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ABSTRACT

Incentives in Competitive Search Equilibrium and Wage Rigidity*

This paper examines competitive search equilibrium when workers' effort choice and 'type' are private information. We derive a modified Hosios Rule determining the allocation of resources, and analyze how private information influences the responsiveness of the unemployment rate to changes in macroeconomic variables. Most importantly, private information increases the responsiveness of the unemployment rate to changes in the general (type- and effort independent) productivity level. If the changes also affect the information structure, the responsiveness of the unemployment rate may be large, even if the changes in expected productivity are small.

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1 Introduction

In any search market, the resource constraint implies that there is a trade-off between high wages and a high exit rate from unemployment. In competitive search equilibrium, a market maker optimally balances this trade-off. The resulting wage, or equivalently, employment rent, ensures that agents on both sides of the market have the correct incentives to enter the market and search for trading partners.

In a pure contractual setting with asymmetric information, rents play a different role. In a standard principal-agent model (Laffont and Tirole, 1993), where output is contractible and the agent has private information about his type, this information makes him better off. He receives information rents. The stronger incentives the principal gives the agent to exert effort, the higher will this information rent typically be. The principal thus faces a trade-off between rent extraction and effort provision, and chooses a wage contract that optimally balances these two considerations.

In this paper we study the interplay between search frictions and incentive contracts in competitive search equilibrium. When the market maker trades off employment rents and a high exit rate from unemployment, he takes into account the fact that employment rents ease the constraints imposed by workers' private information and thereby enhance efficiency. We derive a modified Hosios Rule which determines the constrained efficient resource allocation. When the value of relaxing the private information constraints is large, employment rents are large, and few resources are used to create new jobs.

We believe that our model sheds some light on the issue of how wages respond to aggregate shocks. Recent studies by Shimer (2004, 2005a) document empirical regularities of the business cycle that the standard matching model of the labor market cannot account for. He finds that fluctuations in the unemployment rate predicted by the model in response to observed productivity shocks are much smaller than actual fluctuations in the unemployment rate. The reason is that wages are too flexible, and thereby absorb too much of the shock. He also finds that a low job creation rate, not a high job destruction rate is responsible for high unemployment rates during recessions. Similar findings are reported in Hall (2004b).

We find that a negative change in productivity, which reduces the productivity of all matches by the same amount, reduces the match surplus and tightens the constraints imposed by the workers' private information. Consequently, employment rents become more important relative to creating new jobs. Therefore, wages typically become less responsive and unemployment more responsive to such shocks than in the standard search model. We interpret such a shock as an increase in input prices (for example oil prices). If, in addition, worker effort is more crucial after a negative change (for instance because effort and other inputs with higher prices are substitutes), the responsiveness of the unemployment rate is further increased. The same may be true if a negative change is associated with (or caused by) more private information to workers.

On this point our paper is related to Hall (2005). Hall assumes that the workers' share of the match surplus is counter-cyclical, and rationalizes this by referring to social norms.[1] Our model delivers a counter-cyclical sharing rule as an optimal response to changes in aggregate variables in the presence of private information.

A related model is found in Shimer and Wright (2004). They consider a competitive search model where firms (not workers) have private information about productivity and workers have private information about effort. They show how private information may distort trade, and thereby increase unemployment. However, the mechanisms in the two papers are very different. We focus on how the allocation of rents between workers and firms may influence the inefficiencies caused by private information. This is absent in Shimer and Wrights paper, who instead focus on the direct effect of inefficiencies created by two-sided private information on the unemployment and vacancy rates.

Another related model is developed in Faig and Jerez (2005). They analyze trade in a retail market with search frictions when buyers have private information about their willingness-to-pay. Although their paper is similar to ours in the sense that they study private information in a competitive search environment, their model and emphasis differ from ours. Moral hazard is absent in their model, and their focus is on welfare analysis. They do not derive the modified Hosios condition, nor do they study the effects of changes in macroeconomic variables.

Several recent studies seek to make the search model consistent with Shimer's and Hall's empirical findings. In Kennan (2004), workers and firms bargain over wages once they meet. Firms have private information in booms,

[1] In Hall (2004a) it is also shown that wage stickiness may be the result of alternative specifications of the bargaining procedure or of self-selection among workers.

but not in recessions, and thus earn information rents in booms. This increases the profits in booms, and thus also unemployment volatility. Nagypál (2004) and Krause and Lubik (2004) allow for on-the-job search in a matching model and show how this may amplify the effects of productivity shocks on the unemployment rate. Menzio (2004) shows that firms with private information may find it in their interest to keep wages fixed if hit by high-frequency shocks. In Rudanko (2005) the effect of risk averse workers and contractual incompleteness on volatility is explored.

Our model is also related to an older literature on efficiency wage models (see for instance Weiss 1980 and Shapiro and Stiglitz 1984), notably those papers that examines the impact of efficiency wages on unemployment volatility. Strand (1992) finds that efficiency wages reduce unemployment volatility. He assumes that firms may be tempted to fire workers after a negative aggregate shock. As a result, firms are reluctant to hire more workers during a boom as this increases the wages necessary to deter shirking. Thus, if productivity differences are relatively small, employment does not change over the cycle. Danthine and Donaldson (1990) argue that efficiency wages may exacerbate the effect of productivity shocks on the unemployment rate if the shocks are short-lived compared to the time it takes to fire shirking workers. Ramey and Watson (1997) analyze how contractual fragility caused by firms' inability to commit to a wage contract may increase the volatility of the unemployment rate. Rocheteau (2001) introduces shirking in a search model and shows that the non-shirking constraint forms a lower bound on wages. Finally, MacLeod, Malcomson and Gomme (1994) find that efficiency wages may lead to wage rigidity if workers face a higher probability of being fired for exogenous reasons during a recession than during a boom.

The paper is organized as follows: Section 2 presents the model. The optimal incentive contracts are derived in section 3 and the labour market equilibrium in section 4. In section 5 we analyze whether private information influences the responsiveness of wages to shocks. In section 6 we discuss, among other things, alternative formulations of the incentive problem. Section 7 concludes.

2 The model

The matching of unemployed workers and vacancies is modelled using the Diamond-Mortensen-Pissarides framework (Diamond 1982, Mortensen 1986, Pissarides 1985) with competitive wage setting. The asymmetric information model builds on the procurement model of Laffont and Tirole (1993), see also Moen and Rosén (2005) for an interpretation of this model in a frictionless labor market.

Agents and information

The economy consists of a continuum of ex ante identical workers and firms. All agents are risk neutral and have the same discount factor r. The measure of workers is normalized to one. Workers leave the market at an exogenous rate s and new workers enter the market as unemployed at the same rate.

Output y of a worker-firm pair is observable and contractible and given by

$$y = \overline{y} + \varepsilon + \gamma e.$$

The variable ε reflects a match-specific productivity term, which is observable to the worker but not to the firm. It is I.I.D. over all worker-firm matches and continuously distributed on some interval $[\underline{\varepsilon}, \overline{\varepsilon}]$ with cumulative distribution function H.[2] The corresponding density function h has an increasing hazard rate. The variable e denotes worker effort, also unobservable to the firm.[3] The utility flow of a worker is given by $\omega = w - \psi(e)$, where $\psi(e)$ denotes the cost of effort. We assume that $\psi(e)$ is increasing and that $\psi'(e)$ is increasing and convex in e.

