_l, \widetilde{R}_h)$ satisfies (14) and (R_l, R_h) satisfies (3), the fact that $\widetilde{R}_l = x_l + w$ necessarily implies that $\widetilde{R}_h < x_h$, which in turn implies that (17) holds with strict inequality. Consider next Case 2ii where $s^*(\widetilde{R}_l, \widetilde{R}_h) = s_{FB}$ (while $\widetilde{R}_l \leq x_l + w$), implying that $U_{s_{FB}}(\widetilde{R}_l, \widetilde{R}_h) = \mu_{s_{FB}}$. If it was true that $\widetilde{R}_h - \widetilde{R}_l \geq x_h - x_l$, we would have $U_s(\widetilde{R}_l, \widetilde{R}_h) \geq \mu_s$ for all $s \geq s_{FB}$ and hence also for all $s \geq s^*(R_l, R_h)$, violating the fact that $(\widetilde{R}_l, \widetilde{R}_h)$ satisfies (14) in conjunction with the fact that (R_l, R_h) satisfies (3). It must consequently be true that $\widetilde{R}_h - \widetilde{R}_l < x_h - x_l$, implying that (17) holds again with strict inequality.

Finally, since $R_l = x_l + w$, the repayment in the high cash-flow state is uniquely pinned down: it is the maximum feasible value of R_h at which the borrower's participation constraint (3) binds. (Existence follows from continuity of all payoffs in R_h.)

Case 3. We finally show that it cannot be true that $s^*(R_l, R_h) < s_{FB}$. Since the argument is analogous to that in Case 2, we shall be brief. Suppose to the contrary that $s^*(R_l, R_h) < s_{FB}$. By Claim 2, we can then construct a new contract $(\widetilde{R}_l, \widetilde{R}_h)$ with $\widetilde{R}_l < R_l$ and $\widetilde{R}_h > R_h$ such that (14) holds while $s^*(R_l, R_h) < s^*(\widetilde{R}_l, \widetilde{R}_h) \leq s_{FB}$. (In fact, as (R_l, R_h) is feasible by construction, it is immediate that $s^*(R_l, R_h) < s_{FB}$ implies the existence of a contract $(\widetilde{R}_l, \widetilde{R}_h)$ with $s^*(\widetilde{R}_l, \widetilde{R}_h) = s_{FB}$.) By construction, the local lender is again strictly better off under $(\widetilde{R}_l, \widetilde{R}_h)$, while by (14) the borrower is not worse off if $V_s(R_l, R_h) \leq 0$ under the original contract for all $s \in [s^*(R_l, R_h), s^*(\widetilde{R}_l, \widetilde{R}_h))$. But this follows immediately as $\mu_s < k$ and $U_s(R_l, R_h) > k$ for all $s^*(R_l, R_h) < s < s_{FB}$.

Summing up, we have shown that the optimal contract (R_l, R_h) is unique, that it satisfies $x_l < R_l < R_h < x_h$, and that $s^*(R_l, R_h) \geq s_{FB}$. In the second-best case $s^*(R_l, R_h) > s_{FB}$ we have additionally shown that $R_l = x_l + w$. The fact that the second-best case applies whenever $w < C_{FB}$ follows immediately from the construction of C_{FB}. Q.E.D.

Proof of Corollary 2. We consider a mean-preserving spread in the project's cash-flow distribution. Denote the cash flows and the success probability after the increase in risk by $\hat{x}_l$, $\hat{x}_h$, and $\hat{p}_s$ for all $s \in S$, where $\hat{x}_l < x_l$ and $\hat{x}_h > x_h$. To preserve the mean, the success probability must change from $p_s = \frac{\mu_s - x_l}{x_h - x_l}$ to $\hat{p}_s = \frac{\mu_s - \hat{x}_l}{\hat{x}_h - \hat{x}_l}$, while s_{FB} remains unchanged. Note that

$$p_s - \hat{p}_s = \frac{\mu_s[(\hat{x}_h - \hat{x}_l) - (x_h - x_l)] + \hat{x}_l x_h - x_l \hat{x}_h}{(x_h - x_l)(\hat{x}_h - \hat{x}_l)} \tag{18}$$

is strictly increasing in s since μ_s is strictly increasing and $\hat{x}_h - \hat{x}_l > x_h - x_l$. Finally, denote the optimal contracts before and after the increase in risk by (r, C) and $(\hat{r}, \hat{C})$ and the associated optimal cutoffs by s^* and $\hat{s}^*$, respectively.

