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FACTOR MODELS: A MONTE
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ABSTRACT

Impulse Response Functions from Structural Dynamic Factor Models: A Monte Carlo Evaluation*

The estimation of structural dynamic factor models (DFMs) for large sets of variables is attracting considerable attention. In this paper we briefly review the underlying theory and then compare the impulse response functions resulting from two alternative estimation methods for the DFM. Finally, as an example, we reconsider the issue of the identification of the driving forces of the US economy, using data for about 150 macroeconomic variables.

JEL Classification: C32, C51 and E52

Keywords: factor models, principal components, structural identification, structural VAR and subspace algorithms

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1 Introduction

Recent work in the macroeconometric literature considers the problem of summarising efficiently a large set of variables and using this summary for a variety of purposes including forecasting. Work in this field has been carried out in a series of recent papers by Stock and Watson (2002, SW), Forni and Reichlin (1998) and Forni et al. (2000,2001, FHLR). Factor analysis has been the main tool used in summarising the large datasets.

The main factor model used in the past to extract dynamic factors from economic time series has been a state space model estimated using maximum likelihood. This model was used in conjunction with the Kalman filter in a number of papers carrying out factor analysis (see, among others, Stock and Watson (1989) and Camba-Mendez et al. (2001)). However, maximum likelihood estimation of a state space model is not practical when the dimension of the model becomes too large due to the computational cost. For the case considered by SW where the number of time series can be greater than the number of observations, maximum likelihood estimation is not practically feasible. For this reason, SW have suggested a principal component based estimator of the factors and shown that it can estimate consistently the factor space asymptotically (but the number of time series has to tend to infinity). In small samples and for a finite number of series, the dynamic element of the principal component analysis is not easy to interpret. FHLR suggested an alternative procedure based on dynamic principal components (see Chapter 9 of Brillinger (1981)). This method incorporates an explicitly dynamic element in the construction of the factors. Finally, Kapetanios and Marcellino (2005, KM) have developed a parametric approach which retains the attractive framework of a state space model but is computationally feasible for very large datasets because it does not use maximum likelihood but linear algebra methods to estimate the state. These methods are collectively known as subspace methods and are widely used in the engineering literature. KM showed that the parametric approach compares well both with static and dynamic principal components based estimators of the factor model.

This paper builds on the framework developed by KM to show how to identify structural shocks and their effects in dynamic factor models (DFMs), similar to what is widely done using structural VAR models. The analysis is similar to that proposed, independently, in a recent paper by Stock and Watson (2005) that extends previous work by Giannone, Reichlin and Sala (2002, GRS) and Forni, Lippi and Reichlin (2002). Identification can be achieved by Choleski-type decompositions, e.g. Sims (1980), by imposing restrictions on the contemporaneous relationships across the variables, e.g. Amisano and Giannini (1997), or by

restricting the long run MA coefficient matrix, e.g. Blanchard and Quah (1989). Our method can be also extended to satisfy more complicated constraints suggested in the structural VAR literature, such as sign or shape restrictions, e.g. Uhlig (2001).

As indicated by Giannone, Reichlin and Sala (2002, GRS) and further developed in Forni, Lippi and Reichlin (2002), the DFM solves several usual problems with structural VARs. In particular, the DFM takes explicitly into account the fact that few shocks are the driving forces of the economy, so that the number of shocks in the model can be much smaller than that of the variables under analysis, while this is not the case in the VAR. On the other hand, the large dataset exploited in the DFM to extract and identify the shocks avoids the omitted variable problem and requires fewer restrictions than in VARs, as will emerge from the following analysis. Finally, the possibility of non-fundamental errors that can prevent a structural interpretation of the shocks in VARs (see Lippi and Reichlin (1994)) is also virtually eliminated in DFMs.

With respect to the method in GRS, which is based on the combined use of dynamic and static principal component factor estimators, our approach is much simpler and takes into explicit account the parametric structure of the DFM. With respect to Stock and Watson (2005), the main innovation is the use of an alternative (parametric) estimator for the DFM.

However, the main novel contribution of this paper is the evaluation of the performance of the structural DFM-based impulse responses in an extended Monte Carlo study. Results are in accordance with intuition. Models with single factors are better estimated and results improve with a larger number of observations and series. With respect to the SW estimator, that underlies GRS's and Stock and Watson's (2005) approaches, our parametric method appears to be more robust. However, in general both methods perform well and the losses from estimating the factors rather than using the true ones are limited.

We also develop a bootstrap method to provide finite sample standard errors around the estimated responses where the full set of data is bootstrapped rather than only the factors as in GRS. This is particularly important for applications with a larger temporal than longitudinal dimension, since in this case the estimated factors cannot be treated as the true factors, see Bai (2003), Bai and Ng (2005) for details. To illustrate the use of the method, as in GRS, we evaluate the main shocks underlying the evolution of the US economy over the 1959-1998 period, using a dataset of about 150 macroeconomic time series extracted from the one in SW. It turns out that one supply and two demand shocks are important, but monetary shocks are not, in line with what GRS found with their approach and a similar

data set.

The paper is organised as follows: Section 2 describes the basic identification methodology within the structural DFM framework. Section 3 presents the Monte Carlo results on the impulse responses estimators. Section 4 discusses the empirical example. Section 5 summarizes and concludes. Appendix 1 summarizes the results in KM on the parametric estimator for the DFM, while Appendix 2 discusses several extensions of the basic identification methodology.

2 Structural Identification of Factor Models

Our approach to identify the structural shocks in a large scale dynamic factor model is very similar to that proposed, independently, by Stock and Watson (2005). Therefore, we just summarize the main results and focus on the basic case. Additional details and extensions are provided in the Appendixes.

Our starting point is a model of the form

$$\begin{aligned}y_t &= Cf_t + w_t, \\f_t &= Af_{t-1} + e_t,\end{aligned}\tag{1}$$

where y_t is an N -dimensional vector of strictly stationary zero-mean variables observed at time t and f_t is a k -dimensional vector of stationary unobserved states (factors) at time t , with k much smaller than N . We assume that (i) w_t , the idiosyncratic errors, are uncorrelated i.i.d. sequences, (ii) the covariance matrix of w_t is diagonal, (iii) e_t are uncorrelated i.i.d. sequences with full rank covariance matrix, and (iv) the eigenvalues of A are less than one in absolute value.

The hypotheses (i) and (ii) can be relaxed when N diverges by constraining the C matrix, as discussed in Appendix 1. Actually, (ii) could be relaxed also with finite N as long as (i) holds and no factors are i.i.d. The hypothesis (iii) can be also relaxed by allowing for example for an MA structure of the errors or a reduced rank covariance matrix. These extensions are discussed in Appendix 2. Hypothesis (iv) is maintained in the whole large scale dynamic factor model literature, since all the properties of the factor estimators have been derived assuming stationary variables, an exception being Bai and Ng (2004). Appendix 2 considers the cases when there is more dynamics either in the factor equations or in those for the y variables, and when the number of structural shocks is smaller than that of factors.

Without additional restrictions, the parameters of this model are still not identifiable, since the model may be rewritten as

$$\begin{aligned} y_t &= CPP^{-1}f_t + w_t \\ Pf_t &= PAP^{-1}Pf_{t-1} + Pe_t \end{aligned} \tag{2}$$

for some nonsingular matrix P . Hence, we make the further assumption that $P = I$, implicitly subsuming this identification issue into the structural identification of the shocks since the factor sequence Pf_t is generated by the error sequence Pe_t .

We wish to identify the structural shocks \tilde{e}_t underlying the errors e_t , which are linear combinations of structural shocks, namely,

$$\tilde{e}_t = Re_t, \tag{3}$$

where for the moment R is a full rank $k \times k$ matrix (and k is the number of factors). This problem is similar to the one addressed in the structural VAR literature, but here we are interested in the very few structural shocks driving all variables, and need not assume that there are as many structural shocks as variables, which is more sensible from an economic theory point of view. A related issue is that we do not concern ourselves with w_t , since the idiosyncratic shocks can be assumed to have little structural meaning.

The MA representation of the model in (1) is

$$y_t = \sum_{i=0}^{\infty} CA^i e_{t-i} + w_t \tag{4}$$

The reduced form long run effect matrix from e_t to y_t is $\sum_{i=0}^{\infty} CA^i = C(I - A)^{-1}$. We denote the unknown structural long run effect matrix by Γ . Following standard practice from structural VARs where long run restrictions are used to identify the shocks we have that

$$C(I - A)^{-1}R^{-1} = \Gamma \tag{5}$$

and

$$R\Sigma R' = I \tag{6}$$

where Σ is the covariance matrix of e_t .

We have R and Γ to solve from the above equations. These are $Nk + k^2$ parameters to be obtained. (5) involves Nk equations and (6) involves $k(k + 1)/2$ equations (due to the symmetry requirement for Σ). So we need to impose $Nk + k^2 - Nk - k(k + 1)/2 =$

$k(k-1)/2$ restrictions on R and/or Γ . Compare this with the standard SVAR result that $2N^2 - N^2 - N(N+1)/2 = N(N-1)/2$ restrictions on the R or Γ matrices are needed. This of course is much more parsimonious since $N \gg k$. Note that these are necessary but not sufficient conditions. Sufficiency requires that the restrictions are linearly independent (see also Amisano and Giannini (1997)).

A more general identification scheme for the shocks could be of the form $H\tilde{e}_t = Re_t$, see e.g. Amisano and Giannini (1997) in the SVAR literature. In this case, the standard identifying equations are of the form

$$\begin{aligned} C(I - A)^{-1}R^{-1}H &= \Gamma \\ H^{-1}R\Sigma R'H^{-1'} &= I \end{aligned} \tag{7}$$

The solution of these nonlinear equations is problematic, as it involves the term HR^{-1} or its inverse and therefore separate identification of R and H is very difficult. Hence, in what follows we assume $H = I$, i.e. focus on the restrictions in (5) and (6).

The above discussion concerns just identified models, i.e. models where the number of restrictions imposed is equal to the number of unknown parameters to be determined. Of course by the nature of the model few restrictions are required but there is a large number of variables through which restrictions could be imposed. Therefore, the case of overidentifying restrictions is relevant. We suggest a very simple way of imposing overidentifying restrictions.

Let a general formulation of the restrictions for the simple identification scheme involving the matrix R be given by the vector function

$$f(R, A, C, \Sigma) = 0. \tag{8}$$

Then we suggest that the objective function

$$f(R, A, C, \Sigma)'f(R, A, C, \Sigma) \tag{9}$$

be numerically minimised with respect to R .¹ Note that when we have overidentifying restrictions it is generally the case that there are too few free parameters which can be used to satisfy (8). Hence, we suggest minimisation of (9). The proper critical values for tests of the restrictions in (8) can be constructed using the bootstrap procedure described below.

¹A simple example of setting up over-identifying restrictions is

$$f(R, A, C, \Sigma) = \begin{pmatrix} C(I - A)^{-1}R^{-1} - \Gamma \\ R\Sigma R' - I \end{pmatrix} = 0$$

where more than $k(k-1)/2$ restrictions have been imposed on R and/or Γ .

For practical implementation, both the factors and the parameters A and C in (1) have to be estimated. From the work of SW, KM and Bai and Ng (2005), under the maintained assumption $P = I$, the factors can be estimated consistently, and the A and C matrices can be also estimated consistently using the estimated rather than the true factors, see also the Appendixes for details. Whether the estimated factors provide a good approximation to the true factors in finite samples is an empirical issue, and we address it in the Monte Carlo experiments in the next Section.

Another important practical aspect is the computation of standard errors around the estimated responses. We propose the following bootstrap procedure, which is illustrated in the empirical example in Section 4. Once parameter estimates have been obtained, the model is bootstrapped by resampling the residuals of the two sets of equations in (1) to produce bootstrap samples of y_t . One can use either the simple bootstrap for resampling the residuals or, to guard against possible serial correlation in the residuals, the block bootstrap. We chose to use the block bootstrap with block length equal to 20 for the empirical application. Each bootstrap sample of y_t is then analysed using the DFM model in (1) combined with the subspace algorithm described in Appendix 1, and estimates of A and C and hence impulse responses are obtained. The set of estimated parameters and responses can be used to construct the empirical distributions of the impulse responses and, e.g., of the statistic for overidentifying restrictions in (9). A further example of the application of the bootstrap to the subspace algorithm context may be found in Camba-Mendez and Kapetanios (2002a).

3 Monte Carlo experiments

In this section we investigate the small sample properties of the estimated DFM-based impulse responses. For simplicity, we concentrate on the properties of the responses to the reduced form errors, e_t , or, in other terms, we set $R = I$ in (3). If these responses are well estimated, then all the quantities involved in (7) will be well estimated (i.e., the matrices A and C) and so the matrix R required for more general structural identification will be also well estimated. In the first subsection we present the Monte Carlo design, in the second one the results.