The firms offer performance contracts linking wages to output, and a worker observes these before he approaches the firm. When a worker and a firm meet, the worker first learns ε and then decides whether to accept the contract. If he rejects the offer he starts searching again, and the vacancy remains vacant.

Matching and asset value equations

Let u denote the unemployment rate and v the vacancy rate in the economy. The number of matches is determined by a concave, constant return to scale matching function $x(u, v)$. Let p denote the matching rate for workers

[2] Note that ε also may reflect idiosyncrasies of the job in question, which can easily be observed by the worker, but not by the firm.

[3] We have thus not included cross effects between effort and worker type. Adding a cross term εe would complicate the expressions, and would not lead to new insights.

and q the matching rate of firms. Since the matching function has constant return to scale, we can write $q = q(p)$, with $q'(p) < 0$.[4] We assume that the matching function is Cobb Douglas, $x(u, v) = Au^{\beta}v^{1-\beta}$, and hence that $p = A^{\frac{1}{\beta}}q^{-\frac{1-\beta}{\beta}}$.

Suppose all matches with a match-specific productivity term that exceeds a cut-off level ε^* lead to employment. Let U denote the expected discounted utility of an unemployed worker and W the expected discounted utility of an employed worker. It follows that

$$(r + s)U = z + p(1 - H(\varepsilon^*))(W - U). \tag{1}$$

where z is utility flow when unemployed. The expected discounted rent for a worker associated with a match is defined as

$$R \equiv (1 - H(\varepsilon^*))(W - U).$$

Below we refer (somewhat imprecisely) to R as employment rents. There is a flow cost, c, associated with maintaining a vacancy. Let V denote the expected discounted value of a firm with a vacancy and J the expected discounted value to a firm that matches with a worker. We can write

$$rV = -c + q(J - V).$$

There is free entry of vacancies, hence the equilibrium value of V is equal to zero. Let S denote the expected discounted surplus of a match, defined as

$$S = J + R.$$

Finally, let Y denote the expected discounted output net of worker effort of a match, given that the match leads to employment. By definition, we have that $Y = J + W$, and hence that

$$S = (1 - H(\varepsilon^*))(Y - U).$$

[4]The probability rates p and q can be written as $p = x(u, v)/u = x(1, \theta) = \widetilde{p}(\theta)$ and $q = x(u, v)/v = x(1/\theta, 1) = \tilde{q}(\theta)$. The matching technology can thus be summarised by a function $q = \tilde{q}(\theta) = \tilde{q}(\widetilde{p}^{-1}(p)) = q(p)$.

3 Optimal incentive contracts

An important building block for deriving the equilibrium is to characterize the solution to the problem of maximizing the match surplus S with respect to the wage contract $w(y)$ under the condition that the expected employment rent to the worker are below a threshold R. (Alternatively, the same contract can be derived as the solution to the problem of maximizing expected firm profit J given that the worker obtains an expected rent R). We do not allow for transfers that take place before the worker learns ε. In principle, the market maker may condition the contract on tenure or past behavior. However, as shown in Baron and Besanko (1984) and Moen and Rosén (2005), the optimal dynamic contract repeats the optimal static contract provided that the firm can commit not to renegotiate. See footnote 6 for further discussion.

In order to derive the optimal wage contract we use the revelation principle. A mechanism is a triple $(\varepsilon^*, e(\varepsilon), w(\varepsilon))$ that obeys the workers' incentive compatible (IC) constraints and the individual rationality (IR) constraints. Let $\varrho(\varepsilon)$ denote the rent to a worker of "type" ε. The incentive compatibility constraint can then be expressed as[5]

$$(r+s)\varrho'(\varepsilon) = \psi'(e(\varepsilon))/\gamma. \tag{2}$$

The individual rationality constraint requires that $\varrho(\varepsilon^*) \geq 0$.

The expected (flow) surplus of a match can be expressed as

$$(r+s)S = \int_{\varepsilon^*}^{\overline{\varepsilon}} [\overline{y} + \varepsilon + \gamma e(\varepsilon) - \psi(e(\varepsilon)) - (r+s)U] dH, \tag{3}$$

[5] Derivation of the incentive compatibility constraint is standard, and can be found in Laffont and Tirole (1993). To repeat, a worker of type ε that reports type $\widetilde{\varepsilon}$ receives a utility flow given by

$$\omega(\varepsilon, \widetilde{\varepsilon}) = w(\widetilde{\varepsilon}) - \psi(e(\widetilde{\varepsilon}) - \frac{\varepsilon - \widetilde{\varepsilon}}{\gamma}).$$

Truth-telling requires that $\varepsilon = \arg\max_{\widetilde{\varepsilon}} \omega(\varepsilon, \widetilde{\varepsilon})$, and from the envelope theorem it follows that the incentive compatibility constraint is given by $\frac{\delta \omega}{\delta \widetilde{\varepsilon}} = \psi'(e(\varepsilon))/\gamma$ (evaluated at $\widetilde{\varepsilon} = \varepsilon$). Since $\varrho(\varepsilon) = \frac{\omega(\varepsilon)}{r+s} - U$ the equation thus follows.

which is maximized subject to the IR constraint, the IC constraint, and the maximum expected rents given to workers:

$$\begin{aligned} \varrho(\varepsilon^*) &= 0 \qquad (4) \\ (r+s)\varrho'(\varepsilon) &= \psi'(e(\varepsilon))/\gamma \\ \int_{\varepsilon^*}^{\overline{\varepsilon}} \varrho(\varepsilon) dH(\varepsilon) &\leq R. \qquad (5) \end{aligned}$$

The associated Hamiltonian is

$$\begin{aligned} \mathcal{H} &= [\overline{y} + \varepsilon + \gamma e(\varepsilon) - \psi(e(\varepsilon)) - (r+s)U] h(\varepsilon) \\ & \quad - \lambda \psi'(e(\varepsilon))/\gamma \ - (r+s)\alpha[\int_{\varepsilon^*}^{\overline{\varepsilon}} \varrho(\varepsilon) dH(\varepsilon) - R]. \end{aligned}$$

The first order conditions for $e(\varepsilon)$ can be written as

$$(\gamma - \psi'(e(\varepsilon)) h(\varepsilon) = \lambda \psi''(e(\varepsilon))/\gamma. \qquad (6)$$

Furthermore,

$$\lambda'(\varepsilon) = \delta\mathcal{H}/\delta\varrho = -(r+s)\alpha h(\varepsilon).$$

Since $\overline{\varepsilon}$ is free it follows that $\lambda(\overline{\varepsilon}) = 0$. Thus, $\lambda = (r+s)\alpha(1 - H(\varepsilon))$. Inserted, this gives

$$\gamma - \psi'(e(\varepsilon)) = (r+s)\alpha \frac{1 - H(\varepsilon)}{h(\varepsilon)} \psi''(e(\varepsilon))/\gamma. \qquad (7)$$

