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ABSTRACT

Investments into Education - Doing as the Parents Did*

Empirical evidence suggests that parents with higher levels of education generally also attach a higher importance to the education of their children. This implies an intergenerational chain transmitting the attitude towards the formation of human capital from one generation to the next. We incorporate this intergenerational chain into an OLG-model with endogenous human capital formation. In absence of any state intervention such an economy might be characterized by multiple steady states. A temporary public investment into human capital formation is then needed for a transition from a steady state with low human capital levels to one with a higher human capital level. Furthermore, it can be shown that even the best steady state is suboptimal when the human capital is privately provided. This inefficiency can be overcome by a permanent public subsidy for education.

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Keywords: education subsidy, human capital formation and indirect reciprocity

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1 Introduction

In modern economies human capital is one of the most important determinants of economic progress and welfare. In contrats to the investment into physical capital the formation of human capital is to a large extent not financed by its owner. Rather, parents and the state cover most of the expenditures on education. The parental engagement has traditionally been explained by credit market imperfections, parental altruism (see e.g. [3] and [4]) and/or an exchange between education expenditures for the children and old-age support for the parents (see e.g. [6] and [5]). As predicted by these theories, it has indeed been found that parental expenditures on the their children's education depend on the family income (see e.g. [16] and [3]).

But these theories are difficult to reconcile with the empirical fact that even for given family income, higher educated parents tend to spend more on the education of their children [16] than parents with lower education. This fact is consistent with the so called 'home environment externality' [13], which states that not only private and public investments into education, but also innate abilities and the 'family environment' determine human capital formation. This strand of literature (see e.g. [4], [11], [13] and [14]) assumes that children's ability to acquire human capital depends on parental levels of education. Higher levels of parental education are assumed to increase the marginal product of investments into the human capital of children. Hence, the higher the level of education of the parents, the more effective investments in human capital become. If parents care about their children, this home environment externality can explain the effect of parents' human capital on the education expenditures. In addition, Eckstein and Zilcha [11] show the suboptimality of private investments into human capital, if a home environment externality exists and if parents only care about the educational level of their offsprings. The source of suboptimality is twofold. First, parents do not take into account the impact of their investment into the education of their children on their children's wages. Second, they do not take into account their impact on the relative effectiveness of their children's investments into the education of their grandchildren.

The empirical literature is inconclusive about whether inheritance of genes or the effect of the home environment leads to the relation between parents' education and expenditures for children's human capital formation (see e.g. [22], [20]). According to the sociological literature, however, there is another reason for this relation, which is unrelated to the impact of parents' education on the production of children's human capital. Rather, it seems that parents have a pervasive influence in shaping young people's attitudes to education. (see e.g. [19], [7] and [8]). More precisely, parents with higher levels of education transmit a more positive attitude towards education to their children (see e.g [19]).[1] In contrast to the 'home environment externality', this parental influence implies that parents directly affect children's preferences, rather than their production of human capital. This intergenerational transmission of attitudes can be viewed as an example of indirect reciprocity, which has been found to be particularly important within family relations (see e.g.[1], [2] and [15]). In contrast to direct reciprocity (see e.g. [10], [21]), we speak of indirect reciprocity when a person does not directly reciprocate to the behavior of another person, but rather reciprocates indirectly to a third party (see e.g. [1], [17], [18] and [12]). In our context people do not directly reciprocate for the education they have received from their own parents, but rather repay it to their children by financing their education. In this way investments into human capital do not only affect the immediate recipient, i.e. the next generation, but also future generations. Hence an intergenerational chain is created - the more education parents have received themselves, the more they are willing to finance the education of their children.

Our paper investigates the impact of this intergenerational chain on welfare and the optimal education policy. We assume that the parents' preference for the human capital of their children depends on parents' own human capital, which was financed for by the grandparents. Using an OLG model we show that this might lead to multiple stable steady states with different production levels. There exists a 'bad' steady state, which is characterized by low incomes and no investments into formal education. To overcome this 'bad' steady state and to get the economy into a 'good' steady state with investments into formal education, a temporary public funding of education is necessary. Furthermore, as in Eckstein and Zilcha [11] an externality is generated. Different to their model, however, the source of the externality is not the parental impact on the relative effectiveness of their children's investments into the human capital of the grandchildren. It is rather the influence parents have on their children's preferences for the grandchildren's education. Due to the existence of this indirect reciprocity mechanism, even the best steady state is suboptimal. A permanent subsidy on human cap-

[1] A similar intergenerational attitude transmission mechanism has been analysed in the context of arts education (see [9])

ital acquisition is required to overcome this inefficiency. It is important to distinguish this aspect from the aforementioned rational for government intervention, as in this case the inefficiency is created directly by the parental influence on the children's preferences and not by the parental impact on the offsprings' production function of human capital.

The paper is organized as follows. In the next section we describe the model, followed by a characterization of the economy with private investments into human capital. In section 4 we analyze the welfare properties of this economy. We show that the private provision of human capital leads to inefficiencies, and we characterize an efficient public subsidy scheme of education. Finally, we draw conclusions. All the proofs are delegated to the appendix.

