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ABSTRACT

Sticky Prices and Indeterminacy*

The aim of the present paper is to analyze the link between price rigidity and indeterminacy. This is done within a cash-in-advance economy from which we know that it exhibits indeterminacy at high degrees of relative risk aversion. I find that price stickiness reduces the scope of these sunspot equilibria: sluggish price adjustment requires degrees of relative risk aversion compatible with indeterminacy that prove too high to square with data.

JEL Classification: E31 and E32

Keywords: Calvo-pricing, cash-in-advance economies and sunspot equilibria

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1 Introduction

While often swept under the rug, a well-established fact in monetary theory is that flexible price cash-in-advance models display sunspot equilibria for weak degrees of intertemporal substitution (see for example Farmer, 1999). This sort of real indeterminacy gives way for macroeconomic instability to be generated by self-fulfilling beliefs. The present paper extends this result to monetary economies that are interspersed with price rigidity. In particular, I show that if price stickiness is modelled as in Calvo (1983) then sunspot equilibria in cash-in-advance economies become less likely, however, indeterminacy cannot be excluded unless prices are completely fixed.

2 The economy

The model constructed here is related in spirit to Farmer (1999). As well, it features aspects of New Keynesian models as discussed by Goodfriend and King (1997) for example. The artificial economy is populated by immortal, atomistic households of measure one who sell labor services and consume the final good. Factor and financial markets as well as the market for final goods are perfectly competitive. Indeterminacy goods are produced by monopolistic competitors that set prices infrequently. Following Carlstrom and Fuerst (2000) or Christiano and Eichenbaum's (1995) limited participation economy, I assume that financial intermediaries take cash deposits from the households. They then loan these resources to intermediate firms for firms then being able to pay the workers in cash. As is typical in limited participation models, money is held to satisfy a cash-in-advance constraint.

The representative household derives utility from the function

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - h_t) \quad u(., .) = \frac{c_t^{1-\sigma}}{1-\sigma} - h_t \quad \beta \in (0, 1], \sigma \in (0, \infty]$$

where β denotes the subjective discount factor, c_t stands for the consumption good and h_t is hours worked. The coefficient σ measures the relative risk-aversion – the inverse of the elasticity of substitution between consumption at different dates. Hours worked enter linearly in the utility function. This reduced-form function follows from assuming that labor is indivisible and that a lottery for employment allocates labor as in Hansen (1985). In the Appendix, I present a version of the model with a less than perfectly elastic supply of labor. E_0 is the expectations operator conditional on information in period 0. People hold wealth in two forms. They can arrange cash-holdings, m_{t+1} , which they carry into period $t + 1$. They can also loan cash, n_t , to financial intermediaries at the beginning of the period. These deposits earn the nominal interest R_t . I denote by Π_t the profit flow from firms and intermediaries, hence, the household's budget constraint is given by

$$m_{t+1} = m_t + W_t h_t - P_t c_t + n_t (R_t - 1) + \Pi_t$$

where P_t is the price level and W_t is the nominal wage. A positive value is assigned to the inconvertible currency by assuming that during the shopping session the household is subject to the cash-in-advance-restriction

$$m_t + W_t h_t - n_t \geq P_t c_t.$$

Optimal asset holdings imply the following Euler equation

$$c_t^{-\sigma} = \beta R_t E_t c_{t+1}^{-\sigma} \frac{P_t}{P_{t+1}} \equiv \beta R_t E_t \frac{c_{t+1}^{-\sigma}}{\pi_{t+1}}. \quad (1)$$

Hence, π_{t+1} denotes the gross rate of inflation. The optimal choice of work effort is captured by the labor supply function

$$1 = c_t^{-\sigma} \frac{W_t}{P_t}. \quad (2)$$

The final goods sector assembles the continuous range of distinct intermediate inputs $i \in [0, 1]$ with the constant returns to scale production function

$$Y_t = \int_0^1 y_{i,t}^{(v-1)/v} di$$

where $v \in (1, \infty]$ denotes the elasticity of substitution between goods. Given aggregate output, the demand for a intermediate good, $y_{i,t}$, is a function of its relative price

$$y_{i,t} = \frac{p_{i,t}}{P_t} Y_t \quad P_t \equiv \int_0^1 p_{i,t}^{1-v} di. \quad (3)$$

