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ABSTRACT

Understanding the Link between Money Growth and Inflation in the Euro Area*

Announced in the autumn of 1998, the monetary policy strategy of the European Central Bank (ECB) quickly became controversial, arguably because the ECB provided neither an explicit representation of the inflation process nor an explanation for why it necessitated the adoption of a two-pillar framework. Several reduced-form empirical models that seek to do so have subsequently been presented in the literature. The hallmark of these models is the hypothesis that inflation can be decomposed into a 'trend', which is explained by a smoothed measure of past money growth, and a deviation from that trend, which is accounted for by the output gap. In this paper we survey this literature, discuss how it relates to the monetary transmission mechanism and extend the inflation equations by introducing cost-push shocks. We find that changes in import prices, oil prices and exchange rates are statistically significant in euro-area inflation equations but that they leave intact the earlier findings that money growth and the output gap matter.

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Introduction

In preparation for the establishment of European Monetary Union in January 1999, the European Central Bank (ECB) decided to adopt a monetary policy strategy consisting of two main elements or 'pillars'. The first of these was *'a prominent role for money with a reference value for the growth of a monetary aggregate'*, subsequently defined to be 4.5 per cent annual growth of M3, and the second *'a broadly-based assessment of the outlook for future price developments'*.[1] From the outset this two-pillar framework was controversial. One explanation for this might have been that the ECB provided neither an explicit representation of the inflation process nor a motivation for why it necessitated a two-pillar framework. Whatever the reasons, many observers misinterpreted the two pillars as combining monetary and inflation targeting, and criticized the framework for being inconsistent and lacking clarity.

Recently several authors have presented empirical models that provide a formal interpretation of the two pillars by incorporating money growth in a reduced-form Phillips-curve model for inflation. The monetary and the economic pillars of the ECB's framework are in these models viewed as reflecting different time perspectives of the determination of inflation. While money growth impacts on inflation in the long run, real economic indicators such as the output gap and cost-push factors influence inflation in the short run. This notion of different time horizons in the determinants of inflation is emphasised by the ECB in its recent review of the monetary policy strategy. For instance, in an article in the June 2003 Monthly Bulletin on the outcome of its evaluation of the strategy, the ECB (2003, p. 87) writes:

> *An important argument in favour of adopting the two-pillar approach relates to the difference in the time perspective for analysing price developments. The inflation process can be broadly decomposed into two components, one associated with the interplay between demand and supply factors at high frequency, and the other connected to more drawn-out and persistent trends. The latter is empirically closely associated with the medium-term trend growth of money.*

Of course, the ECB is not the only central bank that finds it useful to distinguish between frequency bands in analysing inflation. In discussing the new monetary concept introduced by the Swiss National Bank in 2000, Jordan, Peytrignet and Rich (2001, p. 48) emphasise the importance of the time horizon in analysing inflation:[2]

> *The SNB … continues to monitor two sets of indicators providing leading information on future price developments …. The first set of indicators is useful for forecasting short-run price developments …. It includes various indicators on the cyclical state of the economy, notably the output gap and supply and demand conditions in the labour market, as well as the real exchange rate of the Swiss franc. The second set of indicators comprises the monetary aggregates, which provide useful leading information on long-run price developments. […] Both sets of indicators are used together with the forecasts from various econometric models to produce a broadly based consensus inflation forecast, which now forms the centre stage of Swiss monetary policy.*

In this paper we review the literature that seeks to formalise this understanding of the inflation process and present some additional evidence on the determination of inflation in the euro area. The paper is organised as follows. The next section starts by providing a highly stylized example of why it may be difficult to detect the impact of money growth on inflation. The essence of the argument is that changes in the average (or 'trend') growth rate of money over some period of time determine the average rate of inflation in the same period. However, changes in this trend occur gradually and may therefore be difficult to identify. Moreover, since only a small part of variations in 'headline' money growth are due to changes in the trend, it is not surprising that 'headline' money growth tends to be insignificant in inflation equations. Thus, econometric work may underestimate the importance of money for inflation. The section goes on to review the papers of Gerlach (2003), Neumann (2003), Neumann and Greiber (2004), Gerlach (2004) and Assenmacher-Wesche and Gerlach (2006), which focus on this mechanism. The following two sections presents the econometric methodology, the data and the results.[3] Extending previous work, we offer new empirical evidence on the determinants of inflation at different time horizons. We find that changes in the exchange rate, the price of oil and import prices are statistically highly significant in euro-area inflation equations but that they do not appear to change the earlier findings that money growth and the output gap matter. While the output gap is informative for frequencies above one year, our analysis shows that the cost-push variables can explain inflation at higher frequencies. The final section concludes.

Modelling the Two Pillars

As a preliminary, it is useful to consider a simple empirical Phillips curve of the form:

$$\pi_t = \alpha_0 + \alpha_\pi \pi_{t-1} + \alpha_g g_{t-1} + \varepsilon_t \qquad (1)$$

according to which inflation, π_t, depends on its own lagged value, the lagged output gap, g_{t-1}, and a residual, ε_t, which can be interpreted as capturing cost-push shocks. Next, we consider the average (or 'trend') rate of inflation in a time span which is sufficiently long for the output gap and the residual to average to zero:

$$\bar{\pi}_t = \alpha_0 / (1 - \alpha_\pi) \qquad (2)$$

where a bar, '–', denotes the average value. Before interpreting equation (2), note that the deviation of inflation from its average rate in the time period in question is given by:

$$(\pi_t - \bar{\pi}_t) = \alpha_\pi (\pi_{t-1} - \bar{\pi}_t) + \alpha_g g_{t-1} + \varepsilon_t\,. \qquad (3)$$

According to equation (2) and provided that the persistence in the inflation process, α_π, is constant, the average rate of inflation over the period considered is captured by the constant in the Phillips curve, α_0. Thus, in periods when inflation was high, such as in the 1970s and the early 1980s, estimates of equation (1) will yield a relatively large constant. Furthermore, in periods when the average rate of inflation was relatively low, such as the late 1990s and early 2000s, the constant will be small. Econometrically, this model may 'explain' movements in inflation quite well if estimated in a sample period in which the average rate of inflation is well-defined and constant. Economically, however, it will be much less satisfactory in that it is unable to account for changes in the average rate of inflation over time. For it to be able to do so, some hypothesis about the determination of α_0 must be added. To anticipate the discussion below, models that seek to motivate the use of the two-pillar strategy argue that the average growth rate of money, perhaps adjusted for the average growth rate of output and changes in velocity, plays an important role in determining α_0.[4] Thus, changes in the average rate of inflation are attributed to changes in monetary conditions.[5]

Equation (3) holds that variations of inflation around its mean are due to movements in the output gap and to supply shocks. Interestingly, if the average rate of inflation is constant in the sample, perhaps because detrended data are used, variables that determine average inflation will be insignificant if included in the regression, leading the econometrician to infer incorrectly that they play no role in the inflation process. With this as background we turn to a discussion of the different models that have been proposed as interpretations of the two-pillar strategy.

Gerlach (2003)

In what may be the first paper that sought to provide a formal description of the two-pillar framework, Gerlach (2003) used a simple exponential filter to compute an estimate, $\tilde{\pi}_t$, of $\bar{\pi}_t$ and to compute a similar estimate, $\tilde{\mu}_t$, of the average or trend growth rate of money $\bar{\mu}_t$. For some time series x_t, the filter is given by:

$$\tilde{x}_t = \tilde{x}_{t-1} + \lambda\left(x_t - \tilde{x}_{t-1}\right) \tag{4}$$

or

$$\tilde{x}_t = \lambda \sum_{j=0}^{\infty} \left(1-\lambda\right)^j x_{t-j} \tag{5}$$

Equation (5) warrants four comments. First, Cogley (2002) shows that depending on the choice of the smoothing parameter, λ, the filter removes the high-frequency variation of a time series. Gerlach (2003) consequently thinks of $\tilde{x}_t$ as an estimate of the low-frequency component of the series. The smoothing parameter captures the speed by which a once-and-for-all change in x_t impacts on $\tilde{x}_t$. In particular, ln(2)/λ captures the half-life of this adjustment. Gerlach (2003) conducts the analysis under the assumption that λ = 0.075, which correspond to a half-life of about 9.2 quarters. Second, there is no reason to assume that this estimate of $\bar{x}_t$ is optimal. In fact, much of the work in the subsequent literature has aimed at constructing better estimates of, or using better methods to model, the low-frequency components of inflation and the other variables involved in the analysis.

Third, since the estimate of the trend is an infinite moving average of the data, it evolves slowly over time. Given the value of the smoothing parameter used by Gerlach (2003), the ratio of the variance of changes in $\tilde{x}_t$ to the variance of changes in x_t is trivially small.[6] Headline data therefore contain little information about fluctuations in the trend.

