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CLUBS AND HOUSEHOLDS

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ABSTRACT

Clubs and Households*

The relationship between our general equilibrium model with multimember households and club models with multiple private goods is investigated. The main distinction in the definitions consists of the equilibrium concepts. As a rule, competitive equilibria among households where no group of consumers can benefit from forming a new household and valuation equilibria prove equivalent in the absence of consumption externalities, but not in their presence. We provide several examples and applications.

JEL Classification: D13, D50, D62 and D71
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1 Introduction

The aim of this inquiry is to clarify the relationship between our general equilibrium model with multi-member households and club models with multiple private goods.[1] The traditional general equilibrium model of a pure exchange economy has treated households as if they were single consumers. The distinction between a household and its members potentially leads to inquiries into household decisions, household formation, household stability, the interaction between the competitive market allocation of private goods and household formation — and to a host of related modeling issues. Household decisions have been widely studied in the empirically oriented literature. Of particular interest for our purposes is the contribution of Chiappori (1988, 1992) who introduced a model of collective rationality (efficient consumption decisions) of multi-member households. Household formation or, more generally, group formation is the main subject of the literature on matching, assignment games, and hedonic coalitions.

One would expect that the prevailing household structure, that is, the partition of the population into households, and the decision criteria of households affect the allocation of resources among consumers. Conversely, economic considerations are likely to influence decisions to form or dissolve households. Therefore, we aim to develop a formal framework that integrates three allocation mechanisms operating at different levels of aggregation. First, individual decisions are made to join or leave households. Second, collective decisions within households determine the consumption plans of household members. Third, competitive exchange across households yields a feasible allocation of resources. By and large, the literature on matching, assignment games, and hedonic coalitions has not achieved this integration.[2] It assumes at most

[1] See in particular Cole and Prescott (1997), Ellickson (1979), Ellickson, Grodal, Scotchmer, and Zame (1999, 2001), Gilles and Scotchmer (1997, 1998), Wooders (1988, 1989, 1997).

[2] A noteworthy exception are Drèze and Greenberg (1980) who combine the concepts of individual stability and price equilibrium, but confine the analysis of their most com-

one private good. Thus, it has no use for competitive commodity markets and cannot investigate the interaction between household formation and the competitive market allocation of private goods.

In Haller (2000) and Gersbach and Haller (2001) we take a first step and incorporate the collective rationality concept of Chiappori (1988, 1992) into a general equilibrium framework. This setting has allowed us to study the interaction between two of the three allocation mechanisms: collective decisions and competitive markets. Haller (2000) assumes an exogenously given household structure. Every household member consumes an individual bundle of private goods and has individual preferences. Preferences permit positive or negative intra-household externalities: Individual welfare can be affected by own consumption and the consumption of fellow household members. Gersbach and Haller (2001) introduce a variable household structure, with household specific preferences: An individual cares about who belongs to her household and who consumes what in her household. Hence, in general, there can be group externalities (related to household composition) as well as consumption externalities (related to household consumption). An allocation consists of two parts, an allocation of commodities to consumers and an allocation of people into households. In Gersbach and Haller (2004) we take further steps towards an endogenous household structure by amending the equilibrium conditions with stability requirements known from the matching literature.

Henceforth, we shall use "household model" as generic term for the kind of models developed and analyzed in Gersbach and Haller (2004) and use "club model" as generic term for a sophisticated club model which also allows for endogenous group formation and competitive market allocation of (multiple) private goods. For the sake of direct comparison, we rule out multiple club memberships and abstract club projects, features which could be incorporated in a refined household model. Then the distinguishing feature of the

prehensive model to an instructive example.

club model is that individuals shop for both club memberships and private consumption. This means that club memberships are priced via admission fees or valuations. Each person makes optimal choices, given her individual budget constraint. In equilibrium, prices are such that markets for memberships and markets for commodities clear. In the household model, actual households make collective consumption decisions for their members, subject to a household budget constraint. In equilibrium, nobody wants to exercise an outside option (like becoming single) at the prevailing market prices and commodity markets clear. In both models, the outcome is an allocation of commodities to individual consumers and a partition of the population into clubs or households. The two models can be considered equivalent, if they yield the same equilibrium outcomes.

A priori, the only difference between the two models lies in the equilibrium concepts. We are going to show that in essence, the two models are equivalent in the absence of consumption externalities. To be precise, the most stringent equilibrium concept for the household model, a competitive equilibrium where no group of consumers can benefit from forming a new household, and the standard equilibrium concept for club models, valuation equilibrium, coincide in the absence of consumption externalities. Moreover, the equilibrium outcomes belong to the strong core, which is a rather general property of valuation equilibria. We further show by means of an example that the equivalence breaks down in the presence of consumption externalities. In the example, a valuation equilibrium allocation which is Pareto-optimal does not exist. However, there exists a competitive equilibrium where no group of consumers can benefit from forming a new household and the equilibrium allocation constitutes a strong Pareto-optimum. Finally, we demonstrate that this allocation can be sustained as valuation equilibrium outcome if club contracts can stipulate restrictions on private consumption of club members.

2 The Basic Model

Here we present the primitive data underlying both the household model and the club model. We consider a finite pure exchange economy with variable household structure. There is a finite and non-empty set of individuals or consumers, I. A (potential) household is any non-empty subset h of the population I. $\mathcal{H} = \{h \subseteq I | h \neq \emptyset\}$ denotes the set of all potential households. For $i \in I$, $\mathcal{H}_i = \{h \subseteq I | i \in h\}$ denotes the set of all potential households which have i as a member. Variable household structure means that household membership is an endogenous outcome. The households that actually form give rise to a **household structure** P, that is, a partition of the population I into non-empty subsets. If P is the prevailing household structure and $i \in I$ is a consumer, then let $P(i)$ denote the unique element of P (unique household in P) to which i belongs.

Commodities. There exists a finite number $\ell \geq 1$ of commodities. Thus the commodity space is $\mathbb{R}^\ell$. Each commodity is formally treated as a private good, possibly with externalities in consumption. Consumer $i \in I$ has consumption set $X_i = \mathbb{R}^\ell_+$ so that the **commodity allocation space** is $\mathcal{X} \equiv \prod_{j \in I} X_j$. Generic elements of $\mathcal{X}$ are denoted $\mathbf{x} = (x_i)$, $\mathbf{y} = (y_i)$. Commodities are denoted by superscripts $k = 1, \ldots, \ell$. For a potential household $h \subseteq I$, $h \neq \emptyset$, set $\mathcal{X}_h = \prod_{i \in h} X_i$, the consumption set for household h. $\mathcal{X}_h$ has generic elements $\mathbf{x_h} = (x_i)_{i \in h}$. If $\mathbf{x} = (x_i)_{i \in I} \in \mathcal{X}$ is a commodity allocation, then consumption for household h is the restriction of $\mathbf{x} = (x_i)_{i \in I}$ to h, $\mathbf{x_h} = (x_i)_{i \in h}$.

