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ABSTRACT

Inflation Bias with Dynamic Phillips Curves*

We generalise the analysis of inflation bias with dynamic Phillips curves in three respects. First, we examine the discretionary (time consistent) solution in cases where the Phillips curve has both a backward looking and forward-looking component. Second, we show that the commitment (time inconsistent) solution does not normally involve zero inflation and output at its natural rate. Instead, with a purely forward-looking Phillips curve and positive discounting, it will involve a dynamic path for inflation in which steady state inflation is below its target. In this sense, we obtain negative inflation bias. Third, we show that the timeless perspective policy has the same steady state as the commitment case, but without any short-term output gains.

JEL Classification: E52, E61, E63 and F41
Keywords: commitment, discretion, inflation bias and timeless perspective policy

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1 Introduction

The textbook model of inflation bias is based on a static Phillips curve. In the Barro-Gordon model, discretionary (time consistent) policy produces positive inflation bias. If the authorities are able to commit in some way, and therefore implement a time inconsistent solution, then they can achieve at best target inflation, but with output at its natural rate. Most macroeconomic models over the last ten or so years have involved dynamic Phillips curves, where current inflation depends on expected future inflation, and possibly also on the past level of inflation. Although there has been a good deal of analysis of inflation bias for these models (see Clarida, Galí, and Gertler (1999), Woodford (2003), and Walsh (2003) for example), we will argue in this paper that it is incomplete. In particular, we show that, if the government discounts at a rate which is greater than the rate at which the private sector discounts expected inflation in the Phillips curve, the time inconsistent/commitment solution leads to time varying paths for output and inflation and to steady state outcomes in which inflation is *below* its target level. More generally, we extend the existing literature by examining discretionary solutions for hybrid backward/forward Phillips curves, and by examining commitment solutions where government and private sector discount rates differ. This discussion will include the particular case of a NAIRU Phillips curve.

The structure of this paper is as follows. We begin by briefly restating the familiar Barro Gordon model as background to our analysis. This involves a Phillips curve in which current inflation depends on pre-set expectations of current inflation. The rest of the paper considers a dynamic Phillips curve. An appendix considers the authority's optimisation problem in the general case where current inflation depends in part on expectations of next period's inflation, and in part on inflation in the previous period. In the text we begin by looking at the case where the Phillips curve is entirely backward looking, and we focus on the role of discounting in influencing inflation bias. We then add a forward looking element, which encompasses the special case considered by Clarida, Galí, and Gertler (1999) in which inflation is purely forward looking. Then we consider the time inconsistent/commitment solution to the policy problem. We show that in a purely forward looking model with positive discounting the steady state inflation bias will be negative, and (with or without discounting) output will be above the natural rate in the short run. We go on to consider a timeless perspective policy, and then examine welfare losses for all three types of policy. All the analysis described so far considers a NAIRU Phillips curve, in which the long run rate of inflation is independent of the level of output. In the final section of the paper we look at a New Keynesian Phillips curve, in which this is not true. We consider the commitment solution in this case and allow the authority's discount rate to be above or below the rate of discounting in the private sector. Our main results stay unchanged, but we lose expositional clarity.

2 Barro-Gordon Model with Static Phillips Curve

We begin with the standard Barro-Gordon model (see Barro and Gordon (1983), and Blanchard and Fischer (1989) for a textbook exposition), partly because it is familiar, and partly to show the relationship between its results and those obtained with a dynamic Phillips curve. The authorities maximise a quadratic discounted loss,

$$\min(\mathcal{L} = \frac{1}{2}\mathcal{E}_0 \sum_{t=0}^{\infty} \beta^t(\omega(\pi_t - \pi^*)^2 + (y_t - y^T)^2)) \tag{1}$$

subject to the Phillips curve in the form:

$$\pi_t = \pi_t^e + \phi y_t \tag{2}$$

where

$$\pi_t^e = \mathcal{E}_{t-1}\pi_t$$

namely, inflation expectations about the period t are set one period earlier, in period $t-1$ so they are predetermined in period t. These assumptions make the Phillips curve essentially static. The Phillips curve assumes that the natural rate of output is zero, but we assume $y^T > 0$, which is a origin of the inflation bias problem.[1] The Hamiltonian for the optimisation problem can be written as

$$\mathcal{H} = \mathcal{E}_0 \sum_{t=0}^{\infty} \left(\frac{1}{2}\beta^t(\omega(\pi_t - \pi^*)^2 + (y_t - y^T)^2) + \psi_t \left(\mathcal{E}_{t-1}\pi_t + \phi y_t - \pi_t \right) \right)$$

and the first order conditions are

$$\frac{\partial H}{\partial \pi_t} = \beta^t \omega \left(\pi_t - \pi^* \right) - \psi_t = 0 \tag{3}$$

$$\frac{\partial \mathcal{H}}{\partial y_t} = \beta^t(y_t - y^T) + \phi\psi_t = 0 \tag{4}$$

$$\frac{\partial \mathcal{H}}{\partial \psi_t} = \mathcal{E}_{t-1}\pi_t + \phi y_t - \pi_t = 0 \tag{5}$$

This is a system of three *static* equations. Rational expectations imply that $\mathcal{E}_{t-1}\pi_t = \pi_t$, so the solution to this problem can be found as:

$$\pi = \pi^* + \frac{y^T}{\omega\phi} \tag{6}$$

$$y = 0 \tag{7}$$

Thus, as soon as the policymaker targets $y^T > 0$ there is a positive inflation bias, whose size is determined by the size of the output target, the degree of inflation aversion of the policymaker, and the size of the coefficient on demand pressure in the Phillips curve.

[1] For example, the natural rate might be below the target rate because of distortions caused by monopolistic competition, as is now standard in the recent literature analysing monetary policy using welfare derived from agents' utility (see Benigno and Woodford (2004) for example).

3 Dynamic Backward Looking Phillips Curve

The backward looking, accelerationist Phillips curve typically takes the form

$$\pi_t = \pi_{t-1} + \phi y_t. \tag{8}$$

This could be derived from (2) if we assume that expectations are static, $\pi_t^e = \pi_{t-1}$, but this is not the only justification for a Phillips curve of this form. As above, we assume that authorities minimise the intertemporal loss function (1), subject to this inflation process. This is a truly dynamic inflation process and to analyse it we need to solve a dynamic optimisation problem. The solution to this problem is given in the Appendix, and the adjustment path towards the steady state in this case is plotted in the first panel of Figure 2. Steady state values are given by:

$$\pi = \pi^* + \frac{(1-\beta)\, y^T}{\omega\phi} \tag{9}$$

$$y = 0 \tag{10}$$

There is an inflation bias, whose size depends on the authorities' rate of discounting. When the loss function is one-period, i.e. $\beta = 0$, then the inflation bias is identical to the one for the static model above (Section 2). If the authorities do not discount the future, i.e. $\beta = 1$, there is no inflation bias. The more policymakers discount the future, the larger is the inflation bias is. The reason why *some* discounting is required for inflation bias to be present is straightforward. Without discounting, the costs of the inflation bias would always count for more in the welfare function than any short term welfare gains generated by higher output. Only with discounting is it possible for these short term output gains to justify positive inflation in the steady state. Note that the amount of inflation bias in the steady state is always less than in the Barro-Gordon case, because the future inflation caused by targeting excess output causes policy to be less expansionary, unless there is complete discounting.[2]

4 Dynamic Forward Looking Phillips Curve and Discretion

A purely backward looking Phillips curve is rarely found in current academic analysis, or indeed policy modelling. It is much more common to find a relationship of the following form

$$\pi_t = \mathcal{E}_t \pi_{t+1} + \phi y_t \tag{11}$$

where expectations are assumed to be rational. This is the main model considered in Clarida, Galí, and Gertler (1999), for example. In fact an equation of this form is different from the New

[2] Levine (1988) effectively obtains this result, but does not draw on its significance. The issue is clearly discussed in Carlin and Soskice (2005).