We first consider the case where $s^* = \hat{s}^* = s_{FB}$. In order to implement the first best, it must hold that

$$k(1 + r) = \frac{k - (1 - p_{s_{FB}})(C + x_l)}{p_{s_{FB}}}. \tag{19}$$

Note that $1 - p_s = \frac{x_h - \mu_s}{x_h - x_l}$ and $p_s/p_{s_{FB}} = \frac{\mu_s - x_l}{\mu_{s_{FB}} - x_l}$. Using these expressions together with (19), we obtain

$$U_s = (1 - p_s)(x_l + C) + p_s k(1 + r) = \frac{1}{k - x_l}(\mu_s [k - (x_l + C)] + kC). \tag{20}$$

Note that $[k - (x_l + C)]/(k - x_l)$ is strictly decreasing in both x_l and C. Using (20), the requirement that $s^* = \hat{s}^* = s_{FB}$ translates to

$$\frac{1}{k - x_l}(\mu_{s_{FB}} [k - (x_l + C)] + kC) = \frac{1}{k - \hat{x}_l}(\mu_{s_{FB}}[k - (\hat{x}_l + \hat{C})] + k\hat{C}). \tag{21}$$

Moreover, in order for the borrower's participation constraint to hold with equality both before and after the increase in risk, the local lender's expected payoff must satisfy

$$\int_{s_{FB}}^{1} \left[\frac{1}{k - x_l} \left[\mu_s [k - (x_l + C)] + kC\right]\right] f(s)ds \tag{22}$$
$$= \int_{s_{FB}}^{1} \left[\frac{1}{k - \hat{x}_l}[\mu_s[k - (\hat{x}_l + \hat{C})] + k\hat{C}]\right] f(s)ds.$$

As μ_s is strictly increasing, (21) and (22) can be jointly satisfied only if

$$\frac{k - (x_l + C)}{k - x_l} = \frac{k - (\hat{x}_l + \hat{C})}{k - \hat{x}_l},$$

which—given that $\hat{x}_l < x_l$—implies that $\hat{C} > C$.

We next consider the case where $s^* > s_{FB}$ and $\hat{s}^* > s_{FB}$, implying that $C = \hat{C} = w$ by Proposition 2. We show that this implies that $\hat{s}^* > s^*$. We argue to a contradiction and assume that $\hat{s}^* \leq s^*$. Consider the contract $(\widetilde{r}, w)$ which prior to the increase in risk implements $\widetilde{s}^* = \hat{s}^*$, i.e.,

$$p_{\hat{s}^*}[k(1 + \widetilde{r}) - w - x_l] + (w + x_l) = \hat{p}_{\hat{s}^*}[k(1 + \hat{r}) - w - \hat{x}_l] + (w + \hat{x}_l). \tag{23}$$

Using the definition of μ_s, we can rewrite (23) as

$$p_{\hat{s}^*}[x_h - k(1+\widetilde{r}) + w] - w = \hat{p}_{\hat{s}^*}[\hat{x}_h - k(1+\hat{r}) + w] - w. \quad (24)$$

We will now show that under $(\widetilde{r}, w)$ the borrower's participation constraint would be slack. Consequently, the local lender could increase $\widetilde{r}$, thereby pushing $\widetilde{s}^*$ strictly below $\hat{s}^*$—and since $\hat{s}^* \le s^*$ also strictly below s^*—until (3) binds.[33] But this would imply that prior to the increase in risk there existed a contract that satisfies the borrower's participation constraint and implements a strictly lower cutoff than (r, w), contradicting the optimality of (r, w). The borrower's participation constraint is slack under $(\widetilde{r}, w)$ if

$$\begin{aligned} & \int_{\hat{s}^*}^{1} [p_s(x_h - k(1+\widetilde{r}) + w) - w] f(s) ds \\ > & \int_{\hat{s}^*}^{1} [\hat{p}_s(\hat{x}_h - k(1+\hat{r}) + w) - w] f(s) ds = \overline{V}, \end{aligned} \quad (25)$$

where the equality follows from the fact that the participation constraint binds under $(\hat{r}, w)$. But (25) is implied by (24) and the fact that $p_s - \hat{p}_s$ is strictly increasing in s.

