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ABSTRACT

Price-Level Determination Under Dispersed Information and Monetary Policy*

This paper considers the determination of aggregate price level under dispersed information. Central Bank sets policy in response to its noisy measure of the price level, and each agent makes its decisions by observing a subset of data. Information revealed to the agents and Bank is determined endogenously. It is shown that the aggregate state of the economy is not revealed perfectly to anybody but this economy behaves as if it is a representative-agent economy in which the representative agent has perfect information while the Bank has partial information. The Bank has information set affects fluctuations in the price level through its effect on policy.

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1 Introduction

It is almost impossible to directly observe everything in a national economy. Many kinds of goods are traded, and different industries face different productivity shocks and demand shocks. Different households consume different basket of goods, and earn different income. Most agents make their decisions by observing only a small subset of economic variables. On the contrary, Central Bank uses a large amount of resources to collect data. This is because the Bank's policy is concerned with the stabilisation of *aggregate* economy, such as aggregate output and aggregate price levels. Compared with individual firms and households, it seems that the Central Bank contributes to aggregating dispersed information. However, the Bank is NOT the only aggregator of information. Prices aggregate information, as is famously shown by Grossman (1981, 1989). Does the Bank play an additional role in information aggregation beyond prices? Which kind of uncertainty remains after the Bank and prices aggregate information? How is the aggregate price level determined when both the Bank and private agents cannot directly observe aggregate states of the economy?

We study those issues by constructing a simple model of price-level determination in an island economy. The model can be interpreted as an island-economy version of the model of price-level determination in Chapter 2 of Woodford (2003). We consider an endowment economy in which there is continuum of islands producing different kinds of goods. Each island is subject to supply and demand shocks. Agents consume only a subset of the variety of goods. They observe information (for example, prices and quantities) about the goods they consume, but they do not observe information about the goods they do not consume. There is a Central Bank in the economy. The Bank collects data and construct (noisy) indicators of aggregate activities, and sets the nominal interest rate in response to its noisy measure of aggregate price level. We study how the aggregate price level is determined in this economy.

It is shown that, the equilibrium of our island economy is very similar to the equilibrium of a representative-agent economy in which the representative agent has perfect information but the Central Bank does not. In other words, even after prices aggregate information, the uncertainty facing the Bank remains in a non-trivial way. As a result, the quality of the Bank's information has important effects on fluctuations in the aggregate price level through monetary policy.

This paper offers a micro-foundation of models of monetary policy under data uncertainty. Typically, the existing literature on monetary policy under data uncertainty takes a reduced-form approach. In other words, much work has been built on models that have micro-foundation for individual optimisation under perfect information, and uncertainty is added to those models in a rather ad-hoc way. For example, Svensson and Woodford (2003) assumes that private agents and the Bank have symmetric but partial information, while Aoki (2003, 2006) and Svensson and Woodford (2004) assume asymmetric information. Compared to those papers, in this paper the structure of uncertainty is determined in equilibrium. The equilibrium properties of our model are similar to those of the reduced-form approach that assumes perfect information by private sector but imperfect information by Central Bank.

In terms of the modelling strategy, this paper is also related to Lorenzoni (2005). He constructs an island-economy model in which agents' expectations about aggregate productivity shocks generate business cycles. Our modelling strategy is similar to his. He focuses on the implications of heterogenous information and strategic interaction among monopolistically competitive firms. Our paper focuses more on monetary policy issues by assuming endowment economy and abstracting from strategic interaction among agents.

We focus on information aggregation by the Bank v.s. prices, and how the Bank's information set affects the fluctuations in the price level.

Section 2 presents the model. Section 3 computes equilibrium price level in three cases. Section 3.2 considers the case of perfect information as a benchmark. Section 3.3 considers a special case in which prices reveals the aggregate state of the economy. Section 3.4 characterises the general case, which is the main part of the paper. Section 4 extends the analysis of Section 3.4. Section 5 concludes.

2 An island-economy model of price-level determination

2.1 Model

The model is an island model with stochastic endowment and demand shock. The economy consists of continuum of islands with mass 1. In each island there is mass 1 of Lucas trees that produce island-specific goods. Stochastic fluctuations in production represent supply shock. In order to motivate demand shock, we assume that in each island there is local government that consumes island-specific goods. Agents in each island visits a discrete number n of islands and consume goods and hold trees in those islands. They do not observe the variables of the islands they do not visit. This assumption captures the idea that information is dispersed. There are two kinds of assets in the economy: the trees and a risk-free nominal bond. Assume that nominal bond is zero in net supply. There is Central Bank in the economy which sets the nominal interest rate on the risk-free bond. The markets in this economy are the markets for each good, each tree and the risk-free bond. The model can be interpreted as an island-economy version of the model of price-level determination in Chapter 2 of Woodford (2003).

Let us present the model in detail. Each island is indexed by $i \in [0, 1]$. In each island, there is measure 1 of agents and Lucas trees. Tree in island i produces island-specific goods i, and its production at time t per unit of tree is denoted by $y_t(i)$. All agents in island i are assumed to be identical. An agent in island i consumes his consumption basket that contains only n kinds of goods. Its consumption basket is denoted by J_i. For simplicity, it is assumed that J_i is constant over time.

The preference of agent i is defined as

$$E_0^i \sum_{t=0}^{\infty} \beta^t \log C_t^i, \quad 0 < \beta < 1, \tag{1}$$

where C_t^i is defined as the following Cobb-Douglas aggregator

$$C_t^i = \frac{1}{n} \prod_{j \in J_i} c_t^i(j)^{1/n}. \tag{2}$$

Operator E^i is the expectation operator conditional on the information set of agents in island i. We will define the information set in detail in Section 2.3. Agents in island i holds trees of island $j \in J_i$ and the risk-free nominal bond.[1]

In order to introduce demand shock in the economy, we assume that there is local government in each island. Assume that the local government in island i consumes a fraction $1 - \gamma_t(i)$ of total output $y_t(i)$. Then, the government expenditure in each period is given by $(1 - \gamma_t(i))y_t(i)$. This represents island-specific demand shock. Furthermore, assume that each local government finances its expenditure by collecting income tax

[1]For simplicity of notation, we assume that the consumption basket and portfolio of assets are both equal to J_i. It is possible to relax this assumption without changing the results below.

$\tau_t(i)y_t(i)$. For simplicity, assume balanced budget:

$$\tau_t(i)p_t(i)y_t(i) = \big(1-\gamma_t(i)\big)p_t(i)y_t(i).$$

This implies $\tau_t(i) = 1-\gamma_t(i)$.

Under this assumption, the flow budget constraint of agent i is given by

$$\sum_{j\in J_i} p_t(j)c_t^i(j) + \sum_{j\in J_i} S_{t+1}^i(j)q_t(j) + B_{t+1}^i = \sum_{j\in J_i} S_t^i(j)\big[q_t(j) + p_t(j)\gamma_t(j)y_t(j)\big] + R_tB_t^i \equiv W_t^i, \tag{3}$$

where $p_t(j)$ is the price of good j, $q_t(j)$ is the price of tree j, B_t^i is the holdings of the nominal bond of agent i at the beginning of period t, R_t is the risk-free nominal interest rate, and $S_t^i(j)$ is the holdings of tree j of agent i. Finally, $p_t(j)\gamma_t(j)y_t(j)$ represents after-tax nominal dividend from holding one unit of tree j. W_t^i represents the total wealth of agent i at time t. At time 0, it is assumed that an agent in island i is endowed with one unit of tree i, that is, $S_0^i(i) = 1$ and $S_0^i(j) = 0, i \neq j$.