Monopolistic competitors produce the intermediate products and have access to the technology

$$y_{i,t} = Ah_{i,t} \quad A > 0.$$

Before hiring workers at the competitive wage W_t , these firms must borrow cash at the short-term rate, R_t , from the financial intermediaries. Assuming that each firm i operates under perfectly competitive input markets, the firm determines its production plan by minimizing costs

$$R_t W_t h_{i,t} \quad \text{s.t.} \quad Ah_{i,t} \geq y_{i,t}.$$

This minimization implies real marginal costs equal

$$\Phi_t = R_t \frac{W_t}{AP_t}. \quad (4)$$

As in Calvo (1983), price adjustment opportunities are accorded to monopolistic firms with probability $1-\theta$: $\theta \rightarrow 0$ corresponds to a world with perfectly flexible prices whereas at $\theta \rightarrow 1$ prices remain fixed forever. Each monopolist's intertemporal profit maximization is to choose the optimal price p_t^* to maximize the expected, discounted profits

$$E_t \sum_{\tau=0}^{\infty} (\beta\theta)^\tau \frac{c_{t+\tau}}{c_t} \frac{p_t^*}{P_{t+\tau}} - \Phi_{t+\tau} y_{i,t,t+\tau}$$

where the household's intertemporal marginal rate of substitution is the appropriate discount factor for future profits and $y_{i,t,t+\tau}$ is the firm's period $t+\tau$

output conditional on the optimal price. The dynamic first-order-condition regarding the monopoly prices is given by

$$p_t^* = \frac{v}{v-1} \frac{E_t \prod_{\tau=0}^{\infty} (\beta\theta)^\tau (c_{t+\tau}/c_t)^{-\sigma} P_{t+\tau}^v Y_{t+\tau} \Phi_{t+\tau}}{E_t \prod_{\tau=0}^{\infty} (\beta\theta)^\tau (c_{t+\tau}/c_t)^{-\sigma} P_{t+\tau}^{v-1} Y_{t+\tau}}. \quad (5)$$

Intermediaries accept cash from the households and receive injections from the central bank. They use these resources to lend to intermediate firms for firms being able to pay the wage bill. The loans must be repaid at the end of the period. Consequently, the intermediary sector faces the following constraint given the two sources of cash:

$$W_t h_t = n_t + M_{t+1}^s - M_t^s.$$

M_t^s stands for nominal money supply.

There is no government consumption. Nominal money supply grows at the constant gross rate $G = 1$, hence, there is no inflation in the steady state.

3 Sunspots

Having formulated the model, I will go about solving it by taking a first-order approximation around the zero-inflation steady state. In doing so, I denote percentage deviations of a variable's steady state by a hat, e.g. $\hat{b}_t \equiv (c_t - c)/c$. In symmetric equilibrium, the monopolists' optimal pricing equations transform into the New Keynesian Phillips curve

$$\hat{b}_t = \beta E_t \hat{b}_{t+1} + \frac{(1-\theta)(1-\beta\theta)}{\theta} \hat{\Phi}_t.$$

The household's first-order conditions reduce to

$$(1-\sigma)E_t \hat{b}_{t+1} = \hat{b}_t - \hat{\Phi}_t.$$

Noting that $\hat{b}_t = \hat{b}_t$, the artificial economy boils down to the second-order functional difference equation

$$\frac{-\beta\theta - (1-\sigma)(1-\theta)(1-\beta\theta)}{\theta} E_t \hat{b}_{t+1} + \frac{1+\beta\theta^2}{\theta} \hat{b}_t = \hat{b}_{t-1}. \quad (6)$$

Since \mathfrak{b}_{t+1} is a non-predetermined variable, unique dynamics require that exactly one root of this equation is outside the unit circle. If both roots are inside the unit circle, the economy is indeterminate. Equation (6) can be rewritten in matrix notation as

$$\begin{pmatrix} E_t \mathfrak{b}_{t+1} \\ \mathfrak{b}_t \end{pmatrix} = \mathbf{M} \begin{pmatrix} \mathfrak{b}_t \\ \mathfrak{b}_{t-1} \end{pmatrix}$$

where \mathbf{M} is a 2×2 matrix whose elements all depend upon the parameters that describe the economy's preferences and technologies. The determinant of \mathbf{M} is given by

$$\text{DetM} = \frac{\theta}{1 - \theta + \beta\theta^2 - \sigma(1 - \theta)(1 - \beta\theta)}$$

and the trace is given by

$$\text{TrM} = \frac{1 + \beta\theta^2}{1 - \theta + \beta\theta^2 - \sigma(1 - \theta)(1 - \beta\theta)}.$$

