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ABSTRACT

Trends in Hours and Economic Growth*

We study long-run trends in aggregate market hours of work and shifts across economic sectors within the context of balanced aggregate growth. We show that a model of many goods and uneven TFP growth in market and home production can rationalize the observed falling or U-shaped aggregate hours and structural change across market sectors. The dynamics of market hours are driven by substitutions between home and market production and depend critically on the existence of many market sectors. Extensions show how the model can explain rising leisure and more complex hours dynamics without violating balanced aggregate growth.

JEL Classification: J21, J22, O14 and O41
Keywords: balanced growth, home production, hours of work, labour supply, marketization and structural transformation

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A feature of modern economic growth is the changing trend in total hours of work. In the early stages of modern growth total hours typically fall. In later stages trends become less clear-cut, with no systematic overall dynamic pattern. In the United States the trend over the last century appears to be an asymmetric U-shape, a steep decline followed by a small rise. In other countries there is a monotonic decline, although one that flattens out as growth progresses. A "stylized fact" of low-frequency fluctuations in market hours is that modern economic growth causes a long-lasting decline, which eventually dissipates. Figure 1 uses data from the website of the Groningen Growth and Development Centre and shows average weekly hours of work for the population of working age. We show data for the United States and the biggest European economies since 1960. The main fact of a declining trend that either slows down or reverses is evident. Even more striking is the decline in hours before 1960, in the earlier stages of economic development. Table 1 is derived from Maddison (1995), and as with figure 1 it shows weekly hours of work for the population of working age. The rate of decline in the late 19th and early 20th century is striking.[1]

The changing trends in aggregate hours that one finds in long runs of data are usually neglected by modern growth theory, which typically assumes a constant rate of labour force growth. A seemingly unrelated feature of modern growth is structural change, the continuous reallocation of labour across sectors of economic activities. Over long periods of time the most striking feature of structural change is the decline of agriculture and the rise of services, with relatively smaller changes in industrial employment. In this paper we propose a unified framework for the study of these two phenomena and posit that they are part of the same economic process: the response of employment to the uneven distribution of technological change across production sectors located in the market and the home.

The introduction of home production is critical in the explanation of changes in overall market hours; perhaps surprisingly, the changing composition of market hours also turns out to be critical in explaining the dynamics of total market hours. Moreover, the introduction of home production improves the explanation of labour reallocations within market sectors. Our model can explain a long-lasting fall in overall market hours as part of a dynamic process that is consistent with Kaldor's aggregate balanced-growth facts and with the transformation from agriculture to services. It is also consistent with a subsequent rise in market hours, which reaches some limit either monotonically or with

[1]The initial decline in market hours was driven by a rapid fall in the number of hours of work by those at work. In the United States this process went on until about 1940. The decline of agricultural employment, however, also contributed to the decline of total hours, through the withdrawal of some workers from the labour force, especially women and children. In more recent times there has been an increase in the labour force participation of women, which increased overall hours in the United States but less so in the European countries, where hours per employee continued to fall. See Durand (1975, esp. ch. 4) and Maddison (1995) for cross-country evidence and Goldin (1995) for female labour supply in the US time series and in other countries.

Table 1: Weekly hours of work, population of working age

Year	USA	France	Italy	UK
1890	35.9	39.7	43.4	35.4
1913	31.4	37.1	40.1	32.5
1929	27.3	32.0	32.0	26.9
1938	20.2	24.7	26.3	28.5
1960	23.8	25.1	24.2	28.5

The numbers shown are for the average weekly number of hours of market work for the working age population, ages 15-64. Source: Maddison (1995) for total hours and Mitchell (1980) and US Historical Statistics for the working age population.

a turning point.[2]

The model that we use to demonstrate these claims has three general uses of time, market work, home work and leisure. Market work produces consumer and capital goods and we refer to it as the supply of labour. Home work produces consumption goods for the individual's own use and the time allocated to it is like market work, but it is not part of the conventional definition of labour supply. We show that because of the uneven distribution of technological change the time allocated to home production is likely to change during the course of economic development. These changes drive the changes in aggregate labour supply. Under plausible conditions the time allocated to home production is likely to increase in the early stages of modern growth but eventually it will decrease. By how much it decreases and what happens after the decrease depends on a number of factors. We devote most of the paper to a benchmark model with only one home-produced good, and show that the eventual decrease in home production time continues indefinitely. But small generalizations to the model, such as the introduction of a second home-produced good or a more general model of leisure, can give richer dynamics for both home production and aggregate labour supply.

The intuition behind our results derives from the key assumption that although market activities at the disaggregation level of agriculture, manufacturing and services produce goods that are poor substitutes for each other, a lot of home production produces services that are close substitutes for services produced in the market. Over time, the composite of the market and home service sectors attracts labour from agriculture and industry because it has lower mean TFP growth rate than other market sectors. We call this force the "structural transformation" force, because it is the force that causes

[2] Home production has been studied extensively in a partial context, starting with Becker (1965) and Gronau (1977). More recently it has been studied in the context of equilibrium business cycles and to some extent in the context of growth (see Gronau, 1997 for a survey, and Parente et al. (2000) and Gollin et al. (2000) for growth-related work). Structural change within the market economy has been studied by many authors. See Kuznets (1966), Baumol (1967) and Fuchs (1980) for early contributions and Echevarria (1997), Kongsamut et al.(2001), Caselli and Coleman (2001) and Ngai and Pissarides (2004) for more recent work.

structural change in the market economy: it is a force for an increase in the employment share of both the market and home service sectors. But if the market service sector has higher TFP growth than the home sector, within the services composite there is a movement of labour from the home to the market. We call this force the "marketization" force. The tension between the two forces, the structural transformation force pushing for a rise in the share of the home sector and the marketization force pushing for a fall, is at the core of our paper. It drives the dynamics of overall market hours, in a way that we explain more fully below. It explains why the share of agriculture and manufacturing employment may decline faster when there is a home service sector: labour from these sectors has two potential destinations, both with low TFP growth rates. And it also explains why the share of market services may rise faster with home production: it receives labour from two sources, market sectors and the home sector.

The dynamics of overall market hours are influenced by the relative importance of the structural transformation and marketization forces during economic growth. In the first stages of modern growth, when agriculture employs a large fraction of the labour force, the movement of labour from agriculture to home production is relatively large and offsets the movement from home production to market services, so home production gains employment. But eventually, as agricultural employment shrinks and the home production sector grows, the marketization force dominates, bringing a fall in the size of the home production sector. Reflecting this reversal, although in the early stages of modern growth aggregate labour supply decreases, eventually it increases, tracing a U pattern.

In more complex versions of our model the late increase may not actually take place, or if it does, it need not continue indefinitely. We briefly discuss one such extension. If in addition to the benchmark structure there is one more service sector each in the market and the home, both of which produce goods that experience low and common TFP growth, both sectors attract labour indefinitely. Activities that might come under this heading are personal care services. The existence of such a sector implies that eventually overall home production time increases, with more productive home production time moving to less productive activities. Moreover, if, for reasons not specified in our model, women are more likely to be engaged in home production than men are, our model can explain the fall in male labour supply and the eventual rise in female labour supply as part of a unified process of economic growth.

Our benchmark model has both home production and leisure time but as the previous intuition made clear, the driving force for the long shifts in labour supply is the home production sector and not leisure. In the benchmark model we make the conventional assumption that leisure time enters the utility function directly, in order to focus on the role of home production in employment dynamics. As in King et al. (1988), our utility function is such that the existence of a steady state that satisfies Kaldor's aggregate facts requires constant leisure time. During periods of transition to an aggregate balanced-growth equilibrium - following for example war or some other major event that disturbs the initial growth equilibrium - changes in leisure also contribute to changes in aggregate labour supply, but these periods cannot explain the long swings in labour supply that is the topic of this paper.

This raises the question whether the big falls in hours of work shown in Table 1 and figure 1 can be explained by a rise in home production alone. Some might argue that leisure time has also been increasing along the steady state. In an extension to our benchmark model we show how a rising leisure time can be obtained even when the economy is on a balanced growth path. The idea behind this extension is to divide leisure time into two components, one that is the pure enjoyment of time, as in conventional theory, and one that is enjoyment of time obtained with the help of some capital input. In the first group there are activities like spending time with friends or playing with one's children. In the second there are activities like watching TV and surfing the net. The enjoyment of time in the second category depends on the capital input that the agent puts into it, so we can think of it as producing a leisure good with capital and labour as inputs. The key difference between this leisure component and home production is that home production produces goods that have close substitutes in the market, like cooked food, whereas leisure production has no close market substitutes. One cannot outsource TV watching time. We show that the extended model implies a rising "leisure production time," which gives a rising overall leisure time.

Although our main objective in this paper is theoretical we also derive explicit quantitative predictions about the evolution of market hours that can be compared with the data. Ramey and Francis (2005) have recently argued that the fall and subsequent rise in market hours in 20th-century United States is primarily reflected in a rise and subsequent fall in home production time, with leisure remaining approximately constant (see section 2 below and in particular figure 2). This is a prediction of our benchmark model for an economy on a balanced-growth path. The evidence of Mokyr (2000) is also consistent with our claim that the fall in market hours early in the 20th century was accompanied by a rise in home production time. He argues that at the beginning of the century in the United States there was an increased demand for home production services: cleaner homes, better-prepared food, and so on, which required more home production time, supplied especially by women.

Our model's explanation of the recent rise in labour supply is different from the one put forward by Greenwood et al. (2005), who also interpret the rise in female labour supply as the result of the fall in home production time, and more in line with the marketization force in Freeman and Schettkat (2005) and Rogerson (2004). Our claim is that marketization takes place because similar goods can be produced more efficiently in the market. Greenwood et al. claim that employment in the home sector is on a falling trend because the sector is substituting from labour to capital as the prices of durable goods fall. In our model the price of durable goods also falls because of higher TFP growth in manufacturing than in services, but the substitution of capital for labour is not the driving force for the decline in home production time. In favour of our explanation is the observation that in the United States people consume more restaurant food than in Europe, where more food is prepared at home. The model of Greenwood et al. should predict that even in the United States, at least as much food is prepared at home but in less time (see Freeman and Schettkat, 2005, for more evidence of this kind).[3]

Section 1 outlines our benchmark model of multi-sector growth when there is home production and leisure. The dynamic properties of employment shares and the existence of an aggregate balanced growth path are derived and discussed in this section. Section 2 discusses a numerical illustration based on US data on structural change and changes in aggregate market hours. Sections 3 and 4 discuss two extensions that give more general results about the dynamic behaviour of home production and aggregate labour supply, one with a richer leisure model and one with more than one home production sector. In the concluding section we discuss some extensions, paying particular attention to the role of taxation and other distortions in the dynamics of labour supply.

1 Home production and leisure in a growth model

We simplify our exposition by assuming that market work takes place in three differentiated sectors and home production takes place in only one sector.[4] Each of the three market sectors captures a distinct feature of production. Sector 1, labelled agriculture, produces a consumption good that does not have close substitutes elsewhere; sector 2, labelled manufacturing, produces the economy's capital stock and another consumption good that also does not have close substitutes in other sectors; sector 3, labelled services, produces only a consumption good that has a close substitute produced in the home. Our labels are obviously not accidental, but we emphasize more the nature of the good produced in each sector rather than the accuracy of their description as agriculture, manufacturing and services.

We derive the equilibrium of our economy from a social planning problem that maximizes the utility function of a representative agent. Equilibrium is defined as a set of dynamic paths for the allocation of capital and time to the three market sectors, home production and leisure, and the allocation of the output of each sector to consumption

[3] A potential econometric test of our alternative hypotheses builds on the behaviour of wages. In a decentralized economy our model would predict that women leave home production and join market production because female wages are rising. In the Greenwood et al. explanation the increased efficiency of home production releases time, which is now supplied to the market, so the impact should be from the increased supply of female labour to wages.