Endowments. Every potential household h is endowed with a commodity bundle $\omega_h > 0$. In general, the aggregate or social endowment depends on the prevailing household structure P and equals $\omega_P = \sum_{h \in P} \omega_h$. The social endowment is independent of the household structure if (and only if) the endowment of each household equals the sum of the individual endowments of its members. We call this condition individual property rights.

(IPR) Individual Property Rights: $\omega_h = \sum_{i \in h} \omega_i$ for all $h \in \mathcal{H}$.

Note that if the social endowment is independent of the household structure, then it equals $\omega_S = \sum_{i \in I} \omega_{\{i\}}$.

Allocations. We define an **allocation** of the economy with variable household structure as a pair $(\mathbf{x}; P)$ where $\mathbf{x} \in \mathcal{X}$ is an allocation of commodities and P is a household structure. The allocation is **feasible**, if $\sum_{i \in I} x_i = \omega_P$.

Preferences. Preferences are household-specific. This means that an individual cares only about the composition of and the consumption in its own household. Different household members may exert different externalities upon others. To formally represent **household-specific preferences**, let us denote $\mathcal{X}^* = \bigcup_{h \in \mathcal{H}} \mathcal{X}_h$ and define $\mathcal{A}_i = \{(\mathbf{x_h}; h) \in \mathcal{X}^* \times \mathcal{H} : h \in \mathcal{H}_i, \mathbf{x_h} \in \mathcal{X}_h\}$ for $i \in I$. We assume that each individual $i \in I$ has a utility representation $U_i : \mathcal{A}_i \to \mathbb{R}$. In the following, we are going to consider the special case of

(ACE) Absence of Consumption Externalities: $U_i(\mathbf{x_h}; h) = V_i(x_i; h)$ for $i \in I, (\mathbf{x_h}; h) \in \mathcal{A}_i, \mathbf{x_h} = (x_j)_{j \in h}$.

In this case, individual i cares only about own consumption and household composition. Still, preferences over one's own consumption may change with household composition and, vice versa, preferences over household composition can depend on own consumption. In the separable case, $U_i(\mathbf{x_h}; h) = u_i(x_i) + v_i(h)$ and preferences over own consumption and preferences over household composition are independent. If $v_i \equiv 0$, then the separable case reduces to **absence of externalities**.

Social Welfare. A feasible allocation $(\mathbf{x}; P)$ is a **weak core allocation**, if there do not exist a non-empty subset J of I, a partition Q of J into households and consumption bundles $y_j \in \mathcal{X}_j$ for $j \in J$ such that $\sum_{j \in J} y_j = \sum_{j \in J} \omega_j$ and $U_j(y_{Q(j)}; Q(j)) > U_j(x_{P(j)}; P(j))$ for all $j \in J$. A feasible allocation $(\mathbf{x}; P)$ is **weakly Pareto-optimal**, if there is no feasible allocation $(\mathbf{y}; Q)$ such that $U_j(y_{Q(j)}; Q(j)) > U_j(x_{P(j)}; P(j))$ for all $j \in J$. The definition of **strong core allocation** and **strong Pareto optimum**, respectively, rules out weak improvements by any coalition J or the grand coalition I, respectively.

Define a **state** of the economy as a triple $(p, \mathbf{x}; P)$ such that $p \in \mathbb{R}^{\ell}$ is a price system and $(\mathbf{x}; P) \in \mathcal{X} \times P$ is an allocation, i.e. $\mathbf{x} = (x_i)_{i \in I}$ is an allocation of commodities and P is an allocation of consumers (a household structure, a partition of the population into households). We say that in state $(p, \mathbf{x}; P)$,

(a) consumer i can benefit from exit, if $P(i) \neq \{i\}$ and there exists $y_i \in B_{\{i\}}(p)$ such that $U_i(y_i; \{i\}) > U_i(\mathbf{x_{P(i)}}; P(i))$;

(b) consumer i can benefit from joining another household g, if $g \in P$, $g \neq P(i)$ and there exists $\mathbf{y_{g \cup \{i\}}} \in B_{g \cup \{i\}}(p)$ such that $U_j(\mathbf{y_{g \cup \{i\}}}; \mathbf{g} \cup \{\mathbf{i}\}) > U_j(\mathbf{x_{P(j)}}; P(j))$ for all $j \in \mathbf{g} \cup \{\mathbf{i}\}$;

(c) a group of consumers h can benefit from forming a new household, if $h \notin P$ and there exists $\mathbf{y_h} \in B_h(p)$ such that $U_j(\mathbf{y_h}; h) > U_j(\mathbf{x_{P(j)}}; P(j))$ for all $j \in h$.

3 Competitive Equilibrium among Households

In order to introduce the equilibrium concept, we consider first a household $h \in P$ and a price system $p \in \mathbb{R}^{\ell}$. For $\mathbf{x_h} = (x_i)_{i \in h} \in \mathcal{X}_h$, denote

$$p * \mathbf{x_h} = p \cdot \left(\sum_{i \in h} x_i \right),$$

the household's aggregate expenditure. Then h's **budget set** is defined as

$$B_h(p) = \{\mathbf{x_h} \in \mathcal{X}_h \ : \ p * \mathbf{x_h} \leq p \cdot \omega_h\}.$$

We next define the **efficient budget set** $EB_h(p)$ as the set of $\mathbf{x_h} \in B_h(p)$ with the property that there is no $\mathbf{y_h} \in B_h(p)$ such that

$U_i(\mathbf{y_h}; h) \geq U_i(\mathbf{x_h}; h)$ for all $i \in h$;

$U_i(\mathbf{y_h}; h) > U_i(\mathbf{x_h}; h)$ for some $i \in h$.

The subsequent equilibrium condition 0, $\mathbf{x_h} \in EB_h(p)$, constitutes the most general form of collective rationality of households in the sense of Chiappori (1988, 1992). For single households, it coincides with the standard condition of utility or preference maximization.

Definition 1 *A state* $(p, \mathbf{x}; P)$ *is a* **competitive equilibrium (among households)** *if* $(\mathbf{x}; P)$ *is a feasible allocation and*

0. $\mathbf{x_h} \in EB_h(p)$ *for all* $h \in P$.

Thus in a competitive equilibrium $(p; \mathbf{x}; P)$, each household makes an efficient choice under its budget constraint and markets clear. Households play a dual role: as collective decision making units and as competitive market participants. Efficient choice by the household refers to the individual consumption and welfare of its members, not merely to the aggregate consumption bundle of the household.

Definition 1 as it stands is also applicable in the case of an exogenously given household structure P, as in Haller (2000). The definition needs to be amended to incorporate endogenous household formation and to achieve the integration of three allocation mechanisms, each operating at a particular level of aggregation: Individual decisions are made to join or leave households. Collective decisions within households determine the consumption plans of household members. Competitive exchange across households achieves a feasible allocation of resources.

