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## MONETARY PERSISTENCE, IMPERFECT COMPETITION AND STAGGERING COMPLEMENTARITIES

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## ABSTRACT

# Monetary Persistence, Imperfect Competition and Staggering Complementarities*

This paper explores the influence of wage and price staggering on monetary persistence. We show that, for plausible parameter values, wage and price staggering are highly complementary in generating monetary persistence. We do so by proposing the new measure "quantitative persistence," after discussing weaknesses of the "contract multiplier," which is generally used to compare persistence. The existence of complementarities means that beyond understanding how wage and price staggering work in isolation, it is important to explore their interactions. Furthermore, our analysis indicates that the degree of monetary persistence generated by wage vis-à-vis price staggering depends on the relative competitiveness of the labour and product markets. We show that the conventional wisdom that wage staggering can generate more persistence than price staggering does not necessarily hold.

JEL Classification: E40, E50 and E52
Keywords: monetary persistence, monetary policy, price staggering and wage staggering

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# 1 Introduction[1]

We show in this paper that, for plausible parameter values, wage and price staggering are highly complementary in generating persistent output effects in response to monetary policy shocks. In other words, the joint effect of wage and price staggering on monetary persistence is larger than the sum of the individual effects. Thus the comparisons between the effects of wage and price staggering, which are so common in the New Keynesian literature, are only of limited usefulness. Clearly, the larger the complementarities between wage and price staggering are, the less important it is to know how wage and price staggering work in isolation and the more important it is to explore their interactions. This result deserves attention because, in practice, it is very common for nominal wages and prices *both* to be set for finite periods of time.

In evaluating the relative effects of wage and price staggering on monetary persistence, as well as their joint effects, the production technology turns out to be important. Since the real effects of temporary monetary shocks work themselves out over the short run, it is natural to assume that firms face diminishing returns to labor - also a primarily short-run phenomenon. We show that the more rapidly diminishing the returns to labor are, the more the relative competitiveness of the product and labor markets matters for the relative monetary persistence generated by wage and price staggering. Our analysis indicates that, for plausible technological parameter values, the relative competitiveness has a sizeable influence on the relative monetary persistence.

In order to understand the complementarities, it is necessary to analyze the individuals effects of wage and price staggering. In the recent New Keynesian literature, a large body of articles argues that wage staggering generates more monetary persistence than price staggering in response to monetary policy shocks (i.e. the real effects of temporary monetary shocks are more persistent when wages are set through overlapping nominal contracts than when prices are set in this way), see e.g. Andersen (1998), Chari et al. (2000), Huang and Liu (2002). This paper calls this conventional wisdom into question. It shows that the relative strength of monetary persistence generated by wage vis-à-vis price staggering depends on the relative competitiveness of the labor and product markets. In particular, the more competitive the product market is relative to the labor market, the more monetary persistence is generated by price relative to wage staggering. We show that if the product market is sufficiently more competitive than the labor market, price staggering makes the real effects of temporary monetary shocks more persistent than does wage staggering. This result is potentially important because, in practice, product markets are often more competitive than labor markets. There are various obvious reasons for this, e.g. employers often find it more costly to switch between employees than consumers find it to switch between products.

[1] We thank Guido Ascari for very helpful comments.

In this context, it turns out to be useful to think carefully about how we measure monetary persistence. The effects of a monetary shock on real economic activity through time (e.g. the effects of a temporary increase in money growth on national output) can be described by the relevant impulse response function (IRF). The "degree of monetary persistence" is a summary statistic of this function. The standard statistic, which is generally used in the New Keynesian literature, is the "contract multiplier," usually defined as the ratio of the response after the contract duration has elapsed to the response in the impact period (see e.g. Huang and Liu, 2002). In other words, this summary statistic measures how much the response dies out within a given span of time.

While the contract multiplier captures one feature of the IRF, it misses other important ones. Suppose, for example, that wage and price staggering were associated with IRFs (of output to a given monetary shock) that differed only by an additive constant. This difference, however large, would not be identified by the contract multiplier, because both IRFs have the same slope at every point in time, and thus the ratio of the response in period 1 and period $t$ would be the same. To capture this difference, it is convenient to use a measure that we call "quantitative persistence:" for a temporary unit shock in period 1, it is the sum of the output responses from period 2 onwards. In words, quantitative persistence measures by how much output changes, in total, after the monetary shock has disappeared. This measure of monetary persistence turns out to be particularly useful in describing how wage and price staggering affect monetary persistence. It is also useful in capturing the complementarities between wage and price staggering in generating monetary persistence.

The paper is organized as follows. Section 2 presents the underlying dynamic general equilibrium models, which are standard New Keynesian models with Calvo staggering. In order to understand the complementarities, it is necessary to look at the relative strength of monetary persistence individually first and jointly thereafter. Section 3 describes, formally and intuitively, how the relative strength of monetary persistence generated by wage vis-à-vis price staggering depends on the relative competitiveness of the labor and product markets. Section 4 derives the complementarities between wage and price staggering in generating monetary persistence. Section 5 relates our results to the existing literature. Section 6 concludes.

# 2 Models of Wage and Price Staggering

Our model economies each contain households, firms and a government. The government prints money and bonds and imposes taxes/transfers on the households.[2] Our models of wage and price staggering are completely standard Calvo (1983) models.

[2]Without loss of generality we assume no government consumption. If we assumed that the government consumes a constant fraction of each good, which is financed via lump-sum taxation, we would obtain a similar dynamic system. Calculations are available on request.

The model is linearized around a zero money growth steady state. Monetary shocks are generated when the monetary authority (government) increases the money supply and the economic agents do not know the shock until it occurs. We will discuss the effects of a one time increase of the money supply by 1%,[3] which is transferred from the monetary authority to the households in a lump-sum manner ("helicopter drop of money").

In the model of wage staggering, there is a continuum of households supplying differentiated labor and the firms produce output by means of all the labor types. These labor types are imperfect substitutes in production (as in Blanchard and Kiyotaki, 1987). The households' wage setting is randomly staggered, with each household having a fixed probability of changing its wage in any given period of time. The wages are set to maximize the households' utility, subject to their budget constraints and labor demand functions. The firms maximize their profits instantaneously with respect to employment and output, subject to their production functions.

In the model of price staggering, firms supply a continuum of goods to the households. These goods are imperfectly substitutes in consumption. The firms' price setting is randomly staggered. The prices are set to maximize the firms' profits, subject to their production functions and their product demand functions. The households maximize their utility instantaneously with respect to consumption, labor, real money balances and bond holdings, subject to their budget constraints.

In the first step, we derive the dynamic system for wage staggering, with the purpose to generate IRFs. Thus we will be able to compare them to IRFs for price staggering, which will be generated from the according dynamic system afterwards.

## 2.1 Wage Staggering

### 2.1.1 Firms

The product market is perfectly competitive. There is a fixed number of identical firms (normalized to unity), producing a homogeneous product. The firms are price-takers. Firms face the following short-run Cobb-Douglas type production function:[4]

$$Y_t(j) = A_t N_t(j)^{1-\alpha} \tag{1}$$

where $j$ is the index for the firm, $Y_t$ is the level of production, $A_t$ is a productivity parameter, $N_t$ is the labor input, and $\alpha$ denotes how significant the diminishing

[3] In most other papers the money growth follows an autoregressive process. We however do not consider autocorrelations of the money supply, as we seek to identify the endogenous persistence generated by the behavior of the model (rather than the persistence of the shocks). As Taylor noted, "leaving all the persistence of inflation to exogenous serial correlation is not a completely satisfactory conclusion" (Taylor, 1999: page 1040).

[4] We use the following terminology. Capital letters are level variables ($Y_t$), lower case letters denote logarithmic variables ($y_t$), lower case letters with a bar ($\bar{y}$) denote the variable at the steady state and lower case variables with a tilde ($\tilde{y}_t$) denote deviations from the steady state.

returns to labor are.[5]

Under perfect competition, prices are set uniformly and are equal to marginal costs:

$$P_t = MC_t^n \tag{2}$$

where $P_t$ is the aggregate price level and $MC_t^n$ are the nominal marginal costs.

### 2.1.2 Households

The aggregate labor input is a Dixit-Stiglitz function of a continuum of individual labor inputs (normalized to unity):

$$N_t = \left[ \int_{h'=0}^{1} N_t \left(h'\right)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dh' \right]^{\frac{\varepsilon_w}{\varepsilon_w - 1}} \tag{3}$$

where $N_t(h)$ is the amount of labor chosen from household $h$ and $\varepsilon_w$ is the elasticity of substitution between different labor types.

Minimizing the firm's labor cost, we obtain its labor demand function for each labor type:

$$N_{t+i}(h) = \left( \frac{W_t^*(h)}{W_{t+i}} \right)^{-\varepsilon_w} N_{t+i} \tag{4}$$

where $W_t^*(h)$ is the optimal wage set by household $h$ in period $t$. The corresponding aggregate wage index $W_{t+i}$ is defined as

$$W_{t+i} = \left[ \int_{h'=0}^{1} W_{t+i} \left(h'\right)^{1-\varepsilon_w} dh' \right]^{\frac{1}{1-\varepsilon_w}} . \tag{5}$$

The household's instantaneous utility is $U(C_{t+i}(h)) - V(N_{t+i}(h)) + Z(M_{t+i}(h)/P_{t+i})$, $U', V', Z' > 0$, $U", V", Z" < 0$, where $C_{t+i}(h)$ is its consumption,[6] $N_{t+i}(h)$ is its employment, and $M_{t+i}(h)/P_{t+i}$ are its real money balances. In each period the wages can be reset with probability $(1 - \theta_w)$.

The household maximizes its utility in a Calvo setting[7]

[5] As the effect of monetary shocks work themselves out over the short run, we assume a fixed amount of capital. Many recent papers assume full mobility of capital. Altig et al. (2005: page 2) comment this approach as "empiricially unrealistic but [it is] defended on the grounds of tractability. The hope is that these assumptions are innocuous and do not affect major model properties. In fact these assumptions matter a lot."

[6] As usual in the literature, we assume complete insurance markets that allow households to share the income risk stemming from staggered wage setting.

[7] We choose a separable utility function with the standard desirable long-run properties.

$$U\left(C_t(h), \frac{M_t(h)}{P_t}, N_t(h)\right) = \frac{C_t^{1-\sigma}(h)}{1-\sigma} + \frac{\left(\frac{M_t(h)}{P_t}\right)^{1-\nu}}{1-\nu} - \frac{N_t^{1+\varphi}(h)}{1+\varphi} \tag{6}$$

subject to its budget constraint:

$$\begin{aligned} & E_t \sum_{i=0}^{\infty} \beta^i \left( C_{t+i} + \frac{R_{t+i}^{-1} B_{t+i} + M_{t+i}}{P_{t+i}} \right) \qquad (7) \\ = \; & E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{W_t(h)}{P_{t+i}} N_{t+i}(h) + \frac{T_{t+i}}{P_{t+i}} + \frac{\Pi_{t+i}}{P_{t+i}} + \frac{B_{t+i-1}}{P_{t+i}} + \frac{M_{t+i-1}}{P_{t+i}} \right] \end{aligned}$$

where $P_t$ is the aggregate price index, $R_{t+i}$=1 + $r_{t+i}$ is the discount factor on its one-period bond holdings $B_{t+i}$, $T_{t+i}$ is its net lump-sum transfers from government, and $\Pi_{t+i}$ is its profit income.