Let (a, b) denote a linear contract of the form $w = a + by$. It is well known that the optimal non-linear contract can be represented by a menu $(a(\epsilon), b(\epsilon))$ of linear contracts (see, e.g., Laffont and Tirole, 1993). For any b, the worker chooses the effort level such that $\psi'(e) = b\gamma$. Using the condition $\psi'(e) = b\gamma$ in equation (7), we obtain

$$b(\epsilon) = 1 - (r+s)\alpha \frac{1 - H(\epsilon)}{h(\epsilon)} \frac{\psi''(e)}{\gamma^2}. \qquad (8)$$

Henceforth, we refer to b as the incentive power of the associated linear contract.[6] The optimal cut-off value ε^* is obtained by setting $\mathcal{H} = 0$:

$$-[\overline{y}+\varepsilon^*+\gamma e(\varepsilon^*)-\psi(e(\varepsilon^*))-(r+s)U]h(\varepsilon^*)+(r+s)\alpha(1-H(\varepsilon^*))\frac{\psi'(e(\varepsilon^*))}{\gamma} = 0. \tag{9}$$

An interesting variable is α, the Lagrangian parameter associated with the employment rent constraint. This variable shows the shadow value of increasing R. For sufficiently high values of R, the constraint (5) does not bind and hence $\alpha = 0$. From the first order conditions (7) and (9) it then follows that $\psi'(e(\varepsilon)) = \gamma$ (full incentives for all types) and that $\overline{y}+\varepsilon^*+\gamma e(\varepsilon^*)-\psi(e(\varepsilon^*)) = (r+s)U$ (net productivity equal outside option for an "ε^*-type" worker). We refer to this as the first best production level. Let R^* denote the lowest value of R such that constraint (5) does not bind. When $\psi'(e(\varepsilon)) = \gamma$ equation (2) implies that $(r+s)\varrho'(\varepsilon) = 1$ and, since $\varrho(\varepsilon^*) = 0$, that $\varrho(\varepsilon) = \frac{\varepsilon-\varepsilon^*}{r+s}$ and hence that[7]

$$R^* = \int_{\varepsilon^*}^{\overline{\varepsilon}} \frac{\varepsilon - \varepsilon^*}{r+s} dH(\varepsilon).$$

For $R > R^*$, the optimal value of S is independent of R, and we write $S = S^*(U)$. We can now show the following Lemma:

Lemma 1 *Suppose $R < R^*$. Then*

a) The cut-off level ε^* is increasing in α (for a given U)
b) α is decreasing in R (for given U)
c) If $\varepsilon^* > \underline{\varepsilon}$, α is decreasing in U. If $\varepsilon^* = \underline{\varepsilon}$, α is independent of U

Proof. See appendix. ■

[6] As the workers' outside option is constant over time, a time dependent contract cannot improve efficiency. To gain intuition for this, suppose to the contrary that the firm offered the worker a contract with an effort level $e_1(\varepsilon)$ for the first t periods, and then effort level $e_2(\varepsilon)$, with $e_1(\varepsilon) \neq e_2(\varepsilon)$ for some ε. The firm can always improve by smoothing the effort levels. From the incentive compatibility constraint it follows that the flow value of the information rent is convex in e (since $\psi'''(e) > 0$). At the same time, the net output flow is concave in e. Smoothing effort levels therefore both increases the net output flow and reduces the information rents, and thereby surely increases efficiency. A mathematical proof is given in Moen and Rosén (2005).

[7] More intuitively, when the worker is given full incentives he is the residual claimant and receives the entire production value in excess of $(r+s)U$.

Denote $\frac{\partial S}{\partial R}$ by S_R, $\frac{\partial S_R}{\partial R}$ by S_{RR} and $\frac{\partial S_R}{\partial U}$ by S_{RU}.. Note that

$$(r+s)S_R = \partial \mathcal{H}/\partial R = (r+s)\alpha.$$

We can then easily prove our first Proposition.

Proposition 1 *For a given U the following holds:*
a) The match surplus $S(R,U)$ is increasing and concave in R for $R < R^*$ ($S_{RR} < 0$). If all types are hired ($\varepsilon^* = \underline{\varepsilon}$), then $S_{RU} = 0$.
b) The effort level $e(\varepsilon)$ is strictly increasing in R for all ε, and the cut-off level ε^* is decreasing in R for $R \leq R^*$.

Proof. a) That $S_{RR} < 0$ follows from Lemma 1b and that $S_R = \alpha$.. That $S_{RU} = 0$, when $\varepsilon^* = \underline{\varepsilon}$ follows from Lemma 1c and that $S_R = \alpha$. b) Since the second order condition for optimal $e(\varepsilon)$ must be satisfied, equation (7) imply that $e(\varepsilon)$ is strictly decreasing in α for all ε. Together with Lemma 1b this proves the first part. The second part follows directly from Lemma 1a and Lemma 1b. ■

As the principal has more rents to dole out, he can afford to give stronger incentives to all worker types, and this increases the match surplus. On the other hand, since the match surplus is concave in effort for any worker type, the marginal value of rents is decreasing and approaches zero as R goes to R^*.

4 Equilibrium

Our equilibrium solution concept is competitive search equilibrium (Moen 1997, Shimer 1996). In competitive search equilibrium, the expected utility of unemployed workers is maximized subject to the resource constraint of the economy (essentially the free entry condition of firms).[8] As in Mortensen and Wright (2002), competitive search equilibrium can be interpreted as follows: A market maker determines the wage contract in his market. Free entry of market makers ensures that the only market maker that survives in the market is the one that maximizes the utility of unemployed workers given, the free entry condition of firms.

[8] See Acemoglu and Shimer (1999) and Moen and Rosén (2004).

As a benchmark, suppose the planner has full information about ε, enabling him to implement the first best production level. In this case, $S = S^*(U)$, as defined above. The competitive search equilibrium p^c, R^c, U^c then solves

$$\begin{aligned} \max_R (r+s)U &= z + pR \\ &\text{S.T.} \\ \frac{c}{q(p)} &= S^*(U) - R. \end{aligned}$$

If R^* is less than the "search rent" R^c, the market maker can implement first best production level even in the presence of private information:

Lemma 2 *If the search rent R^c exceeds the information rent R^*, the market maker can implement first best production level.*

Proof. Omitted ■

In the remainder of the paper we assume that $R^* > R^c$. This assumption is discussed in some detail in section 6.2. First best production level can also be obtained when $R^* > R^c$ if the market maker can cross subsidize entry, by collecting an entry fee from workers and a subsidy for vacancies.

Proposition 2 *(Irrelevance of private information) Suppose the market maker can collect an entry fee from the workers, and subsidize vacancies that enter their market. Then the first best competitive search equilibrium is always feasible.*

Proof. See Appendix. ■

Cross subsidization between workers and firms breaks the link between the workers' rent when employed and the firms' incentives to enter the market. Thus, the market maker can solve for the optimal trade-off between wages and job finding rate without influencing worker productivity once hired. A similar result is derived in Faig and Jarez (2005).