[4] The three-sector economy of this paper can easily be generalized to many sectors along the lines of Ngai and Pissarides (2004). It is also possible to extend the model to one of more than one home sector (see sections 3 and 4).

and capital. The utility function of the representative agent is

$$U = \int_0^\infty e^{-\rho t} \left[\ln \phi(.) + v\left(1-l\right)\right] dt \tag{1}$$

$$\phi\left(.\right) = \left(\sum \omega_j c_j^{(\varepsilon-1)/\varepsilon}\right)^{\varepsilon/(\varepsilon-1)} \qquad j = a, m, sh \tag{2}$$

$$c_{sh} = \left[\left(1-\psi\right) c_s^{(\sigma-1)/\sigma} + \psi c_h^{(\sigma-1)/\sigma}\right]^{\sigma/(\sigma-1)} \tag{3}$$

$$\varepsilon, \sigma, \rho, \omega_j, > 0, \sum \omega_j = 1; \psi \in (0,1), l \in (0,1), c_i \geq 0, \forall i.$$

$$R1 : \varepsilon < 1 < \sigma$$

where $v\left(.\right)$ is the utility of leisure, with $v' > 0, v'' < 0$, and $v' \to \infty$ as $l \to 1$, $\phi(.)$ is a CES utility function over final consumption goods and c_{sh} is a composite service good, which is the outcome of a CES combination of market and home goods. Subscript a stands for agriculture, m for manufacturing, s for market-produced service goods and h for home-produced service goods. l is the fraction of total time allocated to market and home work. Restriction $R1$ on ε and σ implies that market and home-produced services are close substitutes for each other but the outputs of the agricultural, manufacturing and composite service sectors are not close substitutes.

The restrictions on the utility function are a combination of sufficient restrictions previously derived by King et al. (1988) and Ngai and Pissarides (2004). King et al. (1988, p.202) show that the following restrictions on utility are sufficient for the existence of a balanced-growth path in a one-sector model:

$$U(c,l) = \frac{c^{1-\theta}}{1-\theta} v\left(1-l\right) \qquad \theta \neq 1$$

$$= \ln c + v\left(1-l\right) \qquad \theta = 1$$

Our utility function in (1) is equivalent to setting $\theta = 1$, a restriction that was shown by Ngai and Pissarides (2004) to be a sufficient condition for the existence of a balanced growth path when there are many sectors with unequal TFP growth rates and $\varepsilon \neq 1$.

Our measure of total time is the total time available to the population who can work. We let l_i denote the share of that time allocated to each of the four production activities ($i = a, m, s, h$) and write

$$\sum l_i = l. \tag{4}$$

Total market employment is $l_a + l_m + l_s \equiv q$, which, in the absence of unemployment, is also the conventional definition of the aggregate supply of labour. Market employment shares are then defined by l_i/q, for $i = a, m, s$. The U-shape fact about the aggregate labour supply is a statement about the evolution of q, whereas structural change refers to changes in the market shares l_i/q.

Production functions are identical in all activities except for their TFP parameters A_i,

$$F^i = A_i F\left(l_i k_i, l_i\right); \quad \dot{A}_i / A_i = \gamma_i \quad i = a, m, s, h, \tag{5}$$

where the production function F is constant returns to scale, has positive and diminishing returns to inputs, and satisfies the Inada conditions, k_i is the capital-labour ratio of each sector and A_i is TFP in each sector i, with growth rate γ_i. We impose the quantitative restrictions

$$R2 : \gamma_a \geq \gamma_m > \gamma_s > \gamma_h,$$

which are justified in the quantitative section of the paper.

All sectors produce consumption goods but only manufacturing produces capital goods:

$$\begin{aligned} c_i &= A_i l_i F\left(k_i, 1\right) \qquad\qquad i = a, s, h & (6)\\ \dot{k} &= A_m l_m F\left(k_m, 1\right) - c_m - \left(\delta + \nu\right) k & (7)\\ \sum l_i k_i &= k, & (8) \end{aligned}$$

where δ is the capital depreciation rate, ν is the population growth rate and k is the ratio of the capital stock to the population.[5]

We obtain optimal allocations by maximizing the utility function in (1) subject to (4)-(8). We distinguish between the "static" conditions that give optimal allocations across sectors and the "dynamic" ones that give optimal allocations over time.

1.1 Optimal sector allocations

The optimal allocation of resources across industrial sectors is obtained from the first-order maximization conditions for c_i, l_i, k_i and l. Free factor mobility implies that both the value of marginal product of factors and the marginal rate of technical substitution between capital and labour are equalized across sectors. These imply equality of the capital-labour ratios across sectors and equality between relative prices and relative TFP levels:

$$\begin{aligned} k_i &= k_m = k/l \quad i = a, s, h, & (9)\\ \frac{\phi_i}{\phi_m} &= p_i = \frac{A_m}{A_i} \quad i = a, s, h. & (10) \end{aligned}$$

We use manufacturing as our numeraire throughout the analysis.

[5] Our assumption of a single capital good used in the market and home does not necessarily imply the transfer of capital goods between the two uses after installation. Although we argue later that the home sector is declining, empirically the capital depreciation rate and population growth rate far exceed the rate of decline of the home sector, so gross investment in the home is always positive.

We can immediately derive one strong result about structural change and home production in this economy. From the conditions for optimal choice of c_s and c_h in (10), we obtain,

$$\varphi \equiv \frac{p_h c_h}{p_s c_s} = \left(\frac{\psi}{1-\psi}\right)^{\sigma} \left(\frac{A_s}{A_h}\right)^{1-\sigma}; \tag{11}$$

and from the production function we derive

$$\frac{l_h}{l_s} = \varphi. \tag{12}$$

So

$$\frac{\dot{l}_s}{l_s} - \frac{\dot{l}_h}{l_h} = (\sigma - 1)(\gamma_s - \gamma_h). \tag{13}$$

With $\sigma > 1$, if TFP in the market sector is rising faster than in the home sector, the home sector will be losing labour to the market sector. It implies that if the TFP growth rate of the market sector remains indefinitely above the TFP growth rate of the home sector, eventually the home sector will vanish and all services will be produced in the market. Thus, our claims about the eventual marketization of all home production come from two quantitative restrictions: $\sigma > 1$ and $\gamma_s > \gamma_h$. We return later in the paper to a discussion of these conditions and to the question whether the model can be generalized to yield a home sector in its asymptotic state.

In order to derive the direction of structural change in other sectors, we define a new variable to represent the ratio of expenditure on the outputs of agriculture and services to expenditure on the consumption of the manufacturing good, $x_i \equiv p_i c_i / c_m$. By definition, $x_m = 1$. Using (10) and (11), we derive in the Appendix,

$$x_a \equiv \frac{p_a c_a}{c_m} = \left(\frac{\omega_a}{\omega_m}\right)^{\varepsilon} \left(\frac{A_m}{A_a}\right)^{1-\varepsilon} \tag{14}$$

$$x_s \equiv \frac{p_s c_s}{c_m} = \left(\frac{\omega_s}{\omega_m}\right)^{\varepsilon} \left(\frac{A_m}{A_s}\right)^{1-\varepsilon} (1+\varphi)^{(\sigma-\varepsilon)/(1-\sigma)} \tag{15}$$

$$x_h \equiv \frac{p_h c_h}{c_m} = \varphi x_s \tag{16}$$

where $\omega_s \equiv \omega_{sh} (1-\psi)^{\frac{\sigma(\varepsilon-1)}{(\sigma-1)\varepsilon}}$, which is the weight of the market service good in the utility function when home production is close to zero. Note also that if home production is not present in the utility function (i.e. $\psi \to 0$), we have $\varphi \to 0$ and $x_h \to 0$, and x_s has the same form as x_a.

The total value of consumption per capita, including the consumption of home produced goods, is

$$c = \sum p_i c_i = X c_m \tag{17}$$

where $X = \sum x_i$. Making use of (9) and (10), the total value of aggregate output per capita is given by:

$$y \equiv \sum p_i F^i = lA_m F(k_m, 1). \tag{18}$$

Using (14)-(18), we obtain

$$\frac{l_i}{l} = \frac{x_i}{X}\frac{c}{y}, \qquad i = a, s, h, \tag{19}$$

$$\frac{l_m}{l} = \frac{1}{X}\frac{c}{y} + \frac{s}{y}, \tag{20}$$

where $s \equiv y - c$ are savings. These equations show that the employment share used in the production of consumption good i is a fraction x_i/X of the aggregate consumption rate, whereas the manufacturing employment share has two parts, one that obeys the same law as the share of other consumption sectors and another that is equal to the savings rate. The first component is employment required to produce the manufacturing consumption good and the second is the employment required to produce the economy's investment goods.

The results for employment shares in (19) imply, after differentiation with respect to time,

$$\frac{\dot{l}_s}{l_s} - \frac{\dot{l}_a}{l_a} = (1-\varepsilon)(\gamma_a - \gamma_s) + \frac{\varphi}{1+\varphi}(\sigma - \varepsilon)(\gamma_s - \gamma_h), \tag{21}$$

$$\frac{\dot{l}_h}{l_h} - \frac{\dot{l}_a}{l_a} = (1-\varepsilon)(\gamma_a - \gamma_h) - \frac{1}{1+\varphi}(\sigma - \varepsilon)(\gamma_s - \gamma_h). \tag{22}$$

Conditions (21) and (22) give an important result. In the absence of home production, the second term of (21) on the right vanishes, and we obtain the result that for $\varepsilon < 1$ employment moves from agriculture, the high TFP-growth sector, to services. But since $\sigma > \varepsilon$ and $\gamma_s > \gamma_h$, the second term in (21) is also positive, and so the speed at which market services attract labour from agriculture is faster. Equation (22) gives a contrasting result. The first term on the right shows a movement of labour from agriculture to the home sector, because as with market services the home sector produces a service good that is a poor substitute for agricultural output. But the second term shows that (at least for $\sigma > \varepsilon$ and $\gamma_s > \gamma_h$) the movement is either mitigated or reversed, because service goods are more efficiently produced in the market. Intuitively, the introduction of a home production sector with small TFP growth rate accelerates the move out of agriculture because the gap between TFP in agriculture and the composite of the destination sectors is now bigger. We can also see that home production has its biggest impact on the decline of agriculture early on in the stages of economic growth. From (11) the second term in (21) becomes progressively smaller over time and vanishes as $t \to \infty$, whereas the second term in (22) becomes progressively larger. The opposite signs in (22) are the reason behind the changing trends in aggregate labour supply as we now show more formally.

Using (19) and (20), the dynamics of l_i depend on the sector-specific component x_i/X and the aggregate components l and c/y. In the remainder of this section we study the dynamics of l_i and the implied aggregate labour supply due to the sector-specific components. The additional dynamics due to changes in the aggregate l and c/y can easily be obtained from (19) and (20) and we postpone discussion to the next section.

Using (11), and (14)-(16), we obtain,

$$\frac{\dot{x}_a}{x_a} - \frac{\dot{X}}{X} = (1-\varepsilon)(\bar{\gamma} - \gamma_a) \tag{23}$$

$$\frac{\dot{x}_s}{x_s} - \frac{\dot{X}}{X} = (1-\varepsilon)(\bar{\gamma} - \gamma_{sh}) + (\sigma - 1)(\gamma_s - \gamma_{sh}) \tag{24}$$

$$\frac{\dot{x}_h}{x_h} - \frac{\dot{X}}{X} = (1-\varepsilon)(\bar{\gamma} - \gamma_{sh}) - (\sigma - 1)(\gamma_{sh} - \gamma_h) \tag{25}$$

$$-\frac{\dot{X}}{X} = (1-\varepsilon)(\bar{\gamma} - \gamma_m) \tag{26}$$

where $\gamma_{sh} = (\gamma_s + \varphi\gamma_h)/(1+\varphi)$ is the TFP growth rate for the service composite (c_{sh}) and $\bar{\gamma}$ is a weighted average of the TFP growth rates in all sectors, with the weight on γ_i equal to x_i/X. Because x_i/X is proportional to the share of employment used to produce consumption goods we call $\bar{\gamma}$ the consumption-weighted TFP growth rate of the economy.