In our static model, the amendments made to reflect endogenous household formation, the freedom of people to join or leave households, assume the form of stability requirements. Specific requirements are that at the going market prices, no individual should benefit from exit; no consumer should benefit from joining another household; no group of consumers should benefit from forming a new household. The first two requirements combined constitute the weak form of individual stability which is a prominent equilibrium concept in the literature on hedonic coalitions; see, e.g., Drèze and Greenberg (1980). The last requirement that at the going market prices, no group of consumers should benefit from forming a new household is the weak counter-part of the stability concept prevalent in the matching literature; see, e.g., Gale and Shapley (1962), Roth and Sotomayor (1990).

In Gersbach and Haller (2004), we consider competitive equilibria at which no individual benefits from exit and call such equilibria Competitive Equilibria with Free Exit (CEFE). In addition, if no consumer can benefit from joining another household, we call the corresponding equilibria Competitive Equilibria with Free Household Formation (CEFH). In that paper, we establish existence of CEFE and study the welfare properties of CEFE. We address existence of CEFH — which need not always exist — and investigate the welfare implications of strengthening the stability requirement from CEFE to CEFH. In Gersbach and Haller (2005) we observe that the most stringent stability requirement, that in equilibrium no group of consumers can benefit from forming a new household has very strong welfare implications. In particular, weak core inclusion obtains: if at a competitive equilibrium, no group of consumers benefits from forming a new household, then the corresponding equilibrium allocation belongs to the weak core. For the sake of completeness, we provide the short proof of the result.

Proposition 1 (Weak Core Inclusion)
Let $(p, \mathbf{x}; P)$ be a competitive equilibrium at which no group of consumers can benefit from forming a new household. Then $(\mathbf{x}; P)$ belongs to the weak core.

PROOF. Let $(p, \mathbf{x}; P)$ be a competitive equilibrium at which no group benefits from forming a new household. Suppose coalition J can strictly improve upon the allocation $(\mathbf{x}; P)$ by means of a partition Q of J and household consumption plans $\mathbf{y_h}, h \in Q$. Now let $h \in Q$. Then $U_i(\mathbf{y_h}; h) > U_i(\mathbf{x_{P(i)}}; P(i))$ for all $i \in h$. If $h \in P$, then $p * \mathbf{y_h} > p\omega_h$, since $\mathbf{x_h} \in EB_h(p)$. If $h \notin P$, then $p * \mathbf{y_h} > p\omega_h$, since group h cannot benefit from forming a new household. But then $p \sum_{i \in J} y_i = \sum_{i \in J} py_i = \sum_{h \in Q} \sum_{i \in h} py_i = \sum_{h \in Q} p * \mathbf{y_h} > \sum_{h \in Q} p\omega_h = p \sum_{h \in Q} \omega_h$, contradicting $\sum_{i \in J} y_i = \sum_{h \in Q} \omega_h$. Hence no coalition J can strictly improve upon the allocation $(\mathbf{x}; P)$. ■■

If one assumes in addition the budget exhaustion property and the redistribution property of Gersbach and Haller (2001), then weak core inclusion can be replaced by strong core inclusion, that is the assertion that no coalition of consumers can weakly improve upon the allocation $(\mathbf{x}; P)$.

4 Valuation Equilibrium

In accordance with the club literature, Absence of Consumption Externalities (ACE) and Individual Property Rights (IPR) are assumed throughout this section.

Definition 2 *A state* $(p, \mathbf{x}; P)$ *is a* **valuation equilibrium** *if* $(\mathbf{x}; P)$ *is a feasible allocation and there exist admission prices or valuations* $V_i(h)$ *for* $i \in I$, $h \in \mathcal{H}_i$, *such that:*

1. $\sum_{i \in h} V_i(h) = 0$ *for* $h \in P$;
2. $\sum_{i \in h} V_i(h) \leq 0$ *for* $h \in \mathcal{H}$, $h \notin P$;
3. $px_i + V_i(P(i)) = p\omega_i$ *for* $i \in I$;
4. *If* $i \in I$, $h \in \mathcal{H}_i$, $y_i \in \mathcal{X}_i$ *with* $U_i(y_i; h) > U_i(x_i; P(i))$, *then* $py_i + V_i(h) > p\omega_i$.

Lemma 1 *Suppose* $(p, \mathbf{x}; P)$ *is a valuation equilibrium. Then there do not exist a household* $g \in \mathcal{H}$ *and consumption bundles* $y_i \in \mathcal{X}_i$ *for* $i \in g$ *such that* $(y_i)_{i \in g} \in B_g(p)$ *and* $U_i(y_i; g) > U_i(x_i; P(i))$ *for all* $i \in g$.

PROOF. Suppose $(p, \mathbf{x}; P)$ is a valuation equilibrium with admission prices $V_i(h)$ for $i \in I$, $h \in \mathcal{H}_i$, and there exist a household $g \in \mathcal{H}$ and consumption bundles $y_i \in \mathcal{X}_i$ for $i \in g$ such that $U_i(y_i, g) > U_i(x_i, P(i))$ for all $i \in g$. Then by equilibrium condition 4, $py_i + V_i(g) > p\omega_i$ for all $i \in g$. Hence, by equilibrium conditions 1 and 2, $\sum_{i \in g} py_i > \sum_{i \in g} p\omega_i$. Therefore, $(y_i)_{i \in g} \notin B_g(p)$. ■■

Proposition 2 *Let each* U_i *be continuous and strictly monotone in* $x_i \in X_i$. *Suppose* $(p, \mathbf{x}; P)$ *is a valuation equilibrium. Then:*

(i) $(p, \mathbf{x}; P)$ *is a competitive equilibrium at which no group of consumers can benefit from forming a new household.*

(ii) $(\mathbf{x}; P)$ *is a strong core allocation.*

PROOF. Let U_i, $i \in I$, be as hypothesized.

(i) Suppose $(p, \mathbf{x}; P)$ is a valuation equilibrium. Then $(\mathbf{x}; P)$ is a feasible allocation. If $h \in P$ and $\mathbf{x_h} \notin EB_h(p)$, then there exist $j \in h$ and $\mathbf{z_h} \in B_h(p)$ with $U_j(z_j; h) > U_j(x_j; h)$ and $U_i(z_i; h) \geq U_i(z_; h)$ for $i \in h$, $i \neq j$. In case $h = \{j\}$, set $y_j = z_j$. In case $h \neq \{j\}$, by the hypothesized continuity and strict monotonicity of the utility functions, $z_j \neq 0$ and there exists $\varepsilon \in (0, 1)$ such that $U_j((1-\varepsilon)z_j; h) > U_j(x_j; h)$ and $U_i(z_i + (\varepsilon/[|h|-1])z_j; h) > U_i(x_i)$ for $i \in h, i \neq j$. Set $y_j = (1-\varepsilon)z_j$ and $y_i = z_i + (\varepsilon/[|h|-1])z_j$ for $i \in h, i \neq j$. In any case, $h \in P$ and $\mathbf{x_h} \notin EB_h(p)$ implies existence of consumption bundles $y_i \in \mathcal{X}_i$ for $i \in h$ such that $(y_i)_{i \in g} \in B_h(p)$ and $U_i(y_i; h) > U_i(x_i; h) = U_i(x_i; P(i))$ for all $i \in h$, which contradicts the assertion of Lemma 1. Hence $\mathbf{x_h} \in EB_h(p)$ for $h \in P$ has to hold and $(p, \mathbf{x}; P)$ is a competitive equilibrium.