A sign-on fee paid by the worker to the firm may play the same role as an entry fee. When the worker has private information, the sign-on fee must be agreed upon before the private information is revealed to the worker.

In what follows we do not allow for cross-subsidization between workers and firms. As the market maker has to obey the individual rationality constraint and the incentive compatibility constraint of workers, he faces a relationship $S(R,U)$ between productivity and worker rents. The constrained competitive search equilibrium then solves

$$\begin{aligned} \max_R (r+s)U &= z + pR & (10) \\ \text{S.T. } \frac{c}{q(p)} &= S(R,U) - R. & (11) \end{aligned}$$

For any given R, there exists a corresponding value of p and U, hence we can write $p = p(R)$ and $U = U(R)$. By definition, $\partial U/\partial R = 0$ in optimum. From equation (10) it follows that

$$el_R p = -1, \tag{12}$$

where $el_R p$ denotes the elasticity of p with respect to R. From equation (11) it follows that

$$el_R[\frac{c}{q(p(R))}] = -(1 - S_R)\frac{R}{S-R}. \tag{13}$$

Substituting in for $el_R p = -1$ gives

$$\begin{aligned} el_R[\frac{c}{q(p(R))}] &= -el_p q(p) el_R p(R) = el_p q(p) \\ &= -\frac{\eta}{1-\eta}, \end{aligned}$$

where $\eta = |el_\theta \widetilde{q}(\theta)|$ denotes the absolute value of the elasticity of q with respect to $\theta = v/u$. [9] Equilibrium in the search market is thus given by

$$(1 - S_R)\frac{R}{S-R} = \frac{\eta}{1-\eta}. \tag{14}$$

[9] To see this, let $p = \widetilde{p}(\theta)$ and $q = \widetilde{q}(\theta)$. Then

$$\begin{aligned} el_p q(p) &= el_p \widetilde{q}(\widetilde{p}^{-1}(p)) \\ &= \frac{el_\theta \widetilde{q}(\theta)}{el_\theta \widetilde{p}(\theta)} \end{aligned}$$

Since $el_\theta \widetilde{q}(\theta) = -\eta$ and $el_\theta \widetilde{p}(\theta) = el_\theta[\theta \widetilde{q}(\theta)] = 1 - \eta$, it follows that $el_p q(p) = -\frac{\eta}{1-\eta}$.

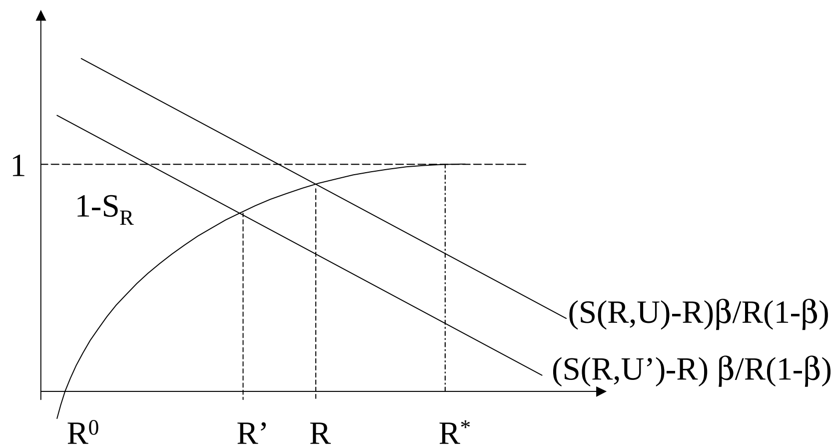

Figure 1

When $S_R = 0$, the equation is identical to the Hosios condition for efficiency in search models (Hosios 1990). We will refer to this equation as the modified Hosios condition.

Proposition 3 *The constrained competitive search equilibrium satisfies the modified Hosios condition (14).*

The modified Hosios condition states that the share of the match surplus that is allocated to the worker increases, as the marginal value of worker rents, S_R, increases. Thus, a smaller fraction of the match surplus is allocated to job creation. With Cobb-Doulgas matching function, $\eta = \beta$, and the modified Hosios condition is

$$(1 - S_R)\frac{R}{S - R} = \frac{\beta}{1 - \beta}. \tag{15}$$

Equilibrium is described in figure 1. At R^0, $1 - S_R(R) = 0$. For values at or below R^0, there is no trade-off between R and p, as an increase in R

increases p. Equilibrium thus has to be to the right of R^0. By assumption, the equilibrium value of R is below R^*.

The decreasing curves show the ratio $\frac{S-R}{R}$ as a function of R for a given value of U. For $R > R^0$, $S_R(R;U) - R$ is decreasing in R, as $S_{RR} < 0$ (Proposition 1a) [10] Note that for a given R, S is decreasing in U. Thus, a positive shift in U shifts the $\frac{S-R}{R}$-curve downwards.

5 Comparative statics

As mentioned in the introduction, an important issue is whether private information influences the responsiveness of wages to economy-wide shocks. We address this question by analyzing how a *change* in parameter values (for instance productivity) changes the unemployment rate.

It is well known that the comparative statics with endogenous cut-off levels are notoriously difficult in search models. Therefore, we first assume that all worker types are hired, i.e. $\varepsilon^* = \underline{\varepsilon}$. In this case we have from Proposition 1a that $S_{RU} = 0$. We return to the case with an interior cut-off level in section 5.3.

We say that private information stabilizes the unemployment rate whenever a negative shift (in, say, productivity) leads to a reduction in S_R and thus to a larger fraction of the match surplus being allocated to job creation. In the opposite case, the private information destabilize the unemployment rate.

In general, a shift in parameters may influence the relationship between S and R. However, some shocks will typically not influence this relationship, and we refer to them as information-neutral shifts. These shifts are:

- Changes in the value of leisure (or unemployment benefits).
- Changes in the matching function.
- Changes in general (type- and effort-independent) productivity, here changes in $\overline{y}$. This may be interpreted as changes in input prices (e.g. oil prices).

[10] By definition, $\frac{dU}{dR} = 0$ in equilibrium. Thus, the slope of the $\frac{S(R;U)-R}{R}$ curve (with U constant) is equal to the slope of the $\frac{S(R,U(R))-R}{R}$-curve at the equilibrium point.

By contrast, shifts in the distribution of ε and the importance of unobservable effort, γ, have a direct influence on the relationship between S and R, and are referred to as information-changing shifts.

5.1 Information-neutral shifts

Information-neutral shifts are straightforward to analyze. The effect of such shifts will depend on its effect on the available match surplus S. If the available surplus decreases during a recession, this will increase S_R for a given sharing rule, and lead to a counter-cyclical sharing rule.

As a warm-up, consider first an increase in z, the value of leisure. For a given wage contract, an increase in z increases the expected discounted utility U for unemployed workers. For a given R this reduces the match surplus S, and the $\frac{S-R}{R}$-curve in figure 1 shifts down. It follows that R falls, and hence that S_R increases in equilibrium. Thus, a smaller share of the surplus is allocated to job creation, increasing the unemployment rate further. Thus, for changes in the value of leisure, private information tends to destabilize the unemployment rate.