2 The model

We assume a competitive economy, in which the output in period t, Y_t, does not only depend on physical capital used in t, K_t, and on labour L_t, but also on human capital, H_t. The economy is endowed with a Cobb-Douglas production technology. The normalized production function is given by

$$y_t = k_t^{\alpha} h_t^{1-\alpha} \tag{1}$$

where $\alpha \in (0,1)$ and $y_t = \frac{Y_t}{L_t}$, $k_t = \frac{K_t}{L_t}$, $h_t = \frac{H_t}{L_t}$. y_t denotes the output per worker in period t , and k_t and h_t are respectively physical and human capital per worker in t.

Every worker is assumed to supply inelastically one unit of labour, and for simplicity the number of workers is constant over time, i.e. $L_t = L$ for all t. Markets are assumed to be perfectly competitive, so that factors earn their marginal product:

$$r_t = f_k^{'}(k_t, h_t) = \alpha \left(\frac{h_t}{k_t}\right)^{1-\alpha} \tag{2}$$

$$w_t = f(k_t, h_t) - k_t f_k^{'}(k_t, h_t) = (1-\alpha) k_t^{\alpha} h_t^{1-\alpha} \tag{3}$$

with r_t being the interest rate and w_t the wage.

The capital stock is assumed to depreciate fully in one period, so that the capital stock in period t are the savings in period $t-1$.

Human capital is produced by formal education, i.e. schooling. We assume however, that even without any formal education everyone acquires some minimum human capital. We normalize human capital such that the minimum human capital is one. Human capital production is given by

$$h_{t+1} = (e_t)^{\beta} + 1 \tag{4}$$

with $\beta \in (0, 1)$. $e_t \geq 0$ denotes the private expenditures into the formal education of a child born in t. Of course, the resulting human capital becomes productive in period $t + 1$.

At each point in time three overlapping generations are alive in the economy.

Generation	Period		
	$t - 1$	t	$t + 1$
(1)	Education		
(2)		Work	
(3)			Retirement

Take a representative individual born at the beginning of period $t - 1$. In this period he belongs to the youngest generation 1 which gets educated. The amount of his education is decided upon by his parent. In the next period t, the individual belongs to the working (parent) generation 2. In this period he works and has one child[2]. He divides his income between consumption in period t, savings for consumption in $t + 1$ and spending for the education of his child. In period $t + 1$, the individual belongs to the retired generation 3 and consumes his savings. At the end of this period, the individual dies.

Only the working generation has to make a decision. Individuals working in time t are assumed to maximize their utility function given by

$$U(c_{2,t}, c_{3,t+1}, h_{t+1}) = \ln c_{2,t} + \gamma \ln c_{3,t+1} + \varphi(h_t) \ln h_{t+1}, \tag{5}$$

where $c_{2,t}$ denotes the immediate consumption of an individual working in period t. $c_{3,t+1}$ is the consumption in the next period $t+1$ when the individual belongs to the retired generation 3. Since we assume full depreciation of the capital stock in one period, the savings in period t are the capital stock in

[2] For simplicity we assume that each adult has only one child, and each child has only one parent.

period $t+1$, and the old generation only consumes the interest on their savings. Therefore, $c_{3,t+1} = k_{t+1} r_{t+1}$. h_{t+1} is the human capital of the child, which becomes effective in period $t+1$. γ and φ measure the individual's attitude towards future old-age consumption and towards the human capital of the child, respectively.

As explained in the introduction, there exists a lot of evidence that the importance parents attach to the education of their children is determined by indirect reciprocity. More precisely, the education a parent has received in his own childhood shapes his willingness to invest into the human capital of his own child. In order to capture this, we introduce an attitude function:

$$\varphi : [1, \infty) \rightarrow \Re_{+}^{0},$$

with $\varphi(h_t)$ denoting the attitude of a parent with a human capital of h_t.[3] We assume that $\varphi(h_t)$ is continuous and differentiable. If the parent has not received any formal education himself, he is not willing to finance any formal education of his child. Furthermore, his attitude towards his child's education is positively correlated with his own human capital h_t, which was financed for by his own parent. These considerations lead to

$$\varphi(1) = 0$$

and

$$\varphi'(h_t) > 0.$$

In the next section we characterize the economy with pure private investments into human capital.

3 Private investments into human capital

Agents working in period t have to decide how much of their wage income w_t they want to spend on instantaneous consumption and on the education of their child. Furthermore, they save in order to finance consumption when they are retired. Recall that due to full depreciation of the capital stock, $c_{3,t+1} = k_{t+1} r_{t+1}$. Recall also that $e_t = (h_{t+1} - 1)^{\frac{1}{\beta}}$.

[3] Recall that even without formal education each individual is endowed with a minimum human capital normalized to 1. Hence, φ is defined for human capital levels not below 1.

The maximization problem of a representative agent working in t can be written as:

$$\max_{c_{2,t},k_{t+1},h_{t+1}} U(c_{2,t},k_{t+1},h_{t+1}) = \ln c_{2,t} + \gamma \ln k_{t+1} r_{t+1} + \varphi(h_t) \ln h_{t+1}$$

$$s.t.\ w_t = c_{2,t} + k_{t+1} + (h_{t+1}-1)^{\frac{1}{\beta}}$$

$$\begin{aligned} h_{t+1} &\geq 1 \\ c_{2,t}, k_{t+1} &\geq 0. \end{aligned}$$

Denote by $\widetilde{k}_{t+1}$, $\widetilde{h}_{t+1}$ the utility maximizing choice of the agent working in period t, when the human capital for the next generation is provided privately. The sequence of utility maximizing choices is denoted by $\{\widetilde{k}_t, \widetilde{h}_t\}_{t=1}^{\infty}$, with $\widetilde{k}_1$ and $\widetilde{h}_1$ being the initial endowment with physical and human capital. The solution is characterized by the following lemma.