Fourth, this filter is one-sided in that it only uses past information to form an estimate of the low-frequency component of the series. This has the notable advantage of making the filter operational in real time.

Using synthetic quarterly data for the euro area for 1980-2001, Gerlach goes on to estimate a version of equation (3) that incorporates two lags of inflation and the contemporaneous output gap among the regressors, and finds that the output gap has a large impact on $\pi_t - \tilde{\pi}_t$, which he interprets as a measure of the high-frequency component of inflation. Next he assumes that the inflation trend is determined in accordance with the quantity theory:

$$\tilde{\pi}_t = \theta_0 + \theta_1(\tilde{\mu}_t - \tilde{\gamma}_t) + \zeta_t \qquad (6)$$

where $\tilde{\mu}_t$ denotes the low-frequency component of money growth and $\tilde{\gamma}_t$ that of output growth, and where $\theta_1 = 1$ if shocks to velocity are uncorrelated with money growth. By combining equations (3) and (6), Gerlach obtains the estimate $\hat{\theta}_1$ = 0.84 (with a standard error of 0.09), which is compatible with the notion that trend money growth determines the average rate of inflation in some time span.

To understand the link between this model and the two-pillar strategy of the ECB, note that the model augments a standard reduced-form empirical Phillips curve with a measure of the low-frequency component of money growth, which is obtained by filtering money growth. Sustained changes in money growth will shift the Phillips curve vertically, generating changes in the steady-state rate of inflation. The ECB's monetary pillar can therefore be best seen as an approach to predict the average rate of inflation over some time period. By contrast, changes in the output gap, which by construction are temporary, will generate variations in inflation around that steady state. The economic analysis, the ECB's second pillar, can thus be seen as a method to predict short-run movements in inflation around its average level at a point in time, which highlights that the two pillars pertain to different time horizons.

Neumann (2003)

Neumann (2003) follows the analysis of Gerlach (2003), but sharpens it in important ways. In particular, he assumes that current inflation depends on inflation expectations formed last period:

$$\pi_t = \pi_t^e + \alpha_g g_{t-1} + \varepsilon_t \tag{7}$$

and adds an expectations-formations mechanism of the form:

$$\pi_t^e = \overline{\pi}_{t-1} + \alpha_e \left(\pi_{t-1} - \overline{\pi}_{t-1}\right), \tag{8}$$

where $\overline{\pi}_t$ is referred to as the 'core' rate of inflation. From a standard money demand relationship, he derives that:

$$\overline{\pi}_t = \overline{\mu}_t - \alpha_\gamma \overline{\gamma}_t . \qquad (9)$$

Thus, the core rate of inflation depends on the 'core' rate of money growth minus the 'core' rate of output growth, multiplied by the income elasticity of money demand, α_γ. Thus, he releases the assumption that the income elasticity of money demand is unity. This leads to an inflation equation of the form:

$$\pi_t = (1-\alpha_e)(\overline{\mu}_{t-1} - \alpha_\gamma \overline{\gamma}_{t-1}) + \alpha_e \pi_{t-1} + \alpha_g g_{t-1} + \varepsilon_t . \qquad (10)$$

To fit this equation, estimates of the core rates of money and output growth are required. To obtain these Neumann (2003) uses the Hodrick-Prescott (HP) filter. Since this filter is two-sided, incorporating both past and future values of the filtered variable, it seems plausible that it leads to better estimates of the low-frequency component than a one-sided filter. Of course, if the purpose is to analyse the role of money growth for inflation in the past, this is desirable. By contrast, if the purpose is to forecast future inflation, a one-sided filter may be better.[7]

A second reason why a one-sided filter may be preferable is that a two-sided filter can lead to inconsistent estimates of the model's parameters. Least squares estimators are only consistent if the error term is uncorrelated with the explanatory variables. In the case of a two-sided estimate of $\overline{\mu}_t$, this requires future observations of μ_t to be uncorrelated with π_t. Thus, there may be no feedback from inflation to money growth. This problem does not arise with a one-sided filter.[8]

Neumann (2003) proceeds to estimate the model on quarterly data for the period 1986-2002 and finds that it fits well, in particular when he controls for the change in velocity observed in the second half of 2001.

Neumann and Greiber (2004)

Neumann and Greiber (2004) extend the analysis in Neumann (2003) by incorporating oil prices as a measure of cost-push shocks, and find them to be highly significant. More importantly, they use four methods to compute core money growth: the Hodrick-Prescott filter, the exponential filter of Cogley (2002), the band-pass filter of Baxter and King (1999) and wavelet analysis.[9] They conclude that these filters lead to similar estimates of core money growth, but conclude that the exponential filter performs less well since it leads to an implausibly high estimate of the income elasticity of money demand.[10] Interestingly, the output gap is only significant when the exponential filter is used, and is therefore dropped from the other regressions. Neumann and Greiber (2004) also study what frequency bands of money growth are most important and conclude that fluctuations of a periodicity of less than eight years do not seem to matter for inflation.

Gerlach (2004)

Gerlach (2004) provides a more refined version of the Gerlach (2003) model. First, the Phillips curve is extended to include expectations of future inflation, π^e_{t+1}:

$$\pi_t = (1-\alpha_\pi)\pi^e_{t+1} + \alpha_\pi \pi_{t-1} + \alpha_g g_{t-1} + \varepsilon_t . \tag{11}$$

Second and as asserted by the ECB, the expected future rate of inflation is assumed to depend on trend money growth, $\tilde{\mu}_t$, using the exponential filter discussed above.[11] Under these assumptions it is possible to derive an inflation equation of the form

$$\pi_t = \beta_1 \mu_{t-1} + \beta_2 g_{t-1} + \beta_3 \pi_{t-1} + \beta_4 g_{t-2} + \beta_5 \pi_{t-2} + e_t , \tag{12}$$

where the β-parameters are functions of the α-parameters in equation (11) and the smoothing parameter λ in equation (5), and where the residuals obey a first-order moving-average process with a coefficient of – (1–λ). The smoothing parameter can consequently be estimated jointly with the other parameters of the model. Fitting the equation on the sample period 1992-2003, Gerlach (2004) does not reject the overriding restrictions and finds that inflation in the euro area appears to be entirely forward looking. The estimated value of the smoothing parameter is 0.09, with a standard error of 0.02, which is close to the value assumed by Gerlach (2003). Furthermore, the data prefer a specification in which past money growth rates, rather than past inflation rates, determine inflation expectations.

Of course, the assumption that money growth determines inflation expectations should be taken with a grain of salt, and the equation is arguably best interpreted as a reduced-form forecasting model. However, it does establish that past low-frequency movements in money growth contain information that is useful for understanding inflation, and that this information is not embedded in past inflation rates.

Assenmacher-Wesche and Gerlach (2006)

The studies reviewed above use relatively simple approaches to construct measures of 'trend', 'core' or the 'low-frequency' components of inflation, money growth and output growth. Moreover, with the exception of Gerlach (2004), these measures are all computed in a step preliminary to estimation. Since the ECB has motivated the two-pillar strategy by arguing that the determinants of inflation vary by time horizon or frequency, Assenmacher-Wesche and Gerlach (2006), AWG in what follows, find it natural to use band spectral regression methods that integrate the filtering and estimation stages. More importantly, the use of advanced econometric techniques makes it possible to study the inflation process in the euro area at specific frequencies and thus permits a better understanding of the role of money in the inflation process.

Since we extend AWG's analysis below, it is useful to review it in some detail. AWG start by decomposing inflation, π_t, into low-frequency (*LF*) and high-frequency (*HF*) components:

$$\pi_t = \pi_t^{LF} + \pi_t^{HF}, \tag{13}$$

and hypothesise that the high-frequency movements of inflation are related to movements in the output gap:

$$\pi_t^{HF} = \alpha_g g_{t-1} + \varepsilon_t^{HF}. \tag{14}$$

By construction, the output gap has no low-frequency variation, which implies that it can at most explain temporary changes in the rate of inflation. Next, AWG assume that the low-frequency variation of inflation can be understood in terms of the quantity theory of money:[12]

$$\pi_t^{LF} = \alpha_\mu \mu_t^{LF} + \alpha_\gamma \gamma_t^{LF} + \alpha_v v_t^{LF} + \varepsilon_t^{LF}, \tag{15}$$

where v_t denotes the rate of change of velocity, which they assume depends on the change of the long-term interest rate, ρ_t:[13]

$$v_t^{LF} = \tilde{\alpha}_\rho \rho_t^{LF} + \varepsilon_t^{v,LF}. \tag{16}$$

Equation (15) warrants three comments. First, at low frequencies, the growth rate of real output is identical to the growth rate of potential. AWG therefore define the output gap as the (logarithmic) difference between current output and the low-frequency component of output. In this paper, we use the Hodrick-Prescott filter to compute potential output and show that the results are not affected by the particular definition of the output gap. Second, under the quantity theory, and provided that money growth is uncorrelated with velocity shocks at low frequencies (that is, μ_t^{LF} and $\varepsilon_t^{v,LF}$ are orthogonal), we expect that $\alpha_\mu = -\alpha_\gamma = 1$. Third, since velocity is defined using the levels of money, output and prices, equation (15) is in fact an identity and can be written $\pi_t^{LF} \equiv \mu_t^{LF} + \gamma_t^{LF} - v_t^{LF}$. It is the assumption that changes in velocity depend on changes in the long-term interest rates that renders the relationship inexact.