Agent i maximises (1) subject to (2) and (3). As is well known, the optimal consumption decision for each good is given by

$$p_t(j)c_t^i(j) = \frac{1}{n}P_t^iC_t^i, \tag{4}$$

where

$$P_t^i = \prod_{j\in J_i} p_t(j)^{1/n}, \tag{5}$$

and the two Euler equations are

$$\frac{1}{C_t^i} = \beta E_t^i\left[\frac{1}{C_{t+1}^i}R_t\frac{P_t^i}{P_{t+1}^i}\right], \tag{6}$$

$$\frac{1}{C_t^i} = \beta E_t^i \left[\frac{1}{C_{t+1}^i} \frac{q_{t+1}(j) + \gamma_{t+1}(j) p_{t+1}(j) y_{t+1}(j)}{q_t(j)} \right] \quad \forall j \in J_i. \tag{7}$$

Equation (4) is an implication of Cobb-Douglas specification (2). Under this assumption, the expenditure share of each good is equal to $1/n$. Finally, the consumption function is given by

$$P_t^i C_t^i = (1 - \beta) W_t^i. \tag{8}$$

Equation (8) is an implication of log utility (1).[2] Under log utility, the agents spends a fraction $1 - \beta$ of its total wealth on current consumption. In Appendix A we will verify (8) holds in equilibrium.

2.2 Market equilibrium

Assume for symmetry that each island receives customers from n islands. In other words, the distribution of demand for each good is uniform. Let I_i be the set of islands that consume good i (and hold tree i). Since there is fixed supply 1 of Lucas trees in each island and nominal bond is zero in net supply, the equilibrium conditions for the asset markets are

$$\sum_{j \in I_i} S_t^j(i) = 1 \quad \forall i \tag{9}$$

and

$$\int_0^1 B_t^i di = 0. \tag{10}$$

Next, we construct the market clearing condition for each good. The total private demand for good i is given by $\frac{1}{n} \sum_{j \in I_i} P_t^j C_t^j$. Since we assumed that the local government

[2] See, for example, Sargent (1987), Chapter 3.

expenditure is given by $(1-\gamma_t(i))y_t(i)$, the market clearing condition is given by

$$p_t(i)\gamma_t(i)y_t(i) = \sum_{j\in I_i} \frac{1}{n} P_t^j C_t^j \quad \forall i, t. \tag{11}$$

In equilibrium, $\{c_t^j(i), p_t(i), q_t(i), S_t^i(j), B_t^i\}$ are determined in order to satisfy the market clearing conditions ((9), (10) and (11)) and the optimality conditions ((4), (6) and (7)), given the sequence of of nominal interest rate that is specified by the Central Bank. Here let us characterise the equilibrium of this economy intuitively. The details of derivation are given in Appendix A. In Appendix A, it is shown that the relative prices of any goods i and j is given by

$$\frac{p_t(i)}{p_t(j)} = \frac{\gamma_t(j)y_t(j)}{\gamma_t(i)y_t(i)}, \tag{12}$$

and the price of tree i is given by

$$q_t(i) = \frac{\beta}{1-\beta} p_t(i)\gamma_t(i)y_t(i). \tag{13}$$

Equations (12) and (13) imply that $q_t(i) = q_t(j)$ for any i, j. Equation (12) stems from the assumption of the Cobb-Douglas preference (2). Under (2), the nominal expenditure share of each goods is all equal to $1/n$. Equation (13) is an implication of log utility.[3] The portfolio decision regarding the trees is indeterminate as long as it satisfies the market clearing condition for tree i

$$\sum_{j\in I_i} S_t^j(i) = 1. \tag{14}$$

This is because all of the dividend stream is equal $(p_t(i)\gamma_t(i)y_t(i) = p_t(j)\gamma_t(j)y_t(j))$

[3] See, for example, Sargent (1987), Chapter 3.

in equilibrium and therefore all the trees are perfect substitutes. In this symmetric equilibrium, the optimal holding of the nominal bond is given by $B_t^i = 0$ for all agents. Therefore, (3), (12) and (14) imply that

$$P_t^i C_t^i = p_t(i)\gamma_t(i)y_t(i). \tag{15}$$

Substituting (15) into the Euler equation (6), we obtain

$$E_t^i \beta \left[\frac{p_t(i)\gamma_t(i)y_t(i)}{p_{t+1}(i)\gamma_{t+1}(i)y_{t+1}(i)} \right] R_t = 1. \tag{16}$$

Equation (16) represents the expectational IS equation for island i.

Finally, let us introduce monetary policy. Assume that the Central Bank visits m islands and collects data. Assume that $m > n$. This means that the Bank visits more islands than the private agents do. This represents the idea that the Bank has a better technology to aggregate data. Let J_b denote the set of islands the Bank visits. By visiting those islands the Central Bank observes $y_t(i), p_t(i), q_t(i)$ for $i \in J_b$. Let us define the observed aggregate price as

$$P_t^o = \prod_{i \in J_b} p_t(i)^{1/m}. \tag{17}$$

The observed aggregate output Y_t^o and the government expenditure Γ_t^o are also defined in the same way. Following Woodford (2003), monetary policy is characterised by an interest rate rule of the form

$$R_t = \Phi(P_t^o), \quad \Phi' > 0. \tag{18}$$

Equations (12), (16), and (18) determine the sequence of price levels in this economy, given the sequences of $y_t(i)$ and $\gamma_t(i)$. Alternatively, it is possible to consider an interest rate that responds to inflation rate. In this case the main results presented below will go through. In order to analyse the price-level determination further, we log-linearise the model around the steady state with constant prices.

2.3 Log-linearised model

We define the steady state of the economy as $\gamma_t(i) = \Gamma$, $y_t(i) = Y$, and $p_t(i) = 1$ for all t and all i. Here the price level is normalised to one without loss of generality. Then equation (16) implies that $R = \beta^{-1}$. In what follows, for any variable x_t, $\hat{x}_t$ is defined as $\hat{x}_t \equiv \log(x_t/x)$, where x is the steady-state value of x_t.

The log-linear approximation of the IS equation (16) for island i is given by

$$\begin{aligned} \hat{R}_t &= E_t^i \hat{p}_{t+1}(i) - \hat{p}_t(i) + E_t^i \big[\{\hat{y}_{t+1}(i) - \hat{y}_t(i)\} + \{\hat{\gamma}_{t+1}(i) - \hat{\gamma}_t(i)\} \big] \qquad (19) \\ &\equiv E_t^i \hat{p}_{t+1}(i) - \hat{p}_t(i) + \hat{r}_t^n(i), \end{aligned}$$

where

$$\hat{r}_t^n(i) \equiv E_t^i \big[\{\hat{y}_{t+1}(i) - \hat{y}_t(i)\} + \{\hat{\gamma}_{t+1}(i) - \hat{\gamma}_t(i)\} \big] \qquad (20)$$

represents the natural interest rate for island i. In other words, $\hat{R}_t = \hat{r}_t^n(i)$ at all times is consistent with $\hat{p}_t(i) = 0$. The log-linear approximation of the relative-price equation (12) is given by

$$\hat{p}_t(i) + \hat{y}_t(i) + \hat{\gamma}_t(i) = \hat{p}_t(j) + \hat{y}_t(j) + \hat{\gamma}_t(j) \quad \forall i, j. \qquad (21)$$

The log-linearised asset price equation is given by

$$\hat{q}_t(i) = \hat{p}_t(i) + \hat{y}_t(i) + \hat{\gamma}_t(i) \quad \forall i. \tag{22}$$

Now we consider how each island-specific variables are related to aggregate variables. We assume that supply shock and demand shock in each island consist of a common part and idiosyncratic part:

$$\hat{y}_t(i) = \hat{Y}_t + \varepsilon_{y,t}(i), \quad \hat{\gamma}_t(i) = \hat{\Gamma}_t + \varepsilon_{\gamma,t}(i). \tag{23}$$