Indeterminacy will occur if and only if

$$-1 < \text{DetM} < 1 \quad \text{and} \quad -1 - \text{DetM} < \text{TrM} < 1 + \text{DetM}.$$

The four inequalities imply respectively

$$\sigma > \frac{1 + \beta\theta^2}{(1 - \theta)(1 - \beta\theta)}, \quad \sigma > \frac{1 - 2\theta + \beta\theta^2}{(1 - \theta)(1 - \beta\theta)}, \quad \sigma > \frac{2(1 + \beta\theta^2)}{(1 - \theta)(1 - \beta\theta)}$$

and $\sigma > 0$. Within the admissible parameter space, the most binding condition among the necessary and sufficient conditions for local indeterminacy turns out to be the third condition. This pins down the critical value of relative risk

$$\sigma^* = \frac{2(1 + \beta\theta^2)}{(1 - \theta)(1 - \beta\theta)} > 0.$$

For values of σ above σ^* both eigenvalues of \mathbf{M} are smaller than one in absolute value. Thus, fluctuations around the equilibria converge, that is the

adjustment dynamics to the steady state are indeterminate. It is for this reason that one is able to construct sunspot equilibria.

How is the critical value of σ related to the other two deep parameters? It is increasing in both β and θ which can be seen from

$$\partial\sigma^*/\partial\beta = \frac{2\theta(1+\theta)}{(1-\theta)(1-\beta\theta)^2} > 0$$

and

$$\partial\sigma^*/\partial\theta = \frac{2(1-\beta+\beta^2\theta^2+\beta\theta^2)}{(1-\theta)^2(1-\beta\theta)^2} > 0.$$

Taking limits, I find

$$\lim_{\theta \rightarrow 0} \sigma^* = 2 \quad \text{and} \quad \lim_{\theta \rightarrow 1} \sigma^* = \infty.$$

Several conclusions can be drawn from these results. First, the minimum value of σ that generates indeterminacy is increasing in the degree of price stickiness. This reduces the previously known parametric space of indeterminacy in cash-in-advance models. Second, if prices are perfectly flexible, the parameter σ 's lower bound for indeterminacy is reached. It equals two and it is independent of β . The two roots become $\mu_1 = (1-\sigma)^{-1}$ and $\mu_2 = 0$. This special case replicates the results in Farmer (1999) or Weder (2005a). Third, with complete price stickiness, indeterminacy is no longer possible in this class of cash-in-advance models since the critical value of σ rises to infinitely high degrees of relative aversion to risk. The equivalent insight can be obtained by noting what happens to the larger roots of (6) as prices become inexorably sticky:

$$\lim_{\theta \rightarrow 1} \mu_1 = \frac{1}{\beta} > 1.$$

To gain further understanding of the impact of price stickiness on indeterminacy, I will calibrate the model. Let us assume that time evolves in discrete units which are specified to be one quarter long. Accordingly, I set

$\beta = 0.99$. At $\theta = 0.25$, $\theta = 0.50$ and $\theta = 0.75$ the critical values become $\sigma^* = 3.76$, $\sigma^* = 9.89$ and $\sigma^* = 48.37$.

Now, what do the above numbers imply regarding the empirical plausibility of sunspot equilibria in cash-in-advance economies with sticky prices? Hansen and Singleton (1983) suggest that the coefficient of relative risk is between zero and two. Kocherlakota (1996, p. 52) states that values of the coefficient "above ten (or for that matter, above five) imply highly implausible behavior on the part of individuals". Several recent empirical studies have assessed the degree of price stickiness. Sbordone (2002) estimates the average time between price changes and finds it to fall in between 2.5 and 3.5 quarters. Bils and Klenow (2004) suggest more frequent adjustments of prices than typical studies. They find that only half of goods prices do last five months or more. Notice that in the model $(1 - \theta)^{-1}$ is the average number of periods for which a firm's price remains fixed. Thus, Sbordone's numbers correspond to model θ between 0.60 and 0.71. Even at her lower number, I find a critical value $\sigma^* = 16.70$ which is outside the empirically plausible range. Phrased alternatively, Kocherlakota's empirical upper bound $\sigma = 5$ requires modest degrees of price stickiness. Concretely θ must be smaller than 0.34 (i.e. prices remain fixed for about 1.5 quarters), for sunspots to matter. Although compatible with Bils and Klenow's findings which suggest a value of θ at about 0.25, the potentiality of indeterminacy is weakened by price rigidities.