Lemma 1 *If $\widetilde{k}_t > 0$ it holds that:*

i) The solution $(\widetilde{k}_{t+1}, \widetilde{h}_{t+1})$ is fulfills the first order conditions

$$\frac{\partial U}{\partial h_{t+1}} = \frac{\varphi(\widetilde{h}_t)}{\widetilde{h}_{t+1}} - \frac{\frac{1}{\beta}\left(\widetilde{h}_{t+1}-1\right)^{\frac{1}{\beta}-1}}{\widetilde{w}_t - \widetilde{k}_{t+1} - \left(\widetilde{h}_{t+1}-1\right)^{\frac{1}{\beta}}} = 0 \tag{6}$$

and

$$\frac{\partial U}{\partial k_{t+1}} = \frac{\gamma}{\widetilde{k}_{t+1}} - \frac{1}{\widetilde{w}_t - \widetilde{k}_{t+1} - \left(\widetilde{h}_{t+1}-1\right)^{\frac{1}{\beta}}} = 0 \tag{7}$$

ii) $\widetilde{k}_{t+1} > 0$.

iii) If $\widetilde{h}_t = 1$, then $\widetilde{h}_{t+1} = 1$.

iv) If $\widetilde{h}_t > 1$, then $\widetilde{h}_{t+1} > 1$.

Proof: see Appendix.

If $\widetilde{k}_1 = 0$, no production, no consumption, and no formal education is possible in any future period. Since this case is not interesting, we restrict the analysis from now on to $\widetilde{k}_1 > 0$.

Since $\varphi(1) = 0$, it can be easily seen that conditions (6) and (7) are always fulfilled by:

$$h^* = 1$$

$$k^* = \left(\frac{\gamma(1-\alpha)}{1+\gamma}\right)^{\frac{1}{1-\alpha}}$$

In this steady state, no formal education takes place, and human capital is at its lowest possible level. The stability properties of this steady state depend on the functional form of the attitude function, $\varphi(h_t)$. The same holds true for the existence and the properties of other steady states. Without a further specification of the attitude function we cannot give a further characterization of the steady states. We will see, however, that our main results concerning the inefficiency of the private provision of human capital and the availability of an optimal public education subsidy do not only hold for steady states, but for any equilibrium path.

But before we analyze the efficiency properties of the economy, we show first that there exists no unlimited expansionary path.

Proposition 2 *There exists a triple h^m, k^m, w^m such that for any initial conditions $\widetilde{k}_1$ and $\widetilde{h}_1$ there exists a t^m such that:*

$$\begin{aligned} \widetilde{h}_t &< h_m \text{ whenever } t > t^m \\ \widetilde{k}_t &< k_m \text{ whenever } t > t^m \\ \widetilde{w}_t &< w_m \text{ whenever } t > t^m \end{aligned}$$

Proof: See Appendix

In this section we have shown that there always exists a steady state without formal education. The existence and the stability of other steady states depends on the properties of the attitude function, which we do not specify further. However, for any attitude function the economy cannot grow forever.

In order to see whether a state financed education system can improve the result of the privately provided human capital formation, we next turn to the welfare analysis.

4 The optimal education subsidy

As we have seen, there always exists a steady state with no investment into human capital and low consumption levels. To overcome this steady state, a temporary government intervention might be necessary, allowing the agents to acquire some formal education. Depending on the exact form of the attitude function, it is possible that after an initial intervention the economy converges to another steady state or to a limit cycle with positive education levels. There also exists the possibility that a permanent state intervention is necessary to prevent the economy to fall back to the state without formal education.

But even if the system without intervention does not converge to the steady state without formal education, it is not clear whether for any given initial condition the private provision of human capital generates an efficient outcome. To investigate the efficiency properties of pure private investment into human capital, we will compare the private investments into human and physical capital with the investments a social planner would make if endowed with the same capital levels.

The social planner chooses the investment in human and physical capital such that he maximizes the weighted sum of utilities of all generations, subject to the resource constraint of the economy.

$$\max_{c_{2,t},c_{3,t+1},h_{t+1}} W = \sum_{t=1}^{\infty} \omega_t \left[\ln c_{2,t} + \gamma \ln c_{3,t+1} + \varphi(h_t) \ln h_{t+1}\right]$$

$$s.t.\ k_t^{\alpha} h_t^{1-\alpha} = k_{t+1} + (h_{t+1} - 1)^{\frac{1}{\beta}} + c_{2,t} + c_{3,t}$$

and

$$\begin{aligned} h_t &\geq 1 \\ k_t &\geq 0 \end{aligned}$$

with ω_t being strictly larger than zero for all t. Denote by $\widehat{k}_{t+1}$ and $\widehat{h}_{t+1}$ the optimal choice of the social planner. The sequence of optimal choices is denoted by $\{\widehat{k}_t, \widehat{h}_t\}_{t=1}^{\infty}$, with $\widehat{k}_1 > 0$ and $\widehat{h}_1 \geq 1$ being the initial endowment with physical and human capital.

