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ABSTRACT

Asset Price Dynamics When Traders Care About Reputation*

What are the equilibrium features of a dynamic financial market where traders care about their reputation for ability? We modify a standard sequential trading model to study a financial market with career concerns. We show that this market cannot be informationally efficient: there is no equilibrium in which prices converge to the true value, even after an infinite sequence of trades. This finding, which stands in sharp contrast with the results for standard financial markets, is due to the fact that our traders face an endogenous incentive to behave in a conformist manner. We show that there exist equilibria where career-concerned agents trade in a conformist manner when prices have risen or fallen sharply. We also show that each asset carries an endogenous reputational benefit or cost, which may lead to systematic mispricing if asset supply is not infinitely elastic.

JEL Classification: C7 and G0

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1 Introduction

The substantial increase in the institutional ownership of corporate equity around the world in recent decades has underscored the importance of studying the effects of institutional trade on asset prices.¹ Institutions, and their employees, may be guided by incentives not fully captured by standard models in finance. For example, consider the case of US mutual funds which make up a significant proportion of institutional investors in US equity markets. An important body of empirical work highlights the fact that mutual funds (e.g. Chevalier and Ellison [12]) and their employees (Chevalier and Ellison [13]) both face *career concerns*: they are interested in enhancing their *reputation* with their respective principals and sometimes indulge in perverse actions (e.g. excessive risk taking) in order to achieve this. Given the importance of institutions in equity markets, it is plausible to expect that such behaviour may affect equilibrium quantities in these markets. What are the equilibrium features of a market in which a large proportion of traders care about their reputation?

While a growing body of literature examines the effects of agency conflicts on asset pricing, the explicit modeling of reputation in financial markets is in its infancy.² Dasgupta and Prat [15] present a two-period micro-founded model of career concerns in financial markets to examine the effect of reputation in enhancing trading volume. However, that analysis is done for a static market: each asset is traded only once.

In this paper, in contrast, we study a multi-period sequential trade market in which some traders care about their reputations. We show that the dynamic properties of this market are very different from those of standard markets.

1.1 Summary of Results

We present the most parsimonious model that captures the essence of our arguments. Much of our model is standard. We present a T -period sequential trade market for a single (Arrow) asset where all transactions occur via uninformed *market makers* who are risk neutral and competitive (following Glosten and Milgrom [20] and Kyle [21]) and quote bid and ask prices to reflect the informational content of order flow. In addition there is a large group of liquidity-driven *noise traders* who trade for exogenous reasons that are unrelated to the liquidation

¹On the New York Stock Exchange the percentage of outstanding corporate equity held by institutional investors has increased from 7.2% in 1950 to 49.8% in 2002 (NYSE Factbook 2003). For OECD countries as a whole, institutional ownership now accounts for around 30% of corporate equity (Nielsen [25]). Allen [2] presents persuasive arguments for the importance of financial institutions to asset pricing.

²For example, Allen and Gorton [4] and Dow and Gorton [19] examine the asset pricing implications of non-reputational agency conflicts. Reputational concerns are implicit in the contractual forms assumed in the general equilibrium models of Cuoco and Kaniel [14] and Vayanos [38].

value of the asset.

Our only innovation is that we introduce a large group of reputationally-concerned traders (whom we call *fund managers*), who trade on behalf of other (inactive) investors. These traders receive a payoff that depends both on the direct profits they produce and on the reputation that they earn with their principals.³ These fund managers can be of two types (smart or dumb) and receive informative signals about the asset liquidation value, where the precision depends on their (unknown) type. In each trading round either a randomly selected fund manager or a noise trader interact with the market maker. The asset payoff is realized at time T and all payments are made.

At time T , every fund manager is evaluated on the basis of all available information, with the exception of the agent's private signal. This implies that each investor can observe the liquidation of the asset and the portfolio choice of his own agent.⁴ This assumption is plausible for relatively sophisticated investors, such as corporate pension plans, investment banks, insurance companies, and hedge fund clients. It may instead be an unrealistic requirement for retail mutual fund investors, who typically have limited knowledge of their fund's portfolio composition.⁵

We present the following results.

1. We begin with an impossibility result. We show that, in this market of career-concerned traders, *prices never converge to true liquidation value, even after an infinite sequence of trades*. If fund managers trade according to their private signal, the price evolves to incorporate such private information. Over time, the price should converge to the true liquidation value. However, as the uncertainty over the liquidation value is resolved, two things happen. First, the fund managers have less opportunity to make trading profits because the price is close to the liquidation value. The expected profit for a fund manager who trades according to his signal is always positive, but it tends to zero as the price becomes more precise. Second, taking a “contrarian” position (e.g. selling when the price has been going up) starts to carry a reputational cost: with high probability, the trade will turn out to be incorrect and the fund manager will “look dumb” in the eyes of (rational) principals. Because of the combination of these two effects, when the price becomes sufficiently precise fund managers begin to behave in a conformist way: their trade stops reflecting their private information. From then on,

³The principals may be line managers at mutual fund companies with oversight over the particular fund manager's activities, or, directly, the investors who have placed their funds with the company.

⁴For all our core results, it is irrelevant whether the investor observes the portfolio choice of other fund manager besides his.

⁵See Prat [28] for a discussion of the role of portfolio disclosure in delegated portfolio management with career concerns.

there is no information aggregation whatsoever and the price stays constant.

2. We then report a positive result. We prove that there exist an equilibrium where fund managers trade sincerely in the beginning, but, as soon as the price hits an upper threshold or a lower threshold, they begin pooling. We characterize the conditions that guarantee the existence of such *truncated sincere equilibria*, and construct examples to illustrate their properties. We also show that as career concerns become increasingly important to fund managers, these truncated sincere equilibria are the most informative equilibria that can exist.
3. We show that, at any given price, there may be a difference between the expected total value of the asset for a regular trader and for a trader with career concerns. The difference arises because managers gain in reputation from buying underpriced assets and selling overpriced ones. We refer to this difference as the *reputational benefit or cost* carried by the asset at that price. If the asset supply is not infinitely elastic, the reputational benefit or cost will affect the equilibrium price. The transaction price can differ from this period's public expectation of liquidation value. Thus, the existence of such a reputational concerns can lead to the *systematic mispricing of assets* in financial markets. We characterize the nature of such systematic mispricing and show that current mispricing depends on past trades and prices.
4. Finally, we examine a number of natural extensions. The baseline model was presented with arbitrary preferences over individual reputation and with binary signals and asset liquidation values. In these extensions, we provide micro-foundations for the reputational preferences of fund managers, and generalize the model to richer payoff and signal spaces to show that our main results are robust to these changes. Further, in our baseline model we assumed that fund managers cared about their absolute reputations and were unaware of their type. We extend the model to demonstrate that as long as self-knowledge is not very accurate, our core results remain unchanged. We also allow for rewards based on relative reputation and show that sincere trading cannot be sustained in equilibrium.

Our results generate testable implications. As summarized above, our model suggests that the incentives of career-concerned fund managers can lead to the systematic mispricing of assets in financial markets – when asset supply is not perfectly elastic, persistently bought (sold) assets will develop a reputational premium (discount), and trade above (below) fair prices. This has implications for long-term returns: Assets that have been persistently bought (sold) by fund managers are likely to experience negative (positive) corrections when uncertainty is resolved, thus achieving low (high) long-term returns.

In empirical work motivated by these results, Dasgupta, Prat, and Verardo [16] use data from the SEC 13-F filings of US portfolio managers from 1983 to 2004 to examine net trades between career-concerned institutional traders and regular investors. They find that the persistence of institutional trading has strong power in predicting the cross-section of stock returns at long horizons. Stocks that have been persistently bought (sold) by institutional investors over three or more quarters significantly underperform (overperform) the market. A strategy of buying stocks that institutions sell for five or more consecutive quarters and shorting ones that they buy for identical periods yields significant excess returns. Using returns from such a portfolio, Dasgupta, Prat, and Verardo estimate intercepts from time-series regressions using the CAPM model, the Fama-French three-factor model, and the four-factor model that includes the Carhart momentum factor. Excess returns (alphas) range from around 5% to 11% per year depending on the chosen pricing model and the holding period. Evidence from this paper also indicates that a vast majority of institutional traders exhibit conformist trading patterns.

Further supporting evidence can be found in the work of Dennis and Strickland [18]. These authors have examined the relationship between price reactions and institutional ownership for a cross section of stocks on single trading days when the market return is high (in absolute value). They find that the magnitude of a firm's return on such days is increasing in the extent of institutional ownership. In particular, they find that the percentage of ownership by mutual funds, the class of institutions for which career concerns are best documented, to be specially important in driving their results. They interpret this as evidence of positive feedback herding by mutual funds. Our theoretical results provide a clean parallel to these empirical findings. We show theoretically that fund managers trade in a conformist manner precisely when prices have risen or fallen sharply, creating a reputational value to buying or selling respectively.⁶

1.2 Related Literature

This paper brings together two influential strands of the literature. The first strand concerns the theory of dynamic financial markets with asymmetrically informed traders (Glosten and Milgrom [20] and Kyle [21]). The second strand focusses on the analysis of career concerns in sequential investment decision-making (Scharfstein and Stein [30]). Models in the first strand consider a full-fledged financial market with endogenously determined prices but do not allow traders to have career concerns. Models in the second strand do the exact opposite: they analyze the role of reputational concerns in a partial equilibrium setting, where prices

⁶See Sias [34] for a recent survey and reconciliation of the growing literature on momentum trading and herding by institutions.

are exogenously fixed.

In the first strand, Glosten and Milgrom [20] have shown that in dynamic financial markets the price must tend to the true liquidation value in the long term. More recently, Avery and Zemsky [8] have shown that statistical information cascades à la Banerjee [9] and Bikhchandani, Hirshleifer, and Welch [10] are impossible in such a market.⁷ After every investment decision, the price adjusts to reflect the expected value of the asset based on information revealed by past trades. Thus, traders with private information stand to make a profit by trading according to their signals. But by doing so, they release additional private information into the public domain. In the long run, the market achieves informational efficiency. Our work shows that the addition of career concerns changes things dramatically. The presence of a reputational motive can make the market informationally inefficient and can generate mispricing.⁸

Other authors (Lee [22] and Chari and Kehoe [11]) have argued that information cascades can occur when prices are endogenous.⁹ However, their arguments hinge on a market breakdown: trade stops altogether. Instead, in our model cascades occur in a financial market with trade.