Application of Lemma 1 to $h \notin P$ yields that no group of consumers can benefit from forming a new household.

(ii) By Theorem 3 of Gilles and Scotchmer (1997), a valuation equilibrium allocation $(\mathbf{x}; P)$ is a strong core allocation. Hence the assertion. ■■

Definition 3 *Let $i \in I$ and $(x_i; h) \in X_i \times \mathcal{H}_i$. We say that consumer i*

(a) *has a* **group preference against** $(x_i; h)$ *if there exists $g \in \mathcal{H}_i$ such that*

$$U_i(x_i; h) < U_i(y_i; g) \tag{1}$$

for all $y_i \in X_i$.

(b) *has a* **group preference for** $(x_i; h)$ *if there exists $g \in \mathcal{H}_i$ such that*

$$U_i(x_i; h) > U_i(y_i; g) \tag{2}$$

for all $y_i \in X_i$. In case (2) holds for a particular $g \in \mathcal{H}_i$, we also say that i **prefers** $(x_i; h)$ **to** g.

Proposition 3 *Let each U_i be continuous and strictly monotone in $x_i \in X_i$. Suppose $(p, \mathbf{x}; P)$ is a competitive equilibrium at which no group of consumers can benefit from forming a new household and no individual i has a group preference against $(x_i; P(i))$. Then $(p, \mathbf{x}; P)$ is a valuation equilibrium and $(\mathbf{x}; P)$ is a strong core allocation.*

PROOF. Let $U_i, i \in I$, and $(p, \mathbf{x}; P)$ be as hypothesized. Because of strong monotonicity, $p \gg 0$. First we are going to define valuations $V_i(h)$ for $h \in \mathcal{H}, i \in h$. For $h \in P$, $i \in h$, set $V_i(h) \equiv p\omega_i - px_i$. For $h \notin P$, let h^* denote the set of $i \in h$ who favor $(x_i, P(i))$ to h. We distinguish two cases.

Case 1: $h^* \neq \emptyset$. For $j \in h \setminus h^*$, set $V_j(h) \equiv 1 + p\omega_j$. For $i \in h^*$, set $V_i(h) \equiv -\Big(1 + \sum_{j \in h \setminus h^*} V_j(h)\Big)$.

Case 2: $h^* = \emptyset$. Take $i \in I$. There exist $y_i, z_i \in X_i$ such that

$$U_i(y_i; h) \leq U_i(x_i; P(i)) \leq U_i(z_i; h), \tag{3}$$

since i does not have a group preference against $(x_i, P(i))$ and i does not favor $(x_i, P(i))$ to h. Now put

$$m_i(h) \equiv \min\{m_i | \max\{U_i(z_i; h) : z_i \in X_i, pz_i \leq m_i\} = U_i(x_i; P(i))\}. \tag{4}$$

Because of (3), $p \gg 0$, and continuity of U_i in its first argument, $m_i(h)$ is well defined. $m_i(h)$ is the smallest expenditure on i's consumption that permits individual i to attain utility $U_i(x_i; P(i))$ as member of household h. Set $V_i(h) \equiv p\omega_i - m_i(h)$.

Next we show that with these valuations $V_i(h)$ for $h \in \mathcal{H}, i \in h$, $(p, \mathbf{x}; P)$ is a valuation equilibrium.

Condition 1:

For $h \in P$, $\sum_{i \in h} V_i(h) = \sum_{i \in h}(p\omega_i - px_i) = p\sum_{i \in h} \omega_i - p\sum_{i \in h} x_i$. Because of IPR, $\sum_{i \in h} \omega_i = \omega_h$. Because of strict monotonicity, $\mathbf{x_h} \in EB_h(p)$ implies $p\sum_{i \in h} x_i = p\omega_h$. Hence $\sum_{i \in h} V_i(h) = p\omega_h - p\sum_{i \in h} x_i = 0$ as asserted.

Condition 2:

- For $h \in \mathcal{H}, h \notin P, h^* \neq \emptyset$, we obtain $\sum_{i \in h} V_i(h) = \sum_{j \in h \setminus h^*} V_j(h) - \sum_{i \in h^*} \left(1 + \sum_{j \in h \setminus h^*} V_j(h)\right) \leq -|h^*| < 0$; hence the assertion.
- For $h \in \mathcal{H}, h \notin P, h^* = \emptyset$, suppose that $\sum_{i \in h} m_i(h) < p\omega_h$. Then set $d(h) \equiv p\omega_h - \sum_{i \in h} m_i(h)$. For each $i \in h$, choose $y_i \in \arg\max\{U_i(z_i; h) : z_i \in X_i, pz_i \leq m_i(h) + d(h)/|h|\}$. By strict monotonicity, $U_i(y_i; h) > U_i(x_i; P(i))$. Further $p\sum_{i \in h} y_i \leq \sum_{i \in h} m_i(h) + d(h) = \sum_{i \in h} m_i(h) + p\omega_h - \sum_{i \in h} m_i(h) = p\omega_h$, that is $\mathbf{y_h} = (y_i)_{i \in h}$ belongs to $B_h(p)$. Consequently, the consumers in h can benefit from forming the new household h — and choosing $\mathbf{y_h} \in B_h(p)$. This contradicts the hypothesis that in the competitive equilibrium $(p, \mathbf{x}; P)$ no group of consumers

can benefit from forming a new household. Hence, to the contrary, $\sum_{i\in h} m_i(h) \geq p\omega_h$. Therefore, $\sum_{i\in h} V_i(h) = \sum_{i\in h}(p\omega_i - m_i(h)) = p\omega_h - \sum_{i\in h} m_i(h) \leq 0$ as asserted.

Condition 3: Since $V_i(h) \equiv p\omega_i - px_i$ for $h \in P$, $i \in h$, it follows $px_i + V_i(P(i)) = p\omega_i$ for $i \in I$ as asserted.

Condition 4: Let $i \in I$, $h \in \mathcal{H}_i$, $y_i \in \mathcal{X}_i$ with $U_i(y_i; h) > U_i(x_i; P(i))$. We have to show that $py_i + V_i(h) > p\omega_i$.