3.1 Experimental design

We adopt the state space model in (1) to generate the data, i.e.,

$$\begin{aligned} y_t &= Cf_t + w_t, \\ f_t &= Af_{t-1} + e_t. \end{aligned} \tag{10}$$

We assume first that we have a single factor, generated by an AR model with 2 different specifications, and then three factors, generated by four different VAR(1) models:

- $A = 0.2$
- $A = 0.6$
- $A = \begin{pmatrix} 0.2 & 0 & 0 \\ 0 & 0.2 & 0 \\ 0 & 0 & 0.2 \end{pmatrix}$
- $A = \begin{pmatrix} 0.6 & 0 & 0 \\ 0 & 0.6 & 0 \\ 0 & 0 & 0.6 \end{pmatrix}$
- $A = \begin{pmatrix} 0.3 & 0.1 & 0.1 \\ 0.2 & 0.3 & 0.1 \\ 0.2 & 0.2 & 0.3 \end{pmatrix}$
- $A = \begin{pmatrix} 0.6 & 0.1 & 0.1 \\ 0.2 & 0.6 & 0.1 \\ 0.2 & 0.2 & 0.6 \end{pmatrix}$

For each case, we present results for the following (N, T) pairs: (50,50), (100,50), (50,100) and (100,100), where N is the number of variables and T the sample size. These are experiment sets A,B,C and D respectively. The six specifications of the A matrix will be denoted experiments 1-6. The error terms are generated as standard normal pseudo-random variables. The covariance matrix for each of these cases is set to be the identity matrix. The matrix C is generated from standard normal random variates for each replication. The assumed number of factors is set equal to the true number of factors for all experiments.

Our analysis involves comparing true and estimated impulse responses. In essence, we are interested in the estimation properties of the sequence $C, CA, CA^2, \dots, CA^i, \dots$. However, the impulse response analysis is conditional on the identification scheme involved in estimating

the factors (see Appendix 1 for details), and essentially we estimate CP^{-1} and PAP^{-1} for some nonsingular matrix P . In the theory section we could set $P = I$ without loss of generality, but for the Monte Carlo study we must be able to recover the P matrix to compare estimated and true impulse responses. This matrix cannot be recovered analytically as it is a complicated function of the matrices of coefficients involved in equation (13) of Appendix 1. However it can be easily estimated for each Monte Carlo replication. The simplest method is to solve the system of equations $C_{[1:k]}P^{-1} = \hat{C}_{[1:k]}$ where $C_{[1:k]}$ denotes the first k rows of the matrix C . This system of equations produces an estimate for P which is used to make estimated and true impulse responses comparable.

It is worth stressing that estimating P does not lessen the importance of our analysis, since we are forcing equality of only k^2 elements of the true and estimated C matrices ($C_{[1:k]}P^{-1} = \hat{C}_{[1:k]}$) out of Nk , with N much larger than k . For example when $N = 100$ and $k = 3$ we are only restricting 9 elements out of 300.

3.2 Results

The results of the Monte Carlo experiments are summarized in Tables 1-4 for the different combinations of N and T . In the first two blocks of each table (identified by the labels E) we report the bias and RMSE of the estimated impulse responses for up to 20 periods ahead obtained using the KM parametric factor estimator described in Appendix 1. We then present the same statistics for the SW principal component estimator of the factors (label S) and for the true factors (label T). The former comparison is interesting since the SW factors underlie the impulse response functions in Giannone et al. (2002) and in Stock and Watson (2005). The latter to evaluate whether estimating the factors rather than using the true ones introduces sizeable distortions in the estimated responses. Finally, we report the correlation between the estimated impulse responses based on the KM factor estimates and the true responses (for horizons longer than 10 so as to have a reasonable amount of observations on which to calculate correlations), and the proportion of Monte Carlo replications where the true and estimated responses (based on the KM factor estimates) have the same sign. These are two important summary measures to evaluate whether the estimated responses follow a similar dynamic pattern as the true ones, or at least share the same sign.²

²For a small proportion of the cases (especially experiments 3 and 4) the estimated impulse responses were very large. As by construction they should not fall outside the range of a standard normal random variable, we reject estimates which exceed 10 in absolute value. In the worst case of experiment 3 around 7% of impulse responses were thus rejected.

A number of conclusions emerge from the Monte Carlo analysis. Let us start with Table 1, i.e. $N = 50$, $T = 50$, and low values for the horizon h , up to 5. First, the biases and RMSEs are larger in the experiments with 3 factors and diagonal VAR coefficient matrices than in single factor models. This difference remains with the SW estimated factors, while it disappears with the true factors. Thus, as one might expect, both factor estimation methods perform well when the same number of series is driven by a lower number of factors, that can therefore be more accurately estimated. This result casts some doubts on the reliability of the DFM-based impulse responses when many factors have to be estimated. For example, Stock and Watson (2005) estimate seven factors, even though the N and T dimensions of their sample is larger and they identify only one shock.

Second, overall estimation is better for less persistent than for more persistent factors, in terms of both bias and RMSE. This property emerges also for the SW factors and for the true factors. A possible explanation for this finding is that larger but stationary roots can lead to larger biases in the estimation of the dynamics in the factor equations, and of the resulting errors.

Finally, comparing the figures for the KM and the SW estimated factors, the former appear to perform better than the latter, in particular in the three factors experiments. Moreover, the performance of the KM estimator appears to be more stable across experiments, while there are some cases (e.g. A3 and C5 in Tables 1 and 3) where the RMSE of the estimated impulse responses is very large for the SW factors.

When the horizon h increases three main comments are in order. First, in general both the bias and the RMSE decrease with h , as might be expected since both the true and estimated responses should converge to their unconditional mean of zero. Second, the ranking of estimation methods is virtually unaffected and both of them yield very similar results to the true factors case. Third, the correlation between the true and estimated responses is high for the single factor experiments, above 0.77, rather high for three persistent factors, above 0.66, but can be low for the three factor experiments with low persistence, above 0.37. However, even in the last case, the fraction of true and estimated responses with the same sign remains above 58%.

The concluding set of comments refers to larger values of N (Table 2), of T (Table 3), and of both N and T (Table 4). First, overall, the properties of the estimated responses in terms of bias and RMSE improve when we consider samples of 100 observations rather than 50, and a less pronounced improvement takes place with more series (100 rather than 50). Second,

the ranking across experiments of the factor estimation methods and the comparison with the true factor case are not affected by larger values of N and/or T . Finally, the correlation between the true and estimated impulse responses increases with both T and N in a number of cases. For example, for experiment A3 and $h = 10$ the correlation is 0.37 increasing to 0.39 for experiment set B3, to 0.49 for C3 and to 0.54 for D3. A similar, but less pronounced, pattern emerges for the estimated fraction of equal sign.

4 An empirical example

In this section we apply the dynamic factor model estimated with the state space method (Appendix 1) to extract the main shocks driving the US economy and evaluate their effects on a set of key macroeconomic variables. A similar analysis was conducted by GRS, and we wish to evaluate whether or not we reach similar conclusions using our technique.

To mimic the behaviour of the US economy, we consider a large balanced dataset of 146 macroeconomic variables, over the period 1959:1-1998:12 for a total of 480 observations. The dataset is extracted by the one in SW, dropping series with missing observations and other data irregularities. A list of all the variables, together with the stationarity transformation applied, is presented in the Data Appendix, see SW for additional details.

Details on the estimation and goodness of fit of the model are available upon request. Here we focus on the estimated factors and their driving shocks, assuming that three factors drive all variables. The latter is a reasonable hypothesis in the light of the results of SW that found that three factors were sufficient to produce good forecasts of several macroeconomic variables (when the factors are estimated as the static principal components of all variables), and of Giannone, Reichlin and Sala (2002) and Forni, Lippi and Reichlin (2002) who found two factors to be sufficient (when estimated using the dynamic principal components). Stock and Watson (2005) suggested that more factors may be needed, up to seven, but most of them seem to be only needed for the many more financial series that they have in their dataset.

As we noted before, once the factors are estimated the choice of the identification scheme is left to the users. Since we do not want to impose any particular a priori hypothesis, we follow the original suggestion in Sims (1980) and consider all possible Choleski decompositions of the variance covariance matrix of the residuals, that correspond to all possible orderings of the three factors. It turns out that the results are virtually invariant to the rotation, as a consequence of the near orthogonality of the residuals of the VAR for the factors. This

is a lucky outcome since it implies that also more general identification procedures would produce similar results.

In Figure 1 we report the responses of industrial production, inflation and the federal fund rate to the shocks, assuming without loss of generality (for the result mentioned above) that the factors are ordered as 1,2,3. It turns out that the first shock has negative effects on all variables. Within an IS-LM / AD-AS framework, these are the expected consequences of a negative demand shock such as a drop in autonomous consumption or investment, that incidentally underlie the US recessions of, respectively, the early '90s and the beginning of the new century. Therefore we interpret this shock as a (negative) demand shock.

The third shock is associated with a decrease of industrial production but an increase of inflation and the interest rate. Using the same interpretative scheme as above, this shock can be interpreted as a (negative) supply shock, such as those hitting the US economy in the '70s.

The second shock is more problematic, in the sense that its effects vary a bit depending on the ordering of the factors. In particular, while the effects on inflation are always positive, those on industrial production can be either slightly positive or slightly negative, and the fed fund follows the same pattern as industrial production. A possible interpretation is that this shock is associated with exchange rate depreciations or expansionary fiscal policy, whose real effects are uncertain in the short run while in general inflation increases, such as those taking place in the '80s in the US (more precisely in the second part of the '80s in the case of the US dollar depreciation).

So far our structural identification of the three shocks driving the factors is based on the responses of three fundamental macroeconomic variables but, since a much larger information set is available, we can now evaluate whether our identification is correct by plotting the responses of other macroeconomic variables to what we have identified as demand and supply shocks.

In Figure 2 we consider the response of employment growth, producer price inflation and a longer term interest rate (the 5-year government bond yield). The response of the three variables to the first (negative) demand shock follows the pattern in Figure 1, namely, both employment and inflation and the interest rate decrease. The responses to the supply shock are also similar: employment decreases, inflation increases, only the longer term interest rate slightly decreases, which can be in line with market expectations of a decrease in interest rates after the shock. The response to the second demand shock is also in line with what we

have found in Figure 1: inflation increases, while employment growth initially decreases and then becomes positive (associated with the behaviour of industrial production), and the long term interest rate follows the same pattern as the short rate.

In Figure 3 we focus on inventories, new orders, and capacity utilizations. The first variable should have an anti-cyclical behaviour while the latter two should be pro-cyclical. Actually, inventories increase with the negative demand and supply shock, while new orders and capacity utilization drop. These two variables increase with the positive demand shock (shock 2), and there is also a minor positive effect on inventories, likely associated with the inverted u- shaped behaviour of production.

In Figure 4 we notice that private consumption expenditures decrease with the negative demand and supply shock, and increase with the positive demand shock. Hours worked in the manufacturing sector decrease with the negative demand shock but not with the negative supply shock, because of a different behaviour of the hourly wage, also plotted in Figure 4, that drops much more in the presence of the supply shock. The same drop takes place in the case of the positive demand shock, which is harder to explain, and is related to a positive effect on hours worked.

In Figure 5 we focus on the stock market index and on the exchange rate with the Deutsche Mark and the Japanese Yen. Our idea that the second demand shock is possibly associated with a devaluation is supported by this figure. The depreciation associated with the negative supply shock seems at odd with the increase in the short term interest rate, but not with the drop in real activity and also in the stock market index. Similarly, the appreciation of the dollar after the negative demand shock can be attributed to the stronger stock market, that likely reflects the non-permanent effects of the demand shock (also confirmed by the overall stable level of the real interest rate, compare Figure 1).

Overall, the pattern of response of the other macroeconomic variables we have considered in Figures 2-5 is in agreement with our identification of the structural shocks on the basis of the response of industrial production, consumer price inflation and the federal fund rate.³ Though the bootstrapped standard errors around these responses are fairly large, their sign and overall coherency across variables and with economic theory is encouraging.

To conclude, it is interesting that none of the three shocks we have identified admit a monetary policy interpretation. This is in line with the findings in Giannone et al. (2002) and Forni et al. (2002), and more generally with the by now common view that monetary

³Notice that the dataset we use to extract the factors includes several other variables, but their level of disaggregation makes it difficult to use them to provide additional checks on the validity of our identification.

shocks have only a limited impact on the economy.

5 Conclusions

Since Sims' (1980) pioneering paper, the adoption of VAR models has been widespread in applied econometrics. The early contributions did not attempt a structural interpretation of the shocks, actually VARs were considered a-theoretical models, but mainly focused on forecasting and the derivation of stylized facts on the dynamic relationships across variables. Later on the potential of structural VARs to provide a sounder economic interpretation of the stylized facts was discovered, and it is nowadays current practice.