Changing β does not alter these results which can be seen from

$$\lim_{\beta \rightarrow 0} \sigma^* = \frac{2}{1 - \theta} > 2 \quad \text{and} \quad \lim_{\beta \rightarrow 1} \sigma^* = \frac{2(1 + \theta^2)}{(\theta - 1)^2} > 2.$$

Even an outlandish $\beta = 0.10$ together with $\theta = 0.60$ implies $\sigma^* = 8.42$ which remains an unreasonably high number.

Overall, price stickiness appears to significantly reduce the possibility of indeterminacy in cash-in-advance models. However, given the ongoing

empirical discussion on the degree of stickiness, the issue of the plausibility of indeterminacy in cash-in-advance economies cannot be completely settled here: the uncertainty about the empirical counterparts of θ and σ remains simply too great.

Quite interestingly, it should be noted that stickiness increases the determinacy region here whereas the opposite is generally found in the New Keynesian indeterminacy literature with money-in-utility setups and interest rate rules (e.g. Lubik and Marzo, 2005). It is my hunch that the opposite results are the consequences of the very different mechanisms from which the indeterminacies arise i.e. how money demand is modelled and what the central bank is doing in these models.¹

4 Dynamics

Lastly, let me comment on some basic business cycle properties of the cash-in-advance economy. Empirically, impulse response dynamics show partiality to a model of output that propagates shocks cyclically en route back to the initial state (e.g. Farmer, 1999). This suggests that the roots of (6) should be complex-conjugate. Can the model laid out here replicate this? Using the case of logarithmic utility as the benchmark determinacy model, the roots of (6) become

$$\mu_1 = \frac{1}{\beta\theta} > 1 \quad \text{and} \quad 0 < \mu_2 = \theta < 1.$$

It is easy to show that output is described by the first-order equation

$$\mathfrak{y}_t = \theta \mathfrak{y}_{t-1}.$$

This means that the whole endogenous persistence arises from the degree of price stickiness. However, this model is not able to generate cycles.

¹I would like to thank Thomas Lubik for pointing this out to me.

Under indeterminacy, cycles are driven by i.i.d. sunspot shocks $u_{t+1} \equiv \mathfrak{b}_{t+1} - E_t \mathfrak{b}_{t+1}$ and the dynamics are captured by the AR(2)-process

$$\mathfrak{b}_{t+1} = \frac{1 + \beta\theta^2}{\beta\theta + (1 - \sigma)(1 - \theta)(1 - \beta\theta)} \mathfrak{b}_t - \frac{\theta}{\beta\theta + (1 - \sigma)(1 - \theta)(1 - \beta\theta)} \mathfrak{b}_{t-1} + u_{t+1}.$$

In Farmer's (1999) flexible price version, the model predicts output dynamics following the first-order process

$$\mathfrak{b}_{t+1} = \frac{1}{1 - \sigma} \mathfrak{b}_t + u_{t+1}$$

which cannot mimic empirically observed impulse response dynamics and cycles. A cyclical response to shocks requires that the associated roots are imaginary. Does introducing price stickiness yield such a result? Mathematically, imaginary roots are equivalent to the condition (derived from matrix M 's eigenvalues)

$$\Delta \equiv 4\theta(\theta - \beta\theta^2 - 1) + (1 + \beta\theta^2)^2 + 4\sigma\theta(1 - \theta - \beta\theta + \beta\theta^2) < 0.$$

Given the parameter restrictions on β , σ and θ it is not possible to find such a solution. In fact Δ is never negative in the defined parameter space. The nonexistence can be seen as follows. Taking the limits, I find that

$$\lim_{\sigma \rightarrow -\infty} \Delta = -\infty \quad \text{and} \quad \lim_{\sigma \rightarrow \infty} \Delta = \infty.$$

The fixed point $\Delta = 0$ is compatible with the unique value of risk aversion

$$\sigma^{**} = -\frac{(1 - 2\theta + \beta\theta^2)^2}{4\theta(1 - \theta)(1 - \beta\theta)} < 0.$$

Of course, $\sigma^{**} < \sigma^*$. Hence, roots are always real in the indeterminacy zone. Furthermore, numerical analysis reveals that the AR(1) root – and it turns out that it is also the larger root – is always negative which means

that artificial output is oscillating around the steady state every period. The model can neither generate hump-shaped impulse response dynamics nor cycles. To sum up, the model is able to bring about enormous persistence – namely for σ slightly above σ^* – but these cycles do not resemble those that we generally label as business cycles.