In the second strand, Scharfstein and Stein [30] have shown that managers who care about their reputation for ability may choose to ignore relevant private information and instead mimic past investment decisions of other managers.¹⁰ This is because a manager who possesses “contrarian” information (for instance he observes a negative signal for an asset that has experienced price growth) jeopardizes his reputation if he decides to trade according

⁷An information cascade is an equilibrium event in which information gets trapped, and agents’ actions no longer reveal *any* of their valuable private information.

⁸Avery and Zemsky [8] also show that if uncertainty is multidimensional (when “event uncertainty” compounds the usual value uncertainty) a *weaker* form of conformism, which they term herd behavior, may occur in financial markets. However, in their model *cascades are absent* and prices always converge to the true liquidation value (see Avery and Zemsky Proposition 2). In contrast, we show that cascades occur even in the absence of event uncertainty, due to the presence of career concerns.

There is almost universal agreement in the literature on the meaning of a *cascade*, which is the definition we have used (see footnote 7). However, there is little agreement on the definition of the term *herds* (for example, substantively different definitions are used by Avery and Zemsky [8], Smith and Sorensen [35], and Chari and Kehoe [11]). In the interest of clarity, throughout this paper we shall restrict attention to cascades *only*.

Recently, Park and Sabourian [27] have explored generalizations of the necessary conditions for herds in Avery and Zemsky’s model. As in Avery and Zemsky, however, cascades cannot arise in their model.

⁹In Lee’s [22] model the existence of a transaction cost to trading can prevent traders with relatively inaccurate signals from trading, thus trapping private information in an illiquid market. In Chari and Kehoe [11], traders have the option of exiting the market (by making an outside investment) and may in equilibrium find it optimal to exit before further information arrives, thus, again, trapping private information.

¹⁰Other more recent papers in this strand include, for example, Avery and Chevalier [7], Prendergast and Stole [29], and Trueman [37].

to his signal.¹¹ Scharfstein and Stein's analysis is carried out in partial equilibrium: Prices play no informational role in such an analysis.

We show that conformist behavior arises in market equilibrium due to the reputational concerns of financial traders. Such conformism endogenously prevents prices from playing their full role in informational revelation. By studying career concerns in financial equilibrium, we are able to study the effects of micro-founded reputation-driven conformism on financial market quantities (prices, informational efficiency, trades, and profits), which leads to relevant predictions on observable market variables, as discussed above.

The rest of the paper is organized as follows. In the next section we present the model. Section 3 discusses the impossibility of full information aggregation. Section 4 characterizes some of the equilibria of this game, and proves that information cascades can arise in equilibrium. Section 5 studies the dynamics of the reputational benefit or cost of the asset and shows that, if the market making sector has market power, the price can incorporate a reputational premium and systematic mispricing can arise. Extensions are examined in section 6. Section 7 concludes.

2 The Model

The economy lasts T discrete periods: $1, 2, \dots, T$. Trade can occur in periods $1, 2, \dots, T - 1$. The market trades an Arrow security, which has equiprobable liquidation value $v = 0$ or 1 , which is revealed at time T .

In practice, the asset could be a bond with maturity date T with a serious possibility of default. It could also be the common stock of a company which is expected to make an announcement of great importance (earnings, merger, etc.) at time T : all traders know that the announcement will occur but they may have different information on the content of the announcement.

There are a large number of fund managers and noise traders. At each period $t \in \{1, 2, \dots, T - 1\}$ either a fund manager or a noise trader enters the market with probabilities $1 - \delta$ and $\delta \in (0, 1)$ respectively. The traders interact with a risk-neutral competitive market maker, and can issue market orders (a_t) to buy ($a_t = 1$) one unit or sell ($a_t = 0$) one unit of the asset. The market maker sets ask (p_t^a) and bid (p_t^b) prices equal to expected value of v conditional on order history. Denote the history of observed orders at the beginning of period t (not including the order at time t) by h_t . Let $p_t = E(v|h_t)$, $p_t^a = E(v|h_t, \text{buy})$, $p_t^b = E(v|h_t, \text{sell})$. A mixed strategy for the manager at time t is denoted by $\alpha_t(h_t, s_t)$, the

¹¹The partial equilibrium reputational model that is closest to ours is Ottaviani and Sorensen [26]. The connection will be discussed in detail in Section 3.

probability that he chooses $a_t = 1$ given history h_t and signal s_t .

The fund manager can be of two types: $\theta \in \{b, g\}$ with $\Pr(\theta = g) = \gamma$. The type is independent of v . If at time t a fund manager appears, he receives a signal $s_t \in \{0, 1\}$ with distribution

$$\Pr(s_t = v|v, \theta) = \sigma_\theta,$$

where

$$\frac{1}{2} \leq \sigma_b < \sigma_g \leq 1.$$

Fund managers do not know their type. Noise traders buy or sell a unit with equal probability independent of v .

The returns obtained by the trader at time t is denoted χ_t and is defined by:

$$\chi_t(a_t, p_t^a, p_t^b, v) = \begin{cases} v - p_t^a & \text{if } a_t = 1 \\ p_t^b - v & \text{if } a_t = 0 \end{cases}$$

If a fund manager traded at time t , his actions are observed at time T . Principals (e.g., line managers in the fund management firm) form a posterior belief about the manager's type based upon the managers actions, which we define to be

$$\hat{\gamma}_t(a_t, h_t, v) = \Pr(\theta_t = g|a_t, h_t, v)$$

Suppose the fund manager at time t receives utility

$$u(a_t, p_t^a, p_t^b, v, h_t) = \beta\pi(\chi_t) + (1 - \beta)r(\hat{\gamma}_t),$$

where $\beta \in (0, 1)$ is a parameter, and the functions π and r measure the direct payoff and reputational payoff and are increasing and piecewise continuous in the relevant arguments.¹²

We implicitly assume that the fund manager in t observes the whole history of trades and prices up to his time, h_t , (along with the current price and his private signal) and that principals observe h_T (and the liquidation value v). The assumption that fund managers and principals are able to observe the history of trades is not indispensable. Our main results would hold even if the fund manager in t could only observe the price p_t in t and his private signal, and the principals could only observe the trade of their fund manager and the liquidation value (indeed, in the equilibria we characterize, strategies only depend on these variables).

We can show that in this setting, in contrast to well-known prior results, that the market cannot be fully informationally efficient *even after an infinite sequence of trades*.

¹²In section 6, we provide microfoundations for such a utility function.

3 The Impossibility of Full Revelation

We first present an example and then discuss the general result.

3.1 An Example

Let π and r be linear. The manager maximizes $\beta\chi_t + (1 - \beta)\hat{\gamma}_t$.

A *sincere equilibrium* is one in which fund managers play according to their signals: $a_t = s_t$ for all t at which a fund manager is active.

If $\beta = 1$, there is a sincere equilibrium (Glosten and Milgrom [20], Avery and Zemsky [8]). Suppose instead that $\beta < 1$.

Proposition 1 *There is no sincere equilibrium.*

The argument is intuitive and we present it in detail here. Suppose there is a sincere equilibrium. Suppose that at t the price is p_t and the manager plays $a_t = s_t$. Let

$$\begin{aligned}\hat{v}_1^t &= \Pr(v = 1 | s_t = 1, h_t) = \frac{\Pr(s_t = 1 | v = 1)}{\Pr(s_t = 1 | h_t)} p_t = \frac{\sigma}{p_t \sigma + (1 - p_t)(1 - \sigma)} p_t \\ \hat{v}_0^t &= \Pr(v = 1 | s_t = 0, h_t) = \frac{\Pr(s_t = 0 | v = 1)}{\Pr(s_t = 0 | h_t)} p_t = \frac{1 - \sigma}{(1 - p_t)\sigma + p_t(1 - \sigma)} p_t\end{aligned}$$

where $\sigma = \gamma\sigma_g + (1 - \gamma)\sigma_b$. The bid-ask prices are

$$\begin{aligned}p_t^a &= \delta_t^b p_t + (1 - \delta_t^b) \hat{v}_1^t \\ p_t^b &= \delta_t^s p_t + (1 - \delta_t^s) \hat{v}_0^t\end{aligned}$$

where $\delta_t^b = \Pr(\text{noise trader} | \text{buy order}, h_t)$, is the probability assigned by the market maker that he faces a noise trader upon receiving a buy order and observing the history of trades. Similarly, $\delta_t^s = \Pr(\text{noise trader} | \text{sell order}, h_t)$. Straightforward calculations show that

$$\begin{aligned}\delta_t^b &= \frac{\frac{1}{2}\delta}{\frac{1}{2}\delta + (1 - \delta)[p_t\sigma + (1 - p_t)(1 - \sigma)]} \\ \delta_t^s &= \frac{\frac{1}{2}\delta}{\frac{1}{2}\delta + (1 - \delta)[p_t(1 - \sigma) + (1 - p_t)\sigma]}\end{aligned}$$

Suppose the current price is p_t and the manager in t observes $s_t = 0$. If he buys, his expected trading profit is $\hat{v}_0^t - p_t^a$, while if he sells it is $p_t^b - \hat{v}_0^t$. Thus the difference between the expected profit of buying and selling is

$$\Delta\pi = 2\hat{v}_0^t - p_t^a - p_t^b.$$

The reputational payoffs in a sincere equilibrium are:

$$\begin{aligned}\hat{\gamma}(a_t = 1, v = 1) &= \frac{\sigma_g}{\sigma} \gamma & \hat{\gamma}(a_t = 0, v = 1) &= \frac{1 - \sigma_g}{1 - \sigma} \gamma \\ \hat{\gamma}(a_t = 1, v = 0) &= \frac{1 - \sigma_g}{1 - \sigma} \gamma & \hat{\gamma}(a_t = 0, v = 0) &= \frac{\sigma_g}{\sigma} \gamma\end{aligned}$$