Given the ranking in $R2$, $\gamma_a \geq \gamma_m > \gamma_s > \gamma_h$, we have $\gamma_a > \bar{\gamma} > \gamma_{sh}$. The second inequality follows from rewriting $\bar{\gamma}$ as a weighted average of γ_a, γ_m, and γ_{sh} with weight on γ_i equal to x_i/X, and $x_{sh} = x_s + x_h$. So, abstracting from changes in l and c/y, the employment shares of agriculture and manufacturing are subject only to a "structural transformation" force that moves labour from high TFP growth sectors to low TFP growth sectors, whereas the employment shares of the two service sectors are subject to two forces, a similar structural transformation force and a marketization force that moves labour from the home sector to the market sector. As a result of the two forces, (23) implies that l_a is falling, (24) implies that l_s is rising, and (25) implies that the dynamics for l_h are not likely to be monotonic. We show in the Appendix that the growth rate of l_h is falling over time. If the growth rate is positive at t_0, then l_h is hump-shaped, rising at first and falling later. Otherwise l_h is falling monotonically. From (25), l_h is hump-shaped if and only if $(1-\varepsilon)(\bar{\gamma} - \gamma_{sh}) > (\sigma - 1)(\gamma_{sh} - \gamma_h)$. Intuitively, the share of home production is growing at some initial period t_0 if the inflow of labour due to the poor substitutability between different types of goods dominates the outflow due to the good substitutability between goods within the service aggregate. This is more likely to take place in the initial stages of modern growth, when agriculture and manufacturing, which lose labour, are large.

The dynamics of manufacturing employment l_m are also non-monotonic if γ_m is below the initial $\bar{\gamma}$. But since $\bar{\gamma}$ converges to γ_s, the only remaining consumption sector in the asymptotic steady state, $\bar{\gamma}$ eventually falls below γ_m and so l_m also eventually falls until it converges to the savings ratio.

Overall labour supply is equal to $q = l - l_h$, so its dynamics parallel the dynamics of home production. Just as the time devoted to home production eventually has to fall, aggregate labour supply eventually has to rise. But reflecting the likely hump-shaped evolution of home production, initially labour supply is likely to fall, giving a U-shaped aggregate labour supply.

1.2 Aggregate balanced growth

We now turn to the optimal intertemporal allocations. Note first that the marginal utility of manufacturing goods is ϕ_m/ϕ. Given that both ϕ and c_{sh} are homogenous of degree one, together with (10) and the definition of c, we obtain,

$$\phi = \sum_{i=a,m,sh} \phi_i c_i = \phi_m c, \tag{27}$$

which implies that the marginal utility of c_m is equal to $1/c$. So the optimal choice of leisure satisfies

$$\frac{v'(1-l)}{1/c} = (1-\alpha_m) A_m F(k_m, 1), \tag{28}$$

where $\alpha_m = k_m F_K(k_m, 1)/F(k_m, 1)$ is the capital share in sector m. This, of course, is a restatement of the familiar condition that the marginal rate of substitution between leisure and consumption (in this case c_m, the numeraire) is equal to wages. Combining (28) with (18), leisure satisfies

$$\frac{c}{y} = \frac{1-\alpha_m}{v'(1-l)\,l} \tag{29}$$

Therefore, as in one-sector models, there is a close relationship between the dynamics of l and the dynamics of c/y.

In order to obtain these dynamics we restrict our production functions to be Cobb-Douglas, $F(k_i, 1) = k_i^{\alpha}$, $\alpha \in (0,1)$. This implies that our Hicks-neutral technology of the preceding sections is also labour-augmenting, which is required for the existence of a balanced growth path. Of course, under the Cobb-Douglas restriction, the α_m in (28) and (29) is equal to the constant α.

We show in the Appendix that the following two dynamic equations hold:

$$\frac{\dot{k}_e}{k_e} = \left[1 - \frac{1-\alpha}{v'(1-l)\,l}\right]\left(\frac{k_e}{l}\right)^{\alpha-1} - \left(\delta + \nu + \frac{\gamma_m}{1-\alpha}\right) \tag{30}$$

and

$$\left[1+\frac{-v''\left(1-l\right)l}{\alpha v'\left(1-l\right)}\right]\frac{\dot{l}}{l}=\frac{\gamma_m+\left(1-\alpha\right)\left(\delta+\nu\right)+\rho}{\alpha}-\left(\frac{1-\alpha}{v'\left(1-l\right)l}\right)\left(\frac{k_e}{l}\right)^{\alpha-1}. \qquad (31)$$

where $k_e \equiv kA_m^{-1/(1-\alpha)}$ is the capital stock per capita in efficiency units. The economy converges to a unique steady state where l and k_e are constant. The unique steady state is saddle-path stable and the saddle path is downward-sloping. We note that the existence of a steady state requires constant TFP growth rate in the capital-producing sector only, i.e. γ_m must be constant. The other TFP growth rates appear only in the dynamics of relative prices and sectoral time allocations and need not be constant. Along the transition, if the initial k_e is smaller than its steady-state level, l is falling (leisure is rising) and the capital stock is rising, and from (29), c/y is also rising. So the model's aggregate transitional dynamics parallel the transitional dynamics of the one-sector Ramsey model, except that our aggregates include the output and consumption of home production. Changes in overall labour supply along the transition are taking place both because of substitutions between home and market production and because of substitutions between leisure and work.

On the steady state path leisure time is constant but the supply of labour is not constant because of changes in the amount of time allocated to home production. But the constancy of both k_e and l implies that the capital-labour ratio in the economy as a whole grows at the rate of labour-augmenting technological growth in manufacturing, $\gamma_m/(1-\alpha)$. From (18) we also see that output per hour in the economy as a whole, y/l, is growing at the same rate. The aggregate capital stock in the market sector is given by

$$k_{market}=\sum_{i\neq h} l_i k_i = qk_m, \qquad (32)$$

and so the market capital-labour ratio, k_{market}/q is simply k_m, which is equal to k/l and grows at rate $\gamma_m/\left(1-\alpha\right)$. Market output in this economy is

$$y_{market}=\sum_{i\neq h} p_i A_i k_i^{\alpha} l_i = qA_m k_m^{\alpha} \qquad (33)$$

and so market output per hour, y_{market}/q is growing at the same constant rate and the capital-output ratio in the market economy is constant. This confirms our claim that our economy satisfies Kaldor's stylized facts of aggregate balanced growth, despite the changes in labour supply.

Since the only additional restriction needed to derive the balanced growth path is that the production functions be Cobb-Douglas, the dynamics derived in the preceding section for employment shares hold on this balanced growth path. So labour supply is likely to exhibit a U-shaped evolution even when the economy's capital-output ratio is constant, and labour supply does not have to reach constancy or near constancy in finite time, even if the economy's aggregate ratios do. Away from the steady state the employment shares are characterized by some additional dynamics, due to changes in l and c/y. Usually, the transitional dynamics will be driven by the economy needing to accumulate more capital per worker than it has at some initial state, and we briefly discuss the implications of these transitions for employment shares and aggregate labour supply.

In a transition with rising capital-labour ratio the adjustment dynamics are characterized by rising leisure time, and so by a falling labour supply. The transitional dynamics are superimposed on the U-shaped steady-state evolution of labour supply. Therefore, on the downward-sloping branch of the U the transitional dynamics reinforce the fall in labour supply, whereas on the upward branch of the U they mitigate the rise. This implies that the transitional dynamics have an asymmetric effect on the steady-state U-shape, increasing the steepness of the falling branch but making the rising branch flatter.

The saving rate falls along the transition. Equations (19) and (20) immediately yield that the impact on the employment shares of consumption sectors is positive. So when there are transitional dynamics the share of time allocated to market services is rising at an even faster rate but the decline of the share of agriculture and home production may slow down.

2 Quantitative Analysis

Suppose now the economy is on the balanced growth path that solves (30) and (31). We have shown in the previous section that the model is qualitatively consistent with facts on structural change and trends in market and home hours under two restrictions, $R1 : \varepsilon < 1 < \sigma$, $R2 : \gamma_a \geq \gamma_m > \gamma_s > \gamma_h$.[6] The objective of this section is to evaluate the quantitative predictions of the model. More specifically, how do the overall trends in hours and employment shares evolve in an economy with parameters derived from US data during 1900-2000?

Our main source of data for the allocation of time between market, home production and leisure is Ramey and Francis (2005). Figure 2 plots the weekly hours spent on each use of time for 1900-2000, with HP and cubic trends. Leisure time is trendless at about 37 hours, with the exception of a recent rise (which is statistically significant only at the 10 per cent level), but market work and home work have statistically significant trends (at the 1 per cent level). The significant trend in market work is the quadratic but in home work the cubic term is also significant, giving a slower rise up to the 1960s than the fall thereafter. Market work starts at about 34 hours and declines to 30 hours in

[6] The ranking of market TFP growth rates is consistent with the ranking of Jorgensen and Gallop (1992). We explain below how we get the low γ_h and how we give values to the market rates.

the post-war period, after which it rises. The trend rise is about 8 per cent.[7] Home production time mirrors changes in market work. The most important feature of these data is that changes in overall labour supply are due to substitutions between home and market production, as in the steady-state of our model. We now investigate how well our model does predicting the trends due to these substitutions.

From (16), (19) and (20) we see that the dynamic evolutions that we are interested in are fully determined by the evolution of the vector (φ, x_a, x_s), given the saving rate along the balanced growth path (η). Our strategy is to choose the initial values for this vector to match some initial allocation of time and then trace its evolution from other extraneous information. However, although this is possible for the sub-vector (x_a, x_s), we do not have enough information to do the same for φ. For this reason we calibrate the entire path of φ to the actual data, and use the information to predict other series of the model. Since φ is the main determinant of the marketization force in our model, we effectively match the marketization by construction. The objective is to see how this force interacts with structural change across different types of goods to yield predictions about aggregate labour supply and structural change within market sectors.

The dynamics of the vector (φ, x_a, x_s) depend on the elasticities of substitution ε and σ and on the differences in TFP growth rates between agriculture and manufacturing $(\gamma_a - \gamma_m)$, between manufacturing and services $(\gamma_m - \gamma_s)$, and between home and market services $(\gamma_s - \gamma_h)$. We begin with the parameters related to home production, $\varphi, \gamma_s - \gamma_h$ and σ.

On the model's steady state φ is equal to the ratio of the time allocated to market services to the time allocated to the home sector. The *Historical Statistics* provide service employment for the years 1899, 1919, and 1929-2000.[8] The ratio of home to market hours in 1900 in Ramey and Francis (2005) is (in our notation) $l_h/q = 0.7$. The service employment share in 1899 from *Historical Statistics* is 0.28, so $l_s = 0.28q$, which yields $\varphi = l_h/l_s = 2.5$ in 1900.

The rate of growth of φ is equal to $(1 - \sigma)(\gamma_s - \gamma_h)$ and although there are independent estimates for σ, there are no estimates for γ_h that we could use. We match φ for the entire period to the ratio l_h/l_s, and choose $\gamma_s - \gamma_h$ to match its rate of growth, given the value that we give to σ. The computed series for φ is shown in figure 3. The average growth rate for the computed series for 1900-2000 is -0.009, so we set $(1 - \sigma)(\gamma_s - \gamma_h) = -0.009$. As assumed by the model, log-linearity in φ is a good approximation.

[7] The rise in figure 2 is slightly less than the one shown in figure 1, which uses different sources and hours are divided by the population of working age. The main difference is that figure 2 excludes full-time equivalent school attendance from the denominator (which has been trending up) and adds an estimate of hours by sole proprietors to the numerator (which has been going down).

[8] We follow Kuznets (1966) to define our three sectors as follows: (1) agriculture includes agriculture, forestry, and fisheries, (2) industry includes mining, manufacturing, construction, utilities, transportation and communication, and (3) services are the rest of the economy.

The parameter σ is the elasticity of substitution between service and home goods. Using micro data Rupert et al. (1995) estimate it in the range $1.67 - 1.8$, and more recently Aguiar and Hurst (2005) estimate it in the range 2.2 to 2.5. We choose $\sigma = 2$ as our benchmark but we also report results for higher (and lower) values to test robustness. The benchmark implies $\gamma_s - \gamma_h = 0.009$. Finally, one other parameter that we need that involves home production is $\eta = \eta^0 (1 - p_h c_h / y)$, where η^0 is the total saving as a fraction of market production only. Our model yields $p_h c_h / y = l_h / l$, so $\eta = \eta^0 / (1 + \varphi l_s / q)$. Using Maddison's (1992) data, the gross saving rate was 0.19 in 1900. This gives $\eta = 0.11$.