Defining

$$\xi_t = \frac{(1-\alpha)\widehat{k}_{t+1}^{\alpha} \widehat{h}_{t+1}^{-\alpha}}{\widehat{k}_{t+1}^{\alpha} \widehat{h}_{t+1}^{1-\alpha} - \widehat{k}_{t+2} - \left(\widehat{h}_{t+2} - 1\right)^{\frac{1}{\beta}} - \widehat{k}_{t+1}\widehat{r}_{t+1}},$$

the socially optimal solution is characterized by the following lemma:

Lemma 3 *If* $\widehat{k}_t > 0$ *it holds that:*

i) The solution of the social planners problem is fulfills the first order conditions

$$\frac{\partial W}{\partial h_{t+1}} = \omega_t \left(\frac{\varphi(\widehat{h}_t)}{\widehat{h}_{t+1}} - \frac{\frac{1}{\beta}\left(\widehat{h}_{t+1}-1\right)^{\frac{1}{\beta}-1}}{\widehat{k}_t^{\alpha}\widehat{h}_t^{1-\alpha} - \widehat{k}_{t+1} - \left(\widehat{h}_{t+1}-1\right)^{\frac{1}{\beta}} - \widehat{k}_t\widehat{r}_t} \right) + \omega_{t+1}\left(\xi_t + \varphi'(\widehat{h}_{t+1})\ln \widehat{h}_{t+2}\right) = 0 \tag{8}$$

and

$$\frac{\partial W}{\partial k_{t+1}} = \omega_t \left(\frac{\gamma}{\widehat{k}_{t+1}} - \frac{1}{\widehat{k}_t^{\alpha}\widehat{h}_t^{1-\alpha} - \widehat{k}_{t+1} - \left(\widehat{h}_{t+1}-1\right)^{\frac{1}{\beta}} - \widehat{k}_t\widehat{r}_t} \right) = 0 \tag{9}$$

ii) $\widehat{k}_{t+1} > 0$, $\widehat{h}_{t+1} > 1$, *and* $\widehat{c}_{2,t} > 0$.

Proof: see Appendix.

Note that since $\widehat{c}_{2,t+1} = \widehat{k}_{t+1}^{\alpha}\widehat{h}_{t+1}^{1-\alpha} - \widehat{k}_{t+2} - \left(\widehat{h}_{t+2}-1\right)^{\frac{1}{\beta}} - \widehat{k}_{t+1}\widehat{r}_{t+1} > 0$, $\xi_t > 0$ for all t. Furthermore, comparing the first order conditions of the private solution as characterized by Lemma 1 with the first order conditions of the optimal solution reveals that the sequence of private decisions, $\{\widetilde{k}_t, \widetilde{h}_t\}_{t=1}^{\infty}$, differs from the sequence of socially optimal choices, $\{\widehat{k}_t, \widehat{h}_t\}_{t=1}^{\infty}$. Hence, the private solution is not optimal. The intuitive reason for this inefficiency can be understood by comparing the first order condition for the private provision of the human capital, (6), with the first order condition of the social planner's problem, (8). The latter incorporates the impact of the child's human capital on the next period's production, leading to the emergence of ξ_t in (8). Furthermore, it also includes the indirect reciprocity impact of the child's human capital on his own future attitude towards the education of the grandchild, leading to the emergence of $\varphi'(\widehat{h}_{t+1})\ln \widehat{h}_{t+2}$ in (8). These two externalities are neglected when human capital is privately provided, leading to an underprovision of human capital.

Can this inefficiency be overcome by public expenditures on human capital formation? Think of a situation where the public finances schools and universities. Even if schools and universities are fully financed by the state, parents still have to take care of the children's costs of living, the costs of supplementary education, the costs of teaching material and other things indirectly connected to the human capital formation of children. Hence, parts of the education expenditures are always paid by parents. Furthermore, in such a system of mixed financing a better education of the children requires higher expenditures of parents as well as of the state. Finally, the education level of the children is largely influenced by the parent's willingness to cover the children's costs of living, even in a system where the state finances schools and universities. To model such a situation where human capital formation is partly privately, partly publicly financed, assume that in each period t private education expenditures are subsidized by the state at a rate s_t. To finance this subsidy, wage income is taxed at a rate τ_t. The balanced budget condition for the state for period t is given by:

$$s_t \left(h_{t+1} - 1\right)^{\frac{1}{\beta}} = \tau_t w_t. \tag{10}$$

We assume that an individual agent takes the tax rate and the subsidy scheme as given when he maximizes his utility. This implies that he does not take into account the balanced budget condition of the state. This assumption seems plausible for a large economy with many agents. With this simplification, the decision problem of a representative agent working in period t can be written as:

$$\max_{c_{2,t}, c_{3,t+1}, h_{t+1}} U(c_{2,t}, c_{3,t+1}, h_{t+1}) = \ln c_{2,t} + \gamma \ln c_{3,t+1} + \varphi(h_t) \ln h_{t+1}$$

$$s.t. (1 - \tau_t) w_t = c_{2,t} + \frac{c_{3,t+1}}{r_{t+1}} + (1 - s_t) \left(h_{t+1} - 1\right)^{\frac{1}{\beta}}$$

and

$$\begin{aligned} h_{t+1} &\geq 1 \\ c_{2,t} &\geq 0 \\ c_{3,t+1} &\geq 0. \end{aligned}$$