Terms $\varepsilon_{y,t}(i)$ and $\varepsilon_{\gamma,t}(i)$ respectively represent idiosyncratic supply and demand shock in island i. Furthermore we assume that those are *i.i.d.* normal across islands and across time, so that $\int_0^1 \varepsilon_{y,t}(i)di = \int_0^1 \varepsilon_{y,t}(i)di = 0$. Then we have

$$\hat{P}_t = \int_0^1 \hat{p}_t(i)di, \quad \hat{Y}_t = \int_0^1 \hat{y}_t(i)di, \quad \hat{\Gamma}_t = \int_0^1 \hat{\gamma}_t(i)di.$$

Furthermore, (21) and (22) imply that

$$\hat{P}_t + \hat{Y}_t + \hat{\Gamma}_t = \hat{p}_t(i) + \hat{y}_t(i) + \hat{\gamma}_t(i) \quad \forall i,$$

and

$$\hat{q}_t(i) = \hat{p}_t(i) + \hat{y}_t(i) + \hat{\gamma}_t(i) = \hat{P}_t + \hat{Y}_t + \hat{\Gamma}_t \equiv \hat{q}_t \quad \forall i. \tag{24}$$

Then the relative price of island i relative to the aggregate price level can be written as

$$\begin{aligned}\hat{p}_t(i) - \hat{P}_t &= \big(\hat{Y}_t - \hat{y}_t(i)\big) + \big(\hat{\Gamma}_t - \hat{\gamma}_t(i)\big) \\ &= -\big(\varepsilon_{y,t}(i) + \varepsilon_{\gamma,t}(i)\big) \\ &\equiv \varepsilon_{p,t}(i). \end{aligned} \tag{25}$$

Therefore the relative price depends only on idiosyncratic demand and supply shock.

Finally, the linearised monetary policy rule is given by

$$\hat{R}_t = \phi_p \hat{P}_t^o, \quad \phi_p > 0. \tag{26}$$

Since we assume that the Central Bank visits m islands in J^b, we can express $\hat{P}_t^o$ as

$$\hat{P}_t^o = \hat{P}_t + \varepsilon_{p,t}, \tag{27}$$

where

$$\begin{aligned}\varepsilon_{p,t} &\equiv -\frac{1}{m}\sum_{i \in J_b} \left[\varepsilon_{y,t}(i) + \varepsilon_{\gamma,t}(i)\right] \\ &\equiv -(\varepsilon_{y,t} + \varepsilon_{\gamma,t})\end{aligned} \tag{28}$$

represents the Central Bank's measurement error of the aggregate price level. Similarly, terms $\varepsilon_{y,t} = \hat{Y}_t^o - \hat{Y}_t$ and $\varepsilon_{\gamma,t} = \hat{\Gamma}_t^o - \hat{\Gamma}_t$ respectively represent the Bank's measurement errors of aggregate supply and demand shock. Note that, since each island has measure zero in the aggregate economy, those measurement errors are independent of $(\hat{P}_t, \hat{Y}_t, \hat{\Gamma}_t)$.

The equilibrium prices $(\hat{P}_t, \hat{p}_t(i))$ and asset prices $\hat{q}_t$ are determined by (19), (24), (25), (26) and (27), given the stochastic processes of exogenous variables ($\hat{Y}_t$, $\hat{y}_t(i)$, $\hat{\Gamma}_t$, $\hat{\gamma}_t(i)$). For concreteness we assume the following stochastic processes for the aggregate shocks.

$$\hat{Y}_t = \delta \hat{Y}_{t-1} + u_{y,t}, \quad 0 < \delta < 1, \tag{29}$$

$$\hat{\Gamma}_t = \eta \hat{\Gamma}_{t-1} + u_{\gamma,t}, \quad 0 < \eta < 1. \tag{30}$$

Furthermore, we assume that all of the innovations in the economy are *i.i.d.* normal. Specifically, we assume that

$$\begin{aligned} \varepsilon_{y,t}(i) \sim N(0, s_y^2), \qquad \varepsilon_{\gamma,t}(i) \sim N(0, s_\gamma^2) \quad \forall i, \\ u_{y,t} \sim N(0, \sigma_y^2), \qquad u_{\gamma,t} \sim N(0, \sigma_\gamma^2). \end{aligned} \tag{31}$$

Finally, let us define the information set of agents in island i. We assume that the agents in island i observes their own variables ($\hat{p}_t(i)$, $\hat{y}_t(i)$, $\hat{\gamma}_t(i)$) and the variables of islands they visit ($\hat{p}_t(j)$, $\hat{y}_t(j)$, $\hat{\gamma}_t(j)$ for $j \in J_i$), and the noisy aggregate data collected by the Central Bank ($\hat{P}_t^o$, $\hat{Y}_t^o$, $\hat{\Gamma}_t^o$). Without loss of generality, assume for simplicity of notation that $i \in J_i$. This means that the agent in island i consumes goods produced in island i. We assume that agents cannot distinguish aggregate and idiosyncratic part of shocks in each variable (23). The information set $\Omega_t(i)$ is defined by

$$\Omega_t(i) = \left\{ \left(\hat{p}_s(j), \hat{y}_s(j), \hat{\gamma}_s(j) \ \text{ for } j \in J_i\right), \ \left(\hat{P}_s^o, \hat{Y}_s^o, \hat{\Gamma}_s^o\right) \right\}_{s=0}^{t}.$$

The expectation operator E_t^i is conditional on $\Omega_t(i)$, the stochastic processes (23) and (27) - (30). Next section completely characterises the equilibrium.

3 Determination of aggregate price level under dispersed information

3.1 Equilibrium given private-sector expectations

Combining equations (25), (26) and (27), the interest-rate rule can be written as

$$\hat{R}_t = \phi_p \hat{p}_t(i) + \phi_p \left\{ -\varepsilon_{p,t}(i) + \varepsilon_{p,t} \right\}. \tag{32}$$

Similar to the representative-agent model of Woodford (2003), the price level is uniquely determined when $\phi_p > 0$. From (19) and (32), $\hat{p}_t(i)$ should satisfy

$$\begin{aligned} \hat{p}_t(i) = {} & \phi^{-1} \hat{r}_t^n(i) + \left(1 - \phi^{-1}\right) \left\{ \varepsilon_{p,t}(i) - \varepsilon_{p,t} \right\} \\ & + E_t^i \sum_{\tau=0}^{\infty} \phi^{-1-\tau} \left[\phi^{-1} \hat{r}_{t+1+\tau}^n(i) + \left(1 - \phi^{-1}\right) \left\{ \varepsilon_{p,t+1+\tau}(i) - \varepsilon_{p,t+1+\tau} \right\} \right], \end{aligned} \tag{33}$$

where $\phi \equiv (1 + \phi_p)$. Equation (33) shows that the price of island i depends on the expectations about its future natural interest rate, future idiosyncratic shocks and future measurement errors of the Central Bank. The expectations are conditional on the information set of island i. This is a generalisation of Chapter 2 of Woodford (2003) to our island economy. Each price $\hat{p}_t(i)$ also has to satisfy the relative-price equation (25).

Terms $\varepsilon_{p,t}(i)$ and $\varepsilon_{p,t}$ on the right hand side of (33) are not directly observable to the agents in island i. However, we can derive the equilibrium price in terms of the observable variables of the agents in island i. We use (25) and (28) to obtain

$$\varepsilon_{p,t}(i) - \varepsilon_{p,t} = \left\{ \hat{Y}_t^o - \hat{y}_t(i) \right\} + \left\{ \hat{\Gamma}_t^o - \hat{\gamma}_t(i) \right\}.$$

Substituting this to equation (33), we obtain the equilibrium price in terms of observables

$$\phi^{-1}\hat{p}_t(i) + (1-\phi^{-1})\left[\hat{q}_t - \hat{Y}_t^o - \hat{\Gamma}_t^o\right]$$
$$= \phi^{-1}\hat{r}_t^n(i) + E_t^i \sum_{\tau=0}^{\infty} \phi^{-1-\tau}\left[\phi^{-1}\hat{r}_{t+1+\tau}^n(i) + (1-\phi^{-1})\left\{\varepsilon_{p,t+1+\tau}(i) - \varepsilon_{p,t+1+\tau}\right\}\right]. \quad (34)$$

Now all terms of (34) are observable to agent i.