The remaining parameters are related to market sectors. We set initial x_a and x_s to match initial employment shares from the *Historical Statistics.* By definition, $n_i = l_i / q$, and given $q = l - l_h = l - \varphi l_s$ we obtain $l_i / l = n_i / (1 + n_s \varphi)$ for $i = a, s, m$. We then use (19) and (20) to derive $x_i = (l_i / l)(l_m / l - \eta)^{-1}$ for $i = a, s$. Using (10), the differences in TFP growth rates are set to match the changes in the prices of agriculture and service goods relative to manufacturing goods.[9] The price data for services starts in 1929. The average annual growth rate for the relative price of services in terms of manufacturing for the period 1929-2000 is 1.02 per cent. For the same period, the relative price of agriculture in terms of manufacturing price is falling at an average rate of 0.88 per cent.[10] These two numbers are consistent with the direct estimates of Jorgenson and Gallop (1992), who calculate an average TFP growth rate for the period 1947-85 of 0.0206 for agriculture and 0.0082 for the private non-farm sector.[11] Within the non-farm sector, TFP growth rates vary but the TFP growth rate for industrial sectors is in general higher than the one for service sectors.[12]

For the elasticity of substitution ε we do not have direct estimates. In Ngai and Pissarides (2004), we obtained an average estimate of $\varepsilon = 0.3$ over the period 1977-2001 from the relationship between changes in relative prices and changes in relative employment, using thirteen 2-digit consumption-goods sectors from the OECD STAN database and input-output tables. Of course, the more aggregative the sector decomposition the smaller the value of ε should be. For our three sectors in this paper we set $\varepsilon = 0.1$ as a benchmark. The calibrated benchmark values are shown in Table 2.

[9] The measurement of both prices and TFP, especially in the earlier period, is fraught with difficulties but we still considered it useful to see how the model behaves quantitatively given the available information. Given the measurement problems, we use the average rate of TFP change for the whole period rather than looking at different sub-periods, even though our balanced growth path allows γ_s and γ_a to change over time.

[10] Source for 1929-1970: *Historical Statistics of the United States: Colonial Times to 1970, Part 1 and 2.* Employment is in series F250-258, the implicit price deflator for services in series E17, and the wholesale price index for industrial commodities and farm products in series E24-25. For 1970-2000, see *Economic Report of the President*, Tables B-62 and B-67.

[11] The numbers are obtained from adding the productivity growth rates due to input quality adjustment from Table 4 to the TFP growth rates in Table 1, 0.0158 for agriculture and 0.0044 for the non-farm sector.

[12] A recent update by Jorgenson and Stiroh (2000) confirmed the ranking for the period 1958-1996.

Table 2: Baseline Parameters, United States, 1900-2000

η	σ	ε	$\gamma_m - \gamma_a$	$\gamma_m - \gamma_s$	$\gamma_s - \gamma_h$
0.11	2	0.1	-0.009	0.01	0.009

With knowledge of the vector (φ, x_a, x_s) we can compute the market employment shares $n_i = l_i/(l_a + l_m + l_s)$ from (19)-(20), given the constant leisure and the savings rate. The results are shown in figure 4 against the HP-filtered data. The fit is very good for all three sectors, especially for services, where we track the rise throughout the period. Overall market hours can be expressed as $q/l = 1 - l_h/l$, and so $q/l = 1 - \varphi(l_s/q)(q/l)$. This gives $(1 - \varphi n_s)(q/l) = 1$, where n_s is the service employment share shown in figure 4. Since l is constant and φ was calibrated to the l_h/l_s time series, the model's prediction of the time path of q is a non-linear transformation of its prediction of the time path of the service employment share, so how well the model does predicting q is dependent on how well it does predicting n_s (and vice versa). We show the predictions for q and l_h, the time devoted to home production, in figure 5. The model predicts the shares of each activity. In order to obtain the hours from the predicted shares we set the value of leisure at the sample mean and assume that the length of the week is 94 hours (the total less time spent on essential activities - see the next section). The predicted path for labour supply turns out to be U-shaped. As we argued previously, the non-monotonicity is due to the tension between the marketization force, shown here by the falling φ, and the structural transformation force, shown by the rising n_s. The model's prediction tracks well the trends in both home and market hours. Since by construction we restricted leisure to constancy the small rise in leisure in the last few years of the sample has to be attributed elsewhere. The model attributes it to market production.

In order to highlight the role of home production in the model's prediction of structural change, we compute again the predictions of market shares in figure 4 by forcing home production to zero (setting $\psi \to 0$). We make use of the calibrated market parameters $(\varepsilon, \gamma_m - \gamma_a, \gamma_m - \gamma_s, \eta^0)$, with $\eta^0 = 0.18$ which matches the average gross saving rate between 1900-2000. Note that in this case $\varphi = 0$ and the initial values for x_a and x_s are adjusted accordingly to match the initial employment shares. Figure 6 compares the predictions with home production and without. It is clear that as we argued intuitively, the predictions of structural change with home production are better. Without home production both the decline of the share of agriculture and the rise of the share of services are slower.

Our results in figures 4 and 5 are robust to variations in the elasticities ε and σ within a plausible range. We consider higher ε elasticities, $\varepsilon = 0.2$ and $\varepsilon = 0.3$ and σ elasticities in the estimated ranges of $\sigma = 1.7$ and $\sigma = 2.5$. We retain the same calibration procedure, namely, calibrate $(\sigma - 1)(\gamma_s - \gamma_h)$ to the average growth rate of φ in the data. Therefore, an increase (decrease) in σ implies a decrease (increase) in $\gamma_s - \gamma_h$.

The two sets of variables that we are interested in are market work, $q = l - l_h$ and employment shares $n_i = l_i/q$, $i = a, s, m$. Equations (23)-(26) represent $\dot{l}_i/l_i$ along the balanced growth path. The second terms for both $\dot{l}_s/l_s$ and $\dot{l}_h/l_h$ can be rewritten as a function of the marketization force $(\sigma - 1)(\gamma_s - \gamma_h)$, which is independent of the choices of (ε, σ). Therefore, the effects of the elasticity parameters on time allocation are through the structural transformation term $(1 - \varepsilon)(\bar{\gamma} - \gamma_i)$, $i = a, m, sh$.

Increasing the elasticity of substitution across goods implies lower $\dot{l}_i/l_i$ for all i. So, market hours decline at a slower rate during the early development stage and rise more rapidly subsequently. The lower $\dot{l}_s/l_s$ and $\dot{l}_h/l_h$ together imply that employment shares in services rise less rapidly. These are confirmed by figure 7, which shows percentage point deviations from the benchmark at different values of the elasticity parameters. The predicted market hours for both $\varepsilon = 0.3$ and $\varepsilon = 0.2$ lie above the benchmark. The predicted service employment shares for both $\varepsilon = 0.3$ and $\varepsilon = 0.2$ lie below the benchmark.

Turning now to the elasticity of substitution within the services aggregate, we find that higher σ implies lower $(\gamma_s - \gamma_h)$, and as a result, lower $(\bar{\gamma} - \gamma_{sh})$ and higher $(\bar{\gamma} - \gamma_i)$, $i = a, m$.[13] Thus, $\dot{l}_s/l_s$ and $\dot{l}_h/l_h$ decrease, but $\dot{l}_a/l_a$ and $\dot{l}_m/l_m$ increase when σ is higher. Therefore, the impact on market hours and service employment shares are similar to a higher ε. These again are confirmed by figure 7. The predicted market hours for $\sigma = 2.5$ ($\sigma = 1.7$) lie above (below) the baseline. The predicted service employment shares for $\sigma = 2.5$ ($\sigma = 1.7$) lie below (above) the baseline.

Considering now the robustness results as a whole, we find that only the high value $\varepsilon = 0.3$ gives results that are not very close to our benchmark. One way of interpreting these results is to treat $\varepsilon = 0.3$ as an upper bound for the substitution elasticity between the three final goods needed to obtain good results from our model. As we have already noted, $\varepsilon = 0.3$ is an average value found for two-digit sectors, which must necessarily be above the value for the higher-level aggregates of this paper.

[13] To see this, rewrite

$$\begin{aligned} \bar{\gamma} - \gamma_{sh} &= \textstyle\sum_{i=a,m} (\gamma_i - \gamma_s) x_i/X + [\varphi/(1+\varphi)](\gamma_s - \gamma_h)(1 + x_a)/X \\ \bar{\gamma} - \gamma_s &= \textstyle\sum_{i=a,m} (\gamma_i - \gamma_s) x_i/X - (\gamma_s - \gamma_h)\varphi x_s/X \end{aligned}$$

The results follow from the fact that higher σ implies lower $(\gamma_s - \gamma_h)$. Note that $\gamma_i - \gamma_s = (\gamma_i - \gamma_m) + (\gamma_m - \gamma_s)$, $i = a, m$, which are fixed.

3 More on the economics of leisure

We have treated leisure so far as in conventional growth and real business cycle models, as time that yields utility directly, without the help of any goods. But a large amount of leisure in time use surveys is enjoyed with the use of some capital or intermediate goods, such as watching TV, surfing the net or talking on the telephone. We generalize our benchmark model by introducing a leisure good c_l that is produced mostly at home using time and capital goods.[14] One important outcome of this extension is that now changes in labour supply may be reflected in changes in leisure time, even if the economy is on a balanced growth path.

We assume that leisure is of two types, one as in the benchmark model and one that is the output of a "production" process that uses capital and labour through a production function that is identical to the one for other goods. We use subscript l for leisure-goods production and let A_l denote its TFP level. We assume that the leisure good (say TV viewing services) is a better substitute for service goods than it is for agricultural and manufacturing goods. But it is not as good a substitute for market services as home production is. This is the main feature that differentiates home production from leisure production. Home production such as cooked food has market-produced close substitutes but leisure production such as TV viewing does not have close substitutes in the market; if an individual hires somebody to do her TV viewing for her the end product will not be a close substitute to watching the TV herself. Yet both cooked food and TV viewing are produced at home with some durable good purchased from the manufacturing sector.

Formally, we assume that the services aggregate now consists of three goods, market services and home production as before, combined into c_{sh} as in the benchmark model, and leisure goods, which are combined with c_{sh} into a grand service good, c_{shl}. We want the elasticity of substitution between c_{sh} and c_l to be bigger than the one between service goods and manufacturing goods (our ε) but smaller than the elasticity of substitution between market and home produced services (our σ). We choose it to be 1, which gives a particularly simple and appealing result on the dynamics of leisure time. But the model also has a solution if the elasticity is bigger or smaller than one.

The utility of goods now is,

$$\phi(.) = \left(\sum \omega_j c_j^{(\varepsilon-1)/\varepsilon}\right)^{\varepsilon/(\varepsilon-1)} \quad j = a, m, shl; \quad c_{shl} = c_{sh}^{1-\xi} c_l^{\xi}, \tag{34}$$

[14] In time use surveys by far the dominant good of the kind that we have in mind is watching TV. See below in this section for some data. Greenwood and Vandenbroucke (2005) also put forward the idea that the dynamics of leisure time are influenced by the complementarities between durables and time. Their approach, however, is different from ours. They claim that leisure has increased because the quality and variety of goods like TV, which are complementary to leisure time, has gone up. Our claim runs along the lines of our previous discussion, people consume more time watching TV and doing other similar things because technological progress elsewhere has increased their consumption of other goods and other goods are poor substitutes for TV watching time.

with c_{sh} modelled as before, as a CES between c_s and c_h with elasticity σ. This specification reduces to the benchmark model when $\xi \to 0$. The static efficiency condition from production and consumption implies that (9) and (10) now hold for the extended set $i = a, s, h, l$. The marketization force is the same as before, so φ, x_a and x_h are the same as before. Define x_l in the same way as the other x_i,

$$x_l \equiv \frac{p_l c_l}{c_m} = \frac{p_l c_l}{p_s c_s} x_s = \frac{\xi}{1-\xi} x_{sh} \tag{35}$$

Given (19), which still holds, we obtain,

$$\frac{l_l}{l_h + l_s} = \frac{x_l}{x_{sh}} = \frac{\xi}{1-\xi}. \tag{36}$$

This is an important result that is due to our unit elasticity assumption for c_{sh} and c_l : the ratio of leisure-production time to service-production time is a constant. The size of the constant depends on the parameter ξ. We show in the appendix that all the other results of the benchmark model still hold, with minor changes because of the existence of an additional sector. Importantly, the aggregates (consumption, income and capital stock) are still defined as before and a balanced growth path with constant capital-output ratio exists. The new element is that on this steady state total leisure is now defined as $1 - l + l_l$, and it may not be constant. Structural change is still in the direction of slow-growing sectors in the market and the marketization force for home production, given $\sigma > 1$, is still present.