The full model is given by:

$$\pi_t = \alpha_g g_{t-1} + \left\{ \alpha_\mu \mu_t^{LF} + \alpha_\gamma \gamma_t^{LF} + \alpha_\rho \rho_t^{LF} \right\} + \varepsilon_t , \qquad (17)$$

where $\varepsilon_t = \varepsilon_t^{LF} + \alpha_v \varepsilon_t^{v,LF} + \varepsilon_t^{HF}$ and $\alpha_\rho \equiv \alpha_v \tilde{\alpha}_\rho$. According to this model, the average rate of inflation during some period is given by the term in curly brackets, { }, that is, by the low-frequency part of money growth relative to real output growth and changes in the long-term interest rate, which AWG think of as the first pillar. Variation in inflation around that average is determined by movements in the output gap, which is seen as shorthand for the second pillar. In analysing and forecasting inflation it is appropriate to consider low-frequency, as opposed to 'headline', movements in money growth.

Since the present paper extends the analysis in AWG, the results are similar and we therefore do not discuss the details of that paper here.[14] In brief, AWG find that money growth relative to output growth matters at low frequencies, and the output gap at high frequencies, for inflation. Furthermore, testing in the frequency domain, they find that these correlations in both cases reflect causality (in the predictive sense).

Estimation

As noted above, AWG show that quantity-theoretic variables (that is, money and real output growth, and changes in long interest rates) determine inflation in the long run, whereas the output gap accounts for short-run variations of the inflation rate. Neither group of variables, however, is able to account for movements in inflation in the euro area at periodicities of less than 1 or 2 years. One likely explanation is that in the very short run inflation is affected by a range of cost-push shocks stemming from exchange rate movements, changes in oil prices or import prices. It is therefore desirable to extend the analysis by incorporating also these factors.

Following AWG, we hypothesise that the high-frequency movements of inflation are related to once-lagged movements in the output gap, g_t, but also include cost-push factors, c_t:

$$\pi_t^{HF} = \alpha_g g_{t-1} + \alpha_c c_t + \varepsilon_t^{HF} . \qquad (18)$$

Since the transmission from the output gap to inflation takes time, we assume a one-quarter lag.[15] Though we hypothesise that cost-push factors influence inflation at high frequencies, at lower frequencies reverse causality seems plausible. Thus, a change in the overall inflation environment is likely to lead to a depreciation of the exchange rate, and thus to rising import prices, including for oil. We therefore expect to find the cost-push variables also to be significant, but not necessarily causal for inflation, in the low-frequency regressions.

The full model is given by:

$$\pi_t = \alpha_g g_{t-1} + \alpha_c c_t + \left\{\alpha_\mu \mu_t^{LF} + \alpha_\gamma \gamma_t^{LF} + \alpha_\rho \rho_t^{LF}\right\} + \varepsilon_t , \qquad (19)$$

where $\varepsilon_t = \varepsilon_t^{LF} + \alpha_v \varepsilon_t^{v,LF} + \varepsilon_t^{HF}$ and $\alpha_\rho \equiv \alpha_v \tilde{\alpha}_\rho$.

The data

The availability of long time series for the euro area is still limited. To investigate cost-push factors we are therefore restricted to the series available in the database for the ECB's area-wide model (Fagan et al. 2005).[16] The sample period runs from 1970Q1 to 2003Q4. As a preliminary step to the formal econometric analysis below we review the raw data. Since the original HICP data are not seasonally adjusted, we first deseasonalised the rate of inflation by removing a frequency band around the seasonal peaks.[17] This obviates the need to model the seasonal dynamics in the regressions below.

The first column of Figure 1 presents a plot of the quarterly rate of inflation using the seasonally adjusted data, the quarterly rate of money growth as measured by M3, the quarterly change in the government bond yield, and the quarterly rate of real income growth, all for the period 1970Q2 to 2003Q4. The figure shows that inflation rose in the early 1970s and remained high and volatile before declining in the early 1980s. Since the mid-1980s, inflation has fluctuated around a roughly constant level. It is evident that the fall in inflation was associated with a gradual decline in money growth over the sample as central banks took measures to disinflate after the sharp increase in inflation during the 1970s. The change in the long-term interest rate lies slightly above its mean in the 1970s and below thereafter, with persistent fluctuations.[18] Finally, real income growth was quite volatile over the sample. However, there is some evidence that the rate of growth of output has declined, as evidenced by the fact that output growth was below average in most quarters in the 1990s. Next we turn to the output gap, using the Hodrick-Prescott (HP) filter to construct a measure of the trend output.[19] The main movements of the output gap are associated with the large recession around 1974 following the first oil shock, and the recession in 1992-3.

The last three panels in Figure 1 show the cost-push variables, which are all measured as log first differences. While the average change of the nominal effective exchange rate was almost zero in the sample considered, the rise and the subsequent fall of the US dollar against the European currencies in the 1980s can be seen by mainly positive changes during the early 1980s and negative changes in the second half of the 1980s.[20] The graph for oil prices, measured in terms of (synthetic) euros, shows several spikes in the early 1970s, and a peak in 1979 corresponding to the second oil shock.[21] The collapse in oil prices in 1986, which was associated with a sharp fall in inflation in the euro area, is also readily apparent. Turning to the rate of change of import prices, we see clearly the impact of changes in oil prices, suggesting that the series contain similar information. In contrast to the oil price series, the rate of change of import prices shows a slight downward trend, corresponding to the reduction in global inflation during the past thirty years.

Long-run and short-run characteristics of the data

To obtain an impression of the cost-push data and their relation to the rate of inflation, Figure 2 plots the low-frequency and high-frequency part of the series separately. The cut-off between low and high frequencies is chosen to be 4 years. While this choice is arbitrary, AWG show that it provides a reasonable distinction between the long run and the short run.

We consider first the low-frequency behaviour of the cost-push variables in the left-hand panel. Note that in the long run neither exchange rate changes nor oil price changes are closely tied to inflation. By contrast, the rate of change of import prices does decline over time in much the same way as inflation. Turning to the high-frequency band in the right-hand panel, we note that there appears to be a strong positive correlation with inflation. This is compatible with the notion that changes in exchange rates, oil prices and import prices exert cost-push effects on inflation. We next proceed to a more formal analysis of the determinants of inflation at different frequencies.

Unit root tests

Figures 1 and 2 suggest that inflation and money growth may be nonstationary but that the other series appear stationary. Since stationary variables, which by definition do not experience permanent shocks, can not impact on nonstationary variables in the long run, it is of interest to explore more formally these potential differences in the time-series characteristics of the data. We do so by performing Augmented Dickey-Fuller (ADF) tests, Elliot, Stock, and Rotenberg (ERS) tests, Phillips and Perron (PP) tests, and the Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test, which in contrast to the other tests considers stationarity as the null hypothesis.[22] The optimal lag length is determined by the Akaike criterion (AIC), under the assumption that it is at most 8 lags. Table 1 shows that inflation and money growth can be regarded as $I(1)$ whereas the other variables apparently are $I(0)$.[23]

The empirical model discussed above implies that money and output growth and the change in the interest rate are informative for inflation in the long run whereas the output gap and cost-push shocks explain short-run fluctuations in the inflation rate. The results in AWG, however, indicate that for inflation movements with a frequency higher than 4 or 6 quarters, the output gap does not add much information. One potential explanation for this finding is that quarterly output data are often to a large extent estimated and thus contain errors. Furthermore, seasonal effects are only imperfectly removed by the seasonal-adjustment procedures. In the econometric work below, we therefore investigate whether movements in inflation with a periodicity between 2 and 6 quarters can be explained by cost-push factors.

Methodology

Engle (1974) demonstrated that a regression model that is valid in the time domain can be transferred into the frequency domain by taking Fourier transforms of the data and estimating it on the transformed variables. If all frequencies are included, the results are identical to those obtained if the model is fitted in the time domain. However, if some frequencies are excluded, it is possible to test whether the model varies across frequencies. Heuristically, one can think of Engle’s method as filtering the data and regressing the components corresponding to certain frequencies on each other. Thus, removing certain frequencies reduces the number of degrees of freedom. Given that the ECB has emphasised that the determinants of inflation differ across frequencies, it is of particular interest to use band spectrum regression since it allows us to investigate this hypothesis. Thus, at low frequencies we expect the quantity-theoretic variables to play a dominant role for inflation, whereas at high frequencies the output gap and the cost-push variables should be of critical importance.

