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ABSTRACT

Strong and Weak Ties in Employment and Crime*

This paper analyses the interplay between social structure and information exchange in two competing activities, crime and labour. We consider a dynamic model in which individuals belong to mutually exclusive two-person groups, referred to as dyads. There are multiple equilibria. If jobs are badly paid and/or crime is profitable, unemployment benefits have to be low enough to prevent workers for staying too long in the unemployment status because they are vulnerable to crime activities. If, instead, jobs are well paid and/or crime is not profitable, unemployment benefits have to be high enough to induce workers to stay unemployed rather to commit crime because they are less vulnerable to crime activities. Also, in segregated neighbourhoods characterized by high interactions between peers, a policy only based on punishment and arrest will not be efficient in reducing crime. It has to be accompanied by other types of policies that take into account social interactions.

JEL Classification: A14, J40 and K42
Keywords: crime, forward-looking agents, labour market and social interaction

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1 Introduction

Social interactions and peer effects have proved to be crucial in various aspects of economic activities, including education, crime, smoking, teenage pregnancy, school dropout, etc. (see Durlauf, 2004, for a survey). In the present paper, we focus on the role of social contacts in crime and our main objective is to show how policies aiming at reducing crime are affected by the mode of socialization between agents.

For that, we distinguish between weak and strong ties in the pattern of social interactions.[1] Following Granovetter (1973), we consider that the strength of a social tie corresponds to the duration of a relationship. We define as *strong tie* a social relationship between two agents that is repeated over time (for example members of the same family or very close friends) and as *weak tie* a transitory social encounter between two persons.[2] We show that different modes of socialization affect differently the agents' incentives to enter either the labor or the crime market. The structure of social interactions thus affects the aggregate crime and employment level in the economy, and has consequences for the design of optimal crime policies.

To be more precise, we consider a model in which individuals belong to mutually exclusive two-person groups, referred to as *dyads*. Dyad members do not change over time so that two individuals belonging to the same dyad hold a *strong tie* with each other. However, each dyad partner can meet other individuals outside the dyad partnership, referred to as *weak ties* or random encounters. By definition, weak ties are transitory and only last for one period.

We then assume that individuals learn about crime opportunities by interacting with active criminals. These interactions can take the form of either strong or weak ties. The process through which individuals learn about crime behavior and opportunities results from a combination of a socialization process that takes place *inside* the family (in the case of strong ties) and a socialization process *outside* the family (in the case of weak ties). Bisin and Verdier (2000) refer to the former as *vertical* socialization and to the latter as *oblique* socialization. Both currently active criminals and potential criminals exert an influence over one another to commit offences by meeting each other. In contrast, we assume that

[1] The impact of labor market outcomes on crime has been modeled in different ways (see in particular the recent contributions of İmrohoroğlu *et al.* 2000, Burdett *et al.* 2003, and Verdier and Zenou 2004) but the role of friends and peers on crime has received so far less attention (exceptions include Sah 1991, Glaeser *et al.*, 1996, Calvó-Armengol and Zenou 2004, Silverman, 2004, and Ballester *et al.* 2004, 2005).

[2] Montgomery (1994) uses a similar model of weak and strong ties in the labor market.

individuals learn about job opportunities exclusively through employment agencies.[3]

We analyze the flows of dyads between states and characterize all the steady-state equilibria of this dynamic economy. For this purpose, we solve for the endogenous individual decisions to switch between the three possible statuses, that is, criminal, unemployed and employed. We work throughout with forward-looking agents, who anticipate fully the impact of their current decisions on their future opportunities and payoffs.

Four equilibria can emerge, that differ in their composition by agent's statuses. In one equilibrium, all agents are unemployed. We also find two polar equilibria composed either of criminals and unemployed agents, or employed and unemployed agents. Finally, a mixed equilibrium exists, where both criminals, employed and unemployed workers coexist. We characterize the ranges of exogenous parameter values for which each of those equilibria emerges. Multiple equilibria only arise for a particular range of values, where both a completely mixed economy and a degenerate economy composed solely of criminals can emerge.

We then analyze how endogenous outcomes (crime, employment and unemployment) respond to variations of the exogenous parameters. This comparative static exercise sheds light on the interplay between the crime market and the labor market, and illuminates the impact of pure labor market interventions or pure crime policies on both markets. Pure labor market interventions consist on modifying the unemployment benefit, while pure crime policies impinge on deterrence. In substance, we show how altering agents' incentives in one market spills over to the other (related) market. Thus, deterrence affects unemployment rates while unemployment benefits influence crime rates. This relationship is not trivial. It depends on the relative gains from crime and the labor market, and also from the social cohesion of the economy.

When jobs are badly paid and/or crime is profitable, we show that the aggregate crime level increases with the unemployment benefit. The labor market regulation has thus an impact on crime rates and, here, the optimal policy to reduce crime consists on decreasing the unemployment insurance. The reason is the following. First, when the unemployment benefit is low, the opportunity cost of searching a job decreases, and workers have more incentives to find jobs quickly. As such, the unemployment spell decreases. Second, note that unemployed workers are more prone to enter in the crime business than employed workers. This is because their rents are lower. A shorter unemployment spells thus reduces the workers' exposure to crime opportunities. Therefore, through its dynamic effect on the duration of unemployment, a reduction in unemployment benefits decreases aggregate crime.

[3]For a model of job information gathering through social contacts, see Calvó-Armengol and Jackson (2004).

Suppose now that workers are well paid and/or crime is not profitable case. With a similar reasoning, we can conclude that a higher unemployment benefit induces workers to stay unemployed longer rather than to commit crime, and the crime rate decreases.

Unemployment, in our model, is not only the “waiting room” for employment (as it is usually perceived) but also for crime. Unemployed workers trade off the costs and benefits from becoming employed or a criminal. In a dynamic setting, the opportunity cost of searching for a good job becomes a crucial determinant of this trade-off. The impact of the unemployment insurance on crime thus depends on the relative values of being employed or criminal. Our analysis suggests that an optimal unemployment benefit policy should discriminate among the different characteristics of local labor markets.

Beyond agents’ incentives, the pattern of social interactions also shapes market outcomes and affects the effectiveness of policy interventions.

Recall that, in our model, crime opportunities are only disseminated through word-of-mouth communication among criminals, and between criminals and jobless agents. The information flows from crime insiders to crime outsiders depend on the frequency of encounters between the two types of agents. If transitory encounters are rare and most interactions take place within strong ties, crime opportunities only flow within dyads composed of a criminal and an unemployed, but almost never across dyads. Crime feeds itself with crime, and there is little osmosis between crime and unemployment. Suppose, instead, that transitory encounters are more frequent. Then, many interactions take the form of occasional weak ties outside best-friend partnerships. Information flows are not circumscribed to the dyad but are very intense across dyads, and crime opportunities spread widely in the society. Unemployed agents, which are also would-be criminals, now face a high chance to undertake an illegal activity. Crime thus feeds itself with both crime and labor because of the strong connection between these two markets.

A first consequence of this observation is that an increase in the frequency of weak ties raises crime but reduces both employment and unemployment. When people spend most of their time in extroverted interactions with outside peer (weak ties), and are not stuck to introverted meetings with the reduced circle of best friends (strong ties), the crime rate soars to very high level while employed falls down sharply. Again, this is because when one is pulled towards crime activities and both his best friend and peers are criminal, it becomes extremely difficult to go back to the labor market. You need that you *and* your best friend are caught, and then you get a job offer, an event that can take quite a long time

This interplay between modes of socialization and crime and labor market outcomes has implications for the design of crime policies. The direct effect of increasing the arrest probability is, of course, to pull criminals outside from crime into unemployment, the doorstep for employment. But, in our model, unemployment is also the "waiting room" for crime, not only for employment. Therefore, the actual decrease in crime following an increase in deterrence depends on the unemployment-to-crime flows. These flows are higher when weak tie encounters dominate strong tie interactions. The impact of higher deterrence is thus relatively moderate under frequent random encounters, compared to the case where agents meet within their circles of close friends.

Acknowledging the fact that homogamy favors socialization (Conley and Topa, 2002), neighborhood segregation usually fosters broad socialization patterns at the neighborhood level, and outside inner family circles. In these cases, a policy based only on punishment and arrest will not be that efficient in reducing crime. It has to be accompanied by other types of policies designed from a community-wide, multiple solution perspective to the crime problem, rather than from a purely individualistic approach.

Related literature It is well-recognized the labor market opportunities have a strong impact on criminal behavior. For instance, there are also sizable and significant effects of unemployment (Raphael and Winter-Ebmer, 2001), wages (Machin and Meghir, 2004) and inequality (Bourguignon *et al.*, 2003) on crime.

More important for our purpose, friends and, more generally, the social environment, is also conducive to criminal behavior. For instance, the positive correlation between self-reported delinquency by adolescent and the number of delinquent friends is among the strongest and most consistent findings in the delinquency literature (Warr, 1996, Matsueda and Anderson, 1998). More precisely, Glaeser *et al.* (1996) find that across crimes, crime committed by younger people has higher degrees of social interaction, while, across cities, for serious crimes in general and for larceny and auto theft in particular, the degree of social interactions is larger in those communities where families are less intact, that is, have more female-headed households. Ludwig *et al.* (2001) and Kling *et al.* (2005) use data from the Moving to Opportunity (MTO) experiment, which relocates families from high- to low-poverty neighborhoods. They show that this experiment reduces juvenile arrests for violent offences by 30 to 50 per cent of the arrest rate for control groups. In their study of a gang located in a black inner-city neighborhood, Levitt and Venkatesh (2000) also find that social/nonpecuniary factors play an important role in criminal decisions and gang activities.

More recently, using a very detailed data-set of friendship networks in the United States from the National Longitudinal Survey of Adolescent Health (AddHealth), Calvó-Armengol *et al.* (2005) test directly the impact of social networks on juvenile crime. They show that, after controlling for observable individual characteristics and unobservable network specific factors, the individual's position in a network is a key determinant of his or her level of criminal activity. A standard deviation increase in an individual's centrality in a network increases the level of individual delinquency by 45 per cent of one standard deviation. Using the same dataset, Patacchini and Zenou (2005) find that conformity is very strong within groups of delinquents and that the higher the taste for conformity of an individual, the lower the deviation from the norm's group. Their results suggest that, for teenagers, the decision to commit crimes is not a simple choice based primarily on individual considerations but is strongly affected by their environment and peers.[4]

The rest of the paper is organized as follows. The model is described in Section 2. We first focus on imperfectly myopic agents. The steady-state equilibrium analysis is in Section 3, while Section 4 is devoted to the comparative statics exercise. Section 5 analyses the case of perfectly forward-looking agents, and both a theoretical analysis and numerical simulations are proposed. All proofs are relegated to an Appendix.

2 The model

Consider a population of individuals of size one. Individual are either employed, unemployed, or involved in criminal activities. Time is continuous and indexed by t, and individuals live for ever.

Dyads We assume that individuals belong to mutually exclusive two-person groups, referred to as *dyads*. We say that two individuals belonging to the same dyad hold a *strong tie* with each other. We assume that dyad members do not change over time. A strong tie is created once and for ever, and can never be broken. We can thus think of strong ties as links between members of the same family, or between very close friends.

Individuals can be in either of three different states: employed, unemployed or criminals. Dyads, which consist of paired individuals, can thus be in six different states:[5]

(i) both members are employed −we denote by d_2 the number of such dyads;

(ii) one member is employed and the other is unemployed (d_1);

[4] Chen and Shapiro (2003) and Bayer *et al.* (2003) find also strong peer effects in crime by investigating the influence that individuals serving time in the same facility have on the subsequent criminal behavior of offenders.

[5] The inner ordering of dyad members does not matter.

(iii) both members are unemployed (d_0);

(iv) one member is unemployed and the other is a criminal (d_{-1});

(v) both members are criminals (d_{-2});

(vi) one member is employed while the other is a criminal ($d_{\pm 1}$).

We exclude this last type of dyad, in which one member is employed and the other is a criminal. This is explained below.