Now, as in the benchmark model and for as long as TFP growth in manufacturing exceeds TFP growth in the service sectors, service employment $l_s + l_h$ is monotonically increasing over time, until in the limit $l_h, l_a \to 0$ and $l_m \to (1 - c/y)$. But now because of (36), l_l is also monotonically increasing over time. Thus, total leisure time, $1 - l + l_l$, is increasing over time, with l constant on the balanced growth path and l_l rising. We address two questions about this dynamic. First, how big is the share of leisure now and how big is it in the asymptotic state? This will give an idea of the dynamics involved. Second, what happens to overall labour supply when there is leisure production.

The answer to the first question depends mainly on the preference parameter ξ. This is because both the the current and asymptotic l_l is a constant fraction $\xi/(1-\xi)$ of service employment. In the American Time Use Surveys (ATUS) of 2003 and 2004 (available online at www.bls.gov/tus/) there is a fairly detailed breakdown of the activities in which people engage in their leisure time. If we include under our leisure production TV watching, sports participation and telephone, mail and email we find that individuals over the age of 15 spend about 21 hours a week in these activities. Total leisure time is about 39 hours and total work time (market and home) 50 hours.[15] Making use of the

[15] The remainder is spent on essential activities like sleep, 74 hours, education, 3.5 hours and unclassified items, 1.5 hours. For earlier results from time use surveys in the United States and elsewhere see Juster and Stafford (1991).

data on home and market production from the same surveys we get an approximate value of $\xi = 1/3$. In the asymptotic steady state our model prediction (on the assumption that the time devoted to the other activities mentioned in the preceding footnote remains the same) is that total work converges to 44 hours and total leisure time to 45 hours. So the prediction is that once the structural transformation and marketization forces run their course, there will be a net shift of 5 hours a week from work to leisure activities. It is also predicted that the shift will take more than 100 years to get close to its asymptotic value.

Labour supply with leisure production is $q = l - l_h - l_l$. Since home production converges to zero and leisure converges to a constant, labour supply must also converge to a constant. Leisure is rising throughout the adjustment to the asymptotic steady state, whereas we have argued that the transformation and marketization forces that drive labour supply in the benchmark first lower labour supply and then increase it. So with leisure production the predicted initial fall in labour supply is faster and due to both the rise in leisure and the rise in home production, whereas in the second phase, when labour supply increases, the rise would be mitigated. Two forces are acting against each other in the second phase, the marketization of home production pushes for a rise in labour supply and the rise in leisure for a fall. With the parameter values used in our benchmark calibrations and ξ set equal to 1/3, the marketization force dominates and labour supply is on a very slowly increasing trend. Moreover, the prediction of a rising labour supply is not very sensitive to the precise value of ξ used.

4 Is the eventual disappearance of home production inevitable?

Home production in our benchmark eventually disappears, partly because it produces a good that is a close substitute to a market good and partly because the rate of TFP growth in the market sector is always bigger than it is in the home sector. The numerical illustration has shown that the decline of the home sector can be very slow, taking hundreds of years.[16] However, the model can be easily generalized to avoid this conclusion altogether, without affecting the main results derived so far.

The generalization requires the introduction of more than one home good. From (11) and (12) we obtain that l_h does not indefinitely fall in relation to l_s either if $\sigma = 1$, or if the ratio of TFP in the market and the home is constant (i.e., if their growth rates are equal). Previous quantitative literature has argued for $\sigma > 1$ at the aggregate level, but this does not exclude the possibility of some home goods having unit elasticity with respect to some market goods. Nevertheless, we now maintain the assumption that

[16] For example, it takes about 300 years for the share of the home sector out of total work (l_h/l) to go down to 10%.

$\sigma > 1$ and consider the implications of equal TFP growth rates.

More specifically, we briefly discuss the implications of dividing the service sector into a progressive component and a stagnant one. The progressive component includes sub-sectors such as business services, trade and catering, which have positive TFP growth rates. The stagnant component includes education, health care and the arts, which have zero or near zero TFP growth rates. We can now define two types of home production sectors, one that produces services that are close substitutes for progressive service goods, such as food preparation, and one that produces substitutes for stagnant services, such as caring for dependants. The TFP growth rate in the progressive home sector is postulated to be below the one in the market sector, and this sector behaves like the one that we modelled in our benchmark case. It eventually vanishes through the marketization of all its output. The stagnant home sector has the same low TFP growth rate as the stagnant market sector, and the ratio of time employed in each remains constant over time. From (11) this ratio depends on preferences and the relative TFP level for the two goods, but from (13) and as long as the growth rates of their TFP levels are equal, their relative employment shares do not change.

In accordance with our technological explanation of structural change, the stagnant service composite will be attracting labour over time from all other sectors, which will be shared equally between the market and home sub-sectors. As before, the progressive market sector will be continually gaining labour from the progressive home sector. Consider the implications for the dynamics of overall labour supply in the benchmark case with constant leisure. The employment share of the progressive home sector will be hump-shaped, as in the benchmark model. The share of the stagnant sector will be rising monotonically. So in the early stages of economic development, when both components are rising, the overall share of the home sector will be rising, and so labour supply falling. Labour supply will also be falling in the very distant stages of economic development, as it approaches the asymptotic steady state, because by that time the progressive home sector will have practically vanished but the share of the stagnant home sector will still be rising. Between the two extremes overall labour supply may be rising or falling, and may have more than one turning point.

The model can be further extended with the introduction of more sectors to give richer dynamics. It is obvious from the discussion with the two home sectors, however, that it is possible to have a monotonically falling labour supply, or one that falls at first, rises at some point but then falls again. The stylized fact of a falling labour supply is, if anything, reinforced by the introduction of a stagnant service sector. The main impact on the dynamics of labour supply take place in later stages, where, as we argued in the introduction, cross-country data suggest less clear-cut dynamics patterns.

5 Conclusions

Our objective of showing that a unified framework can simultaneously account for balanced aggregate growth, structural change between agriculture, industry and services and a changing trend in aggregate hours of work has been accomplished. Our prediction of the coexistence of a changing trend in hours on the one hand and balanced aggregate growth on the other is new to a model of economic growth and is the result of studying structural change in a model with home production. The assumptions that drive our results are (a) market goods are poor substitutes with each other but home-produced goods have close substitutes in the market, and (b) agriculture and industry have higher rates of total factor productivity growth than do services, but within the services group market services have higher rates of TFP growth than home services. On the aggregate economy's balanced growth path the dynamics of aggregate market hours are driven by the dynamics of home production, but off the steady state there are transitional dynamics with leisure time rising and the supply of labour falling. We have also shown that an extension which refines the use of leisure time and pays attention to the fact that most leisure time is spent with some capital good, such as a TV set, has the implication that leisure time is also rising over time on the balanced growth path.

Quantitative analysis shows that our model matches well the dynamics of US hours and market employment shares since 1900. In particular, we explain the fall in market hours up to the post-war period and the more recent rise. Of course, we are not suggesting that no other factor can contribute to the explanation of the dynamics of market hours of work. Some might think it implausible that the rapid fall in average hours of work for working men from 1900 to 1940 is matched one-for-one with a rise in home production for this group. A model with rising leisure, as for example in our extension with leisure capital goods, is probably more appropriate for this group, although time use surveys show that men do spend more time on home production when they work less in the market. The rise of female employment since 1960, however, is more likely to be primarily the outcome of the marketization of home production, the primary force emphasized in this paper.

We abstracted from all distortions to competitive market allocations. But labour markets, even in the United States, are subject to many distortions which could influence the allocation of time between market and home. Several authors have focused on the role of taxation, but the evidence so far does not favour a tax explanation of the trends in labour supply. In Prescott's (2004) work the marginal tax rate in both the United States and United Kingdom was the same in the early 1970s and the early 1990s, yet market hours increased in the United States and decreased in the United Kingdom. Using similar methodologies Mulligan (2002) and Ramey and Francis (2005) find that the distortion required to explain the dynamics of market hours in the 20th century does not mimic the imputed tax distortion in the data. Econometric studies also fail to find significant tax effects on the dynamics of aggregate labour supply, though some impact exists (Nickell, 2004).

European data show the same general patterns for market hours of work as in the United States, but more recently with some delay in the marketization of home production. We did not discuss in any detail reasons for these differences; the distortions discussed in the preceding paragraph seem to be more relevant in this comparison. Future work needs to enrich the technological explanation of change that we have emphasized in this paper with the introduction of taxes and other distortions to economic activity (see again Freeman and Schettkat, 2005, Prescott, 2004, Rogerson, 2004, and Messina, 2005, for related work).

References

[1] Aguiar, M., and E. Hurst (2005), "Life Cycles Prices and Consumption." Mimeo, University of Chicago.

[2] Baumol, W. (1967). "Macroeconomics of Unbalanced Growth: The Anatomy of Urban Crisis," *American Economic Review* 57: 415-26.

[3] Caselli, F. and W.J. Coleman II (2001). "The U.S. Structural Transformation and Regional Convergence: A Reinterpretation," *Journal of Political Economy* 109: 584-616.

[4] Durand, J. (1975). *The Labour Force in Economic Development: A Comparison of International Census Data, 1946-1966.* Princeton, NJ: Princeton University Press.

[5] Echevarria, C. (1997). "Changes in Sectoral Composition Associated with Economic Growth." *International Economic Review* 38 (2): 431-452.

[6] Freeman, R. and R. Schettkat (2005), "Marketization of Household Production and the EU-US Gap in Work', *Economic Policy*, January pp. 5-50.

[7] Fuchs, V. (1980). "Economic Growth and the Rise of Service Employment." NBER working paper No. 486.

[8] Goldin, C. (1995). "The U-Shaped Female Labour Force Function in Economic Development and Economic History." In T. P. Schultz, ed., *Investment in Women's Human Capital and Economic Development.* Chicago, IL: University of Chicago Press, pp.61-90.

[9] Gollin, D., S. Parente, and R. Rogerson, (2000). "Farm Work, Home Work, and International Productivity Differences." Forthcoming in *The Review of Economic Dynamics.*

[10] Greenwood, J. and G. Vandenbroucke, (2005). "Hours Worked: Long-Run Trends." Research Report No. 10, Economie d'avant garde, University of Rochester.

[11] Greenwood, J., A. Seshadri and M. Yorukoglu (2005). "Engines of Liberation." *Review of Economic Studies* 72, 109-133.

[12] Gronau, R. (1977). "Leisure, Home Production, and Work – the Theory of the Allocation of Time Revisited." *Journal of Political Economy* 85: 1099-1123.

[13] Gronau, R. (1997). "The Theory of Home Production: The Past Ten Years." *Journal of Labour Economics*, v15: 197-205.

[14] Jorgenson, D. and F. Gallop (1992). "Productivity Growth in U.S. Agriculture: A Postwar Perspective." *American Journal of Agricultural Economics 74:* 745-750.

[15] Jorgenson, D. and K. Stiroh (2000). "U.S. Economic Growth at the Industry Level." *American Economic Review* 90 (2): 161-167.

[16] Juster, F and F. Stafford (1991). "The Allocation of Time: Empirical Findings, Behavioral Models, and Problems of Measurement." *Journal of Economic Literature* 29: 471-522.

[17] King, R., C. Plosser and S. Rebelo (1988). "Production, Growth and Business Cycles I. The Basic Neoclassical Model." *Journal of Monetary Economics* 211: 195-232.

[18] Kongsamut, P., S. Rebelo and D. Xie (2001). "Beyond Balanced Growth," *Review of Economic Studies* 68: 869-882.