Define

$$\Delta \hat{\gamma} = \left(\frac{\sigma_g}{\sigma} - \frac{1 - \sigma_g}{1 - \sigma} \right) \gamma.$$

The expected reputational benefit of choosing $a_t = 1$ instead of $a_t = 0$ when $s_t = 0$ is

$$\begin{aligned}\Delta r &= \Pr(v = 1 | s_t = 0, h_t) (\hat{\gamma}(a_t = 1, v = 1) - \hat{\gamma}(a_t = 0, v = 1)) \\ &\quad + \Pr(v = 0 | s_t = 0, h_t) (\hat{\gamma}(a_t = 1, v = 0) - \hat{\gamma}(a_t = 0, v = 0)) \\ &= \hat{v}_0^t \left(\frac{\sigma_g}{\sigma} - \frac{1 - \sigma_g}{1 - \sigma} \right) \gamma + (1 - \hat{v}_0^t) \left(\frac{1 - \sigma_g}{1 - \sigma} - \frac{\sigma_g}{\sigma} \right) \gamma \\ &= (2\hat{v}_0^t - 1) \Delta \hat{\gamma}\end{aligned}$$

It is a best response to play $a_t = 0$ instead of $a_t = 1$ when $s_t = 0$ when

$$\beta \Delta \pi + (1 - \beta) \Delta r < 0.$$

Let the price rise to one. Notice that as $p_t \rightarrow 1$, $\hat{v}_0^t \rightarrow 1$ and $\hat{v}_1^t \rightarrow 1$. That is, regardless of whether the manager has received the high or the low signal, the accumulated information in prices convinces him that the expected liquidation value is 1. Also notice that δ_t^b and δ_t^s are bounded, and thus as $p_t \rightarrow 1$, it must also be true that $p_t^a \rightarrow 1$ and $p_t^b \rightarrow 1$. Now, taking limits we have

$$\begin{aligned}\lim_{p_t \rightarrow 1} \Delta \pi &= \lim_{p_t \rightarrow 1} 2\hat{v}_0^t - p_t^a - p_t^b = 2 - 1 - 1 = 0 \\ \lim_{p_t \rightarrow 1} \Delta r &= \lim_{p_t \rightarrow 1} (2\hat{v}_0^t - 1) \Delta \hat{\gamma} = (2 - 1) \Delta \hat{\gamma} > 0\end{aligned}$$

Hence, for p_t high enough it is a best response for a fund manager with $s_t = 0$ to play $a_t = 1$. Thus, there can be no sincere equilibrium.

This example sheds light on why this market may fail to aggregate information efficiently. But proving that there is no sincere equilibrium is not sufficient. There may be informative equilibria that are not fully sincere. To show the inevitability of information cascades, we need to argue that there exists no equilibrium that leads to fully revealing prices even after an infinite sequence of trades by informed fund managers. This is done in the following section.

3.2 The General Result

Let us revert to the general problem. The expected utility of the t -period fund manager is

$$E(u(a_t, p_t^a, p_t^b, v, h_t) | s_t) = \sum_{v=0,1} \Pr(v | h_t, s_t) \left(\beta \pi \left(\chi \left(a_t, p_t^a, p_t^b, v \right) \right) + (1 - \beta) r \left(\hat{\gamma} \left(a_t, v, h_t \right) \right) \right)$$

Define

$$\begin{aligned} E(\Delta u(p_t^a, p_t^b, v, h_t) | s_t) &= E(u(1, p_t^a, p_t^b, v, h_t) | s_t) - E(u(0, p_t^a, p_t^b, v, h_t) | s_t) \\ &= \sum_{v=0,1} \Pr(v | h_t, s_t) \left(\beta \Delta \pi \left(\chi \left(p_t^a, p_t^b, v \right) \right) + (1 - \beta) \Delta r \left(\hat{\gamma} \left(v, h_t \right) \right) \right) \end{aligned}$$

where

$$\Delta \pi \left(\chi \left(p_t^a, p_t^b, v \right) \right) = \pi(v - p_t^a) - \pi(p_t^b - v)$$

and

$$\Delta r \left(\hat{\gamma} \left(v, h_t \right) \right) = r \left(\hat{\gamma} \left(1, v, h_t \right) \right) - r \left(\hat{\gamma} \left(0, v, h_t \right) \right)$$

We restrict attention to *non-perverse equilibria*. We say that an equilibrium is *perverse* if for some period t and some history h_t which occurs with positive probability on the equilibrium path, the fund manager at t is more likely to buy if he has a negative signal than if he has a positive signal ($\alpha_t(h_t, s_t = 0) > \alpha_t(h_t, s_t = 1)$). Thus, we are not excluding perverse behavior off the equilibrium path. Note that perverse equilibria are extremely implausible in a financial context because perverse behavior after a history h_t implies that the bid-ask spread is strictly negative conditional on h_t .

We are now ready to state our main result. There exists no non-perverse equilibrium in which the price p_t converges to the true liquidation value v as $t \rightarrow \infty$. In particular, there exists an upper and a lower threshold, such that if the price crosses one of these bounds, trade is entirely devoid of information for the rest of the game:

Proposition 2 *In any non-perverse equilibrium there exists $(\underline{p}, \bar{p}) \in (0, 1)^2$ such that if $p_t > \bar{p}$ or $p_t < \underline{p}$ then the actions of fund managers from period t onwards cannot provide information about their private signals.*

The proposition is proved in several steps. In any putative informative non-perverse equilibrium, we first show that as the price approaches either 0 or 1 the expected trading profit goes to zero. Second, we analyze the reputational incentives in such a putative equilibrium. As the price goes to one, the fund manager faces a positive and non-infinitesimal expected reputational benefit if he chooses to buy rather than to sell. Conversely, when the price goes to zero, he faces a positive and non-infinitesimal benefit if he sells rather than buying. Putting

together the profit motive and the reputational motive, we conclude that if the price is high enough or low enough the fund manager will ignore his private information. From then on, the market is stuck in an information cascade. No additional private information is revealed. But this means that we could not have been in an informative non-perverse equilibrium.

The proof is involved and is presented in detail in the appendix. Our result bears a connection to Ottaviani and Sorensen [26], who provide a general analysis of reputational cheap talk and show that full information transmission is generically impossible. Of particular interest to the present paper is their Proposition 9, where they consider a sequence of experts providing reports on a common state of the world and they show that informational herding must occur. Our result is different because: (a) we show the necessity of informational cascades (while Ottaviani and Sorensen prove that herding must occur but they cannot exclude that the true value is revealed in the limit); (b) our experts have a profit motive as well as a reputational motive; and (c) most importantly, our model is embedded in a financial market.

4 Price Dynamics: Information Cascades in Equilibrium

We have just demonstrated a negative result: non-perverse equilibria never result in the long-run convergence of prices to true value. A natural question remains: What equilibria can arise in this sequential trade market with reputationally sensitive traders? We deal with this question here.

To simplify our construction of equilibria, we assume that the functions π and r are linear, though the spirit of our arguments extend to more general functions. In all other respects we use the general specification introduced for the main result.

We define a *truncated sincere equilibrium* as follows. There exists $\bar{p} \in (0, \frac{1}{2})$ such that:

1. A manager with $s_t = 1$ buys if $p_t \geq \bar{p}$ and sells if $p_t < \bar{p}$;
2. A manager with $s_t = 0$ buys if $p_t > 1 - \bar{p}$ and sells if $p_t \leq 1 - \bar{p}$.

Notice that a crucial property of the truncated sincere equilibrium is that at high and low prices fund manager trade without regard to their private information. Having thus earlier ruled out equilibria where prices converge to true value after even an infinite sequence of trades, we shall now explicitly demonstrate that conformist behaviour can arise in equilibrium, giving rise to an information cascade. We can prove:

Proposition 3 *Given $(\gamma, \sigma_b, \sigma_g, \delta)$, there exists a $\bar{\beta} > 0$ such that for all $\beta < \bar{\beta}$ a truncated sincere equilibrium exists where \bar{p} is the unique $p_t \in (0, 1)$ that solves*

$$\beta \Delta \pi_1(p_t) + (1 - \beta) \Delta r_1(p_t) = 0,$$

where $\Delta\pi_1(p_t)$ and $\Delta r_1(p_t)$ are defined under sincere strategies. In addition, for β small enough, there exists no non-perverse equilibrium in which trading reveals information when $p_t \notin [\bar{p}, 1 - \bar{p}]$.

The formal proof is involved and is presented in the appendix. We note here only that the proof is constructed under the following natural off-equilibrium beliefs: if a fund manager trades in a contrarian way in a region of p_t where equilibrium strategies dictate conformism, he is assumed to have traded sincerely. Here, we construct an example based on proposition 3 which provides some intuition behind the proof of the result.¹³

Suppose that $\delta = 0.25$, $\gamma = 0.5$, $\beta = 0.4$, $\sigma_g = 1$ and $\sigma_b = 0.5$. The expected total payoff difference between buying and selling (Δu_1) depends on p_t and on the equilibrium strategies followed, α_1^t and α_0^t . Using the formulas presented above, it is not hard to see that the expected payoff difference between buying and selling in the sincere part of the equilibrium, $\Delta u_1(p_t, 1, 0)$, crosses 0 exactly once in $p_t \in [0, 1]$ at $\bar{p} \simeq 0.144$. In order to compute a truncated sincere equilibrium, we must also compute $\Delta u_1(p_t, 0, 0)$ and $\Delta u_1(p_t, 1, 1)$, that is the payoff differences in the lower and upper conformism regions.

Figure 1 shows, from the point of view of a manager who observes $s_t = 1$, the expected payoff differences for the relevant regions, that is $\Delta u_1(p_t, 0, 0)$ in the conformist-sell region $p_t \in [0, 0.144)$, $\Delta u_1(p_t, 1, 0)$ in the sincere region (*thick line*) $p_t \in [0.144, 0.856]$, and $\Delta u_1(p_t, 1, 1)$ in the conformist-buy region $p_t \in (0.856, 1]$.

The key point is that the total benefit is positive if and only if $p \geq \bar{p}$, indicating that the best response of a manager with $s_t = 1$ is indeed to buy if and only if $p \geq \bar{p}$. A few remarks are in order.