Aggregate state Denoting by e_t, u_t and c_t respectively the employment rate, the unemployment rate, and the crime rate at time t, where $c_t, e_t, u_t \in [0, 1]$, we have:

$$\begin{cases} e_t = 2d_{2,t} + d_{1,t} \\ u_t = 2d_{0,t} + d_{1,t} + d_{-1,t} \\ c_t = 2d_{-2,t} + d_{-1,t} \end{cases} \tag{1}$$

The population normalization condition can then be written as

$$e_t + c_t + u_t = 1 \tag{2}$$

or, alternatively,

$$d_{-2,t} + d_{-1,t} + d_{0,t} + d_{1,t} + d_{2,t} = \frac{1}{2} \tag{3}$$

Social interactions We assume that individuals randomly meet by pairs repeatedly through time. Matching can take place between dyad partners or not. At each period, any given individual is matched with his dyad partner with probability $1 - \omega$, while he is matched randomly to any other individual in the population with complementary probability ω.

We refer to matchings inside the dyad partnership as *strong ties*, and to matchings outside the dyad partnership as *weak ties* or random encounters.[6]

Therefore, each individual is born with a strong tie and will spend all his life with this person (think for example of a brother, a sister or a best mate). The identity of the strong tie partner is fixed for ever. However, throughout his life, each individual also meets other people that are not as close as his strong tie, the weak ties. These interactions are transitory, and the identity of the weak tie partner changes at each encounter.

Within each matched pair, information is exchanged, as explained below.

[6]Another way of interpreting ω and $1 - \omega$ is to view them as the time spent per period with weak and strong ties respectively. Indeed, if each individual has one unit of time per period then, on average, he spends ω units of time with one individual outside the dyad (weak tie) and $1 - \omega$ units of time with his dyad partner (strong tie).

Information transmission Unemployed workers hear of job vacancies at exogenous rate λ while employed workers lose their job at exogenous rate δ. All jobs and all workers are identical (unskilled labor).

Symmetrically, delinquents are aware of some criminal activity at exogenous rate α. Delinquents pass this information on to their current matched partner, be it a strong tie or a weak tie. Information about crime is thus essentially obtained through friends and relative (i.e. strong and weak ties), whereas information about jobs flows mainly through formal methods (without the help of any ties).

This information transmission protocol defines a Markov process. The state variable is the relative size of each type of dyad. Transitions depend on the labor market and crime turnover, and on the nature of social interactions as captured by ω.

We assume that, during a small interval of time t and $t + dt$, at most one dyad partner hears of an exogenous offer, and that both members of a dyad cannot change their status at the same time.

For example, two unemployed workers cannot at the same time either find a job or become criminal, i.e. during t and $t + dt$, the probability assigned to a transition from a d_0-dyad to either a d_2-dyad or a $d_{-2}-$dyad is zero. Similarly, two employed workers (d_2-dyad) or two criminals ($d_{-2}-$dyad) cannot become both unemployed, i.e. switch to a d_0-dyad during t and $t + dt$. Of course, this does not imply that a d_0-dyad can never become a d_2-dyad. It just means that a d_2-dyad cannot switch to a d_0-dyad during t and $t + dt$; it just takes more time.

This applies to all the other dyads mentioned above.

Incentives The material payoff is w for an employed worker, and b for an unemployed worker, where b refers to the unemployment insurance benefit. We assume that $w > b$.[7]

Unemployed workers decide between becoming a criminal, staying unemployed, or becoming employed. Individuals are forward-looking with respect to their future status when taking this decision, and anticipate the impact of current decisions on their future opportunities and payoffs. Yet, they are myopic with respect to the status of their current partner, which they treat as a default state. In Section 5 we relax this assumption and extend the analysis to the case where individuals are forward-looking both for their own status and that of their strong tie partner.

[7] We keep the wage w fixed throughout. A partial equilibrium approach renders the analysis more tractable without affecting qualitatively the results. See Burdett *et al.* (2003) for a general equilibrium analysis where the wage is endogenous.

In the long-run, individual values for each possible dyad outcome are given by the following Bellman equations.

Each Bellman equation is written for the individual with the first subscript. For example, V_{01} is the lifetime expected utility of an unemployed worker whose strong tie is currently employed, while V_{10} corresponds to the lifetime expected utility of an employed worker with an unemployed dyad partner.

We have:

$$rV_{11} = w + \delta \left(V_{01} - V_{11} \right) \tag{4}$$

$$rV_{01} = b + \lambda \left(V_{11} - V_{01} \right) \tag{5}$$

$$rV_{10} = w + \delta \left(V_{00} - V_{10} \right) \tag{6}$$

$$rV_{00} = b + \lambda\phi \left(V_{10} - V_{00} \right) + q(c)\psi \left(V_{-10} - V_{00} \right). \tag{7}$$

In these expressions, r is the interest rate, $q(c) = \omega c \alpha$ denotes the rate at which individuals hear from a crime opportunity from a weak tie.

The parameters ϕ and ψ account for the *endogenous decisions* to accept a job and a crime opportunity, respectively. These are binary values, that is, $\phi, \psi \in \{0, 1\}$, where $\phi = 1$ (resp. $\psi = 1$) when an unemployed worker accepts a job (resp. becomes a criminal), and $\phi = 0$ (resp. $\psi = 0$) otherwise.

We assume that $w > b$ so that $V_{11} > V_{01}$.[8] We have:

$$rV_{-10} = g + p \left(F_{00} - V_{-10} \right),$$

where g denotes the proceeds from crime, p the rate at which criminal are caught and:

$$F_{00} = -f + V_{00},$$

where $f > 0$ is the penalty associated with jail.

So, when a criminal is caught, he spends some time in prison $(-f)$ and then with probability 1 gets out. Observe that these equations are written under the assumption that, once released, a criminal becomes automatically unemployed. The time spent in prison is also taken short enough so that the strong tie's status does not change meanwhile.

[8] Substracting (5) to (4) gives $(r + \delta + \lambda) (V_{11} - V_{01}) = w - b$.

Combining, we obtain:

$$rV_{-10} = g - pf + p\left(V_{00} - V_{-10}\right). \tag{8}$$

We also have:

$$rV_{0-1} = b + h(c)\psi_c\left(V_{-1-1} - V_{0-1}\right), \tag{9}$$

where $h(c) = (1 - \omega + \omega c)\alpha$ is the probability to hear from a crime opportunity either by a weak or a strong tie (α is the rate at which 'potential' criminals hear from a crime opportunity). The expected payoffs from crime for a criminal associated with a criminal is V_{-1-1}.

The parameter ψ_c is the probability to become a criminal because of a peer-pressure that favors conformism. Again, this is a binary variable, where $\psi_c = 1$ when peer-pressure induces to become a criminal, and $\psi_c = 0$, otherwise.

We have:

$$rV_{-1-1} = g + p\left(F_{0-1} - V_{-1-1}\right)$$

Also:

$$F_{-1-1} = -f + V_{0-1}.$$

The expected lifetime utility of a criminal (whether his strong tie is unemployed or criminal) consists of today's expected gain g plus the expected probability to be caught p times the expected utility loss.

In our definition, a criminal (identified by -1) is not necessary someone who is committing a crime but someone who is actively (full time) searching for a crime opportunity and will commit a crime if he obtains the information on a crime opportunity (this occurs at rate α). As a result, this individual will never obtain a job since he is not looking for it.

Observe that, contrary to a "criminal", an unemployed worker whose strong tie is unemployed (his utility is V_{00}) does not always accept a crime opportunity, only when $\psi = 1$.

Combining:

$$rV_{-1-1} = g - pf + p\left(V_{0-1} - V_{-1-1}\right). \tag{10}$$

Observe also that strong ties and thus conformity matter a lot in this model.

Indeed, as stated above, employed workers are incompatible with criminals, and vice-versa because strong ties influence each other. Conformism prevents individuals to switching to polar activities than that of their strong tie. As a result, no employed-criminal dyad of the type $+1-1$ can ever form.

Furthermore, the behavior of an unemployed worker is totally different whether his strong tie is employed, unemployed or criminal. Indeed, if an unemployed worker has a strong tie who is employed (dyad 01), he will always accept a job opportunity (see (5)) whereas if his strong tie is unemployed (dyad 00), he will accept a job opportunity with probability ϕ (see (7)). Similarly, an unemployed whose strong tie is criminal (dyad $0-1$) will always accept to commit a crime (see (9)), whereas the same unemployed whose strong tie is unemployed (dyad 00) will accept a crime opportunity with probability ψ (see (7)).

All the assumptions are compatible with a model in which individual's behavior is strongly influenced by his friends and relatives (peers).

Flows of dyads between states It is readily checked that the net flow of dyads from each state between t and $t+1$ is given by:[9]

$$\begin{cases} \dot{d}_{2,t} = \lambda d_{1,t} - 2\delta d_{2,t} \\ \dot{d}_{1,t} = 2\phi\lambda d_{0,t} - (\delta+\lambda)\, d_{1,t} + 2\delta d_{2,t} \\ \dot{d}_{0,t} = p d_{-1,t} - 2\left[\psi q(c_t) + \phi\lambda\right] d_{0,t} + \delta d_{1,t} \\ \dot{d}_{-1,t} = 2p d_{-2,t} - \left[p + h(c_t)\psi_c\right] d_{-1,t} + 2\psi q(c) d_{0,t} \\ \dot{d}_{-2,t} = -2p d_{-2,t} + h(c_t)\psi_c d_{-1,t} \end{cases} \tag{11}$$

Graphically,

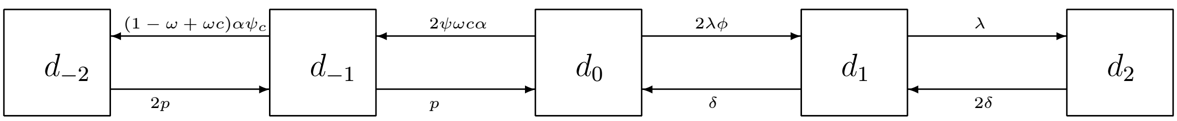

Figure 1

These dynamic equations reflect the flows across dyads.

For instance, in the first equation, the variation of dyads composed of two employed workers ($\dot{d}_{2,t}$) is equal to the number of d_1-dyads in which the unemployed worker has found a job minus the number of d_2-dyads in which one the two employed has lost his job. In the last equation, the variation of dyads composed of two criminals ($\dot{d}_{-2,t}$) is equal to the number of $d_{-1}-$dyads in which the unemployed has become a criminal (through either his strong tie with probability $\alpha(1-\omega)$ or his weak tie with probability $\alpha\omega c$) minus the number of $d_{-2}-$dyads in which one the two criminals has been caught.

[9] Recall that $q(c_t) = \alpha\omega c_t$, while $h(c_t) = \alpha(1-\omega+\omega c_t)$.

All the other equations have a similar interpretation.

Note that this dynamic system would be relatively easy to analyze if it weren't for the fact that flows between d_{-2}, d_{-1} and d_0 depend on $c = 2d_{-2} + d_{-1}$, an endogenously determined parameter.

Observe that the assumption stated above that both members of a dyad cannot lose their status at the same time is reflected in the flows described by (11).

Take for example the dyad d_{-2}. To switch to a d_0-dyad, it will take at least two periods. During the first period, there is a probability $2p$ for a $d_{-2}-$dyad to become a $d_{-1}-$dyad (indeed it has to be that only one of them has been caught, that is, either the first or the second member of the $d_{-2}-$dyad; this occurs with probability $p + p = 2p$). Then, for the second period, there is a probability p (since now there is only one member who is criminal) for a $d_{-1}-$dyad to become a d_0-dyad. What is crucial in our analysis is that members of the same dyad (strong ties) always stay together throughout their life. So, for example, if a $d_{-1}-$dyad becomes after some periods a d_1-dyad, the members of this dyad are exactly the same; they have just changed their status.

Observe also that the encounter of weak ties is "localized".

Take for example an unemployed who belongs to a $d_{-1}-$dyad. The only random encounters (weak ties) he can meet is among the pool of criminals and unemployed, and therefore will only obtain information about crime but not job opportunities (this is our definition of active criminal).

What we capture here is the fact that the type of random encounters is strongly influenced by the status of the partner in the dyad (conformism, as in Akerlof 1987). There is indeed a complementarity between strong and weak ties since someone's strong tie influences the nature of his relationship with random encounters. This is consistent with our assumption that criminals and employed workers are never partners and thus do not form strong ties with each other leading to $d_{\pm1}-$dyads.