[19] Kuznets, S. (1966). *Modern Economic Growth: Rate, Structure, and Spread.* New Haven, Conn.: Yale University Press.

[20] Maddison, A., (1992). "A long-run perspective on saving," *Scandinavian Journal of Economics*, 84: 181-196.

[21] Maddison, A. (1995). *Monitoring the World Economy 1892-1992.* Development Centre studies, OECD.

[22] Messina, J. (2005). "The Role of Product Market Regulations in the Process of Structural Change" mimeo, European Central Bank, forthcoming *European Economic Review.*

[23] Mitchell, B.R. (1980). *European Historical Statistics, 1750-1975.* London: Macmillan Press.

[24] Mokyr, J. (2000). "Why Was There More Work for Mother? Technological Change and the Household, 1880-1930." *Journal of Economic History*, Vol. 60, No. 1, pp. 1-40.

[25] Mulligan, C. B. (2002). "A century of Labor Market Distortions", Working Paper 8774, National Bureau of Economic research, Cambridge, Massachusetts.

[26] Ngai, L. R. and C. Pissarides (2004). "Structural Change in a Multi-Sector Model of Growth." CEPR Discussion Paper 4763.

[27] Nickell, S. (2004). "Employment and Taxes", CEP Discussion Paper No 634.

[28] Parente, S., Rogerson, R., Wright, R., (2000). "Homework In Development Economics: Household Production and the Wealth of Nations". *Journal of Political Economy* 108, 680-688.

[29] Prescott, E., (2004). "Why Do Americans Work So Much More than Europeans?" Quarterly Review of the Federal Reserve Bank of Minneapolis, July, 2-13.

[30] Ramey, V. and N. Francis. (2005). "A Century of Work and Leisure." Working Paper.

[31] Rogerson, R. (2004). "Structural Transformation and the Deterioration of European Labour Market Outcomes." Working Paper.

[32] Rupert, P., R. Rogerson and R. Wright. (1995)."Estimating Substitution Elasticities in Household Production Models." *Economic Theory* 6, 179-193.

[33] US Bureau of the Census (1975). *Historical Statistics of the United States, Colonial Times to 1970.* Bicentennial Edition, Part 1 and Part 2. US Government Printing Office, Washington, DC.

Appendix

Claim 1 *Derivation of* x_i, $i = a, s, h$.

Proof. The utility function and (10) immediately give the expressions for x_a and x_h. To derive x_s, first derive c_{sh}/c_s using the utility function and (10),

$$\begin{aligned} c_{sh} &= c_s\left(\partial c_{sh}/\partial c_s\right) + c_h\left(\partial c_{sh}/\partial c_h\right) = \left(\partial c_{sh}/\partial c_s\right)\left(c_s + p_h c_h/p_s\right) \\ &= (1-\psi)\left(c_{sh}/c_s\right)^{1/\sigma}(1+\varphi)\, c_s \end{aligned}$$

so

$$c_{sh}/c_s = \left[(1-\psi)(1+\varphi)\right]^{\sigma/(\sigma-1)},$$

and so from (10) again

$$\begin{aligned} p_s &= \frac{\omega_{sh}}{\omega_m}\left(\frac{c_m}{c_{sh}}\right)^{1/\varepsilon}(1-\psi)\left(\frac{c_{sh}}{c_s}\right)^{1/\sigma} \\ &= \frac{\omega_{sh}}{\omega_m}\left(\frac{c_m}{c_s}\right)^{1/\varepsilon}(1-\psi)\left(\frac{c_{sh}}{c_s}\right)^{(1/\sigma-1/\varepsilon)}, \end{aligned}$$

which together with c_{sh}/c_s implies,

$$\begin{aligned} x_s &\equiv \left(p_s c_s/c_m\right) = \left(\omega_{sh}/\omega_m\right)^{\varepsilon}\left(A_m/A_s\right)^{1-\varepsilon}(1-\psi)^{\varepsilon}\left(c_{sh}/c_s\right)^{(\varepsilon-\sigma)/\sigma} \\ &= \left(\omega_{sh}/\omega_m\right)^{\varepsilon}\left(A_m/A_s\right)^{1-\varepsilon}(1-\psi)^{\sigma(\varepsilon-1)/(\sigma-1)}(1+\varphi)^{(\varepsilon-\sigma)/(\sigma-1)}. \end{aligned}$$

■

Claim 2 *The growth rate of* x_h/X *is decreasing over time. If it is positive at* t_0*, then* x_h/X *is hump-shaped. Otherwise,* x_h/X *falls monotonically.*

Proof. Let $g_h(t)$ denotes growth rate of x_h/X at time t, given

$$g_h(t) = (1-\varepsilon)\left(\bar{\gamma} - \gamma_{sh}\right) - (\sigma-1)\left(\gamma_{sh} - \gamma_h\right),$$

first note that $\lim_{t\to\infty} x_s/X = 1$ implies $\lim_{t\to\infty}\bar{\gamma} = \gamma_s$, and $\lim_{t\to\infty}\varphi = 0$ implies $\lim_{t\to\infty}\gamma_{sh} = \gamma_s$, so

$$\lim_{t\to\infty} g_{x_h}(t) = (1-\sigma)\left(\gamma_s - \gamma_h\right) < 0.$$

Therefore, eventually x_h/X starts to fall towards zero. Given $\gamma_{sh} = \left(\gamma_s + \varphi\gamma_h\right)/(1+\varphi)$, using the definition of φ,

$$d\gamma_{sh}/dt = (\sigma-1)\left(\gamma_s - \gamma_h\right)^2 \varphi (1+\varphi)^{-2} > 0,$$

Let $x_{sh} = x_s + x_h = (1+\varphi)\, x_s$, using the definition of $\bar{\gamma}$, equations (23) to (26) imply

$$\begin{aligned} d\bar{\gamma}/dt - d\gamma_{sh}/dt &= \textstyle\sum_{i=a,m,sh} (x_i\gamma_i/X) \left(\dot{x}_i/x_i - \dot{X}/X\right) - (1 - x_{sh}/X)\, d\gamma_{sh}/dt \\ &= (1-\varepsilon) \textstyle\sum_{i=a,m,sh} (x_i\gamma_i/X)\,(\bar{\gamma} - \gamma_i) - (1 - x_{sh}/X)\, d\gamma_{sh}/dt \\ &= -(1-\varepsilon) \textstyle\sum_{i=a,m,sh} (x_i/X)\,(\gamma_i - \bar{\gamma})^2 - [(1 + x_a)/X]\, d\gamma_{sh}/dt < 0. \end{aligned}$$

Together they imply $g_h'(t) < 0$. So x_h/X is hump-shaped over time if $g_h(t_0) > 0$. Otherwise, x_h/X is falling monotonically. ■

Claim 3 *The equilibrium l and $k_e \equiv kA_m^{-1/(1-\alpha)}$ converge to a unique steady state.*

Proof. Using (9) and (10), the feasibility condition can be rewritten as follows:

$$\begin{aligned} \dot{k} &= l_m A_m k_m^\alpha - c_m - (\delta+\nu)\, k \\ &= \left(l - \textstyle\sum_{i=a,s,h} l_i\right) A_m k_m^\alpha - c_m - (\delta+\nu)\, k \\ &= l A_m k_m^\alpha - \textstyle\sum_{i=a,s,h} p_i A_i k_i^\alpha l_i - c_m - (\delta+\nu) k \\ &= l^{1-\alpha} A_m k^\alpha - c - (\delta+\nu)\, k \end{aligned}$$

From (28),

$$\frac{c}{k} = \left(\frac{1-\alpha}{v'(1-l)\, l}\right)\left(\frac{k_e}{l}\right)^{\alpha-1},$$

so we have,

$$\frac{\dot{k}_e}{k_e} = \left[1 - \frac{1-\alpha}{v'(1-l)\, l}\right]\left(\frac{k_e}{l}\right)^{\alpha-1} - D_k, \tag{37}$$

where $D_k = \delta + \nu + \gamma_m/(1-\alpha)$. Given that the marginal utility of c_m is equal to $1/c$, optimal saving implies,

$$\dot{c}/c = \alpha A_m k_m^{\alpha-1} - (\delta + \rho + \nu).$$

Finally, differentiation of (28) w.r.t. time yields,

$$\frac{\dot{c}}{c} + \frac{-v''(1-l)\, l}{v'(1-l)}\left(\frac{\dot{l}}{l}\right) = \gamma_m + \alpha\left(\frac{\dot{k}}{k} - \frac{\dot{l}}{l}\right)$$

which simplifies to,

$$\left[1 + \frac{-v''(1-l)\, l}{\alpha v'(1-l)}\right]\frac{\dot{l}}{l} = D_l - \frac{1-\alpha}{v'(1-l)\, l}\left(\frac{k_e}{l}\right)^{\alpha-1}, \tag{38}$$

where $D_l = [\gamma_m + (1-\alpha)(\delta+\nu) + \rho]/\alpha$. There exists steady state l and k_e satisfying:

$$\begin{aligned}
\dot{l} &= 0 : \frac{1-\alpha}{v'(1-l)\,l}\left(\frac{k_e}{l}\right)^{\alpha-1} = D_l \\
\dot{k}_e &= 0 : \left[1-\frac{1-\alpha}{v'(1-l)\,l}\right]\left(\frac{k_e}{l}\right)^{\alpha-1} = D_k
\end{aligned}$$

Solving the two equations yields,

$$\left(\frac{c}{y}\right)^{*} = \frac{1-\alpha}{v'(1-l)\,l} = \frac{D_l}{D_l+D_k}$$

Substitution back to $\dot{l}=0$ gives the unique steady state

$$\begin{aligned}
l^{*} &: v'(1-l)\,l = (1-\alpha)(D_l+D_k)/D_l \\
k_e^{*} &= l^{*}(D_l+D_k)^{1/(\alpha-1)}. \qquad (39)
\end{aligned}$$

To show convergence, note that $\dot{l}=0$ is downward sloping and $\dot{k}_e=0$ is upward sloping in $k_e - l$ space. Also, as $k_e \to 0$, we have $l \longrightarrow 1$ along $\dot{l}=0$, and $l \to \bar{l} < 1$ along $\dot{k}_e = 0$, where $\bar{l}$ satisfies $v'(1-l)\,l = 1-\alpha$. Finally, using (37) and (38) increasing k_e implies higher $\dot{l}$ and lower $\dot{k}_e$. We can now construct a phase diagram in $k_e - l$ space. The $\dot{l}=0$ is downward sloping and unstable and the $\dot{k}_e=0$ is upward sloping and stable, so there is a unique convergent downward-sloping saddle path. ■

Claim 4 *If $\gamma_h < \gamma_s < \gamma_a$, then in the asymptotic steady state,*

$$l_m/l = s/y, \quad l_s/l = c/y, \quad l_a/l = l_h/l = 0.$$

Proof. From (19) and (20), the proof is completed if $\lim_{t\to\infty} x_s/X = 1$. Given $\gamma_s > \gamma_h$, (11) implies $\lim_{t\to\infty}\varphi = 0$. Given $\gamma_s < \gamma_a$, (14) and (15) imply

$$\lim_{t\to\infty}(x_a/x_s) = (\omega_a/\omega_s)^{\varepsilon}(A_s/A_a)^{1-\varepsilon}(1+\varphi)^{(\sigma-\varepsilon)/(\sigma-1)} = 0.$$

Finally, using (16) and $X = \sum x_i$,

$$\lim_{t\to\infty}(x_s/X) = \lim_{t\to\infty}\left[(1+x_a)/x_s + 1 + \varphi\right]^{-1} = 1.$$