Equation (34) has an interesting implication for information aggregation by prices. Note that the relative price equation (21) implies that the relationship between any individual price $\hat{p}_t(i)$ and P_t^o are given by

$$\hat{p}_t(i) + \hat{y}_t(i) + \hat{\gamma}_t(i) = \hat{P}_t^o + \hat{Y}_t^o + \hat{\Gamma}_t^o. \quad (35)$$

We can use this equation to substitute out all of the idiosyncratic terms in (34) and express (34) in terms of $\hat{P}^o$. Furthermore, under the assumptions on the stochastic processes (23), (29) and (30) and the *i.i.d.* assumptions on the innovations, equation (34) becomes[4]

$$(1-\phi^{-1})\hat{P}_t^o + \phi^{-1}\hat{q}_t = E_t^i\big[A_y\hat{Y}_t + A_\gamma\hat{\Gamma}_t\big], \quad (36)$$

where

$$A_y \equiv \frac{\delta\phi^{-1}(1-\phi^{-1})}{1-\delta\phi^{-1}}, \quad A_\gamma \equiv \frac{\eta\phi^{-1}(1-\phi^{-1})}{1-\eta\phi^{-1}}.$$

Another expression of (36) is

$$\hat{P}_t^o + \phi^{-1}\big(\hat{Y}_t^o + \hat{\Gamma}_t^o\big) = E_t^i\big[A_y\hat{Y}_t + A_\gamma\hat{\Gamma}_t\big], \quad (37)$$

[4]Here we used the definition of the natural rate (20).

The right hand side of equation (36) must be identical across agents because the left hand side is common for all agents. This implies

$$E_t^i\left[A_y\hat{Y}_t + A_\gamma\hat{\Gamma}_t\right] = E_t^j\left[A_y\hat{Y}_t + A_\gamma\hat{\Gamma}_t\right] \quad \forall i, j.$$

We will analyse this equation in detail later. This is a result of information aggregation by prices. Even if each individual has asymmetric information, prices aggregate each information so that the information regarding the right hand side of (36) becomes identical for all agents. However, it is important to remark that this does *not* mean that $E_t^i\hat{Y}_t = E_t^j\hat{Y}_t$ and $E_t^i\hat{\Gamma}_t = E_t^j\hat{\Gamma}_t$. Further decomposition of the right hand side of (36) does depend on each information.

3.2 Special case 1: perfect information

As a special case, it is useful to consider the economy under perfect information. Assume that all of the agents can observe all shocks. This means that all of the agents can observe $\hat{Y}_t$ and $\hat{\Gamma}_t$. Then $E_t^i\hat{Y}_t = \hat{Y}_t$ and $E_t^i\hat{\Gamma}_t = \hat{\Gamma}_t$ for all i. Substituting these into (37), the equilibrium is given by

$$\hat{P}_t^o + \phi^{-1}\left(\hat{Y}_t^o + \hat{\Gamma}_t^o\right) = A_y\hat{Y}_t + A_\gamma\hat{\Gamma}_t. \tag{38}$$

The relative price of each good is given by (35). The economy is essentially identical to the one considered in Chapter 2 of Woodford (2003). When information is perfect, the model behaves like the one with the representative household facing only aggregate shocks.

3.3 Special case 2: fully-revealing equilibrium

As the second special case, consider the economy with only supply shock ($\hat{\gamma}_t(i) = \hat{\Gamma}_t = 0$), and assume that this is common knowledge.[5] In this case, (37) is given by

$$\hat{P}_t^o + \phi^{-1}\hat{Y}_t^o = A_y E_t^i \hat{Y}_t. \tag{39}$$

We will show that the equilibrium is given by

$$\hat{P}_t^o + \phi^{-1}\hat{Y}_t^o = A_y \hat{Y}_t. \tag{40}$$

Compare this with (38). This is an example of fully-revealing rational expectations equilibrium. In this equilibrium, the aggregate variables ($\hat{Y}_t$ and $\hat{P}_t$) are all revealed.

Now we verify that (40) is indeed the equilibrium. On one hand, given that the equilibrium $\hat{P}_t^o$ is determined by (40), $\hat{Y}_t$ can be identified even if it is not directly observable to any agents. This is because the left hand side of (40) is observable to all agents. Therefore we have

$$E_t^i \hat{Y}_t = \hat{Y}_t \quad \forall i. \tag{41}$$

On the other hand, when the private-sector expectations are given by (41), we can substitute this equation into (39) to compute the equilibrium. Then the equilibrium is given by (40). Therefore expectations and equilibrium are consistent with each other. The price of each goods is then given by

$$\hat{p}_t(i) = \hat{P}_t^o + \left\{ \hat{Y}_t^o - \hat{y}_t(i) \right\}.$$

[5] Alternatively we can assume that $\hat{y}_t(i) = \hat{Y}_t = 0$. The properties of the equilibrium analysed below remain exactly the same.

In this equilibrium, the Central Bank has no additional role in information aggregation beyond the market mechanism because the market mechanism fully reveals information about aggregate supply shock.

We have assumed that the Bank responds to its noisy measure of the aggregate price level (equation (26)). However, in this special case, it is possible for the central Bank to respond to the true aggregate price level ($\hat{P}_t$) even if it is not directly observable. To see this, assume that the equilibrium is fully revealing so that $\hat{P}_t$ and $\hat{Y}_t$ are identified (we will verify it later), and that monetary policy is given by

$$\hat{R}_t = \phi_p \hat{P}_t. \tag{42}$$

Under (42), equilibrium is given by

$$\hat{P}_t + \phi^{-1}\hat{Y}_t = A_y \hat{Y}_t. \tag{43}$$

Compare this with (40). Now the both sides of equation (43) are not directly observable. However, (43) implies that

$$\hat{P}_t + \hat{Y}_t = (1 + A_y - \phi^{-1})\hat{Y}_t,$$

and since $\hat{P}_t + \hat{Y}_t = \hat{P}_t^o + \hat{Y}_t^o$ in this economy,[6] we obtain

$$\hat{P}_t^o + \hat{Y}_t^o = (1 + A_y - \phi^{-1})\hat{Y}_t.$$

Now the left hand side of this equation is observable and therefore $\hat{Y}_t$ can be fully

[6] In order to derive this, set $\hat{\gamma}_t(i) = \hat{\Gamma}_t = 0$ in equation (35) and integrate over [0,1].

identified in the equilibrium. Then the aggregate price level $\hat{P}_t$ is also identified by (43). Now we have verified that $\hat{P}_t$ is fully revealed and therefore the Bank can choose the policy according to (42) even if the Bank cannot visit all of the island (i,e., $\hat{P}_t$ is not directly observable). Notice again that the ability for the Central Bank to collect the aggregate data does not affect the nature of the equilibrium. The market mechanism is enough to aggregate information.

3.4 General case

Now we analyse the general case where we have both demand and supply shocks. Again equation (36) must hold in equilibrium for any i. We have two unobservable aggregate variables, namely, $\hat{Y}_t$ and $\hat{\Gamma}_t$. In this general case, equilibrium is not fully revealing, and it is necessary to solve the optimal filtering problem of agents. It is convenient to construct the 'summary' indicators for the agents in island i. As assumed in Section 3.2, the agents in island i observes the variables of the islands they visit and the noisy aggregate indicators released by the Central Bank.