Then, more interestingly, consider a negative shift in $\overline{y}$. This reduces both Y and U. Since $\partial U/\partial R = 0$ in equilibrium, it follows from (1) that $(r+s)dU = p(dW - dU)$ and $dW = dY$. Thus,

$$\frac{\partial U}{\partial \overline{y}} = \frac{p}{r+s+p}\frac{\partial Y}{\partial \overline{y}} < \frac{\partial Y}{\partial \overline{y}}. \tag{16}$$

Thus, after a negative shift in $\overline{y}$, S falls for a given R. It follows that the $\frac{S-R}{R}$ curve shifts down, and hence that R falls and S_R increases. Thus, a larger share of the surplus is allocated to the worker after the negative shift, and hence destabilizes the unemployment rate.

Finally, consider a shift in the matching technology measured by A. A negative shift in A increases the unemployment rate. At the same time U shifts down, and hence $\frac{S-R}{R}$ shifts up. It follows that R increases and S_R decreases. Thus, private information stabilizes the unemployment rate after a shift in A. The same holds for an increase in the cost of search, c.

Proposition 4 *Consider a shock to the economy. Then the following holds*

a) Private information destabilizes the unemployment rate after a shift in the value of leisure.

b) Private information destabilizes the unemployment rate after contract-independent changes in productivity (the same for all worker "types")

c) Private information stabilizes the unemployment rate after change in the matching technology and in the cost of search.

Proof. See Appendix ■

To elaborate more on the intuition, consider an information-neutral change that reduces the match surplus. For a given sharing rule, this implies that the rents to the worker falls. Hence the marginal value of worker rents increases, and an optimal response to this is to increase the share of the surplus allocated to the worker. If the direct effect of the change is increased unemployment, the private information increases the magnitude of the changes in the unemployment rate.

5.2 Information-changing shifts

Consider first the effects of shifts in the importance of unobservable effort. We want a negative shift in the production function to be associated with increased importance of effort. If input prices drive the shock, this may be interpreted as worker effort and other inputs being substitutes. We therefore rewrite the production function slightly to

$$y = \widetilde{y} + \varepsilon + \gamma(e - e^0), \tag{17}$$

where $e^0 > e^*$, the equilibrium value of e. This is equivalent to our initial formulation with $\widetilde{y} = \overline{y} + \gamma e^0$. Suppose that a negative shock is driven by an increase in γ (the importance that the worker exert effort).

Proposition 5 *Consider a positive shift in γ. Suppose $\psi''/(\psi')^2$ is non-increasing in e, then the private information destabilizes the unemployment rate*

Proof. See Appendix ■

The destabilizing effects may be particularly strong in this case, as it consists of two components. An increase in γ increases S_R *for a given value of* R, which is not the case in the shifts studied previously. This comes *in addition* to the effects of a reduction in S induced by the fall in productivity.

The restriction imposed on $\psi(e)$ is rather mild, and is satisfied for most convex functions. For instance, any polynomial of the form $\psi(e) = e^n$ $(n > 1)$ satisfies this condition, as well as the exponential function $\exp e$.

Consider then a shift in the distribution of ε. To this end, write the match-specific productivity term as $a = k\varepsilon$, where ε is as before and k a scalar. We study the effects of an increase in k. On the one hand, an increase in k increases the amount of private information workers posses. For a given contract this increases worker rents. Thus, for a given R the incentive power of the contract has to be reduced. This tends to increase the marginal value of effort and thus S_R, the marginal value of worker rents. On the other hand, an increase in k implies that more rents are needed to increase worker incentives, which tends to reduce the value of S_R. It turns out that if the private information problems are moderate, the first effect dominates, and an increase in k increases S_R. If private information problems are more severe, an increase in k may reduce S_R. To get sharper result, assume that the cost of effort function $\psi(e)$ is quadratic, and define $\overline{b}$ as

$$\overline{b} = \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} b(\varepsilon)/(\overline{\varepsilon} - \underline{\varepsilon})d\varepsilon.$$

The following then holds:

Lemma 3 *For a constant U, the following holds: If $\overline{b}$ is above $1/2$ initially, an increase in k increases S_R. If $\overline{b}$ is below $1/2$ initially, an increase in k decreases S_R.*

Proof. See Appendix ■

Note that if R is close to R^*, then b is close to 1 for all ε, and $\overline{b}$ is certainly above $1/2$.

What then about the effects caused by changes in U? From the previous section we know that an increase in U, *cet. par.*, increases S_R. As long as $\varepsilon^* = \underline{\varepsilon}$, an increase in k is associated with a fall in U, as the effort level falls. In this case, the effects through U and through the change in the contract moves in the same direction after a shock if $b < 1/2$. However, if $\varepsilon^* > \underline{\varepsilon}$, an increase in k increases the average ε among hired workers for a given cut-off. Hence, U may increase. In this case the two effects go in the same direction if $b > 1/2$.

The important insight here is that changes in workers' private information, either in γ (unobserved effort) or k may give rise to substantial changes in the unemployment level even if the changes in U and average productivity are fairly small.

5.3 Effects through the cut-off level

So far we have assumed that all types are hired. In this subsection we briefly discuss the effect of the same shifts on the cut-off level when $\varepsilon^* > \underline{\varepsilon}$. To facilitate reading we repeat the first-order condition for optimal cut-off level ε^*.

$$\overline{y} + \varepsilon^* + \gamma e - \psi(e(\varepsilon^*)) - (r+s)U = (r+s)\alpha \frac{(1-H(\varepsilon^*))}{h(\varepsilon^*)} \frac{\psi'(e(\varepsilon^*))}{\gamma}. \quad (18)$$

The left-hand side denotes the net productivity loss of increasing ε^* while the right-hand side represents the gain in terms of reduced worker rents (which has shadow flow value $(r+s)\alpha$). The following Proposition holds:

Proposition 6 *For information-neutral productivity shocks, a fall in $\overline{y}$ or a rise in z increases the cut-off level ε^*. A fall in A decreases the cut-off level.*

Proof. See Appendix ■

A fall in $\overline{y}$ implies that the left-hand side falls (since $\overline{y}$ falls more than $(r+s)U$). This tends to increase the cut-off level ε^*. Furthermore, we know that an increase in α also increases ε^*, (from Lemma 1a) A similar argument holds for shifts in z and A.

Thus, in all cases the effects through the cut-off level seems to exacerbate our previous findings. For instance, a negative shift in $\overline{y}$ will increase the cut-off level, and further destabilize the unemployment level. However, there is one caveat here: As ε^* shifts up after a fall in $\overline{y}$, this tends to dampen the increase in S_R, and S_R may even fall. However, this typically happens when the increase in ε^* (and thus its adverse effect on the unemployment rate) is large.