- In the case $h = P(i)$, $py_i + V_i(h) \leq p\omega_i$ implies $py_i + p\omega_i - px_i \leq p\omega_i$ and, consequently, $py_i \leq px_i$, which together with $U_i(y_i; P(i)) > U_i(x_i; P(i))$ contradicts $\mathbf{x_h} \in EB_h(p)$. Therefore, $py_i + V_i(h) > p\omega_i$ has to hold.
- In the case $h \notin P; h^* \neq \emptyset$, the inequality $U_i(y_i; h) > U_i(x_i; P(i))$ implies $i \in h \setminus h^*$ and, consequently, $py_i + V_i(h) = py_i + 1 + p\omega_j > p\omega_i$.
- In the case $h \notin P; h^* = \emptyset$, the inequality $U_i(y_i; h) > U_i(x_i; P(i))$ implies $py_i > m_i(h)$ and $py_i + V_i(h) > m_i(h) + V_i(h) = m_i(h) + p\omega_i - m_i(h) = p\omega_i$. Thus $py_i + V_i(h) > p\omega_i$ holds in all cases.

This completes the proof that $(p, \mathbf{x}; P)$ is a valuation equilibrium. $(\mathbf{x}; P)$ is a strong core allocation by Proposition 2 (ii). ■■

Remark. Without the hypothesis that "no individual i has a group preference against $(x_i, P(i))$", the conclusion needs not hold. Namely, consider $I = \{1, 2\}$, $\ell = 1$, and utility functions $U_i(x_i, \{i\}) = x_i$; $U_1(x_1, \{1, 2\}) = 2 + x_1$; $U_2(x_2, \{1, 2\}) = -1 + x_2$. The endowments are $\omega_1 = \omega_2 = 1$. Then $x_1 = x_2 = 1$, $p = 1$, $P = \{\{1\}, \{2\}\}$ constitute a competitive equilibrium where consumer 2 cannot benefit from formation of household $h = \{1, 2\}$. Moreover, consumer 1 has a group preference against $(\{1\}, 1)$. In particular, $U_1(0; \{1, 2\}) > U_1(1; \{1\})$. Hence valuations supporting $(p, \mathbf{x}; P)$ as a valuation equilibrium would have to satisfy $V_1(\{1,2\}) > 1$, by equilibrium condition 4. Consequently, $U_2(\omega_2 + V_1(\{1,2\}); \{1,2\}) > U_2(x_2; \{2\})$. Hence equilibrium condition 4 would require $p(\omega_2 + V_1(\{1,2\})) + V_2(\{1,2\}) > p\omega_2$ or, since $p = 1$, $V_1(\{1,2\}) + V_2(\{1,2\}) > 0$, a violation of equilibrium condition 2. In other words: To prevent both consumers from choosing household $\{1,2\}$ over their respective households in P, the valuations (admission prices) $V_i(\{1,2\})$ would have to be so high that $\{1,2\}$ becomes a profitable club. Thus, $(p, \mathbf{x}; P)$ cannot be a valuation equilibrium.

Notice that **essentiality** in the sense of Mas-Collel (1990) rules out that a consumer i has group preferences for or against any $(x_i; h) \in X_i \times \mathcal{H}_i$. Therefore, our example violates essentiality. It also violates the property

$$U_i(\omega_i; \{i\}) > U_i(0; h) \text{ for all } i \in I, h \in \mathcal{H}_i$$

which corresponds to the property that "endowments are large relative to the value of club goods" in Gilles and Scotchmer (1997).

5 Consumption Externalities

The club literature, as a rule, assumes that preferences are selfish, or in our terminology, that consumption externalities are absent (ACE). In contrast, our general model of households allows for the possibility that individuals care about the composition and the consumption of each member of the household to which they belong.

Propositions 2 and 3 imply that in the absence of consumption externalities, the household model and the club model are essentially equivalent: Competitive equilibria where no group of consumers benefits from forming a new household and valuation equilibria coincide. In addition, the corresponding equilibrium allocations are strong core allocations. In the presence of consumption externalities, a valuation equilibrium may no longer be Pareto-optimal and need not be a competitive equilibrium even when there exist Pareto-optimal competitive equilibria.

Any extension of the concept of a valuation equilibrium to instances with arbitrary household-specific preferences should replace equilibrium condition 4 with a condition which implies the following:

5a. If $i \in I$, $y_i \in X_i$ with $U_i((y_i, (x_j)_{j \in P(i), j \neq i}); P(i)) > U_i(\mathbf{x_{P(i)}}; P(i))$, then $py_i + V_i(P(i)) > p\omega_i$.

5b. If $i \in I$, $y_i \in X_i$ with $U_i(y_i; \{i\}) > U_i(\mathbf{x_{P(i)}}; P(i))$, then $py_i + V_i(\{i\}) > p\omega_i$.

The subsequent example only utilizes equilibrium conditions 1,2,3,5a and 5b and, therefore, applies to any conceivable generalization of the valuation equilibrium concept. The example substantiates the claim that with consumption externalities, a valuation equilibrium need not be Pareto-optimal and need not be a competitive equilibrium even when there exist Pareto-optimal competitive equilibria.

5.1 Example

Let $I = \{1, 2\}$, $\ell = 2$, $\omega_1 = \omega_2 = (4, 1), \omega_{\{1,2\}} = \omega_1 + \omega_2 = (8, 2)$. A consumption bundle for individual i is denoted (x_i, y_i) within the example. The specific utility representations are

$U_i(x_i, y_i) = x_i y_i$ for $i = 1, 2$; $(x_1, y_i) \in X_i$;

$U_1((x_1, y_1), (x_2, y_2)); \{1, 2\}) = Ax_1 y_1$
for $((x_1, y_1), (x_2, y_2)) \in X_1 \times X_2$;

$U_2((x_1, y_1), (x_2, y_2)); \{1, 2\}) = Bx_2 y_2 \cdot (1 + x_1)^{-1}$
for $((x_1, y_1), (x_2, y_2)) \in X_1 \times X_2$,

with $A > 1, B > 9$. Further put for $((x_1, y_1), (x_2, y_2)) \in X_1 \times X_2$:

$W((x_1, y_1), (x_2, y_2)) \equiv U_1((x_1, y_1), (x_2, y_2)); \{1,2\}) \cdot U_2((x_1, y_1), (x_2, y_2)); \{1,2\})$. Then $\ln W = \ln A + \ln B + \ln x_1 + \ln y_1 + \ln x_2 + \ln y_2 - \ln(1 + x_1)$.

Now consider the state $(\widehat{p}, \widehat{\mathbf{x}}; \widehat{P})$ given by the price system $\widehat{p} = (\widehat{p}_1, \widehat{p}_2) = (1, 6)$, the commodity allocation $\widehat{\mathbf{x}} = ((\widehat{x}_1, \widehat{y}_1), (\widehat{x}_2, \widehat{y}_2)) = ((2, 1), (6, 1))$, and the household structure $\widehat{P} = \{I\} = \{\{1, 2\}\}$.