Taking into account (3), the system (11) reduces to a four-dimensional dynamic system in $d_{2,t}, d_{1,t}, d_{-1,t}$ and $d_{-2,t}$ given by:

$$\begin{cases} \overset{\bullet}{d}_{2,t} = \lambda d_{1,t} - 2\delta d_{2,t} \\ \overset{\bullet}{d}_{1,t} = 2\phi\lambda\left[1/2 - d_{-2,t} - d_{-1,t} - d_{1,t} - d_{2,t}\right] - (\delta + \lambda)\, d_{1,t} + 2\delta d_{2,t} \\ \overset{\bullet}{d}_{-1,t} = 2p d_{-2,t} - \left[p + h(c_t)\psi_c\right] d_{-1,t} + 2\psi q(c)\left[1/2 - d_{-2,t} - d_{-1,t} - d_{1,t} - d_{2,t}\right] \\ \overset{\bullet}{d}_{-2,t} = -2p d_{-2,t} + h(c_t)\psi_c d_{-1,t} \end{cases}$$

where, using (1):

$$\begin{cases} e_t = 2d_{2,t} + d_{1,t} \\ c_t = 2d_{-2,t} + d_{-1,t} \end{cases}$$

3 Steady-state equilibrium analysis

Steady-state dyad flows At a steady-state $(d_2^*, d_1^*, d_0^*, d_{-1}^*, d_{-2}^*)$, each of the net flow in (11) is equal to zero. Setting these net flows equal to zero leads to the following relationships:

$$d_2^* = \frac{\lambda}{2\delta} d_1^*, \qquad d_1^* = \frac{2\phi\lambda}{\delta} d_0^* \tag{12}$$

$$d_{-2}^* = \frac{(1 - \omega + \omega c^*)\, \psi_c \alpha}{2p} d_{-1}^*, \qquad d_{-1}^* = \frac{2\psi\omega c^* \alpha}{p} d_0^* \tag{13}$$

where

$$e^* = 2d_2^* + d_1^* \text{ and } c^* = 2d_{-2}^* + d_{-1}^* \tag{14}$$

$$d_0^* = \frac{1}{2} - d_2^* - d_1^* - d_{-1}^* - d_{-2}^* \tag{15}$$

Definition 1 *A steady-state dyad flows equilibrium is a seven-tuple* $(d_2^*, d_1^*, d_0^*, d_{-1}^*, d_{-2}^*, u^*, c^*)$ *such that equations (12), (13), (14) and (15) are satisfied.*

The following result identifies all steady-state equilibria and provides conditions for their existence.

Proposition 1 (steady-state) *There are four different steady-state dyad flows equilibria:*

(i) A full-unemployment steady-state $\mathcal{U}$ *with* $c^* = e^* = 0$ *when* $\phi = 0$ *and* $\psi, \psi_c \in \{0, 1\}$.

(ii) A crime-free steady-state $\mathcal{E}$ *with* $c^* = 0$ *and* $e^* > 0$ *when* $\phi = 1$ *and* $\psi, \psi_c \in \{0, 1\}$.

(iii) A no-employment steady-state $\mathcal{C}$ *with* $e^* = 0$ *and* $c^* > 0$ *when either* $\phi = 0, \psi = 1, \psi_c = 0$ *and* $p < \alpha\omega$, *or* $\phi = 0, \psi = \psi_c = 1$ *and* $p < \alpha[\omega + \sqrt{\omega(4 - 3\omega)}]/2$.

(iv) A mixed steady-state $\mathcal{M}$ *with* $e^* > 0$ *and* $c^* > 0$ *when either* $\phi = \psi = 1$, $\psi_c = 0$ *and* $p\,(1 + \lambda/\delta)^2 < \alpha\omega$, *or* $\phi = \psi = \psi_c = 1$ *and* $p < X(\omega)$, *where* $X(\omega)$ *is a uniquely defined function* $X : [0, 1] \to [0, \alpha]$.[10]

[10] We have $X(\omega) = \alpha\omega x(\omega)$, where $x(\omega)$ is the unique positive root of the following three-degree polynomial

$$x^3 - \left[\frac{1}{\omega} - \frac{\lambda}{\delta}\left(2 + \frac{\lambda}{\delta}\right)\right] x - \frac{1}{\omega} - 1.$$

This proposition states that four steady-state equilibria may emerge. The proof consists on two lemmata. Lemma 1 characterizes the values for the seven-tuple $(d_2^*, d_1^*, d_0^*, d_{-1}^* d_{-2}^*, u^*, c^*)$ for each possible steady-state $\mathcal{U}$, $\mathcal{E}$, $\mathcal{C}$ and $\mathcal{M}$. These values all derive from the positive root of a quadratic equation on d_0. Lemma 2 then provides conditions for this root to belong to $[0, 1/2]$, the range of possible values for d_0^*.

At $\mathcal{U}$, the economy is populated only with dyads of two unemployed agents, that is, $d_0^* = 1/2$. At $\mathcal{E}$, there are three types of dyads, d_0^*, d_1^* and d_2^*. These two equilibria can exist for any value of p. This is not true for the other steady-state.

Consider for instance the no-employment steady-state $\mathcal{C}$. This steady-state arises for low enough values of the apprehension probability. Indeed, when p is too high, crime is reduced and employment becomes positive. The upper bound on p for $\mathcal{C}$ to arise is $X(\omega)$, which increases with ω, the frequency of random encounters. Indeed, when ω increases, individuals diversify their crime information sources, and the no-employment steady-state $\mathcal{C}$ can be sustained under higher punishment levels.

The intuition is the same for the mixed steady-state $\mathcal{M}$.

Observe that our dynamic system has multiple steady-state. When $\phi = 0$ and $\psi = 1$, both $\mathcal{U}$ and $\mathcal{C}$ can coexist, while when $\phi = \psi = 1$, both $\mathcal{E}$ and $\mathcal{M}$ can coexist.

Unemployed workers' decisions Proposition 1 characterizes all the steady-state dyad flows for the dynamic process displayed in Figure 1 as a function of the exogenous parameters $\delta, \lambda, \alpha, p$ and ω for this dynamic process. These steady-state dyad flows also depend on the unemployed workers' decisions captured by the binary variables ϕ, ψ and ψ_c, and that enter the Markov transitions of the dynamic process.

We now solve for the whole economy equilibrium by endogeneizing the parameters ϕ, ψ and ψ_c. This amounts to solving the seven Bellman equations (14) to (10) defined in Section 2. The solutions to these Bellman equations depend on the exogenous parameters of the economy that correspond to the contemporaneous payoffs arising from the different actions, b, w, g and f, and on the exogenous parameters for the dyad dynamics that shape future payoffs, $\delta, \lambda, \alpha, p$ and ω.

Recall that we conduct a partial equilibrium analysis where the wage w is treated as exogenous. This simplifying assumption allows to pin down precisely the impact of the network, captured through ω, on crime behavior and labor market outcomes. Burdett *et al.* (2003) offers a general equilibrium analysis where the wage is endogenous, but which does

See details in Lemma 2, Appendix 1. That the image of $X(\omega)$ is $[0, \alpha]$ is established in Lemma 5, Appendix 2.

not contemplate the role of networks.

The seven Bellman equations can be split into two different groups. Some equations can be solved two by two. These are equations (4) and (5) for V_{11} and V_{01}, and equations (9) and (10) for V_{-1-1} and V_{0-1}. The three remaining equations, (6), (7) and (8) form a system of three equations with three unknowns .

We first solve two pairs of Bellman equations. From (4) and (5) we easily obtain:

$$V_{11} - V_{01} = \frac{w - b}{r + \lambda + \delta},$$

whereas (9) and (10) yield to:

$$V_{-1-1} - V_{0-1} = \frac{g - pf - b}{r + p + \psi_c h(c^*)},$$

where $h(c^*) = \alpha\,(1 - \omega + \omega c^*)$.

Thus

$$\psi_c = \begin{cases} 1 & \Leftrightarrow & V_{-1-1} \geq V_{0-1} & \Leftrightarrow & g - pf \geq b \\ 0 & \Leftrightarrow & V_{-1-1} < V_{0-1} & \Leftrightarrow & g - pf < b \end{cases} \tag{16}$$

As it can be seen, all the results are independent of ϕ and ψ. This is a side-effect of conformism, according to which individuals are strongly influenced by their strong ties in their decisions to commit crime.

We now solve for the last block of equations (6), (7) and (8), which will give us the different values of ϕ and ψ. Substracting equations two by two, we get:

$$\phi = \begin{cases} 1 & \Leftrightarrow & V_{10} \geq V_{00} & \Leftrightarrow & (w - b)\,(r + p) \geq q(c)\psi\,(g - pf - w) \\ 0 & \Leftrightarrow & V_{10} < V_{00} & \Leftrightarrow & (w - b)\,(r + p) < q(c)\psi\,(g - pf - w) \end{cases} \tag{17}$$

$$\psi = \begin{cases} 1 & \Leftrightarrow & V_{-10} \geq V_{00} & \Leftrightarrow & (r + \delta + \lambda\phi)\,(g - pf) \geq \lambda\phi w + (r + \delta)\,b \\ 0 & \Leftrightarrow & V_{-10} < V_{00} & \Leftrightarrow & (r + \delta + \lambda\phi)\,(g - pf) < \lambda\phi w + (r + \delta)\,b \end{cases} \tag{18}$$

This is very intuitive. Recall that $\phi = 1$ (resp. $\psi = 1$) when unemployed take a job offer (resp. a crime opportunity).

An unemployed worker whose strong tie is unemployed will accept a job opportunity if the wage net of unemployment benefit is large enough compared to crime proceeds net of wage. These conditions are similar to the ones found in the search literature (see in particular Burdett *et al.* 2003) where w and $g - pf$ correspond to the reservation wage and crime gain, respectively.

Denote by $c^*(1,1,1)$ and $c^*(0,1,1)$ the equilibrium crime levels when $\phi = \psi = \psi_c = 1$, and $\phi = 0, \psi = \psi_c = 1$, respectively. Note that $c^*\,(0,1,1) > c^*\,(1,1,1)$.[11]

[11] See Lemma 4 in the Appendix.

Proposition 2 (equilibria) *Let $w > b$ and $p < \min\{X(\omega), \alpha[\omega + \sqrt{\omega(4-3\omega)}]/2\}$.*

A. Suppose first that $w > g - pf$. Then,

(i) if $b \leq g - pf - [w - (g - pf)]\,\lambda/\,(r + \delta)$, both the mixed and the crime-free steady-state equilibria $\mathcal{M}$ and $\mathcal{E}$ coexist.

(ii) if $g - pf - [w - (g - pf)]\,\lambda/\,(r + \delta) < b < w$, there exists a unique (no crime) steady-state equilibrium $\mathcal{E}$.

B. Assume now that $w \leq g - pf$. Then,

(i) if $b < w + [w - (g - pf)]\,\alpha\omega c^*(0,1,1)/\,(r + p)$, both the mixed and the crime-free steady-state equilibria $\mathcal{M}$ and $\mathcal{E}$ coexist.

(ii) if $w + [w - (g - pf)]\,\alpha\omega c^*(0,1,1)/\,(r + p) \leq b < w + [w - (g - pf)]\,\alpha\omega c^*(1,1,1)/\,(r + p)$, there are multiple equilibria, since the mixed, the crime-free and the no-employment steady-state equilibria $\mathcal{M}$, $\mathcal{E}$ and $\mathcal{C}$ coexist.

(iii) if $w + [w - (g - pf)]\,\alpha\omega c^*(1,1,1)/\,(r + p) \leq b < w$, there exists a unique (no-employment) steady-state equilibrium $\mathcal{C}$.

This result completely characterizes the steady-state equilibria, illustrated in Figure 2.

[*Insert Figure* 2 *here*]

This characterization suggests very different policy implications depending on the values of the wage w, the proceeds from crime g and the punishment cost pf. We distinguish two cases.

On one hand, when $w \leq g - pf$, the unemployment benefit has to be low enough to prevent workers from staying too long in the unemployment status (d_0) because they are vulnerable to crime activities. In this case, unemployed agents are more likely to take crime opportunities (instead of entering the labor market) since the returns from crime high relative to labor outcomes.

On the contrary, when $w > g - pf$, the unemployment benefit has to be high enough to induce workers to stay unemployed rather than committing commit crime. They are now less prone to choose crime activities and more attracted by the labor market.

More generally, our results suggest that the planner can use the unemployment benefit to control unemployment spells. Indeed, because of the intra-dyad pressure for conformism, unemployment plays the role in our model of a "waiting room" for employment (as it is usually perceived) but also for crime. While unemployed, agents receive offers both from the crime sector and from the labor market. These two types of offers arrive at different rates and deliver different payoffs.