1. The graph displays a small positive jump at $p \simeq 0.144$. This represents the “switch” in payoff differences between the conformist part of the equilibrium and the sincere part. The jump derives from the combination of changes both in $\Delta\pi$ and in Δr .

In general, the excess profits from buying vs selling will be *higher* in the conformist section of the equilibrium than in the sincere part. This is because separation worsens the bid-ask spread (which is zero in the conformist part). Thus, in making the transition from conformist play at $p < 0.144$ to sincere play at $p \geq 0.144$ the fund manager faces a *negative* impact on profits.

On the other hand, the reputational difference between buying and selling will be *lower* under conformist strategies at $p \simeq 0.144$ than under sincere strategies. This is because

¹³If β is high, the most informative equilibrium has a more complex structure. There exist regions in which a “contrarian” fund manager plays a mixed strategy. However, there are still two price thresholds such that no information is revealed outside of them.

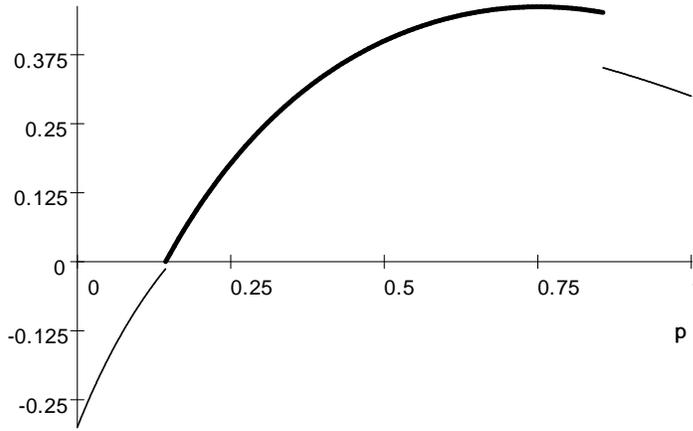


Figure 1: Excess payoff to buying in a truncated sincere equilibrium ($s_t = 1$)

separation enhances the reputational benefit of ex post correct trades. Thus, in making the transition from conformist play at $p < 0.144$ to sincere play at $p \geq 0.144$ the fund manager faces a *positive* impact on reputational rewards.

The “small β ” property highlighted in Proposition 3 guarantees that the reputational gains will overwhelm the loss in profits, thus guaranteeing an equilibrium switch from conformist to sincere play at $p \simeq 0.144$ when career concerns are sufficiently important.

2. Note that this example illustrates that in practice, β does not need to be particularly small for a truncated sincere equilibrium to exist and for conformism to arise in equilibrium Proposition 3 identified sufficient conditions only.
3. When $p = 1 - \bar{p}$ the total benefit line displays a negative jump due to the combination of negative effects on profit and reputation. However, it is easy to check that the both functions must still be positive. If p is high, a manager with $s_t = 1$ has a two-fold reason to buy rather than sell.

5 Reputational Premium and Systematic Mispricing

From the viewpoint of fund managers the expected total value of the asset can differ from expected liquidation value. The difference captures the expected *reputational benefit or cost*

that the fund manager incurs if he buys or sells the asset. Intuitively, an asset that has seen recent price rises (falls) as a result of persistent buying (selling) is likely to be underpriced (overpriced): buying (selling) such an asset may enhance the manager's reputation.

Consider a fund manager at time t who observes signal s_t and forms expectation $\hat{v}_{s_t}^t$. Let $w_{s_t}^t$ be the price at which the manager is indifferent between buying and selling. We refer to $w_{s_t}^t$ as the fund manager's *expected total value of the asset*. It is the solution of

$$\begin{aligned} & \beta (\hat{v}_{s_t}^t - w_{s_t}^t) + (1 - \beta) (\hat{v}_{s_t}^t \hat{\gamma}^t(a_t = 1, v = 1) + (1 - \hat{v}_{s_t}^t) \hat{\gamma}^t(a_t = 1, v = 0)) \\ = & \beta (w_{s_t}^t - \hat{v}_{s_t}^t) + (1 - \beta) (\hat{v}_{s_t}^t \hat{\gamma}^t(a_t = 0, v = 1) + (1 - \hat{v}_{s_t}^t) \hat{\gamma}^t(a_t = 0, v = 0)) \end{aligned}$$

If the fund manager has no career concerns ($\beta = 1$), we simply have that $w_{s_t}^t = \hat{v}_{s_t}^t$: the expected total value is just the expected liquidation value. However, if $\beta < 1$ there may be a wedge between the two values, which we indicate with

$$\rho_{s_t}^t = w_{s_t}^t - \hat{v}_{s_t}^t = \frac{1 - \beta}{2\beta} \left(\begin{array}{l} \hat{v}_{s_t}^t (\hat{\gamma}^t(a_t = 1, v = 1) - \hat{\gamma}^t(a_t = 0, v = 1)) \\ + (1 - \hat{v}_{s_t}^t) (\hat{\gamma}^t(a_t = 1, v = 0) - \hat{\gamma}^t(a_t = 0, v = 0)) \end{array} \right).$$

The value $\rho_{s_t}^t$ depends on the manager's private signal. It is interesting to look at the average value of $\rho_{s_t}^t$ for all managers. Recall that the ex ante probability that the manager receives signal $s_t = 1$ is $\sigma p_t + (1 - \sigma)(1 - p_t)$. Then, let

$$\begin{aligned} \rho^t &= (\sigma p_t + (1 - \sigma)(1 - p_t)) \rho_1^t + (1 - \sigma p_t - (1 - \sigma)(1 - p_t)) \rho_0^t \\ &= \frac{1 - \beta}{2\beta} \left(\begin{array}{l} p_t (\hat{\gamma}^t(a_t = 1, v = 1) - \hat{\gamma}^t(a_t = 0, v = 1)) + \\ (1 - p_t) (\hat{\gamma}^t(a_t = 1, v = 0) - \hat{\gamma}^t(a_t = 0, v = 0)) \end{array} \right) \end{aligned}$$

We refer to ρ^t as the *reputational benefit or cost* of the asset for the manager in t . We can then see that in a truncated sincere equilibrium the reputational payoff is

$$\rho^t = \begin{cases} \frac{1 - \beta}{2\beta} \left(p_t \left(\frac{\sigma_g}{\sigma} - 1 \right) + (1 - p_t) \left(\frac{1 - \sigma_g}{1 - \sigma} - 1 \right) \right) \gamma & \text{if } p \in [0, \bar{p}] \\ \frac{1 - \beta}{2\beta} (2p_t - 1) \left(\frac{\sigma_g}{\sigma} - \frac{1 - \sigma_g}{1 - \sigma} \right) \gamma & \text{if } p \in [\bar{p}, 1 - \bar{p}] \\ \frac{1 - \beta}{2\beta} \left(p_t \left(1 - \frac{1 - \sigma_g}{1 - \sigma} \right) + (1 - p_t) \left(1 - \frac{\sigma_g}{\sigma} \right) \right) \gamma & \text{if } p \in [1 - \bar{p}, 1] \end{cases}$$

It is then easy to show that:

Proposition 4 *In a truncated sincere equilibrium, the reputational benefit is positive if and only if $p_t \geq \frac{1}{2}$. It is strictly increasing in p_t except possibly at the truncation points.*

There is a systematic difference between the valuation of the asset for traders with career concerns and traders without. The difference is increasing in the price, except possibly at a

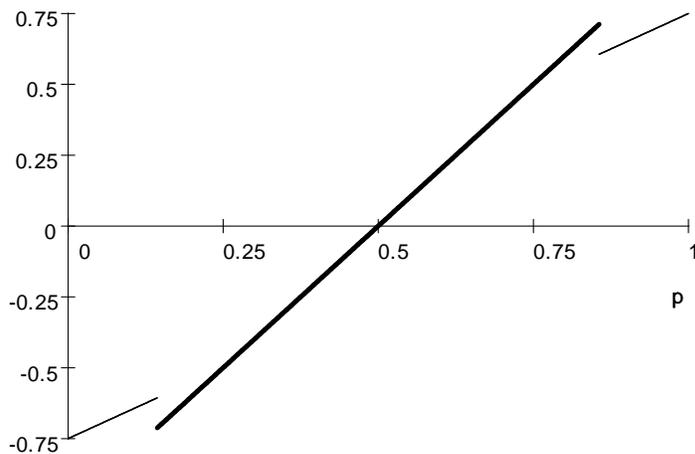


Figure 2: Reputational benefit in a truncated sincere equilibrium

countable number of points. This point is well illustrated by plotting the above function for the example computed in the previous section. This is shown in Figure 2.

So far, this difference in valuation has not affected the equilibrium price. This is because, by utilizing the Glosten-Milgrom set-up, we have implicitly assumed that the price is always equal to the valuation of the side of the market that has no career concerns. Is this assumption reasonable?

In practice, we should expect a good proportion of traders (if not the majority) to have career concerns. If those traders's valuations of the asset are systematically different from the objective expected value of the asset, we should observe systematic mispricing.

Ideally, we should use a formal model of the non-career concerned side of the market that takes into account credit constraints and generates an imperfectly elastic demand function. However, this difficult task is clearly outside the scope of this paper. Instead, we limit ourselves to studying what is perhaps the polar opposite of the Glosten-Milgrom case: we assume that in every period there is only one trader without career concerns who can sell the asset. While this is a stark and very unrealistic way of capturing the idea that the reputational benefit translates into a price premium, we believe that the underlying intuition is valid for any microstructure model where the side of the market without career concerns does not have a demand that is infinitely more elastic than the side with career concerns.

Specifically, we assume that in every period t the fund manager faces one short-lived

monopolistic trader who has an asset to sell and a large number of buyers. The opposite case (many sellers, one buyer) would be similar.

The monopolistic seller operates only in period t . If he does not sell the asset at t , he will keep it until T when he will receive the liquidation value v . The monopolistic seller does not know the liquidation value, and infers it from the market by observing past order flow and also from the current period order flow.