The household's decision can be decomposed into two optimization problems. First, the "wage contracting problem" which only takes place with probability $(1-\theta)$ in each period. Here the utility function is maximized with respect to the optimal wage. Second, the "intra-contract problem" in which the contract wage is given and the household maximizes its utility with respect to its other endogenous variables (consumption, money and bond holdings) each period.

Solving the wage contracting problem, we obtain the following optimal wage[8]:

$$w_t^*(h) = \mu_w + (1-\beta\theta) E_t \sum_{i=0}^{\infty} (\beta\theta)^i \left( \ln\left[ -\frac{V_N(N_{t+i}(h))}{U_C(C_{t+i})} \right] + p_{t+i} \right) \tag{8}$$

where $w_t^*(h)$ is the logarithm of the re-set wage and $\mu_w = (\varepsilon_w / (\varepsilon_w - 1))$ is the steady

state mark-up over the marginal rate of substitution and $V_N$, $U_c$ are the first derivatives of the utility function with respect to labor and consumption. $-V_N(N_{t+i}(h))/U_c(C_{t+i})$ denotes the marginal rate of substitution between labor and consumption.

For the intra-contract problem we obtain the following general first order conditions:

$$U_{C_t} = \beta R_t E_t \left( U_{C_{t+1}} \frac{P_t}{P_{t+1}} \right) \tag{9}$$

and

$$\frac{U_{M_t}}{U_{C_t}} = 1 - R_t^{-1} \tag{10}$$

[8] The derivations of these and further results are given in the appendix.

where $U_{C_t}$, $U_{M_t}$ denote the first derivatives of the utility function with respect to consumption and money holdings in period $t$.

Log-linearizing the consumption function and money demand function derived from the household's decision problem, we obtain:

$$c_t = E_t(c_{t+1}) - \frac{1}{\sigma}(r_t - E_t(\pi_{t+1}) + \rho) \tag{11}$$

and

$$\tilde{m}_t - \tilde{p}_t = \frac{\sigma}{\nu}\tilde{c}_t - \eta \tilde{r}_t \tag{12}$$

where $\eta = (1/\bar{r}\nu)$ and[9] $\bar{r}$ is the steady state interest rate, and $\rho = -\ln\beta$ is the time discount rate.

Finally, we close the system with a goods market clearing condition (13), a production function (14) and a money supply equation (15):

$$y_t = c_t \tag{13}$$

$$y_t = a_t + (1-\alpha)\, n_t \tag{14}$$

$$m_t = m_{t-1} + \Delta m_t. \tag{15}$$

### 2.1.3 Dynamic System

For the wage-staggering model, the intertemporal output response to the monetary shock can be derived from equations (2), (8), (11), (12), (13), (14) and (15), yielding

$$\begin{aligned} & E_t(\left[1 + \frac{1}{\sigma}\frac{\alpha}{1-\alpha}\right]\tilde{y}_{t+1}) + \frac{1}{\sigma}E_t(\tilde{w}_{t+1}) + \frac{1}{\sigma\eta}(\tilde{m}_t - \tilde{p}_t) \\ = \; & (1 + \frac{1}{\sigma\eta} + \frac{1}{\sigma}\frac{\alpha}{1-\alpha})\tilde{y}_t + \frac{1}{\sigma}\tilde{w}_t \end{aligned} \tag{16}$$

$$\begin{aligned} \beta\theta_w E_t \tilde{w}_{t+1} = \; & [(1+\beta)\,\theta_w]\,\tilde{w}_t - \theta w_{t-1} \\ & - \frac{1}{1+\varphi\epsilon_w}(1-\theta_w)(1-\beta\theta_w)\left[\sigma + \frac{\varphi+\alpha}{(1-\alpha)}\right]\tilde{y}_t \end{aligned} \tag{17}$$

where $(\tilde{m}_t - \tilde{p}_t)$ are the real money balances. The first equation expresses an IS type relation between the deviations of real money holdings, wages and output from the

[9] When we have a one-off monetary shock, the interest elasticity is not of further relevance for the IRFs of the dynamic system. For the calculations below, we assumed $\sigma = \nu = 1$.

steady state. The second equation expresses the wage dynamics in dependence of the output deviations.[10]

## 2.2 Price Staggering

The labor market is perfectly competitive; labor is a homogeneous factor; households and firms are wage-takers. There is a continuum of goods and a fixed number of identical households (normalized to unity). Each household maximizes its utility with respect to consumption of all the goods, labor, and real money balances, subject to its budget constraint.

### 2.2.1 Firms

Minimizing the cost of consumption of the different product varieties for a given consumption bundle,

$$Y_t = \left[ \int_{j'=0}^{1} Y_t(j')^{\frac{\varepsilon_p - 1}{\varepsilon_p}} \, dj' \right]^{\frac{\varepsilon_p}{\varepsilon_p - 1}} \tag{18}$$

we obtain the following product demand function:

$$Y_{t+i}(j) = \left( \frac{P_t^*(j)}{P_{t+i}} \right)^{-\varepsilon_p} Y_{t+i} \tag{19}$$

where $P_t^*(j)$ is the wage set by firm $j$. The corresponding aggregate price index is $P_{t+i}$ is defined as

$$P_{t+i} = \left[ \int_{j'=0}^{1} P_{t+i}(j')^{1-\varepsilon_p} \, dj' \right]^{\frac{1}{1-\varepsilon_p}} . \tag{20}$$

In each period the firm resets its price with probability $(1 - \theta_p)$. Thus the firm maximizes its profit

$$\max_{\{P_t^*(j)\}} E_t \sum_{i=0}^{\infty} (\beta\theta_p)^i \left( P_t(j) \, Y_t(j) - N_t(j) \, W_t \right) \tag{21}$$

subject to its production function

$$Y_t(j) = A_t N_t(j)^{1-\alpha} \tag{22}$$

[10] The wage can also be expressed in terms of prices, by using the relationship from the production function (1): $\tilde{w}_t = \tilde{p}_t - (\alpha / (1-\alpha)) \, \tilde{y}_t$. Thus the two equations can be re-written in terms of prices instead of wages. Further note that equation (15) holds.

and to its product demand function (19).

Solving this problem we obtain the following price setting equation:

$$p_t^* = \mu_p + (1-\beta\theta_p) E_t \sum_{i=0}^{\infty} (\beta\theta_p)^i mc^n_{t,t+i} \tag{23}$$

where $\mu_p = (\varepsilon_p / (\varepsilon_p - 1))$ is the steady state mark-up over marginal costs and $mc^n_{t,t+i}$ are the nominal marginal costs in period $t+i$ when prices were set in period $t$.

### 2.2.2 Households

As households are wage takers in the price-staggering model, their optimality problem reduces to the intra-contract optimization problem of the wage-staggering model above, with the difference that they optimize with respect to their labor supply and all other endogenous variables:

$$\begin{aligned} & \max_{\{C_{t+i}, B_{t+i}, M_{t+i}, N_{t+i}\}} E_t \sum_{i=0}^{\infty} \beta^i \left[ U\left( C_{t+i}(h), \frac{M_{t+i}(h)}{P_{t+i}}, N_{t+i}(h) \right) \right] \\ = & \left[ E_t \sum_{i=0}^{\infty} \beta^i \left( \frac{C_{t+i}^{1-\sigma}(h)}{1-\sigma} + \frac{\left( \frac{M_{t+i}(h)}{P_{t+i}} \right)^{1-\nu}}{1-\nu} - \frac{N_{t+i}^{1+\varphi}(h)}{1+\varphi} \right) \right] \end{aligned} \tag{24}$$

subject to its budget constraint

$$\begin{aligned} & E_t \sum_{i=0}^{\infty} \beta^i \left( C_{t+i} + \frac{R_t^{-1} B_{t+i} + M_{t+i}}{P_{t+i}} \right) \\ = & E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{W_t(h)}{P_{t+i}} N_{t+i} + \frac{T_{t+i}}{P_{t+i}} + \frac{\Pi_{t+i}}{P_{t+i}} + \frac{B_{t+i-1}}{P_{t+i}} + \frac{M_{t+i-1}}{P_{t+i}} \right] \end{aligned} \tag{25}$$

This yields the following labor supply function (in logs), in addition to (11) and (12):

$$w_t - p_t = \sigma c_t + \varphi n_t. \tag{26}$$

### 2.2.3 Dynamic System

In the price-staggering model, the associated intertemporal output response to the monetary shock is described by the following two equations, which can be derived from (1), (11), (12), (23), (26), (13), (14) and (15), yielding:

$$E_t \tilde{y}_{t+1} + \frac{1}{\sigma} E_t \tilde{p}_{t+1} + \frac{1}{\sigma\eta} (\tilde{m}_t - \tilde{p}_t) = \left(1 + \frac{1}{\sigma\eta}\right) \tilde{y}_t + \frac{1}{\sigma} \tilde{p}_t \tag{27}$$

$$\beta E_t \tilde{p}_{t+1} = (1+\beta)\,\tilde{p}_t - \tilde{p}_{t-1} - \kappa \tilde{y}_t \tag{28}$$

where $\kappa = [(1-\theta_p)(1-\beta\theta_p)(1-\alpha)] / [\theta_p [1+\alpha(\varepsilon_p - 1)]]$. Furthermore equation (15) holds.

# 3 The Effect of Competition on Monetary Persistence

We consider monetary persistence in response to a simple, one-off money growth shock. In particular, suppose that money growth is initially zero, then in period 1 it increases to some positive constant (normalized to unity), and thereafter it returns to zero. By "monetary persistence" we mean the effects of this shock on national output *after* period 1 (i.e. from period 2 onwards).

## 3.1 The Conventional Case

We simulate the impulse response functions (IRFs) of the deviation of output from the steady state under wage and price staggering with respect to a one-off 1% money growth shock,[11] for the following standard parameter values[12]:

| | | |
|---|---|---|
| $\theta_w = 0.75$ | $\theta_p = 0.75$ | $\alpha = 0.3$ |
| $\varphi = 1$ | $\nu = 1$ | $\sigma = 1$ |
| $\beta = 0.99$ | $\varepsilon_w = 10$ | $\varepsilon_p = 10$ |

The values for $\theta_w$ and $\theta_p$ imply that prices or wages are set every four quarters, on average.[13] Since there are diminishing returns to labor in the short run (over which the monetary shocks work themselves out), we set $\alpha = 0.3$, which is the standard value (corresponding to a 70% labor share of income under perfect competition). By setting $\sigma = 1$, we obtain a logarithmic utility function for consumption. Furthermore, we choose $\nu = 1$. The disutility of labor is quadratic ($\varphi = 1$). By setting $\beta = 0.99$, we obtain a quarterly real discount rate of 1%, i.e. about 4% a year, as it is standard in the literature.

The value for $\varepsilon_p$ implies a steady state mark-up of about 11% over marginal costs, whereas the interpretation for $\varepsilon_w$ is somewhat more difficult, it is the mark-up over marginal rate of substitution between work and consumption.[14] For the moment we

[11] The nominal money supply increases by one percent in period 1.