When the relative gains from the labor market dominate those from crime (that is, $w > g - pf$) the planner can decrease the opportunity cost to stay unemployed by raising the unemployment benefit. By doing so, unemployed workers can experience longer unemployment spells, thus increasing their chance to be employed (instead of becoming delinquents).

On the contrary, when crime payoffs are higher than expected wages (that is, $w \leq g - pf$), a lower unemployment benefit discourage unemployed workers to search longer for (relatively more) profitable crime opportunities.

Our results thus shed some light on the crucial debate around the unemployment benefit policy (see for example the survey by Atkinson and Micklewright, 1991). Some think that it has to be low enough to induce workers to accept low-paid jobs whereas others postulate that it has to be high enough to help unemployed workers to survive. Here the implications are different since labor and crime activities affect each other. It is only in areas where job opportunities are low (badly paid) and crime profitable (for example drug activities) that one has to set a low level of unemployment benefit. By doing so, the planner can optimally control the time spent in the "waiting room" by unemployed workers.

4 Comparative statics

Let us perform some comparative statics analysis for the steady-state equilibrium $\mathcal{M}$.

We first analyze the response of the employment, unemployment and crime rates to variations of the deterrence effort p and the labor market turnover δ and λ.

Proposition 3 *Let $w > b$ and consider the equilibrium $\mathcal{M}$. Then,*

(i) The number of d_0^*, d_1^*, d_2^*-dyads and the employment rate e^* increase with p and λ, and decrease with α and δ; they also decrease with ω on $[0, \overline{\omega}] \subset [0, 1]$, for some given upper bound $0 < \overline{\omega} < 1$.

(ii) The crime rate c^* increases with δ and decreases with λ.

(iii) The sum $c^* + u^*$ of the crime and the unemployment rate decreases with p and λ, and increases with α and δ; it also increases with ω on $[0, \overline{\omega}]$.

First, when punishment p increases,[12] dyads d^*_{-2} and d^*_{-1} are destroyed at a faster rate since criminals are caught more often. The number of d^*_0-dyads thus increases. As a result, the relative chance to find a job is higher (since individuals are more often in a d^*_0-dyad), and thus the number of d^*_1, d^*_2-dyads increases. Altogether, the employment rate e^* rises.

Second, increasing p has an ambiguous effect on the crime rate c^* and on the unemployment rate u^*. Indeed, when p increases two forces are at work. There is a *direct negative effect* on crime since dyads d^*_{-2} and d^*_{-1} are destroyed at a faster rate. This triggers an increase in the number of d^*_0-dyads, unemployed agents akin both to undertake a criminal activity or to take a job. But, then, this larger pool of would-be criminals can favor a quicker return of individuals to criminal activities. This is the *indirect positive effect*. The simulations reported in the next section show that the direct negative effect always predominates, and that the indirect positive effect gets weaker as p increases. In short, the crime rate decreases with p at a faster rate.

The simulations also show that d_{-1} varies non-linearly with p, first increases and then decreases.[13] This non-linear variation of d^*_{-1} is the cause of the sign ambiguity of c^* in p, and the need to resort to numerical simulations to sort it out.[14] This non-linear variation, together with the (numerically documented) monotonicity of c^* in p, implies that the composition of the crime pool in $d^*_{-1}, d^*_{-2}-$dyads depends on the punishment rate p. For low punishment rates p, active criminals have almost always strong ties with other criminals, and $d^*_{-2}-$prevail. When criminals are rarely apprehended, both dyad members end up involved in illicit activities because they share crime opportunities with each other, and they rarely quit the crime market. Instead, when the punishment rate p is high, active criminals are most often embedded in mixed $d^*_{-1}-$dyads, and have unemployment strong ties. In this case, criminals are often apprehended, and the situation where both strongly tied agents are active criminals does not last long. Therefore, the extent to which a high p decreases the crime rate depends on the number information sources on crime opportunities that are available outside the strong dyadic partner. When ω is low, few sporadic weak-tie encounters occur, and an increase in p decreases c significantly. Instead, when ω is high, transitory encounters (weak ties) constitute valuable information sources that dampen the effect of the apprehen-

[12]Recall that p is *not* a probability but a rate, so its value is between 0 and $+\infty$.

[13]These simulations correspond to the case where agents are forward-looking with respect to their own state (as here) as well as that of their dyad partner. See the next section for details.

[14]In fact, when p increases, the sum $d^*_{-1}+d^*_{-2}$ decreases. This is simply a consequence of (i) in Proposition 3 and of (15). But this does not allow us to sign the direction of movements of d^*_{-1} and $d^*_{-2}-$dyads separately. Therefore, one cannot exclude the case where d^*_{-2} increases and d^*_{-1} decreases in such a way that $c^* = 2d^*_{-2} + d^*_{-1}$ increases, but $u^* = d^*_{-1} + 2d^*_0 + d^*_1$, $d^*_{-1} + d^*_2$ and $c^* + u^*$ all decrease.

sion probability. In this case, an increase in p has a lower impact on c. Section 5.2 examines the connections between c, p and ω in more details.

Third, and not surprisingly, an increase in the job destruction rate δ, or a decrease in the job acquisition rate λ or the crime opportunity rate α, reduces the employment rate e^* and increases the crime rate c^*. This result is however interesting since it links the business cycle to crime rates so that in booms (resp. downturns) crime is reduced (resp. increased), a fact largely documented (Freeman 1999 is a good survey).

Finally, we now investigate the impact of weak and strong ties on crime and unemployment.

In our model, individuals belong to mutually exclusive groups, the dyads. The intra-dyad interaction is a tight and permanent relationship, the *strong tie*. If all social relations were only strong ties, the population would be fragmented into two-agents clusters with no communication whatsoever among them. Agents also interact outside their dyads. Such inter-dyad interactions, transitory by nature, are referred to as *weak ties*. Weak tie interactions spread information across dyads. In our model, the information that travels across dyads corresponds to criminal opportunities. The parameter ω measures the proportion of social interaction that occurs outside the dyad, the inter-dyad interactions. High values of ω thus correspond to frequent encounters between unemployed and delinquents. When ω is low, criminals form impermeable and clustered social groups. When ω is high, instead, the social cohesion among criminals is lower, and delinquents and unemployed are in close contact with each other.

The impact of ω on crime is ambiguous. First, increasing ω induces more transitions from unemployment to crime, as dyads d_{-1} and d_{-2} are created at a faster rate. Observe that transitory encounters between criminals and unemployed dwellers in the form of weak ties are indeed the ones that pull d_0-dyads to $d_{-1}-$dyads. This is a *positive direct effect*. But, because of this positive direct effect, the number of d_0-dyads may itself decrease. In fact, Proposition 3 (i) above establishes that d_0^* varies negatively with ω, at least in some range $[0, \overline{\omega}]$. This corresponding reduction in would-be criminals then favors a subsequent decrease in crime. This is a *negative indirect effect*. Simulations in the next section show that the direct positive effect predominates, while the indirect negative effects gets stronger as ω increases. In short, c increases with ω with decreasing marginal returns.

5 Extension: forward-looking dyads

So far, we assumed that agents in a dyad were imperfectly forward-looking since they were forward looking for themselves but not for their strong tie in the dyad. We now relax this assumption by assuming that all agents are perfectly forward looking both for themselves and for their strong tie.

5.1 Theory

Let us first write the different Bellman equations when all agents are forward-looking for their dyad. Let us start with the dyads who are participating in the labor market. We can now write (4), (5), (6) and (7) as:

$$rV_{11} = w + \delta\left(V_{01} - V_{11}\right) + \delta\left(V_{10} - V_{11}\right) \tag{19}$$

$$rV_{01} = b + \lambda\phi_{01}\left(V_{11} - V_{01}\right) + \delta\left(V_{00} - V_{01}\right) \tag{20}$$

$$rV_{10} = w + \delta\left(V_{00} - V_{10}\right) + \lambda\phi_{10}\left(V_{11} - V_{10}\right) \tag{21}$$

$$rV_{00} = b + \lambda\phi_{e0}\left(V_{10} - V_{00}\right) + q(c)\psi_{c0}\left(V_{-10} - V_{00}\right) + \lambda\phi_{0e}\left(V_{01} - V_{00}\right) + q(c)\psi_{0c}\left(V_{0-1} - V_{00}\right) \tag{22}$$

In equation (19), the two dyad partners are employed and we take the point of view of the first individual. He earns a wage w today and may lose his job at rate δ. However, since he is perfectly forward-looking, he takes into account the fact that his dyad partner can also lose his job at rate δ. Equations (20), (21) and (22) are interpreted similarly.

Observe that each individual, when he takes into account the employment decision of his friend, he takes this decision as if it was his own. Take for example (20). The individual is unemployed today while his friend is employed; he accepts a job if and only if $V_{11} > V_{01}$. Take now (21). The individual is employed and his friend is unemployed. When the first individual considers the employment decision of his friend, he takes this decision as if it was his own, that is, he knows his friend will always accept a job if only if $V_{11} > V_{01}$. The same reasoning applies for ϕ_{e0} and ϕ_{0e}. As a result, we have:

$$\phi_{01} = \phi_{10} = \phi_c \text{ and } \phi_{e0} = \phi_{0e} = \phi.$$

We now rewrite (8), (9) and (10), the Bellman equations for perfectly forward-looking agents participating in the crime market. We obtain:

$$rV_{-10} = g - pf + p\left(V_{00} - V_{-10}\right) + h(c)\psi_{-10}\left(V_{-1-1} - V_{-10}\right) \tag{23}$$

$$rV_{0-1} = b + h(c)\psi_{0-1}\left(V_{-1-1} - V_{0-1}\right) + p\left(V_{00} - V_{0-1}\right) \tag{24}$$

$$rV_{-1-1} = g - p\,f + p\left(V_{0-1} - V_{-1-1}\right) + p\left(V_{-10} - V_{-1-1}\right) \tag{25}$$

As above, because each individual, when taking into account the crime decision of his friend, decides as if it was himself, we have

$$\psi_{-10} = \psi_{0-1} = \psi_c \text{ and } \psi_{c0} = \psi_{0c} = \psi$$

This is also consistent with the previous section when agents were partially forward looking. Because the analysis is quite complicated, we assume in this section that w is large enough compared to b so that it is always worthwhile to take a job, that is $V_{11} \geq V_{01}$ and $V_{10} \geq V_{00}$, or equivalently $\phi_c = \phi = 1$. This also avoid to deal with uninteresting trivial equilibria such as $\mathcal{U}$ and $\mathcal{C}$ where nobody is employed (i.e. $e^* = 0$).

The Bellman equations can now be written as:

$$rV_{11} = w + \delta\left(V_{01} - V_{11}\right) + \delta\left(V_{10} - V_{11}\right) \tag{26}$$

$$rV_{01} = b + \lambda\left(V_{11} - V_{01}\right) + \delta\left(V_{00} - V_{01}\right) \tag{27}$$

$$rV_{10} = w + \delta\left(V_{00} - V_{10}\right) + \lambda\left(V_{11} - V_{10}\right) \tag{28}$$

$$rV_{00} = b + \lambda\left(V_{10} - V_{00}\right) + \lambda\left(V_{01} - V_{00}\right) + q(c)\psi\left(V_{-10} - V_{00}\right) + q(c)\psi\left(V_{0-1} - V_{00}\right) \tag{29}$$

$$rV_{-10} = g - pf + p\left(V_{00} - V_{-10}\right) + h(c)\psi_c\left(V_{-1-1} - V_{-10}\right) \tag{30}$$

$$rV_{0-1} = b + h(c)\psi_c\left(V_{-1-1} - V_{0-1}\right) + p\left(V_{00} - V_{0-1}\right) \tag{31}$$

$$rV_{-1-1} = g - p\,f + p\left(V_{0-1} - V_{-1-1}\right) + p\left(V_{-10} - V_{-1-1}\right) \tag{32}$$

The main problem is that all these equations are linked together, mainly through V_{00}. As a result, we cannot, as in the previous sections, solve independently different blocks of equations.

However, we can still characterize the equilibria. We have indeed the following proposition, which is the counterpart of Proposition 1 when $\phi = 1$.

Proposition 4 *There are two possible steady-state equilibria and we have:*

(i) When $\psi = 0$ and $\psi_c \in \{0, 1\}$ or $\psi = 1$ and $\psi_c = 0$, there exists a unique crime-free steady-state equilibrium $\mathcal{E}$ with $c^* = d^*_{-2} = d^*_{-1} = 0$, and $u^* = \delta / (\delta + \lambda)$.