Finally, note that from (5) we have that $\hat{C}_{FB} > C_{FB}$ due to $\hat{x}_l < x_l$. Hence, the only remaining case is where $C = \hat{C} = w$ and $\hat{s}^* > s^* = s_{FB}$. Q.E.D.

Proof of Proposition 4. We first prove an auxiliary result.

Claim. *Take any contract (R_l, R_h) with $R_l = x_l + w$ and $R_h > R_l$ and a different contract $(\widetilde{R}_l, \widetilde{R}_h) \neq (R_l, R_h)$ satisfying $V_s(\widetilde{R}_l, \widetilde{R}_h) \le V_s(R_l, R_h)$ for some $s = \widetilde{s} < 1$. Then it holds that $V_s(\widetilde{R}_l, \widetilde{R}_h) < V_s(R_l, R_h)$ (and therefore that $U_s(\widetilde{R}_l, \widetilde{R}_h) > U_s(R_l, R_h)$) for all $s > \widetilde{s}$.*

Proof. We can rewrite the condition that $V_s(\widetilde{R}_l, \widetilde{R}_h) \le V_s(R_l, R_h)$ at $s = \widetilde{s}$ as

$$(\widetilde{R}_l - R_l) + p_s[(\widetilde{R}_h - \widetilde{R}_l) - (R_h - R_l)] \ge 0. \quad (26)$$

Since $R_l = x_l + w$ we have that $\widetilde{R}_l - R_l \le 0$. Hence, for (26) to hold it must be true that $\widetilde{R}_h - \widetilde{R}_l \ge R_h - R_l$. There are two cases: i) if $\widetilde{R}_l = R_l$, then (26) and $\widetilde{R}_h \neq R_h$ together imply that $\widetilde{R}_h > R_h$ and therefore that $(\widetilde{R}_h - \widetilde{R}_l) - (R_h - R_l) > 0$; ii) if $\widetilde{R}_l < R_l$ it follows directly from (26) that $(\widetilde{R}_h - \widetilde{R}_l) - (R_h - R_l) > 0$. Given that p_s is strictly increasing in s, this implies in either case it holds that $V_s(\widetilde{R}_l, \widetilde{R}_h) < V_s(R_l, R_h)$ for all $s > \widetilde{s}$. Q.E.D.

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[33] Existence of such a contract follows from continuity of the borrower's payoff in $\widetilde{r}$. Note that we need only consider a marginal adjustment, thereby ensuring that the resulting cutoff does not fall below s_{FB}.