We face a further modeling choice. If there are noise traders who submit market orders, a monopolistic seller can set an infinitely high price and make infinite profits. We could assume that noise traders submit limit orders (with stochastic limits), but we choose to simplify the analysis by assuming that there are no noise traders. The results can be taken as the limit of a model in which there is a vanishing probability of a noise trader who submits a limit order.

We assume that the price charged by the seller cannot go above 1. If the price is greater than 1, new assets will be issued.

A characterization of the equilibrium set is difficult. The monopolistic seller faces an informed buyer (the type of the buyer is s_t) and must choose whether to exclude the low type ($s_t = 0$). This, in turn, reflects on investors' beliefs and makes the analysis extremely complex.

We offer a partial characterization of equilibrium, restricted to the set of prices for which play is sincere. To do this, we define the average price \bar{P}_t as the expected price at which a transaction occurs at time t , that is

$$\bar{P}_t = \Pr(\text{buy}|p_t) p_t^a + \Pr(\text{sell}|p_t) p_t^b.$$

Reinterpreting p_t as the expectation of the liquidation value v of the asset conditional on publicly available information¹⁴, we can prove the following:

Proposition 5 *Suppose that in equilibrium, at a certain p_t , the fund manager trades sincerely. Then, if $p_t > \frac{1}{2}$ there is overpricing: $\bar{P}_t > p_t$.*

Proof. If the equilibrium is sincere, the expected benefit of a manager with $s_t = 1$ of buying rather than selling is

$$\beta \Delta \pi_1(p) + (1 - \beta) \Delta r_1(p)$$

where

$$\begin{aligned} \Delta \pi_1(p) &= 2\hat{v}_1(p) - p^a(p) - p^b(p); \\ \Delta r_1(p) &= (2\hat{v}_1(p) - 1) \left(\frac{\sigma_g}{\sigma} - \frac{1 - \sigma_g}{1 - \sigma} \right) \gamma; \end{aligned}$$

¹⁴The last period transaction price will no longer be identical to such expectations with a monopolistic market maker.

The bid price is set competitively at $p^b(p) = \hat{v}_0(p)$. The monopolistic seller sets $p^a(p)$ so that the manager's expected benefit is zero:

$$\beta (2\hat{v}_1(p) - p^a(p) - \hat{v}_0(p)) + (1 - \beta) (2\hat{v}_1(p) - 1) \left(\frac{\sigma_g}{\sigma} - \frac{1 - \sigma_g}{1 - \sigma} \right) \gamma = 0.$$

If $p_t > \frac{1}{2}$, the second addend is strictly positive. Hence, we must have

$$p^a(p) + \hat{v}_0(p) > 2\hat{v}_1(p).$$

But, because $\hat{v}_1(p) > \hat{v}_0(p)$, we have $p^a(p) > \hat{v}_1(p)$. Hence,

$$\bar{P}_t = \Pr(s_t = 1) p_t^a + \Pr(s_t = 0) \hat{v}_0(p) > \Pr(s_t = 1) \hat{v}_1(p) + \Pr(s_t = 0) \hat{v}_0(p) = p$$

■

In the appendix, we provide an example to illustrate overpricing. In what follows, we consider variations on the baseline model and the robustness of our main result.

6 Extensions

6.1 A More General Set-Up

The baseline model was presented with binary states and signals. Here, we generalize our analysis to include many states and signals.

Let $V = \{v_1, v_2, \dots, v_N\}$ be a discrete space of states and $[s_{\min}, s_{\max}]$ be a continuous space of signals. Denote v_1 by v_{\min} and v_N by v_{\max} . Let $k(v)$ be the prior probability mass function of v (which is independent of θ) and assume that $k(v) > 0$ for all v .

A fund manager of type θ receives a signal distributed according to $f_\theta(s|v)$, with the following properties:

A1 Full support: $f_g(s|v) > 0$ for all s and v .

A2 Monotone Likelihood Ratio Property (MLRP): For every pair $\{s'', s'\}$ and $\{v'', v'\}$, such that $s'' > s'$ and $v'' > v'$,

$$\frac{f_g(s''|v'')}{f_g(s'|v'')} > \frac{f_g(s''|v')}{f_g(s'|v')}.$$

A3 Garbling: Let $\bar{f}(s) \equiv \sum_v f_g(s|v) k(v)$ for every s . Define

$$f_b(s|v) = \tau f_g(s|v) + (1 - \tau) \bar{f}(s),$$

where $\tau \in (0, 1)$ is a parameter that captures the informativeness of a bad manager's signal compared to a good manager's.

The first assumption (A1) is crucial. It implies that the signal is never fully informative: for all s and v : $\Pr(v|s) < 1$. If a manager knows he has the truth, he would follow his signal even if all his predecessors had traded in the opposite direction.

Since the signal space is no longer binary, it is now important to allow for the possibility that the manager does not trade. Let $a_t = \frac{1}{2}$ denote the decision not to trade.

First, note that if there are no career concerns ($\beta = 1$) there exists a fully informative equilibrium (see Avery and Zemsky [8]). When instead career concerns are present, we shall see that full information revelation is impossible.

As before, we focus on non-perverse equilibria. Let $\alpha(a_t|h_t, s_t)$ denote the probability that a manager who observes s_t after history h_t selects action a_t . We say that an equilibrium is non-perverse if $\alpha(1|h_t, s_t)$ and $\alpha(0|h_t, s_t)$ are respectively non-decreasing and non-increasing in s_t for all h_t .

Proposition 6 *There exists no non-perverse equilibrium in which $\lim_{t \rightarrow \infty} p_t = v$ for all v .*

The proof is in the appendix. Here we provide some intuition for the result. Given the MLRP, it is easy to see that the manager would be willing to randomize between actions at a maximum of two signals. Call these s_H and s_L , so that any non-perverse equilibrium must be characterized the the following strategies: buy if $s > s_H$, sell if $s < s_L$, and do not trade if $s \in (s_L, s_H)$.

If the equilibrium is to reveal information, both s_H and s_L cannot be at the boundary. Suppose $s_L > s_{\min}$. In what follows, if $s_H = s_{\max}$, simply replace s_H by s_L , and substitute “buying” with “not trading.” Then, a manager who buys will reveal that his signal $s \geq s_H$ and the manager who sells will reveal that his signal $s \leq s_L$. In a non-perverse equilibrium, as $p_t \rightarrow v_{\min}$, from the manager’s perspective it becomes inevitable that the ex post evaluation will be carried out in the state $v = v_{\min}$. But at such a state, revealing oneself to have received a signal in the range $[s_{\min}, s_L]$ is strictly better for one’s reputation than revealing oneself to have received a signal outside that range, due to the strict MLRP. Since profits are irrelevant in the limit as prices converge to true value, selling becomes optimal regardless of signal, and the manager does not follow the proposed equilibrium strategies.

6.2 Self Knowledge

The baseline analysis was carried out under the assumption that the manager did not know his type. What happens if the fund managers have some self-knowledge? We now allow the fund manager to receive two signals: the now familiar s_t and a new signal z_t , with $\Pr(z_t = \theta|\theta) = \rho$.

If they know their types perfectly ($\rho \rightarrow 1$), one can show that information cascades will not occur. A high-type manager will trade sincerely even if most people before him have made trades of the opposite sign. However, if we assume that fund managers only have some limited information about their types ($\rho \rightarrow \frac{1}{2}$), we still have a failure in information aggregation:

Proposition 7 *If self-knowledge is not too accurate, there exists no equilibrium in which a fund manager with $z_t = g$ plays sincerely.*¹⁵

As usual, the proof is given in the appendix. However, the intuition is simple. As long as the received signal about his type leaves some residual uncertainty, as public information about the state becomes sufficiently precise, a manager with a “good” type-signal and a “contrarian” value-signal becomes convinced that he has simply received two incorrect signals. Thus, just as in the baseline case, it becomes optimal for him to ignore his signals and behave in a conformist manner.

6.3 Relative Reputation

Suppose that the reputational component of the fund manager’s payoff depends on her relative reputation. The payoff is now

$$\beta\pi(\chi_t) + (1 - \beta)r_t(\hat{\gamma}_1, \dots, \hat{\gamma}_T).$$

We assume that r is still continuous and differentiable in its components and that, for fund manager t ,

$$\begin{aligned} \frac{\partial r_t}{\partial \hat{\gamma}_t} &> 0 \\ \frac{\partial r_t}{\partial \hat{\gamma}_\tau} &\leq 0 \quad \text{for } \tau \neq t. \end{aligned}$$

This formulation encompasses the standard “benchmarking” setting in which the reputational payoff is an increasing (and perhaps convex) function of the difference between the reputation of a particular manager and the average reputation of all managers:

$$r_t(\hat{\gamma}_1, \dots, \hat{\gamma}_T) = R\left(\hat{\gamma}_t - \frac{\sum_{\tau=1}^T \hat{\gamma}_\tau}{T}\right).$$

Proposition 8 *There is no sincere equilibrium.*

¹⁵We could actually show the stronger result (analogous to Proposition 2) that there exists no informative equilibrium.

Proof. Suppose a sincere equilibrium exists and consider the fund manager in the last period, T . As this agent's action does not affect the reputations of all other agents, the analysis is identical to the case without relative performance. Thus, this agent will not trade sincerely for extreme prices. ■

One may think that being evaluated on a relative basis will encourage contrarian behavior and lead to perfect information aggregation. This intuition comes from small numbers of managers, while ours is an asymptotic result. If there are incentives to play in a contrarian manner, some managers may indeed play in a contrarian manner early on in the game. As for the profit motive, the incentive for contrarian play will become smaller as time passes and information becomes more precise. In the long term, the intuition of our baseline result is still valid.

6.4 Micro-founded Career Concerns

The analysis to date has assumed that managers attach positive value to being able to impress their principals. While this assumption is consistent with empirical evidence as discussed in the introduction, we have not provided theoretical foundations for managerial career concerns. There are many ways to do so. We present a particularly simple example.¹⁶

There are two periods, $i = 1, 2$, after which the world ends. In each period, an asset with liquidation value $v_i \in \{0, 1\}$ is traded. At the end of period i , v_i is revealed to all. The payoffs v_1 and v_2 are iid. Within each period, there is a long sequence of T trading rounds.