Define the summary indicators for the agents in island i as

$$\bar{Y}_t^o(i) \equiv \frac{1}{n+m}\left[\sum_{j\in J_i}\hat{y}_t(j) + m\hat{Y}_t^o\right], \quad \bar{\Gamma}_t^o(i) \equiv \frac{1}{n+m}\left[\sum_{j\in J_i}\hat{\gamma}_t(j) + m\hat{\Gamma}_t^o\right] \tag{44}$$

Using (23), equation (44) can be written as

$$\bar{Y}_t^o(i) = \hat{Y}_t + \bar{\varepsilon}_{y,t}(i), \quad \bar{\Gamma}_t^o(i) = \hat{\Gamma}_t + \bar{\varepsilon}_{\gamma,t}(i). \tag{45}$$

where

$$\bar{\varepsilon}_{y,t}(i) \equiv \frac{1}{n+m}\sum_{j\in J_i\cup J_b}\varepsilon_{y,t}(j), \quad \bar{\varepsilon}_{\gamma,t}(i) \equiv \frac{1}{n+m}\sum_{j\in J_i\cup J_b}\varepsilon_{\gamma,t}(j).$$

Terms $\bar{\varepsilon}_{y,t}$ and $\bar{\varepsilon}_{\gamma,t}$ can be interpreted as the measurement errors in those summary indicators. Under assumption (31), we have

$$\bar{\varepsilon}_{y,t}(i) \sim N\left(0,\ \left(s_y/(n+m)\right)^2\right), \quad \bar{\varepsilon}_{\gamma,t}(i) \sim N\left(0,\ \left(s_\gamma/(n+m)\right)^2\right). \tag{46}$$

Under the assumption that idiosyncratic part of shocks are *i.i.d* across islands, we can regard $\bar{Y}_t^o(i)$ as a single observable variable rather than treating $y_t(j)$ and $\hat{Y}_t^o$ separately.

To summarise, the equilibrium price level and relative prices are given by equation (37) and (35), and the expectations $(E_t^i\hat{Y}_t, E_t^i\hat{\Gamma}_t)$ are derived by the Kalman filter. The observation equations of the filtering problem are given by (37) and (45), and the transition equations of hidden states $(\hat{Y}_t, \hat{\Gamma}_t)$ are given by (29) and (30). Expectations and equilibrium must be consistent with each other, so the filtering and equilibrium are determined simultaneously.

Note that, in this case, there is no fully revealing rational expectations equilibrium unlike the special case considered in Section 3.2. The intuition for this result is simple. Suppose information is fully revealed so that $E_t^i\hat{Y}_t = \hat{Y}_t, E_t^i\hat{\Gamma}_t = \hat{\Gamma}_t$. Given those expectations, the equilibrium is determined as

$$\hat{P}_t^o + \phi^{-1}\big(\hat{Y}_t^o + \hat{\Gamma}_t^o\big) = A_y\hat{Y}_t + A_\gamma\hat{\Gamma}_t.$$

The observation equations of the agent in island i are given by the above equation and (45). However, from those equations the agent cannot identify $\hat{Y}_t$ and $\hat{\Gamma}_t$ separately.

However, the equilibrium of this economy is very similar to the fully-revealing equilibrium. Indeed, we will show that the equilibrium is given by

$$\hat{P}_t^o + \phi^{-1}\big(\hat{Y}_t^o + \hat{\Gamma}_t^o\big) = A_y\hat{Y}_t + A_\gamma\hat{\Gamma}_t. \tag{47}$$

Here it is important to stress that this does not imply that $E_t^i \hat{Y}_t = \hat{Y}_t$ and $E_t^i \hat{\Gamma}_t = \hat{\Gamma}_t$. It only implies that

$$E_t^i \left[A_y \hat{Y}_t + A_\gamma \hat{\Gamma}_t \right] = A_y \hat{Y}_t + A_\gamma \hat{\Gamma}_t \quad \forall i. \tag{48}$$

In other words, the sum of the two shocks are fully revealed but each shock is not. Now let us verify that (47) is indeed the equilibrium. Suppose that equilibrium is given by (47). Then, the rational expectations of $A_y \hat{Y}_t + A_\gamma \hat{\Gamma}_t$ is given by (48). When expectations are given by (48), then the equilibrium condition (37) implies (47). The details of the proof and the derivation of Kalman filter with respect to $E_t^i \hat{Y}_t$ and $E_t^i \hat{\Gamma}_t$ are given in Appendix B.

What does this imply? Compare (47) with (38). This economy behaves like the economy under perfect information. However, information is not fully revealed.[7] However, the price mechanism aggregates the information to the level

$$E_t^i \left[A_y \hat{Y}_t + A_\gamma \hat{\Gamma}_t \right] = E_t^j \left[A_y \hat{Y}_t + A_\gamma \hat{\Gamma}_t \right] = A_y \hat{Y}_t + A_\gamma \hat{\Gamma}_t \quad \forall i, j.$$

This is as if the uncertainty about aggregate shocks facing the private agents is fully removed. Even though each of $\hat{Y}_t$ and $\hat{\Gamma}_t$ are not revealed, $A_y \hat{Y}_t + A_\gamma \hat{\Gamma}_t$ is revealed and this is what matters to the determination of the price level.

On the contrary, the situation facing the Bank is somewhat different. In the special case analysed in Section 3.3, the Central Bank can identify $\hat{Y}_t$ and $\hat{P}_t$ separately. However, in this case, the Bank cannot identify $\hat{Y}$, $\hat{\Gamma}_t$ and $\hat{P}_t$ separately.[8] Therefore the uncertainty about the aggregate shocks facing the Bank still remains.

It is interesting to compare our results of this section with the existing literature

[7]Equations (B.4) and (B.5) in Appendix B show that $E_t^i \hat{Y}_t \neq \hat{Y}_t$ and $E_t^i \hat{\Gamma}_t \neq \hat{\Gamma}_t$. Furthermore, $E_t^i \hat{Y}_t \neq E_t^j \hat{Y}_t$ and $E_t^i \hat{\Gamma}_t \neq E_t^j \hat{\Gamma}_t$ for $i \neq j$ because different agents receive different information.

[8]Nor can private agents identify these variables, but it does not matter for their decisions.

on monetary policy under data uncertainty. Much of the existing literature takes a reduced-form approach, namely, uncertainty regarding aggregate shocks is added into the otherwise standard monetary model with the representative agent. The information sets of the Central Bank and private agents are assumed in a rather ad-hoc way. For example, Aoki (2003, 2006) and Svensson and Woodford (2004) assume that the private agents have perfect information about the aggregate shocks but the Bank does not. On the contrary Svensson and Woodford (2003) assumes symmetric partial information between the Bank and the private agents. The island model we consider here behaves similarly to the models considered in Aoki (2003, 2006) and Svensson and Woodford (2004). In the equilibrium, information is almost perfectly revealed to private agents. Even though the two shocks are not identified separately, a linear combination of the demand and supply shocks is revealed in the equilibrium. However, the fact that those two shocks are not identified has an important implications for monetary policy, because the Bank must respond to those shocks differently. In the next section, we will consider such an example.

4 An application: price-level determination with Central Bank filtering

In the previous two sections, the Bank's policy is given by (26) that just responds to its noisy measure of the aggregate price level. In this section, we briefly discuss an example in which the Bank's policy depends on its estimate of the current state of the economy. This example is motivated by the fact that, in reality, the Bank need to respond to various economic shocks. As is discussed in Woodford (2003), keeping track of the natural interest rate is one of the important features of good monetary policy that seeks price stability. Therefore, let us consider the following simple rule

$$\hat{R}_t^n = \phi_p \hat{P}_t^o + E_t^b \hat{r}_t^n, \tag{49}$$

where $E_t^b \hat{r}_t$ represents the Bank's estimate of the aggregate natural rate, defined by

$$\hat{r}_t^n \equiv (\delta - 1)\hat{Y}_t + (\eta - 1)\hat{\Gamma}_t. \tag{50}$$