[12] In addition, the elasticity of substitution at the labor market is varied, which is discussed later.

[13] This is in line with the empirical evidence surveyed by Taylor (1999). In a very recent study Stahl (2005) shows that an average price duration of one year, before a new increase takes place, is a fairly consistent pattern for the German metal working industry.

[14] For a discussion of the role of the marginal rate of substitution, see e.g. Gali et al. (2003).

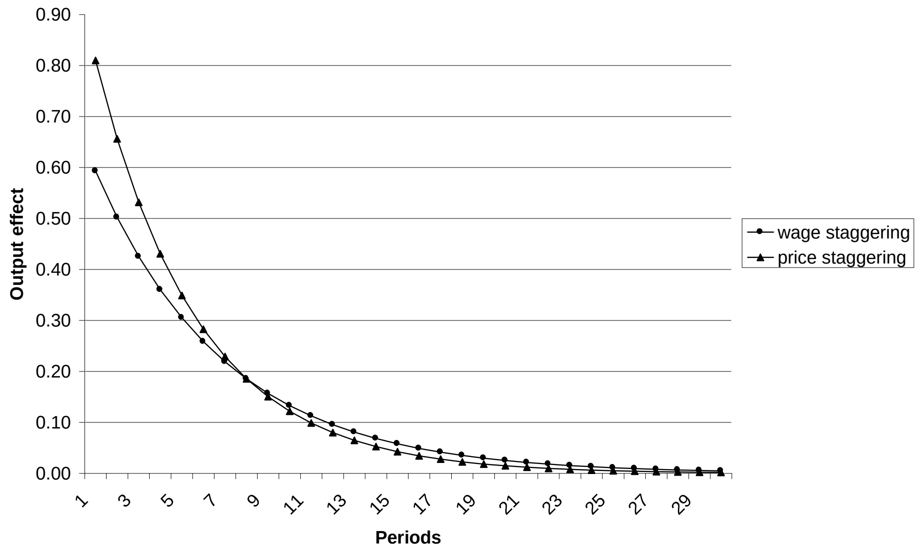


Figure 1: The Conventional Case

assume that $\varepsilon_w = \varepsilon_p$ and set them both to 10, as it is common in the literature (see e.g. Kim, 2003), although there is no empirical literature that would give explicit support for this assumption.

Under this standard assumption, we obtain the conventional finding of the existing literature, namely that the output response dies out more slowly under wage staggering than under price staggering. Existing studies in general use the contract multiplier to measure persistence (see e.g. Huang and Liu, 2002),[15] dividing the output effect in the fourth period (as the average contract duration is 4 when setting either $\theta_w$ or $\theta_p$ to 0.75) by the output effect during the impact period. For the described calibration we get a contract multiplier of 53% for price staggering, whereas it is 72% for wage staggering (see Figure 1 for an optical inspection).

## 3.2 Competition and Persistence

### 3.2.1 Numerical Results

For simplicity, we capture the degree of competition in the product and labor markets by the elasticities of substitution among products (in household consumption) and among labor types (in firm production), respectively. The greater the product elasticity of substitution, the lower is the mark-up of prices over marginal cost (Lerner's

[15] Chari et al. (2000, p. 1152) use a somewhat different version of the contract multiplier, defined as: "half life of output in the model with staggered price setting to the half life of output under synchronized price setting."

index of monopoly power); the greater the labor elasticity of substitution, the lower is the mark-up of wages over the marginal rate of substitution between labor and consumption.

For a variety of reasons, product markets are commonly more competitive than labor markets. This is certainly true under centralized wage bargaining, since centralized price bargaining is relatively uncommon. But even in the absence of centralized wage bargaining, wage setting often tends to be more centralized than price setting: workers of comparable types in an enterprise or firm often set their wages at the same time, whereas such synchronization generally does not apply to substitutable products across the economy. Consequently, firms' costs of switching among standard labor types tends to be substantially greater than consumers' costs of switching among standard product types.

Microeconomic evidence shows that the elasticities of substitution among different labor types are quite low. Griffin's (1992)[16] estimate for the elasticity of substitution between white males and females as well as for white males and black males are e.g. roughly 3.[17] Thus we set the elasticity of substituion to 2 and 4, respectively.[18] The elasticities of substitution that are used for different product types in the literature have a very wide span too. We are aware of a range from 6 (Sbodorne, 2002) to 10 (Chari et al., 2000) or 11 (Galì, 2003), which would mean mark-ups between 10 and 20% over the marginal costs.

It turns out that the relative degrees of competition in the product and labor markets (viz., the relative elasticities of substitution[19]) play an important role in determining the relative magnitudes of monetary persistence generated by wage and price staggering. To show this, we plotted the output responses for different labor elasticities of substitution ($\varepsilon_w = 2$, $\varepsilon_w = 4$) that may be empirically more realistic (see Figure 2).

The impulse response function of the price-staggering model ($\varepsilon_p = 10$) starts at a much higher level than the one for the wage staggering function. It dies out at about the same speed than the one of the wage-staggering model with $\varepsilon_w = 2$ and somewhat faster as the one with $\varepsilon_w = 4$.

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[16] Griffin (1992) used firm-level data for 555 large firms listed on the New York Stock Exchange.

[17] Based on an estimation with a translog cost system with capital included and with federal contractors. See Griffin (1992).

[18] We are in line with Huang and Liu (2002), who - in contrast to many other authors - use different values for the elasticities of substitution of wage and price staggering. They set $\varepsilon_w$ equal to 2, 4 and 6 alternatively.

[19] In the context of our model, the elasticity of substitution among labor types depends on what constitutes a wage-setting cohort. If workers with comparable human capital set their wages at the same time, then the corresponding elasticity of substitution among different cohorts will be relatively small. On the other hand, if wage-setting cohorts are chosen randomly across occupations, then the corresponding elasticity will of course be relatively high.

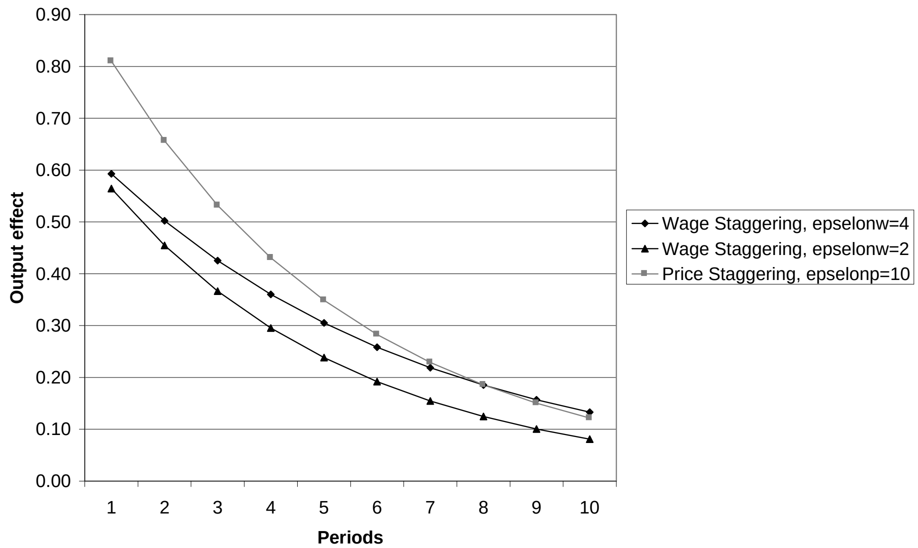


Figure 2: Output IRFs for Different Market Structures

#### 3.2.2 An Alternative Measure of Monetary Persistence

When we set $\varepsilon_p = 10$ and $\varepsilon_w = 4$, the contract multiplier for wage staggering is 61% and thus well above the 53% for price staggering. Again, even with a significant difference in the market structure in the product and labor market, the conventional wisdom seems to hold: wage staggering generates more output persistence than price staggering in terms of the contract multiplier. Nevertheless, the optical inspection of Figure 2 calls this result into question. Although the output IRF for wage staggering dies out more slowly (see contract multiplier), it starts at a much lower level. The contract multiplier captures the relative change in the slope of the IRFs, but not the relative positions of these IRFs. If the wage and price staggering IRFs had the same slope, but the wagesetting IRF were much lower, then the wage and price-setting responses would have the same contract multiplier, but we would clearly like to say that the output response under wage setting is more persistent (in some sense) than that under price setting.

On this account, we propose a new output persistence measure. Our main measure of monetary persistence will be what we have called *quantitative persistence*: the sum of all output changes from period 2 onwards, due to a one-off monetary shock which is normalized to a unit shock:

$$\psi = \sum_{t=2}^{\infty} \tilde{y}_t \tag{29}$$

where $\tilde{y}_t$ is the difference between output in the presence and absence of the shock (deviations from the steady state).

This expression would have to be rewritten if we assume an exogenenous serial correlation of the money supply, as it is done in most papers. Then we would have to subtract the effects, resulting from the additional increase in the money supply due to the serial correlation.

When $\varepsilon_p = \varepsilon_w = 10$, then the quantitative persistence measure is 5.37 for wage staggering and 3.46 for price staggering. Thus the qualitative result of the "contract multiplier" that wage staggering is more persistent than price staggering is confirmed when both markets have the same competitive structure.

For $\varepsilon_p = 10$ and $\varepsilon_w = 4$, the quantitative persistence measure is 3.46 for price staggering and 3.28 for wage staggering. Thus the degree of persistence is similar, albeit somewhat bigger for price staggering. This result is more in line with the optical inspection of Figure 2, which shows two impulse response functions with a similar output effect. As a consequence, the conventional result that wage staggering is always a lot more persistent than wage staggering is already questioned.

For $\varepsilon_w = 2$ the contract multiplier drops to 52%. Thus it indicates equivalence of wage and price staggering. The visual inspection of Figure 2 shows that the contract multiplier tells a completely counter-intuitive story. Both IRFs die out at about the same speed,[20] but the IRF for price staggering starts at a much higher level. From our point of view it would be hard to claim that the two IRFs are equivalent in terms of output persistence. The quantitative persistence captures the difference appropriately and falls to 2.34 for $\varepsilon_w = 2$, whereas it is 3.46 for price staggering. As a consequence, the quantitative persistence measure signals that price staggering would be almost 50% more persistent than wage staggering.

Figure 3 depicts the persistence from price staggering to wage staggering (as a quotient, in terms of quantitative persistence) when we fix $\varepsilon_p = 10$ and change the labor elasticity of substitution ($\varepsilon_w$) in the wage-staggering model (the labor elasticity of substitution varies from 1 to 10, corresponding to a range of $\varepsilon_p - \varepsilon_w$ from -9 to 0, as shown in Figure 3). It can be seen that the labor elasticity of substitution ($\varepsilon_w$) has to be about 5.5 units smaller than the product elasticity of substitution ($\varepsilon_p$) to obtain the same "quantitative persistence" for both staggering types (quotient is equal to 1). The more competitive the product market is relative to the labor market, the greater is the persistence from price staggering relative to wage staggering.