Keynesian Phillips curve that is typically derived from microfoundations, which takes the form

$$\pi_t = \beta \mathcal{E}_t \pi_{t+1} + \phi y_t \tag{12}$$

where β is a household discount factor. (See Woodford (2003), for example, for a derivation.) As McCallum (2002) has stressed, this microfounded Phillips curve is inconsistent with the NAIRU hypothesis. With a New Keynesian Phillips curve, there is a (very steep) long run trade-off between output and inflation; in steady state $\pi = \frac{\phi}{(1-\beta)} y$.

For the moment we restrict ourselves to a NAIRU Phillips curve (which, as McCallum notes, is deeply embedded in the macroeconomic literature and in policy analysis), because it allows us to focus on the issue of inflation bias in its clearest form. We consider the New Keynesian Phillips curve in the final section of the paper. As we shall see, the points we make here continue to be present with a New Keynesian Phillips curve, but adding a long run inflation output trade-off creates additional complexity with no additional insight.

In empirical work, this purely forward looking model has had mixed success. Many authors have worked with a Phillips curve that involves both forward and backward looking elements i.e.

$$\pi_t = \alpha \mathcal{E}_t \pi_{t+1} + (1-\alpha)\pi_{t-1} + \phi y_t \tag{13}$$

where $0 \leq \alpha \leq 1$. There is, as yet, no agreement on the value of α. Gali and Gertler (1999), Benigno and Lopez-Salido (2002) find a predominantly forward-looking specification, while Mehra (2004) finds an extremely backward-looking specification of the Phillips curve. Mankiw (2001) argues that stylised empirical facts are inconsistent with predominantly forward-looking Phillips Curve. For this reason, we examine the model for any α. This generalises the results shown in Clarida, Galí, and Gertler (1999), who only consider the purely forward looking case, $\alpha = 1$.

The solution to this problem, using the same objective function as before, is complex to derive (except for the special case of $\alpha = 1$), and is described in detail in the Appendix. The upper panel of Figure 2 shows the steady state level of inflation bias for all values of α, and for several values of discounting[3]. The values for $\alpha = 0$ correspond to equation (9). As long as $\alpha < 1$, the dynamic paths are qualitatively similar to those in Figure 1, panel I. However, when $\alpha = 1$, the steady state value of inflation bias is independent of the level of discounting, and is given by

$$\pi = \pi^* + \frac{y^T}{\omega\phi} \tag{14}$$

$$y = 0 \tag{15}$$

[3]Here and everywhere below we choose parameters, and in particular the discount rate, for presentational clarity and not necessarily realism.

(This is the result derived by Clarida, Galí, and Gertler (1999)). This solution is also equal to the one in the Barro-Gordon case. Furthermore, the solution is not dynamic, in the sense that inflation will immediately *jump* to this value and stay there, and output will always be at the natural rate. The intuition behind this is as follows. In the static model, there must be no incentive for the authorities to trade-off more output for higher inflation. Only if inflation is positive, and equal to the level above, will this be the case. In the dynamic, discretionary case, the authorities should have no incentive to deviate from the optimal path. Once again, this will only be true if inflation is at its inflation bias level. This must be true for all periods, and so the solution will be constant. As the solution is constant, there is no role for discounting.

5 Dynamic Forward Looking Phillips Curve and Commitment

In the static model, the optimal time inconsistent solution involves zero inflation, and output at its natural rate. However, as the Appendix shows formally, the steady state optimal solution for our dynamic model in the purely forward looking case is given by

$$\pi = \pi^* - \frac{(1-\beta)}{\beta}\frac{y^T}{\omega\phi} \tag{16}$$

$$y = 0 \tag{17}$$

Output in steady state must be equal to its natural rate. However, as long as the discount factor β is below one, steady state inflation will be below target i.e. there is a *negative inflation* bias, the size of which increases with discounting. Only in the case of no discounting do we get the solution from the static model, with no inflation bias. Furthermore, the optimal solution is no longer constant, but follows the dynamic path discussed below.

The intuition for why there is no inflation bias when there is no discounting is identical to the case of the backward looking Phillips curve. The surprising result is that, with discounting, steady state inflation is always negative. The reason follows from a simple property of (11), which is that positive output is associated with *falling* inflation along any rational expectations saddle path. Panel II of Figure 1 plots the optimal path towards the negative steady state. Output is above zero in the short term, but the cost of this is falling inflation. Even though steady state inflation is not zero, the cost of this is offset by the short-term output gains, which are discounted by less. To see why steady state inflation has to be negative, imagine a hypothetical path in which steady state inflation was zero, as well as output. But this would mean that inflation would have to be higher in the short run. The gains from the better steady state would be more than offset by higher losses in short run (see Figure 1, panel III). This is why it will be optimal for the government to promise *negative* inflation in the future.

This optimum path is dynamically time inconsistent. After the initial period, the policy maker has an incentive to re-optimise. The dynamic path shown above would only occur if the policy maker had some commitment mechanism that ensured it did not re-optimise. Whether the authorities could find a commitment mechanism that could sustain the dynamic path described above remains an open question. It is important to recognise that this time inconsistent dynamic path is better than an alternative path along which inflation is constant at its target level and output is allways at its natural rate.

We can summarise our results using the bottom panel of Figure 2, which plots steady state inflation against a parameter α, which measures how forward looking the Phillips curve is. The two lines reflect the discretionary and commitment cases. If $\alpha = 0$, we have a completely backward looking Phillips curve, and the two solutions coincide. As α increases (and with some discounting), steady state inflation falls under commitment, but rises under discretion. (Note from the upper panel of Figure 2 that, in the discretionary case, inflation bias can initially fall with α for very large degree of discounting.) At some value of α the commitment solution for inflation bias becomes negative. Interestingly, there is a level of α, in our case $\alpha = \beta/(1+\beta)$, at which inflation bias on the commitment solution is exactly zero.

6 Dynamic Forward Looking Phillips Curve and Timeless Perspective Policy

Woodford (1999) introduced the 'timeless perspective' policy as an alternative optimisation solution: see Appendix D for brief discussion and derivation. This policy is derived from the dynamic first-order conditions for the fully optimal solution by simple change in *timing* of those variables that bring time-inconsistency. As a result, the steady state *levels* of the economy under a timeless perspective policy remain the same as the ones for the commitment policy, i.e. there is negative inflation bias with discounting. However, the dynamic path towards it is very different. As discussed in Woodford (2003) and derived in Appendix, the timeless perspective policy generates sluggish adjustment of an instrument. For the entirely forward-looking Phillips curve, the level of output at any time t is completely determined by its level in the previous period $t-1$. If we start with $y_0 = 0$, as we implicitly assumed in all examples above (but where this assumption did not play any role), then output will have to remain at zero for all time and inflation will jump immediately to its steady state level, which is negative. Thus, the losses from having inflation bias in the long run, are not compensated by having zero (or close to zero) inflation in at least one of the periods, and output closer to target in the short term.

With a predominantly forward looking Phillips curve ($\alpha = 0.9$ for example) the optimal path is characterised by *rising* output (tending towards zero) and negative inflation, see Figure 1, panel IV. This is a consequence of the new instrument equation for output, which requires that the difference in output is negatively related to the level of inflation. When inflation is negative, output must rise.

discount factor β :	0.01	0.1	0.25	0.5	0.75	0.9	0.99
Loss from Discretion	1.85	2.04	2.44	3.67	7.33	18.3	183.3
Loss from Commitment	0.001	0.02	0.09	0.38	1.37	4.38	49.4
Loss from Timeless Perspective Policy	1.3E+4	120.5	16.7	3.67	2.59	5.16	50.0

Table 1: Forward-looking Phillips curve: effect of discount factor on the welfare loss.