It is assumed that the Bank announces $E_t^b \hat{r}_t^n$, therefore it is observable to the private agents. When the policy is given by (49), the equation that corresponds to (34) is

$$\begin{aligned} &\phi^{-1}\hat{p}_t(i) + (1-\phi^{-1})\left[\hat{q}_t - \hat{Y}_t^o - \hat{\Gamma}_t^o\right] \\ =&\phi^{-1}\left(\hat{r}_t^n(i) - E_t^b \hat{r}_t^n\right) \\ &+ E_t^i \sum_{\tau=0}^{\infty} \phi^{-1-\tau}\left[\phi^{-1}\left(\hat{r}_{t+1+\tau}^n(i) - E_{t+1+\tau}^b \hat{r}_{t+1+\tau}^n\right) + (1-\phi^{-1})\left\{\varepsilon_{p,t+1+\tau}(i) - \varepsilon_{p,t+1+\tau}\right\}\right]. \end{aligned} \tag{51}$$

Notice that the relationship between each island-specific natural rate and the aggregate natural rate is given by

$$\hat{r}_t^n(i) = E_t^i \hat{r}_t^n + (E_t^i \hat{Y}_t - \hat{Y}_t) + (E_t^i \hat{\Gamma}_t - \hat{\Gamma}_t) - \varepsilon_{y,t}(i) - \varepsilon_{\gamma,t}(i). \tag{52}$$

Using (52) and taking exactly the same step as Section 3.1, equation (51) can be written as

$$\begin{aligned} &\hat{P}_t^o + \phi^{-1}\left[\hat{Y}_t^o + \hat{\Gamma}_t^o\right] \\ =&\phi^{-1}\left(\hat{r}_t^n - E_t^b \hat{r}_t^n\right) + E_t^i \sum_{\tau=0}^{\infty} \phi^{-1-\tau}\left[\phi^{-1}\left(\hat{r}_{t+1+\tau}^n - E_{t+1+\tau}^b \hat{r}_{t+1+\tau}^n\right)\right] + \phi^{-1}\left(E_t^i \hat{Y}_t + E_t^i \hat{\Gamma}_t\right). \end{aligned} \tag{53}$$

Again it is useful to consider the perfect information case as a benchmark. Under perfect information both by private agents and the Central Bank, we have

$$E_t^b \hat{r}_t^n = E_t^i \hat{r}_t^n = \hat{r}_t^n \quad \forall t, i,$$

and

$$E_t^i \hat{Y}_t = \hat{Y}_t, \quad E_t^i \hat{\Gamma}_t = \hat{\Gamma}_t.$$

In this case, equation (53) implies

$$\hat{P}_t^o + \phi^{-1} \left[\hat{Y}_t^o + \hat{\Gamma}_t^o \right] = \phi^{-1} \left(\hat{Y}_t + \hat{\Gamma}_t \right)$$

As a further special case, when monetary policy is given by

$$\hat{R}_t^n = \phi_p \hat{P}_t + E_t^b \hat{r}_t^n,$$

the price level is completely stabilised:

$$\hat{P}_t = 0.$$

This is again consistent with Chapter 2 of Woodford (2003) that uses the representative household framework.

Now consider imperfect information. Here, let us consider the simplest example in which both of the Bank and private agents have perfect information about the past state of the economy but imperfect information about the current states, as in Aoki (2003). More specifically, we assume that the Bank can collect the time-t data from only a finite number of islands at time t, but can collect all time $t-1$ data. In other words, it takes two periods for the Bank to visit all of the islands. This assumption captures data revision process — the Bank has better information regarding the past state of the economy. It is assumed that the Bank publishes its data so that the private agents also share this information. That implies that $\hat{Y}_{t-1}$ and $\hat{\Gamma}_{t-1}$ are both known to the Central Bank and private agents at time t. Under this assumption, since

$$E^b_{t+\tau}\hat{r}^n_{t+\tau} = \delta^\tau(\delta-1)\hat{Y}_{t+\tau-1} + \eta^\tau(\eta-1)\hat{\Gamma}_{t+\tau-1} + E^b_{t+\tau}\varepsilon_{y,t+\tau} + E^b_{t+\tau}\varepsilon_{\gamma,t+\tau}, \quad \tau \geq 1,$$

and since $\varepsilon_{y,t+\tau}$ and $\varepsilon_{\gamma,t+\tau}$ are white noise, the private-sector expectation about the Bank's future estimate of the natural rate is given by

$$E^i_t E^b_{t+\tau}\hat{r}^n_{t+\tau} = E^i_t E^b_t E^b_{t+\tau}\hat{r}^n_{t+\tau} = E^i_t\left[\delta^\tau(\delta-1)\hat{Y}_{t-1} + \eta^\tau(\eta-1)\hat{\Gamma}_{t-1}\right] = E^i_t\hat{r}^n_{t+\tau}.$$

Therefore, equation (53) becomes[9]

$$\hat{P}^o_t + \phi^{-1}\left[\hat{Y}^o_t + \hat{\Gamma}^o_t + E^b_t\hat{r}^n_t\right] = \phi^{-1}\left[E^i_t[\delta\hat{Y}_t + \eta\hat{\Gamma}_t]\right]. \tag{54}$$

By taking exactly the same step as in Section 3.4, we can show that equilibrium given $E^b_t\hat{r}^n_t$ is given by

$$\hat{P}^o_t + \phi^{-1}(\hat{Y}^o_t + \hat{\Gamma}^o_t + E^b_t\hat{r}^n_t) = \phi^{-1}(\delta\hat{Y}_t + \eta\hat{\Gamma}_t). \tag{55}$$

Again, a linear combination $\delta\hat{Y}_t+\eta\hat{\Gamma}_t$ is revealed in equilibrium so that it is similar to the equilibrium of the model in which the representative household has perfect information. This equilibrium is very similar to that considered in Aoki (2003) which is based on a reduced-form approach. However, as in Section 3.4, the agents cannot identify $\hat{Y}_t$ and

[9] Here we used (50) to substitute out $\hat{r}^n_t$.

$\hat{\Gamma}_t$ separately.

Finally, we compute the equilibrium and the Bank's filtering $E_t^b \hat{r}_t^n$. Those are determined simultaneously. The observation equations for the Bank are given by

$$Z_t^b \equiv \hat{P}_t^o + \phi^{-1}(\hat{Y}_t^o + \hat{\Gamma}_t^o + E_t^b \hat{r}_t^n) = \phi^{-1}(\delta \hat{Y}_t + \eta \hat{\Gamma}_t), \tag{56}$$

$$\hat{Y}_t^o = \hat{Y}_t + \varepsilon_{y,t}, \tag{57}$$

$$\hat{\Gamma}_t^o = \hat{\Gamma}_t + \varepsilon_{\gamma,t}, \tag{58}$$

where $E_t^b \hat{r}_t^n \equiv E_t^b \big[(\delta - 1)\hat{Y}_t + (\eta - 1)\hat{\Gamma}_t\big]$.

For simplicity of notation, for any variable X_t define 'tilde' variables as

$$\tilde{X}_t \equiv X_t - E_{t-1}^b X_t.$$

For example, $\widetilde{E_t^b Y_t} \equiv E_t^b \hat{Y}_t - E_{t-1}^b \hat{Y}_t$. Taking exactly the same procedure as the the private-sector filtering derived in Appendix B, the Bank's filtering takes the form:

$$\widetilde{E_t^b Y_t} = \kappa_y^b \tilde{Y}_t^o + \kappa_\gamma^b \tilde{\Gamma}_t^o + \kappa_z^b \tilde{Z}_t, \tag{59}$$

where κ_y^b, κ_γ^b, κ_z^b are the Kalman gain of the Bank's filtering problem. From (56) we have

$$\widetilde{E_t^b \Gamma_t} = \frac{1}{\eta}\left(\phi \tilde{Z}_t - \delta \widetilde{E_t^b Y_t}\right).$$

Then the Bank's estimate of the natural rate is given by

$$\widetilde{E_t^b \hat{r}_t^n} = \frac{\delta - \eta}{\eta} \widetilde{E_t^b Y_t} + \frac{\eta - 1}{\eta} \phi \tilde{Z}_t. \tag{60}$$