Engle's estimator requires stationary time series. Since inflation and money growth in contrast to the other variables are nonstationary, we use a two-step approach. First we test for cointegration between money growth and inflation using the Johansen procedure. Cointegration is a long-run property and refers to the relationship between the time series at frequency zero. Thus, a finding that money growth and inflation are cointegrated implies that in the long run the two variables are related. AWG follow an alternative strategy and estimate directly the relationship between inflation and money growth at the zero frequency using the band spectral estimator proposed by Phillips (1991) for nonstationary time series. The advantage of the strategy pursued here is that it is considerably easier to implement.

Results

Cointegration results

Table 2 shows the results from the cointegration analysis of the nonstationary variables, inflation and money growth.[24] The trace statistic indicates the presence of a single cointegrating relation at the 95 per cent significance level. Since the coefficient on money growth is close to one, we test the hypothesis of a unit coefficient with a likelihood-ratio test and find that the restriction is not rejected

($\chi^2(1) = 0.35$, $p = 0.56$). The loading coefficients, α, imply that inflation increases if the linear combination $\mu_{t-1} - \pi_{t-1}$ is positive. In contrast, there is no adjustment from money growth towards equilibrium.[25] We therefore impose the unit restriction on money growth and continue the analysis of the long-run determinants of inflation using the difference between inflation and money growth as the dependent variable.[26] Since output growth, changes in interest rates, the lagged output gap and changes in cost-push variables appear stationary, standard frequency domain methods can be applied.

Band spectrum regression: Low frequencies

Next we turn to the estimates that are obtained using the band spectral regression method of Engle (1974). In Table 3 we consider the results at low frequencies, defined as the frequency band corresponding to periodicities of between 4 years and infinity. As discussed above, the dependent variable in these regressions is the difference between inflation and money growth. The first column shows the results for a regression in which only the quantity-theoretic variables are included. We obtain a coefficient on output growth that is significantly different from zero but not from minus unity. The coefficient on the change in the interest rate is positive and highly significant. This suggests that higher interest rates reduce the demand for money so that, given the rate of money growth, inflation rises.

Of course, the finding that inflation, money growth, output growth and changes in interest rates are closely linked at low frequencies could simply reflect a money demand relationship. If so, the correlation between money growth and inflation would reflect causality from inflation to money. To explore whether this is indeed the case, we next investigate the feedback patterns, relying on the notion of causality introduced by Granger (1969). In the bivariate case, a variable is said to cause inflation if it contains information about future inflation that is not contained in past realisations of inflation. The extent and direction of causality can differ between frequency bands (Granger and Lin, 1995).[27] The empirical analysis below is based on the frequency-wise measure of causality suggested by Geweke (1982) and Hosoya (1991). Since it is important to account for possible feedback from third variables, we include, in addition to money growth and inflation, the other quantity-theoretic variables in the analysis.[28] For causality tests the lag length should neither be too short, since this may cut off significant coefficients, nor too long, since in this case the tests may lack power. We perform the tests with a lag length of 12.[29]

Figure 3 shows the frequency-wise measure of causality, where on the x-axis we have plotted the frequency, ω, measured in fractions of π (so that periodicity, measured in quarters, is given by $2\pi/\omega$).[30] At low frequencies money growth clearly causes inflation while there may be causality from inflation to money growth at periodicities higher than 1 year (that is, 0.5π).[31]

In the second column of Table 3 we include the output gap and all the cost-push variables in addition to the quantity-theoretic variables in the low-frequency regression. The results show again that output growth is significantly different from zero and that the hypothesis that the coefficient is minus unity, as the quantity theory would lead us to expect, cannot be rejected. Furthermore, the output gap is insignificant, indicating that it does not account for inflation at low frequencies. Turning to the cost-push variables, we note that, except for the change in the exchange rate, they are insignificant, as is the change in the interest rate.

Since the three cost-push variables are likely to be correlated, we consider them individually in columns 3 to 5. We find that the parameter on output growth remains significant and close to minus unity. Furthermore, the parameter on the change in the interest rate is positive and significant, except in column 5 in which import prices are used to capture cost-push shocks. Interestingly, the cost-push variables are all significant when included separately.

Again, we investigate the pattern of causality between inflation and the variables that we expect to determine inflation in the medium to short run.[32] Since including all variables together is not possible, we include one cost-push variable at a time. Moreover, when examining the role of the output gap we do not consider output growth since these variables are correlated at higher frequencies. Figure 4 shows that the output gap causes inflation at business cycle frequencies of 0.075π (which corresponds to a periodicity of about 7 years). In contrast, inflation causes the output gap mainly at the annual frequency.

Rapid money growth in the euro area is likely to lead to both inflation and exchange rate depreciation. Since the cost-push variables all depend on the evolution of the exchange rate, the finding that they are significant at low frequencies may therefore reflect the impact of the exchange rate rather than an independent causal effect. Figure 5 shows that the causality measure from exchange rate changes to inflation peaks at $\omega = 0.9\pi$, which corresponds to a periodicity of 2.2 quarters. In contrast, the causality measure from inflation to the exchange rate is high at the zero frequency and again at high frequencies. A similar pattern of causality from inflation at the zero and high frequencies, and to inflation at high frequencies, is present also for changes in oil prices and import prices. It is difficult to know whether these findings for the highest frequencies are structural or spurious.[33]

Band spectrum regression: High frequencies

Next we turn to the high frequency regressions. Table 4 shows the results when the short run is defined as including frequencies between 0.5 and 4 years.[34] By excluding the zero frequency, inflation and money growth become stationary. However, because they are correlated at the zero frequency (as evidenced by the finding of cointegration), and because of leakage from the zero frequency into all other frequencies, they are correlated also at high frequencies. Thus, this leakage introduces correlation between the regressor and the error term at high frequencies. We therefore use instrumental variables estimation and instrument money growth with its first lag.[35] Interestingly and in contrast to the results for the low frequencies, the quantity-theoretic variables are insignificant at high frequencies. The regression in column 1, in which all cost-push variables are included, shows that changes in import prices, which by construction capture oil price shocks and exchange rate changes, help to account for high-frequency movements in inflation. In column 2 to 4 we include a single cost-push variable at the time, and find again that import prices appear to be more important than the other variables (as evidenced by the $\bar{R}^2$). When they are included, however, the output gap is not significant.

Because the regressions in Table 4 include many insignificant parameters, we re-estimate the regressions by excluding the quantity-theoretic variables. Since there is no longer any need to take into account the correlation of money growth with the error term, we use the Engle (1974) estimator. The results, which are shown in Table 5, are similar to those in Table 4 except that the output gap is significant in all regressions. Again it appears that import price changes contain more information for inflation than changes in exchange rates or in oil prices.

In the analysis below we have defined the high-frequency band as fluctuations of a periodicity of between 0.5 and 4 years and have found the output gap to be highly significant. Next we re-estimate the regressions in Table 5, but consider increasingly narrower frequency bands in order to assess the role of the cost-push variables in accounting for the highest-frequency movements in inflation. Thus, Table 6 shows the results under the assumption that the high-frequency band contains periodicities of 0.5 to 1.5 years. While the output gap is insignificant in these regressions, the cost-push variables remain significant if they are considered individually. These findings suggest that they contain information about inflation at higher frequencies than the output gap.

Overall, our results confirm the findings in AWG that the quantity-theoretic variables explain inflation at low frequencies, whereas the output gap provides information about inflation at business cycle frequencies. The novel finding here is that for frequencies from 0.5 to 1.5 years the cost-push factors, in particular import prices, are important for understanding the dynamics of inflation.

Two-pillar Phillips curve

So far we have studied the determination of inflation in different frequency bands. Next we return to the issue of how to model headline inflation (that is, inflation at all frequencies) and consider three sets of explanatory variables. The first of these are the quantity-theoretic variables. Since the econometric analysis above indicated that only the low-frequency variation of these variables mattered, we use only the frequency band of 4 years to infinity. We think of these variables as the empirical counterpart to the ECB’s 'first pillar' or the 'monetary analysis'. The second set is given by the output gap. Since it did not appear important at the highest frequencies, we focus on the frequency band of 1.5 to 4 years. The third set of regressors contains the variables capturing cost-push shocks, for which we focus on the frequency band of 0.5 to 1.5 years. Of course, we interpret the output gap and the cost-push variables as constituting the 'second pillar' of the 'economic analysis'. This leads to an inflation equation of the form:

$$\pi_t = \beta_0 + \left\{\beta_\mu \mu_t^{4-\infty} + \beta_\rho \rho_t^{4-\infty} + \beta_\gamma \gamma_t^{4-\infty}\right\} + \beta_g g_{t-1}^{1.5-4} + \beta_c c_t^{0.5-1.5} + \varepsilon_t, \qquad (20)$$

which can be thought of as a more refined version of the 'Two-Pillar Phillips Curve' studied by Gerlach (2003, 2004) and AWG.