7 Welfare Losses

Different equilibrium levels and different adjustment paths generate different welfare losses for the three policies we consider. The biggest difference in welfare between the three policies is achieved in the case with an entirely forward-looking Phillips curve[4]. In this case, the level of loss depends on discounting, and this dependence is summarised in Table 1.

A discretionary policy is always worse than a timeless perspective policy[5] for $\beta > 1/2$, see Appendix F. When the discount factor is one, the loss from all policies is infinite, as there is always a loss from having $y = 0$ in equilibrium and it is summed in an undiscounted manner. The difference between the loss from a commitment policy and the loss from a timeless perspective policy for $\beta = 1$ is however finite and it is

$$\mathcal{L}^{C}_{\beta=1} - \mathcal{L}^{TP}_{\beta=1} = \frac{\left(y^T\right)^2}{4}\left(1 - \sqrt{1 + \frac{4}{\phi^2\omega}}\right) < 0$$

This is because under commitment, there is a short run gain from higher output, but there is no gain under the timeless perspective policy.

8 New Keynesian Phillips Curve

Until now, we have focused on Phillips curves that have the NAIRU property. Dynamic solutions involving the New Keynsian Phillips curve have been analysed extensively in Woodford (2003), but only for the case where the discount rate of the private sector and the policy maker are equal (i.e. the policy maker is benevolant). There are number of reasons to explore cases in which discount rates differ. For example, agents in the economy may have hetrogenous preferences, and policy makers may be partisan. Alternatively electoral dynamics (or the contracts of central bankers) might induce policy makers to adopt a much higher discount rate compared to the private sector.

We show in Appendix G, that in the case where the household discount factor is γ (in equation like (12)), $\gamma \neq \beta$, we obtain the following expressions for the steady state output and

[4] For the backward-looking Phillips curve all three policies are the same.

[5] It has been discussed in the literature that stochastic equilibria under the timeless perspective policy can become worse than discretion with smaller β (Blake (2001), Jensen and McCallum (2002)). Here we look at the level bias rather than the stabilisation bias.

inflation for a commitment policy:

$$\pi = \pi^* + \frac{(\beta-\gamma)\left(y^T - \frac{(1-\gamma)}{\phi}\pi^*\right)}{\omega\phi\beta\left(1+\frac{(\beta-\gamma)(1-\gamma)}{\omega\phi^2\beta}\right)}$$

$$y = \frac{\left(\pi^* + \frac{(\beta-\gamma)}{\omega\phi\beta}y^T\right)}{\left(\frac{\phi}{(1-\gamma)} + \frac{(\beta-\gamma)}{\omega\phi\beta}\right)}$$

These results suggest that in the case where the authority's discount rate β is below the private sector's discount rate γ the equilibrium inflation is below its target, unless the inflation target is very high. In a sense our previous result with a NAIRU-consistent Phillips curve is a special case of this, where $\gamma = 1$. In contrast, if $\beta > \gamma$, we have positive steady state inflation. Only if the two discount rates are equal do we get zero inflation in steady state, which is the case considered by Woodford (2003). If steady state inflation is not zero, then steady state output will also be non-zero, because the New Keynesian Phillips curve does not have the NAIRU property. In all cases the dynamics towards this new equilibrium is the same as that presented in Figure 1 for the NAIRU-consistent Phillips Curve.

9 Conclusion

The received wisdom about inflation bias is based on an essentially static model, whereas the current literature works with Phillips curves which are dynamic in structure. We have examined both discretionary and commitment policies for Phillips curves with the a NAIRU property that can be either backward looking, forward looking or both. In the case of discretionary policy, wehave generalised earlier results that were based on a purely forward looking model. If inflation has a backward-looking element, then the outcome of discretionary policy depends on the rate of discount.

We have also shown that, with discounting, the optimal time inconsistent path is dynamic, with initially positive inflation and output above its natural rate, followed by a steady state in which inflation may be below target (if the Phillips curve is sufficiently forward-looking). In this sense, inflation bias can indeed be negative. Even with no discounting, the commitment policy involves a short term in which output is above its natural rate and inflation is above target (although in the long run, of course, inflation will be zero in the absence of discounting).

The timeless perspective policy shares the same steady state as the commitment policy, so that inflation can be below target if there is discounting, providing that the Phillips curve is sufficiently forward-looking. However, if we start from a position in which output is at its natural rate, under the timeless perspective policy, then output stays at this level. That means that, providing the Phillips curve is sufficiently forwardlooking, there will be the negative inflation bias, without even any short run benefit of higher output.