The gap between $\varepsilon_w$ and $\varepsilon_p$ that is necessary to generate the same output persistence by wage and price staggering depends of course on the base value for $\varepsilon_p$. The smaller $\varepsilon_p$, the smaller has to be the gap to obtain the same quantitative persistence. If we assume for example that the elasticity of substitution in the product market is 6, which appears to be the lower bound in the literature, then the two models would show the same persistence if the elasticity of substitution in the labor market would be 2.5.[21]

[20] As measured by the contract multiplier.

[21] In both cases the quantitative persistence measure would be about 2.6.

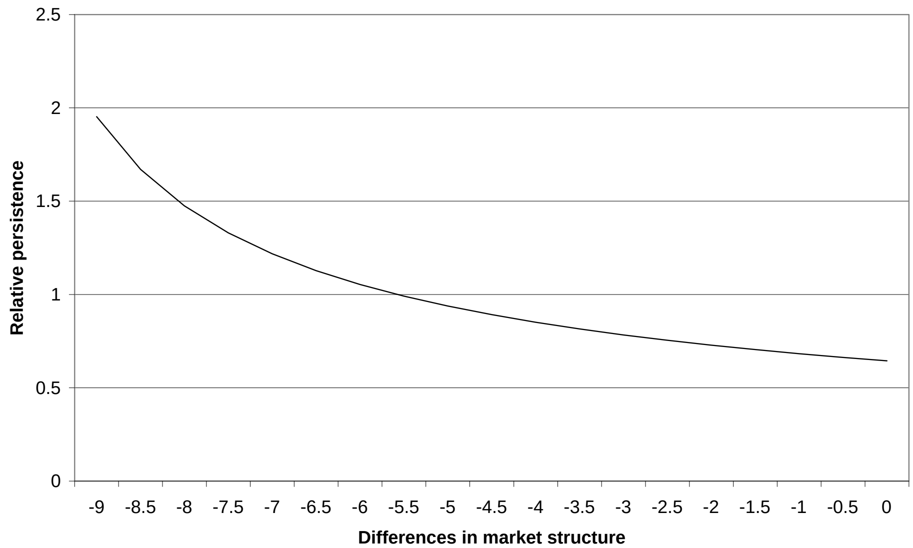


Figure 3: Competition and Relative Persistence (The elasticity of substituion for different product types is fixed to 10. The abscissa denotes $\varepsilon_p - \varepsilon_w$.)

## 3.3 Intuition

### 3.3.1 The Conventional Intuition

The conventional intuition on why monetary persistence is greater under wage staggering than under price staggering may be summarized as follows.[22] Suppose that there are constant returns to labor. Under *price staggering* households set their wages as mark-up over the current marginal rate of substitution.[23] As the households' wage decision is synchronized, wages adjust quickly. They even overshoot their new steady state level, since the positive output effect during the initial periods after the shock increases the marginal disutility of labor and thus raises the marginal rate of substitution between work and consumption. In response, firms raise their prices quickly, since these prices are a constant mark-up over current and future marginal costs (due to constant returns to labor). However prices adjust less quickly than they would do in the absence of price staggering.

Under *wage staggering,* a positive monetary shock raises employment and, with it, the disutility of labor, and thus each household has an incentive to push the wage up. But an increase in the individual wage also raises the household's wage relative to other wagesetting cohorts, leading to a fall in the demand for the household's labor. These wage adjustments are moderate, however, since households dislike fluctuations in their working hours (as the marginal disutility of labor rises with hours employed).

[22] See Huang and Liu (2002) for a more detailed description.

[23] Under perfect competition, naturally, wages are equal to the marginal rate of substitution.

Thus, in contrast to the price-staggering model, there is a gradual rise in wages, rather than overshooting. This leads to slower price adjustments by firms,[24] even though prices can be adjusted instantaneously. The slower price adjustment leaves more room for output deviations from the steady state.

Consequently wage staggering delivers more output persistence than does price staggering.

### 3.3.2 Intuition on How Diminishing Returns Affect Monetary Persistence

We have argued that monetary persistence is a short-run phenomenon, over which returns to labor are generally diminishing. In this context, marginal costs are clearly no longer constant across firms, but depend on the firms' employment.[25]

When there is a positive monetary shock in the *price-staggering model*, then (as above) households adjust their wages upwards instantaneously and wages overshoot their long-run equilibrium. This leads to a rise in average marginal costs for the economy. Thus each firm has an incentive to raise its price. When it does, its price rises relative to other prices and its marginal costs rise relative to other marginal costs.[26] Due to these variations in firm-specific marginal costs, the firm's price increase will be less than it would have been if all firms had the same marginal cost schedule under contant returns to labor. (The faster the returns to labor diminish, the more moderate the price adjustment will be.) Thus the adjustment path from the old to the new steady takes a longer time.[27] This extends the duration of the deviation of output from the steady state, i.e. it magnifies output persistence.[28]

Under *wage staggering*, decreasing returns to labor lead to larger deviations of prices from the old steady state in the impact period than constant returns. The reason is that prices are a mark-up over marginal costs, the marginal costs depend on the deviation of output from the steady state (under diminishing returns), and output responds to the monetary shock.[29] Because of the instantaneous inflation jump during

[24] When we assume no productivity shocks the deviations of the marginal costs from the steady state would be equal to the deviations of the wages from the steady state $\tilde{m}c_t = \tilde{w}_t$. The firm sets prices equal to marginal costs ($\tilde{p}_t = \tilde{m}c_t$).

[25] Mathematically: $\tilde{p}_t = \tilde{m}c_t = \tilde{w}_t + (\alpha / (1-\alpha))\, \tilde{y}_t$.

[26] In mathematical terms: $\tilde{m}c^r_{t,t+i} = \tilde{m}c^r_{t+i} - (\varepsilon\alpha / (1-\alpha)) \left(p^*_t - \tilde{p}_{t+i}\right)$, where $\tilde{m}c^n_{t,t+i}$ is the deviation of the firm-specific nominal marginal costs from the steady state and $\tilde{m}c^n_{t+i}$ is the one of the average economy wide average nominal marginal costs.

[27] Mathematically this can be seen in the following Phillips curve relationship, by setting $\alpha$ to different values $\beta E_t \tilde{\pi}_{t+1} = \tilde{\pi}_t - [(1-\theta_p)(1-\beta\theta_p)(1-\alpha)] / [\theta_p [1 + \alpha(\varepsilon_p - 1)]]\, \tilde{y}_t$.

[28] Note that there is a second countervailing effect. Under decreasing returns to labor the average marginal costs in the economy rise steeper when there is a positive output effect. As a consequence, the overall output effect in the economy is reduced, as we have even more pro-cyclical average marginal costs than under constant returns to labor. Nevertheless, this second effect is dominated by the first one under usual calibrations.

[29] Mathematically, $\tilde{p}_t = \tilde{w}_t + (\alpha / (1-\alpha))\, \tilde{y}_t$. When $\alpha = 0$ (constant returns to labor), we obtain $\tilde{p}_t = \tilde{w}_t$.

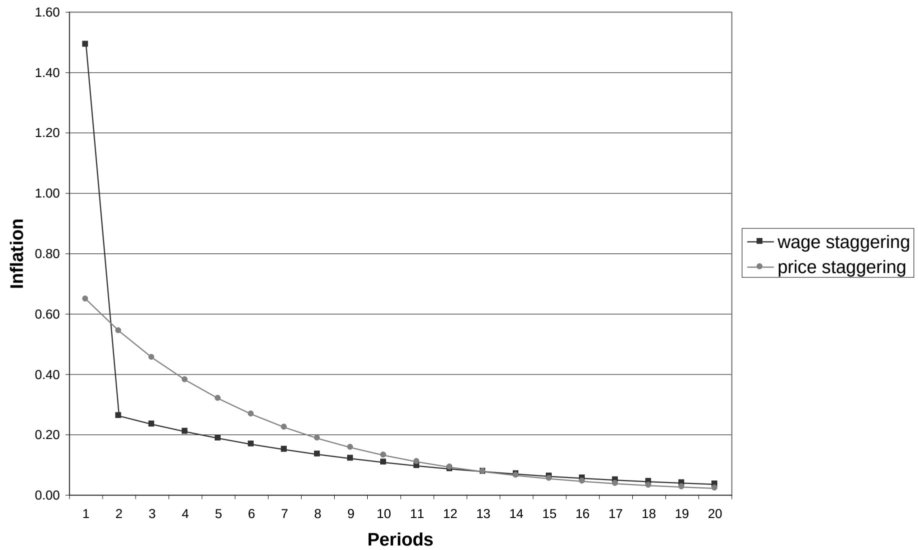


Figure 4: Inflation Persistence

the impact period (see Figure 4), the room for output adjustments will be reduced considerably and thus the wage-staggering mechanism will generate less persistence in terms of "quantitative persistence" than under constant returns to labor.

Although the New Keynesian literature often claims that wage staggering generates more plausible impulse response functions of output with respect to monetary shocks, our analysis sounds a cautionary note. First, as noted, the wage staggering generates more output persistence only when the elasticities of substitution for labor and products are sufficiently close. Secondly, wage staggering has a lower inflation persistence than price staggering, either in terms of the contract multiplier or in terms of quantitative persistence (see Figure 4).

The intuition above shows why the existing literature - resting on the assumption of constant returns to labor - concludes that wage staggering generates more output persistence than price staggering. If the marginal disutility of labor function is assumed to be increasing with output, whereas the marginal cost curve is assumed to be flat and thus independent of the firm-specific output, then wage staggering turns out to lead to more output persistence than price staggering. But in the presence of diminishing returns to labor - which is appropriate in the context of monetary persistence - the output effects of the wage-staggering mechanism are weakened and thus the conventional result need no longer hold.

### 3.3.3 Intuition on How Competitiveness Affects Monetary Persistence

We now explain intuitively how the relative competitiveness of the labor and product markets influences monetary persistence. We measure relative competitiveness in terms of the relative elasticities of substitution among products and labor types. The greater the elasticity of substitution, the smaller is the individual wage rise (in the wage-staggering model) or price rise (in the price-staggering model) relative to the market average, in response to a positive monetary shock. Since demand fluctuations are undesirable for households and firms with repect to their utility and profit maximization, the degree of wage/price adjustment will be more muted.[30] As result, the output response is more persistent.

This means that relative competitiveness matters for persistence. The more competitive the product market relative to the labor market, the greater is the monetary persistence generated by price staggering relative to that generated by wage staggering.

# 4 The Complementarity between Wage and Price Staggering

Finally, consider an economy where households and firms set *both* prices and wages in a staggered fashion. Specifically, households set their wages as mark-up over the current and future individual marginal rate of substitution and prices, firms set their prices as mark-up over their current and future firm-specific marginal costs. Consequently, there is an *intertemporal wage-price spiral:* the slower wages adjust, the slower prices adjust, and vice versa.