Consider then information-changing shifts, and first a change in γ. With the re-specification of the production function (equation (17)), the cut-off level is given by

$$\widetilde{y}+\varepsilon^{*}+\gamma(e(\varepsilon^{*})-e^{0})-\psi(e(\varepsilon^{*}))-(r+s)U=(r+s)\alpha\frac{(1-H(\varepsilon^{*}))}{h(\varepsilon^{*})}\frac{\psi'(e(\varepsilon^{*}))}{\gamma}. \tag{19}$$

An increase in γ influences the cut-off level in several ways. For a given incentive power $b(\varepsilon)$ of the contract, the right-hand side increases in γ for a given ε^* (α increases and $\frac{\psi'(e(\varepsilon^*))}{\gamma}$ stays constant), which tends to increase ε^*. An increase in $b(\varepsilon)$ tends to increase ε^* further. Furthermore, for a given e, worker productivity falls, particularly for low-type workers (and more than the fall in U), which tends to reduce the net productivity loss of increasing ε^*. This also tends to increase ε^*. However, e also increases, particularly for low-type workers, making the results more uncertain.

We have not been able to show robust results for changes in the amount of private information, measured by k. Intuition suggests that the cut-off level should increase, as it become more important to obtain good matches. Furthermore, if $\bar{b}$ is relatively large, α goes up for a given ε^*, which will also tend to increase ε^*. However, it is hard to prove these results analytically.

6 Generalization and discussion

In this section we first discuss alternative formulations of the agency problem between firms and workers. Then we discuss the requirement that $R^* > R^c$ in some detail.

6.1 Alternative formulations of the incentive problem

The shirking model

In the shirking model (Shapiro and Stiglitz, 1984), workers are identical, but both worker effort and output are private information to the worker. Effort is either 0 or 1, and output is y if the worker exerts effort, and zero otherwise. The effort cost of high effort is ψ. Let g denote the probability rate that a shirking worker is detected, in which case he is fired. The non-shirking condition is then given by

$$\psi \leq gR.$$

That is, the cost of effort should be less than the probability rate of being detected when shirking, times the cost of losing the job. Let $R^{ns} = \psi/g$

denote the lowest rent that prevents the worker from shirking. Define the constrained competitive search equilibrium as the allocation that maximizes U given the non-shirking constraint. It follows that $R = \max[R^c, R^{ns}]$.[11]

Suppose we are in a region where the non-shirking constraint binds. A fall in y then has no impact on R. Since the match surplus S decreases, this requires that a larger fraction of the match surplus is given out as employment rents. Thus, shirking destabilizes the economy.

Non-pecuniary aspects of employment

Suppose workers obtain non-pecuniary gains from the employment relationship, and that these gains are private information to the workers and thus cannot be contracted upon. In all other respects the agents have symmetric information.

To be more specific, suppose the utility flow of a match for a worker who is paid a wage w is equal to $w + \tau$, where τ can take a high value τ^h or a low value τ^l. We assume that τ is I.I.D. over all worker-firm pairs. Worker productivity is the same for all workers and equal to y.

Efficient matching requires that a match leads to employment whenever $S(\tau) \geq 0$. Workers, by contrast, only accept jobs for which $R(\tau) \geq 0$. Suppose that initially, $R(\tau^l) \geq 0$ in the unconstrained equilibrium. Thus, both types of workers accept the job and there are no information problems.

Consider a fall in y. For a given sharing rule this leads to a fall in R.. Thus, after a shock, we may have that $R(\tau^l) < 0 < S(\tau^l)$ if the same sharing rule is applied. In order to motivate workers to accept a match after a low realization of τ, the market maker may increase the share of the surplus that is allocated to the workers so that workers accept all job offers. This will increase the workers' share of the surplus and thus destabilize the unemployment rate.

6.2 More on the requirement that $R^* > R^c$

In the analysis above we have assumed that $R^* > R^c$. Here we discuss requirements making this condition to hold. The first thing to note is that if not all workers are hired in competitive search equilibrium with first best production level, then $R^* > R^c$.

[11] Note that S_R is not defined at $R = R^{ns}$. Thus, we cannot set $\alpha = S_R$ in this case.

Lemma 4 *If $\overline{y}+\underline{\varepsilon}+\gamma e(\underline{\varepsilon})-\psi(e(\underline{\varepsilon}))<U^c$, then $R^*>R^c$.*

Proof. The proof is done by contradiction. Suppose $R^c \geq R^*$. In this equilibrium, let ε^* denote the (optimal) cut-off productivity, given by the equation $\overline{y}+\varepsilon^*+\gamma e(\varepsilon^*)-\psi(e(\varepsilon^*))=U^c$. The marginal worker must be paid a wage equal to his productivity. Furthermore, as $w'(y)=1$ for all other workers, first best production level implies zero profit to the firm. Thus, no vacancies enter the market and no workers are employed. This is inconsistent with equilibrium. ■

It follows that as long as the distribution of ε is non-degenerate, first best production level is infeasible, provided that the search frictions measured by the search costs c are sufficiently small:

Corollary 1 *If the search cost c is sufficiently small, $R^*>R^c$ (provided that the distribution of ε is not degenerate).*

Proof. In competitive search equilibrium, $p\to\infty$ as $c\to 0$.. As a result, the optimal cut-off approaches $\overline{\varepsilon}$. From Lemma 4 it then follows that $R^*>R^c$. ■

However, it may well be that $R^c<R^*$ even if all worker "types" are hired. Suppose $a=k\varepsilon$, where ε is as before and k a scalar. where a is a stochastic variable and k a scalar. Then the following holds:

Lemma 5 *For any given combination of parameters and any distribution H of ε with finite support, there exists an interval $k\in(\underline{k},\overline{k})$ such that for any k the following holds: 1) $R^*>R^c$, and 2) the cut-off level is equal to $\underline{\varepsilon}$.*

Proof. See Appendix. ■

7 Conclusion

In this paper we derive competitive search equilibrium when workers have private information regarding effort and "type". Wage contracts are used to enhance efficiency. We then investigate the effects of economy-wide shocks on unemployment- and vacancy rates.

In the standard model, the planner trades off a high wage (or rents associated with employment) and high exit rates from unemployment. Private information brings in an additional effect: Worker rents ease the constraints imposed by the workers' private information, thereby enhancing efficiency. We derive a modified Hosios Rule determining the allocation of resources. When information problems are more severe, fewer resources are used to create vacancies.

Shocks to the economy may change the productivity-enhancing value of worker rents, and this influences the responsiveness of the wage- and unemployment rates. We find that private information reduces the responsiveness of the unemployment rate to changes in the matching technology. However, it increases the responsiveness of the unemployment rate to changes in the deterministic part of the production function or in the value of leisure. The responsiveness of the unemployment rate to changes in the information structure may be large, even if the changes in expected productivity are small.