Table 7 shows the results, which are obtained by OLS. In the first column we tabulate the estimates of a conventional regression in which all frequencies are included. While the coefficient on money growth is significant, it is significantly smaller than unity. Furthermore, the coefficient on output growth is far away from the expected value of minus unity and only significant at the 6 per cent level. A Wald test clearly rejects the hypothesis that money and output growth have coefficients of unity and minus unity, respectively.[36] Changes in oil prices are significant with the wrong sign, whereas the coefficient on changes in import prices is large, positive and significant.

The estimates in column 1 appear difficult to reconcile with the ECB's views of the inflation process. But those views emphasise that the determinants of inflation vary across frequencies and we therefore proceed by estimating the equation using different frequency bands for the regressors. The results are shown in columns 2 to 5 in the same table. Note that since the frequency bands are not overlapping, the regressors in one frequency band are orthogonal to those in another.[37] The parameters on the quantity-theoretic variables and the output gap do therefore not change as we consider the different measures of cost-push shocks. Consequently, we comment on the parameters by frequency band.

In the lowest frequency band, the quantity-theoretic variables are all highly significant, and the parameters on money and real output growth are close to unity and minus unity, respectively. Indeed, a Wald test does not reject this hypothesis. However, while money growth is statistically highly significant, it is likely to explain at most a small fraction of the variance of quarter-to-quarter changes in inflation since low-frequency variables evolve gradually over time.

Turning to the economic analysis, in the intermediate frequency band the output gap is highly significant, and in the highest frequency band changes in oil prices and, in particular, import prices help explain inflation. The fact that the significance of these variables depends on what frequency bands are considered is expected, given the ECB's view that the inflation process varies across frequencies.

Sensitivity analysis

The inflation equations presented in Table 7 are estimated under the assumption that the low-frequency band contains fluctuations of periodicities of between 4 years and infinity, the intermediate frequency band (in which the output gap is relevant) periodicities of between 1.5 and 4 years, and the high frequency band periodicities of between 0.5 and 1.5 years. Next we explore whether better results could be obtained by separating between the different bands at other frequencies. To this end, we re-estimate the two-pillar Phillips curve in equation (20), but vary the exact frequency at which we distinguish between low frequency and intermediate frequencies from 2 to 8 years, and the frequency at which we distinguish between intermediate and high frequencies from 0.5 to 2 years.

To judge the fit, we use the resulting R^2s. The frequency bands for which these maxima are attained are listed in Table 8. While the quantity-theoretic variables explain inflation at a frequency above 5.5 years, the output gap has explanatory power at business cycle frequencies between approximately 1.5 to 5.5 years (depending on what cost-push variable is considered), and the cost-push factors matter at the remaining frequencies. Not surprisingly, it appears that changes in oil prices provide information on the highest-frequency movements in inflation. Since the use of the optimal frequency bands identified in Table 8 leads to parameter estimates that do not differ much from those reported in Table 7, except of course for the higher R^2, in the interest in brevity we do not report them.

Conclusions

In this paper we have reviewed several recent papers that have sought to provide an empirical representation of the ECB's views of the determination of inflation in the euro area. We have argued that these views can be given a natural interpretation in terms of an augmented Phillips curve in which the parameters vary across frequencies. Thus, the 'monetary' analysis is seen as a way of understanding the determination of inflation at low, and the 'economic' analysis at high, frequencies. Furthermore, we have presented empirical evidence that suggests that euro-area inflation can be decomposed into three frequency bands, each with a separate set of determinants. At the lowest frequencies the quantity-theoretic variables – money and output growth, and changes in long-term interest rates as determinants of changes in velocity – play important roles in determining inflation. In the intermediate frequency band the output gap appears to be the main causal variable, while at still higher frequencies cost-push shocks, in particular import prices, are of primary significance.

This view of the inflation process warrants three comments. First, although low-frequency factors are evolving more slowly over time than high-frequency variables and therefore have more gradual and long-lived effects on inflation, at any point in time all three sets of variables are operational. The fact that money growth is important only at low frequencies does consequently not mean that it can be disregarded when analysing current price pressures.

Second, monetary analysis is sometimes thought of as a way of cross-checking the findings of the economic analysis of inflation, implying that these approaches are substitutes. The interpretation of the monetary and economic analysis as pertaining to different frequency bands suggests, by contrast, that they should be thought of as complements. The two pillars are thus best seen as a way to 'build up' an overall assessment of inflation at the current juncture.

Third, while our results confirm that money growth contains information about inflation pressures, they are silent on the question whether this information is best incorporated in the policy analysis using a two-pillar framework or by integrating the pillars in a single analysis of inflation. The choice of a monetary policy strategy has two objectives: to frame the internal decision-making process on the basis of an analysis of all relevant information, and to facilitate the communication of policy decisions to the public. Judging how the two-pillar strategy satisfies these criteria is beyond the scope of this paper.

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Endnotes

[1] See the ECB's press releases of 13 October 1998 and 1 December 1998, which are available at www.ecb.int.

[2] The Deutsche Bundesbank (2005) also argues that low-frequency fluctuations in money growth impact on the long-term evolution of inflation, in contrast to high-frequency swings, which are much less informative about price developments.

[3] Jaeger (2003) also analyses the relation of money growth and inflation across different frequencies but he includes the money gap, not the low-frequency part of money, in his inflation equation.

[4] It has been shown for different countries and time periods that money growth and inflation are closely related, see Benati (2005), Fitzgerald (1999) or Haug and Dewald (2004).

[5] Two-pillar Phillips-curve models do not explain why monetary conditions may have changed. For the Federal Reserve, Ireland (2005) argues that changes in the target level of inflation have been caused by a series of negative supply shocks in the 1970s and positive shocks in the 1990s.

[6] For instance, using the data of Gerlach (2004), the ratio is 0.014 for inflation and 0.006 for money growth.

[7] Bruggemann et al. (2005) show that also a two-sided filter is feasible for real-time analysis of monetary data when they are augmented with forecasts from money demand models.

[8] See for instance Ashley and Verbrugge (2005).

[9] See Crowley (2005) for a discussion of wavelet analysis in economics.

[10] The discussion in Gerlach (2004) suggests that this result arises because filtered real output growth is correlated with the output gap.

[11] See European Central Bank (2001, p. 42) for a discussion of money growth and inflation expectations.

[12] Lucas (1980) presents frequency-domain evidence for US data in support of this proposition.

[13] Reynard (2005) shows that accounting for changes in velocity is critical for understanding the relationship between money growth and inflation in the euro area and in the US since the 1970s.

[14] In another paper, Assenmacher-Wesche and Gerlach (2005) cast the two-pillar framework into a state-space framework in which trend money growth and trend output growth (and consequently the output gap which is defined as actual output less trend output) are treated as unobservables.

[15] Experimentation with lagged and contemporaneous output gap and cost-push factors showed that a lagged output gap and contemporaneous cost-push variables provided the best fit.

[16] M3 is not included in data set for the ECB's area-wide model. The monetary data used were provided separately by the ECB.

[17] See AWG for a detailed discussion of the adjustment.

[18] The interest rate is expressed at a quarterly rate by defining it as $0.25\ln(1+i/100)$, where i is the interest rate in per cent per annum. This makes it comparable to inflation, which is also measured at a quarterly rate.

[19] The HP filter is applied with the usual smoothing parameter of $\lambda = 1600$ for quarterly data. Kaiser and Maravall (2001) show that the HP filter can be viewed as a

band-pass filter. By setting $\lambda = 1600$, fluctuations with a periodicity of more than 40 quarters are retained.

[20] Since the original exchange rate data showed a large spike in the first quarter of 1990 which possibly is related to statistical effects due to German reunification, we removed this spike by regressing the series on a dummy. The results are not affected by this adjustment.

[21] The oil price data is the US dollar price per barrel for UK Brent oil from the International Financial Statistics of the International Monetary Fund. The exchange rate used to convert the series into euros is the euro/US dollar exchange rate from 1999 on and the ECU/US dollar exchange rate from 1979 to 1998. For the time before 1979 an exchange rate series has been constructed by using the exchange rates for the Belgian franc, the Danish crown, the Dutch guilder, the French franc, the German mark, the Irish punt, the Italian Lira and pound sterling with the ECU basket weights from 1975, see European Navigator (2004).

[22] The unit root tests are discussed in Maddala and Kim (1998).

[23] However, the results are not all completely unambiguous. For instance, the PP test accepts trend stationarity for inflation and money growth, and the KPSS test rejects stationarity for output growth and import prices.

[24] The model is specified with a constant that is restricted to lie in the cointegration space. The results are insensitive to other assumptions regarding the deterministic components in the model.