CLAIM 1. *The state $(\widehat{p}, \widehat{\mathbf{x}}; \widehat{P})$ is a competitive equilibrium at which no group of consumers can benefit from forming a new household.*

First observe that $\widehat{\mathbf{x}} = \widehat{\mathbf{x}}_{\mathbf{I}}$ is the unique solution of the problem

$$\max W(\mathbf{z}_{\mathbf{I}}) \text{ subject to } \mathbf{z}_{\mathbf{I}} \in B_I(\widehat{p}),$$

which implies $\widehat{\mathbf{x}} = \widehat{\mathbf{x}}_{\mathbf{I}} \in EB_I(\widehat{p})$. Second, $(\widehat{\mathbf{x}}; \widehat{P})$ is a feasible allocation. Hence $(\widehat{p}, \widehat{\mathbf{x}}; \widehat{P})$ is a competitive equilibrium. Further, the equilibrium utilities are $U_1(\widehat{\mathbf{x}}; I) = 2A, U_2(\widehat{\mathbf{x}}; I) = 2B$ whereas $\max\{U_j(z_j; \{1\}) | z_j \in B_{\{j\}}(\widehat{p})\} = 25/6$ for $j = 1, 2$. Since $A > 3, B > 9$ and $h = \{1\}, \{2\}$ are the only potential new households, this implies that no group of consumers can benefit from forming a new household.

CLAIM 2. *The allocation $(\widehat{\mathbf{x}}; \widehat{P})$ is a strong Pareto optimum.*

Namely, let $(\mathbf{x}'; P')$ be any feasible allocation, $\mathbf{x}' = ((x_1', y_1'), (x_2', y_2'))$. We want to show that $(\mathbf{x}'; P')$ does not weakly dominate the allocation $(\widehat{\mathbf{x}}; \widehat{P})$. We can restrict ourselves to $P' = \widehat{P}$ because

$$U_i(\mathbf{x}'; P') \leq U_i(\mathbf{x}'; \widehat{P}) \text{ for } i = 1, 2.$$

For $P' = \{I\} = \widehat{P}$ this holds trivially true. For $P' = \{\{1\}, \{2\}\}$, it holds true since $A > 1$, feasibility of $\mathbf{x}'$ implies $x_1' \leq 8$, and $B > 9$.

It remains to show that $(\mathbf{x}'; \widehat{P})$ does not weakly dominate the allocation $(\widehat{\mathbf{x}}; \widehat{P})$. Since $(\mathbf{x}'; \widehat{P})$ is feasible and $\omega_{\widehat{P}} = \omega_I$, one obtains $\mathbf{x}' \in B_I(\widehat{p})$. Since $\widehat{\mathbf{x}}$ is the unique maximizer of W on $B_I(\widehat{p})$, with $W(\widehat{\mathbf{x}}) > 0$, $(\mathbf{x}'; \widehat{P})$ cannot weakly dominate $(\widehat{\mathbf{x}}; \widehat{P})$.

CLAIM 3. *If $(p^*, \mathbf{x}^*; P^*)$ be a valuation equilibrium, then $(\mathbf{x}^*; P^*)$ fails to be weakly Pareto-optimal.*

Let $(p^*, \mathbf{x}^*; P^*)$ be a valuation equilibrium which is supported by valuations $V_1(\{1\}), V_1(\{1,2\}), V_2(\{2\}), V_2(\{1,2\})$. We will show that $(\mathbf{x}^*; P^*)$ cannot be weakly Pareto-optimal. By equilibrium conditions 1 and 2, $V_i(\{i\}) \leq 0$ for $i = 1,2$. If for some consumer i, $U_i(\mathbf{x}^*_{\mathbf{P^*(i)}}; P^*(i)) = 0$, then
$U_i(\omega_i; \{i\}) > U_i(\mathbf{x}^*_{\mathbf{P^*(i)}}; P^*(i))$ while $p\omega_i + V_i(\{i\}) \leq p\omega_i$, contradicting equilibrium condition 5b. Hence $U_i(\mathbf{x}^*_{\mathbf{P^*(i)}}; P^*(i)) > 0$ for $i = 1,2$. Consequently, $(x_i^*, y_i^*) \gg 0$ for $i = 1,2$. If $P^* = \{\{1\},\{2\}\}$, then $(\mathbf{x}^*; P^*)$ is not weakly Pareto-optimal because

$$U_i(\mathbf{x}^*; P^*) < U_i(\mathbf{x}^*; \widehat{P}) \text{ for } i = 1,2.$$

These inequalities follow from $(x_i^*, y_i^*) \gg 0$ for $i = 1,2$, $A > 1$, $y_2^* < 8$, $B > 9$. If $P^* = \widehat{P}$, then $(x_i^*, y_i^*) \gg 0$ for $i = 1,2$, the separable form of the consumption externality, and equilibrium conditions 3 and 5a require that each (x_i^*, y_i^*) solves the problem

$$\max x_i y_i \text{ subject to } (x_i, y_i) \in X_i,\ p^* \cdot (x_i, y_i) + V_i(\{1,2\}) \leq p^* \omega_i$$

and further that $p^* = \lambda \cdot (1,4)$, $(x_1^*, y_1^*) = \mu_1 \cdot (8,2)$, $(x_2^*, y_2^*) = \mu_2 \cdot (8,2)$ for some $\lambda > 0, \mu_1 > 0, \mu_2 > 0$. But the gradients of $U_1(\cdot\,; I)$ and $U_2(\cdot\,; I)$ with respect to $\mathbf{z} \in \mathcal{X}_I = \mathcal{X}$ differ at $\mathbf{x}^*_{\mathbf{I}} = \mathbf{x}^* \gg 0$. Therefore, a strict Pareto-improvement is possible by means of a slight change of the commodity allocation, which shows that $(\mathbf{x}^*; P^*) = (\mathbf{x}^*; \widehat{P}))$ is not weakly Pareto-optimal in this case either. This completes the analysis of the example.

5.2 Restrictions on Private Consumption in Clubs

Many clubs have statutes which lay out a governance structure and regulate the conduct of their members to some degree. For instance, club rules may stipulate that members participate in certain events and rituals, contribute labor, provide personal equipment, dress appropriately, or follow dietary restrictions.

Clubs may have good reasons to restrict the consumption possibilities of their members. By imposing individual lower or upper bounds on the consumption of certain goods, the club may be capable of internalizing consumption externalities — provided its members are not lured away by less restrictive clubs. In the foregoing example, consumer 1 in household $I = \{1, 2\}$ will choose $(\widehat{x}_1, \widehat{y}_1)$ when restricted to $x_1 \leq 2$ and given income $\widehat{p} \cdot (\widehat{x}_1, \widehat{y}_1)$ to spend on own consumption. But of course, consumer 1 would prefer a situation with the same individual budget and no restrictions. Therefore, the observation that a suitable restriction can induce the consumer to make a particular choice is only interesting if such a club persists in a valuation equilibrium of a model where alternative clubs, with less severe or no restrictions, are admissible. Accordingly, for $\gamma \in \mathbb{R}_+ \cup \{\infty\}$, let $< \{1, 2\}, \gamma > = < I, \gamma >$ denote the club consisting of members 1 and 2 which imposes the restriction $x_1 \leq \gamma$ on consumer 1.