We can restrict ourselves to the case $s^* > s_{FB}$ which, by Proposition 2, implies that $R_l = x_l + w$. Suppose first the borrower makes a new offer $(\widetilde{R}_l, \widetilde{R}_h)$. For this offer to be profitable for the borrower, it must hold that $s^*(\widetilde{R}_l, \widetilde{R}_h) \leq s^*(R_l, R_h)$. By the definition of $s^*(\widetilde{R}_l, \widetilde{R}_h)$, this implies that $k = U_{s^*(\widetilde{R}_l,\widetilde{R}_h)}(\widetilde{R}_l, \widetilde{R}_h) > U_{s^*(\widetilde{R}_l,\widetilde{R}_h)}(R_l, R_h)$ and therefore that $V_{s^*(\widetilde{R}_l,\widetilde{R}_h)}(\widetilde{R}_l, \widetilde{R}_h) \leq V_{s^*(\widetilde{R}_l,\widetilde{R}_h)}(R_l, R_h)$. By Claim 1, it then follows that $V_s(\widetilde{R}_l, \widetilde{R}_h) < V_s(R_l, R_h)$ and therefore that $U_s(\widetilde{R}_l, \widetilde{R}_h) > U_s(R_l, R_h)$ for all $s > s^*(\widetilde{R}_l, \widetilde{R}_h)$. Hence, the local lender strictly prefers $(\widetilde{R}_l, \widetilde{R}_h)$ to (R_l, R_h) for all $s > s^*(\widetilde{R}_l, \widetilde{R}_h)$ and thus even for values $s \geq s^*(R_l, R_h)$ for which she would have accepted the borrower under the original contract (R_l, R_h). In consequence, the local lender's new expected payoff is $U(\widetilde{R}_l, \widetilde{R}_h) > U(R_l, R_h)$, while the borrower's new expected payoff is $V(\widetilde{R}_l, \widetilde{R}_h)$. As (R_l, R_h) maximizes the local lender's expected payoff subject to leaving the borrower exactly $\overline{V}$, this immediately implies that the borrower's expected payoff under $(\widetilde{R}_l, \widetilde{R}_h)$ is $V(\widetilde{R}_l, \widetilde{R}_h) < V(R_l, R_h) = \overline{V}$, which in turn implies offering $(\widetilde{R}_l, \widetilde{R}_h)$ cannot be profitable for the borrower. The argument for the case in which the local lender makes a new offer, which results in a signaling game, is analogous. Q.E.D.

Proof of Proposition 5. Before we can prove Proposition 5, we must first verify that some key results from our base model extend to the current setting. We first extend the argument from Lemma 2.

Claim 1. *For any $q > 0$, borrowers who are initially attracted by the local lender but rejected after the project evaluation cannot obtain financing elsewhere.*

Proof. Recall that it is now only with probability $q > 0$ that the local lender has a more precise success probability estimate p_s (i.e., she "observes s"), while with probability $1 - q$ her success probability estimate is the same as that of the transaction lenders, namely, p. Moreover, our assumption that only the local lender knows her own success probability estimate implies that only she knows whether her actual success probability estimate is more precise.

The expected NPV of a project that has been rejected by the local lender depends, among other things, on the local lender's decision if she does *not* observe s. As can be easily shown, if in this case the local lender is indifferent between accepting and rejecting, then under the optimal contract she must accept with probability one. To see this, note first that $\int_0^1 U_s(R_l, R_h) f(s) ds = k$, where substituting $U_s(R_l, R_h) = \mu_s - V_s(R_l, R_h)$ yields $\int_0^1 V_s(R_l, R_h) f(s) ds = \mu - k > 0$. Suppose now that under the optimal contract (R_l, R_h) the local lender randomizes between accepting and rejecting if she is indifferent and does not observe s. But since $\int_0^1 V_s(R_l, R_h) f(s) ds > 0$, the borrower's participation constraint could be relaxed if the local lender accepted with probability one, implying there exists another contract with a higher repayment that satisfies the borrower's participation constraint with equality while making the local lender strictly better off, contracting the optimality of (R_l, R_h).

We first consider the case in which the local lender accepts the borrower if she does not observe s. We argue to a contradiction and assume a borrower who is initially attracted by the local lender but then rejected can obtain funding from transaction lenders, i.e., $\int_0^{s^*}(\mu_s - k)\frac{f(s)}{F(s^*)}ds > 0$. The borrower's expected payoff is then

$$q\left[\int_{s^*}^{1}[\mu_s - U_s(R_l, R_h)]f(s)ds + \int_0^{s^*}(\mu_s - k)f(s)ds\right] \tag{27}$$
$$+(1-q)\int_0^1[\mu_s - U_s(R_l, R_h)]f(s)ds.$$

The requirement that (27) be greater than or equal to $\mu - k > 0$ transforms to

$$q\int_{s^*}^{1}[k - U_s(R_l, R_h)]f(s)ds \geq (1-q)\int_0^1[U_s(R_l, R_h) - k]f(s)ds. \tag{28}$$

But the fact that the local lender accepts the borrower if she does not observe s implies that $\int_0^1 U_s(R_l, R_h)f(s)ds \geq k$, which in conjunction with $U_s(R_l, R_h) > k$ for all $s > s^*$ violates (28).