Substituting equation (59) into (60), we can write $\widetilde{E_t^b \hat{r}_t^n}$ in terms of observables as

$$\widetilde{E_t^b \hat{r}_t^n} = \left(\frac{\delta - \eta}{\eta}\right) \left(\kappa_y^b \tilde{Y}_t^o + \kappa_\gamma^b \tilde{\Gamma}_t^o\right) + \left(\frac{\delta - \eta}{\eta} \kappa_z^b + \frac{\eta - 1}{\eta} \phi\right) \tilde{Z}_t. \tag{61}$$

Now we can substitute (61) into equilibrium equation (55) and solve that equation for $\tilde{P}_t^o$ as:

$$\tilde{P}_t^o = -\phi^{-1} \left[\left\{1 + \frac{\delta - \eta}{\eta} \kappa_y^b\right\} \tilde{Y}_t^o + \left\{1 + \frac{\delta - \eta}{\eta} \kappa_\gamma^b\right\} \tilde{\Gamma}_t^o \right.$$
$$\left. + \left\{\frac{\eta - 1}{\eta} + \phi^{-1} \frac{\delta - \eta}{\eta} \kappa_z^b\right\} (\delta \tilde{Y}_t + \eta \tilde{\Gamma}_t) \right]. \tag{62}$$

Equation (62) gives a complete characterisation of the aggregate price level in our example. The equilibrium price level depends explicitly on the Bank's indicators $(\hat{Y}_t^o, \hat{\Gamma}_t^o)$ through its effects on $E_t^b \hat{r}_t^n$. When $\hat{Y}_t^o$ and $\hat{\Gamma}_t^o$ become noisy, the Bank's estimate of the aggregate natural rate becomes noisy, and this fact destabilises the aggregate price level. In the extreme case in which the Bank's data collection procedure becomes very efficient (i.e., $(\hat{Y}_t^o, \hat{\Gamma}_t^o)$ converges to $(\hat{Y}_t, \hat{\Gamma}_t)$), we observe that $\kappa_y^b \to 1$, $\kappa_\gamma^b \to 0$, $\kappa_z^b \to 0$. Then equation (62) implies that

$$\hat{P}_t^o \to \hat{P}_t \to 0,$$

This is consistent with Aoki (2003).[10]

[10]More precisely, equation (62) becomes $\tilde{P}_t \to 0$. $\hat{P}_t \to 0$ implies $\tilde{P}_t \to 0$. See, also Aoki (2003).

5 Conclusion

This paper analyses the determination of aggregate price level in an endowment economy with dispersed information. Our model endogenously determines what uncertainty the Central Bank and private agents face after prices aggregate information in equilibrium.

The next step is to extend our model to a production economy. This extension would enable us to relate our model to the literature on 'heterogenous information and effects of monetary policy'. This line of literature has its root on Lucas (1972). More recently, Amato and Shin (2003), Hellwig (2002), Lorenzoni (2005), Ui (2003) and Woodford (2002) emphasise the role of strategic interactions among firms and higher-order expectations. While they analyse the behaviour of firms rigorously, they use some reduced-form assumptions on the information set of the aggregate demand side and central Bank.[11] For example, Amato and Shin (2003) assume that firms have imperfect information while the Bank and the households have perfect information. It would be interesting to examine whether those assumptions will arise in equilibrium when we extend our model to a production economy. Also, as is analysed in Amato and Shin (2003), the role of public information would be an interesting topic in our framework.

Finally, it would be interesting to examine the robustness of our results. In this paper we used specific functional forms: log utility (1) and Cobb-Douglas consumption index (2). Under those preferences, the substitution and income effects are cancelled out perfectly, and that fact enables us to compute equilibrium relative prices and asset prices analytically. An implication of those assumption is that the asset price, $\hat{q}_t$, is a very powerful aggregator of information. In our economy, the asset price alone reveals information on aggregate nominal expenditure (net of government expenditure) $\hat{P}_t + \hat{\Gamma}_t +$

[11] Lorenzoni (2005) is an exception. He focuses more on the relationship between agents' expectations about productivity shock and business cycles.

$\hat{Y}_t$. Also, the fact that q_t is identical across islands means that the Lucas trees perfectly insure the private agents against idiosyncratic shocks. It would be interesting how much of our results would survive under a more general settings.

Appendix A Construction of equilibrium

Here we show the equilibrium of the model presented in Section 3.1. Suppose agent i maximises (1) subject to (2) and (3) and the initial condition $S_0^i(i) = 1$. Then, in equilibrium,

(a) the relative price of two goods i, j is given by (12) and asset prices are all equal;

(b) for all agents i, the optimal holdings of nominal bond is $B_t^i = 0$;

(c) for all agents i, the optimal portfolio of trees is indeterminate but satisfies (14);

(d) the price of tree j is given by (13);

(e) consumption function of agent i is given by (8).

What we need to show is that (a)-(e) satisfy the first order conditions (4)-(7), budget constraint (3) and the market equilibrium conditions (9)-(11). Under (a)-(c), we can express the consumption function (8) as

$$P_t^i C_t^i = (1-\beta)\left[q_t(i) + p_t(i)\gamma_t(i)y_t(i)\right]. \tag{A.1}$$

Under (a)-(c) and the initial condition $S_0^i(i) = 1$, the budget constraint (3) becomes (after using (4))

$$P_t^i C_t^i = p_t(i)\gamma_t(i)y_t(i). \tag{A.2}$$

Substitute (A.2) into (A.1), we obtain

$$q_t(i) = \frac{\beta}{1-\beta} p_t(i)\gamma_t(i)y_t(i), \tag{A.3}$$

which is equation (13).

Now we show that (a)-(e) satisfy the first order conditions and the market clearing conditions. It is obvious that under (b) the market clearing for the nominal bond is satisfied. That is, $\int_0^1 B_t^i di = 0$ where the net supply of bond is equal to zero. How about the market clearing for the trees? When $q_t(i) = q_t(j)$ for all i, j, all trees become perfect substitutes, so $S_t^i(j)$ becomes indeterminate as long as it satisfies $\sum_{j \in J_i} S_t^i(j) = 1$.[12] It is possible to construct $S_t^i(j)$ such that $\sum_{j \in J_i} S_t^i(j) = 1$ and the market clearing condition $\sum_{j \in I_i} S_t^j(i) = 1$. One such example is $S_t^i(j) = 1/n$ for all i, j. This clearly satisfies (9).

Next we consider the goods-market clearing. The market clearing condition for good i is given by equation (11):

$$p_t(i)\gamma_t(i)y_t(i) = \sum_{j \in I_i} \frac{1}{n} P_t^j C_t^j. \tag{11}$$

When consumption is given by (A.2) for all agents, equation (11) becomes

$$p_t(i)\gamma_t(i)y_t(i) = \sum_{j \in I_i} \frac{1}{n} p_t(j)\gamma_t(j)y_t(j). \tag{A.4}$$

Now it is clear that (a) satisfies (A.4).

Finally, we need to show that (A.1) satisfies the first-order condition (7):

$$\frac{1}{C_t^i} = \beta E_t^i \left[\frac{1}{C_{t+1}^i} \frac{q_{t+1}(j) + \gamma_{t+1}(j)p_{t+1}(j)y_{t+1}(j)}{q_t(j)} \right] \quad \forall j \in J_i. \tag{7}$$

[12]Later, we will show that the consumption function with $\sum_{j \in J_i} S_t^i(j) = 1$ and $B_t^i = 0$ satisfies the first order condition.