[25] This corresponds to the finding that in the long run money growth causes inflation, which we discuss below.

[26] Superconsistency of the cointegration parameters assures that the asymptotic distribution of the other parameters is not affected.

[27] Though the component of a series in a certain frequency band cannot be estimated without the use of a two-sided filter which destroys the temporal aspect of the causal definition, it is possible to deduce causal relationships at different frequencies without estimation of the series' components, as is done in the band spectrum regressions.

[28] We follow the approach suggested by Hosoya (2000) to test for causality in a multivariate system.

[29] The approach is described in more detail in AWG.

[30] While we elsewhere in the paper let π denote of the rate of inflation, in discussing frequencies we let it denote the irrational number defined by the ratio of the circumference of a circle to its diameter.

[31] Breitung and Candelon (2005) argue that in a bivariate cointegrated system the causality measure approximately follows an F-distribution with 2 degrees of freedom. It is not clear, however, if the critical value provides a good approximation for a multivariate system where some variables are $I(1)$ and others $I(0)$. We therefore do not include a critical value in Figures 3 to 5.

[32] Since inflation is nonstationary whereas the other variables are stationary, the latter cannot cause inflation at the zero frequency. Since excluding the zero frequency makes a nonstationary variable stationary, causality at other frequencies is possible.

[33] For instance, one would expect that the exact construction of the synthetic euro area data (for instance, whether the quarterly data is to be thought of as an average

for the quarter, or taken at a point in time within the quarter) would matter for the highest frequencies.

[34] The smallest fluctuation distinguishable in quarterly data is one cycle every two periods, which is also called the Nyquist frequency.

[35] To obtain the results in Table 4 we use the generalized instrumental variables estimator proposed by Corbae, Ouliaris and Phillips (1994).

[36] Since inflation and money growth are nonstationary, the coefficient on money growth follows a Dickey-Fuller distribution. The coefficients on the other variables are distributed normally. As the Wald test assumes asymptotic normality of the coefficients, it is not strictly valid.

[37] This follows from the fact that trigonometric functions underlying the Fourier transformation of the data are orthogonal for different frequencies.

Tables and Figures

Table 1 Unit root tests

	ADF	*PP*	*ERS*	*KPSS*	*AIC lag*
Test with constant					
Inflation	-1.03	-1.70	-1.02	1.77*	5
Money growth	-1.95	-2.83	-0.59	1.81*	5
Output growth	-5.44*	-7.71*	-3.91*	0.55*	1
Interest rate changes	-6.16*	-6.59*	-6.14*	0.39	1
Output gap	-4.72*	-3.63*	-3.64*	0.05	4
Exchange rate changes	-6.64*	-8.42*	-6.62*	0.08	1
Oil price changes	-8.27*	-10.45*	-8.23*	0.24	1
Import price changes	-4.76*	-4.75*	-4.64*	1.57*	1
Test with trend and constant					
Inflation	-2.60	-4.25*	-1.60	1.16*	5
Money growth	-2.42	-5.94*	-2.55	0.21*	5
Output growth	-5.94*	-8.36*	-5.63*	0.09	1
Interest rate changes	-6.32*	-6.73*	-6.30*	0.07	1
Output gap	-4.77*	-3.73*	-4.24*	0.04	4
Exchange rate changes	-6.61*	-8.39*	-6.64*	0.08	1
Oil price changes	-8.40*	-10.53*	-8.32*	0.10	1
Import price changes	-5.49*	-5.36*	-4.92*	0.14	1
Test of 1st differences					
Inflation	-6.60*	-16.79*	-4.79*	0.06	4
Money growth	-5.18*	-21.21*	-1.32	0.08	8
Output growth	-6.56*	-24.48*	-1.20	0.04	7
Interest rate changes	-8.34*	-15.73*	-8.16*	0.03	5
Output gap	-6.13*	-9.31*	-5.07*	0.04	4
Exchange rate changes	-12.94*	-17.04*	-12.75*	0.01	1
Oil price changes	-7.71*	-26.19*	-1.00	0.02	5
Import price changes	-6.78*	-12.66*	-6.02*	0.05	7

Note: The last column indicates the number of lags included in the test, which were chosen by the AIC criterion. The 5% critical values for the tests including a constant only are -2.89 for the Augmented Dickey-Fuller (ADF) and the Phillips-Perron (PP) test, -1.95 for the Elliot, Stock and Rotenberg (ERS) test and 0.46 for the Kwiatkowski, Phillips, Schmidt and Shin (KPSS) test. The 5% critical values for the test including a constant and a trend are -3.45 for the ADF and the PP test, -2.89 for the ERS and 0.15 for the KPSS test. The tests of the first differences include a constant but no trend. The sample period is 1970Q2 to 2004Q4. An asterisk, '*', indicates the rejection of the null hypothesis.

Table 2 Cointegration analysis

Rank	*Eigenvalues*	*Trace test*	*95% critical values*
$r = 0$	0.136	20.81	20.26
$r = 1$	0.014	1.78	9.16

Parameters	*Inflation*	*Money growth*	*Constant*
β unrestricted	1 (-)	-1.093 (0.149)	0.023 (0.003)
β restricted	1 (-)	-1 (-)	0.021 (0.001)
α (for restricted β)	-0.200 (0.048)	0.133 (0.071)	--

Note: The model includes a constant that is restricted to lie in the cointegration space. The sample period is 1970Q2 to 2004Q4. The Akaike criterion suggests the inclusion of 5 lags. A likelihood-ratio test does not reject the overidentifying restriction on the cointegration vector, β ($\chi(1) = 0.35$ and a p-value of 0.56). The adjustment coefficients are denoted with α and r is the rank of the long-run matrix, Π.

Table 3 Band spectrum regressions: Low-frequency band (4 years to infinity)

	Dependent variable: $\pi_t - \mu_t$				
Quantity-theoretic variables					
Output growth	-1.07**	-0.87**	-0.92**	-1.21**	-0.99**
	(0.19)	(0.27)	(0.18)	(0.19)	(0.20)
Interest rate change	4.05**	1.97	3.50**	2.91*	1.95
	(1.07)	(1.18)	(1.19)	(1.35)	(1.40)
Output gap (lagged 1 quarter)		0.02	0.02	-0.08	-0.02
		(0.10)	(0.07)	(0.06)	(0.06)
Cost-push variables					
Exchange rate changes		0.11*	0.13**		
		(0.05)	(0.05)		
Oil price changes (x 10^2)		-0.31		2.65*	
		(2.06)		(1.18)	
Import price changes		0.09			0.11**
		(0.05)			(0.04)
$\bar{R}^2$	0.52	0.53	0.56	0.51	0.55

Note: All regressions include a constant which is not shown. Standard errors in parentheses; * indicates significance at the 5% level, ** significance at the 1% level. The sample period is 1970Q2 to 2003Q4. The number of degrees of freedom is 14 for the regression in the first column, 10 for the second column and 12 for the other columns.

Table 4 Band spectrum regressions: High-frequency band: 0.5 to 4 years

	Dependent variable: π_t			
Quantity-theoretic variables				
Money growth	-0.07	0.41	0.30	-0.16
	(0.72)	(0.63)	(0.59)	(0.77)
Output growth	-0.01	0.12	0.11	0.03
	(0.14)	(0.12)	(0.11)	(0.15)
Interest rate change	0.13	0.55	0.38	0.11
	(0.16)	(0.30)	(0.35)	(0.26)
Output gap (lagged 1 quarter)	0.13	0.30*	0.27*	0.10
	(0.16)	(0.12)	(0.12)	(0.17)
Cost-push variables				
Exchange rate changes	0.01	0.03**		
	(0.01)	(0.01)		
Oil price changes (x 10^2)	0.08		0.38	
	(0.20)		(0.24)	
Import price changes	0.07**			0.08**
	(0.02)			(0.03)
$\bar{R}^2$	0.44	-0.07	0.09	0.42

Note: All regressions include a constant which is not shown. Newey-West (1987) corrected standard errors in parentheses; * indicates significance at the 5% level, ** significance at the 1% level. The sample period is 1970Q2 to 2003Q4. Money growth is instrumented with its first lag. The number of degrees of freedom is 111 for the regression in the first column and 113 for the other regressions.

Table 5 Band spectrum regressions: High-frequency band: 0.5 to 4 years

	Dependent variable: π_t			
Output gap (lagged 1 quarter)	0.14**	0.21**	0.20**	0.13**
	(0.04)	(0.05)	(0.05)	(0.04)
Cost-push shocks				
Exchange rate changes	0.005	0.03**		
	(0.01)	(0.01)		
Oil price changes (x 10^2)	0.07		0.52**	
	(0.14)		(0.09)	
Import price changes	0.07**			0.08**
	(0.01)			(0.01)
$\overline{R}^2$	0.44	0.28	0.31	0.45

Note: All regressions include a constant which is not shown. Newey-West (1987) corrected standard errors in parentheses; * indicates significance at the 5% level, ** significance at the 1% level. The sample period is 1970Q2 to 2003Q4. The number of degrees of freedom is 114 for the regression in the first column and 116 for the other regressions.