■

Leisure production

We define our aggregate c and y as in (17) and (18). The marginal utility of c_m is equal to $1/c$ as in the benchmark model using (27), and the optimal choice of l satisfies (28) as in the benchmark model. As a result, the dynamic equations for $k_e \equiv kA_m^{-1/(1-\alpha)}$ and l are the same as the benchmark model. Therefore, there is a unique steady state where l and k_e are constant. Along this steady state, market capital and output still satisfy (32) and (33), thus, the market capital-output ratio is constant and market output per worker is growing at constant rate $\gamma_m/(1-\alpha)$. We now turn to time allocation across all $i = a, s, h, l$. The main task is to solve out x_s in the generalized model. The optimal consumption choices for c_m and c_s imply,

$$\begin{aligned} p_s &= \frac{\omega_{shl}}{\omega_m}\left(\frac{c_m}{c_{shl}}\right)^{1/\varepsilon}(1-\xi)\left(\frac{c_{shl}}{c_{sh}}\right)(1-\psi)\left(\frac{c_{sh}}{c_s}\right)^{1/\sigma} \\ &= D\left(\frac{c_m}{c_s}\right)^{1/\varepsilon}\left(\frac{c_s}{c_{sh}}\right)^{1/\varepsilon-1/\sigma}\left(\frac{c_{sh}}{c_{shl}}\right)^{1/\varepsilon-1} \\ &= D\left(\frac{c_m}{c_s}\right)^{1/\varepsilon}\left(\frac{c_s}{c_{sh}}\right)^{1/\varepsilon-1/\sigma-\xi(1/\varepsilon-1)}\left(\frac{c_s}{c_l}\right)^{\xi(1/\varepsilon-1)} \end{aligned}$$

where $D = \frac{\omega_{shl}}{\omega_m}(1-\xi)(1-\psi)$. Substituting the expression for c_{sh}/c_s from claim 1, and c_s/c_l from (35), we obtain,

$$x_s = \frac{p_s c_s}{c_m} = Ep_s^{1-\varepsilon}(1+\varphi)^{[\varepsilon-\sigma+\sigma\xi(1-\varepsilon)]/(\sigma-1)}\left[(1+\varphi)\frac{p_s}{p_l}\right]^{-\xi(1-\varepsilon)},$$

where $E = D^{\varepsilon}(1-\psi)^{[\varepsilon-\sigma+\sigma\xi(1-\varepsilon)]/(\sigma-1)}\left(\frac{\xi}{1-\xi}\right)^{-\xi(1-\varepsilon)}$. Simplifying we obtain,

$$x_s = Ep_s^{(1-\xi)(1-\varepsilon)}(1+\varphi)^{[\varepsilon-\sigma+\xi(1-\varepsilon)]/(\sigma-1)}p_l^{\xi(1-\varepsilon)},$$

which reduces to the expression for x_s in the benchmark as $\xi \to 0$. With the results on x_i, the level of l_i continues to satisfy (19).

We now study the dynamics of l_i. Let γ_{shl} be the weighted average TFP growth of the service composite good c_{shl}. Using (35), define $x_{shl} = x_{sh} + x_l = x_{sh}/(1-\xi)$ so,

$$\gamma_{shl} = \frac{x_{sh}}{x_{shl}}\gamma_{sh} + \frac{x_l}{x_{shl}}\gamma_l = (1-\xi)\gamma_{sh} + \xi\gamma_l.$$

$\bar{\gamma}$ is again a weighted average of γ_i with weight x_i, $i = a, m, shl$. Our previous restrictions are $\gamma_a \geq \gamma_m > \gamma_s > \gamma_h$ and $\varepsilon < 1 < \sigma$. We now further restrict $\gamma_m > \gamma_l$, which together imply $\gamma_a > \bar{\gamma} > \gamma_{shl}$,and $\gamma_{shl}(t)$ is strictly increasing over time. The growth rate of x_a is the same as before, and the growth rate of x_s, using the expression for γ_{shl}, is

$$\begin{aligned} \frac{\dot{x}_s}{x_s} &= (1-\varepsilon)[\gamma_m - (1-\xi)\gamma_s - \xi\gamma_l] + [\sigma - \varepsilon - \xi(1-\varepsilon)](\gamma_s - \gamma_h)\varphi/(1+\varphi) \\ &= (1-\varepsilon)(\gamma_m - \gamma_{shl}) + (\sigma-1)(\gamma_s - \gamma_{sh}), \end{aligned}$$

which is identical to the benchmark with γ_{shl} replacing γ_{sh}. The dynamics of l_l and l_s, are,

$$\begin{aligned}\frac{\dot{x}_l}{x_l}-\frac{\dot{X}}{X} &= (1-\varepsilon)(\bar{\gamma}-\gamma_{shl}),\\ \frac{\dot{x}_s}{x_s}-\frac{\dot{X}}{X} &= (1-\varepsilon)(\bar{\gamma}-\gamma_{shl})+(\sigma-1)(\gamma_s-\gamma_{sh}).\end{aligned}$$

Both l_l and l_s rise along the balanced growth path due to the poor substitution across the three goods $i = a, m, shl$ and the ranking of their TFP growth rates. But there is an additional marketization force for the rise in l_s. Therefore compared to the benchmark, any quantitative difference in results is due to the new level of $\bar{\gamma}$ and the presence of γ_{shl}. Asymptotically, $\varphi \to 0$, l_a and l_h converge to zero, l_m converges to the saving rate, and both l_l and l_s converge to a constant where $l_l/l_s = \xi/(1-\xi)$. The dynamics of l_h follow

$$g_h(t) \equiv \frac{\dot{x}_h}{x_h}-\frac{\dot{X}}{X} = (1-\varepsilon)(\bar{\gamma}-\gamma_{shl})-(\sigma-1)(\gamma_{sh}-\gamma_h). \tag{40}$$

As in claim 2, $g_h'(t) < 0$ and $\lim_{t\to\infty} g_h(t) = (1-\sigma)(\gamma_s-\gamma_h) < 0$. So l_h is hump-shaped if $g_h(t_0) > 0$. Define total non-market hours by $l_n = l_l + l_h$, and let

$$x_n = x_h + x_l = \varphi + \frac{\xi}{1-\xi}(1+\varphi)x_s = \varphi_n x_s; \qquad \varphi_n = \frac{\xi+\varphi}{1-\xi}.$$

So,

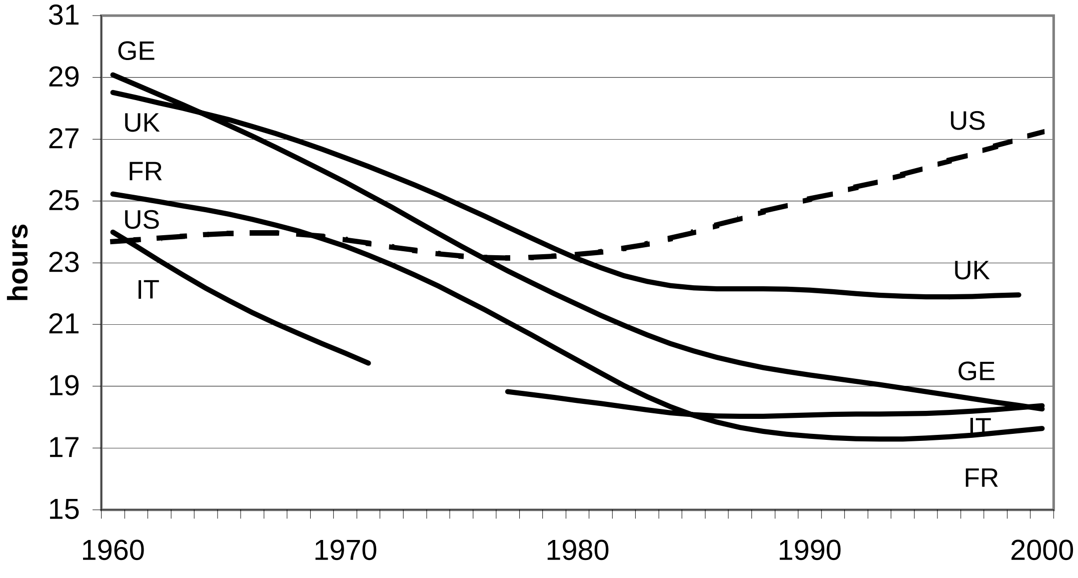
$$\begin{aligned}g_n(t) &\equiv \frac{\dot{x}_n}{x_n}-\frac{\dot{X}}{X} = \frac{x_h}{x_n}\left(\frac{\dot{x}_h}{x_h}-\frac{\dot{X}}{X}\right)+\frac{x_l}{x_n}\left(\frac{\dot{x}_l}{x_l}-\frac{\dot{X}}{X}\right) = \frac{x_h g_h + x_l g_l}{x_n},\\ &= (1-\varepsilon)(\bar{\gamma}-\gamma_{shl})-\frac{\varphi}{\varphi_n}(\sigma-1)(\gamma_{sh}-\gamma_h),\end{aligned}$$

so the dynamics of l_n follow closely those of l_h if ξ is small enough, i.e. hump-shaped. For sufficiently large ξ, the dynamics of l_n follows l_l, i.e. increase monotonically. The presence of leisure is also weakening the marketization effect. Asymptotically,

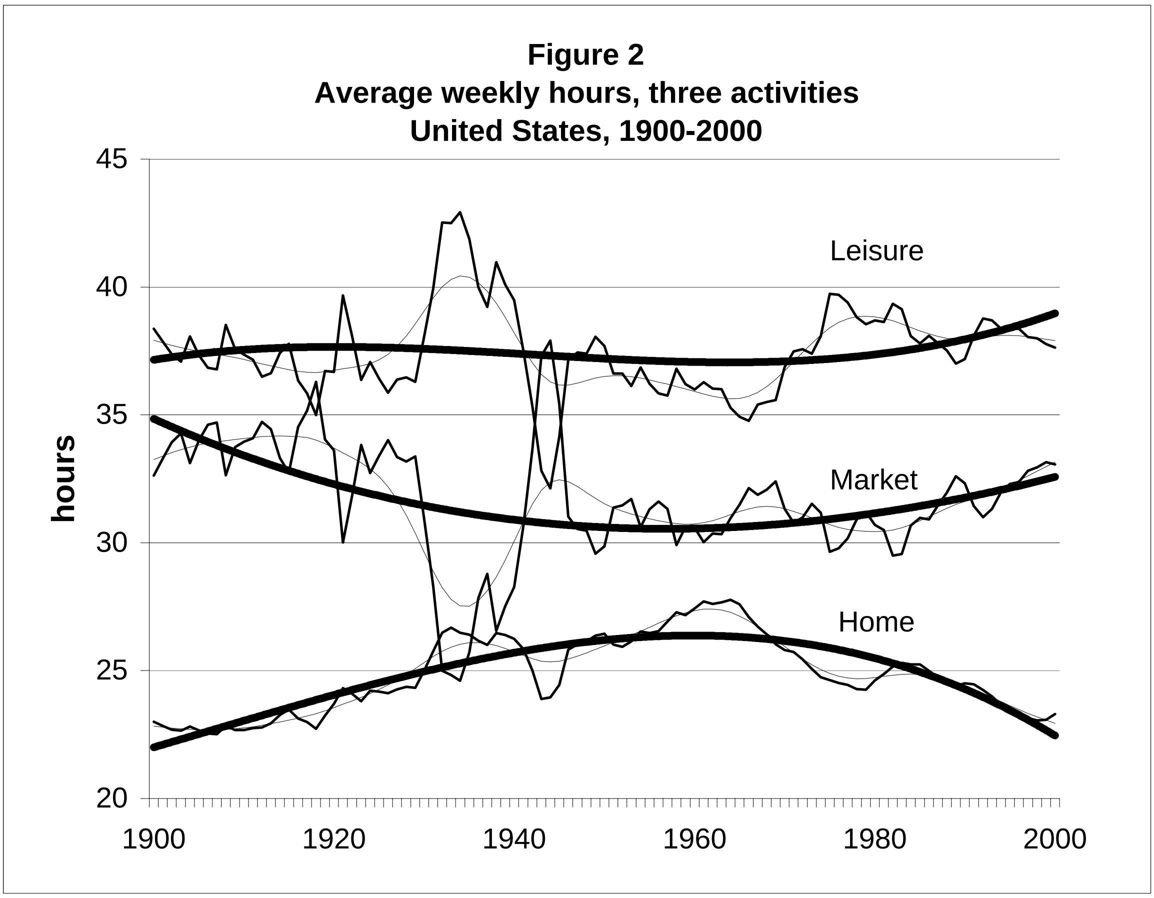
$$\lim_{t\to\infty} g_n(t) = 0,$$

so in the limit, total non-market time converges to a constant.