The dynamic system for joint wage and price staggering is

$$E_t\tilde{y}_{t+1}+\frac{1}{\sigma}E_t\tilde{p}_{t+1}+\frac{1}{\sigma\eta}\left(\tilde{m}_t-\tilde{p}_t\right)=\left(1+\frac{1}{\sigma\eta}\right)\tilde{y}_t+\frac{1}{\sigma}\tilde{p}_t \tag{30}$$

$$\beta E_t\tilde{p}_{t+1}=\left(1+\beta+\delta\right)\tilde{p}_t-\tilde{p}_{t-1}-\delta\tilde{w}_t^n+\delta\frac{\alpha}{1-\alpha}\tilde{y}_t \tag{31}$$

$$\beta\theta_w E_t\tilde{w}_{t+1} = -\theta\tilde{w}_{t-1}(1+\beta\theta_w^2-\frac{1}{1+\varphi\epsilon_w}(1-\theta_w)(1-\beta\theta_w)\varphi\epsilon_w)\tilde{w}_t \tag{32}$$

$$-\frac{1}{1+\varphi\epsilon_w}(1-\theta_w)(1-\beta\theta_w)((\sigma+\varphi\frac{1}{(1-\alpha)})\tilde{y}_t+\tilde{p}_t) \tag{33}$$

and the money growth equation (15) holds.

[30] Firms face the following demand schedule $Y_{t+i}\left(j\right)=\left(P_t^*\left(j\right)/P_{t+i}\right)^{-\varepsilon_p}Y_{t+i}$ and the labor demand looks as follows $N_{t+i}\left(h\right)=\left(W_t^*\left(h\right)/W_{t+i}\right)^{-\varepsilon_w}N_{t+i}$.

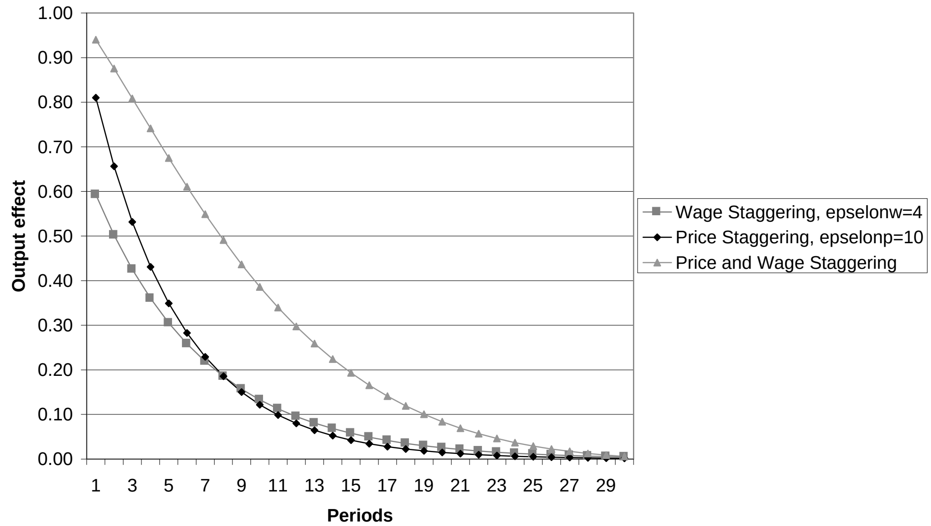


Figure 5: IRFs from Wage Staggering, Price Staggering and Both Types

In this context, we inquire whether wage and price staggering are complementary in their influence on monetary persistence, i.e. whether their joint effect on persistence is greater than the sum of the individual effects. Specifically, we measure the degree of complementarity ($\upsilon$) by dividing the joint effect of wage and price staggering ($\psi_{w+p}$) by the sum of individual effects of the two types of staggering ($\psi_p + \psi_w$):

$$\upsilon = \frac{\psi_{w+p}}{\psi_p + \psi_w}. \tag{34}$$

Values bigger than 1 signal that wage and price staggering are complementary, whereas they are substitutes for values smaller than 1.

When we set $\varepsilon_p = 10$ and $\varepsilon_w = 4$ (and use the same calibration as before, Figure 5 shows the impulse response functions of the three models), we get a quantitative persistence measure of 7.75 for joint staggering, which gives us a complementarity measure of $\upsilon = 1.15$. Thus joint wage and price staggering is 15% more output persistent than the sum of the two staggering mechanisms.[31]

It can be shown that the complementarity depends on the existence of decreasing returns to labor. In our numerical simulations, wage and price staggering are not complementary under constant returns to labor, and they become complementary only once $\alpha$ is larger than 0.15 (see Figure 6).

[31] As the contract multiplier is 53% for price staggering and 61% for wage staggering, it would be impossible to have complementarities.

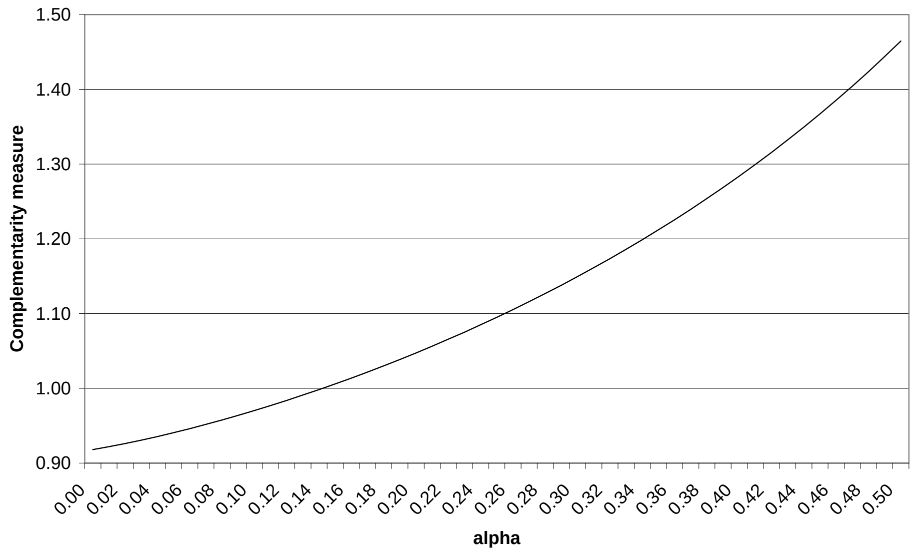


Figure 6: Complementarities between Wage and Price Staggering

# 5 Relation to the Literature

There is a relatively large body of literature on the relative degree of monetary persistence arising from wage and price staggering under Taylor contracts, but relatively little under Calvo contracts (the focus of this paper).

As noted, the recent literature on Taylor contracts concludes that wage staggering generates more monetary persistence than price staggering. In Andersen's (1998) model output responses from wage staggering are always longer lived than from price staggering. In Huang and Liu's (2002) paper the output responses from price staggering are dampened oscillatory, whereas the output IRFs from wage staggering are not.[32] The oscillatory output response to monetary shocks for Taylor contracts under the standard numerical calibrations in dynamic stochastic general equilibrium (DSGE) models (Kiley, 1997, Chari et al., 2000, Huang and Liu, 2002) is considered an important weakness of the Taylor model.

Some authors have sought to overcome persistence problems by incorporating real rigidities in price-staggering models. Edge (2002) assumes firm-specific factor inputs to restore the equivalence of wage and price staggering, i.e. that each household is coupled with a firm, hiring its labor and capital out to that firm only.[33] Jeanne (1998) introduces a real wage rigidity, as unions may be concerned about a fair division of income between labor and capital. Kiley (1997) analyzes the effect of several real

[32] Erceg (1997) uses both types of staggering, which can account for a strong contract multiplier.

[33] The basic idea to slow down price adjustments with real rigidities in a DSGE model with nominal rigidities was first proposed by Kimball (1995) and implemented by Rotemberg (1996). In a unifying framework Ascari (2003) shows that labor immobility plays a key role in generating persistence.

rigidities to increase the persistence of price staggering, such as countercyclical markups.[34] The basic insight goes back to Blanchard and Fischer (1989) and Ball and Romer (1990), who argue that it is necessary to flatten the supply side in order to prevent procyclical marginal costs, which would lead to fast price adjustments and thus low persistence.

Taylor (1999) observed that "there needs to be some neighborhood effects between price setters, so that one firm pays attention to the price decision of the next firm and the most recent firm, thereby linking the price decision of one firm to another and causing the persistence effects". This phenomenon applies to our price-staggering model. Under decreasing returns to labor, firms pay more attention to their relative price from a purely profit-maximizing perspective. If the firm specific price is too far above the average market price,[35] there will be undesirable fluctuations in firm-specific demand.[36]

Regarding Calvo contracts (as in our paper), various contributions examine how realistically Calvo wage and/or price staggering can replicate empirical impulse response functions or how optimal monetary policy has to be conducted in such a framework.[37] To the best of our knowledge, however, the only study that explicitly discusses the differences in persistence generated by Calvo wage and price staggering is Kim (2003). He states that in contrast to Taylor contracts, Calvo wage and price staggering can both generate persistence (no oscillatory movements). But similar to the studies for Taylor staggering, he concludes that wage staggering is generally better able to generate persistence. We confirm the first result, but have doubts about the second because it hinges on two important implausible assumptions: (i) in the basic version of Kim's model (section 2.2.1) the capital stock adjusts flexibly and instantaneously (which we have argued is unlikely to occur over the time span relevant for monetary persistence)[38] and (ii) Kim (2003) assumes the same elasticity of substitution for different product and labor types, whereas we argue that product markets are generally more competitive than labor markets.

The inability to explain sufficient inflation persistence is known to be a major

[34] Kiley (1997) therefore used the ideas of a model from Gali (1994).

[35] See Sbodorne (2002) for an equivalent mathematical derivation.

[36] Thus our result is somewhat contrasting to Kiley's (1997), who claims that increasing returns to labor flatten marginal costs and thus increase persistence. The effect we describe above cannot kick in, as Kiley (1997) uses a first order Taylor approximation to remove firm-specific subscripts. Further differences are that his model incorporates capital accumulation and uses Taylor contracts.

[37] To mention just a few examples: Rotemberg and Woodford (1998) try to match empirical impulse response functions with a Calvo price staggering model. Christiano et al. (2005) have the same objective. Gali (2003) derives impulse response functions from Calvo price staggering and discusses optimal monetary policy. Erceg et al. (2000) use a model with Calvo wage and price staggering that is similar in spirit to ours. They do not discuss the issue of monetary persistence, but optimal monetary policy.

[38] Eichenbaum and Fisher (2004) find out that a fixed-capital version fits the empirical evidence better. A discussion of this issue can be found in Altig et al. (2005). This and other very recent papers (e.g. Woodford, 2005) model firm-specific capital endogenously.

weakness of New Keynesian models (see, for example, Mankiw, 2001). Our paper contributes to this literature by showing and explaining the intuition why wage staggering under decreasing returns has a low inflation persistence, either measured in terms of the contract multiplier or in terms of "quantitative persistence."

The role of the elasticity of substitution has been mentioned in the literature (Ascari 2003, Huang and Liu, 2002[39]), but the influence of *relative competitiveness* in the labor and product markets on the relative monetary persistence generated by wage and price staggering has not been analyzed. This paper does so numerically and intuitively. Furthermore, the existing literature uses the contract multiplier to measure output persistence from numerical impulse response functions (see e.g. Huang and Liu, 2002, Kim 2003). The weaknesses of this measure have not been discussed to date. This paper does so and introduces the quantitative persistence measure to address this problem. The complementarity of wage and price staggering in generating persistence has not been examined either in the literature; our "quantitative persistence" measure enables us to do so in a meaningful way.