CLAIM 4. $(\widehat{p}, \widehat{\mathbf{x}}; \{< \{1, 2\}, 2 >\})$ *is a valuation equilibrium in a model where all clubs,* $\{1\}, \{2\}$ *and* $< \{1, 2\}, \gamma >$, $\gamma \in \mathbb{R}_+ \cup \{\infty\}$, *are admissible.*

The proof is given in the appendix. The claim indicates how certain consumption externalities can be internalized in a carefully chosen extension of the club model. Scotchmer (2004) examines an illustrative example of purchase clubs where local public goods are privately provided by club members and the individual contributions of such goods at proprietary prices are modeled as membership characteristics. To what extent suitably modified club models have the capacity to internalize arbitrary consumption externalities remains an open question which we leave for future research.

6 Final Remarks

For the sake of direct comparison, we have assumed common primitive data for the household and the club model and drawn the distinction in terms of the equilibrium concepts. One of the primitive concepts are households defined as coalitions of named individuals where in principle, each individual can be its own type, that is, *everybody is special.* Accordingly, in our definition, valuations are individualized: Identical individuals in identical households can be charged different admission fees. In contrast, club profiles in the literature specify the number of each member type so that in principle, *everybody is replaceable,* and valuations are anonymous. If in a type or replica economy, a valuation equilibrium is supported by anonymous valuations, then the conclusion of Proposition 2 still holds. Conversely, under the hypothesis of Proposition 3, equal treatment (in terms of utility) of identical individuals in identical households must hold in the competitive equilibrium at hand. Consequently, the valuations constructed in the proof satisfy anonymity. Therefore, the conclusion of the proposition also holds when valuations are required to be anonymous.

Our definition of a household or club h presumes that this group is feasible within the existing population I, that is $h \subseteq I$. The club literature is divided between authors who allow memberships in infeasible clubs as hypothetical alternatives and those who consider only feasible alternatives. The first version amounts to a more demanding definition of equilibrium. The argument in its favor is based on an analogy to consumer sovereignty in conventional pure exchange economies: The consumer is free to choose any consumption bundle from the budget set, regardless of whether or not this choice is compatible with the resources available in the economy. In analogy, the consumer should feel free to select any affordable hypothetical club regardless of whether such a club can be formed within the existing population. The counter-argument is that markets are not necessarily complete: In certain economies certain commodities or commodity bundles are not available or not tradeable, and not priced in the market. The same applies to infeasible clubs. Proposition 2 still holds if one applies the more demanding definition in the club model, with a consistent extension of preferences. Propositions 2 and 3 continue to hold, if infeasible groups are considered both in the household and the club model.

In our previous work on household formation, we rarely consider the most stringent version of a competitive equilibrium, which requires that no group of consumers can benefit from forming a new household. The conclusion of 2 continues to hold if the weaker equilibrium concepts, CEFE and CEFH, are employed. In Proposition 3, however, the hypothesis of a competitive equilibrium in which a group of consumers can benefit from forming a new household cannot be replaced by the weaker conditions CEFE or CEFH.

The formal analysis of the present paper assumes individual property rights (IPR) as does most of the club literature. Haller (2000) and Gersbach and Haller (2001) do not rely on this assumption. A violation of IPR can represent, for example, a reduced form of household production.

7 Appendix

PROOF OF CLAIM 4: PRELIMINARIES. At price system $\widehat{p}$, household I has aggregate income $\widehat{p}\omega_I = 20$. Let $\chi \in [0,1]$ denote consumer 1's expenditure share in household I so that at the price system $\widehat{p}$, $m_1 = m_1(\chi) = 20\chi$ is spent on consumption of member 1 and $m_2 = m_2(\chi) = 20(1-\chi)$ is spent on consumption of member 2. Specifically at state $(\widehat{p}, \widehat{\mathbf{x}}; \widehat{P})$, the amount $m_1 = \widehat{p}\cdot(\widehat{x}_1, \widehat{y}_1) = 8$ is spent on consumption of member 1 and, therefore, $\chi = \widehat{\chi} = 0.4$.

For any $\chi = [0,1]$ and $i = 1,2$, let $(x_i(\chi), y_i(\chi))$ denote the solution of the problem

$$\max\ x_i y_i \text{ subject to } (x_i, y_i) \in X_i,\ \widehat{p}\cdot(x_i, y_i) \le m_i(\chi).$$

For $\chi^0 \equiv \sqrt{12}/10$, $U_1(\mathbf{x_I}(\chi^{\mathbf{0}}); I) = U_1(\widehat{\mathbf{x}}_{\mathbf{I}}; I) = 2A$ and $x_1(\chi^0) = 10\chi^0 = \sqrt{12}$. Further note that the solutions of the quadratic equation $\gamma(20-\gamma) = 12$ or $\gamma^2 - 20\gamma + 12 = 0$ are $\widetilde{\gamma}_1 = 10 - \sqrt{88}$ and $\widetilde{\gamma}_2 = 10 + \sqrt{88}$. Then $\gamma(20-\gamma)/6 \ge 2$ if and only if $\gamma \in [\widetilde{\gamma}_1, \widetilde{\gamma}_2]$.

VALUATIONS. Now we are ready to specify valuations which support $(\widehat{p}, \widehat{\mathbf{x}}; \{<\{1,2\}, 2>\})$ as a valuation equilibrium. Let us set $V_i(\{i\}) = 0$ for $i = 1,2$. For "clubs" of the form $<\{1,2\}, \gamma>$, we distinguish three cases.

Case 1: $\gamma \ge x_1(\chi^0)$. We provide consumer 1 with income $m_1(\chi^0) = 20\chi^0 = 2\sqrt{12}$ available for consumption, by setting $V_1(<I,\gamma>) = \widehat{p}\omega_1 - m_1(\chi^0) = 10 - 2\sqrt{12}$, $V_2(<I,\gamma>) = -V_1(<I,\gamma>)$.

Case 2: $\widetilde{\gamma}_1 \le \gamma < x_1(\chi^0)$. If consumer 1 has income $m_1(\chi^0) = 20\chi^0 = 2\sqrt{12}$ available for consumption, then his ideal choice $(x_1(\chi^0), y_1(\chi^0))$ is no longer available. He will choose $x_1 = \gamma$, $y_1 = (m_1(\chi^0) - \gamma)/6$ and achieve utility $Ax_1y_1 = A\gamma(m_1(\chi^0) - \gamma)/6 < 2A$. He attains the utility level $2A$ only if he has an income $m_1 > m_1(\chi^0)$ — which yields maximal utility $A\gamma(m_1 - \gamma)/6$. Since $x_1(\chi^0) < \widetilde{\gamma}_2$, $\gamma \in [\widetilde{\gamma}_1, \widetilde{\gamma}_2]$. Hence $A\gamma(20-\gamma)/6 \ge 2A$. Therefore, there exists a unique $m_i^\gamma \in [m_1(\chi^0), 20]$ such that $A\gamma(m_1^\gamma - \gamma)/6 = 2A$. We provide consumer 1 with income m_1^γ available for consumption, by setting $V_1(< I, \gamma >) = \widehat{p}\omega_1 - m_1^\gamma$, $V_2(< I, \gamma >) = -V_1(< I, \gamma >)$.