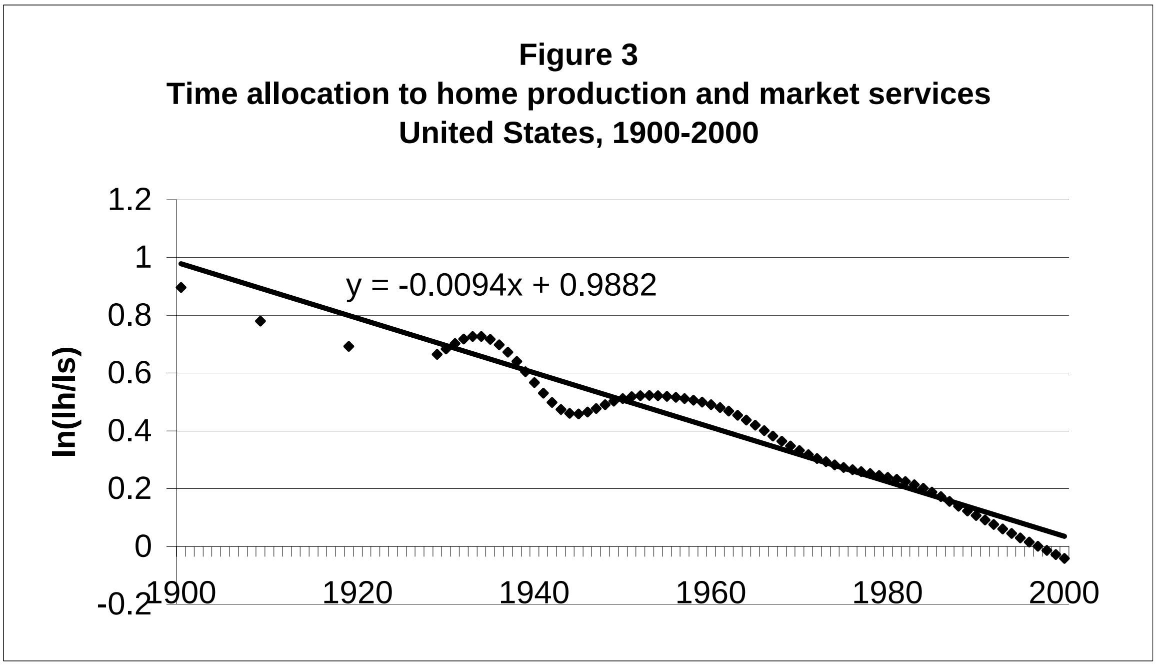
Figure 1
Average weekly hours of work, working age population (ages 15-64), five countries, 1960-2000



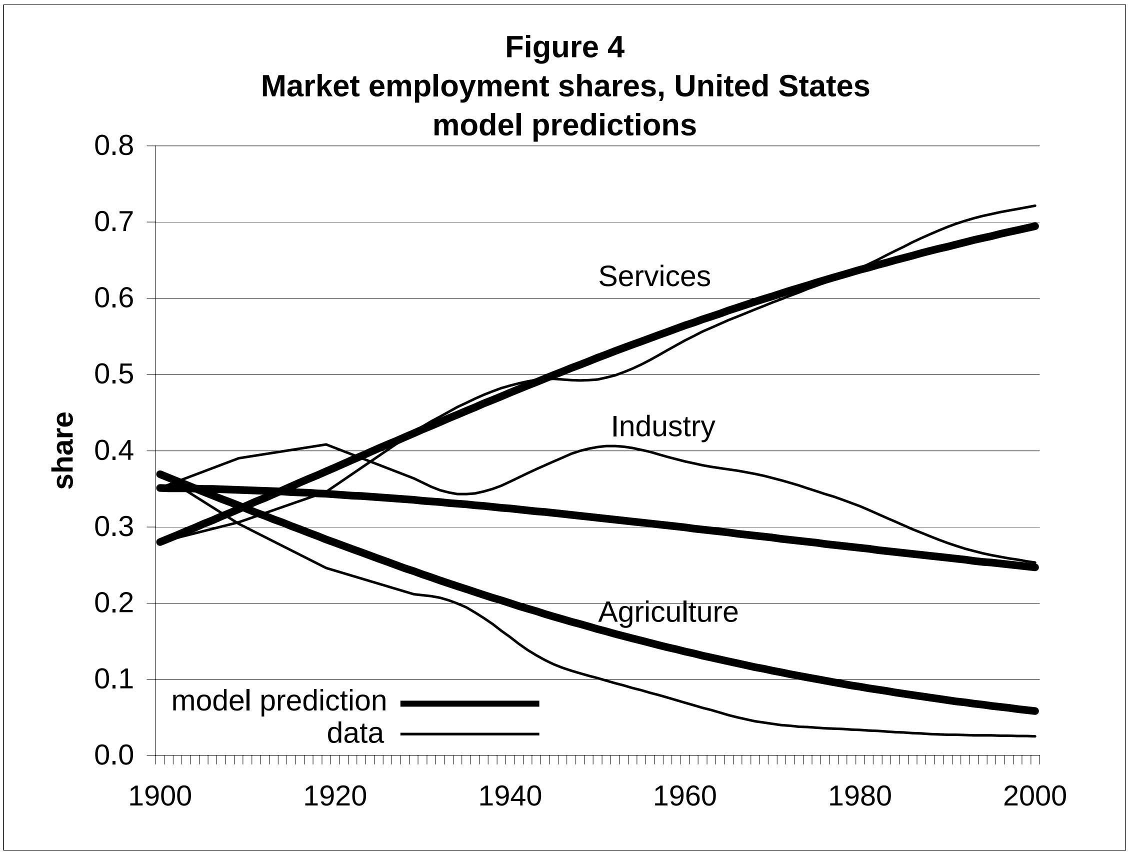
HP filtered data. All filtered data in this and subsequent graphs uses a smoothing parameter of 100.
Sources: Total hours, Groningen Growth and Development Centre
Working age population, OECD



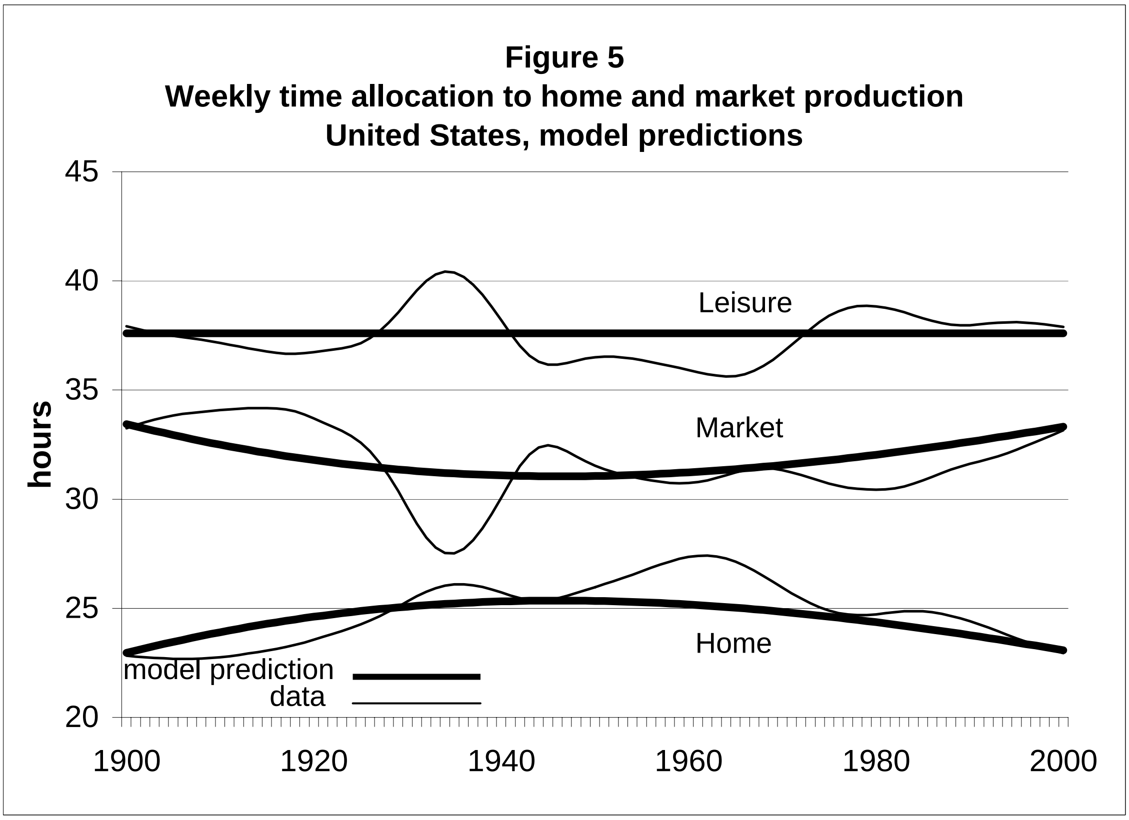
Average weekly hours of market work, home work and leisure for the population aged 10-64 excluding full-time equivalent enrollment in school. Actual data, HP and cubic trend.
Leisure is calculated as total effective weekly hours (94 hours) *minus* market and home hours.
Source: Ramey and Francis (2005).



Source: Ramey and Francis (2005) for market and home hours, HP filtered.
US *Historical Statistics* and BEA for service employment share, HP filtered.



Definitions: Agriculture includes agriculture, forestry and fisheries, industry includes mining, manufacturing, construction, utilities, transportation and communication and services all others
Source: US *Historical Statistics* and BEA, HP filtered.



Model prediction of leisure is set at the mean sample value
Source: See notes to figure 2; data are HP filtered.

Figure 6
Predictions of market employment shares with and without home production, United States

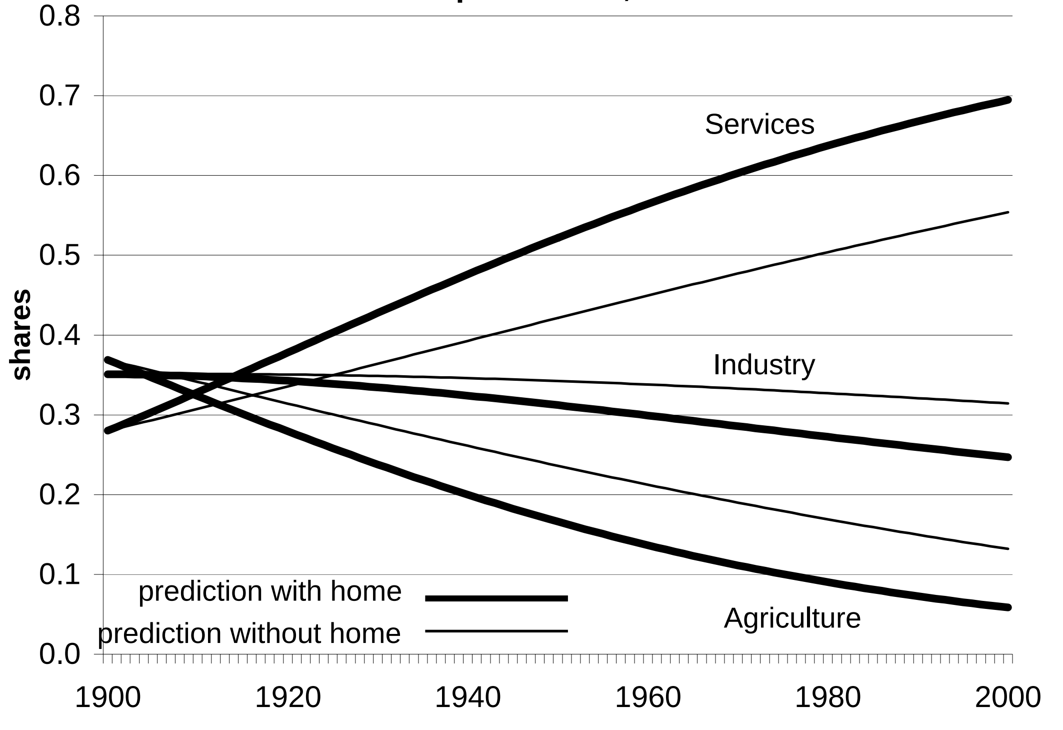


Figure 7
Sensitivity analysis: deviations from the benchmark