5 Concluding remarks

I have introduced Calvo-style price rigidity into a cash-in-advance model and show that increasing price rigidity shrinks the parametric space of sunspot equilibria. How does this result come about? Recall that the sunspot cycle in the cash-in-advance model begins by people increasing today's consumption in response to random sunspot signals. This rise in consumption pushes up the inflation rate via the New Keynesian Phillips curve. As a consequence, the nominal interest rate goes up and – since the nominal interest rate operates like an inflation tax on holding money given that people must satisfy their cash-in-advance restriction – this rise decreases tomorrow's consumption. If the degree of relative risk aversion is sufficiently high, the expectations are self-fulfilling, i.e. the beliefs are compatible with the pattern of prices. On the other hand, if prices are sticky, the sunspot-related increase in inflation is smaller and the interest rate rises by less which curbs the drop in future consumption. Sticky prices therefore counter the sunspot mechanism in cash-in-advance economies.

Given the results here, I suspect that introducing money and sticky prices into a real business cycle model with indeterminacy arising from increasing returns to scale will lead to similar insights. Weder (2005b) is a first step into this direction of research.

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6 Appendix

In the main text, I have assumed that utility is linear in labor. This was primarily done to obtain results that are readily comparable to Farmer (1999). This Appendix presents the local stability properties of the economy when labor supply is less than perfectly elastic.

I assume that periodic utility becomes

$$u(c_t, 1 - h_t) = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{h_t^{1+\chi}}{1+\chi} \quad \sigma \in (0, \infty], \chi \in [0, \infty]$$

which includes the main text's model for $\chi = 0$. The parameter χ stands for the inverse of the labor supply elasticity. This model transforms into

$$\frac{-\beta\theta - (1-\sigma)(1-\theta)(1-\beta\theta)}{\theta} E_t \mathbf{b}_{t+1} + \frac{1 + \beta\theta^2 + \chi(1-\theta)(1-\beta\theta)}{\theta} \mathbf{b}_t = \mathbf{b}_{t-1}$$

or

$$\begin{bmatrix} E_t \mathbf{b}_{t+1} \\ \mathbf{b}_t \end{bmatrix} = \mathbb{M} \begin{bmatrix} \mathbf{b}_t \\ \mathbf{b}_{t-1} \end{bmatrix}.$$

The determinant of \mathbb{M} remains unchanged, however, the trace is now given by

$$\text{Tr} \mathbb{M} = \frac{1 + \beta\theta^2 + \chi(1-\theta)(1-\beta\theta)}{1 - \theta + \beta\theta^2 - \sigma(1-\theta)(1-\beta\theta)}.$$

Indeterminacy will occur if and only if

$$-1 < \text{Det} \mathbb{M} < 1 \quad \text{and} \quad -1 - \text{Det} \mathbb{M} < \text{Tr} \mathbb{M} < 1 + \text{Det} \mathbb{M}.$$

The first two inequalities are the same as before, yet, the other two become

$$\sigma > \frac{2(1 + \beta\theta^2) + \chi(1-\theta)(1-\beta\theta)}{(1-\theta)(1-\beta\theta)} \quad \text{and} \quad \sigma > -\chi.$$

The new critical value becomes

$$\mathbf{e}^* = \frac{2(1 + \beta\theta^2) + \chi(1-\theta)(1-\beta\theta)}{(1-\theta)(1-\beta\theta)}$$

at which the two roots are

$$\rho_1 = -1 \quad \text{and} \quad 0 < \rho_2 = \frac{\theta}{1 + \theta + \beta\theta^2 + \chi(1 - \theta)(1 - \beta\theta)} < 1.$$

Once labor supply is less than perfectly elastic, the indeterminacy zone shrinks. This parallels the result in real business cycle models with indeterminacy arising from increasing returns to scale (see for example Wen, 1998). In the flexible price version of this economy, the condition for indeterminacy becomes

$$\sigma > 2 + \chi.$$

Survey data suggest fairly small labor supply elasticities usually around 0.25 which would put χ at 4. Thus, σ must exceed 6 in the flexible price economy for sunspot equilibria to appear. Indeterminacy vanishes if labor supply is perfectly inelastic.