We next consider the other case in which the local lender accepts the borrower only if she observes s and if $s \geq s^*$. Again, we argue to a contradiction and assume that

$$\frac{(1-q)(\mu - k) + q\int_0^{s^*}(\mu_s - k)f(s)ds}{(1-q) + qF(s^*)} > 0, \tag{29}$$

i.e., the expected NPV conditional on being rejected by the local lender is positive, implying a rejected borrower can obtain funding from transaction lenders. The borrower's expected payoff is then

$$q\int_{s^*}^{1}[\mu_s - U_s(R_l, R_h)]f(s)ds + (1-q)(\mu - k) + q\int_0^{s^*}(\mu_s - k)f(s)ds. \tag{30}$$

The requirement that (30) be greater than or equal to $\mu - k > 0$ transforms to

$$\int_{s^*}^{1}[U_s(R_l, R_h) - k]f(s)ds < 0,$$

which is again violated as $U_s(R_l, R_h) > k$ for all $s > s^*$. Q.E.D.

Given Claim 1, the borrower's expected payoff in case the local lender accepts him if she does not observe s is

$$V(R_l, R_h) := q\int_{s^*}^{1}V_s(R_l, R_h)f(s)ds + (1-q)\int_0^1 V_s(R_l, R_h)f(s)ds, \tag{31}$$

while his expected payoff in case the local lender rejects him if she does not observe s is

$$V(R_l, R_h) := q \int_{s^*}^{1} V_s(R_l, R_h) f(s) ds. \tag{32}$$

The local lender's problem is to maximize

$$U(R_l, R_h) := q \int_{s^*}^{1} [U_s(R_l, R_h) - k] f(s) ds + (1-q) \max\{0, \int_{0}^{1} [U_s(R_l, R_h) - k] f(s) ds\}, \tag{33}$$

which incorporates the optimality of her subsequent credit decision, subject to the borrower's participation constraint $V(R_l, R_h) \geq \overline{V} = \mu - k$. By standard arguments, the borrower's participation constraint must again bind at the optimum.

We now show that the results from Propositions 1 and 2—which characterize the optimal contract if the borrower's participation constraint can be satisfied—extend to the current setting.

Claim 2. *Propositions 1 and 2 carry over to the extended model in Section 5 with the single modifcation that the definition of C_{FB} changes to*

$$C_{FB} = \frac{(k - x_l)(\mu - k)}{q \int_{s_{FB}}^{1} (\mu_s - k) f(s) ds + (1-q)(\mu - k)}. \tag{34}$$

Proof. We begin with the definition of C_{FB}. Note first that we can again use the fact that

$$V_s(R_l, R_h) = C_{FB} \frac{\mu_s - k}{k - x_l}, \tag{35}$$

which is obtained by substituting $s^* = s_{FB}$ and the definition of $p_{s_{FB}}$ in (1). Observe next that given that $\mu - k > 0$ we have that $p_{s_{FB}} > \int_0^1 p_s f(s) ds$ such that $U_{s_{FB}}(R_l, R_h) = k$ implies $\int_0^1 U_s(R_l, R_h) f(s) ds > k$. In words, if the first best is attainable, then the local lender accepts the borrower if she does not observe s. We then obtain (34) from the borrower's binding participation constraint, where we use $V(R_l, R_h)$ as defined in (31). Finally, from $s^* = s_{FB}$ the repayment in the good state $R_h = k(1 + r_{FB})$ is still uniquely determined by (6), though we can now substitute C_{FB} from (34). Accordingly, for all $w \geq C_{FB}$ the optimal contract is unique and implements the first-best credit decision.