Appendix

Proof of Lemma 1

a) It is convenient to rewrite the cut-off equation (9) as

$$\overline{y}+\varepsilon^{*}-(r+s)U=(r+s)\alpha\frac{1-H(\varepsilon^{*})}{h(\varepsilon^{*})}\frac{\psi'(e(\varepsilon^{*}))}{\gamma}-(\gamma e(\varepsilon^{*})-\psi(e(\varepsilon^{*})))$$

Denote the left-hand side by $X_L(\varepsilon)$ and the right-hand side by $X_R(\varepsilon;\alpha)$. Obviously $X_L'(\varepsilon)=1$.. As the second order condition must be satisfied locally, $X_L(\varepsilon)$ crosses $X_R(\varepsilon)$ from below. It is therefore sufficient to show that around $\varepsilon=\varepsilon^{*}$an increase in α shifts $X_R(\varepsilon;\alpha)$ up.

$$\begin{aligned}\frac{\partial X_R(\varepsilon^{*};\alpha)}{\partial\alpha} &= \frac{1-H(\varepsilon^{*})}{h(\varepsilon^{*})}\frac{\psi'(e(\varepsilon^{*}))}{\gamma}+(r+s)\alpha\frac{1-H(\varepsilon^{*})}{h(\varepsilon^{*})}\frac{\psi''(e(\varepsilon^{*}))}{\gamma}\frac{de}{d\alpha}\\ &\quad -(\gamma-\psi'(e(\varepsilon^{*})))\frac{de}{d\alpha}\end{aligned}$$

Equation (7) implies that $\gamma-\psi'(e)=(r+s)\alpha\frac{1-H}{h}\frac{\psi''}{\gamma}$. Hence the two last terms cancel out, and

$$\frac{\partial X_R(\varepsilon^{*};\alpha)}{\partial\alpha}=(r+s)\frac{1-H(\varepsilon^{*})}{h(\varepsilon^{*})}\frac{\psi'(e(\varepsilon^{*}))}{\gamma}>0,$$

completing the proof of result a).

b) Worker rent given type ε' is given by

$$(r+s)\varrho(\varepsilon')=\int_{\varepsilon^{*}}^{\varepsilon'}\frac{\psi'(e(\varepsilon))}{\gamma}dH,$$

and thus the expected rent of a match is given by

$$(r+s)\varrho=\int_{\varepsilon^{*}}^{\overline{\varepsilon}}\int_{\varepsilon^{*}}^{\varepsilon'}\frac{\psi'(e(\varepsilon))}{\gamma}dH.$$

From result (a) we know that ε^{*} is increasing in α. Furthermore, it follows from (7) and the second order condition for optimal $e(\varepsilon)$ that $e(\varepsilon)$ is strictly increasing in α for all ε. Suppose that α is increasing in R. In this case an increase in R implies that (i) e decreases and (ii) ε^{*} increases. But then it follows that ϱ falls, a contradiction as long as $R<R^{*}$.

c) U only enters the contract through the cut-off equation (9), which can be written as

$$\overline{y} + \varepsilon^* - (r+s)U + (\gamma e(\varepsilon^*) - \psi(e(\varepsilon^*))) = (r+s)\alpha \frac{1 - H(\varepsilon^*)}{h(\varepsilon^*)} \frac{\psi'(e(\varepsilon^*))}{\gamma} \quad (20)$$

We first show that an increase in U leads to a fall in α if and only if it leads to an increase in ε^*. Consider an increase in ε^*. This leads to a lower value of ϱ, as we are integrating over a shorter interval. Thus, the rent-constraint allows for more incentive-powered contracts. As a result, the shadow value of R (i.e., α) falls. If ε^* falls, the opposite holds, and α increases.

Suppose then that ε^* decreases in U. For a given α, an increase in U reduces the gain from hiring workers (reduces the left-hand side of 20), and ε^* increases. From a) we know that ε^* is increasing in α. Thus, for ε^* to fall, α decreases. However, we have just shown that ε^* and α move in opposite directions, and we have thus derived a contradiction. Hence, if $\varepsilon^* > \underline{\varepsilon}$, α is decreasing in U.

As U only enters the contract through the cut-off equation (9), α is independent of U when $\varepsilon^* = \underline{\varepsilon}$.

Proof of Proposition 2

We know that efficiency can be obtained if $R^c \geq R^*$. For $R^c < R^*$, the first best production level can be obtained as follows. When the worker and the firm meet, the worker receives an expected rent R^* so that first best production is ensured. To obtain the optimal vacancy rate, the market maker gives the vacancies a subsidy $D = q(p^c)(R^* - R^c)$ when entering the search market. As a result, the expected value of entering is $q(Y^* - R^* - U^c) + D = q(Y^* - R^* - U^c) + q(R^* - R^c) = q(Y^* - R^c - U^c)$. Hence, the correct number of firms enter. The unemployed workers are charged a fee $T = p^c(R^* - R^c)$ when entering. Since $qv = x = pu$, this scheme balances the budget. QED

Proof of Proposition 4

It is convenient to rewrite (15) as

$$(1 - S_R(R, U; X))R - \frac{\beta}{1-\beta}(S(R, U; X) - R) = Q(R, U; X) = 0$$

where $X = \overline{y}, z, A, c$. Total differentiations gives

$$\frac{dR}{dX} = -\frac{\partial Q(R,U;X)}{\partial X} / \frac{\partial Q(R,U;X)}{\partial R}$$

Now,

$$\frac{\partial Q(R,U;X)}{\partial R} = 1 - S_R - S_{RR}R - \frac{\beta}{1-\beta}(S_R - 1)$$

Since $S_{RR} < 0$ (Proposition 1a) and $1 - S_R > 0$, we know that $\frac{\partial Q(R,U;X)}{\partial R} > 0$. Furthermore, since S_R is decreasing in R it is sufficient to prove that: a) $\frac{\partial Q}{\partial z} > 0$, b)$\frac{\partial Q}{\partial \overline{y}} < 0$, c) $\frac{\partial Q}{\partial A} > 0$ and $\frac{\partial Q}{\partial c} < 0$. First recall that $S_{RU} = 0$ (Proposition1a), $S_U < 0$, $S_U \frac{\partial U}{\partial \overline{y}} + S_Y \frac{\partial Y}{\partial \overline{y}} > 0$ (equation (16), $S_U = -1$, and $S_Y = 1$), Also note that $S_{Rz} = S_{R\overline{y}} = S_{RA} = S_{Rc} = 0$ (proof analogous to that of $S_{RU} = 0$).

$$\frac{\partial Q(R,U;z)}{\partial z} = -\frac{\beta}{1-\beta} S_U \frac{\partial U}{\partial z} > 0$$

$$\frac{\partial Q(R,U;\overline{y})}{\partial \overline{y}} = -\frac{\beta}{1-\beta}(S_U \frac{\partial U}{\partial \overline{y}} + S_Y \frac{\partial Y}{\partial \overline{y}}) < 0$$

$$\frac{\partial Q(R,U;A)}{\partial A} = -\frac{\beta}{1-\beta} S_U \frac{\partial U}{\partial A} > 0$$

$$\frac{\partial Q(R,U;c)}{\partial c} = -\frac{\beta}{1-\beta} S_U \frac{dU}{dc} < 0$$

Q.E.D.

Proof of Proposition 5

Worker rent for any given type ε' is given by

$$(r+s)\varrho(\varepsilon') = \int_{\underline{\varepsilon}}^{\varepsilon'} \frac{\psi'(e(\varepsilon))}{\gamma} dH$$

Suppose an increase in γ increases $b(\varepsilon) = \psi'(e(\varepsilon))/\gamma$ for all ε for a given α. This increases ϱ, and for a given R this implies that α increases. It is thus sufficient to show that $b(\varepsilon)$ increases in γ for a given α.