The evolution of large scale dynamic factor models closely parallels that of VARs. The early contributions mainly focused either on theoretical econometric issues or provided empirical applications where a structural interpretation is not needed, e.g. forecasting or the construction of coincident indicators. Recent papers by Giannone et al. (2002), Forni et al. (2002), Stock and Watson (2005), Bernanke, Boivin and Eliasch (2005) tackle the structural interpretation issue. Our theoretical results are in line with theirs, but we suggest to compute the responses to structural shocks using an alternative parametric estimator for the parameters of the underlying factor model.

The extensive set of Monte Carlo experiments we have conducted indicate that in general the biases in the estimated impulse response functions are small, the sign of the responses is correct in a large fraction of cases, and the correlation between the true and estimated responses is usually high. The SW factors, that underlie the structural analysis in Giannone et al. (2002) and Stock and Watson (2005), yield similar or worse results with respect to our KM (subspace based) factors, and the latter also appear to be more robust across experiments. The comparison with the true factors highlights that their estimation introduces only small biases in the subsequent analysis.

Finally, the empirical application with US data shows the rather simple implementation of our approach, even when compared with structural VARs, and the robustness of the results to the choice of the factor estimation method. In particular, as GRS, we find that monetary shocks do not play a major role as driving forces of the US economy.

We therefore conclude that structural parametric large scale dynamic factor models provide a powerful and useful tool for empirical analysis. They allow to go beyond the limitations of structural VARs and can be considered as their natural evolution.

References

- [1] Amisano G. and C. Giannini (1997), “Topics in Structural VAR Econometrics”, *Springer-Verlag*.
- [2] Bai, J. and S. Ng (2002), “Determining the number of factors in approximate factor models”, *Econometrica*, 70, 191-223.
- [3] Bai, J. and S. Ng (2004), “A panic attack on unit roots and cointegration”, *Econometrica*, 72, 1127-1177.
- [4] Bai, J. (2003), “Inferential theory for factor models of large dimension”, Forthcoming in *Econometrica*.
- [5] Bai, J. and S. Ng (2005), “Confidence intervals for diffusion index forecasts and inference for factor-augmented regressions”, *mimeo*.
- [6] Bauer, D. (1998), “Some Asymptotic Theory for the Estimation of Linear Systems Using Maximum Likelihood Methods or Subspace Algorithms”, Ph.D. Thesis.
- [7] Bernanke, B., Boivin, J. and Elias, P. (2005), “Measuring the effects of monetary policy: a factor-augmented vector autoregressive (FAVAR) approach”, *Quarterly Journal of Economics*, 120, 387-422.
- [8] Blanchard, O. and D. Quah (1989). “The dynamic effects of aggregate demand and supply disturbances” *American Economic Review*, 79, 655-73.
- [9] Brillinger D.R. (1981), *Time Series: Data Analysis and Theory*, *McGraw-Hill*.
- [10] Camba-Mendez G., Kapetanios G., Smith R. J. and M. R. Weale (2001), “An Automatic Leading Indicator of Economic Activity: Forecasting GDP Growth for European Countries”, *Econometrics Journal*, 4(1), S56-90.
- [11] Camba-Mendez G. and G. Kapetanios (2002), “Estimating the Rank of the Spectral Density Matrix”, Forthcoming in *Journal of Time Series Analysis*.
- [12] Camba-Mendez G. and G. Kapetanios (2002a), “Statistical Testing Procedures and Information Criteria in Rank Determination for System Identification”, Forthcoming in *IEEE Transactions on Automatic Control*.

- [13] Camba-Mendez G. , Kapetanios G., Smith R. J. and M. R. Weale (2002b), “Tests of Rank in Reduced Rank Regression Models”, Forthcoming in *Journal of Business and Economic Statistics*.
- [14] Deistler, M. and E. J. Hannan (1988), “The Statistical Analysis of Linear Systems”, John Wiley.
- [15] Forni, M. and L. Reichlin (1998), “Let’s get real: A dynamic factor analytical approach to disaggregated business cycle”, *Review of Economic Studies*, 65, 453-474.
- [16] Forni, M., Lippi, M. and L. Reichlin (2002), “Opening the black box: Structural factor models versus Structural VARs”, mimeo.
- [17] Forni, M., Hallin, M., Lippi, M. and L. Reichlin (2000), “The generalised factor model: identification and estimation”, *The Review of Economic and Statistics*, 82, 540-554.
- [18] Forni, M., Hallin, M., Lippi, M. and L. Reichlin (2001), “The generalized dynamic factor model: consistency and rates”, Forthcoming in *Journal of Econometrics*.
- [19] Giannone, D. Reichlin, L. and L. Sala (2002), “Tracking Greenspan: systematic and unsystematic monetary policy revisited”, mimeo.
- [20] Golub G. H and C. F. Van Loan (1996), Matrix Computations, *John Hopkins University Press*.
- [21] Kapetanios, G. and M. Marcellino (2005), “A Comparison of Estimation Methods for Dynamic Factor Models of Large Dimensions”, mimeo (under revision for *Journal of Time Series Analysis*).
- [22] Larimore, W.E. (1983), “System Identification, Reduced Order Filters and Modelling via Canonical Variate Analysis”, *Proc. 1983 Amer. Control Conference*, pp. 445-451.
- [23] Lippi, M. and L. Reichlin (1994), “VAR analysis, nonfundamental representation, Blaschke matrices”, *Journal of Econometrics*, 63, 307-25.
- [24] Magnus, J. and H. Neudecker (1988), “Matrix Differential Calculus with Applications to Statistics and Econometrics”, John Wiley.
- [25] Van Overschee, P. and B. De Moor (1996), “Subspace Identification for Linear Systems”, Kluwer Academic Publishers.

- [26] Quah, D. and T.J. Sargent (1993), “A dynamic index model for large cross-sections”, ch. 7 in Stock, J.H. and M.W. Watson (eds.), *Business cycles, indicators and forecasting*, University of Chicago Press for the NBER.
- [27] Sims, C.A. (1980), “Macroeconomics and reality”, *Econometrica*, 48, 1-48.
- [28] Stock, J.H. and M.W. Watson (1989), “New indexes of coincident and leading economic indicators”, *NBER Macroeconomic Annual*, 351-393.
- [29] Stock, J.H. and M.W. Watson (2002), “Macroeconomic Forecasting Using Diffusion Indexes”, *Journal of Business and Economic Statistics*, 20, 147-62
- [30] Stock, J.H. and M.W. Watson (2005), “Implications of dynamic factor models for VAR analysis”, mimeo.
- [31] Uhlig, H. (2001), “What are the effects of monetary policy on output? Results from an agnostic identification procedure”, mimeo.

6 Appendix 1. The KM factor estimator

In this Appendix we review the subspace algorithm based factor estimator proposed by Kapetanios and Marcellino (2005 KM). We then discuss two extensions of the basic set-up, namely, correlated idiosyncratic errors and panels with a larger cross sectional than time series dimension.

6.1 The basic state space model

Following Deistler and Hannan (1988), we consider the following state space model.

$$\begin{aligned}y_t &= Cf_t + w_t, \quad t = 1, \dots, T \\f_t &= Af_{t-1} + B^*e_{t-1},\end{aligned}\tag{11}$$

where y_t is an N -dimensional vector of strictly stationary zero-mean variables observed at time t ; f_t is a k -dimensional vector of stationary unobserved states (factors) at time t , with k much smaller than N and the eigenvalues of A smaller than one in absolute value; and w_t and e_t are multivariate, mutually uncorrelated, white noise sequences of dimension, respectively, N and k . We assume further that w_t are contemporaneously uncorrelated.

Notice that the common factors and the idiosyncratic errors are identified because the former are persistent and the latter are not. This assumption can be released if N diverges, as well as lack of contemporaneous correlation of the idiosyncratic errors, but in this case additional conditions are required on the C matrix. Both issues are discussed in the next subsection. Notice also that in this model the common factors are identified only up to a nonsingular rotation, and the implications of this feature for structural identification are considered in Section 3.

The main aim of the analysis is to obtain good estimates of the states f_t , for $t = 1, \dots, T$. The procedure that we describe below generates consistent estimators for the (space spanned by the) factors even for the finite N case. This is possible because the driving shocks in the factor equations in (11) are dated $t - 1$. This implies that the structural shocks affect the y variables through the factors with a one period delay, while the idiosyncratic shocks can have contemporaneous effects. KM demonstrate that if e_t drives the factors rather than e_{t-1} it is still possible to consistently estimate Af_{t-1} , and in Appendix 2 we show that this suffices to obtain consistent estimators of the impulse responses. The Monte Carlo experiments of Section 4 provide support for the finite sample performance of the estimators also in this modified model.

As a further generalization of the model in (11), the factors could have a dynamic effect on the variables, i.e., the C matrix could be a finite matrix polynomial in the lag operator, $C(L)$. In this case, following SW, the model can be re-written in the static version (11) but with an extended set of factors, whose driving errors have a reduced rank variance covariance matrix. An example is discussed in Appendix 2, and evidence in favour of the good performance of the factor estimator also in this context is provided in KM.

Deistler and Hannan (1988, Chapter 1) show that the state space model in (11) is equivalent to the prediction error representation

$$\begin{aligned} y_t &= Cf_t + u_t, \quad t = 1, \dots, T \\ f_t &= Af_{t-1} + Bu_{t-1}. \end{aligned} \tag{12}$$

where u_t is white noise. This form is used by KM for the derivation of the estimation algorithm.

As we have mentioned in the Introduction, maximum likelihood techniques, possibly using the Kalman filter, may be used to estimate the parameters of the model under some identification scheme. Yet, for large datasets this is very computationally intensive. Quah and Sargent (1993) developed an EM algorithm that allows to consider up to 50-60 variables, but it is still so time-consuming that it is not feasible to evaluate its performance in a simulation experiment. A convenient solution is provided by subspace algorithms that avoid expensive iterative techniques and instead rely on matrix algebraic methods to provide estimates for the factors as well as the parameters of the state space representation.

There are many subspace algorithms, and vary in many respects, but a unifying characteristic is their view of the state as the interface between the past and the future in the sense that the best linear prediction of the future of the observed series is a linear function of the state. A review of existing subspace algorithms is given by Bauer (1998) in an econometric context. Another review with an engineering perspective may be found in Van Overschee and De Moor (1996).

The starting point of most subspace algorithms is the following representation of the system which follows from the prediction error state space representation in (12).

$$Y_t^f = \mathcal{O}KY_t^p + \mathcal{E}E_t^f \tag{13}$$

where $Y_t^f = (y_t, y_{t+1}, y_{t+2}, \dots)'$, $Y_t^p = (y_{t-1}, y_{t-2}, \dots)'$, $E_t^f = (u_t, u_{t+1}, \dots)'$, $\mathcal{O} = [C', A'C', (A^2)'C', \dots]'$,

$\mathcal{K} = [B, (A - BC)B, (A - BC)^2B, \dots]$, and

$$\mathcal{E} = \begin{pmatrix} I & 0 & \dots & 0 \\ CB & I & \ddots & \vdots \\ CAB & \ddots & \ddots & 0 \\ \vdots & & CB & I \end{pmatrix}.$$

The derivation of this representation is simple once we note that (i) $Y_t^f = \mathcal{O}f_t + \mathcal{E}E_t^f$ and (ii) $f_t = \mathcal{K}Y_t^p$. The best linear predictor of the future of the series at time t is given by $\mathcal{O}KY_t^p$. The state is given in this context by $\mathcal{K}Y_t^p$ at time t . The task is therefore to provide an estimate for \mathcal{K} . Obviously, the above representation involves infinite dimensional vectors.

In practice, truncation is used to end up with finite sample approximations given by $Y_{s,t}^f = (y'_t, y'_{t+1}, y'_{t+2}, \dots, y'_{t+s-1})'$ and $Y_{p,t}^p = (y'_{t-1}, y'_{t-2}, \dots, y'_{t-p})'$. Then an estimate of $\mathcal{F} = \mathcal{O}\mathcal{K}$ may be obtained by regressing $Y_{s,t}^f$ on $Y_{p,t}^p$. Following that, the most popular subspace algorithms use a singular value decomposition (SVD) of an appropriately weighted version of the least squares estimate of \mathcal{F} , denoted by $\hat{\mathcal{F}}$. In particular the algorithm we will use, due to Larimore (1983), applies an SVD to $\hat{\Gamma}^f \hat{\mathcal{F}} \hat{\Gamma}^p$, where $\hat{\Gamma}^f$, and $\hat{\Gamma}^p$ are the sample covariances of $Y_{s,t}^f$ and $Y_{p,t}^p$ respectively. These weights are used to determine the importance of certain directions in $\hat{\mathcal{F}}$. Then, the estimate of \mathcal{K} is given by

$$\hat{\mathcal{K}} = \hat{S}_k^{1/2} \hat{V}_k' \hat{\Gamma}^{p-1/2}$$

where $\hat{U} \hat{S} \hat{V}'$ represents the SVD of $\hat{\Gamma}^{f-1/2} \hat{\mathcal{F}} \hat{\Gamma}^{p1/2}$, \hat{V}_k denotes the matrix containing the first k columns of \hat{V} and \hat{S}_k denotes the heading $k \times k$ submatrix of \hat{S} . \hat{S} contains the singular values of $\hat{\Gamma}^{f-1/2} \hat{\mathcal{F}} \hat{\Gamma}^{p1/2}$ in decreasing order. Then, the factor estimates are given by $\hat{\mathcal{K}}Y_t^p$. KM refer to this method as SSS.