Case 3: $\gamma < \widetilde{\gamma}_1$. Put $V_1(< I, \gamma >) = -10$ and
$V_2(< I, \gamma >) = -V_1(< I, \gamma >) = 10 = \widehat{p}\omega_2$.

VERIFICATION OF EQUILIBRIUM CONDITIONS. Next we show that with these valuations, $(\widehat{p}, \widehat{\mathbf{x}}; \{< \{1, 2\}, 2 >\})$ is a valuation equilibrium.

Conditions 1 and 2: $V_2(< I, \gamma >) = -V_1(< I, \gamma >)$ for all $\gamma \in \mathbb{R}_+ \cup \{\infty\}$ and $V_i(\{i\}) = 0$ for $i = 1, 2$ yield the assertion.

Condition 3: For $\gamma = 2$, $\widetilde{\gamma}_1 \leq \gamma < x_1(\chi^0)$. First consider $i = 1$. Under the constraint $x_1 \leq 2$, $\widehat{p} \cdot (\widehat{x}_1, \widehat{y}_1) = 8$ is exactly the minimum amount that consumer 1 needs to spend on consumption to attain utility level $2A$. Hence $m_1^\gamma = \widehat{p} \cdot (\widehat{x}_1, \widehat{y}_1)$. Since $V_1(< I, \gamma >) = \widehat{p}\omega_1 - m_1^\gamma$ in case 2, the required condition follows:

$$V_1(< I, \gamma >) + \widehat{p} \cdot (\widehat{x}_1, \widehat{y}_1) = V_1(< I, \gamma >) + m_1^\gamma = \widehat{p}\omega_1. \tag{5}$$

Second, consider $i = 2$. Since $V_1(< I, \gamma >) + V_2(< I, \gamma >) = 0$ and $(\widehat{x}_1, \widehat{y}_1) + (\widehat{x}_2, \widehat{y}_2) = \omega_{\widehat{P}} = \omega_1 + \omega_2$, the required condition is implied by (5):

$$\left.\begin{array}{rl} & V_2(< I, \gamma >) + \widehat{p} \cdot (\widehat{x}_2, \widehat{y}_2) \\ = & V_1(< I, \gamma >) + V_2(< I, \gamma >) + \widehat{p} \cdot (\widehat{x}_1, \widehat{y}_1) + \widehat{p} \cdot (\widehat{x}_2, \widehat{y}_2) \\ - & [V_1(< I, \gamma >) + \widehat{p} \cdot (\widehat{x}_1, \widehat{y}_1)] \\ = & \widehat{p}\omega_1 + \widehat{p}\omega_2 - \widehat{p}\omega_1 \\ = & \widehat{p}\omega_2. \end{array}\right\} \tag{6}$$

Condition 4: If $i \in I$, $h = \{i\}$, and $z_i \in \mathcal{X}_i$ with $U_i(z_i; h) > U_i(\widehat{\mathbf{x}}; I)$, then $\widehat{p}z_i + V_i(h) = \widehat{p}z_i > \widehat{p}\omega_i$, since $V_i(h) = 0$ and group h cannot benefit from forming a new household.

If $h = I$ and $\gamma \in \mathbb{R}_+ \cup \{\infty\}$, consider the club $< I, \gamma >$. In cases 1 and 2, consumer 1 achieves utility level $U_1(\widehat{\mathbf{x}}; I) = 2A$ when he spends the amount $\widehat{p}\omega_1 - V_1(< I, \gamma >)$ optimally on his own consumption. To achieve a higher utility level, he would have to consume $z_1 \in X_1$ such that $\widehat{p}z_1 > \widehat{p}\omega_1 - V_1(< I, \gamma >)$ and, hence, $\widehat{p}z_1 + V_1(< I, \gamma >) > [\widehat{p}\omega_1 - V_1(< I, \gamma >)] + V_1(< I, \gamma >) = \widehat{p}\omega_1$. Under the premise that consumer 1 spends the amount $\widehat{p}\omega_1 - V_1(< I, \gamma >)$ optimally on his consumption and achieves utility level $2A$, consumer 2 can achieve at most utility level $2B$ when she spends the amount
$\widehat{p}\omega_2 - V_2(< I, \gamma >)$ optimally on her consumption. This follows from $\widehat{p}\omega_1 - V_1(< I, \gamma >) + \widehat{p}\omega_2 - V_2(< I, \gamma >) = \widehat{p}\omega_I$, $U_1(\widehat{\mathbf{x}}; I) = 2A$, $U_2(\widehat{\mathbf{x}}; I) = 2B$, and $\widehat{\mathbf{x}}_{\mathbf{I}} \in EB_I(\widehat{p})$. To achieve *ceteris paribus* a utility level greater than $2B$, she would have to consume $z_2 \in X_2$ such that $\widehat{p}z_2 > \widehat{p}\omega_2 - V_2(< I, \gamma >)$ and, consequently, $\widehat{p}z_2 + V_2(< I, \gamma >) > [\widehat{p}\omega_2 - V_2(< I, \gamma >)] + V_2(< I, \gamma >) = \widehat{p}\omega_2$.

In case 3, consumer 1 cannot reach the utility level $2A$ when he spends $\widehat{p}\omega_1 - V_1(< I, \gamma >) = 20$, the entire household income, on own consumption. To exceed the utility level $2A$, consumer 1 would have to consume $z_1 \in X_1$ such that $\widehat{p}z_1 > \widehat{p}\omega_1 - V_1(< I, \gamma >)$ and, hence,
$\widehat{p}z_1 + V_1(< I, \gamma >) > [\widehat{p}\omega_1 - V_1(< I, \gamma >)] + V_1(< I, \gamma >) = \widehat{p}\omega_1$.
Consumer 2 attains zero utility when she spends $\widehat{p}\omega_2 - V_2(< I, \gamma >) = 0$ on own consumption. To exceed the utility level $2B$, she would have to consume $z_2 \in X_2$ such that $\widehat{p}z_2 > \widehat{p}\omega_2 - V_2(< I, \gamma >)$ and, consequently,
$\widehat{p}z_2 + V_2(< I, \gamma >) > [\widehat{p}\omega_2 - V_2(< I, \gamma >)] + V_2(< I, \gamma >) = \widehat{p}\omega_2$. ■■

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