(ii) Assume that $p < X(\omega)$. When $\psi = \psi_c = 1$, there exists both a crime-free $\mathcal{E}$ and a mixed steady-state equilibrium $\mathcal{M}$ given by:

$$u^* = 2\left[\frac{\lambda}{\delta} - (1-\omega)\frac{\alpha}{p}\right] d_0^* + \frac{p}{\alpha\omega} \qquad \text{and} \qquad c^* = \frac{1}{2d_0^*}\left(\frac{p}{\alpha\omega}\right)^2 - \frac{1}{\omega}\left(1 + \frac{p}{\alpha}\right) + 1, \tag{33}$$

where d_0^* is the unique positive solution to:

$$\left[\frac{\lambda}{\delta}\left(2 + \frac{\lambda}{\delta}\right) - (1-\omega)\frac{\alpha}{p}\right] d_0^{*2} - \frac{1}{2\omega} d_0^* + \frac{1}{4}\left(\frac{p}{\alpha\omega}\right)^2 = 0 \tag{34}$$

5.2 Numerical simulations

We now calibrate the model. Here is the baseline calibration.

The period is the month. The interest rate is $r = 0.00333$, which implies the standard 4% annual interest rate.

For the labor market, we assume a wage of $w = 2.5$ and the unemployment benefit is set to $b = 1.25$, so that the replacement rate is $b/w = 0.5$ (Postel-Vinay and Robin, 2002, and Burdett *et al.*, 2004). The job destruction rate is equal to $\delta = 0.03$, that is, workers keep on average their job a little bit more than 33 months or nearly 3 years (Postel-Vinay and Robin, 2002). The job acquisition rate is equal to $\lambda = 0.077$, that is, the unemployment duration is on average a little bit more than a year. In the crime market, the penalty is equal to $f = 0.25$ and the gain from crime is $g = 2.5$, which means that the gain is exactly the monthly wage while the penalty amounts to roughly one fourth of the monthly wage (İmrohoroğlu *et al.*, 2000, 2004). Individuals hear from a crime opportunity at rate $\alpha = 1$, that is, one every month on average. The rate at which criminals are caught is set to $p = 0.05$, meaning that, on average, criminals are active during 20 months.

We have thus adopted parameter values that are consistent with a neighborhood experiencing difficulties where jobs do not last very long while unemployment is relatively long and gains from work are at least as high as those of crime.

We focus here on the network aspects of the crime market, captured by ω, the percentage of weak ties. In the benchmark case, we start with $\omega = 0.05$. We then vary this parameter to see how it affects the key endogenous variables.

Table 1 gives the results for the benchmark case when equilibria $\mathcal{M}$ and $\mathcal{E}$ coexist (as described by Proposition 4 (ii)).

We have calibrated this neighborhood to display the salient features of a difficult neighborhood: high unemployment rate (19.28%), high crime rate (35.76%) and relatively low employment rate (44.96%) at the equilibrium $\mathcal{M}$. At the crime-free equilibrium $\mathcal{E}$, instead, both unemployment and employment are higher (28.04% and 71.96%, respectively). At $\mathcal{M}$, 55.04% of people are not working while this number is "only" 28.04% in equilibrium $\mathcal{E}$.

At equilibrium $\mathcal{M}$, the labor and crime figures are mainly due to the low percentage of dyads d_0^*, d_{-1}^*-dyads, with either two unemployed or one unemployed and one criminal best friends, and the high percentage of d_1^*, d_2^*, d_{-2}^*-dyads, with either one unemployed and one employed, two employed of two criminals best friends. The rate $h\left(c\right)^*$ at which unemployed hear from a crime opportunity either from a weak or a strong tie, and the rate $q(c^*)$ at which crime opportunities only come from a weak tie, are $h(c^*) = 0.9679$ and $q(c^*) = 0.0179$. This means that individuals hear from a crime opportunity mainly from strong ties, and the number of d_{-2}^*-dyads is disproportionate with respect to that of d_{-1}^* dyads. Criminals' best friends are almost always criminals themselves.

Table 1: Benchmark Case

	Equilibrium $\mathcal{M}$	Equilibrium $\mathcal{E}$
$u^*(\%)$	19.28	28.04
c^* (%)	35.76	0
e^* (%)	44.96	71.96
d_0^*	0.0246	0.0393
d_1^*	0.1261	0.2018
d_2^*	0.1618	0.2589
d_{-1}^*	0.0175	0
d_{-2}^*	0.1700	0
$q(c^*)$	0.0179	0
$h(c^*)$	0.9679	0.95
V_{11}	669.15	648.10
V_{01}	658.99	636.77
V_{10}	670.32	648.10
V_{00}	664.46	636.77
V_{-10}	691.20	–
V_{0-1}	689.99	–
V_{-1-1}	692.39	–
r=0.0033, w=2.5, b=1.25, δ=0.03, λ=0.077, α=1, f=0.25, g=2.5, p=0.05, ω=0.05		
$X(\omega)=\alpha\omega x(\omega)$=0.183		

We now focus on equilibrium $\mathcal{M}$, and see how variations in ω (weak ties) and p (the rate at which criminals are caught) affect the key variables of this model.

Let us start with the most interesting aspect of our model, the effect of ω on crime, unemployment and employment. Figure 3 displays results that are consistent with Proposition 3 for e^*, and that further allow us to disentangle the separate effect on c^* and u^*. We find that an increase in ω raises crime but reduces both employment and unemployment. This is quite intuitive since when ω increases, individuals hear more about crime opportunities from their weak ties and since they always accept a crime opportunity at an equilibrium $\mathcal{M}$,[15] dyads d_{-2}^* and d_{-1}^* become more numerous while the other employment and unemployment dyads are reduced. When people spend most of their time with their peers (weak ties), the crime rate soars to very high level while employed falls down sharply. Again, this is because

[15]Note that $V_{-10} > V_{00}$ and $V_{-1-1} > V_{0-1}$ in Table 1.

when one is pulled towards crime activities and both his best friend and peers are criminal, it becomes extremely difficult to go back to the labor market. You need that you *and* your best friend are caught and then obtain a job offer. This can take quite a long time. Recall that, here, individuals are perfectly forward looking also for their dyad partner, so that they take into account the fact that an increase in ω also increases the chance for their best friend to become criminal. This exacerbates the effect of weak ties on crime.

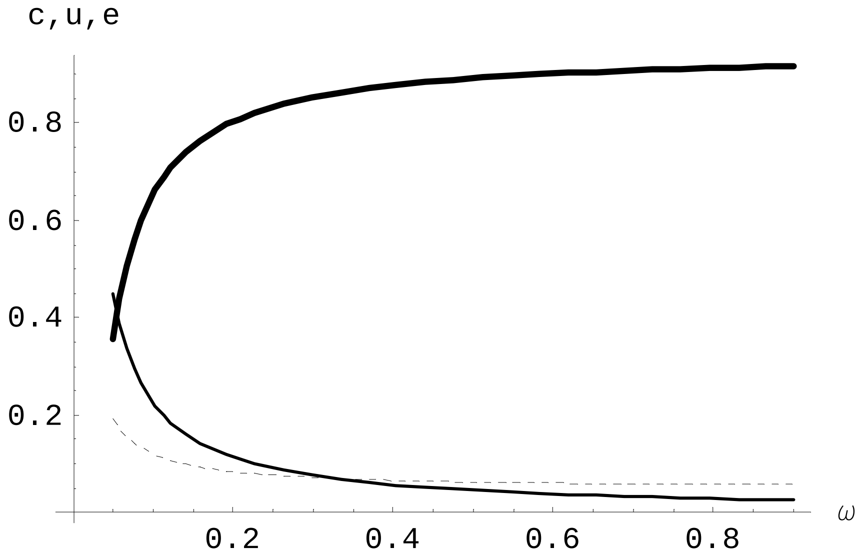

Figure 3: Impact of weak ties ω on crime c^* (solid thick curve), unemployment u^* (dashed curve) and employment e^* (solid thin curve)

Concerning p, the rate at which criminals are caught (Figure 4a), we obtain results consistent with the crime literature and our Proposition 3. More punishment reduces crime and increases both unemployment and employment. The relatively surprising result on unemployment is due to the fact that, in our model, an individual cannot be unemployed and commit crime at the same time, so when p increases, $d^*_{-2}, d^*_{-1}-$dyads are destroyed at a faster rate, which in turn implies that more d^*_0-dyads are created.

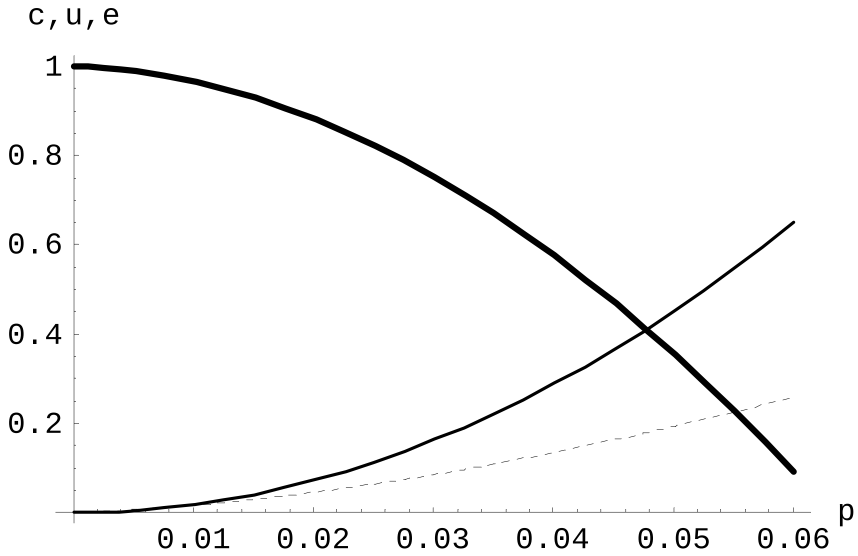

Figure 4a: Impact of arrest rate p on crime c^* (solid thick curve), unemployment u^* (dashed curve) and employment e^* (solid thin curve)

Another interesting result is the non-linearity of d_{-1}^*-dyads following an increase in p (Figure 4b). Indeed, for both low and high p, there are few d_{-1}^*-dyads because, in the former, criminals are rarely caught so, once a d_{-2}^*-dyad is formed, it is rarely destroyed while, in the latter, the overall crime rate is low, and so is the number of d_{-1}^*-dyads.

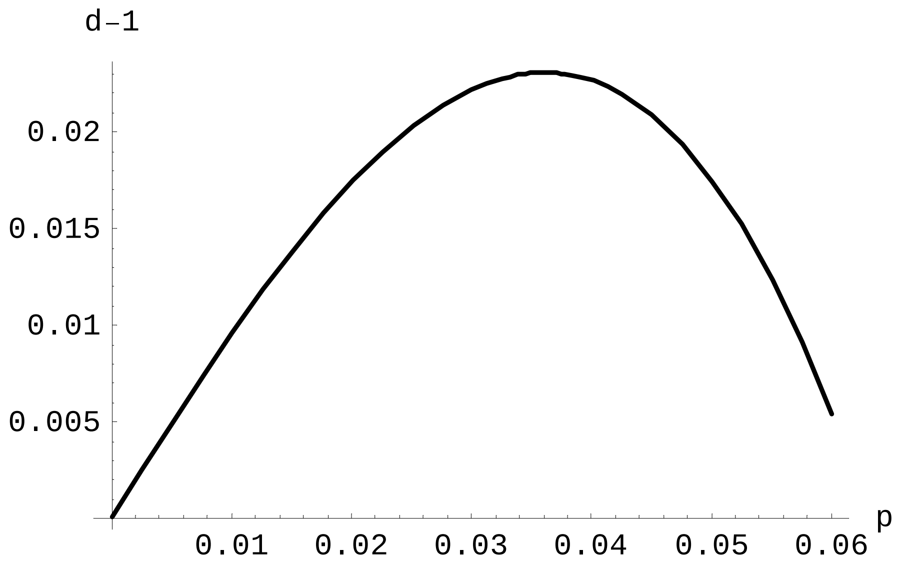

Figure 4b: Impact of arrest rate p on $d_{-1}-$dyads.

We know from Figure 3 that crime increases with weak ties ω, while Figure 4a indicates that raising p decreases crime. We now investigate the interplay between p and ω, and their connection to the crime rate.