To give a concrete example, there could be a bond in each period: one issued at 0 with maturity date T , one issued at T with maturity date $2T$. The default probability of a bond is independently distributed. Alternatively, it could be the stock of the company that makes an earnings announcement at T and another one at $2T$ (however, one would need to ensure that the signal a fund manager receives in the first period provides no information on the second period's announcement).

Suppose there are N ($\ll T$) identical fund management companies, which exist in N identical but non-overlapping regions each being the sole employer of fund managers in their region. Each such fund management company employs exactly one fund manager, who must be from its own region. We assume that the company acts in the best interest of its clients (investors), and thus maximizes trading profits net of costs.

Each region has a large number of potential fund managers. In period i , each fund manager receives a signal about v_i . The signal structure within each period is as in the main model above. That is, managers can be of type $\theta = g$ or $\theta = b$, and a manager of type $\theta = g$

¹⁶A fully micro-founded analysis with endogenous contracting is provided in Dasgupta and Prat [15]. In that paper, career concerns are considered at the firm-wide level.

receives more accurate signals about both v_1 and v_2 . The proportion of good managers in each region is γ_1 in the first period and γ_2 in the second period. γ_1 is known at the beginning of period 1, while γ_2 is revealed at the beginning of period 2.

There are also many noise traders. During each period, and in each trading round, either a noise trader or the fund manager (who has not traded yet in that period) is chosen to trade with a risk neutral market competitive maker who sets prices to reflect information contained in order flow. If the noise trader is selected, he buys or sells one unit of the prevailing asset with equal probability. If a manager is chosen he may buy or sell a unit of the asset as he chooses. Since $T \gg N$, each manager gets to trade once in each period with probability 1.¹⁷ The selection process is symmetric within period and independent across periods. Hence, uninformed traders have the same probability of facing a noise trader or a fund manager in every period.

In period 1, the company employs a manager, with whom it is randomly matched. The fund manager receives a fixed fee y for every period in which he is hired. His total fee is y if he only works in the first period and $2y$ if he works in both periods. There are no long-term contracts, but the manager can also receive a share of the first-period profits: $\hat{\alpha}\chi$ where $\hat{\alpha} \in [0, 1]$. At the beginning of period 2, the company is able to observe the manager's actions and their outcomes from period 1, and can choose to retain him or fire him and be randomly matched with another manager chosen from the region.¹⁸

Managerial career concerns now arise endogenously. The second period has many continuation equilibria because the fund manager is indifferent. We focus on the efficient equilibrium in which he buys when $s_t = 1$ and sells when $s_t = 0$.

The expected profit in the second period is then an increasing function of the ability of the second-period manager. In order to maximize profits, the company will retain the first-period manager at beginning of period 2 if $\hat{\gamma} \geq \gamma_2$ and she will replace him with a randomly selected manager if $\hat{\gamma} < \gamma_2$.

The manager in period 1 therefore must maximize

$$\hat{\alpha}\chi_1(a_1, p_t) + \Pr(\hat{\gamma}(a_1, v_1) \geq \gamma_2)y$$

Dividing the payoff by $\hat{\alpha} + y > 0$, we have that the manager maximizes in period 1

$$\beta\chi_1(p_t) + (1 - \beta)r(\hat{\gamma}(a_1, v_1))$$

¹⁷For finite T and N , there is a small probability that not all fund managers get to trade before time T . Formally, we could assume an exogenous retention rule for the remaining fund managers (for instance, they all get fired). As $T \rightarrow \infty$, the probability of facing such exogenous termination vanishes, which is the case we discuss here.

¹⁸The analysis would be more complicated if the manager received a share of the second-period profit as well. However, Dasgupta and Prat [15] show that, if contracts are endogenous, the fund manager's compensation is independent of his return in the last period.

where $\beta = \frac{\hat{\alpha}}{\hat{\alpha} + y}$, and $r(\hat{\gamma}(a_1, v_1)) = \Pr(\hat{\gamma}(a_1, v_1) \geq \gamma_2)$, which is a special case of the payoffs that we have considered throughout the paper.

6.5 Informed Individual Traders

To date we have restricted attention to a market in which uninformed market makers are faced with either noise traders or reputationally sensitive traders. A natural added member of the marketplace would be informed individual investors, who do not face career concerns. How would the results change if informed individual investors operated in our baseline model?

It is clear that informed individuals devoid of career concerns would trade sincerely, and thus, in the presence of such traders prices would eventually converge to true value. However, the basic intuition of the main result is unchanged in this case: once prices were close enough to true value, career concerned institutional traders would begin to ignore their own information. Thus convergence to true value would take place much more slowly than in the case without fund managers, and the extent of slowdown in convergence would depend on the proportion of career concerned traders in the market. In addition, conformist trading by institutional traders would still occur in the presence of informed individual traders, in keeping with empirical findings (e.g., Dennis and Strickland [18]).

7 Conclusion

The central message of this paper is that the presence of (even small amounts of) reputational concerns will prevent institutional traders from trading sincerely when prices become close enough to liquidation value. Such a tendency to neglect valuable private information is an endogenous (and pervasive) obstacle to the convergence of prices to liquidation values in the long run, and can be the basis of herd-like behavior by institutional traders, along the lines already documented in the empirical literature.

Further, we have argued that the presence of reputational incentives can drive a wedge between the expected liquidation value of an asset and its total value to fund managers. We have characterized the properties of such reputational benefits, and presented an example of how such reputational premia can be incorporated into prices, thus leading to the systematic mispricing of assets. While we have carefully related our central results on conformist trading to existing theoretical explanations in the introduction, it remains for us to do the same for our explorations on mispricing. We now proceed to do so.

While the classical no-trade arguments of Milgrom and Stokey [23] and Tirole [36] preclude bubbles in markets with asymmetric information and rational agents in general, a number of papers construct examples of bubbles while examining the role of higher order beliefs in

asset pricing. Allen, Morris, and Postlewaite [5] build on the no-trade theorems to develop necessary conditions for the existence of bubbles, and provide examples of economies in which bubbles can exist. Morris, Postlewaite, and Shin [24] illustrate the connection between bubbles and higher order uncertainty. Prices are biased statistics of true value in the recent work of Allen, Morris, and Shin [6]. There are also a large number of papers in which bubbles arise and persist because some traders are irrational (e.g. DeLong, Shleifer, Summers, and Waldmann [17], Shleifer and Vishny [32], Abreu and Brunnermeier [1], and Scheinkman and Xiong [31], amongst others).

More closely related to our work, a few papers construct examples of bubbles based on agency conflicts. Most notably, Allen and Gorton [4] develop a model in which prices can diverge from fundamentals due to churning by portfolio managers. In their model, bad fund managers buy bubble stocks at prices above their known liquidation value in the hope of reselling them before they die — at even higher prices — to other bad fund managers. Their behavior is the result of an option-like payoff structure under which profits are shared with managers but losses are not. Churning thus creates the possibility of short-term speculative profits. A related principal-agent conflict leads to bubbles in Allen and Gale [3]. In contrast to both of these papers, in our example, mispricing arises without option-like payoffs purely due to reputational concerns of financial traders.

8 Appendix

8.1 Proof of Proposition 2

We proceed by proving two preliminary results.

Lemma 9 *There exists a function $f : [0, 1] \times [0, 1] \rightarrow [0, 1]$ such that $\Pr(v = 1 | h_t, s_t) = f(p_t, s_t)$ and f satisfies the following properties:*

- (a) $f(p_t, s_t)$ is strictly increasing and continuous in p_t .
- (b) $f(1, s_t) = 1 = 1 - f(0, s_t)$
- (c) $f(p_t, 1) > f(p_t, 0)$

Proof.

$$\begin{aligned}
\Pr(v = 1|h_t, s_t) &= \frac{\Pr(s_t, h_t|v = 1) \Pr(v = 1)}{\Pr(s_t, h_t)} \\
&= \frac{\Pr(s_t|v = 1) \Pr(v = 1)}{\Pr(s_t, h_t)} \Pr(h_t|v = 1) \\
&= \frac{\Pr(s_t|v = 1) \Pr(v = 1)}{\Pr(s_t, h_t)} \frac{\Pr(v = 1|h_t) \Pr(h_t)}{\Pr(v = 1)} \\
&= \frac{\Pr(s_t|v = 1) \Pr(h_t)}{\Pr(s_t, h_t)} p_t
\end{aligned}$$

$$\Pr(s_t, h_t) = \Pr(h_t)[p_t \Pr(s_t|v = 1) + (1 - p_t) \Pr(s_t|v = 0)]$$

Thus

$$\Pr(v = 1|h_t, s_t) = \frac{p_t \Pr(s_t|v = 1)}{p_t \Pr(s_t|v = 1) + (1 - p_t) \Pr(s_t|v = 0)} = f(p_t, s_t)$$

Now parts (a) and (b) follow immediately. To see part (c) note that

$$\begin{aligned}
f(p_t, 1) &= \frac{p_t}{p_t + (1 - p_t) \frac{\Pr(s_t=1|v=0)}{\Pr(s_t=1|v=1)}} \\
&= \frac{p_t}{p_t + (1 - p_t) \frac{1-\sigma}{\sigma}}
\end{aligned}$$

where $\sigma = \gamma\sigma_g + (1 - \gamma)\sigma_b$. Similarly

$$f(p_t, 0) = \frac{p_t}{p_t + (1 - p_t) \frac{\sigma}{1-\sigma}}$$

Since $\sigma_g > \sigma_b > \frac{1}{2}$, $\sigma > \frac{1}{2}$. Thus $\frac{1-\sigma}{\sigma} < 1 < \frac{\sigma}{1-\sigma}$. This then implies $f(p_t, 1) > f(p_t, 0)$ which completes the proof of the lemma. ■

The mixed strategy of manager t in this market will generally depend on both this history he observes and his signal. We denote this by $\alpha_{s_t}^t(h_t)$. A mixed strategy equilibrium is a sequence $\{\alpha_{s_t}^t(h_t)\}_{t=1}^{T-1}$. For notational convenience, we often omit the history argument and we denote the t fund manager's strategy as $(\alpha_0^t, \alpha_1^t) \in [0, 1]^2$.