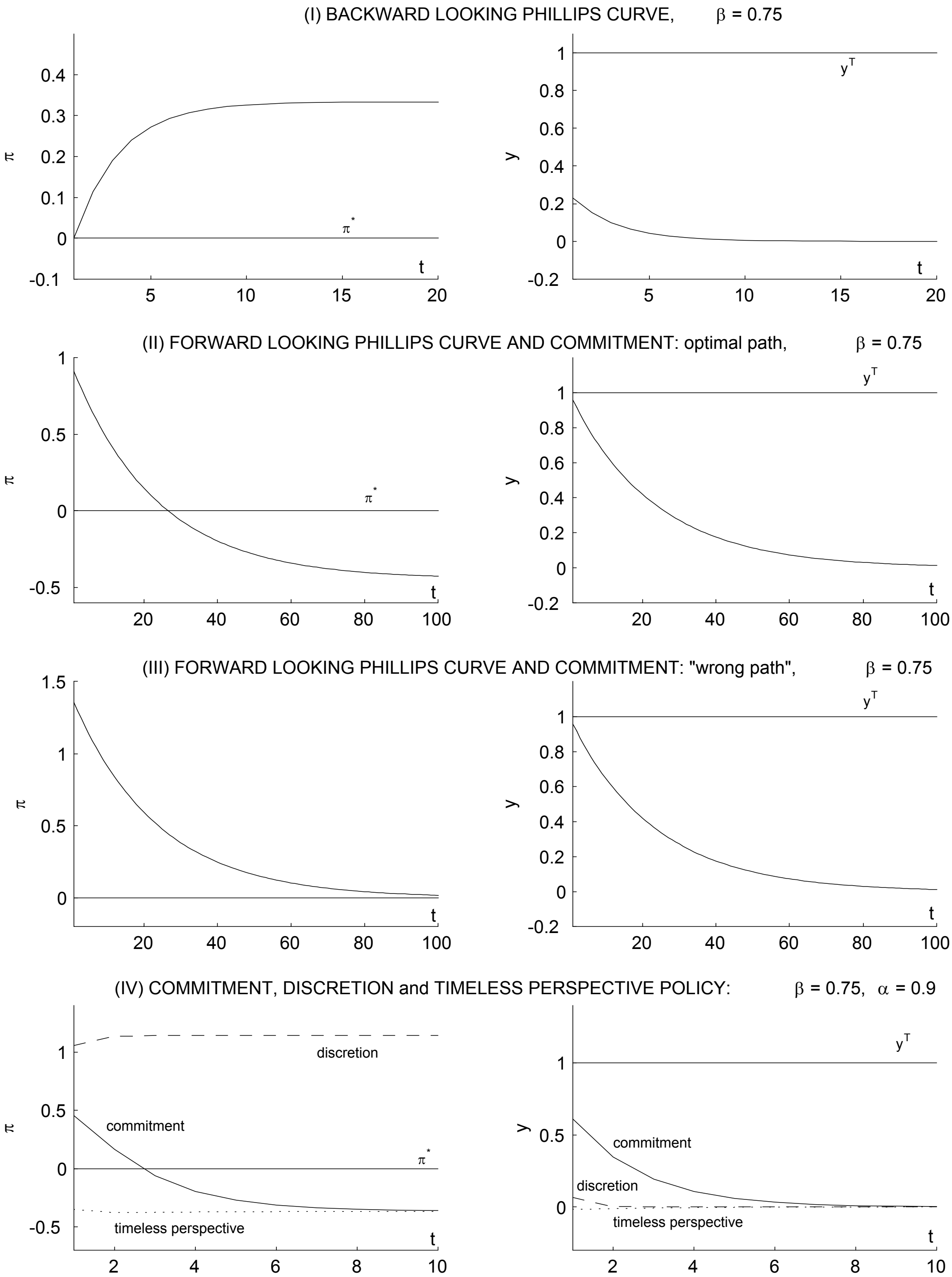

Figure 1: Dynamic Adjustment to Equilibrium

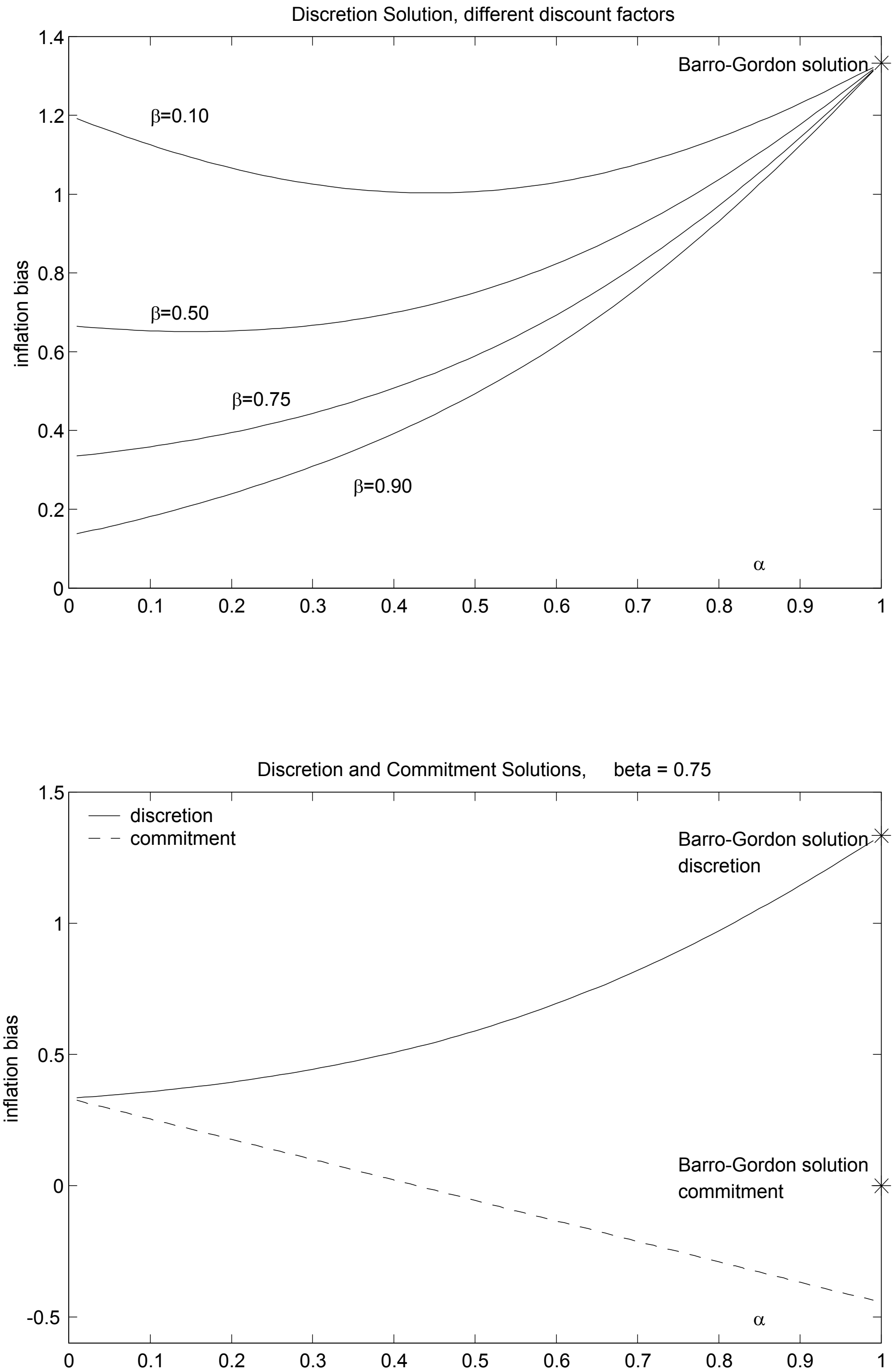

Figure 2: Inflation Bias

A The Model and its Canonical Form

The maximisation problem is stated as (1), and we consider a hybrid Phillips curve of the form 13. Note that this equation can be presented in a matrix form as

$$\begin{bmatrix} 1 & 0 \\ 0 & \alpha \end{bmatrix} \begin{bmatrix} \pi_t \\ \mathcal{E}_t \pi_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -(1-\alpha) & 1 \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ \mathcal{E}_{t-1} \pi_t \end{bmatrix} + \begin{bmatrix} 0 \\ \phi \end{bmatrix} y_t \tag{18}$$

where we assume rational expectations and

$$\pi_t = \mathcal{E}_{t-1} \pi_t$$

i.e. that today's realised inflation is equal to the one which was expected for today when people set expectations one period before today.

This describes two processes, one involving a non-predetermined (jump) variable $\mathcal{E}_{t-1}\pi_t$ and the other by a state (predetermined) variable, past inflation π_{t-1}. Using the notation of Blanchard and Kahn (1980), equation (18), which explains evolution of both variables over time, can be written in a canonical form:

$$A \begin{bmatrix} Y_{t+1} \\ X_{t+1} \end{bmatrix} = B \begin{bmatrix} Y_t \\ X_t \end{bmatrix} + U_t \tag{19}$$

This distinction between predetermined and jump variables becomes important in Section C where we solve the problem under time-consistency, where expectational variables are treated as exogenous.

B Fully Optimal Solution (Commitment)

A fully-optimal (thus time-inconsistent) solution, can be found by using Pontryagin's Maximum Principle. We form the following Hamiltonian

$$\begin{aligned} \mathcal{H} = \mathcal{E}_0 \sum_{t=0}^{\infty} (\frac{1}{2}\beta^t(\omega(\underbrace{\mathcal{E}_{t-1}\pi_t}_{X_t} - \pi^*)^2 + (\underbrace{y_t}_{U_t} - y^T)^2) + \xi_{t+1}(\underbrace{\pi_t}_{Y_{t+1}} - \underbrace{\mathcal{E}_{t-1}\pi_t}_{X_t}) \\ + \psi_{t+1}(\underbrace{\mathcal{E}_{t-1}\pi_t}_{X_t} - \phi \underbrace{y_t}_{U_t} - \alpha \underbrace{\mathcal{E}_t \pi_{t+1}}_{X_{t+1}} - (1-\alpha)\underbrace{\pi_{t-1}}_{Y_t})) \end{aligned}$$

where labels X and Y under variables indicate the treatment of variables in constraints and U is an instrument[6]. Here ψ_{t+1} is a predetermined Lagrange multiplier, associated with constraint for a jump variable $\mathcal{E}_t\pi_{t+1}$, and ξ_{t+1} is non-predetermined Lagrange multiplier which is associated with constraint for a predetermined π_t (see Currie and Levine (1993)).