We next turn to the case where the first best is not attainable, i.e., $s^*(R_l, R_h) > s_{FB}$. Here the key argument in the Proof of Propositions 1 and 2 was that if the (alleged) optimal contract (R_l, R_h) does not have the properties asserted in the propositions, then one can construct a "flatter" contract $(\widetilde{R}_l, \widetilde{R}_h)$ that also satisfies the borrower's participation constraint while making the local lender strictly better off. Recall from Claim 1 that if the lender is indifferent between accepting and rejecting if she does not observe s, then under the optimal contract (R_l, R_h) she must accept with probability one. This in turn implies that if the local lender rejects the borrower under the optimal contract (R_l, R_h) in case she does not observe s, then she must strictly prefer to do so. Since all payoffs are continuous, it is then straightforward to show that the arguments in the Proof of Propositions 1 and 2 fully extend to the current case. (In fact, we only need to multiply all expected payoffs by q.)

We next consider the case where under (R_l, R_h) the local lender accepts the borrower if she does not observe s. We show that the arguments from the Proof of Proposition 1 and 2 also extend to this case, albeit with some minor modifications. Note first that Claim 1 clearly extends as it only concerns $V_s(\cdot)$. We next show that Claim 2 also extends. Given some (R_l, R_h) with $R_l < x_l + w$, we choose $(\widetilde{R}_l, \widetilde{R}_h)$ with $\widetilde{R}_l > R_l$ and $\widetilde{R}_h < R_h$ such that

$$
\begin{aligned}
& q \int_{s^*(R_l,R_h)}^{1} \left[V_s(\widetilde{R}_l, \widetilde{R}_h) - V_s(R_l, R_h) \right] f(s) ds \\
& +(1-q) \int_0^1 \left[V(\widetilde{R}_l, \widetilde{R}_h) - V_s(R_l, R_h) \right] f(s) ds = 0.
\end{aligned} \tag{36}
$$

In words, *if* under $(\widetilde{R}_l, \widetilde{R}_h)$ the cutoff s^* remains unchanged and the borrower is accepted if the local lender does not observe s, then the borrower's expected payoff under $(\widetilde{R}_l, \widetilde{R}_h)$ is the same as under (R_l, R_h). We next now show that $s^*(\widetilde{R}_l, \widetilde{R}_h) < s^*(R_l, R_h)$. Clearly, this is true if

$$
p_{s^*(R_l,R_h)} [(R_h - R_l) - (\widetilde{R}_h - \widetilde{R}_l)] - (\widetilde{R}_l - R_l) < 0,
$$

which—substituting from (36) and using the fact that $\widetilde{R}_l - R_l > 0$ and $R_h - R_l > \widetilde{R}_h - \widetilde{R}_l$—holds if

$$
p_{s^*(R_l,R_h)} < q \int_{s^*(R_l,R_h)}^{1} p_s f(s) ds + (1-q) \int_0^1 p_s f(s) ds. \tag{37}
$$

Given that $U_s(R_l, R_h)$ is strictly increasing in s and $U_{s^*(R_l,R_h)}(R_l, R_h) = k$, the fact that $\int_0^1 U_s(R_l, R_h) f(s) ds \geq k$ (as the local lender accepts the borrower if she does not observe s) implies that $\int_0^1 p_s f(s) ds \geq p_{s^*(R_l,R_h)}$. Together with the fact that p_s is strictly increasing, this implies (37). It remains to show that under $(\widetilde{R}_l, \widetilde{R}_h)$ it is indeed true that the local lender accepts the borrower if she does not observe s. Given that it is true under (R_l, R_h), it certainly true if $\int_0^1 U_s(\widetilde{R}_l, \widetilde{R}_h)) f(s) ds > \int_0^1 U_s(R_l, R_h) f(s) ds$, i.e., if

$$
[(R_h - R_l) - (\widetilde{R}_h - \widetilde{R}_l)] \int_0^1 p_s f(s) ds < \widetilde{R}_l - R_l,
$$

which is implied by (37).

Having extended Claim 2 from the Proof of Propositions 1 and 2, the rest of the argument is straightforward. It is easy to show that under the optimal contract it cannot be the case that

$s^* < s_{FB}$. The argument in Case 2 of the Proof of Propositions 1 and 2 where $s^*(R_l, R_h) > s_{FB}$ proceeds again by contradiction, using the usual construction of a "flatter" contract $(\widetilde{R}_l, \widetilde{R}_h)$. The only deviation from the original argument is that now $(\widetilde{R}_l, \widetilde{R}_h)$ must satisfy the modified requirement in (36). Q.E.D.