Denote by $\overline{k}_{t+1}$ and $\overline{h}_{t+1}$ the utility maximizing choice of the agent working in period t, when the human capital formation is subsidized. The sequence of utility maximizing choices is denoted by $\{\overline{k}_t, \overline{h}_t\}_{t=1}^{\infty}$, with $\overline{k}_1 > 0$ and $\overline{h}_1 \geq 1$ being the initial endowment of the economy with physical and human capital. Using the budget constraint to insert for $c_{2,t}$ the first order conditions are:

$$\frac{\partial U}{\partial h_{t+1}} = \frac{\varphi(\overline{h}_t)}{\overline{h}_{t+1}} - \frac{(1-s_t)\frac{1}{\beta}\left(\overline{h}_{t+1}-1\right)^{\frac{1}{\beta}-1}}{(1-\tau_t)\overline{w}_t - \overline{k}_{t+1} - (1-s_t)\left(\overline{h}_{t+1}-1\right)^{\frac{1}{\beta}}} = 0 \qquad (11)$$

$$\frac{\partial U}{\partial k_{t+1}} = \frac{\gamma}{\overline{k}_{t+1}} - \frac{1}{(1-\tau_t)\overline{w}_t - \overline{k}_{t+1} - (1-s_t)\left(\overline{h}_{t+1}-1\right)^{\frac{1}{\beta}}} = 0. \qquad (12)$$

Applying the same reasoning as in the proof of lemma 1 it is easy to see that the first order conditions characterize the solution.

Is it possible to find a sequence of subsidy schemes $\{s_t, \tau_t\}_{t=1}^{\infty}$ such that the sequence of socially optimal choices is induced? For given initial endowment with physical and human capital such a sequence would have to induce a sequence of individual choices $\{\overline{k}_{t+1}, \overline{h}_{t+1}\}_{t=1}^{\infty}$ such hat $\overline{k}_{t+1} = \widehat{k}_{t+1}$ and $\overline{h}_{t+1} = \widehat{h}_{t+1}$ for all periods. Furthermore, the sequence of schemes would have to respect the balanced budget condition (10) in all periods.

The following proposition shows that there exists indeed a sequence of subsidy schemes that induces an optimal outcome.

Proposition 4 *For equal initial conditions $\overline{h}_1 = \widehat{h}_1$ and $\overline{k}_1 = \widehat{k}_1 > 0$ it holds that:*

i) The sequence of subsidy schemes $\{s_t, \tau_t\}_{t=1}^{\infty}$ defined by

$$s_t = \frac{\beta}{(\gamma+1)} \frac{\left(\widehat{w}_t - \left(\widehat{h}_{t+1}-1\right)^{\frac{1}{\beta}}\right)}{\left(\widehat{h}_{t+1}-1\right)^{\frac{1}{\beta}-1}} \frac{\omega_{t+1}}{\omega_t} \left(\xi_t + \varphi'(\widehat{h}_{t+1}) \ln \widehat{h}_{t+2}\right) \qquad (13)$$

and

$$\tau_t = s_t \frac{(\widehat{h}_{t+1}-1)^{\frac{1}{\beta}}}{\widehat{w}_t} \qquad (14)$$

induces a sequence of choices $\{\overline{k}_{t+1}, \overline{h}_{t+1}\}_{t=1}^{\infty}$ such that $\overline{k}_{t+1} = \widehat{k}_{t+1}$ and $\overline{h}_{t+1} = \widehat{h}_{t+1}$ in all t.

ii) $\{s_t, \tau_t\}_{t=1}^{\infty}$ *respects the balanced budget condition in all periods.*
iii) For all t*,* $0 < s_t < 1$.

Proof: see Appendix

The above proposition shows that an appropriate subsidy scheme can ensure efficiency. The optimal subsidy rate is always strictly larger than zero, so a permanent subsidy is necessary to achieve efficiency. The optimal rate in period t, however, depends on the optimal values of human and physical capital in periods t, $t+1$, and $t+2$. Since nothing guarantees that these optimal human and physical capital values are constant over time, s_t might vary over time accordingly.

5 Conclusions

We have shown that the private allocation of resources leads to inefficient human capital formation. This result rests on the empirical fact that intergenerational family relations are characterized by indirect reciprocity. The willingness of the parents to finance the human capital formation of their children depends on the investments their parents have made into their education. Taking this into account, an economy might exhibit multiple steady states. The economy might get trapped in a low education steady state, where a low education level of the parents leads to negligence of the children's education, reproducing the low education level in the next generation. To overcome such a 'bad' steady state, a temporary subsidy on education is required.

However, even if the economy is not trapped in such a bad steady state, the purely private financing of the education system leads to inefficiencies. This inefficiency can be overcome by a permanent public support for the education of children. This conclusion requires some qualifications. First, if the economy is not in a steady state, the efficient tax and subsidy rates might change from period to period. For political reasons as well as for lack of information, it may be difficult to make these necessary adjustments. Second, the optimal subsidy rate depends on the weight the social planner puts on the different generations. Hence, there is room for intergenerational conflicts. Finally, our model is based on the assumption that labor supply is fixed. Hence, the taxation of wage income does not create any excess burden on the labor market. If labor supply is elastic and if a non-distortive tax is not available, a trade-off exists between the inefficiency created by the tax system and the inefficiency due to the externalities in the human capital formation.

Notwithstanding these qualifications, it can be concluded that the broadening of the model of human behavior to allow for more complex intergenerational relations leads to inefficiencies that have been neglected so far. The analysis thus gives further support for government intervention to support an optimal investment into the education of our children in order to achieve a maximum amount of welfare.