Inspecting the transitions in Figure 1, and holding c constant, a low (resp. high) level of ω implies that there are few (resp. many) transitions from d_0 to d_{-1}, but many (resp. few) from d_{-1} to d_{-2}. When ω is low, an individual has thus little chance to become a criminal unless his dyad partner is already a criminal. Recall that crime opportunities are only disseminated through word-of-mouth from crime market insiders to outsiders. Therefore, when most interactions occur within dyads, crime opportunities can only reach an unemployed agent provided his strong tie dyadic partner is criminal himself. In this case, when two best friends are unemployed (dyad d_0), lacking information on crime opportunities, they are pulled towards the labor market. On the contrary, when one of the two friends is a criminal (dyad d_{-1}), they are pulled towards the crime market because the criminal provides information on crime to his best friend, and a the dyad changes to d_{-2}. Consequently, when ω is low, crime feeds itself with crime only, and there is little osmosis between crime and unemployment.

Consider now the case when ω is high. With more weak tie encounters, the crime and the labor market are closer to each other. Even for an unemployed agent in a d_0-dyad, the chance to become a criminal is high since frequent weak tie encounters provide information on crime opportunities. In that case, crime feeds itself with both crime and labor because of the strong connection between these two markets.

Figure 5a show that, when ω increases, the ratio dyads d_{-2} to crime c decreases, implying that the ratio dyads d_{-1} to crime c increases.

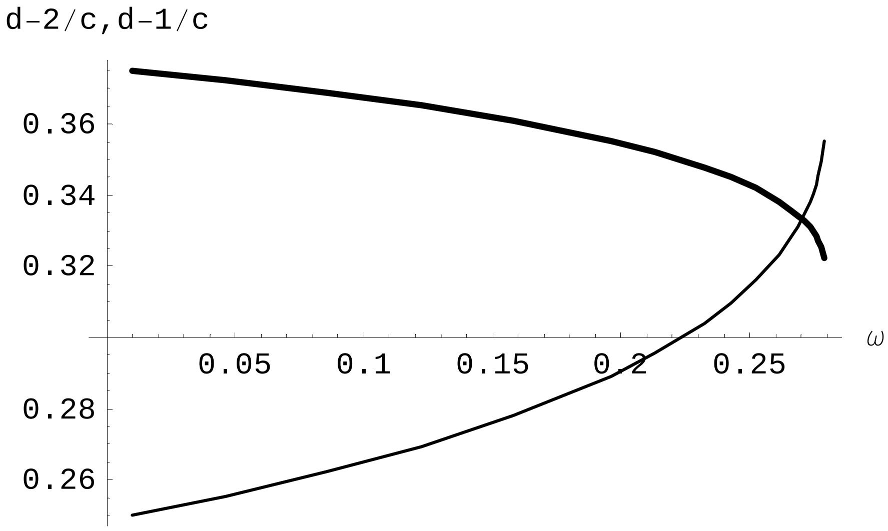

Figure 5a: Impact of weak ties ω on dyads per crime for dyads d_{-2} (d_{-2}/c; solid thick curve) and dyads d_{-1} (d_{-1}/c; solid thin curve).

This has of course crucial implication in terms of policy, that is, the impact of arrest rate p on crime. Because of our discussion above, the efficacy of this policy strongly depends on the level of ω. Figure 1 shows that, whatever the value of ω, increasing p empties dyads d_{-2} (at speed $2p$) and dyads d_{-1} (at speed p), and therefore crime c decreases.

When ω is low, once dyads d_{-2} and d_{-1} are destroyed and dyads d_0 are formed, there is no way back to crime since there are very few transitions between dyads d_0 and d_{-1} (both because ω *and* c are low). In that case, increasing p has a strong negative impact on crime. Consider now the case when ω has a high value. Increasing p still increases the number of d_0-dyads in the waiting room for employment or crime. Now, since ω is high, the transition from d_0 to d_{-1} is fast, and this huge inflow to crime thwarts the direct impact of the increase in p. In that case, a sole policy of p is not very efficient in reducing crime. It has to be accompanied by another policy based on ω.

Figure 5b show the variation of c in p for different values of ω. Consistent with our intuition above, an increase in p is much more efficient in reducing crime when weak ties are not frequent ($\omega = 0.05$) than when they are very frequent ($\omega = 0.5$ or $\omega = 0.95$). Indeed, switching from $p = 0$ to $p = 0.04$ reduces crime rate from 100% to 50% when $\omega = 0.05$, while when $\omega = 0.5$ or $\omega = 0.95$, the reduction is from 100% to roughly 95%.

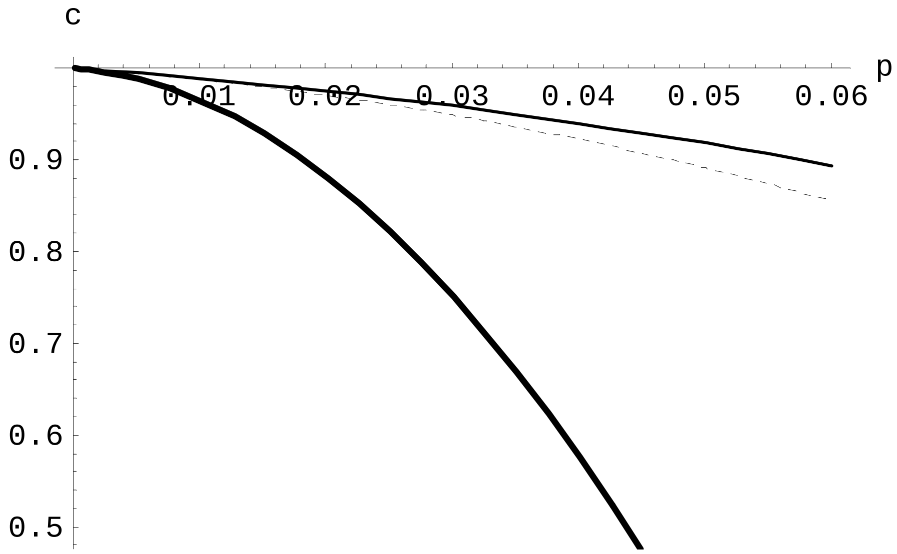

Figure 5b: Impact of arrest rate p on crime c^* for low ($\omega = 0.05$; solid thick curve), medium ($\omega = 0.5$; dashed curve) and high-values ($\omega = 0.95$; solid thin curve) of weak ties.

In segregated neighborhoods characterized by high interactions between peers (high ω), a policy only based on punishment and arrest will not be that efficient in reducing crime. It has to be accompanied by other types of policies. As suggested by Akerlof (1997) and empirically confirmed by Patacchini and Zenou (2005), the decision to commit crimes is not a simple choice based primarily on individual considerations but is strongly affected by environment and peers. This suggests that one should adopt a community-wide, multiple solution approach to the crime problem rather than a purely individualistic approach.

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Appendix

Proof of Proposition 1: We establish the proof in two steps. First, Lemma 1 characterizes all steady-state dyad flows. Lemma 2 then provides conditions for their existence.

Lemma 1 *There exists at most four different steady-state equilibria: (i) a full-unemployment equilibrium $\mathcal{U}$ such that $c^* = e^* = 0$, (ii) a crime-free equilibrium $\mathcal{E}$ such that $c^* = 0$ and $e^* > 0$, (iii) a mixed equilibrium $\mathcal{M}$ such that $c^* > 0$ and $e^* > 0$, and (iv) a no-employment equilibrium $\mathcal{C}$ such that $c^* > 0$ and $e^* = 0$.*

Proof. By combining (12) to (14), we easily obtain:

$$e^* = \frac{2\lambda(\lambda+\delta)\phi}{\delta^2} d_0^* \tag{35}$$

$$c^* = [(1-\omega+\omega c^*)\psi_c\alpha + p]\frac{2\psi\omega c^*\alpha}{p^2} d_0^* \tag{36}$$

We consider four different cases.

(i) If $\phi = \psi = 0$, $\psi_c \in \{0,1\}$, or $\phi = 0, \psi = 1$, $\psi_c \in \{0,1\}$, then $c^* = e^* = 0$, and equations (35) and (36) are satisfied. Furthermore, using (12) and (13), this implies that $d_1^* = d_2^* = d_{-1}^* = d_{-2}^* = 0$ and, using (15), we have $d_0^* = 1/2$. This is referred to as steady-state $\mathcal{U}$.

(ii) If $\phi = 1, \psi = 0$, $\psi_c \in \{0,1\}$ or $\phi = \psi = 1$, $\psi_c \in \{0,1\}$, then $c^* = 0$ and $e^* > 0$, and equation (36) is satisfied. We obtain the steady-state $\mathcal{E}$.

(iii) If $\phi = \psi = 1$, $\psi_c \in \{0,1\}$, $\mathcal{E}$ is not the only possible steady-state; we also obtain a steady-state where both $e^* > 0$ and $c^* > 0$, which we refer to as steady-state $\mathcal{M}$. More precisely, define $Z = (1-\omega)/\omega$, $B = p/\alpha\omega$ and $\theta = \lambda/\delta$. By combining (12) to (14), we obtain

$$d_1^* = 2\theta d_0^*, \qquad d_2^* = \theta^2 d_0^* \tag{37}$$

$$d_{-1}^* = \frac{2c^*}{B} d_0^*, \qquad d_{-2}^* = \frac{(Z+c^*)c^*}{B^2}\psi_c d_0^* \tag{38}$$

$$e^* = 2\theta(1+\theta) d_0^*, \qquad c^*\psi_c = \frac{B^2}{2d_0^*} - B - Z\psi_c \tag{39}$$

We distinguish two subcases.

First, let $\psi_c = 0$. Then, (39) implies that $2d_0^* = B$. Plugging into (37) and (38), we get an exact expression for d_1^*, d_2^*, d_{-1}^* and d_{-2}^* which, together with (15), gives:

$$c^* = \frac{1}{2}\left[1 - B(1+\theta)^2\right]. \tag{40}$$

Second, let $\psi_c = 1$. Using (15), (37) and (38) we get:

$$2d_0^* = 1 - 4\theta d_0^* - 2\theta^2 d_0^* - c^* - \frac{2c^*}{B}d_0^*,$$

where we use the expression for c^* in (14). Using the expression of c^* as a function of d_0^* in (39), and with some algebra, we conclude that d_0^* solves $\Phi_\theta(d_0^*) = 0$ where $\Phi_\theta(x)$ is the following second-order polynomial:

$$\Phi_\theta(x) = \left[\theta(2+\theta) - \frac{Z}{B}\right]x^2 - \frac{(1+Z)}{2}x + \left(\frac{B}{2}\right)^2. \tag{41}$$

(iv) If $\phi = 0, \psi = 1$, $\psi_c \in \{0,1\}$, $\mathcal{U}$ is not the only possible steady-state; we also obtain a steady-state where both $e^* = 0$ and $c^* > 0$, which we refer to as steady-state $\mathcal{C}$. It is easy to check that the equations for this steady-state $\mathcal{C}$ follow, *ceteris paribus*, from those established in (iii) for the steady-state $\mathcal{M}$ by setting $\theta = 0$.

More precisely, when $\psi_c = 0$, we get $2d_0^* = B$ and $c^* = (1-B)/2$.

Instead, when $\psi_c = 1$, d_0^* solves $\Phi_0(d_0^*) = 0$ where $\Phi_0(x)$ is the following second-order polynomial:

$$\Phi_0(x) = -\frac{Z}{B}x^2 - \frac{(1+Z)}{2}x + \left(\frac{B}{2}\right)^2. \tag{42}$$

∎

Lemma 2

(i) *The steady-state $\mathcal{U}$ exists when $\phi = 0$ and $\psi, \psi_c \in \{0,1\}$.*

(ii) *The steady-state $\mathcal{E}$ exists when $\phi = 1$ and $\psi, \psi_c \in \{0,1\}$.*

(iii) *The steady-state $\mathcal{M}$ exists when either $\phi = \psi = 1$, $\psi_c = 0$ and $p(1+\lambda/\delta)^2 < \alpha\omega$, or $\phi = \psi = \psi_c = 1$ and $p < X(\omega)$ where $X(\omega)$ is a uniquely defined function $X: [0,1] \to \mathbb{R}_+$, with $X(0) = 0$ and $X(1) = \sqrt{1-\theta(2+\theta)}$ if $\theta(2+\theta) \leq 1$ and $X(1) = 0$, otherwise.*

(*iv*) *The steady-state* $\mathcal{C}$ *exists when either* $\phi = 0, \psi = 1$, $\psi_c = 0$ *and* $p < \alpha\omega$, *or* $\phi = 0, \psi = \psi_c = 1$ *and* $p < \alpha[\omega + \sqrt{\omega(4 - 3\omega)}]/2$.