Since $q_t(i) = q_t(j)$ and $p_t(i)\gamma_t(i)y_t(i) = p_t(j)\gamma_t(j)y_t(j)$ for all i, j we only need to check that

$$\frac{1}{P_t^i C_t^i} = \beta E_t \left[\frac{1}{P_{t+1}^i C_{t+1}^i} \frac{q_{t+1}(i) + \gamma_{t+1}(i)p_{t+1}(i)y_{t+1}(i)}{q_t(i)} \right] \tag{A.5}$$

is satisfied. Under (A.1) and the portfolio decisions (b) and (c), $P_{t+1}^i C_{t+1}^i$ is given by

$$\begin{aligned} P_{t+1}^i C_{t+1}^i &= (1-\beta)W_{t+1}^i \\ &= (1-\beta)\beta W_t^i \frac{q_{t+1}(i) + p_{t+1}(i)\gamma_{t+1}(i)y_{t+1}(i)}{q_t(i)}. \end{aligned} \tag{A.6}$$

When $P_{t+1}^i C_{t+1}^i$ is given by (A.6) it is easily shown that the first order condition (A.5) is satisfied.

Now we have verified that (a)-(e) are all consistent with the first order conditions and the market clearing conditions. Lastly, the aggregate price level is determined to satisfy the other first order condition (6), which is analysed in Section 3 in detail.

Appendix B Optimal filtering

In this Appendix, we derive optimal filtering discussed in Section 3.4. Suppose that the equilibrium is given by (47). Then the observation equations are given by (47) and (45). Following Hamilton (1994) we write the state-space representation of the system for agent i as

$$\xi_t = \mathbf{F}\xi_{t-1} + v_t, \tag{B.1}$$

$$Z_t(i) = \mathbf{H}'\xi_t + w_t(i). \tag{B.2}$$

where

$$\xi_t \equiv \begin{bmatrix} \hat{Y}_t \\ \hat{\Gamma}_t \end{bmatrix}, \quad v_t \equiv \begin{bmatrix} u_{y,t} \\ u_{\gamma,t} \end{bmatrix}, \quad Z_t(i) \equiv \begin{bmatrix} \bar{Y}_t^o(i) \\ \bar{\Gamma}_t^o(i) \\ \hat{P}_t^o + \phi^{-1}\big(\hat{Y}_t^o + \hat{\Gamma}_t^o\big) \end{bmatrix}, \quad w_t(i) \equiv \begin{bmatrix} \bar{\varepsilon}_{y,t}(i) \\ \bar{\varepsilon}_{\gamma,t}(i) \\ 0 \end{bmatrix}$$

$$\mathbf{F} \equiv \begin{bmatrix} \delta & 0 \\ 0 & \eta \end{bmatrix}, \quad \mathbf{H}' \equiv \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ A_y & A_\gamma \end{bmatrix}$$

Denote the variance-covariance matrices as

$$\mathbf{V} \equiv E[v_t v_t'], \quad \mathbf{W} \equiv E[w_t w_t'].$$

Denote $\xi^i_{t|t}$ as the optimal predictor of ξ_t conditional on date t information of agent i, and $\xi^i_{t|t-1}$ as its prior mean. Then $\xi^i_{t|t}$ is given by the following Kalman filter (see Hamilton (1994)):

$$\xi^i_{t|t} = \xi^i_{t|t-1} + \mathbf{D}_{t|t}\mathbf{H}(\mathbf{H}'\mathbf{D}_{t|t}\mathbf{H} + \mathbf{W})^{-1}(Z_t(i) - \mathbf{H}'\xi^i_{t|t-1}) \tag{B.3}$$

where the transition for $\mathbf{D}_{t|t-1}$ is given by

$$\mathbf{D}_{t+1|t} = \mathbf{F}\left[\mathbf{D}_{t|t-1} - \mathbf{D}_{t|t-1}\mathbf{H}(\mathbf{H}'\mathbf{D}_{t|t-1}\mathbf{H} + \mathbf{W})^{-1}\mathbf{H}'\mathbf{D}_{t|t-1}\right]\mathbf{F}' + \mathbf{V}.$$

Then the optimal predictor of $\hat{Y}_t$ is given by

$$\begin{aligned} E^i_t\hat{Y}_t = E^i_{t-1}\hat{Y}_t &+ \kappa_y\left(\bar{Y}^o_t(i) - E^i_{t-1}\bar{Y}^o_t(i)\right) + \kappa_\gamma\left(\bar{\Gamma}^o_t(i) - E^i_{t-1}\bar{\Gamma}^o_t(i)\right) \\ &+ \kappa_z\left\{\hat{P}^o_t + \phi^{-1}\big(\hat{Y}^o_t + \hat{\Gamma}^o_t\big) - E^i_{t-1}\big[\hat{P}^o_t + \phi^{-1}\big(\hat{Y}^o_t + \hat{\Gamma}^o_t\big)\big]\right\}, \end{aligned}$$

where κ_y, κ_γ, κ_z are (1,1), (1,2), (1,3) elements of $\mathbf{D}_{t|t}\mathbf{H}(\mathbf{H}'\mathbf{D}_{t|t}\mathbf{H}+\mathbf{W})^{-1}$ in equation (B.3). By substituting (47) into this equation, we obtain

$$E_t^i\hat{Y}_t = E_{t-1}^i\hat{Y}_t + \kappa_y\left(\bar{Y}_t^o(i) - E_{t-1}^i\bar{Y}_t^o(i)\right) + \kappa_\gamma\left(\bar{\Gamma}_t^o(i) - E_{t-1}^i\bar{\Gamma}_t^o(i)\right) \\ + \kappa_z\left\{(A_y\hat{Y}_t + A_\gamma\hat{\Gamma}_t) - E_{t-1}^i(A_y\hat{Y}_t + A_\gamma\hat{\Gamma}_t)\right\}. \quad \text{(B.4)}$$

The optimal predictor of $\hat{\Gamma}_t$ can be derived in a similar expression to (B.4). Instead of doing this, we notice that equation (47) implies that

$$\begin{aligned} & E_t^i\hat{\Gamma}_t - E_{t-1}^i\hat{\Gamma}_t \\ &= \frac{1}{A_\gamma}\left[\left\{\hat{P}_t^o + \phi^{-1}\big(\hat{Y}_t^o + \hat{\Gamma}_t^o\big) - E_{t-1}^i\big[\hat{P}_t^o + \phi^{-1}\big(\hat{Y}_t^o + \hat{\Gamma}_t^o\big)\big]\right\} - A_y(E_t^i\hat{Y}_t - E_{t-1}^i\hat{Y}_t)\right] \\ &= \frac{1}{A_\gamma}\left[\left\{(A_y\hat{Y}_t + A_\gamma\hat{\Gamma}_t) - E_{t-1}^i(A_y\hat{Y}_t + A_\gamma\hat{\Gamma}_t)\right\} - A_y(E_t^i\hat{Y}_t - E_{t-1}^i\hat{Y}_t)\right], \end{aligned} \quad \text{(B.5)}$$

where $E_t^i\hat{Y}_t$ is given by (B.4), and again we substituted out the term $\hat{P}_t^o + \phi^{-1}\big(\hat{Y}_t^o + \hat{\Gamma}_t^o\big)$. Equations (B.4) and (B.5) are the rational expectations of $\hat{Y}_t$ and $\hat{\Gamma}_t$ under our equilibrium guess (47). Now we can compute the equilibrium when the expectations are given by (B.4) and (B.5). Recall that the equilibrium is given by (37). Substituting (B.5) into (37), we indeed obtain

$$\begin{aligned} \hat{P}_t^o + \phi^{-1}\big(\hat{Y}_t^o + \hat{\Gamma}_t^o\big) &= A_yE_t^i\hat{Y}_t + A_\gamma E_t^i\hat{\Gamma}_t \\ &= A_y\hat{Y}_t + A_\gamma\hat{\Gamma}_t, \end{aligned}$$

which is equation (47). What we have shown is that when equilibrium is given by (47) then the rational expectations are given by (B.4) and (B.5), and when the rational expectations about $\hat{Y}_t$ and $\hat{\Gamma}_t$ are given by (B.4) and (B.5) then the equilibrium is given by (47). Therefore expectations and equilibrium are consistent with each other.

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