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6 Appendix

6.1 Proof of Lemma 1

Note first that $\widetilde{h}_{t+1}$ and $\widetilde{k}_{t+1}$ have to be finite for all finite values of $(\widetilde{k}_t, \widetilde{h}_t)$. The utility function is strictly quasiconcave, implying a unique solution, which might be either interior (in which case the first order conditions hold) or at the lower bounds. By (3) $\widetilde{w}_t > 0$ whenever $\widetilde{k}_t > 0$. Furthermore, $\widetilde{w}_t - \widetilde{k}_{t+1} = \widetilde{c}_{2,t} + \left(\widetilde{h}_{t+1} - 1\right)^{\frac{1}{\beta}} \geq \widetilde{c}_{2,t} > 0$ due to the INADA condition of the utility function with respect to the consumption levels. This implies that $\frac{\partial U}{\partial k_{t+1}} = \infty$ at $k_{t+1} = 0$. This requires that the condition (7) as well as ii) must hold.

As for the solution for the human capital, note first that for $\widetilde{h}_t = 1$, $\frac{\partial U}{\partial h_{t+1}} = 0$ at $h_{t+1} = 1$. This gives iii) and that condition (6) holds in this case.

If $\widetilde{h}_t > 1$, $\frac{\partial U}{\partial h_{t+1}} = \infty$ at $h_{t+1} = 1$, implying $\widetilde{h}_{t+1} > 1$. This gives iv) and that condition (6) holds also for $\widetilde{h}_t > 1$, which completes the proof. ■

6.2 Proof of Proposition 2

We first introduce the following dynamic system, denoted as upper bound economy and by superscript b, which will be useful for the proof:

$$h^b_{t+1} = (1-\alpha)^{\beta}(k^b_t)^{\alpha\beta}(h^b_t)^{(1-\alpha)\beta} + 1 \tag{15}$$

$$k^b_{t+1} = (1-\alpha)(k^b_t)^{\alpha}(h^b_t)^{(1-\alpha)} \tag{16}$$

The proof now proceeds in three steps. In the first step, we will show that for the same initial conditions for human and physical capital the path of the upper bound economy provides an upper bound for the path of the economy we analyze. In the second step, we will show that the upper bound economy exhibits a globally stable steady state, to which the system converges from any initial conditions. In the third step we will use this steady state to finalize the proof.

Step 1: If the indirect reciprocity economy and the upper bound economy start at the same initial conditions $k_1^b = \widetilde{k}_1 > 0$ and $h_1^b = \widetilde{h}_1$, it holds that:

$$\begin{aligned} k_t^b &\geq \widetilde{k}_t \text{ for all } t > 1 \\ h_t^b &\geq \widetilde{h}_t \text{ for all } t > 1. \end{aligned}$$

The proof is made by induction. For the same initial conditions $k_1^b = \widetilde{k}_1$ and $h_1^b = \widetilde{h}_1$, the definition of the upper bound economy, (3), and (4) give

$$k_2^b = (1-\alpha)(k_1^b)^\alpha (h_1^b)^{(1-\alpha)} = (1-\alpha)(\widetilde{k}_1)^\alpha (\widetilde{h}_1)^{(1-\alpha)} = \widetilde{w}_1 \geq \widetilde{k}_2$$

and

$$\begin{aligned} h_2^b &= (1-\alpha)^\beta (k_1^b)^{\alpha\beta} (h_1^b)^{(1-\alpha)\beta} + 1 \\ &= (1-\alpha)^\beta (\widetilde{k}_1)^{\alpha\beta} (\widetilde{h}_t^b)^{(1-\alpha)\beta} + 1 \\ &= (\widetilde{w}_1)^\beta + 1 \geq (\widetilde{e}_1)^\beta + 1 = \widetilde{h}_2. \end{aligned}$$

So $k_2^b \geq \widetilde{k}_2$ and $h_2^b \geq \widetilde{h}_2$. It is obvious that h_{t+1}^b and k_{t+1}^b are monotonically increasing in h_t^b and k_t^b. This implies that $k_{t+1}^b \geq \widetilde{k}_{t+1}$ and $h_{t+1}^b \geq \widetilde{h}_{t+1}$ whenever $k_t^b \geq \widetilde{k}_t$ and $h_t^b \geq \widetilde{h}_t$, which completes the proof of Step 1.

Step 2: For any initial condition $k_1^b > 0$ [4] the upper bound economy converges to a unique stable state k^{b*}, h^{b*} with $k^{b*} > 0$ and $h^{b*} > 1$.

To show this, note first that $k_1^b > 0$ implies that $k_t^b > 0$ and $h_t^b > 1$ for all $t > 1$. From the definition of the upper bound economy we get

$$h_{t+1}^b = \left(k_{t+1}^b\right)^\beta + 1,$$

implying that

$$h_t^b = \left(k_t^b\right)^\beta + 1.$$

Hence, equation of motion of the upper bound economy is characterized by:

$$k_{t+1}^b = (1-\alpha)\,(k_t^b)^\alpha \left(\left(k_t^b\right)^\beta + 1\right)^{(1-\alpha)}. \tag{17}$$

Differentiating we get

$$\frac{\partial k_{t+1}^b}{\partial k_t^b} = \left[(1-\alpha)\,(k_t^b)^{(\alpha-1)}\left((k_t^b)^\beta + 1\right)^{(-\alpha)}\right]\left[(\alpha\,(1-\beta)+\beta)\,(k_t^b)^\beta + \alpha 1\right] > 0$$