Table 6 Band spectrum regressions: High-frequency band: 0.5 to 1.5 year

	Dependent variable: π_t			
Output gap (lagged 1 quarter)	0.07 (0.05)	0.06 (0.05)	0.05 (0.05)	0.07 (0.05)
<u>*Cost-push shocks*</u>				
Exchange rate changes	0.01 (0.01)	0.02^{*} (0.01)		
Oil price changes (x 10^2)	0.14 (0.13)		0.41^{**} (0.11)	
Import price changes	0.05^{**} (0.02)			0.07^{**} (0.02)
$\bar{R}^2$	0.14	0.05	0.08	0.15

Note: All regressions include a constant which is not shown. Newey-West (1987) corrected standard errors in parentheses; * indicates significance at the 5% level, ** significance at the 1% level. The sample period is 1970Q2 to 2003Q4. The number of degrees of freedom is 63 for the regression in the first column and 65 for the other regressions.

Table 7 Two-Pillar Phillips Curves

Quantity-theoretic variables	*All frequencies*		*Periodicity: 4 to ∞ years*		
Money growth	0.49**	0.89**	0.89**	0.89**	0.89**
	(0.05)	(0.06)	(0.06)	(0.06)	(0.06)
Output growth	-0.18	-0.97**	-0.97**	-0.97**	-0.97**
	(0.10)	(0.23)	(0.23)	(0.23)	(0.23)
Interest rate change	0.41	4.45**	4.45**	4.45**	4.45**
	(0.52)	(1.32)	(1.32)	(1.32)	(1.32)
	All frequencies		*Periodicity: 1.5 to 4 years*		
Output gap (lagged 1 quarter)	0.04	0.25**	0.25**	0.25**	0.25**
	(0.86)	(0.08)	(0.08)	(0.08)	(0.08)
Cost-push shocks	*All frequencies*		*Periodicity: 0.5 to 1.5 year*		
Exchange rate changes	-0.01	0.01	0.02		
	(0.02)	(0.01)	(0.01)		
Oil price changes (x 10^2)	-0.93**	0.14		0.41**	
	(0.25)	(0.18)		(0.15)	
Import price changes	0.21**	0.05*			0.07**
	(0.03)	(0.02)			(0.02)
$\overline{R}^2$	0.72	0.78	0.78	0.78	0.78

Note: The dependent variable is the inflation rate. All regressions include a constant which is not shown. Standard errors in parentheses; * indicates significance at the 5% level, ** significance at the 1% level. The sample period is 1970Q2 to 2003Q4. The first column shows the results from a conventional OLS regression. Columns 2 to 4 include the low-frequency part of money growth, output growth and the interest rate change and the high-frequency part of the output gap. Newey-West corrected standard errors are reported.

Table 8 Optimal frequency bands

	Cost push variable	*Output gap*	*Quantity theoretic variables*
Exchange rate changes	0.50-1.68	1.73-5.33	5.45-∞
Oil price changes	0.50-1.21	1.23-5.45	5.57-∞
Import price changes	0.50-1.98	2.00-5.33	5.45-∞

The table gives the optimal frequency band in years.