[6] We could write $\mathcal{H} = \mathcal{E}_0 \sum_{t=0}^{\infty} \left(\frac{1}{2}\beta^t(\omega(\pi_t - \pi^*)^2 + (y_t - y^T)^2) + \eta_{t+1}(\pi_t - \phi y_t - \alpha \pi_{t+1} - (1-\alpha)\pi_{t-1})\right)$ without distinction of the nature of the two inflation variables. We would only take partial derivatives with respect π_t and y_t and write the first order conditions. This would lead to the same quantitative result for fully optimal problem, but the approach in the text is more general and allows easy comparison with the discretionary policy.

The first order conditions are:

$$\begin{aligned}
\frac{\partial \mathcal{H}}{\partial X_t} &= \frac{\partial \mathcal{H}}{\partial \mathcal{E}_{t-1}\pi_t} = \beta^t \omega(\mathcal{E}_{t-1}\pi_t - \pi^*) + \psi_{t+1} - \alpha\psi_t - \xi_{t+1} = 0 \\
\frac{\partial \mathcal{H}}{\partial Y_t} &= \frac{\partial \mathcal{H}}{\partial \pi_{t-1}} = -(1-\alpha)\psi_{t+1} + \xi_t = 0, \\
\frac{\partial \mathcal{H}}{\partial U_t} &= \frac{\partial \mathcal{H}}{\partial y_t} = \beta^t (y_t - y^T) - \phi\psi_{t+1} = 0 \\
\frac{\partial \mathcal{H}}{\partial \psi_{t+1}} &= \mathcal{E}_{t-1}\pi_t - \mathcal{E}_t\pi_{t+1} - (1-\alpha)\pi_{t-1} - \phi y_t = 0 \\
\frac{\partial \mathcal{H}}{\partial \xi_{t+1}} &= \pi_t - \mathcal{E}_{t-1}\pi_t = 0
\end{aligned}$$

This system collapses to the following equations, where we denoted $\beta^{-t}\psi_t = \mu_t$, $\beta^{-t}\xi_t = \lambda_t$:

$$0 = \omega(\mathcal{E}_{t-1}\pi_t - \pi^*) + \beta\mu_{t+1} - \alpha\mu_t - \beta\lambda_{t+1}, \qquad t > 0 \tag{20}$$

$$0 = -(1-\alpha)\beta\mu_{t+1} + \lambda_t, \qquad t \geq 0 \tag{21}$$

$$0 = y_t - y^T - \phi\beta\mu_{t+1}, \qquad t \geq 0 \tag{22}$$

$$0 = \mathcal{E}_{t-1}\pi_t - \alpha\mathcal{E}_t\pi_{t+1} - (1-\alpha)\pi_{t-1} - \phi y^T - \phi^2\beta\mu_{t+1}, \qquad t \geq 0 \tag{23}$$

The steady state values can be found as:

$$\pi = \pi^* + \left(1 - \beta - \alpha\frac{(1-\beta^2)}{\beta}\right)\frac{y^T}{\omega\phi} \tag{24}$$

$$\xi = -(1-\alpha)\frac{y^T}{\phi} \tag{25}$$

$$y = 0 \tag{26}$$

$$\psi = -\frac{y^T}{\phi\beta} \tag{27}$$

The dynamics towards equilibrium can be obtained by solving the system (20)-(23) numerically and imposing initial conditions $\pi_0 = \bar{\pi}, \psi_0 = 0$ and terminal conditions $\pi_\infty < \infty$, $\xi_\infty < \infty$. These four boundary conditions ensure a unique solution for this fourth order system.

C Time-Consistent Solution (Discretion)

Under discretion the policy maker does not pre-commit and re-optimises every period. This is known to the private sector and it sets expectations accordingly. The solution must satisfy the Bellman principle and solves the Bellman equation:

$$\mathcal{L}_t = \frac{1}{2}\left(\omega(\mathcal{E}_{t-1}\pi_t - \pi^*)^2 + (y_t - y^T)^2\right) + \beta\mathcal{L}_{t+1}(\mathcal{E}_t\pi_{t+1}, y_{t+1}).$$

As every period all players do the same (re-) optimisation routine the solution should be time-invariant, or time-consistent. Note that objective function depends on the forward-looking variable π_t, which is determined endogenously and depends on expected future values of π_t. The assumption of time-consistency implies that the formation of expectations is a time-invariant (linear) function

$$\mathcal{E}_{t-1}\pi_t = -J\pi_{t-1} - Ky_t - L_y y^T - L_\pi \pi^* \tag{28}$$

of all predetermined variables (including constants).

One solution routine (see Oudiz and Sachs (1985) among many others) suggests that constraint (28) should be substituted in the cost criterion which is then minimised to obtain optimal linear feedback function for the instrument (as this must be time-consistent solution). This update for a feedback reaction function is used to calculate expectations (28) using the Blanchard and Kahn (1980) formula. If this procedure converges, we obtain the time-consistent (discretionary) solution. Cohen and Michel (1988) discuss that discretionary solution can be treated as Stackelberg feedback solution with intra-period precommitment of the leading policymaker. With similar technical complexity, this solution routine is more illustrative and it is also easy to see how the polar cases can be obtained from the general solution. Additionally, the algebra is comparable with the algebra for the fully optimal solution. Therefore, we use the latter approach.

We form the following Hamiltonian:

$$\mathcal{H} = \mathcal{E}_0 \sum_{t=0}^{\infty} \left(\frac{1}{2}\beta^t (\omega(\underset{X_t}{\mathcal{E}_{t-1}\pi_t} - \pi^*)^2 + (\underset{U_t}{y_t} - y^T)^2) + \xi_{t+1} \left(\underset{Y_{t+1}}{\pi_t} - \underset{X_t}{\mathcal{E}_{t-1}\pi_t} \right) \right) \tag{29}$$

where the second constraint (from (19)) is not taken into account as the policymaker ignores the feedback from the private sector. Note that in (29) the Lagrange multiplier ξ is non-predetermined variable. Again, labels X, Y and U under variables indicate the treatment of variables.

C.1 Finding the Policy Reaction Function

The first order conditions lead to the following system:

$$\frac{\partial \mathcal{H}}{\partial U_t} + \frac{\partial \mathcal{H}}{\partial X_t}\frac{\partial X_t}{\partial U_t} = \beta^t (y_t - y_t^T) - K\left(\beta^t \omega(\mathcal{E}_{t-1}\pi_t - \pi^*) - \xi_{t+1}\right) = 0 \tag{30}$$

$$\frac{\partial \mathcal{H}}{\partial Y_t} + \frac{\partial \mathcal{H}}{\partial X_t}\frac{\partial X_t}{\partial Y_t} = -J\left(\beta^t \omega(\mathcal{E}_{t-1}\pi_t - \pi^*) - \xi_{t+1}\right) + \xi_t = 0 \tag{31}$$

$$\frac{\partial \mathcal{H}}{\partial \xi_{t+1}} = \pi_t - \mathcal{E}_{t-1}\pi_t = 0 \tag{32}$$

where formula (28) was used to determine partial derivatives of X_t. Substitute output from equation (30) and expectations from (32) into (28) and (31) and obtain the following system of the two dynamic equations:

$$J\omega\pi_t - J\beta\xi_{t+1} = J\omega\pi^* + \xi_t \tag{33}$$

$$\left(1 + K^2\omega\right)\pi_t - K^2\beta\xi_{t+1} = -J\pi_{t-1} - \left(K + L_y\right)y^T + \left(K^2\omega - L_\pi\right)\pi^* \tag{34}$$

for predetermined variable π and non-predetermined variable ξ. The steady state of this system is (we also write it for output, using (30)):

$$\pi = \frac{\left(K^2\omega - (1 + J\beta)L_\pi\right)\pi^* - (1 + J\beta)(K + L_y)y^T}{((J+1)(J\beta+1) + K^2\omega)} \tag{35}$$

$$\xi = -\frac{J\omega(1 + J + L_\pi)\pi^* + J\omega(K + L_y)y^T}{((J+1)(J\beta+1) + K^2\omega)} \tag{36}$$

$$y = \frac{((J+1)(J\beta+1) - K\omega L_y)y^T - K\omega(L_\pi + J + 1)\pi^*}{((J+1)(J\beta+1) + K^2\omega)} \tag{37}$$

For future references, note that $y = 0$ if and only if

$$((J+1)(J\beta+1) - K\omega L_y) = 0 \tag{38}$$

$$K\omega(L_\pi + J + 1) = 0 \tag{39}$$

$$\left((J+1)(J\beta+1) + K^2\omega\right) \neq 0 \tag{40}$$

these requirements will be used below.