[4] Recall that we restrict our analysis to the nontrivial case of $\widetilde{k}_1 > 0$, which of course implies that $k_1^b > 0$.

and

$$\frac{\partial^2 k_{t+1}^b}{\partial k_t^b \partial k_t^b} = \left[-(1-\alpha)^2 (k_t^b)^{\alpha-2}\left((k_t^b)^\beta+1\right)^{(-1-\alpha)}\right] \left[\alpha(1-\beta)+\beta(1-\beta)(k_t^b)^{2\beta}+(1-\beta)(2\alpha+\beta)(k_t^b)^\beta+\alpha\right]<0$$

since

$$-(1-\alpha)^2 (k_t^b)^{\alpha-2}\left((k_t^b)^\beta+1\right)^{(-1-\alpha)} < 0$$
$$\alpha(1-\beta)+\beta(1-\beta)(k_t^b)^{2\beta} > 0$$
$$(1-\beta)(2\alpha+\beta)(k_t^b)^\beta > 0$$
$$\alpha > 0.$$

Hence, the equation of motion (17) is strictly monotone and concave in k_t. This and the fact that the system has a steady state at $k^b = 0$ implies that there is at most one other steady state with $k^b > 0$.

To investigate the possibility of steady states with $k^{b*} > 0$, we set $k_t^b = k_{t+1}^b = k^{b*}$ in (17) and get:

$$k^{b*} = (1-\alpha)(k^{b*})^\alpha[(k^{b*})^\beta+1]^{(1-\alpha)},$$

implying:

$$(k^{b*})^{\frac{1}{(1-\alpha)}} = (1-\alpha)(k^{b*})^{\frac{\alpha}{(1-\alpha)}}[(k^{b*})^\beta+1].$$

Dividing by $(k^{b*})^{\frac{1}{(1-\alpha)}}$ leads to

$$1 = (1-\alpha)(k^{b*})^{(-1)}[(k^{b*})^\beta+1]$$
$$1 = (1-\alpha)[(k^{b*})^{(\beta-1)}+(k^{b*})^{(-1)}]. \quad (18)$$

The right hand side of equation (18) is continuos and strictly monotonically decreasing in k^{b*}. Furthermore,

$$\lim_{k^{b*}\to 0}(1-\alpha)[(k^{b*})^{(\beta-1)}+(k^{b*})^{(-1)}] = \infty$$
$$\lim_{k^{b*}\to \infty}(1-\alpha)[(k^{b*})^{(\beta-1)}+(k^{b*})^{(-1)}] = 0.$$

Hence, there exists a unique $k^{b*} > 0$ fulfilling (18) characterizing the second steady state of the upper bound economy. The steady state value of h is given by

$$h^{b*} = (k^{b*})^\beta + 1 > 1$$

Recall that the equation of motion (18) is strictly monotone and concave. Hence, $k_{t+1}^b > k_t^b$ whenever in $k_t^b < k^{b*}$ and $k_{t+1}^b < k_t^b$ whenever in $k_t^b > k^{b*}$. Therefore, the upper bound economy converges to the steady state k^{b*}, h^{b*} whenever $k_1^b > 0$.

Step 3: For any initial conditions $\widetilde{k}_1 \geq 0$ and $\widetilde{h}_1 \geq 0$ there exists a t^m such that:

$$\begin{aligned} \widetilde{h}_t &< h^{b*} + 1 \text{ whenever } t > t^m \\ \widetilde{k}_t &< k^{b*} + 1 \text{ whenever } t > t^m \\ \widetilde{w}_t &< (1-\alpha)(k^{b*}+1)^{\alpha}(h^{b*}+1)^{1-\alpha} \text{ whenever } t > t^m \end{aligned}$$

Step 3) follows immediately from Step 1), Step 2), and the wage equation 3). ■

6.3 Proof of Lemma 3

Note first that $\widehat{h}_{t+1}$ and $\widehat{k}_{t+1}$ have to be finite for all finite values of $(\widehat{k}_t, \widehat{h}_t)$. The solution might be either interior (in which case the first order conditions hold) or at the lower bounds. Since $\widehat{k}_t > 0$, production takes place in period t. Combining this fact with the INADA condition of the individual utility functions implies that $\widehat{k}_t^{\alpha}\widehat{h}_t^{1-\alpha} - \widehat{k}_{t+1} - \left(\widehat{h}_{t+1} - 1\right)^{\frac{1}{\beta}} - \widehat{k}_t\widehat{r}_t = c_{2,t} > 0$. Therefore, $\frac{\partial W}{\partial k_{t+1}} = \infty$ at $k_{t+1} = 0$. Hence, $\widehat{k}_{t+1} > 0$, and condition (9) holds.