# 6 Concluding Thoughts

This paper shows that the relative degree of competition in the labor and product markets plays a central role in determining the relative monetary persistence arising from wage and price staggering. The more competitive a market is, the more persistent will be the output responses to a monetary shock arising from the wage or price inertia in that market. The intuition is that deviating too much from the optimal price or wage will lead to bigger demand changes in the labor or product markets if there is more competition (i.e. the elasticity of substitution is bigger). Consequently, more competition leads to a dampened wage and price adjustment, which leaves more room for deviations of the output from the steady state.

Finally, we find that wage and price staggering have complementary effects on monetary persistence. We show this in terms of a new measure of monetary persistence, our "quantitative persistence" statistic. The existence of complementarities means that beyond understanding how wage and price staggering work in isolation, it is very important to explore their interactions.

[39] Both studies use Taylor contracts.

# 7 Literature

Altig, D., Christiano, L., Eichenbaum, M., and Linde, J. (2005). Firm-Specific Capital, Nominal Rigidities and the Business Cycle. *NBER Working Paper*; No. 11034; January 2005.

Andersen, T. M. (1998). Persistency in Sticky Price Models. *European Economic Review*; 42; 593-603.

Ascari, G. (2003). Price/Wage Staggering and Persistence: a Unifying Framework. *Journal of Economic Surveys*; Vol. 17; No. 4; 511-540.

Ball, L., and Romer, D. (1990). Real Rigidities and the Non-Neutrality of Money. *Review of Economic Studies*; 57; 183-203.

Blanchard, O. J., and Fischer, S. (1989). Lectures on Macroeconomics. MIT Press, Cambridge; 1989.

Blanchard, O. J., and Kiyotaki, N. (1987). Monopolistic Competition and the Effects of Aggregate Demand. *American Economic Review*; 77; 647-666.

Calvo, G. A. (1983). Staggered Prices in a Utility Maximizing Framework. *Journal of Monetary Economics*; 12; 383-398.

Chari, V. V., Kehoe, P. J., and McGrattan, E. R. (2000). Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem. *Econometrica* 2000; Vol. 68; No. 5 (September 2000); 1151-1179.

Christiano, L. J., Eichenbaum, M., and Evans, C.L. (2005). Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. *Journal of Political Economy*; Vol. 113; No. 1; 1-45.

Clarida, R. H., Gali, J., and Gertler, M. (1999). The Science of Monetary Policy: a New Keynesian Perspective. *Journal of Economic Literature*, Vol. 37, No. 4, 1661-1707.

Dixit, A. K., and Stiglitz, J. E. (1977). Monopolistic Competition and Optimum Product Diversity. *American Economic Review*; 67; 297-308.

Edge, R. M. (2002). The Equivalence of Wage and Price Staggering in Monetary Business Cycle Models. *Review of Economic Dynamics*; 5; 559-585.

Eichenbaum, M., and Fischer, J. D. M. (2004). Evaluating the Calvo Model of Sticky Prices. *NBER Working Paper*; No. 10617; July 2004.

Erceg, C. J. (1997). Nominal Wage Rigidities and the Propagation of Monetary Disturbances. Board of Governors of the Federal Reserve System; *International Finance Discussion Papers*; No. 590; September 1997.

Erceg, C. J., Henderson, D. W., and Levin, A. T. (2000) Optimal Monetary Policy with Staggered Wage and Price Contracts. *Journal of Monetary Economics*, 46, 281-313.

Gali, J. (2003). New perspectives on Monetary Policy, Inflation, and the Business Cycle. In: Dewatriport, M, Hansen, LP, Turnovsky, SJ (Eds), *Advances in Economics and Econometrics*: *Theory and Applications*; eighth World Congress. 2003; 151-197.

Gali, J. (1994). Monopolistic Competition, Business Cycles, and the Composition of Aggregate Demand. *Journal of Economic Theory*; 63; 73-96.

Gali, J., Gertler, M., and López-Salido, D. (2003). Markups, Gaps, and the Welfare Costs of Business Fluctuations. *CEPR Working Paper*; No. 4134; 2003.

Griffin, P. (1992). The Impact of Affirmative Action on Labor Demand: a Test of Some Implications of the Le Chatelier Principle. *Review of Economics and Statistics*; 74; 251-260.

Huang, K. X. D., and Zheng L. (2002). Staggered Price-Setting, Staggered Wage Setting, and Business Cycle Persistence. *Journal of Monetary Economics*; 49; 405-433.

Jeanne, O. (1998). Generating Real Persistent Effects of Monetary Shocks: How Much Nominal Rigidity Do We Really Need? *European Economic Review*; Vol. 42; No. 6; 1009-1032.

Kiley, M. T. (1997). Staggered Price Setting and Real Rigidity. *Federal Reserve Board; Working Paper*; 1997.

Kim, J. (2003) Contract Multiplier Revisited: Solving the Persistence Problem in a Model with Staggered Contracts. *Federal Reserve Bank of Minneapolis*; February 2003.

Kimball, M. S. (1995). The Quantitative Analytics of the Basic Neomonetarist Model. *Journal of Money, Credit, and Banking*; 27; 1241-1277.

Mankiw, G. N. (2001). The Inexplorable and Mysterious Tradeoff Between Inflation and Unemployment. *The Economic Journal*; Vol. 111; 45-61.

Rotemberg, J. J. (1996). Prices, Output, and Hours: An Empirical Analysis Based on a Sticky Price Model. *Journal of Monetary Economics*; 37; 505-533.

Rotemberg, J. J., and Woodford, M. (1998) An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy: Expanded Version. *NBER Technical Working Papers*; No. 0233; 1998.

Sbodorne, A. N. (2002). Prices and Unit Labor Costs: a New Test of Price Stickiness. *Journal of Monetary Economics*; 49; 265-292.

Stahl, H. Time-Dependent or State-Dependent Price Setting? - Micro-Evidence from German Metal-Working Industries -. *Deutsche Bundesbank*; *mimeo*; 11 July 2005.

Taylor, J. B. (1999). Staggered Price and Wage Setting in Macroeconomics. In: Taylor, J. B., Woodford, M. (Eds.). *Handbook of Macroeconomics.* Vol. 1B.; 1999; 1009-1050.

Taylor, J. B. (1980). Aggregate Dynamics and Staggered Contracts. *Journal of Political Economy*; 88; 1-23.

Woodford, M. (2005). Firm-Specific Capital and the New-Keynesian Phillips Curve. *NBER Working Paper*; No. 11149; February 2005.

# 8 Technical Appendix

## 8.1 Wage-Staggering Model

### 8.1.1 Household's Optimization Problem

The representative household optimizes the following utility function:

$$\max_{\{C_{t+i}, W_t(h), B_{t+i}, M_{t+i}\}} E_t \sum_{i=0}^{\infty} \beta^i \left[ U\left(C_{t+i}(h)\right) - V\left(N_{t+i}(h)\right) + Z\left(\frac{M_{t+i}(h)}{P_{t+i}}\right)\right] \tag{35}$$

subject to its budget constraint:

$$\begin{aligned} & E_t \sum_{i=0}^{\infty} \beta^i \left( C_{t+i} + \frac{R_{t+i}^{-1} B_{t+i} + M_{t+i}}{P_{t+i}} \right) \\ = \; & E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{W_t(h)}{P_{t+i}} N_{t+i}(h) + \frac{T_{t+i}}{P_{t+i}} + \frac{\Pi_{t+i}}{P_{t+i}} + \frac{B_{t+i-1}}{P_{t+i}} + \frac{M_{t+i-1}}{P_{t+i}} \right] \end{aligned} \tag{36}$$

and its labor demand function:

$$N_{t+i}(h) = \left( \frac{W_t^*(h)}{W_{t+i}} \right)^{-\varepsilon_w} N_{t+i}, \;\; i = 1, ..., N-1 \tag{37}$$

where $P_t$ is the aggregate price index, $R_{t+i}=1 + r_{t+i}$ is the nominal interest factor on its bond holdings $B_{t+i}$, $T_{t+i}$ is its net lump-sum transfers from government, and $\Pi_{t+i}$ is its profit income.

The problem can be decomposed in a wage-contracting problem where the wage is optimized with respect to all endogeneous variables and a intra-contract period problem where the wage is taken as given and the optimal level of money, bond holdings, and consumption is chosen.

**Wage-Contracting Problem:** Every time the household can change its wages, it has to solve the following optimization problem:

$$\max_{\{\mathsf{W}_t^*(h)\}} E_t \sum_{i=0}^{\infty} (\beta\theta_w)^i \left[ U\left(C_{t+i}(h)\right) - V\left(N_{t+i}(h)\right) + Z\left(\frac{M_{t+i}(h)}{P_{t+i}}\right)\right] \tag{38}$$

s.t.

$$\begin{aligned} & E_t \sum_{i=0}^{\infty} \beta^i \left( C_{t+i} + \frac{R_t^{-1} B_{t+i} + M_{t+i}}{P_{t+i}} \right) \\ = \; & E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{W_t(h)}{P_{t+i}} N_{t+i} + \frac{T_{t+i}}{P_{t+i}} + \frac{\Pi_{t+i}}{P_{t+i}} + \frac{B_{t+i-1}}{P_{t+i}} + \frac{M_{t+i-1}}{P_{t+i}} \right] \end{aligned} \tag{39}$$

and

$$N_{t+i}(h) = \left( \frac{W_t^*(h)}{W_{t+i}} \right)^{-\epsilon_w} N_{t+i} \tag{40}$$

Since the product market is perfectly competitive, profit income is zero: $\frac{\Pi_{t+i}}{P_{t+i}} = 0$. Furthermore, for simplicity, we assume that the government refunds its seigniorage from money and bond creation to the households in the form of lump-sum transfers:

$$\frac{R_t^{-1} B_{t+i} + M_{t+i}}{P_{t+i}} = \frac{T_{t+i}}{P_{t+i}} + \frac{B_{t+i-1}}{P_t} + \frac{M_{t+i-1}}{P_t}. \tag{41}$$

Then the household's budget constraint reduces to

$$E_t \sum_{i=0}^{\infty} \beta^i C_{t+i} = E_t \sum_{i=0}^{\infty} \beta^i \frac{W_t(h)}{P_{t+i}} N_{t+i}. \tag{42}$$

For analytical tractability, we make the usual assumption that households can insure themselves against idiosyncratic consumption shocks.[40] Thus:

$$P_{t+i} C_{t+i} = W_{t+i} N_{t+i}. \tag{43}$$

By substituting (40) and (43) into the utility function and taking the first derivative with respect to the wage, we obtain the following optimal wage:

$$W_t^*(h) = \frac{\epsilon_w}{\epsilon_w - 1} \frac{E_t \sum_{i=0}^{\infty} (\beta\theta)^i \left[ - V_N \left( N_{t+i}^d(h) \right) \right] N_{t+i}^d(h)}{E_t \sum_{i=0}^{\infty} (\beta\theta)^i \left[ \frac{U_c(C_{t+i})}{P_{t+i}} \right] N_{t+i}^d(h)} \tag{44}$$