Consider a set of equilibrium strategies $(\alpha_0^t, \alpha_1^t) \in [0, 1]^2$. We can now compute the posteriors regarding fund managers. The posterior belief is given by

$$\hat{\gamma}(a_t, v, h_t) = \frac{\Pr(a_t|\theta = g, v, h_t) \gamma}{\Pr(a_t|\theta = g, v, h_t) \gamma + \Pr(a_t|\theta = b, v, h_t) (1 - \gamma)}$$

Note that

$$\Pr(a_t = 1|\theta = g, v = 1, h_t) = \alpha_1(h_t) \sigma_g + \alpha_0(h_t) (1 - \sigma_g)$$

and similarly for the other realizations of $\hat{\gamma}$. Therefore, we can write

$$\begin{aligned}\hat{\gamma}(a_t = 1, v = 1, h_t) &= \frac{\alpha_1(h_t)\sigma_g + \alpha_0(h_t)(1 - \sigma_g)}{\alpha_1(h_t)\sigma + \alpha_0(h_t)(1 - \sigma)}\gamma \\ \hat{\gamma}(a_t = 1, v = 0, h_t) &= \frac{\alpha_1(h_t)(1 - \sigma_g) + \alpha_0(h_t)\sigma_g}{\alpha_1(h_t)(1 - \sigma) + \alpha_0(h_t)\sigma}\gamma \\ \hat{\gamma}(a_t = 0, v = 1, h_t) &= \frac{(1 - \alpha_1(h_t))\sigma_g + (1 - \alpha_0(h_t))(1 - \sigma_g)}{(1 - \alpha_1(h_t))\sigma + (1 - \alpha_0(h_t))(1 - \sigma)}\gamma \\ \hat{\gamma}(a_t = 0, v = 0, h_t) &= \frac{(1 - \alpha_1(h_t))(1 - \sigma_g) + (1 - \alpha_0(h_t))\sigma_g}{(1 - \alpha_1(h_t))(1 - \sigma) + (1 - \alpha_0(h_t))\sigma}\gamma\end{aligned}$$

We now show that in all non-perverse equilibria either the manager with the high signal or the manager with the low signal play a pure strategy.

Lemma 10 *There are no mixed strategy equilibria in which $0 < \alpha_0^t < \alpha_1^t < 1$ for any t .*

Proof. Consider a putative equilibrium in which $1 > \alpha_0^t > 0$, i.e. the agent at time t who receives signal zero is exactly indifferent between buying and selling. We will show that in this equilibrium, it must be the case that the agent who receives signal 1 at time t must strictly prefer to buy rather than sell. Consider the expected direct payoff difference between buying and selling: $\sum_{v=0,1} \Pr(v|h_t, s_t) \Delta\pi(\chi(p_t^a, p_t^b, v))$. This can be written as

$$f(p_t, s_t) \left(\pi(1 - p_t^a) - \pi(p_t^b - 1) \right) + (1 - f(p_t, s_t)) \left(\pi(0 - p_t^a) - \pi(p_t^b - 0) \right)$$

Since $\pi(1 - p_t^a) - \pi(p_t^b - 1) > 0 > \pi(0 - p_t^a) - \pi(p_t^b - 0)$, and by Lemma 9 $f(p_t, 1) > f(p_t, 0)$, it is clear that

$$\sum_{v=0,1} \Pr(v|h_t, s_t = 1) \Delta\pi(\chi(p_t^a, p_t^b, v)) > \sum_{v=0,1} \Pr(v|h_t, s_t = 0) \Delta\pi(\chi(p_t^a, p_t^b, v))$$

Now consider the expected reputational payoff difference between buying and selling:

$$\sum_{v=0,1} \Pr(v|h_t, s_t) \Delta r(\hat{\gamma}(v, h_t))$$

This can be expressed as:

$$f(p_t, s_t) [r(\hat{\gamma}(1, h_t, 1)) - r(\hat{\gamma}(0, h_t, 1))] + (1 - f(p_t, s_t)) [r(\hat{\gamma}(1, h_t, 0)) - r(\hat{\gamma}(0, h_t, 0))]$$

Notice that $\hat{\gamma}(1, h_t, 1) > \hat{\gamma}(0, h_t, 1)$. To see why consider the expressions above. Suppose

$$\frac{\alpha_1\sigma_g + \alpha_0(1 - \sigma_g)}{\alpha_1\sigma + \alpha_0(1 - \sigma)} < \frac{(1 - \alpha_1)\sigma_g + (1 - \alpha_0)(1 - \sigma_g)}{(1 - \alpha_1)\sigma + (1 - \alpha_0)(1 - \sigma)}$$

Algebraic manipulation shows that this implies that $(\sigma_g - \sigma)(\alpha_1 - \alpha_0) < 0$ which is a contradiction since $\sigma_g - \sigma > 0$ and $\alpha_1 - \alpha_0 > 0$. Similarly it is also true that $\hat{\gamma}(1, h_t, 0) < \hat{\gamma}(0, h_t, 0)$. To see why, consider again the expressions above. Suppose $\frac{\alpha_1(1-\sigma_g)+\alpha_0\sigma_g}{\alpha_1(1-\sigma)+\alpha_0\sigma} > \frac{(1-\alpha_1)(1-\sigma_g)+(1-\alpha_0)\sigma_g}{(1-\alpha_1)(1-\sigma)+(1-\alpha_0)\sigma}$. This implies that $-(\sigma_g - \sigma)(\alpha_1 - \alpha_0) > 0$ which is a contradiction. Thus, $r(\hat{\gamma}(1, h_t, 1)) - r(\hat{\gamma}(0, h_t, 1)) > 0 > r(\hat{\gamma}(1, h_t, 0)) - r(\hat{\gamma}(0, h_t, 0))$. Given Lemma 9, we know that $f(p_t, 1) > f(p_t, 0)$, and thus

$$\sum_{v=0,1} \Pr(v|h_t, s_t = 1) \Delta r(\hat{\gamma}(v, h_t)) > \sum_{v=0,1} \Pr(v|h_t, s_t = 0) \Delta r(\hat{\gamma}(v, h_t))$$

Finally, the arguments above imply that

$$\sum_{v=0,1} \Pr(v|h_t, s_t = 1) (\beta \Delta \pi + (1 - \beta) \Delta r) > \sum_{v=0,1} \Pr(v|h_t, s_t = 0) (\beta \Delta \pi + (1 - \beta) \Delta r) = 0$$

Thus, if $0 < \alpha^0(h_t) < 1$, then $\alpha^1(h_t) = 1$. An identical argument establishes that if $0 < \alpha^1(h_t) < 1$, then $\alpha^0(h_t) = 0$. This completes the proof of the lemma. ■

Let $\{\alpha_{s_t}(h_t)\}_{t=1}^{T-1}$ be any non-perverse perfect Bayesian equilibrium of this game: $\alpha_1(h_t) > \alpha_0(h_t)$ for all h_t .

Lemma 11 *For every $\varepsilon > 0$ there exists $\bar{p}_1(\varepsilon)$ and $\bar{p}_2(\varepsilon)$ such that (for all h_t):*

$$\Delta \pi \geq -\varepsilon \text{ for all } p_t > \bar{p}_2(\varepsilon)$$

and

$$\Delta \pi \leq \varepsilon \text{ for all } p_t < \bar{p}_1(\varepsilon)$$

Proof. With terms defined as above, first consider the expected direct payoff difference between buying and selling. By a slight abuse of notation, we can write:

$$\begin{aligned} \Delta \pi &= \sum_{v=0,1} \Pr(v|h_t, s_t) \left(\pi(v - p_t^a) - \pi(p_t^b - v) \right) \\ &= f(p_t, s_t) \left(\pi(1 - p_t^a) - \pi(p_t^b - 1) \right) + (1 - f(p_t, s_t)) \left(\pi(0 - p_t^a) - \pi(p_t^b - 0) \right) \end{aligned}$$

Note that this function is *bounded above* by

$$\begin{aligned} U_\pi &= \sum_{v=0,1} \Pr(v|h_t, s_t) (\pi(v) - \pi(-v)) \\ &= f(p_t, s_t) (\pi(1) - \pi(-1)) \end{aligned}$$

And it is *bounded below* by

$$\begin{aligned} L_\pi &= \sum_{v=0,1} \Pr(v|h_t, s_t) (\pi(v - 1) - \pi(1 - v)) \\ &= (1 - f(p_t, s_t)) (\pi(-1) - \pi(1)) \end{aligned}$$

It is apparent that $U_\pi > 0$. From Lemma 9 we also conclude that U_π strictly increasing in p_t and $\lim_{p_t \rightarrow 0} U_\pi = 0$. Similarly, $L_\pi < 0$, strictly decreasing in p_t and $\lim_{p_t \rightarrow 1} L_\pi = 0$.

This means that for every $\varepsilon > 0$ there exists $\bar{p}_1(\varepsilon)$ and $\bar{p}_2(\varepsilon)$ such that

$$\Delta\pi \geq -\varepsilon \text{ for all } p_t > \bar{p}_2(\varepsilon)$$

and

$$\Delta\pi \leq \varepsilon \text{ for all } p_t < \bar{p}_1(\varepsilon)$$

■

Second, consider the expected difference in reputational payoffs between buying and selling. Again, abusing notation, we write:

$$\begin{aligned} \Delta r &= f(p_t, s_t) (r(\hat{\gamma}(a_t = 1, v = 1, h_t)) - r(\hat{\gamma}(a_t = 0, v = 1, h_t))) \\ &\quad + (1 - f(p_t, s_t)) (r(\hat{\gamma}(a_t = 1, v = 0, h_t)) - r(\hat{\gamma}(a_t = 0, v = 0, h_t))) \end{aligned}$$

Lemma 12 *For every number $\epsilon > 0$ (but not too large), there exist $\tilde{p}_1(\epsilon)$ and $\tilde{p}_2(\epsilon)$ such that (for all h_t):*

$$\Delta r \geq \epsilon \text{ for all } p_t > \tilde{p}_2(\epsilon)$$

and

$$\Delta r \leq -\epsilon \text{ for all } p_t < \tilde{p}_1(\epsilon).$$

Proof. From Lemma 10 we know that there cannot be an equilibrium in which $0 < \alpha_0(h_t) < \alpha_1(h_t) < 1$. Thus, either $0 = \alpha_0(h_t) < \alpha_1(h_t) < 1$ or $0 < \alpha_0(h_t) < \alpha_1(h_t) = 1$.