Proof.

(*i*) and (*ii*) are straightforward from Lemma 1.

(*iii*) We know from Lemma 1 that steady-state $\mathcal{M}$ exists when either $\phi = \psi = 1$, $\psi_c = 0$, or $\phi = \psi = \psi_c = 1$. We now check that in both cases $c^* > 0$ and $0 < d_0^* < 1/2$.

When $\phi = \psi = 1$, $\psi_c = 0$, we get $2d_0^* = B$ while c^* is given by (40). In this case, the conditions $c^* > 0$ and $0 < d_0^* < 1/2$ amount to imposing that $B(1+\theta)^2 < 1$, that is, $p(1+\lambda/\delta)^2 < \alpha\omega$.

When $\phi = \psi = \psi_c = 1$, we check whether there exists some $0 < d_0^* < 1/2$ such that $\Phi_\theta(d_0^*) = 0$, where $\Phi_\theta(\cdot)$ is given by (41). We have $\Phi_\theta(0) = (B/2)^2 > 0$ and $\Phi_\theta'(0) = -(1+Z)/2 < 0$. Therefore, (41) has a unique positive root smaller than 1/2 if and only if

$$\Phi_\theta(1/2) = \frac{1}{4}\left[B^2 - (1+Z) - \frac{Z}{B} + \theta(2+\theta)\right] < 0,$$

equivalent to $B^3 - [1 + Z - \theta(2+\theta)]B - Z < 0$.

Let us consider the following three-degree polynomial:

$$f(x) = x^3 - [1 + Z - \theta(2+\theta)]x - Z.$$

We show that this polynomial has a unique positive root, that we denote by $x(\omega)$. Suppose, first, that $1 + Z \leq \theta(2+\theta)$. Then $f'(x) \geq 0$ and $f(0) = -Z < 0$, implying that $f(x)$ has one positive real root and two complex conjugates. Suppose now that $1 + Z > \theta(2+\theta)$. Then, $f''(x) > 0$ for $x \in \mathbb{R}^+$ and $f(0) = -Z < 0$, implying that $f(x)$ has three real roots but only one is positive.

Therefore, $d_0^* < 1/2$ if and only if $B < x(\omega)$, which is equivalent to $p < \omega\alpha x(\omega) = X(\omega)$. Observe that $d_0^* < 1/2$ guarantees that $c^* > 0$.

(*iv*) The conditions for the existence of steady-state $\mathcal{C}$ follow, *ceteris paribus*, from those for the existence of steady-state $\mathcal{M}$ established above by setting $\theta = 0$.

When $\phi = 0, \psi = 1$, $\psi_c = 0$ the condition becomes $B < 1$, that is, $p < \alpha\omega$.

When $\phi = 0, \psi = 1$, $\psi_c = 1$ we write

$$\Phi_0(1/2) = \frac{1}{4}\left[B^2 - (1+Z) - \frac{Z}{B}\right] = \frac{1}{4}(1 + \frac{1}{B})(B^2 - B - Z) < 0.$$

The unique positive solution to $x^2 - x - Z = 0$ is $\left[1 + \sqrt{(4-3\omega)/\omega}\right]/2$. Then, $d_0^* < 1/2$ if and only if $B < \left[1 + \sqrt{(4-3\omega)/\omega}\right]/2$, equivalent to:

$$p < \frac{\alpha}{2}[\omega + \sqrt{\omega(4-3\omega)}]$$

Again, $d_0^* < 1/2$ guarantees that $c^* > 0$.
▮

Proposition 1 follows from the two previous lemmata. ■

Proof of Proposition 2:

Lemma 3 *Suppose that $w > b$. Then,*

(i) The steady-state $\mathcal{U}$ never arises at equilibrium.

(ii) The steady-state $\mathcal{E}$ always arises at equilibrium.

(iii) The steady-state $\mathcal{M}$ arises at equilibrium if:

$$g - pf < w \qquad \text{and} \qquad b \leq \left[1 + \frac{\lambda}{r+\delta}\right](g - pf) - \frac{\lambda}{r+\delta}w,$$

or

$$w \leq g - pf \qquad \text{and} \qquad b \leq \left[1 + \frac{\alpha\omega c^*(1,1,1)}{r+p}\right] w - \frac{\alpha\omega c^*(1,1,1)}{r+p}(g - pf).$$

(iv) The steady-state $\mathcal{C}$ arises at equilibrium if

$$\left[1 + \frac{\alpha\omega c^*(0,1,1)}{r+p}\right] w - \frac{\alpha\omega c^*(0,1,1)}{r+p}(g - pf) < b \leq g - pf.$$

Proof.

(i) From Lemma 2, steady-state $\mathcal{U}$ requires that $\phi = 0$. Using (17) and noting that $q(c) = \alpha\omega c = 0$ at $\mathcal{U}$, $\phi = 1$ at equilibrium is equivalent to $(w - b)(r + p) \leq 0$, which is impossible given our assumption that $w > b$.

(ii) From Lemma 2, steady-state $\mathcal{E}$ obtains when $\phi = 1$, and $\psi, \psi_c \in \{0, 1\}$. We provide conditions such that these values arise at equilibrium. Note that $q(c) = 0$ at $\mathcal{E}$.

• Case $\phi = 1$, $\psi = 0$, $\psi_c \in \{0, 1\}$. Using (17) and (18), the conditions for $\phi = 1$ and $\psi = 0$ are respectively given by $(w - b)(r + p) \geq 0$ and:

$$(r + \delta + \lambda)(g - pf) < \lambda w + (r + \delta) b$$

The first condition is always true since $w > b$. Thus, $\mathcal{E}$ arises at equilibrium when:

$$b > \left(1 + \frac{\lambda}{r+\delta}\right)(g - pf) - \frac{\lambda}{r+\delta}w.$$

• Case $\phi = \psi = 1$, $\psi_c = 0$. Using (17) and (18), the conditions for $\phi = \psi = 1$, $\psi_c = 0$ are respectively given by $(w - b)(r + p) \geq 0$ and:

$$(r + \delta + \lambda)(g - pf) \geq \lambda w + (r + \delta) b$$

$$g - pf < b$$

Again, the first condition follows from $w > b$. The two last conditions give:

$$g - pf < b \leq \left(1 + \frac{\lambda}{r + \delta}\right)(g - pf) - \frac{\lambda}{r + \delta} w$$

But this implies that $g - pf > w$, which is incompatible with the fact that $w > b$ and $b > g - pf$.

• Case $\phi = \psi = \psi_c = 1$. Now the conditions become:

$$b \leq \min\{g - pf; \left(1 + \frac{\lambda}{r + \delta}\right)(g - pf) - \frac{\lambda}{r + \delta} w\}. \tag{43}$$

Suppose first that

$$\left(1 + \frac{\lambda}{r + \delta}\right)(g - pf) - \frac{\lambda}{r + \delta} w > g - pf,$$

which is equivalent to $g - pf > w$. Then, (43) is equivalent to $b \leq g - pf$, which, given that $g - pf > w$, follows from our assumption that $w > b$.

Suppose now that $w > g - pf$. Then, (43) is equivalent to

$$b \leq \left(1 + \frac{\lambda}{r + \delta}\right)(g - pf) - \frac{\lambda}{r + \delta} w.$$

Combining the case $\phi = 1$, $\psi = 0$, $\psi_c \in \{0, 1\}$ with the case $\phi = \psi = \psi_c = 1$, we conclude that $\mathcal{E}$ can always arise at equilibrium under the assumption that $w > b$.

(iii) From Lemma 2, the steady-state $\mathcal{M}$ obtains when either $\phi = \psi = 1$, $\psi_c = 0$ or $\phi = \psi = \psi_c = 1$.

• Case $\phi = \psi = 1$, $\psi_c = 0$. In what follows, c^* denotes $c^*(1, 1, 0)$. Using (17) and (18), the conditions for $\phi = \psi = 1$, $\psi_c = 0$ are respectively given by:

$$(w - b)(r + p) \geq q(c^*)(g - pf - w)$$

$$(r + \delta + \lambda)(g - pf) \geq \lambda w + (r + \delta) b$$

$$g - pf < b$$

By combining these equations, we obtain:

$$g-pf<b\leq \min\left\{\left[1+\frac{q(c^*)}{r+p}\right]w-\frac{q(c^*)}{r+p}\left(g-pf\right),-\frac{\lambda}{r+\delta}w+\left(1+\frac{\lambda}{r+\delta}\right)\left(g-pf\right)\right\}$$

Suppose first that:

$$\left[1+\frac{q(c^*)}{r+p}\right]w-\frac{q(c^*)}{r+p}\left(g-pf\right)<-\frac{\lambda}{r+\delta}w+\left(1+\frac{\lambda}{r+\delta}\right)\left(g-pf\right),$$

equivalent to $w<g-pf$. Then, for the condition above to hold, it has to be that:

$$g-pf<\left[1+\frac{q(c^*)}{r+p}\right]w-\frac{q(c^*)}{r+p}\left(g-pf\right),$$

equivalent to $w>g-pf$. We get a contradiction.

Take now the case when

$$-\frac{\lambda}{r+\delta}w+\left(1+\frac{\lambda}{r+\delta}\right)\left(g-pf\right)<\left[1+\frac{q(c^*)}{r+p}\right]w-\frac{q(c^*)}{r+p}\left(g-pf\right)$$

By the same reasoning, we obtain a contradiction.

- Case $\phi=\psi=\psi_c=1$. In what follows, c^* denotes $c^*\left(1,1,1\right)$. Using (17) and (18), we get:

$$\left(w-b\right)\left(r+p\right)\geq q(c^*)\left(g-pf-w\right)$$

$$\left(r+\delta+\lambda\right)\left(g-pf\right)\geq \lambda w+\left(r+\delta\right)b$$

$$g-pf>b$$

By combining these equations and by observing that in that case $q(c^*)=\omega\alpha c^*(1,1,1)$, we obtain the following condition:

$$b\leq \min\left\{\begin{array}{c} g-pf,\left[1+\frac{\alpha\omega c^*(1,1.1)}{r+p}\right]w-\frac{\alpha\omega c^*(1,1,1)}{r+p}\left(g-pf\right), \\ \left(1+\frac{\lambda}{r+\delta}\right)\left(g-pf\right)-\frac{\lambda}{r+\delta}w \end{array}\right\}$$

which guarantees the existence of equilibrium $\mathcal{M}$. It is readily checked that, when $w>g-pf$, this condition is equivalent to

$$b\leq\left(1+\frac{\lambda}{r+\delta}\right)\left(g-pf\right)-\frac{\lambda}{r+\delta}w,$$

Instead, when $w\leq g-pf$, this is equivalent to

$$b\leq\left[1+\frac{\alpha\omega c^*\left(1,1.1\right)}{r+p}\right]w-\frac{\alpha\omega c^*\left(1,1,1\right)}{r+p}\left(g-pf\right).$$

(iv) From Lemma 2, the steady-state $\mathcal{C}$ obtains when either $\phi = 0, \psi = 1, \ \psi_c = 0$ or $\phi = 0, \psi = \psi_c = 1$.

• Case $\phi = 0, \psi = 1, \psi_c = 0$. In what follows, c^* denotes $c^*(0,1,0)$. Using (17) and (18), we get:

$$(w-b)(r+p) < q(c^*)(g-pf-w)$$

$$(r+\delta+\lambda)(g-pf) \geq \lambda w + (r+\delta) b$$

$$g - pf < b$$

From the first two conditions we deduce that

$$\left[1+\frac{\alpha\omega c^*}{r+p}\right] w - \frac{\alpha\omega c^*}{r+p}(g-pf) < b \leq -\frac{\lambda}{r+\delta} w + \left(1+\frac{\lambda}{r+\delta}\right)(g-pf),$$

which implies that $w < g - pf$, incompatible with both $g - pf < b < w$.