As for the solution for the human capital, note again that since $\widehat{k}_t > 0$, production takes place in period t. Again, combining this fact with the Inada condition of the individual utility functions implies that $\widehat{k}_{t+1}^{\alpha}\widehat{h}_{t+1}^{1-\alpha} - \widehat{k}_{t+2} - \left(\widehat{h}_{t+2} - 1\right)^{\frac{1}{\beta}} - \widehat{k}_{t+1}r_{t+1} = c_{2,t+1} > 0$. Hence $\frac{\partial W}{\partial h_{t+1}} = \infty$ at $h_{t+1} = 1$, implying $\widehat{h}_{t+1} > 1$. This gives condition (8), which completes the proof. ■

6.4 Proof of Proposition 4

i) By combining (11), (12) and the balanced budget condition of the government (10) one gets:

$$\frac{\varphi(\overline{h}_t)}{\overline{h}_{t+1}} - (1-s_t)\frac{(\gamma+1)}{\beta}\frac{\left(\overline{h}_{t+1}-1\right)^{\frac{1}{\beta}-1}}{\left(\overline{w}_t - \left(\overline{h}_{t+1}-1\right)^{\frac{1}{\beta}}\right)} = 0. \tag{19}$$

Furthermore combining (8) and (9) gives:

$$\frac{\varphi(\widehat{h}_t)}{\widehat{h}_{t+1}} - \frac{(\gamma+1)}{\beta}\frac{\left(\widehat{h}_{t+1}-1\right)^{\frac{1}{\beta}-1}}{\widehat{w}_t - \left(\widehat{h}_{t+1}-1\right)^{\frac{1}{\beta}}} + \frac{\omega_{t+1}}{\omega_t}\left(\xi_t + \varphi'(\widehat{h}_{t+1})\ln \widehat{h}_{t+2}\right) = 0. \quad (20)$$

In order to establish the optimal subsidy rate which ensures that $\overline{h}_{t+1} = \widehat{h}_{t+1}$ we set (19) equal to (20), furthermore set $\overline{h}_{t+1} = \widehat{h}_{t+1}$ and solve for s_t.

$$\begin{aligned} & \frac{\varphi(\widehat{h}_t)}{\widehat{h}_{t+1}} - (1-s_t)\frac{(\gamma+1)}{\beta}\frac{\left(\widehat{h}_{t+1}-1\right)^{\frac{1}{\beta}-1}}{\widehat{w}_t - \left(\widehat{h}_{t+1}-1\right)^{\frac{1}{\beta}}} \\ = \quad & \frac{\varphi(\widehat{h}_t)}{\widehat{h}_{t+1}} - \frac{(\gamma+1)}{\beta}\frac{\left(\widehat{h}_{t+1}-1\right)^{\frac{1}{\beta}-1}}{\widehat{w}_t - \left(\widehat{h}_{t+1}-1\right)^{\frac{1}{\beta}}} + \frac{\omega_{t+1}}{\omega_t}\left(\xi_t + \varphi'(\widehat{h}_{t+1})\ln \widehat{h}_{t+2}\right). \end{aligned}$$

This can be written as:

$$\begin{aligned} & \frac{\varphi(\widehat{h}_t)}{\widehat{h}_{t+1}} - \frac{(\gamma+1)}{\beta}\frac{\left(\widehat{h}_{t+1}-1\right)^{\frac{1}{\beta}-1}}{\widehat{w}_t - \left(\widehat{h}_{t+1}-1\right)^{\frac{1}{\beta}}} + s_t\frac{(\gamma+1)}{\beta}\frac{\left(\widehat{h}_{t+1}-1\right)^{\frac{1}{\beta}-1}}{\widehat{w}_t - \left(\widehat{h}_{t+1}-1\right)^{\frac{1}{\beta}}} \\ = \quad & \frac{\varphi(\widehat{h}_t)}{\widehat{h}_{t+1}} - \frac{(\gamma+1)}{\beta}\frac{\left(\widehat{h}_{t+1}-1\right)^{\frac{1}{\beta}-1}}{\widehat{w}_t - \left(\widehat{h}_{t+1}-1\right)^{\frac{1}{\beta}}} + \frac{\omega_{t+1}}{\omega_t}\left(\xi_t + \varphi'(\widehat{h}_{t+1})\ln \widehat{h}_{t+2}\right). \end{aligned}$$

From which it follows:

$$s_t\frac{(\gamma+1)}{\beta}\frac{\left(\widehat{h}_{t+1}-1\right)^{\frac{1}{\beta}-1}}{\widehat{w}_t - \left(\widehat{h}_{t+1}-1\right)^{\frac{1}{\beta}}} = \frac{\omega_{t+1}}{\omega_t}\left(\xi_t + \varphi'(\widehat{h}_{t+1})\ln \widehat{h}_{t+2}\right).$$

Solving for s_t gives:

$$s_t = \frac{\beta}{(\gamma+1)}\frac{\widehat{w}_t - \left(\widehat{h}_{t+1}-1\right)^{\frac{1}{\beta}}}{\left(\widehat{h}_{t+1}-1\right)^{\frac{1}{\beta}-1}}\frac{\omega_{t+1}}{\omega_t}\left(\xi_t + \varphi'(\widehat{h}_{t+1})\ln \widehat{h}_{t+2}\right).$$

ii) It is obvious that (14) implies that the balanced budget condition is fulfilled whenever $\overline{h}_{t+1} = \widehat{h}_{t+1}$.

iii) Since $\xi_t > 0$, $\varphi'(\widehat{h}_{t+1}) > 0$, $\widehat{h}_{t+2} > 1$, and $\widehat{w}_t - \left(\widehat{h}_{t+1} - 1\right)^{\frac{1}{\beta}} = c_{2,t} + k_{t+1} > 0$, the optimal subsidy rate $s_t > 0$.

On the other hand, if $s_t = 1$ the price parents have to pay for the human capital of the children would be zero. Therefore, demand for education would be infinite, which is of course not feasible. Hence, the optimal subsidy rate must fulfill $0 < s_t < 1$. ■