System (33)-(34) has the following two eigenvalues:

$$r_1 = -\frac{1}{2J\beta}\left(J^2\beta + K^2\omega + 1 - \sqrt{\left(J^2\beta + K^2\omega + 1\right)^2 - 4\beta J^2}\right) \tag{41}$$

$$= -\frac{2J}{\left(J^2\beta + K^2\omega + 1 + \sqrt{\left(J^2\beta + K^2\omega + 1\right)^2 - 4\beta J^2}\right)} \tag{42}$$

$$r_2 = -\frac{1}{2J\beta}\left(J^2\beta + K^2\omega + 1 + \sqrt{\left(J^2\beta + K^2\omega + 1\right)^2 - 4\beta J^2}\right) \tag{43}$$

where the second line of representation for r_1 will be convenient to use for small J.

One eigenvalue should be inside the unit circle (the one which corresponds to predetermined variable π) and the other should be outside the unit circle (the one which corresponds to non-predetermined variable ξ). In what follows we assume that $|r_1| < 1$ but the rest can be rewritten for r_2 if needed.

Solution to the system (33)-(34) can be written as (Blanchard and Kahn (1980) formula)

$$\pi_t = a\pi_{t-1} + by^T + c\pi^*$$

$$\xi_t = d\pi_{t-1} + ey^T + f\pi^*$$

where

$$\begin{aligned}
a &= r_1 \\
b &= -\frac{(1-a)(1+J\beta)(K+L_y)}{((J+1)(J\beta+1)+K^2\omega)} \\
c &= \frac{(1-a)\left(K^2\omega-(1+J\beta)L_\pi\right)}{((J+1)(J\beta+1)+K^2\omega)} \\
d &= -\frac{\omega J^2}{(J\beta r_1+(K^2\omega+1))} \\
e &= \frac{(d(1+J\beta)-J\omega)(K+L_y)}{((J+1)(J\beta+1)+K^2\omega)} \\
f &= \frac{-d\left(K^2\omega-(1+J\beta)L_\pi\right)-J\omega(1+J+L_\pi)}{((J+1)(J\beta+1)+K^2\omega)}
\end{aligned}$$

We can also obtain the formula for output:

$$\xi_{t+1} = da\pi_{t-1} + (db+e)\,y^T_{t-1} + (dc+f)\,\pi^*_{t-1}$$

$$\begin{aligned}
y_t &= y^T_t + K\omega(\pi_t-\pi^*) - K\beta\xi_{t+1} \\
&= y^T_t + K\omega(\pi_t-\pi^*) - K\beta\left(da\pi_{t-1} + (db+e)\,y^T + (dc+f)\,\pi^*\right) \\
&= Ka(\omega-\beta d)\,\pi_{t-1} + (1+K\omega b - K\beta(db+e))\,y^T + K(\omega c-\omega-\beta(dc+f))\,\pi^* \\
&= -F\pi_{t-1} - G_\pi\pi^* - G_y y^T \qquad (44)
\end{aligned}$$

Where we denoted

$$F = -Ka(\omega-\beta d) \qquad (45)$$

$$G_\pi = -K(\omega c-\omega-\beta dc-\beta f) \qquad (46)$$

$$G_y = -(1+K\omega b-K\beta db-K\beta e) \qquad (47)$$

Therefore, we finally obtained a linear (time-consistent) representation of a rule for output, given K, J, L_y, L_π which determine expectations π^e_t.

C.2 Finding Solution for Expectations

Our economy evolves according to equation (18). Suppose that output is given as

$$y_t = -F\pi_{t-1} - G_\pi\pi^* - G_y y^T \qquad (48)$$

Substitute it into (18) and obtain the system of two equations:

$$\begin{bmatrix} \pi_t \\ \mathcal{E}_t\pi_{t+1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \frac{1}{\alpha}(\alpha+F\phi-1) & \frac{1}{\alpha} \end{bmatrix} \begin{bmatrix} \pi_{t-1} \\ \mathcal{E}_{t-1}\pi_t \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{\alpha}\phi G_y y^T + \frac{1}{\alpha}\phi G_\pi\pi^* \end{bmatrix} \qquad (49)$$

The eigenvalues of this system are:

$$k_1 = \frac{1}{2\alpha}\left(1 - \sqrt{(2\alpha-1)^2 + 4F\alpha\phi}\right) \tag{50}$$

$$k_2 = \frac{1}{2\alpha}\left(\frac{1}{2} + \sqrt{(2\alpha-1)^2 + 4F\alpha\phi}\right) \tag{51}$$

Again, we need one explosive root, corresponding to inflation expectations and one non-explosive root corresponding to realised inflation. Suppose $|k_1| < 1$. Then the solution of system (49) can be written as

$$\pi_t = g\pi_{t-1} + h\pi^* + ky^T \tag{52}$$

$$\mathcal{E}_{t-1}\pi_t = -N\pi_{t-1} - N_\pi\pi^* - N_y y^T \tag{53}$$

where

$$g = k_1 \tag{54}$$

$$h = -(1-k_1)\frac{G_\pi}{F} \tag{55}$$

$$k = -(1-k_1)\frac{G_y}{F} \tag{56}$$

$$N = -\frac{\phi F - (1-\alpha)}{(\alpha k_1 - 1)} \tag{57}$$

$$N_\pi = \frac{(\alpha(k_1-1) - F\phi)}{(\alpha k_1 - 1)}\frac{G_\pi}{F} \tag{58}$$

$$N_y = \frac{(\alpha(k_1-1) - F\phi)}{(\alpha k_1 - 1)}\frac{G_y}{F} \tag{59}$$

Representation (53) can be taken one lead forward (as did Oudiz and Sachs (1985)):

$$\begin{aligned}\mathcal{E}_t\pi_{t+1} &= -N\pi_t - N_\pi\pi^* - N_y y^T \\ &= -N\mathcal{E}_{t-1}\pi_t - N_\pi\pi^* - N_y y^T \\ &= -\frac{(1-\alpha)}{\alpha}\pi_{t-1} + \frac{1}{\alpha}\mathcal{E}_{t-1}\pi_t - \frac{\phi}{\alpha}y_t\end{aligned}$$

where the second line exploits $\pi_t = \mathcal{E}_{t-1}\pi_t$, and the third line exploits the second equation of the original system (18). The last two lines can be used in order to obtain formula for $\mathcal{E}_{t-1}\pi_t$:

$$\begin{aligned}\mathcal{E}_{t-1}\pi_t &= \frac{(1-\alpha)}{(1+\alpha N)}\pi_{t-1} + \frac{\phi}{(1+\alpha N)}y_t - \frac{\alpha N_\pi}{(1+\alpha N)}\pi^* - \frac{\alpha N_y}{(1+\alpha N)}y^T \\ &= -J\pi_{t-1} - Ky_t - L_y y^T - L_\pi\pi^*\end{aligned}$$

where in the last line we introduced conventional notation and obtained the final formula for expectations. We, therefore, obtained relationships:

$$J = -\frac{(1-\alpha)}{(1+\alpha N)} \tag{60}$$

$$K = -\frac{\phi}{(1+\alpha N)} \tag{61}$$

$$L_y = \frac{\alpha N_y}{(1+\alpha N)} \tag{62}$$

$$L_\pi = \frac{\alpha N_\pi}{(1+\alpha N)} \tag{63}$$

C.3 Iterative Procedure

To find time-consistent solution we need to iterate between the two steps explained above. Suppose we start with initial approximation of coefficient F, G_π and G_y in expression (44) for y_t. We then compute k_1 using (50) and checking that $|k_1| < 1$ and compute N, N_π and N_y using (57)-(59). Next we compute K, J, L_y and L_π from (60)-(63) and use them, first, to compute r_1 from (41) (and checking that $|r_1| < 1$) and, second, to compute an update of F, G_π and G_y from (45)-(47). We iterate until the fixed point is found.