Figure 1 Data

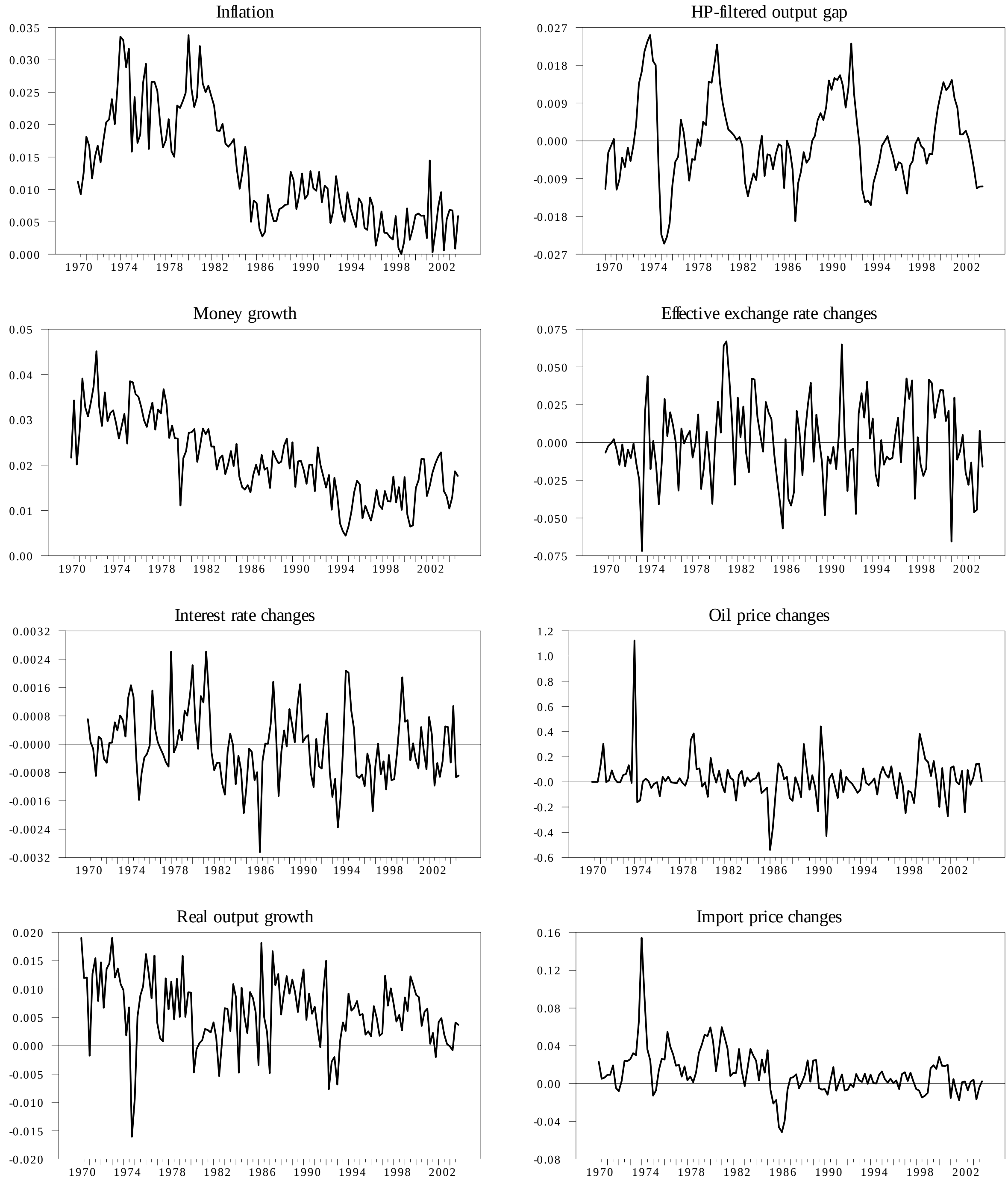

Figure 2 Changes in cost-push variables and inflation at low and high frequency

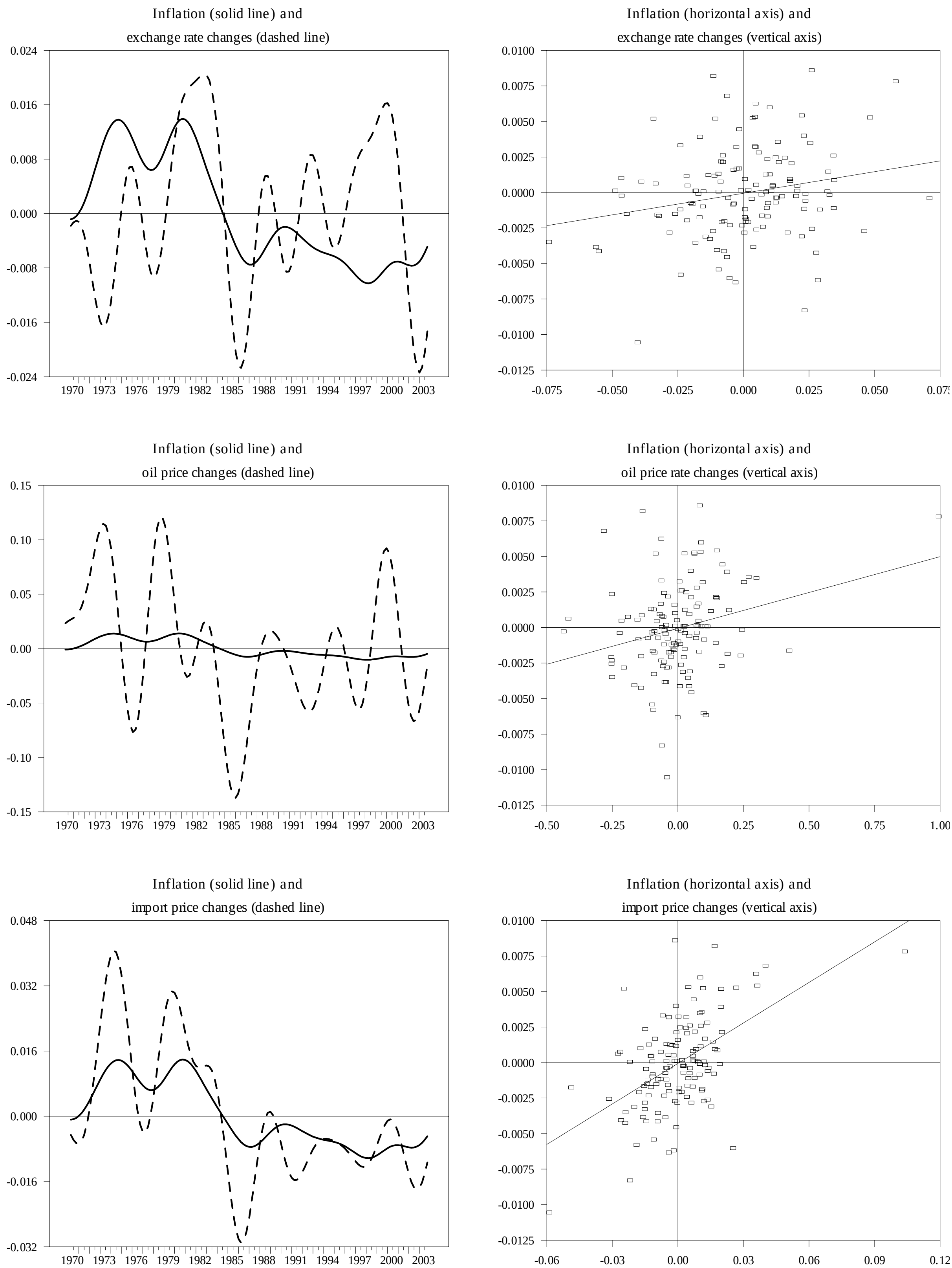

Figure 3 Money growth: frequency-wise causality measure

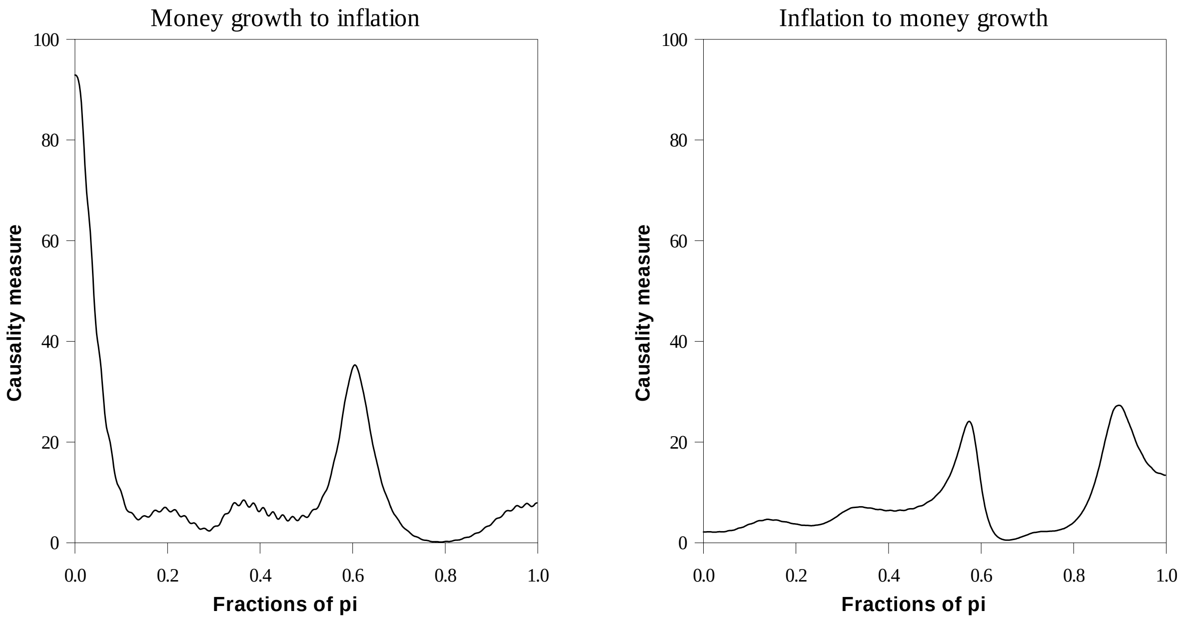

Figure 4 Output gap: frequency-wise causality measure

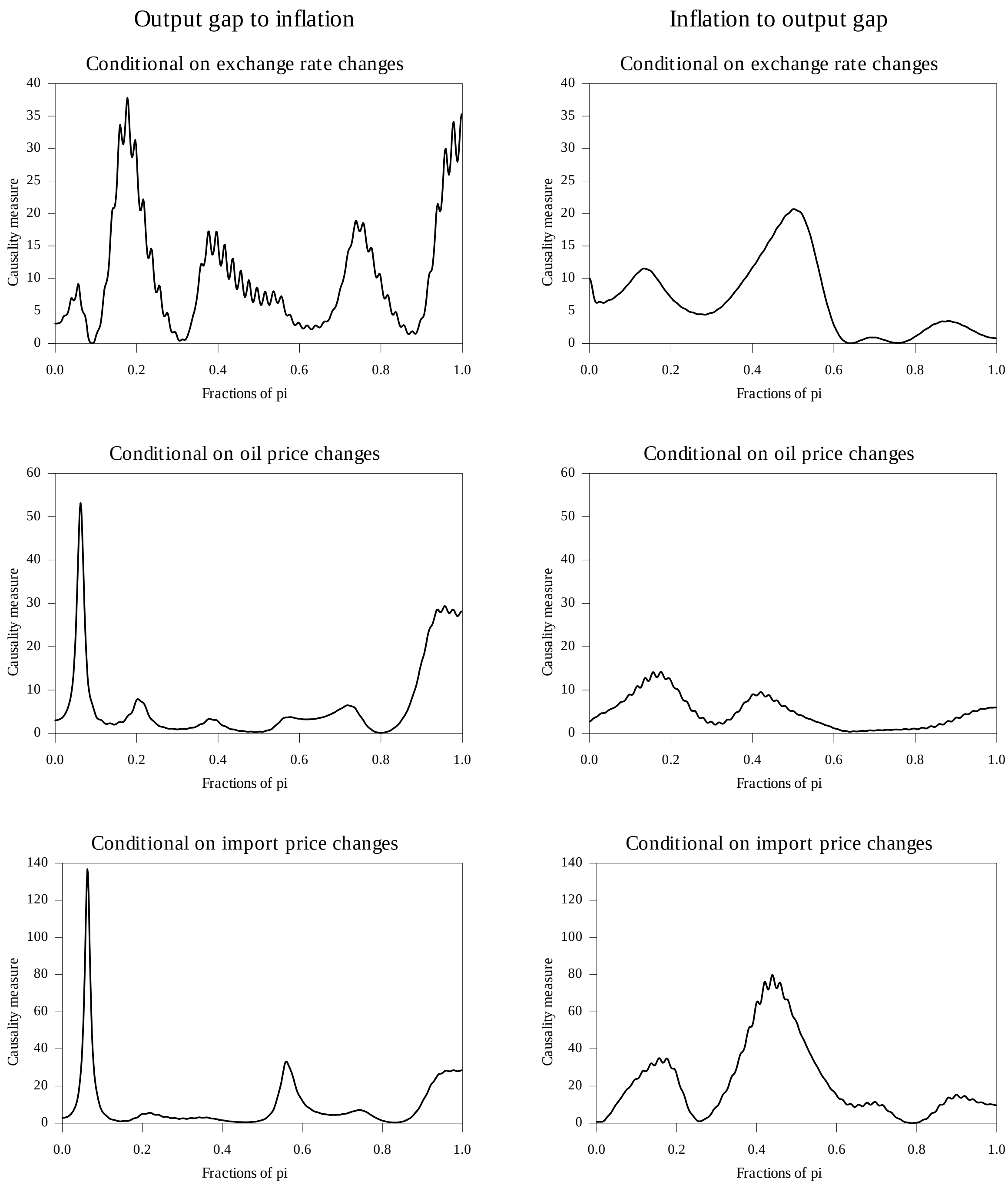

Figure 5 Cost-push variables: frequency-wise causality measure