For what follows it is important to note that the choice of the weighting matrices is important but not crucial for the asymptotic properties of the estimation method. They are only required to be nonsingular. Therefore, an alternative suggestion is to simply use identity matrices instead of the covariance matrices, as in our Monte Carlo study.

KM show that the factor estimates obtained using the state space methodology are consistent in probability for the space spanned by the true factors and derive rates of convergence. Consistent estimation is possible if p increases at a rate given by $\ln(T)^\alpha$ for some sufficiently large α . We have found that a range of α between 1.05 and 1.5 for this parameter provides a satisfactory performance. We have used the value $\alpha = 1.25$ for the simulation experiments.

For consistency s is also required to be set so as to satisfy $sN \geq k$. As N is usually going to be very large for the applications we have in mind, this restriction is not binding. In the simulation experiments and in the empirical example we have carried out below we set $s = 1$, since it is important to use no future information in the computation of the impulse response functions.

Once estimates of the factors have been obtained, and if estimates of the parameters (including the factor loadings) are subsequently required, least squares methods may be used to obtain such estimates. These estimators have been proved to be \sqrt{T} -consistent and asymptotically normal in Bauer (1998), and Bai (2003) provides a similar result when using the SW factor estimators.

6.2 Correlation in the idiosyncratic errors

In this subsection we discuss the case of cross-sectional and/or serial correlation of the idiosyncratic errors. This extension can be rather simply handled within the state space method. Basically, the idiosyncratic errors can be treated as additional pseudo-factors that enter only a few of the variables via restrictions on the matrix of loadings C . These pseudo-factors can be serially correlated processes or not depending on A .

The problem becomes one of distinguishing common factors and pseudo-factors, i.e., cross-sectionally correlated idiosyncratic errors. This is virtually impossible for finite N , while when N diverges a common factor is one which enters an infinite number of series, i.e., the column of the, now infinite dimensional, matrix C associated with a common factor will have an infinity of non-zero entries, and likewise a pseudo-factor will only have a finite number of non-zero entries in the respective column of C . Let k_1 denote the number of common factors thus defined and k_2 the number of pseudo-factors. Note that k_2 may tend to infinity but not faster than N . Then, following Forni et al. (2000), we make the following assumption.

Assumption 1 *The matrix OK in (13) has k_1 singular values tending to infinity as N tends to infinity and k_2 non-zero finite singular values.*

For example, the condition in the assumption is satisfied if k_1 common factors enter a non zero fraction, bN , $0 < b < 1$, of the N series y_t , in the state space model given by (11), while $k_2(N)$ pseudo-factors enter a vanishing proportion of the series y_t , i.e. each such factors enter $c(N)N$ of the series y_t where $\lim_{N \rightarrow \infty} c(N)N = 0$ and $k_2(N)$ is at most $O(N)$.

KM show that the factor estimates remain consistent in this case as long as N grows slower than $T^{1/6}$. Under this condition, the method for estimating the true number of factors proposed by Bai and Ng (2002) can be extended to this framework, see KM for details.

6.3 Dealing with Large Datasets

Up to now we have outlined a method for estimating factors which requires the number of observations to be larger than the number of elements in Y_t^p . Given the work of SW and FHLR this is rather restrictive. The problem in the SSS framework arises because the least squares estimate of the matrix $\mathcal{OK} = \mathcal{F}$ in (13) cannot be computed due to the rank deficiency of $Y^{p'}Y^p$ where $Y^p = (Y_{p,1}^p, \dots, Y_{p,T}^p)'$. KM therefore suggested a modification of the methodology to allow the number of series be larger than the number of observations.

In particular, they proposed a singular value decomposition (SVD) on $Y^{f'}Y^p(Y^{p'}Y^p)^+ = \hat{U}\hat{S}\hat{V}'$ where $Y^f = (Y_{s,1}^f, \dots, Y_{s,T}^f)'$. Then the estimated factors are given by $\hat{\mathcal{K}}Y_t^p$ where $\hat{\mathcal{K}}$ is obtained as before but using the SVD of $Y^{f'}Y^p(Y^{p'}Y^p)^+$. Both weighting matrices can be set equal to the identity matrix also in this case. This decomposition has the property that when $Np < T$ it reduces to the method in the previous subsection. Moreover, the simulation results in KM indicate that it has good finite sample properties.

7 Appendix 2. Structural Identification of Factor Models: Extensions

In this Appendix we consider a few extensions of the basic set-up presented in Section 2. First, we evaluate what happens when e_t does not belong to the space spanned by the lags of y_t , as in (1). Then, we evaluate the effects of more dynamics in the y equations, in the f equations, and the possible presence of fewer shocks than factors.

When e_t does not belong to the space spanned by the lags of y_t , as in (1), we cannot directly estimate consistently the factors using the KM estimator, but KM show that we can estimate consistently Af_{t-1} . Yet, we can still estimate consistently the A and C matrices. Actually, we can rewrite the model (1) as

$$\begin{aligned} y_t &= C(Af_{t-1}) + (Ce_t + w_t) \\ Af_{t-1} &= A(Af_{t-2}) + Ae_{t-1} \end{aligned} \tag{14}$$

or

$$\begin{aligned} y_t &= Cg_t + (Ce_t + w_t) \\ g_t &= Ag_{t-1} + Ae_{t-1} \end{aligned} \tag{15}$$

where $g_t = Af_{t-1}$. The model (15) has the same structure as (12), and the errors in the y and g equations are i.i.d. and uncorrelated. Therefore, g_t can be consistently estimated by the SSS approach, as well as the A and C matrices. Note further that the estimated covariance matrix of the residual of the estimated factor vector autoregression is consistent for $A\Sigma A'$. Hence it is straightforward to obtain an estimate for Σ by pre and post multiplying the estimated covariance matrix by \hat{A}^{-1} and \hat{A}'^{-1} respectively, where \hat{A} denotes the estimate of A .

Three additional interesting extensions to be evaluated are the presence of more dynamics in the y equations, in the f equations, and the possible presence of fewer shocks than factors. All these extensions can be considered within the framework we have presented in Section 2, but with a reduced rank covariance matrix for the errors.

For example, the model could be

$$\begin{aligned} y_t &= C_1f_t + C_2f_{t-1} + w_t \\ f_t &= A_1f_{t-1} + A_2f_{t-2} + Q_1e_t \end{aligned} \tag{16}$$

where the rank of Q_1 is k . This model can be cast in the (1) form as

$$\begin{aligned} y_t &= Cs_t + w_t \\ s_t &= As_{t-1} + Qu_t \end{aligned} \tag{17}$$

where

$$\begin{aligned} C &= (C_1 \ C_2); \quad s_t = \begin{pmatrix} f_t \\ f_{t-1} \end{pmatrix}; \quad A = \begin{pmatrix} A_1 & A_2 \\ I & 0 \end{pmatrix}; \\ Q &= \begin{pmatrix} Q_1 & 0 \\ 0 & 0 \end{pmatrix}; \quad u_t = \begin{pmatrix} e_t \\ 0 \end{pmatrix} = Se_t \quad \text{where } S = \begin{pmatrix} I \\ 0 \end{pmatrix} \end{aligned}$$

All shock identification schemes discussed in Section 2 may be straightforwardly modified. We first discuss a simple identification scheme based on a Cholesky decomposition of the covariance matrix of the errors, assuming that we know the true rank, r , of the covariance matrix. We discuss methods for determining this rank later.

Let the estimated covariance of u_t be denoted by $\hat{\Sigma}$. The eigenvalue-eigenvector decomposition is given by $\hat{\Sigma} = \hat{V}\hat{\Lambda}\hat{V}'$ where $\hat{\Lambda} = \text{diag}(\lambda_1, \dots, \lambda_{2k})$ where $\lambda_1 > \dots > \lambda_{2k}$ are the ordered eigenvalues of $\hat{\Sigma}$. Then, we define the reduced rank estimator for the covariance matrix as $\hat{\Sigma}^* = \hat{V}\hat{\Lambda}^*\hat{V}'$ where $\hat{\Lambda}^* = \text{diag}(\lambda_1, \dots, \lambda_k, 0, \dots, 0)$. $\hat{\Sigma}^*$ is a positive semidefinite (PSD) matrix and a Choleski decomposition can be used to give $\hat{\Sigma}^* = \hat{P}\hat{P}'$ where \hat{P} is a lower triangular matrix.⁴ Impulse response analysis readily follows for the k shocks with unit variance.

For more general identification schemes further analysis is required. The relationship between the structural shocks and the errors is now given by $u_t = Se_t = S\tilde{R}\tilde{e}_t = R\tilde{e}_t$ where R is a $2k \times k$ matrix. Unlike the full rank case where the transformation could be equivalently cast in terms of R or R^{-1} , in the present case R is not a square matrix. We examine the case where identification is carried out using restrictions on the structural long run effect matrix Γ . Equation (5) becomes

$$C(I - A)^{-1}R = \Gamma \quad (18)$$

where Γ is a $N \times k$ matrix. Equation (6) becomes

$$\hat{\Sigma}^* = RR' \quad (19)$$

Note that (19) now involves $k(k+1)/2$ linearly independent equations rather than $2k(2k+1)/2$. Hence, overall there are $Nk + k(k+1)/2$ equations and $Nk + 2k^2$ unknowns resulting in the need for $2k^2 - k(k+1)/2$ restrictions to be placed on Γ . Other identification schemes are of course possible.

Finally, we discuss briefly methods for determining the rank of the covariance matrix of the reduced form errors. A first heuristic approach may be based on the visual inspection of the eigenvalues or singular values of the estimated covariance matrix. A sudden step change in adjacent ordered eigenvalues may be indicative of the rank of the covariance matrix. A more rigorous method may be based on a test of rank. Camba-Mendez and Kapetanios (2002) provide a conservative testing procedure for determining the rank of the spectral density matrix for stationary processes. This test can be readily modified to deal with covariance matrices. However, the asymptotic distribution of the estimated covariance

⁴Standard econometric packages like e.g. GAUSS or MATLAB do not give the Choleski decomposition for PSD matrices, via standard functions, but this can be straightforwardly programmed following e.g. algorithm 4.2.11 of Golub and Van Loan (1996).

matrix is needed. As this is cumbersome to obtain the bootstrap may be used. Camba-Mendez and Kapetanios (2002a) discuss the application of the bootstrap to state space models estimated using subspace algorithms. Other issues concerning application of the bootstrap to tests of rank may be found in Camba-Mendez et al. (2002b).