• Case $\phi = 0$ and $\psi = \psi_c = 1$. In what follows, c^* denotes $c^*(0,1,1)$. Using (17) and (18), we get:

$$(w-b)(r+p) < q(c^*)(g-pf-w)$$

$$(r+\delta+\lambda)(g-pf) \geq \lambda w + (r+\delta) b$$

$$g - pf \geq b$$

these three conditions can be written as:

$$\left[1+\frac{\alpha\omega c^*}{r+p}\right] w - \frac{\alpha\omega c^*}{r+p}(g-pf) < b \leq \min\left\{g-pf, -\frac{\lambda}{r+\delta} w + \left(1+\frac{\lambda}{r+\delta}\right)(g-pf)\right\} \tag{44}$$

Take first the case when

$$g-pf < -\frac{\lambda}{r+\delta} w + \left(1+\frac{\lambda}{r+\delta}\right)(g-pf)$$

which is equivalent to $w < g - pf$. (44) can then be written as:

$$w\left[1+\frac{\alpha\omega c^*}{r+p}\right] - (g-pf)\frac{\alpha\omega c^*}{r+p} < b \leq g - pf,$$

compatible with the fact that $w < g - pf$.

Take now the case when $w > g - pf$. (44) can now be written as

$$\left[1+\frac{\alpha\omega c^*}{r+p}\right] w - \frac{\alpha\omega c^*}{r+p}(g-pf) < b \leq -\frac{\lambda}{r+\delta} w + \left(1+\frac{\lambda}{r+\delta}\right)(g-pf)$$

which implies that $g - pf > w$, a contradiction. ∎

Note also that the steady-state $\mathcal{M}$ arises at equilibrium only when $(\phi, \psi, \psi_c) = (1,1,1)$ but never $(\phi, \psi, \psi_c) = (1,1,0)$. Also, the steady-state $\mathcal{C}$ arises at equilibrium only when $(\phi, \psi, \psi_c) = (0,1,1)$ but never $(\phi, \psi, \psi_c) = (0,1,0)$. Proposition 2 then follows. ■

The following lemmata is helpful to draw Figure 2.

Lemma 4 *We have $c^*(0,1,1) > c^*(1,1,1)$*

Proof. We know from the proof of Lemma 1 that:

$$c^*(0,1,1) = \frac{B^2}{2d_0^*(0,1,1)} - B - Z \text{ and } c^*(1,1,1) = \frac{B^2}{2d_0^*(1,1,1)} - B - Z.$$

Observe that $\Phi_\theta(x) = \theta(2+\theta)x^2 + \Phi_0(x)$, so that:

$$\Phi_\theta(d_0^*(1,1,1)) = 0 = \theta(2+\theta)\left(d_0^*(1,1,1)\right)^2 + \Phi_0\left(d_0^*(1,1,1)\right).$$

This implies that $\Phi_0\left(d_0^*(1,1,1)\right) < 0$. But since $\Phi'\left(d_0^*(0,1,1)\right) < 0$ and $\Phi_0\left(d_0^*(0,1,1)\right) = 0$, we have that $d_0^*(1,1,1) > d_0^*(0,1,1)$. This, in turn, implies that $c^*(0,1,1) > c^*(1,1,1)$. ▮

Proof of Proposition 3

We first establish a useful result.

Lemma 5 *If $p < X(\omega)$ then $p \leq \alpha$.*

Proof. The condition $p \leq \alpha$ is equivalent to $B \leq 1/\omega$. From the proof of Lemma 2, we know that $p < X(\omega)$ is equivalent to $B < x(\omega)$, where $x(\omega)$ is the unique positive real root of the following three-degree polynomial:

$$f(x) = x^3 - [1 + Z - \theta(2+\theta)]\,x - Z.$$

We show that $x(\omega) < 1/\omega$. Given that the three-degree polynomial $f(x)$ is increasing to the right of $x(\omega)$, this is equivalent to showing that $f(1/\omega) > 0$. Recalling that $Z = (1-\omega)/\omega$, some algebra gives:

$$f(1/\omega) > 0 \Leftrightarrow \omega^3 + \omega^2[\theta(2+\theta) - 1] - \omega + 1 > 0.$$

But

$$\omega^3 + \omega^2[\theta(2+\theta) - 1] - \omega + 1 = \omega^2\theta(2+\theta) + (\omega+1)(\omega-1)^2 > 0, \text{ for all } \omega \in [0,1].$$

Therefore, $B < x(\omega)$ implies that $B \leq 1/\omega$. ▮

We can now prove Proposition 3

(i) Let us show that $\partial d_0^*/\partial p > 0$. Note that by (12) and (13), this implies that $\partial d_1^*/\partial p > 0$ and $\partial d_2^*/\partial p > 0$, and (14) implies that $\partial e^*/\partial p > 0$. In turn, (2) implies that $\partial\left(c^* + u^*\right)/\partial p < 0$.

The equilibrium crime level in equilibrium $\mathcal{M}$ defined by (39) is:

$$c^* = \frac{B^2}{2d_0^*} - B - Z,$$

where d_0^* is given by the solution to $\Phi_\theta(d_0^*) = 0$. Observing that $B = p/\alpha\omega$, it is equivalent to analyze the variation of d_0^* and c^* with respect to B than with p. The implicit function theorem applied to (41) gives:

$$\frac{\partial d_0^*}{\partial B} = -\frac{\left(Z/B^2\right) d_0^{*2} + B/2}{\Phi_\theta'(d_0^*)} > 0.$$

But we know that d_0^* is the unique positive positive root of $\Phi_\theta(x)$, a polynomial that grows without bound with x. Therefore, $\Phi_\theta'(d_0^*) < 0$, and the result follows.

(ii) Observing that $2\theta(1+\theta) = 2\lambda/\delta + 2\lambda^2/\delta^2$, the implicit function theorem applied to (41) gives:

$$\frac{\partial d_0^*}{\partial \delta} = \frac{d_0^{*2}}{\Phi_\theta'(d_0^*)}\left(\frac{2\lambda}{\delta^2} + \frac{4\lambda^2}{\delta^3}\right) < 0 \qquad \text{and} \qquad \frac{\partial d_0^*}{\partial \lambda} = -\frac{d_0^{*2}}{\Phi_\theta'(d_0^*)}\left(\frac{2}{\delta} + \frac{4\lambda}{\delta^2}\right) > 0.$$

Again, (12) and (13) imply that $\partial d_1^*/\partial\lambda > 0, \partial d_2^*/\partial\lambda > 0, \partial d_1^*/\partial\delta < 0$, and $\partial d_2^*/\partial\delta < 0$, from which we deduce that $\partial e^*/\partial\lambda > 0$ and $\partial e^*/\partial\delta < 0$. Similar calculations yield $\partial d_0^*/\partial\alpha < 0, \partial d_1^*/\partial\alpha < 0, \partial d_2^*/\partial\alpha < 0$ and $\partial e^*/\partial\alpha < 0$.

Differentiating c^* in (39), we have:

$$\frac{\partial c^*}{\partial \delta} = -\frac{B^2}{2\left(d_0^*\right)^2}\frac{\partial d_0^*}{\partial \delta} > 0 \qquad \text{and} \qquad \frac{\partial c^*}{\partial \lambda} = -\frac{B^2}{2\left(d_0^*\right)^2}\frac{\partial d_0^*}{\partial \lambda} < 0.$$

(iii) We now show that $\partial d_0^*/\partial\omega > 0$, and so that $\partial d_1^*/\partial\omega > 0, \partial d_2^*/\partial\omega > 0$, and $\partial e^*/\partial\omega > 0$. Using the notation $p' = p/\alpha$ (41) gives:

$$\frac{\partial d_0^*}{\partial \omega} = -\frac{1}{\Phi_\theta'(d_0^*)}\left[\frac{1}{p'}d_0^{*2} + \frac{1}{2\omega^2}d_0^* - \frac{(p')^2}{2\omega^3}\right]$$

Since the denominator is negative, we analyze the sign of the numerator, denoted $\Xi(d_0^*)$. We have:

$$\Xi(0) = -\frac{p'^{\,2}}{2\omega^3} < 0, \Xi'(d_0^*) = \frac{2}{p'}d_0^* + \frac{1}{2\omega^2} > 0 \text{ and } \Xi''(d_0^*) = \frac{2}{p'} > 0.$$

Note that Lemma 5 implies that $p' < 1$. We show that $\Xi(1/2) < 0$. This is equivalent to

$$g(\omega) \equiv 4\omega^3 p' \Xi(1/2) = \omega^3 + p'\omega - 2p'^{\,3} < 0.$$

We have $g(0) < 0$. It is easy to see that $g(1) = 1 + p' - 2p'^3 > 0$, for all $p' \in (0, 1)$. Therefore, there exists some $0 < \overline{\omega} < 1$ such that $\partial d_0^*/\partial\omega < 0$ for all $\omega < \overline{\omega}$. ■

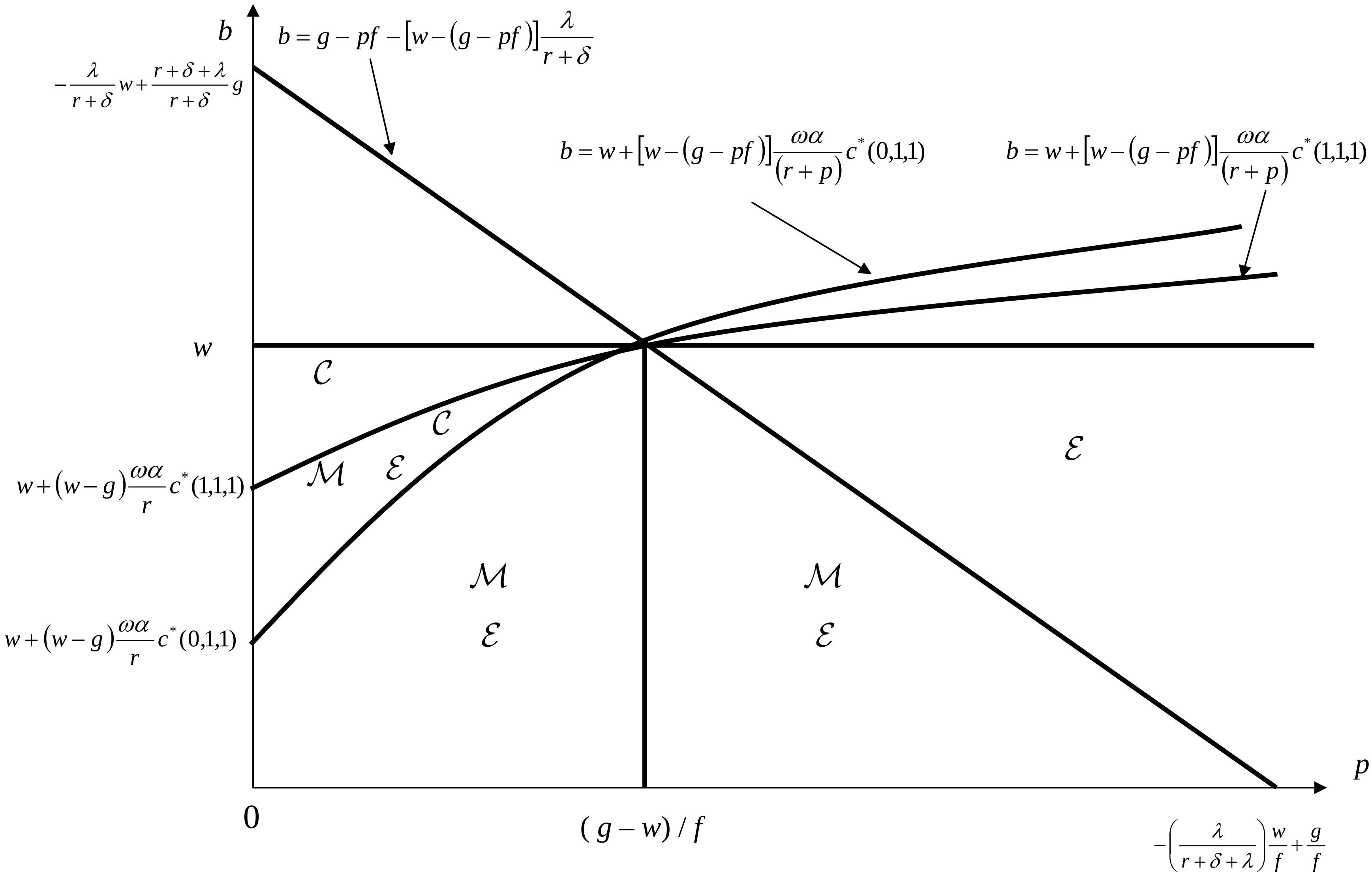

Figure 2: Steady-State Equilibria