In logs:

$$\begin{aligned} w_t^*(h) &= \ln \left( \frac{\epsilon_w}{\epsilon_w - 1} \right) + \ln \left( E_t \sum_{i=0}^{\infty} (\beta\theta)^i \left[ - V_N \left( N_{t+i}^d(h) \right) \right] N_{t+i}^d(h) \right) \\ &\quad - \ln \left( E_t \sum_{i=0}^{\infty} (\beta\theta)^i \left[ \frac{U_C(C_{t+i})}{P_{t+i}} \right] N_{t+i}^d(h) \right) \end{aligned} \tag{45}$$

We log-linearize as follows:

$$w_t^*(h) \approx \bar{w}^*(h) + \sum_{i=0}^{\infty} \left[ \begin{array}{c} \left[ \frac{\partial w_t^*(i)}{\partial V_N} \frac{\partial V_N}{\partial \ln V_N} \right]_{equ} \left[ \begin{array}{c} \ln \left[ -V_N \left( N_{t+i}^d(h) \right) \right] - \\ \ln \left[ -\bar{V}_N (N^d(h)) \right] \end{array} \right] \\ - \left[ \frac{\partial w_t^*(i)}{\partial U_c} \frac{\partial U_c}{\partial \ln U_c} \right]_{equ} \left[ \begin{array}{c} \log U_c(C_{t+i}) - \\ \log \bar{U}_c(C) \end{array} \right] \\ - \left[ \frac{\partial w_t^*(i)}{\partial P_t} \frac{\partial P_t}{\partial p_t} \right]_{equ} (p_{t+i} - \bar{p}) \end{array} \right] \tag{46}$$

[40] For a more detailed description see e.g. Erceg et al. (2000).

which yields:

$$w_t^*(h) \approx \mu_w + (1-\beta\theta)E_t \sum_{i=0}^{\infty} (\beta\theta)^i \left( \ln \left[ -V_N(N_{t+i}^d(h)) \right] - \ln U_c\left(C_{t+i}\right) + p_{t+i} \right) \quad (47)$$

or put differently:

$$w_t^*(h) \approx \mu_w + (1-\beta\theta)E_t \sum_{i=0}^{\infty} (\beta\theta)^i \left( \ln MRS_{t,t+i} + p_{t+i} \right) \quad (48)$$

where $\mu_w = (\varepsilon_w / (\varepsilon_w - 1))$ is the steady state mark-up and $MRS_{t,t+i} = -V_N(N_{t+i}^d(h))/U_c\left(C_{t+i}\right)$ is the marginal rate of substitution in period $t+i$ of households who set their wages in period $t$.

We can rewrite the individual marginal rate of substitution in terms of the average economy-wide marginal rate of substitution, by using the specific utility function (6):

$$\begin{aligned} MRS_{t,t+i} &= \left( \frac{W_{t+i}N_{t+i}}{P_{t+i}} \right)^{\sigma} \left( \left( \frac{W_t^*(h)}{W_{t+i}} \right)^{-\epsilon_w} N_{t+i} \right)^{\varphi} && (49a) \\ &= \left( \frac{W_{t+i}N_{t+i}}{P_{t+i}} \right)^{\sigma} \left( \left( \frac{W_{t+i}}{W_{t+i}} \right)^{-\epsilon_w} N_{t+i} \right)^{\varphi} \left( \frac{W_t^*(h)^{-\epsilon_w}}{W_{t+i}^{-\epsilon_w}} \right)^{\varphi} && (49b) \\ &= MRS_{t+i} \left( \frac{W_t^*(h)^{-\epsilon_w}}{W_{t+i}^{-\epsilon_w}} \right)^{\varphi} && (49c) \end{aligned}$$

where $MRS_{t+i}$ is the average marginal rate of substitution in the economy.

Using (48), we obtain the following equation:

$$w_t^*(h) = \mu_w + (1-\beta\theta)E_t \sum_{i=0}^{\infty} (\beta\theta)^i \left( mrs_{t+i} - \varphi\epsilon_w \left( w_t^*(h) - w_{t+i} \right) + p_{t+i} \right). \quad (50)$$

Using the following approximate relationship for the aggregate wage index:

$$w_t = \theta w_{t-1} + (1-\theta) w_t^* \quad (51)$$

we obtain:

$$w_t = \theta w_{t-1} + (1-\theta) \frac{1}{1+\varphi\epsilon_w} \left[ \mu_w + (1-\beta\theta)E_t \sum_{i=0}^{\infty} (\beta\theta)^i \left( mrs_{t+i} + \varphi\epsilon_w w_{t+i} + p_{t+i} \right) \right] \quad (52)$$

where $mrs_{t+i}$ is the logarithm of $MRS_{t+i}$.

By iterating by one period forward and multiplying with $\beta\theta_w$:

$$\beta\theta_w E w_{t+1} = \beta\theta_w^2 w_t + (1-\theta)\frac{1}{1+\varphi\epsilon_w}\left[\begin{array}{c}\beta\theta_w\mu_w + (1-\beta\theta)E_t\sum_{i=0}^{\infty}(\beta\theta)^{i+1} \\ \left(mrs_{t+i+1}+\varphi\epsilon_w w_{t+i+1}+p_{t+i+1}\right)\end{array}\right] \tag{53}$$

Thus:

$$\begin{aligned} w_t - \beta\theta_w E_t w_{t+1} &= \theta_w w_{t-1} - \beta\theta_w^2 w_t + \frac{1}{(1+\varphi\epsilon_w)}(1-\theta_w)(1-\beta\theta_w) \\ &\quad \left(mrs_t + \varphi\epsilon_w w_t + p_t\right) + (1-\theta)\frac{1}{1+\varphi\epsilon_w}\left(\mu_w - \beta\theta_w\mu_w\right) \end{aligned} \tag{54}$$

where $\mu_w$ (constant) can be dropped when we take deviations from the steady state.

**Intra-Contract Period Problem:** In each period the households have to choose on the optimal allocation of bonds, money holdings, and consumption. Thus the representative household maximizes its utility

$$\max_{\{C_{t+i},B_{t+i},M_{t+i}\}} E_t\sum_{i=0}^{\infty}\beta^i\left[U\left(C_{t+i}(h)\right) - V\left(N_{t+i}(h)\right) + Z\left(\frac{M_{t+i}(h)}{P_{t+i}}\right)\right] \tag{55}$$

subject to its budget constraint:

$$\begin{aligned} & E_t\sum_{i=0}^{\infty}\beta^i\left(C_{t+i} + \frac{R_t^{-1}B_{t+i} + M_{t+i}}{P_{t+i}}\right) \\ = \;& E_t\sum_{i=0}^{\infty}\beta^i\left[\frac{W_t(h)}{P_{t+i}}N_{t+i}(h) + \frac{T_{t+i}}{P_{t+i}} + \frac{\Pi_{t+i}}{P_{t+i}} + \frac{B_{t+i-1}}{P_{t+i}} + \frac{M_{t+i-1}}{P_{t+i}}\right] \end{aligned} \tag{56}$$

We obtain the following first order conditions via a Langrangian

$$U_{C_t} = \beta R_t E_t\left(U_{C_{t+1}}\frac{P_t}{P_{t+1}}\right) \tag{57}$$

$$\frac{U_{M_t}}{U_{C_t}} = 1 - R_t^{-1}. \tag{58}$$

We optimize the following utility function

$$U\left(C_t(h), \frac{M_t(h)}{P_t}, N_t(h)\right) = \frac{C_t^{1-\sigma}(h)}{1-\sigma} + \frac{\left(\frac{M_t(h)}{P_t}\right)^{1-\nu}}{1-\nu} - \frac{N_t^{1+\varphi}(h)}{1+\varphi} \tag{59}$$

subject to its budget constraint

$$E_t \sum_{i=0}^{\infty} \beta^i \left( C_{t+i} + \frac{R_t^{-1} B_{t+i} + M_{t+i}}{P_{t+i}} \right) \tag{60}$$
$$= E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{W_t(h)}{P_{t+i}} N_{t+i} + \frac{T_{t+i}}{P_{t+i}} + \frac{\Pi_{t+i}}{P_{t+i}} + \frac{B_{t+i-1}}{P_{t+i}} + \frac{M_{t+i-1}}{P_{t+i}} \right]$$

$$\frac{\partial L}{\partial C_t} = C_t^{-\sigma} - \lambda_t = 0 \tag{61}$$

$$\frac{\partial L}{\partial C_{t+1}} = \beta C_{t+1}^{-\sigma} - \beta \lambda_{t+1} = 0 \tag{62}$$

$$\frac{\partial L}{\partial B_t} = R_t^{-1} \frac{1}{P_t} \lambda_t - \beta \frac{1}{P_{t+1}} \lambda_{t+1} = 0 \tag{63}$$

$$\frac{\partial L}{\partial M_t} = \frac{E_t M_t^{-\nu}}{E_t P_t^{1-\nu}} - \lambda_t \frac{1}{P_t} + \beta \lambda_{t+1} \frac{1}{P_{t+1}}. \tag{64}$$

Combining conditions (61), (62), and (63), we obtain the following consumption Euler equation:

$$1 = \beta R_t \left[ \left( \frac{E_t C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{E_t P_{t+1}} \right) \right] \tag{65}$$

We use a first order Taylor approximation:

$$R_t \left[ \left( \frac{E_t C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) \right] = (1 + r_t) E_t \left[ \exp\left( -\sigma \Delta c_{t+1} - \pi_{t+1} \right) \right] \tag{66a}$$
$$\cong (1 + r_t) \left[ 1 - E_t \sigma \Delta c_{t+1} - E_t \pi_{t+1} \right] \tag{66b}$$
$$\cong (1 + r_t) - E_t \sigma \Delta c_{t+1} - E_t \pi_{t+1}. \tag{66c}$$

This delivers us equation (11)..

When we plug (61) and (62) into (64), we obtain the following money demand equation:

$$\frac{\frac{E_t M_t^{-\nu}}{E_t P_t^{-\nu}}}{C_t^{-\sigma}} - 1 + \beta \frac{C_{t+1}^{-\sigma}}{C_t^{-\sigma}} \frac{P_t}{P_{t+1}} = 0 \tag{67}$$

When we use the Euler consumption equation (65), we obtain:

$$\frac{\left(\frac{M_t}{P_t}\right)^{-\nu}}{C_t^{-\sigma}} = 1 - \frac{1}{1+r_t} \tag{68}$$

$$\frac{M_t}{P_t} = \left(\frac{1+r_t}{r_t} C_t^{\sigma}\right)^{\frac{1}{\nu}} \tag{69}$$

In logarithmic terms:

$$m_t - p_t = \frac{1}{\nu}\left(-\ln \frac{r_t}{1+r_t} + \sigma \ln C_t\right) \tag{70a}$$

$$= \frac{1}{\nu}\left(-\ln\left(1 - \frac{1}{e^{\ln(1+r_t)}}\right) + \sigma \ln C_t\right). \tag{70b}$$

We log-linearize and use $(1 + r_t) \cong r_t$ for values close enough to zero:

$$\tilde{m}_t - \tilde{p}_t \cong \frac{\sigma}{\nu}\tilde{c}_t + \frac{1}{\nu}\frac{1}{1-\frac{1}{e^{\bar{r}}}}\frac{1}{e^{2-\bar{r}}}e^{\bar{r}}\tilde{r}_t \tag{71a}$$

$$= \frac{\sigma}{\nu}\tilde{c}_t + \frac{1}{\nu}\frac{1}{e^{\bar{r}}-1}\tilde{r}_t \tag{71b}$$

$$\cong \frac{\sigma}{\nu}\tilde{c}_t + \frac{1}{\nu}\frac{1}{\bar{r}}\tilde{r}_t \tag{71c}$$

### 8.1.2 The Firms' Problem

In the wage-staggering model firms are price takers. Thus the prices are equal to the nominal marginal costs.[41]

$$p_{t+i} = mc_{t+i}^{n}. \tag{72}$$

## 8.2 Price Staggering Model

### 8.2.1 Household's Optimization Problem

In contrast to the pure wage staggering model households maximize their utility also with respect to the working time in the price staggering model, as they do not have any wage setting power and thus they are wage takers.