Table 1: Results for Estimated Impulse Responses for experiments A, N=50, T=50

	Exp	Horizon							
		1	2	3	4	5	10	15	20
Bias (E) ^a	A1	-0.001	-0.000	0.000	0.000	0.000	0.000	0.000	0.000
	A2	0.002	0.001	0.000	-0.000	-0.001	-0.001	-0.000	-0.000
	A3	0.033	-0.001	-0.000	0.002	0.002	0.000	0.000	0.000
	A4	-0.012	-0.020	-0.022	-0.019	-0.014	-0.000	0.000	-0.000
	A5	0.003	0.014	0.012	0.011	0.008	0.002	0.001	0.000
	A6	0.029	0.022	0.012	0.005	0.001	-0.003	-0.001	-0.000
RMSE (E)	A1	0.244	0.083	0.033	0.014	0.006	0.000	0.000	0.000
	A2	0.569	0.357	0.235	0.161	0.113	0.025	0.007	0.002
	A3	0.677	0.437	0.210	0.067	0.025	0.001	0.000	0.000
	A4	0.748	0.777	0.777	0.694	0.557	0.095	0.027	0.009
	A5	0.896	0.552	0.366	0.247	0.174	0.038	0.010	0.003
	A6	0.930	0.887	0.831	0.767	0.701	0.440	0.293	0.206
Bias (S)	A1	-0.001	-0.000	0.000	0.000	0.000	0.000	0.000	0.000
	A2	0.001	0.001	-0.000	-0.001	-0.001	-0.001	-0.000	-0.000
	A3	-0.663	0.040	0.014	-0.002	-0.000	0.000	0.000	0.000
	A4	-0.034	-0.031	-0.020	-0.012	-0.007	-0.001	-0.001	-0.000
	A5	-0.047	-0.023	-0.023	-0.016	-0.013	-0.001	0.000	0.000
	A6	0.275	0.347	0.337	0.296	0.247	0.079	0.023	0.007
RMSE (S)	A1	0.248	0.086	0.034	0.015	0.007	0.000	0.000	0.000
	A2	0.576	0.365	0.243	0.168	0.119	0.028	0.008	0.003
	A3	21.355	0.994	0.362	0.151	0.051	0.001	0.000	0.000
	A4	1.060	0.956	0.756	0.591	0.468	0.154	0.047	0.014
	A5	2.383	1.956	1.469	1.059	0.749	0.124	0.022	0.005
	A6	7.320	9.781	9.827	8.828	7.504	2.581	0.850	0.327
Bias (T)	A1	-0.002	-0.000	0.000	0.000	0.000	0.000	0.000	0.000
	A2	0.005	0.003	0.001	0.000	-0.000	-0.000	-0.000	-0.000
	A3	-0.000	0.002	0.001	0.001	0.001	0.000	0.000	0.000
	A4	0.007	0.007	0.006	0.004	0.003	0.002	0.001	0.000
	A5	-0.001	0.004	0.002	0.001	0.001	0.000	0.000	0.000
	A6	0.010	0.008	0.004	0.000	-0.002	-0.005	-0.004	-0.003
RMSE (T)	A1	0.254	0.088	0.035	0.015	0.007	0.000	0.000	0.000
	A2	0.590	0.376	0.252	0.175	0.124	0.029	0.009	0.003
	A3	0.330	0.134	0.061	0.029	0.015	0.001	0.000	0.000
	A4	0.581	0.394	0.284	0.210	0.158	0.043	0.014	0.006
	A5	0.458	0.263	0.165	0.106	0.071	0.012	0.003	0.001
	A6	0.678	0.582	0.522	0.470	0.424	0.267	0.187	0.140
Corr (E)	A1						0.777	0.777	0.777
	A2						0.986	0.983	0.982
	A3						0.373	0.370	0.369
	A4						0.709	0.678	0.665
	A5						0.578	0.578	0.579
	A6						0.734	0.744	0.746
Sign (E)	A1	0.890	1.000	0.890	1.000	0.890	1.000	0.890	1.000
	A2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	A3	0.690	0.629	0.580	0.574	0.567	0.586	0.605	0.595
	A4	0.870	0.796	0.735	0.693	0.659	0.578	0.566	0.560
	A5	0.786	0.782	0.768	0.770	0.755	0.744	0.746	0.760
	A6	0.857	0.837	0.845	0.846	0.852	0.844	0.828	0.831

^aBias (E) and RMSE (E) denote Bias and RMSE of estimated impulse responses (IR) using estimated subspace factors. Bias (S) and RMSE (S) denote Bias and RMSE of estimated IR using estimated principal component factors. Bias (T) and RMSE (T) denote Bias and RMSE of estimated IR using true factors. Corr. (E) denotes correlation between estimated IR using estimated subspace factors and true IR. Sign (E) denotes estimated probability that estimated IR using estimated subspace factors and true IR have the same sign.

Table 2: Results on Estimated Impulse Responses for experiment set B, N=100, T=50

	Exp	Horizon							
		1	2	3	4	5	10	15	20
Bias (E) ^a	B1	-0.010	-0.003	-0.001	-0.000	-0.000	0.000	0.000	0.000
	B2	0.003	0.004	0.003	0.002	0.002	0.000	0.000	0.000
	B3	0.034	0.014	0.003	0.003	0.001	0.000	0.000	0.000
	B4	-0.026	-0.029	-0.027	-0.023	-0.019	-0.008	-0.003	-0.001
	B5	-0.001	-0.010	-0.009	-0.008	-0.006	-0.002	-0.000	-0.000
	B6	0.020	0.022	0.020	0.017	0.015	0.008	0.004	0.003
RMSE (E)	B1	0.241	0.083	0.034	0.016	0.008	0.000	0.000	0.000
	B2	0.577	0.357	0.233	0.157	0.110	0.023	0.006	0.002
	B3	0.726	0.460	0.135	0.068	0.033	0.005	0.001	0.000
	B4	0.782	0.673	0.570	0.482	0.408	0.164	0.055	0.015
	B5	0.826	0.531	0.343	0.225	0.151	0.028	0.008	0.003
	B6	0.858	0.793	0.709	0.626	0.551	0.300	0.182	0.122
Bias (S)	B1	-0.010	-0.004	-0.001	-0.001	-0.000	0.000	0.000	0.000
	B2	0.003	0.004	0.003	0.002	0.002	0.000	0.000	0.000
	B3	0.054	0.021	0.005	0.003	0.002	0.000	0.000	0.000
	B4	-0.018	-0.026	-0.027	-0.024	-0.020	-0.008	-0.003	-0.001
	B5	0.025	0.006	-0.000	-0.003	-0.003	-0.001	-0.000	-0.000
	B6	0.006	0.008	0.007	0.007	0.007	0.005	0.003	0.002
RMSE (S)	B1	0.245	0.086	0.036	0.017	0.008	0.000	0.000	0.000
	B2	0.584	0.366	0.241	0.165	0.116	0.026	0.007	0.002
	B3	1.652	0.593	0.166	0.089	0.051	0.009	0.002	0.000
	B4	0.774	0.673	0.589	0.513	0.443	0.183	0.062	0.018
	B5	0.877	0.557	0.364	0.242	0.164	0.030	0.009	0.003
	B6	0.824	0.754	0.674	0.596	0.526	0.293	0.185	0.129
Bias (T)	B1	-0.011	-0.004	-0.001	-0.001	-0.000	0.000	0.000	0.000
	B2	0.003	0.004	0.003	0.003	0.002	0.001	0.000	0.000
	B3	0.013	0.002	0.000	0.000	-0.000	-0.000	-0.000	-0.000
	B4	0.005	0.003	0.001	-0.000	-0.001	-0.000	-0.000	-0.000
	B5	-0.003	-0.005	-0.006	-0.005	-0.004	-0.001	-0.000	-0.000
	B6	-0.003	0.001	0.002	0.003	0.004	0.004	0.003	0.003
RMSE (T)	B1	0.248	0.088	0.037	0.017	0.009	0.001	0.000	0.000
	B2	0.593	0.373	0.247	0.169	0.119	0.027	0.008	0.003
	B3	0.356	0.140	0.064	0.031	0.017	0.002	0.000	0.000
	B4	0.634	0.429	0.300	0.214	0.157	0.041	0.013	0.005
	B5	0.477	0.292	0.190	0.128	0.090	0.022	0.008	0.003
	B6	0.680	0.564	0.495	0.439	0.391	0.230	0.152	0.110
Corr (E)	B1						0.816	0.816	0.816
	B2						0.988	0.985	0.985
	B3						0.394	0.392	0.391
	B4						0.717	0.688	0.676
	B5						0.593	0.592	0.592
	B6						0.748	0.750	0.749
Sign (E)	B1	0.910	1.000	0.910	1.000	0.910	1.000	0.910	1.000
	B2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	B3	0.709	0.651	0.597	0.596	0.584	0.590	0.575	0.580
	B4	0.880	0.802	0.752	0.700	0.675	0.582	0.554	0.587
	B5	0.789	0.790	0.781	0.771	0.773	0.738	0.750	0.748
	B6	0.865	0.840	0.856	0.851	0.844	0.839	0.826	0.819

^aSee notes in Table 1

Table 3: Results on Estimated Impulse Responses for experiment set C, N=50, T=100

	Exp	Horizon							
		1	2	3	4	5	10	15	20
Bias (E) ^a	C1	0.002	0.001	0.000	0.000	-0.000	-0.000	-0.000	-0.000
	C2	0.001	0.000	-0.000	-0.001	-0.001	-0.000	-0.000	-0.000
	C3	-0.000	0.002	0.001	-0.000	-0.000	-0.000	-0.000	-0.000
	C4	0.012	0.014	0.013	0.011	0.009	0.002	0.000	0.000
	C5	0.035	0.022	0.012	0.008	0.005	0.001	0.000	0.000
	C6	-0.002	-0.006	-0.008	-0.009	-0.010	-0.008	-0.005	-0.004
RMSE (E)	C1	0.225	0.066	0.022	0.008	0.003	0.000	0.000	0.000
	C2	0.575	0.349	0.219	0.141	0.093	0.015	0.004	0.001
	C3	0.406	0.147	0.049	0.018	0.007	0.000	0.000	0.000
	C4	0.610	0.419	0.299	0.214	0.152	0.028	0.005	0.001
	C5	0.815	0.490	0.292	0.179	0.114	0.016	0.003	0.001
	C6	0.703	0.614	0.553	0.497	0.446	0.260	0.160	0.105
Bias (S)	C1	0.002	0.001	0.000	0.000	-0.000	-0.000	-0.000	-0.000
	C2	0.001	0.000	-0.000	-0.001	-0.001	-0.000	-0.000	-0.000
	C3	0.005	0.004	0.001	0.000	-0.000	-0.000	-0.000	-0.000
	C4	0.013	0.015	0.014	0.012	0.009	0.002	0.000	0.000
	C5	-0.535	-0.158	-0.064	-0.024	-0.008	0.001	0.000	0.000
	C6	-0.026	-0.029	-0.028	-0.026	-0.023	-0.014	-0.008	-0.005
RMSE (S)	C1	0.227	0.067	0.023	0.008	0.003	0.000	0.000	0.000
	C2	0.579	0.353	0.223	0.144	0.095	0.016	0.004	0.001
	C3	0.457	0.151	0.049	0.017	0.007	0.000	0.000	0.000
	C4	0.607	0.412	0.291	0.208	0.148	0.028	0.006	0.001
	C5	20.723	7.393	3.274	1.391	0.598	0.021	0.003	0.001
	C6	1.251	1.157	0.980	0.831	0.712	0.357	0.195	0.117
Bias (T)	C1	0.002	0.001	0.000	0.000	0.000	-0.000	-0.000	-0.000
	C2	0.001	0.001	0.000	-0.000	-0.000	-0.000	-0.000	0.000
	C3	0.004	0.004	0.001	0.000	0.000	-0.000	-0.000	-0.000
	C4	-0.001	0.001	0.001	0.001	0.001	0.001	0.000	0.000
	C5	-0.003	0.000	0.001	0.001	0.001	0.000	-0.000	0.000
	C6	-0.000	-0.002	-0.004	-0.005	-0.006	-0.005	-0.004	-0.003
RMSE (T)	C1	0.230	0.068	0.023	0.008	0.003	0.000	0.000	0.000
	C2	0.591	0.365	0.232	0.151	0.101	0.018	0.005	0.002
	C3	0.270	0.090	0.033	0.014	0.006	0.000	0.000	0.000
	C4	0.593	0.375	0.248	0.168	0.116	0.021	0.005	0.001
	C5	0.432	0.256	0.157	0.097	0.062	0.008	0.002	0.000
	C6	0.663	0.559	0.502	0.453	0.408	0.243	0.154	0.104
Corr (E)	C1						0.936	0.936	0.936
	C2						0.993	0.992	0.992
	C3						0.488	0.484	0.482
	C4						0.826	0.802	0.791
	C5						0.746	0.749	0.750
	C6						0.876	0.881	0.882
Sign (E)	C1	0.970	1.000	0.970	1.000	0.970	1.000	0.970	1.000
	C2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	C3	0.747	0.671	0.600	0.565	0.547	0.552	0.549	0.562
	C4	0.929	0.868	0.814	0.772	0.744	0.626	0.583	0.576
	C5	0.857	0.872	0.875	0.873	0.867	0.851	0.843	0.844
	C6	0.911	0.905	0.911	0.926	0.929	0.923	0.917	0.918

^aSee notes in Table 1

Table 4: Results on Estimated Impulse Responses for experiment set D, N=100, T=100