D Timeless Perspective Policy

The required zeros in the initial conditions of the predetermined Lagrange multipliers for the commitment solution, and the implied time-inconsistency, can be explained using the observation that the system (20)-(23) is written for the time index $t = 1, 2, \ldots$ but equation (20) needs to be rewritten for $t = 0$ with $\mu_0 = 0$ while the rest of the system stays the same. We explicitly have one term less in (20) in the initial period. The private sector therefore observes that the policy maker in the initial period does something different from what it promises to do later on. Restarting the optimization at a later date yields the same first order conditions, an implied new policy in the first period and dynamic inconsistency.

A potential resolution to this problem, suggested by Woodford (1999), is to design a policy such that the first order conditions and hence the optimal policy would *look the same* for the private sector for every period of time, including $t = 0$. Formally, it means that we eliminate the predetermined Lagrange multipliers from the system and therefore have no associated problems with zero initial conditions. The resulting control rule would be time invariant.

We derive the timeless perspective policy by eliminating μ from the system (20)-(23). We use equation (22) to solve for predetermined $\mu_{t+1} = \frac{1}{\phi\beta}\left(y_t - y^T\right)$, and substitute μ into the remaining equations of the system. Note that when we substituted $\mu_t = \frac{1}{\phi\beta}\left(y_{t-1} - y^T\right)$ in equation (20) we assumed that relationship which is valid for predetermined μ for time $t+1$, is also valid at time $t = 0$, so that it is *here* that we introduce the timeless perspective policy rule.

We finally obtain the following dynamic system:

$$y_t = \beta(1-\alpha)y_{t+1} + \frac{\alpha}{\beta}y_{t-1} - \phi\omega(\pi_t - \pi^*) + \left(1 - \beta - \alpha\frac{1-\beta^2}{\beta}\right)y^T \tag{64}$$

$$\pi_t = \alpha\pi_{t+1} + (1-\alpha)\pi_{t-1} + \phi y_t \tag{65}$$

The steady state of this system is the same the one as for commitment, i.e. there is a steady state inflation bias (24).

It is apparent that timeless perspective policy introduces instrument persistence. It becomes important, therefore, to know the initial value for output, y_0. For all previously considered policies this information was not needed, as there were no constraints on the use of instrument. The dynamics towards equilibrium can be obtained by solving the system (64)-(65) numerically and imposing initial conditions $\pi_0 = \bar{\pi}, y_0 = \bar{y}$ and terminal conditions $\pi_\infty < \infty$, $y_\infty < \infty$. These four boundary conditions ensure a unique solution for this system of two equations of the second order.

E Polar Cases of General Solution

E.1 Backward-looking Phillips Curve

If the Phillips curve is backward-looking then $\alpha = 0$ and there should not be any difference between time consistent and fully optimal solutions, as there are no non-predtermined variables, which are treated differently by these two approaches. We demonstrate this

For $\alpha = 0$ the dynamic system (20)-(23) for the *fully optimal policy* collapses to the two dynamic equations:

$$\beta\mu_{t+1} = \omega(\pi_t - \pi^*) + \mu_t$$
$$\pi_t = \phi^2\mu_t + \pi_{t-1} + \phi\left(y^T + \phi\mu_t\right)$$

which eigenvalues are

$$r_1 = \frac{1}{2\beta}\left(\beta + \phi^2\omega + 1 - \sqrt{\left(\beta + \phi^2\omega + 1\right)^2 - 4\beta}\right) \tag{66}$$

$$r_2 = \frac{1}{2\beta}\left(\beta + \phi^2\omega + 1 + \sqrt{\left(\beta + \phi^2\omega + 1\right)^2 - 4\beta}\right) \tag{67}$$

where $|r_1| < 1$

The dynamic solution can be presented in the form of feedback rules (Blanchard and Kahn

(1980) formula):

$$\pi_t = r_1\pi_{t-1} + (1-r_1)\left(\pi^* + \frac{(1-\beta)\,y^T}{\omega\phi}\right) \tag{68}$$

$$y_t = \frac{\phi\,(\omega-\beta b r_1)}{(\omega\phi^2+1)}\left(\pi^* - \pi_{t-1} + \frac{(1-\beta)\,y^T}{\omega\phi}\right) \tag{69}$$

$$= -F\pi_{t-1} - G_\pi\pi^* - G_y y^T$$

$$\mu_t = -b\,(\pi^* - \pi_{t-1}) - \left(1 + \frac{b\,(1-\beta)}{\omega}\right)\frac{y^T}{\phi} \tag{70}$$

where the second representation for y_t is needed in order to make this solution comparable with derived below time consistent solution. Here

$$b = \frac{1}{2\beta\phi^2}\left(\left(1-\beta+\phi^2\omega\right) - \sqrt{\left(1+\beta+\phi^2\omega\right)^2 - 4\beta}\right)$$

and

$$F = \frac{\phi\,(\omega-\beta b r_1)}{(\omega\phi^2+1)} = \frac{r_1}{2\phi}\left(\left(\beta+\phi^2\omega-1\right) + \sqrt{\left(1+\beta+\phi^2\omega\right)^2 - 4\beta}\right) \tag{71}$$

$$G_\pi = -F \tag{72}$$

$$G_y = -\frac{(1-\beta)}{\omega\phi}F \tag{73}$$

The steady state values are obtained either directly from (20)-(23), or from (68)-(70).

For the same case of $\alpha = 0$ the under the *discretionary time consistent policy*, from (60)-(63) it immediately follows that

$$J = -1,\ K = -\phi,\ L_y = L_\pi = 0$$

Eigenvalue r_1 from (41) becomes

$$r_1 = \frac{1}{2\beta}\left(\beta+\phi^2\omega+1 - \sqrt{\left(\beta+\phi^2\omega+1\right)^2 - 4\beta}\right)$$

which is identical to the eigenvalue (66) for the time inconsistent policy.

The steady state position can be determined from (35)-(37) and it is exactly the same as for the time-inconsistent policy.

The feedback coefficients for the policy rule (45)-(47) become

$$F = \phi a\,(\omega-\beta d) = \frac{a}{2\phi}\left(\beta+\phi^2\omega-1+\sqrt{\left(\beta+\phi^2\omega+1\right)^2 - 4\beta}\right) \tag{74}$$

$$G_\pi = \phi a\,(\beta d-\omega) = -F \tag{75}$$

$$G_y = -\frac{(1-\beta)}{\phi\omega}\phi a\,(\omega-d\beta) = -\frac{(1-\beta)}{\phi\omega}F \tag{76}$$

which coincide with formulae (71)-(73) above.