We can now finally prove Proposition 5. The local lender is able to attract the borrower if there exists a contract such that the borrower's expected payoff $V(R_l, R_h)$ is at least $\overline{V} = \mu - k$. Formally, the local lender can attract the borrower if and only if

$$\max_{R_l, R_h} V(R_l, R_h) \geq \mu - k. \tag{38}$$

We first show that the left-hand side of (38) is strictly increasing in q, which establishes the existence of a cutoff $\widehat{q} \in [0, 1]$. Clearly, this is the case if, holding (R_l, R_h) fixed, $V(R_l, R_h)$ is increasing in q. If $V(R_l, R_h)$ is determined by (32), this is obviously true. Suppose next that $V(R_l, R_h)$ is determined by (31). Differentiating $V(R_l, R_h)$ with respect to q shows that the borrower's expected payoff is increasing in q if

$$\int_0^{s^*} V_s(R_l, R_h) f(s) ds \leq 0. \tag{39}$$

Using the fact that $k \geq \mu - V(R_l, R_h)$ in conjunction (31), the condition that the local lender accepts the borrower if she does not observe s transforms to $q \int_0^{s^*} V_s(R_l, R_h) f(s) ds \leq 0$, which in turn implies that (39) is satisfied.

We next consider how the uniquely optimal contract varies with q for $q \geq \widehat{q}$. From the definition of C_{FB} in (34) we know that if $s^* = s_{FB}$ is feasible for some $q' < 1$, then it is also feasible for all higher $q > q'$. Moreover, by (34) the corresponding optimal contract prescribes for all $q > q'$ a strictly lower collateral requirement C_{FB}, which must be matched by an increase in r_{FB} to preserve $s^* = s_{FB}$.

We finally consider the case where the first best cannot be attained. From our previous arguments, we know that as q increases conditional on $q \geq \widehat{q}$, the borrower's expected payoff must, for any given contract, increase correspondingly irrespective of whether the local lender accepts or rejects the borrower if she does not observe s. If under the previously optimal contract the local lender (at least) weakly prefers to accept the borrower if she does not observe s, then following a marginal increase in r she does so strictly. From continuity of the borrower's expected payoff, this implies that following an increase in q, the local lender optimally raises the loan rate r (while leaving $C = w$ since $s^* > s_{FB}$). The argument is the same if under the previously optimal contract the local lender strictly prefers to reject the borrower if she does not observe s, implying she still does so after a marginal increase in r. Q.E.D.

Proof of Proposition 6. It is straightforward to extend the argument from the Proof of Proposition 5 to show that

$$C_{FB} := \frac{(k - x_l)(\mu - k - \kappa)}{q \int_{s_{FB}}^{1} (\mu_s - k) f(s) ds + (1-q)(\mu - k)}, \tag{40}$$

which is strictly decreasing in κ.[34] Consequently, if the competitive pressure from transaction lenders increases (lower κ), the optimal collateral requirement increases accordingly. Of course, r_{FB} must decrease. If the first best cannot be attained, implying that $C = w$, it follows immediately that r must decrease, which in turn implies s^* must increase.

As for the second part of the claim, differentiating (40) with respect to q and κ, while noting that $\int_{s_{FB}}^{1} (\mu_s - k) f(s) ds > \mu - k$ from the definition of s_{FB}, yields $d^2 C_{FB}/(dq d\kappa) > 0$. Of course, if the first best is unattainable we invariantly have that $C = w$. Q.E.D.

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[34] This condition is obtained from inserting $V(R_l, R_h) = q \int_{s^*}^{1} V_s(R_l, R_h) f(s) ds + (1-q) \int_0^1 V_s(R_l, R_h) f(s) ds$ and $V_s(R_l, R_h) = C_{FB} \frac{\mu_s - k}{k - x_l}$, where the latter condition follows from $s^* = s_{FB}$, into the binding participation constraint $V(R_l, R_h) = \mu - k - \kappa$.

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