The first order condition (7) reads

$$\gamma[1 - \psi'(e(\varepsilon))/\gamma - (r+s)\alpha \frac{1-H(\varepsilon)}{h(\varepsilon)} \psi''(e(\varepsilon))/\gamma^2] = 0$$

For a given ε, suppose $b(\varepsilon) = \psi'(e(\varepsilon))/\gamma$ stays constant. It is sufficient to show that this will increase the left-hand side of the first order condition for all ε. The second order conditions then ensure that $e(\varepsilon)$ must increase further, i.e., that $\psi'(e(\varepsilon))/\gamma$ increases.

Substituting $b(\varepsilon) = \psi'(e(\varepsilon))/\gamma$ into the first order condition gives

$$\gamma[1 - b(\varepsilon) - (r+s)\alpha\frac{1-H(\varepsilon)}{h(\varepsilon)}\frac{\psi''(e(\varepsilon))b(\varepsilon)}{\psi'(e(\varepsilon))^2}] = 0$$

For a given b, e is increasing in γ, and it follows that the left-hand side is increasing in e provided that $\frac{\psi''(e(\varepsilon))}{\psi'(e(\varepsilon))^2}$ is decreasing. QED

In addition, an increase in γ reduces output, and as for a reduction in $\overline{y}$ this will reduce R for a given α. This will further increase α.

Proof of Lemma 3

With a match-specific productivity term $a = k\varepsilon$, where ε is as before and k a scalar the analogue to (2) and (8) are

$$(r+s)\varrho'(\varepsilon) = k\psi'(e(\varepsilon))/\gamma$$

and

$$b(\epsilon) = 1 - k(r+s)\alpha\frac{1-H(\epsilon)}{h(\epsilon)}\frac{\psi''(e)}{\gamma^2}. \tag{21}$$

Worker rent for any given type ε' is given by

$$(r+s)\varrho(\varepsilon') = \int_{\epsilon^*}^{\varepsilon'} k\frac{\psi'(e(\varepsilon))}{\gamma}dH = \int_{\underline{\varepsilon}}^{\varepsilon'} kb(\varepsilon)dH(\epsilon),$$

and the expected employment rents by

$$(r+s)\varrho = \int_{\underline{\varepsilon}}^{\overline{\varepsilon}}\int_{\underline{\varepsilon}}^{\varepsilon'} kb(\epsilon)d\epsilon dH(\epsilon).$$

Using integration by parts the above integral can be rewritten as

$$\begin{aligned}(r+s)\varrho &= -|_{\underline{\varepsilon}}^{\overline{\varepsilon}}(1-H)\int_{\underline{\varepsilon}}^{\epsilon'} kb(\epsilon)d\epsilon + \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} kb(\epsilon)(1-H)d\epsilon \\ &= \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} kb(\epsilon)(1-H)d\epsilon. \end{aligned} \tag{22}$$

From equations (21) and (22) it follows that

$$(r+s)\varrho = \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} k[1-k(r+s)\alpha\frac{1-H(\epsilon)}{h(\epsilon)}\frac{\psi''(e)}{\gamma^2}](1-H(\epsilon))d\epsilon.$$

Given that $\psi'''(e)=0$, the derivative with respect to k is

$$\frac{d(r+s)\varrho}{dk} =$$

$$\begin{aligned} & \int_{\underline{\varepsilon}}^{\overline{\varepsilon}} [1-k(r+s)\alpha\frac{1-H(\epsilon)}{h(\epsilon)}\frac{\psi''(e)}{\gamma^2} - k(r+s)\alpha\frac{1-H(\epsilon)}{h(\epsilon)}\frac{\psi''(e)}{\gamma^2}](1-H(\epsilon))d\epsilon \\ = & \int_{\epsilon^*}^{\overline{\varepsilon}} [2b(\epsilon)-1](1-H(\epsilon))d\epsilon. \end{aligned}$$

Hence, an increase in k increases (reduces) rents if $\bar{b} > (<)\ 1/2$. QED

Proof of Proposition 6

It is convenient to repeat the cut-off equation (9)

$$-[\overline{y}+\varepsilon^*+\gamma e(\varepsilon^*)-\psi(e(\varepsilon^*))-(r+s)U]h(\varepsilon^*)+(r+s)\alpha(1-H(\varepsilon^*))\frac{\psi'(e(\varepsilon^*))}{\gamma} = 0.$$

Denote the left-hand side by $X_L(\varepsilon;\overline{y},z,A)$. As the second order condition must be satisfied locally, we know that $\partial X_L/\partial\varepsilon < 0$ around $\varepsilon = \varepsilon^*$. It is therefore sufficient to show that around $\varepsilon = \varepsilon^*$ X_L decreases in $\overline{y}$ but increase in z and A.

$$\frac{\partial X_L(\varepsilon;\overline{y},z,A)}{\partial \overline{y}} = -1+(r+s)\frac{\partial U}{\partial \overline{y}}+(r+s)\frac{(1-H(\varepsilon^*))}{h(\varepsilon^*)}\frac{\psi'(e(\varepsilon^*))}{\gamma}\frac{\partial \alpha}{\partial \overline{y}}.$$

As equation (16) implies that $-1+(r+s)\frac{\partial U}{\partial \overline{y}} < 0$, and Proposition 4 that $\frac{\partial \alpha}{\partial \overline{y}} < 0$, $\frac{\partial X_L}{\partial \overline{y}} < 0$.

$$\frac{\partial X_L(\varepsilon;\overline{y},z,A)}{\partial z} = (r+s)\frac{\partial U}{\partial z}+(r+s)\frac{(1-H(\varepsilon^*))}{h(\varepsilon^*)}\frac{\psi'(e(\varepsilon^*))}{\gamma}\frac{\partial \alpha}{\partial z}.$$

As $\frac{\partial \alpha}{\partial z} > 0$ (Proposition 4) it follows that $\frac{\partial X_L}{\partial z} > 0$.

$$\frac{\partial X_L(\varepsilon; \overline{y}, z, A)}{\partial A} = (r+s)\frac{\partial U}{\partial A} + (r+s)\frac{(1-H(\varepsilon^*))}{h(\varepsilon^*)}\frac{\psi'(e(\varepsilon^*))}{\gamma}\frac{\partial \alpha}{\partial A}.$$

Since $\frac{\partial U}{\partial A} > 0$ and $\frac{\partial \alpha}{\partial A} > 0$ (Proposition 4) we have that $\frac{\partial X_L}{\partial A} > 0$.

Proof of Lemma 5

For $k = 0$, all workers are hired and first best is obtained. In this case, an increase in k does not influence U. We have to show that the market maker starts reducing the incentive power of the contract before he increases the cut-off. As $R^c > 0$, $\overline{y} + k\underline{\varepsilon} + \gamma e^* - \psi'(e^*) > U^c$ must hold at the point where $R^c = R^*$. At this point, increasing the cut-off level has a first-order effect on expected output. Reducing the incentive power of the contract slightly only gives a second-order effect on expected output. It thus follows that the market maker reduces the incentive power of the contract before he increases the cut-off level (i.e., for a lower value of k).

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