E.2 Forward-looking Phillips Curve and Commitment

For $\alpha = 1$ the dynamic system (20)-(23) collapses to the following system:

$$\beta\mu_{t+1} = \mu_t - \omega(\pi_t - \pi^*) \tag{77}$$

$$\mathcal{E}_t\pi_{t+1} - \beta\phi^2\mu_{t+1} = \pi_t - \phi y^T \tag{78}$$

$$y_t = y^T + \phi\beta\mu_{t+1} \tag{79}$$

which solution can be written as

$$\begin{aligned}
\mu_t &= c\mu_{t-1} - \frac{(1-c)\,y^T}{\beta\phi}, && t > 0 \\
\pi_t &= \frac{1-\beta c}{\omega}\mu_t + \pi^* + \frac{(1-c)\,y^T}{\omega\phi}, && t \geq 0 \\
y_t &= \beta\phi c\mu_t + cy^T, && t \geq 0
\end{aligned}$$

where parameter c is non-explosive eigenvalue of the dynamic system:

$$c = \frac{1}{2\beta}\left(\left(1+\beta+\beta\phi^2\omega\right) - \sqrt{\left(1+\beta+\beta\phi^2\omega\right)^2 - 4\beta}\right) < 1 \tag{80}$$

The steady state values are determined from (77)-(78) and they are presented in the main text of the paper.

E.3 Forward-looking Phillips Curve and Discretion

Solution for $\alpha = 1$ can be obtained straightforwardly from the following formulae. Formulae (60), (42), (45), (57) applied consequently, suggest that

$$J = 0,\ r_1 = 0,\ F = 0,\ N = 0.$$

From formulae (61), (38), (39) we obtain that

$$K = -\phi,\ L_\pi = -1,\ L_y = -\frac{1}{\phi\omega}.$$

Formulae (46) and (47) yield

$$G_\pi = 0,\ G_y = 0$$

and, finally, formulae (35)-(37) give the steady state inflation, output and the Lagrange multiplier in the main text.

Note that we did not take limits anywhere, so the time-consistent solution with $\alpha = 1$ can be obtained directly by imposing linear feedback form on expectation formation and on the policymaker's reaction function.

E.4 Forward-looking Phillips Curve and Timeless Perspective Policy

The system (64)-(65) collapses for $\alpha = 1$ into the following system:

$$y_t = \frac{1}{\beta} y_{t-1} - \phi\omega(\pi_t - \pi^*) - \frac{1-\beta}{\beta} y^T \tag{81}$$

$$\pi_t = \pi_{t+1} + \phi y_t \tag{82}$$

A unique solution to this system has a time-consistent representation:

$$\begin{aligned} y_t &= c y_{t-1} \\ \pi_t &= \frac{1-\beta c}{\beta\phi\omega} y_{t-1} - \frac{1-\beta}{\phi\omega\beta} y^T + \pi^* \end{aligned}$$

where c is defined in (80).

Note that if $y_0 = 0$, then $y_t \equiv 0$ and $\pi_t \equiv \pi^* - \frac{1-\beta}{\phi\omega\beta} y^T$ for any $t \geq 0$.

F Welfare Loss

The biggest difference in welfare between the three policies is achieved in the case with entirely forward-looking Phillips curve. The levels of loss are computed by substituting adjustment path into the loss criterion (1). For $\alpha = 1$ we obtain

$$\begin{aligned} \mathcal{L}^D &= \frac{(y^T)^2}{2} \frac{\left(1+\omega\phi^2\right)}{(1-\beta)\,\omega\phi^2} = \frac{\left(y^T\right)^2}{2\,(1-\beta)} \left(\frac{1}{\omega\phi^2} + 1\right) \\ \mathcal{L}^C &= \frac{\left(y^T\right)^2}{2} \left(\frac{(1-\beta) - \sqrt{\left(\beta + \phi^2\omega\beta + 1\right)^2 - 4\beta}}{2\omega\phi^2\beta^2} + \frac{(1+\beta)}{2\beta\,(1-\beta)} \right) \\ &= \frac{\omega\phi^2\beta\left(y^T\right)^2}{(1-\beta)} \left((1-\beta)^2 + \omega\phi^2\beta\,(1+\beta) + (1-\beta)\sqrt{\left(\beta + \phi^2\omega\beta + 1\right)^2 - 4\beta} \right)^{-1} \\ \mathcal{L}^{TP} &= \frac{(y^T)^2}{2} \left(\frac{(1-\beta)}{\phi^2\omega\beta^2} + \frac{1}{(1-\beta)} \right) = \frac{\left(y^T\right)^2}{2\,(1-\beta)} \left(\frac{1}{\phi^2\omega} \frac{(1-\beta)^2}{\beta^2} + 1 \right) \end{aligned}$$

where the second line for $\mathcal{L}^C$ is convenient when taking limits.

The pairwise differences between losses are

$$\begin{aligned} \mathcal{L}^C - \mathcal{L}^{TP} &= \frac{\left(y^T\right)^2}{4\omega\phi^2\beta^2} \left(\omega\phi^2\beta + \beta - 1 - \sqrt{4\omega\phi^2\beta + \left(\omega\phi^2\beta + \beta - 1\right)^2} \right) < 0, \text{ for all } \beta \\ \mathcal{L}^C - \mathcal{L}^D &= \frac{\left(y^T\right)^2}{4\omega\phi^2\beta^2} \left(1 + \omega\phi^2\beta - \beta - \sqrt{4\omega\phi^2\beta^2 + \left(1 + \omega\phi^2\beta - \beta\right)^2} - \frac{2\beta^2}{(1-\beta)} \right) < 0, \text{ for all } \beta \\ \mathcal{L}^D - \mathcal{L}^{TP} &= \frac{\left(y^T\right)^2 (2\beta - 1)}{2\,(1-\beta)\,\omega\beta^2\phi^2} < 0, \text{ for } \beta > \frac{1}{2}. \end{aligned}$$

G New-Keynesian Phillips Curve and Commitment

We solve the problem of minimisation loss (1) subject to the New Keynesian Phillips Curve:

$$\pi_t = \gamma\pi_{t+1} + \phi y_t$$

The first order conditions (as above, or as in Woodford (2003), p. 472) are:

$$0 = \omega(\pi_t - \pi^*) + \delta\psi_{t+1} - \gamma\psi_t$$
$$0 = (y_t - y^T) - \delta\phi\psi_{t+1}$$
$$0 = \pi_t - \phi y_t - \gamma\pi_{t+1}$$

The steady state can be found as:

$$\pi = \pi^* + \frac{(\beta-\gamma)\left(y^T - \frac{(1-\gamma)}{\phi}\pi^*\right)}{\omega\phi\beta\left(1 + \frac{(\beta-\gamma)(1-\gamma)}{\omega\phi^2\beta}\right)}$$
$$y = \frac{\left(\pi^* + \frac{(\beta-\gamma)}{\omega\phi\beta}y^T\right)}{\left(\frac{\phi}{(1-\gamma)} + \frac{(\beta-\gamma)}{\omega\phi\beta}\right)}$$
$$\psi = \frac{(1-\gamma)}{\beta\phi^2}\pi - \frac{y^T}{\beta\phi}$$

We can compute the derivative

$$\frac{\partial\pi}{\partial\gamma} = -\frac{\left((\gamma-\beta) + \phi^2\omega\right)y^T - \left(1 - (\gamma-\beta)\right)\omega\phi\pi^*}{\omega^2\phi^3\beta\left(1 + \frac{(\beta-\gamma)(1-\gamma)}{\omega\phi^2\beta}\right)^2}$$

It is clear that if $\gamma > \beta$ then $\frac{\partial\pi}{\partial\gamma} < 0$ unless π^* is very high. As $\pi = \pi^*$ if $\gamma = \beta$ then bias becomes negative for $\gamma > \beta$.

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