[41] The market clearing conditions will be shown after the derivation of the first order conditions of the price-staggering model.

$$\max_{\{C_{t+i},B_{t+i},\mathsf{M}_{t+i},N_{t+i}\}} E_t \sum_{i=0}^{\infty} \beta^i \left[ U\left( C_{t+i}(h), \frac{M_{t+i}(h)}{P_{t+i}}, N_{t+i}(h) \right) \right]$$

$$= \left[ E_t \sum_{i=0}^{\infty} \beta^i \left( \frac{C_{t+i}^{1-\sigma}(h)}{1-\sigma} + \frac{\left( \frac{M_{t+i}(h)}{P_{t+i}} \right)^{1-\nu}}{1-\nu} - \frac{N_{t+i}^{1+\varphi}(h)}{1+\varphi} \right) \right] \tag{73}$$

subject to its budget constraint

$$E_t \sum_{i=0}^{\infty} \beta^i \left( C_{t+i} + \frac{R_t^{-1} B_{t+i} + M_{t+i}}{P_{t+i}} \right) \tag{74}$$

$$= E_t \sum_{i=0}^{\infty} \beta^i \left[ \frac{W_t(h)}{P_{t+i}} N_{t+i} + \frac{T_{t+i}}{P_{t+i}} + \frac{\Pi_{t+i}}{P_{t+i}} + \frac{B_{t+i-1}}{P_{t+i}} + \frac{M_{t+i-1}}{P_{t+i}} \right].$$

This yields the same two following first order conditions as before (for the derivation see wage-staggering model):

$$1 = \beta R_t \left[ \left( \frac{E_t C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{E_t P_{t+1}} \right) \right] \tag{75}$$

$$\frac{M_t}{P_t} = \left( \frac{1+r_t}{r_t} C_t^{\sigma} \right)^{\frac{1}{\nu}}. \tag{76}$$

The consumption Euler equation and the money demand equation can be log-linearized as in the wage-staggering model.

In addition, we get the following labor supply equation when we take the first derivative with respect to the utility function and use equation (61):

$$\frac{W_{t+i}}{P_{t+i}} = -\frac{N_{t+i}^{\varphi}}{C_{t+i}^{\sigma}} = MRS_{t+i} \tag{77}$$

or alternatively in logs:

$$w_{t+i} - p_{t+i} = c_{t+i}^{\varphi} + n_{t+i}^{\sigma} = mrs_{t+i}. \tag{78}$$

When households are wage takers, the real wage is always equal to the marginal rate of substitution.

### 8.2.2 Firms' Maximization Problem

The firms' maximization problem in the price-staggering model is similar to the households optimization problem in the wage-staggering model. The firms maximize

$$\max_{\{P_t^*(j)\}} E_t \sum_{i=0}^{\infty} (\beta\theta_p)^i \left(P_t^*(j) Y_t(j) - N_t(j) W_t(j)\right) \tag{79}$$

subject to the production function

$$Y_t(j) = A_t N_t(j)^{1-\alpha} \tag{80}$$

and

$$Y_{t+i}(j) = \left(\frac{P_t^*(j)}{P_{t+i}}\right)^{-\varepsilon_p} Y_{t+i}. \tag{81}$$

We get the following first order condition:

$$E_t \sum_{i=0}^{\infty} (\beta\theta_p)^i \left((1-\varepsilon_p) P_t^*(j) Y_{t,t+i} + \frac{\varepsilon_p}{1-\alpha} W_{t+i} N_{t,t+i}(j)\right) = 0. \tag{82}$$

where $_{t,t+i}$ indicates the value of the variable in period $t+k$ when prices were set in period $t$.

This can be rewritten as:

$$E_t \sum_{i=0}^{\infty} (\beta\theta_p)^i \left(Y_{t,t+i}\left(P_t^*(j) - \frac{\varepsilon_p}{\varepsilon_p - 1} MC_{t,t+i}^n\right)\right) = 0. \tag{83}$$

Thus we obtain:

$$P_t^*(j) = \frac{\varepsilon_p}{\varepsilon_p - 1} \frac{E_t \sum_{i=0}^{\infty} (\beta\theta_p)^i Y_{t,t+i} MC_{t,t+i}^n}{E_t \sum_{i=0}^{\infty} (\beta\theta_p)^i Y_{t,t+i}}. \tag{84}$$

By using a first order Taylor approximation:

$$p_t^*(j) \approx \mu_p + (1-\beta\theta) E_t \sum_{i=0}^{\infty} (\beta\theta)^i mc_{t,t+i}^n \tag{85}$$

where $\mu_p = \frac{\varepsilon_p}{\varepsilon_p - 1}$ is the steady state mark-up over nominal marginal costs.

With decreasing returns to labor the firm specific marginal cost are not necessarily equal to the economy-wide average marginal costs.

Firm-specific real marginal costs:

$$MC^r_{t,t+i}{=}\frac{W_{t+i}/P_{t+i}}{(1-\alpha)(Y_{t,t+i}/N_{t,t+i})}. \tag{86}$$

Average real marginal costs in the economy:

$$MC^r_{t+i} = \frac{W_{t+i}/P_{t+i}}{(1-\alpha)(Y_{t+i}/N_{t+i})}. \tag{87}$$

Using (86) and (87) and reformulating:

$$MC^r_{t,t+i} = MC^r_{t+i}\left(\frac{P^*_t}{P_{t+k}}\right)^{\frac{-\varepsilon\alpha}{1-\alpha}}. \tag{88}$$

When we log-linearize, we obtain

$$\tilde{m}c^r_{t,t+i}{=}\ \tilde{m}c^r_{t+i}{-}\frac{\varepsilon_p\alpha}{1-\alpha}\left(p^*_t{-}\tilde{p}_{t+i}\right). \tag{89}$$

Plugging in and reformulating:

$$\tilde{p}^*_t(h)\left(1+\frac{\varepsilon\alpha}{1-\alpha}\right) = (1-\beta\theta)E_t\sum_{i=0}^{\infty}(\beta\theta)^i\left(\tilde{m}c^r_{t+i}{+}\frac{1-\alpha+\varepsilon\alpha}{1-\alpha}\tilde{p}_{t+k}\right). \tag{90}$$

where $\hat{m}c^r_{t+k}$ is the deviation from the steady state of the average economy-wide real marginal costs.

We use the approximate relation

$$\tilde{p}_t{=}\theta\tilde{p}_{t-1}{+}(1-\theta)p^*_t \tag{91}$$

and use the same forward iteration as for wage setting, we obtain the following Phillips curve relationship:

$$\tilde{\pi}_t{=}\beta E_t\tilde{\pi}_{t+1} + \frac{(1-\theta)\,(1-\beta\theta)\,(1-\alpha)}{\theta\,[1+\alpha\,(\varepsilon-1)]}\tilde{m}c^r_t. \tag{92}$$

When we log-linearize the average economy-wide marginal costs from above, we obtain:

$$\tilde{m}c^r_t{=}\ \tilde{w}_t{-}\tilde{p}_t{+}\frac{\alpha}{1-\alpha}\tilde{y}_t{-}\frac{1}{1-\alpha}\tilde{a}_t \tag{93}$$

where the productivity term can be skipped, when we assume that there are no productivity shocks ($\tilde{a}_t = 0$).

## 8.3 Closing the System

The following conditions hold for all three models (price staggering, wage staggering, and both types of staggering).

**Goods Market Clearing:** In this simple version of the model we have the following goods market clearing condition:

$$Y_t = C_t. \tag{94}$$

Or in logarithms:

$$y_t = c_t. \tag{95}$$

Thus we can derive the following equation from the Euler consumption equation (65):

$$\tilde{y}_t = E_t(\tilde{y}_{t+1}) - \frac{1}{\sigma}(r_t - E_t(\pi_{t+1}) + rr_t) \tag{96}$$

where $rr_t$ is the natural rate interest. When we plug in the money demand function (12), we obtain (30), which is the same for all three dynamic systems:

$$E_t\tilde{y}_{t+1} + \frac{1}{\sigma}E_t\tilde{p}_{t+1} + \frac{1}{\sigma\eta}(\tilde{m}_t - \tilde{p}_t) = \left(1 + \frac{1}{\sigma\eta}\right)\tilde{y}_t + \frac{1}{\sigma}\tilde{p}_t. \tag{97}$$

**Production Function:** Furthermore, to close the system, we have to use the production function (1). Up to a first order approximation[42] it can be shown that:

$$y_t = a_t + (1 - \alpha)\, n_t. \tag{98}$$

Thus the following relationships for deviations of the marginal rate of substitution from the steady state can be derived, when we take deviations from the steady state, assume no productivity shocks ($\tilde{a}_t = 0$) and use equation (78) for the marginal rate of substitution:

$$m\tilde{r}s_t = (\sigma + \varphi\frac{1}{(1 - \alpha)})\tilde{y}_t. \tag{99}$$

Furthermore, using (93), with $\tilde{a}_t = 0$, the following equation is valid:

$$\tilde{p}_t = \tilde{m}c_t^n = \tilde{w}_t + \frac{\alpha}{1 - \alpha}\tilde{y}_t. \tag{100}$$

**Money Supply Equation:** Furthermore the following condition holds:

$$m_t = m_{t-1} + \Delta m_t. \tag{101}$$

[42] The derivation is available on request.

## 8.4 The Three Dynamic Systems

We can define all three dynamic systems by using the equations above. As mentioned, equation (96), which is derived from the Euler consumption equation and which can be rewritten as an IS-type equation (97) [43], holds in all three cases.

For the wage staggering model, we use the wage dynamics equation (54), take derivations from the steady state, use (99) and express prices in terms of wages (54) to obtain equation (17).

For the price-staggering model equations (78), (92) and (93) are used to obtain (28).

For the wage- and price-staggering model equations (92) and (93) are used to derive the Phillips curve relationship. Furthermore, when taking deviations from the steady state (54) and using (99), the wage dynamics equation (32) can be obtained.

[43] The only difference is that we expressed the IS-type equation in terms of wages in the wage-staggering model.