	Exp	Horizon							
		1	2	3	4	5	10	15	20
Bias (E) ^a	D1	-0.004	-0.001	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
	D2	0.001	0.001	0.001	0.000	0.000	-0.000	-0.000	-0.000
	D3	0.010	0.001	-0.000	-0.000	-0.000	-0.000	0.000	0.000
	D4	-0.021	-0.024	-0.021	-0.017	-0.012	-0.002	-0.000	-0.000
	D5	-0.009	-0.005	-0.006	-0.005	-0.005	-0.001	-0.000	-0.000
	D6	-0.013	-0.015	-0.014	-0.013	-0.011	-0.006	-0.003	-0.002
RMSE (E)	D1	0.222	0.063	0.020	0.007	0.003	0.000	0.000	0.000
	D2	0.586	0.354	0.221	0.141	0.092	0.014	0.003	0.001
	D3	0.379	0.139	0.049	0.018	0.007	0.000	0.000	0.000
	D4	0.749	0.643	0.523	0.403	0.300	0.054	0.009	0.002
	D5	0.787	0.473	0.279	0.160	0.094	0.011	0.002	0.000
	D6	0.733	0.639	0.567	0.503	0.444	0.238	0.138	0.086
Bias (S)	D1	-0.004	-0.001	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
	D2	0.001	0.001	0.001	0.000	0.000	-0.000	-0.000	-0.000
	D3	0.012	0.002	0.000	-0.000	-0.000	-0.000	0.000	0.000
	D4	-0.006	-0.006	-0.004	-0.003	-0.001	0.000	0.000	0.000
	D5	-0.032	-0.019	-0.013	-0.009	-0.006	-0.001	-0.000	-0.000
	D6	-0.012	-0.013	-0.012	-0.010	-0.009	-0.004	-0.002	-0.001
RMSE (S)	D1	0.224	0.065	0.021	0.007	0.003	0.000	0.000	0.000
	D2	0.590	0.359	0.225	0.144	0.095	0.015	0.003	0.001
	D3	0.407	0.161	0.059	0.022	0.008	0.000	0.000	0.000
	D4	0.723	0.588	0.458	0.341	0.246	0.042	0.007	0.002
	D5	0.854	0.490	0.283	0.163	0.097	0.011	0.002	0.000
	D6	0.751	0.713	0.674	0.616	0.551	0.288	0.157	0.094
Bias (T)	D1	-0.005	-0.002	-0.000	-0.000	-0.000	-0.000	-0.000	-0.000
	D2	0.003	0.002	0.002	0.001	0.001	0.000	-0.000	-0.000
	D3	0.008	0.001	0.000	0.000	0.000	0.000	0.000	0.000
	D4	-0.006	-0.008	-0.007	-0.005	-0.004	-0.001	-0.000	-0.000
	D5	-0.007	-0.007	-0.006	-0.004	-0.003	-0.001	-0.000	-0.000
	D6	0.002	0.002	0.001	-0.000	-0.001	-0.003	-0.003	-0.003
RMSE (T)	D1	0.227	0.066	0.022	0.008	0.003	0.000	0.000	0.000
	D2	0.597	0.366	0.230	0.148	0.098	0.015	0.003	0.001
	D3	0.278	0.094	0.034	0.013	0.005	0.000	0.000	0.000
	D4	0.616	0.394	0.261	0.177	0.122	0.023	0.005	0.001
	D5	0.443	0.268	0.167	0.106	0.069	0.011	0.002	0.001
	D6	0.679	0.557	0.488	0.433	0.384	0.216	0.130	0.083
Corr (E)	D1						0.954	0.954	0.954
	D2						0.995	0.993	0.993
	D3						0.539	0.537	0.536
	D4						0.827	0.804	0.794
	D5						0.765	0.766	0.766
	D6						0.868	0.869	0.868
Sign (E)	D1	0.979	1.000	0.979	1.000	0.979	1.000	0.979	1.000
	D2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
	D3	0.770	0.710	0.652	0.639	0.626	0.604	0.621	0.619
	D4	0.926	0.871	0.820	0.785	0.745	0.631	0.586	0.574
	D5	0.864	0.875	0.881	0.873	0.868	0.844	0.844	0.849
	D6	0.915	0.900	0.912	0.911	0.898	0.900	0.902	0.898

^aSee notes in Table 1

Figure 1

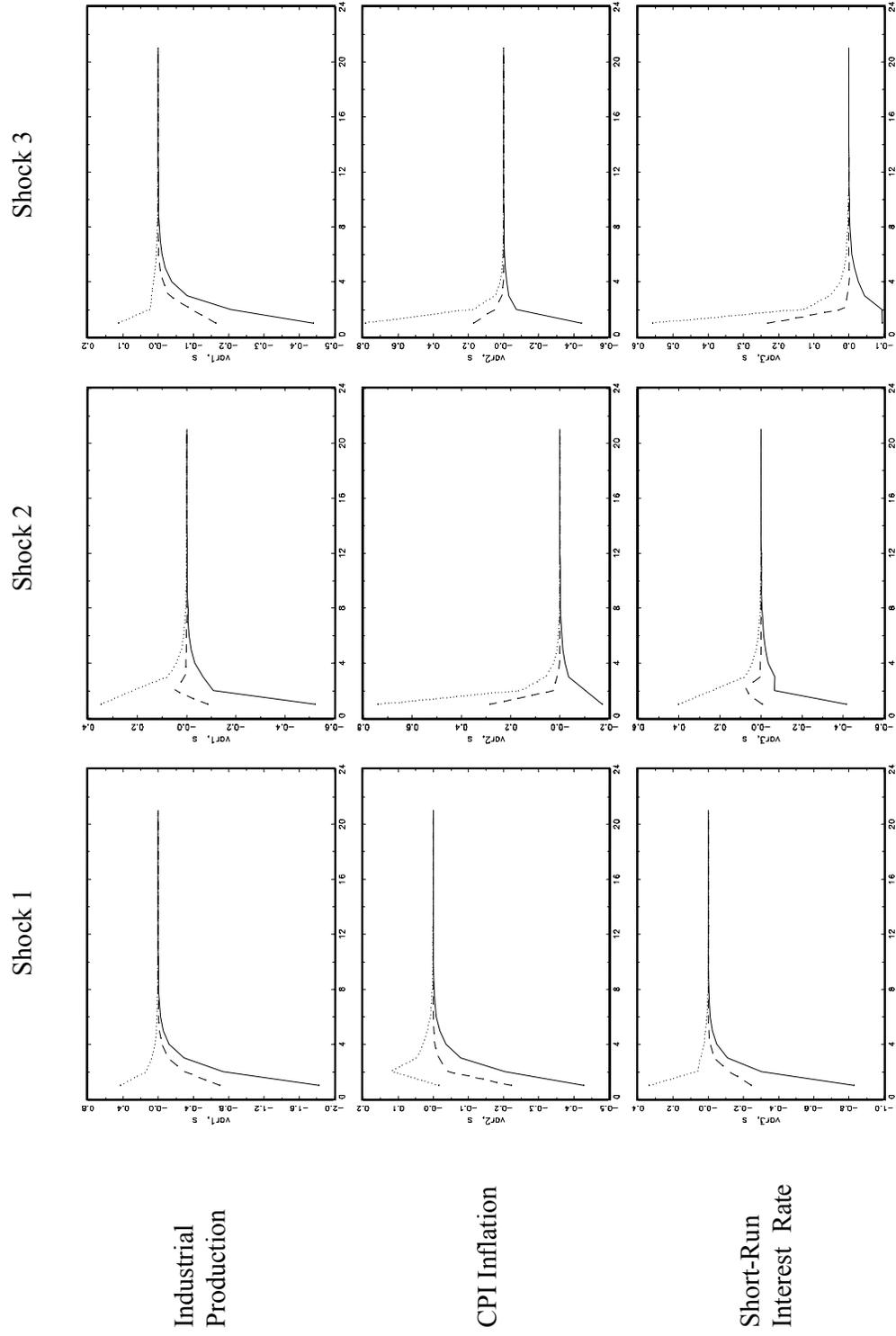


Figure 2

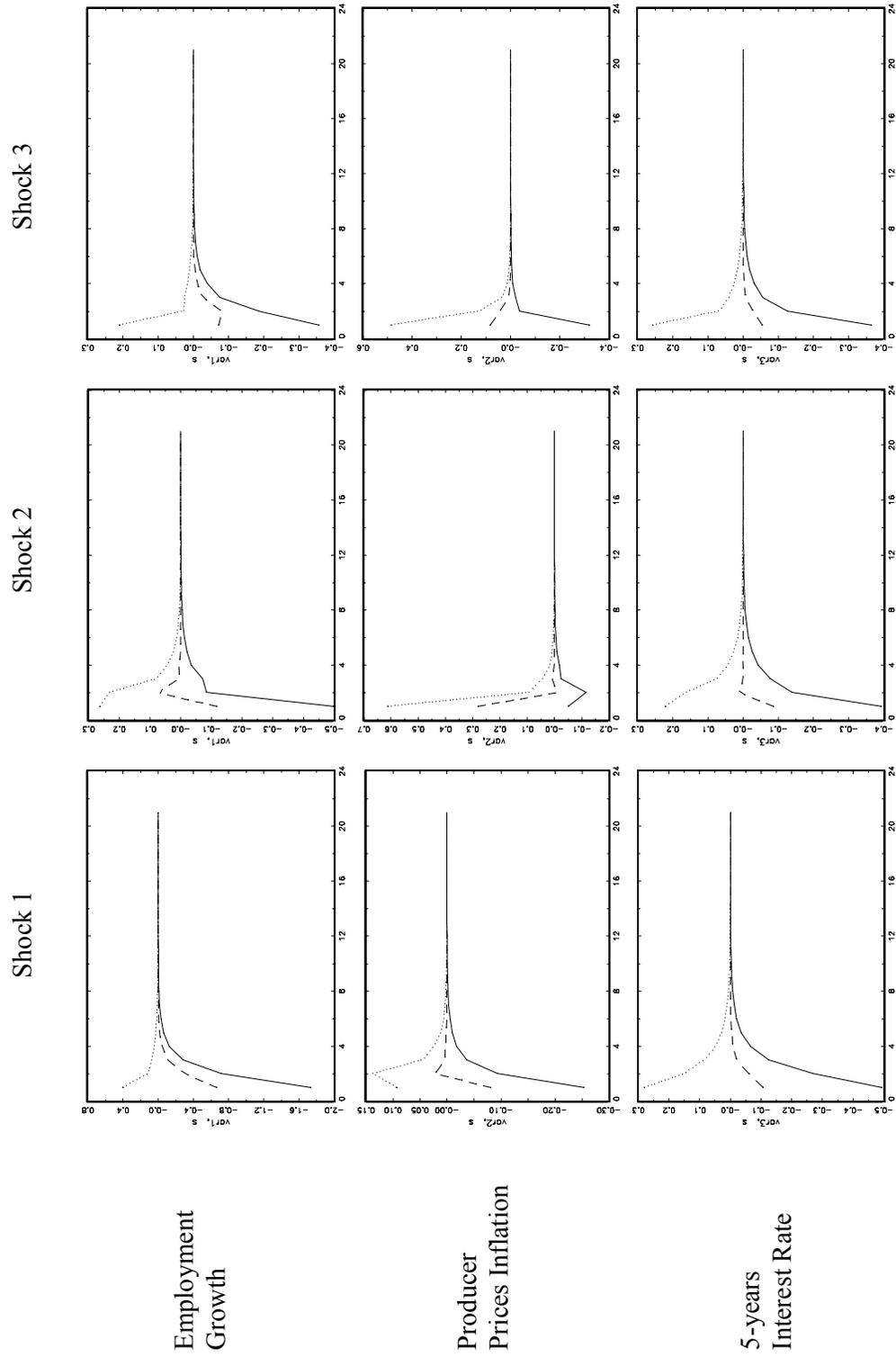


Figure 3

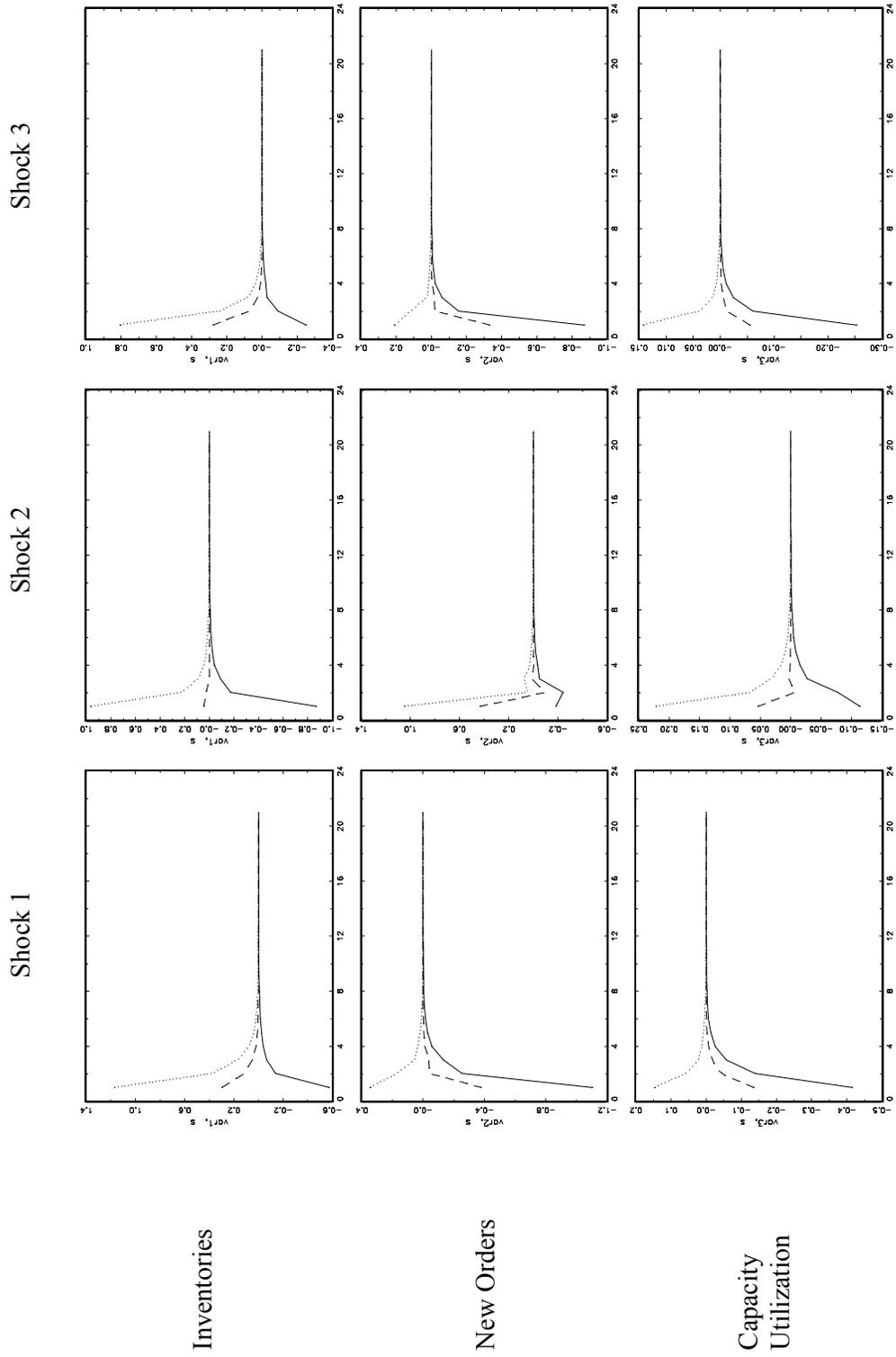


Figure 4

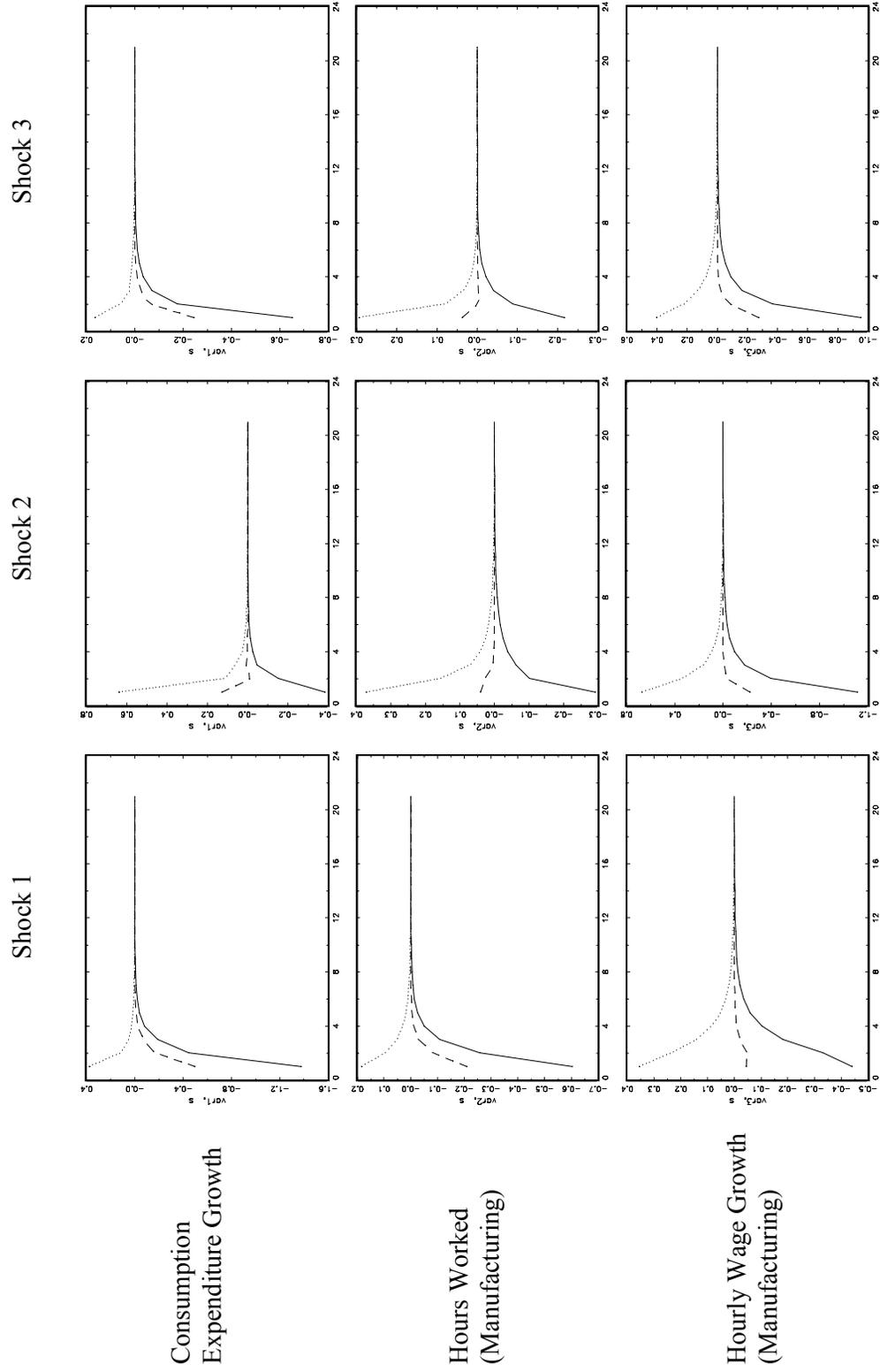
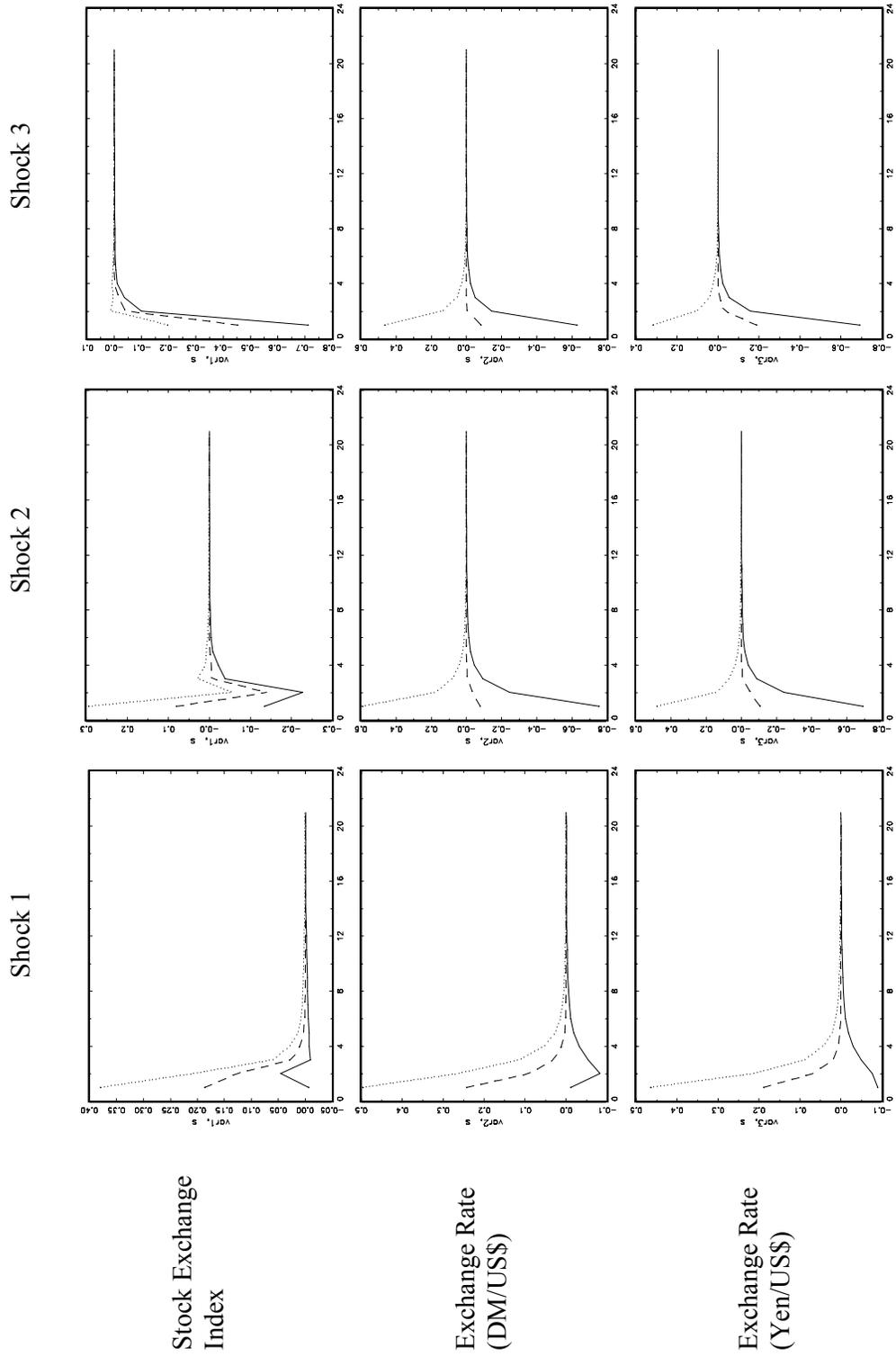


Figure 5



DATA APPENDIX

This appendix lists the variables used in the empirical analysis,
with a short description and the transformation applied.

The transformation codes are: 1 = no transformation; 2 = first difference; 3= second difference;
4 = logarithm; 5 = first difference of logarithm; 6 = second difference of logarithm.

<i>Variable</i>	<i>Transf</i>
1 INDUSTRIAL PRODUCTION: TOTAL INDEX(1992=100,SA)	5
2 INDUSTRIAL PRODUCTION: PRODUCTS,TOTAL(1992=100,SA)	5
3 INDUSTRIAL PRODUCTION: FINAL PRODUCTS(1992=100,SA)	5
4 INDUSTRIAL PRODUCTION: CONSUMER GOODS(1992=100,SA)	5
5 INDUSTRIAL PRODUCTION: DURABLE CONSUMER GOODS(1992=100,SA)	5
6 INDUSTRIAL PRODUCTION: NONDURABLE CONSUMER GOODS(1992=100,SA)	5
7 INDUSTRIAL PRODUCTION: BUSINESS EQUIPMENT(1992=100,SA)	5
8 INDUSTRIAL PRODUCTION: INTERMEDIATE PRODUCTS(1992=100,SA)	5
9 INDUSTRIAL PRODUCTION: MATERIALS(1992=100,SA)	5
10 INDUSTRIAL PRODUCTION: NONDURABLE GOODS MATERIALS(1992=100,SA)	5
11 INDUSTRIAL PRODUCTION: MANUFACTURING(1992=100,SA)	5
12 INDUSTRIAL PRODUCTION: DURABLE MANUFACTURING(1992=100,SA)	5
13 INDUSTRIAL PRODUCTION: NONDURABLE MANUFACTURING(1992=100,SA)	5
14 INDUSTRIAL PRODUCTION: MINING(1992=100,SA)	5
15 INDUSTRIAL PRODUCTION: UTILITIES(1992=100,SA)	5
16 CAPACITY UTIL RATE: MANUFACTURING,TOTAL(%OF CAPACITY,SA)(FRB)	1
17 PURCHASING MANAGERS' INDEX (SA)	1
18 NAPM PRODUCTION INDEX (PERCENT)	1
19 PERSONAL INCOME (CHAINED) (BIL 92\$, SAAR)	5
20 INDEX OF HELP-WANTED ADVERTISING IN NEWSPAPERS "(1967=100;SA)"	5
21 EMPLOYMENT: "RATIO;" HELP-WANTED ADS:NO.UNEMPLOYED CLF	4
22 CIVILIAN LABOR FORCE:EMPLOYED,TOTAL (THOUS.,SA)	5
23 CIVILIAN LABOR FORCE:EMPLOYED,NONAGRIC.INDUSTRIES(THOUS.,SA)	5
24 UNEMPLOYMENT RATE:ALL WORKERS,16 YEARS & OVER(% ,SA)	1
25 UNEMPLOY.BY DURATION: AVERAGE(MEAN)DURATION IN WEEKS(SA)	1
26 UNEMPLOY.BY DURATION: PERSONS UNEMPL.LESS THAN 5WKS(THOUS.,SA)	1
27 UNEMPLOY.BY DURATION: PERSONS UNEMPL.5 TO 14 WKS(THOUS.,SA)	1
28 UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 WKS +(THOUS.,SA)	1
29 UNEMPLOY.BY DURATION: PERSONS UNEMPL.15 TO 26 WKS(THOUS.,SA)	1
30 EMPLOYEES ON NONAG.PAYROLLS:TOTAL(THOUS.,SA)	5
31 EMPLOYEES ON NONAG.PAYROLLS:TOTAL,PRIVATE (THOUS,SA)	5
32 EMPLOYEES ON NONAG.PAYROLLS:GOODS-PRODUCING(THOUS.,SA)	5
33 EMPLOYEES ON NONAG.PAYROLLS:CONTRACT CONSTRUCTION(THOUS.,SA)	5
34 EMPLOYEES ON NONAG.PAYROLLS:MANUFACTURING(THOUS.,SA)	5
35 EMPLOYEES ON NONAG.PAYROLLS:DURABLE GOODS(THOUS.,SA)	5
36 EMPLOYEES ON NONAG.PAYROLLS:NONDURABLE GOODS(THOUS.,SA)	5
37 EMPLOYEES ON NONAG.PAYROLLS:SERVICE-PRODUCING(THOUS.,SA)	5
38 EMPLOYEES ON NONAG.PAYROLLS:WHOLESALE & RETAIL TRADE (THOUS.,SA)	5
39 EMPLOYEES ON NONAG.PAYROLLS:FINANCE,INSUR.&REAL ESTATE (THOUS.,SA)	5
40 EMPLOYEES ON NONAG.PAYROLLS:SERVICES(THOUS.,SA)	5
41 EMPLOYEES ON NONAG.PAYROLLS:GOVERNMENT(THOUS.,SA)	5
42 AVG. WEEKLY HRS. OF PRODUCTION WKRS.: MANUFACTURING (SA)	1
43 AVG. WEEKLY HRS. OF PROD. WKRS.:MFG., OVERTIME HRS. (SA)	1
44 NAPM employment index (percent)	1
45 MANUFACTURING & TRADE: TOTAL(MIL OF CHAINED 1992 DOLLARS)(SA)	5
46 MANUFACTURING & "TRADE: MANUFACTURING;TOTAL	5

47 MANUFACTURING & "TRADE: MFG;" DURABLE GOODS	5
48 MANUFACT.& "TRADE:MFG;NONDURABLE" GOODS	5
49 MERCHANT WHOLESALERS: TOTAL (MIL OF CHAINED 1992 DOLLARS)(SA)	5
50 MERCHANT WHOLESALERS:DURABLE GOODS TOTAL	5
51 MERCHANT WHOLESALERS:NONDURABLE GOODS	5
52 RETAILTRADE: TOTAL (MIL OF CHAINED 1992 DOLLARS)(SA)	5
53 RETAILTRADE: NONDURABLE GOODS (MIL OF 1992 DOLLARS)(SA)	5
54 PERSONAL CONSUMPTION EXPEND (CHAINED)-TOTAL(BIL 92\$,SAAR)	5
55 PERSONAL CONSUMPTION EXPEND (CHAINED)-TOTAL DURABLES(BIL 92\$,SAAR)	5
56 PERSONAL CONSUMPTION EXPEND (CHAINED)-NONDURABLES(BIL 92\$,SAAR)	5
57 PERSONAL CONSUMPTION EXPEND (CHAINED)-SERVICES(BIL 92\$,SAAR)	5
58 PERSONAL CONS EXPEND (CHAINED)-NEW CARS (BIL 92\$,SAAR)	5
59 HOUSING "STARTS: NONFARM(1947-58);TOTAL" FARM&NONFARM(1959-)(THOUS.,SA	4
60 HOUSING STARTS: NORTHEAST (THOUS.U.)S.A.	4
61 HOUSING STARTS: MIDWEST (THOUS.U.)S.A.	4
62 HOUSING STARTS: SOUTH (THOUS.U.)S.A.	4
63 HOUSING STARTS: WEST (THOUS.U.)S.A.	4
64 HOUSING AUTHORIZED:TOTAL NEW PRIV HOUSING UNITS (THOUS.,SAAR)	4
65 MOBILE HOMES: MANUFACTURERS' SHIPMENTS(THOUS.OF UNITS,SAAR)	4
66 MANUFACTURING & TRADE INVENTORIES:TOTAL(MIL OF CHAINED 1992)(SA)	5
67 INVENTORIES,BUSINESS,MFG(MIL OF CHAINED 1992 DOLLARS, SA)	5
68 INVENTORIES,BUSINESS DURABLES(MIL OF CHAINED 1992 DOLLARS, SA)	5
69 INVENTORIES,BUSINESS,NONDURABLES(MIL OF CHAINED 1992 DOLLARS, SA)	5
70 MANUFACTURING & TRADE INV:MERCHANT WHOLESALERS	5
71 MANUFACTURING & TRADE INV:RETAIL TRADE (MIL OF CHAINED 1992 DOLLARS)(SA)	5
72 RATIO FOR MFG & TRADE:INVENTORY/SALES (CHAINED 1992 DOLLARS, SA)	2
73 RATIO FOR MFG & "TRADE:MFG;INVENTORY/SALES"(87\$)(S.A.)	2
74 RATIO FOR MFG & "TRADE:WHOLESALE;INVENTORY/SALES(87\$)(S.A.)"	2
75 RATIO FOR MFG & TRADE:RETAIL"TRADE;INVENTORY/SALES(87\$)(S.A.)"	2
76 NAPM INVENTORIES INDEX (PERCENT)	1
77 NAPM NEW ORDERS INDEX (PERCENT)	1
78 NAPM VENDOR DELIVERIES INDEX (PERCENT)	1
79 NEW ORDERS (NET)-CONSUMER GOODS & MATERIALS, 1992 DOLLARS(BCI)	5
80 NEW ORDERS, DURABLE GOODS INDUSTRIES, 1992 DOLLARS(BCI)	5
81 NEW ORDERS, NONDEFENSE CAPITAL GOODS,IN 1992 DOLLARS(BCI)	5
82 MFG NEW ORDERS:ALL MANUFACTURING INDUSTRIES,TOTAL(MIL\$,SA)	5
83 MFG NEW ORDERS:MFG INDUSTRIES WITH UNFILLED ORDERS(MIL\$,SA)	5
84 MFG NEW ORDERS:DURABLE GOODS INDUSTRIES, TOTAL(MIL\$,SA)	5
85 MFG NEW ORDERS:DURABLE GOODS INDUST WITH UNFILLED ORDERS(MIL\$,SA)	5
86 MFG NEW ORDERS:NONDURABLE GOODS INDUSTRIES, TOTAL (MIL\$,SA)	5
87 MFG NEW ORDERS:NONDURABLE GDS IND.WITH UNFILLED ORDERS(MIL\$,SA)	5
88 MFG UNFILLED ORDERS: ALL MANUFACTURING INDUSTRIES,TOTAL(MIL\$,SA)	5
89 MFG UNFILLED ORDERS: DURABLE GOODS INDUSTRIES,TOTAL(MIL\$,SA)	5
90 MFG UNFILLED ORDERS: NONDURABLE GOODS INDUSTRIES, TOTAL(MIL\$,SA)	5
91 CONTRACTS & ORDERS FOR PLANT & EQUIPMENT (BIL\$,SA)	5
92 CONTRACTS & ORDERS FOR PLANT & EQUIPMENT IN 1992 DOLLARS(BCI)	5
93 NYSE COMMON STOCK PRICE INDEX: COMPOSITE (12/31/65=50)	5
94 S&P'S COMMON STOCK PRICE INDEX: COMPOSITE (1941-43=10)	5
95 S&P'S COMMON STOCK PRICE INDEX: INDUSTRIALS(1941-43=10)	5
96 S&P'S COMMON STOCK PRICE INDEX: CAPITAL GOODS (1941-43=10)	5
97 S&P'S COMMON STOCK PRICE INDEX: UTILITIES (1941-43=10)	5
98 S&P'S COMPOSITE COMMON STOCK: DIVIDEND YIELD(% PER ANNUM)	1
99 S&P'S COMPOSITE COMMON STOCK: PRICE-EARNINGS RATIO(% ,NSA)	1
100 UNITED "STATES;EFFECTIVE" EXCHANGE RATE(MERM)(INDEX NO.)	5

101 FOREIGN EXCHANGE RATE: GERMANY(DEUTSCHE MARK PER U.S.\$)	5
102 FOREIGN EXCHANGE RATE: SWITZERLAND(SWISS FRANC PER U.S.\$)	5
103 FOREIGN EXCHANGE RATE: JAPAN (YEN PER U.S.\$)	5
104 FOREIGN EXCHANGE RATE: CANADA(CANADIAN \$ PER U.S.\$)	5
105 INTEREST RATE: U.S.TREASURY CONST MATURITIES,5-YR.(% PER ANN,NSA)	2
106 INTEREST RATE: U.S.TREASURY CONST MATURITIES,10-YR.(% PER ANN,NSA)	2
107 BOND YIELD: MOODY'S AAA CORPORATE(%PER ANNUM)	2
108 BOND YIELD: MOODY'S BAA CORPORATE(%PER ANNUM)	2
109 SECONDARY MARKET YIELDS ON FHA MORTGAGES(%PER ANNUM)	2
110 Spread FYCP -FYFF	1
111 Spread FYGM3-FYFF	1
112 Spread FYGM6-FYFF	1
113 Spread FYGT1-FYFF	1
114 Spread FYGT5-FYFF	1
115 Spread FYGT10-FYFF	1
116 Spread FYAAAC-FYFF	1
117 Spread FYBAAC - FYFF	1
118 Spread FYFHA-FYFF	1
119 MONEY STOCK:M1(CURR,TRAV.CKS,DEM DEP,OTHER CK'ABLE DEP)(BIL\$,SA)	6
120 MONEY STOCK:M2(M1+O'NITE RPS,EURO\$,G/P&B/D MMMFS&SAV&SM TIME DEP)	6
121 MONEY STOCK:M3(M2+LG TIME DEP,TERM RP'S&INST ONLY MMMFS)(BIL\$,SA)	6
122 MONEY SUPPLY-M2 IN 1992 DOLLARS (BCI)	5
123 MONETARY BASE,ADJ FOR RESERVE REQUIREMENT CHANGES(MIL\$,SA)	6
124 DEPOSITORY INST RESERVES:TOTAL,ADJ FOR RESERVE REQ CHGS(MIL\$,SA)	6
125 DEPOSITORY INST RESERVES:NONBORROW+EXT CR,ADJ RES REQ CGS(MIL\$,SA)	6
126 NAPM COMMODITY PRICES INDEX (PERCENT)	1
127 PRODUCER PRICE INDEX: FINISHED GOODS(82=100,SA)	6
128 PRODUCER PRICE INDEX: FINISHED CONSUMER GOODS(82=100,SA)	6
129 INDEX OF SENSITIVE MATERIALS PRICES (1990=100)(BCI-99A)	6
130 CPI-U: ALL ITEMS(82-84=100,SA)	6
131 CPI-U: APPAREL & UPKEEP(82-84=100,SA)	6
132 CPI-U: TRANSPORTATION(82-84=100,SA)	6
133 CPI-U: MEDICAL CARE(82-84=100,SA)	6
134 CPI-U: COMMODITIES(82-84=100,SA)	6
135 CPI-U: DURABLES(82-84=100,SA)	6
136 CPI-U: SERVICES(82-84=100,SA)	6
137 CPI-U: ALL ITEMS LESS FOOD (82-84=100,SA)	6
138 CPI-U: ALL ITEMS LESS SHELTER (82-84=100,SA)	6
139 CPI-U: ALL ITEMS LESS MIDICAL CARE (82-84=100,SA)	6
140 PCE,IMPL PR DEFL:PCE (1987=100)	6
141 PCE,IMPL PR "DEFL:PCE;" DURABLES (1987=100)	6
142 PCE,IMPL PR "DEFL:PCE;" NONDURABLES (1987=100)	6
143 PCE,IMPL PR "DEFL:PCE;" SERVICES (1987=100)	6
144 AVG HR EARNINGS OF CONSTR WKRS: CONSTRUCTION (\$,SA)	6
145 AVG HR EARNINGS OF PROD WKRS: MANUFACTURING (\$,SA)	6
146 U. OF MICH. INDEX OF CONSUMER EXPECTATIONS(BCD-83)	1