If $0 = \alpha_0(h_t) < \alpha_1(h_t) < 1$, the difference

$$r(\hat{\gamma}(a_t = 1, v = 1, h_t)) - r(\hat{\gamma}(a_t = 0, v = 1, h_t))$$

reduces to

$$r\left(\frac{\sigma_g}{\sigma}\gamma\right) - r\left(\frac{(1 - \alpha_1(h_t))\sigma_g + (1 - \sigma_g)}{(1 - \alpha_1(h_t))\sigma + (1 - \sigma)}\gamma\right),$$

which lies in the interval:

$$\left[r\left(\frac{\sigma_g}{\sigma}\gamma\right) - r(\gamma), r\left(\frac{\sigma_g}{\sigma}\gamma\right) - r\left(\frac{1 - \sigma_g}{1 - \sigma}\gamma\right) \right]$$

On the other hand if $0 < \alpha_0(h_t) < \alpha_1(h_t) = 1$, then the difference reduces to

$$r\left(\frac{\sigma_g + \alpha_0(h_t)(1 - \sigma_g)}{\sigma + \alpha_0(h_t)(1 - \sigma)}\gamma\right) - r\left(\frac{1 - \sigma_g}{1 - \sigma}\gamma\right)$$

which lies in the interval:

$$\left[r(\gamma) - r\left(\frac{1 - \sigma_g}{1 - \sigma}\gamma\right), r\left(\frac{\sigma_g}{\sigma}\gamma\right) - r\left(\frac{1 - \sigma_g}{1 - \sigma}\gamma\right) \right]$$

Now, defining

$$U_r^1 = r \left(\frac{\sigma_g}{\sigma} \gamma \right) - r \left(\frac{1 - \sigma_g}{1 - \sigma} \gamma \right) > 0$$

and

$$L_r^1 = \min \left[r(\gamma) - r \left(\frac{1 - \sigma_g}{1 - \sigma} \gamma \right), r \left(\frac{\sigma_g}{\sigma} \gamma \right) - r(\gamma) \right]$$

we note that in all non-perverse equilibria, the difference

$$r(\hat{\gamma}(a_t = 1, v = 1, h_t)) - r(\hat{\gamma}(a_t = 0, v = 1, h_t))$$

is bounded below by L_r^1 and bounded above by U_r^1 .

Now consider the case where $v = 0$. With a procedure (that we omit) analogous to the one used above, we see that the difference

$$r(\hat{\gamma}(a_t = 1, v = 0, h_t)) - r(\hat{\gamma}(a_t = 0, v = 0, h_t))$$

is bounded below by

$$L_r^0 = r \left(\frac{1 - \sigma_g}{1 - \sigma} \gamma \right) - r \left(\frac{\sigma_g}{\sigma} \gamma \right) < 0,$$

and it is bounded above by

$$U_r^0 = \max \left[r(\gamma) - r \left(\frac{\sigma_g}{\sigma} \gamma \right), r \left(\frac{1 - \sigma_g}{1 - \sigma} \gamma \right) - r(\gamma) \right] < 0.$$

Thus Δr is bounded above by

$$f(p_t, s_t)U_r^1 + (1 - f(p_t, s_t))U_r^0$$

and bounded below by

$$f(p_t, s_t)L_r^1 + (1 - f(p_t, s_t))L_r^0$$

Finally, we note that $U_r^1 = -L_r^0 > 0$ and $L_r^1 = -U_r^0 > 0$. Thus, Δr is bounded above by

$$U_r = f(p_t, s_t)U_r^1 - (1 - f(p_t, s_t))L_r^1$$

and bounded below by

$$L_r = f(p_t, s_t)L_r^1 - (1 - f(p_t, s_t))U_r^1$$

Now, utilizing Lemma 9 above, we know that U_r strictly increasing in p_t and $\lim_{p_t \rightarrow 0} U_r = -L_r^1 < 0$. Similarly, L_r , strictly decreasing in p_t and $\lim_{p_t \rightarrow 1} L_r = L_r^1 > 0$.

Thus, for every number $0 < \epsilon < L_r^1$ there exists a price $\tilde{p}_2(\epsilon) < 1$ such that for $p_t > \tilde{p}_2(\epsilon)$, $\Delta r \geq \epsilon$.

The part of the proof concerning $\tilde{p}_1(\epsilon)$ is analogous and it is omitted. ■

Fix any number $\epsilon \in (0, L_r^1)$. By Lemma 12, we know there exists $\tilde{p}_2(\epsilon) < 1$ such that for $p_t > \tilde{p}_2(\epsilon)$, $\Delta r \geq \epsilon$. Now consider another number, $\rho = \frac{1-\beta}{\beta}(\epsilon - \delta)$ (where $\delta \in (0, \epsilon)$). By Lemma 11, we know that there exists $\bar{p}_2(\rho) < 1$ such that if $p_t > \bar{p}_2(\rho)$, then $\Delta\pi \geq -\rho$. Thus for $p_t > \max[\tilde{p}_2(\epsilon), \bar{p}_2(\rho)]$,

$$\Delta u = \beta\Delta\pi + (1-\beta)\Delta r \geq \beta \left[-\frac{1-\beta}{\beta}(\epsilon - \delta) \right] + (1-\beta)\epsilon = (1-\beta)\delta > 0$$

Thus, for any price greater than $\max[\tilde{p}_2(\epsilon), \bar{p}_2(\rho)]$ the fund manager would always choose to buy. The case for selling is symmetric.

8.2 Proof of Proposition 3

Suppose that the manager plays according to (1) and (2). above. Given the symmetry of the game and the proposed equilibrium, we restrict attention to $p_t \in [0, \frac{1}{2}]$. It is easy to see that a manager with $s_t = 0$ always wants to sell. Thus we can set $\alpha_0^t = 0$ for the relevant price range.

To simplify notation, let $\alpha_1^t = \alpha$ denote the probability that a manager with $s_t = 1$ buys. Further, we define:

$$\begin{aligned} \Pr(s_t = 1|v = 1, h_t) &= \sigma \\ \Pr(s_t = 1|h_t) &= \Sigma_t \equiv \sigma p_t + (1-\sigma)(1-p_t) \end{aligned}$$

Under the equilibrium strategies, it is easy to compute the following prices:

$$\begin{aligned} p_t^a &= \frac{\delta \frac{1}{2} + (1-\delta)\alpha\sigma}{\delta \frac{1}{2} + (1-\delta)\alpha\Sigma_t} p_t \\ p_t^b &= \frac{\delta \frac{1}{2} + (1-\delta)(1-\alpha\sigma)}{\delta \frac{1}{2} + (1-\delta)(1-\alpha\Sigma_t)} p_t, \end{aligned}$$

and the following beliefs,

$$\begin{aligned} \hat{\gamma}(a_t = 1, v = 1) &= \frac{\sigma_g}{\sigma} \gamma & \hat{\gamma}(a_t = 0, v = 1) &= \frac{(1-\alpha)\sigma_g + 1 - \sigma_g}{(1-\alpha)\sigma + 1 - \sigma} \gamma \\ \hat{\gamma}(a_t = 1, v = 0) &= \frac{1 - \sigma_g}{1 - \sigma} \gamma & \hat{\gamma}(a_t = 0, v = 0) &= \frac{(1-\alpha)(1 - \sigma_g) + \sigma_g}{(1-\alpha)(1 - \sigma) + \sigma} \gamma \end{aligned}$$

Notice that in computing $\hat{\gamma}(a_t = 1, v = 1)$ and $\hat{\gamma}(a_t = 1, v = 0)$ we have not restricted attention to $\alpha > 0$. We are thus implicitly imposing the following off-equilibrium beliefs: if a manager trades in a contrarian manner in a region where equilibrium strategies require conformism, then he is assumed to have played sincerely. The beliefs used here are natural, in that they would be the on-equilibrium beliefs that would occur if a small proportion of managers always traded sincerely for exogenous reasons.

Now, consider a manager with $s_t = 1$. His expected trading profit from buying instead of sell is

$$\Delta\pi_1(p_t, \alpha) = 2\hat{v}_1^t - p_t^a - p_t^b$$

His expected reputational payoff from buying instead of selling is

$$\begin{aligned}\Delta r_1(p_t, \alpha) &= \hat{v}_1^t (\hat{\gamma}(a_t = 1, v = 1) - \hat{\gamma}(a_t = 0, v = 1)) \\ &\quad + (1 - \hat{v}_1) (\hat{\gamma}(a_t = 1, v = 0) - \hat{\gamma}(a_t = 0, v = 0)).\end{aligned}$$

Finally, define the total differential payoff from buying instead of selling as

$$\Delta u_1(p_t, \alpha) = \beta \Delta\pi_1(p_t, \alpha) + (1 - \beta) \Delta r_1(p_t, \alpha).$$

In order to show the result, we need to show that for a given $(\gamma, \sigma_b, \sigma_g, \delta)$, there exists a $\bar{\beta} > 0$ such that for all $\beta < \bar{\beta}$, $\Delta u_1(p_t, 1) \geq 0$ for $p_t \geq \bar{p}$ and $\Delta u_1(p_t, 0) \leq 0$ for $p_t \leq \bar{p}$. The first step is captured in the following lemma:

Lemma 13 *Given $(\gamma, \sigma_b, \sigma_g, \delta)$, there exists a $\bar{\beta}_1 > 0$ such that for all $\beta < \bar{\beta}_1$, for every α , Δu_1 is increasing in p_t .*

Proof. Given $(\gamma, \sigma_b, \sigma_g)$, we have

$$\frac{\partial}{\partial p_t} \Delta r_1(p_t, \alpha) = \frac{\partial}{\partial p_t} \hat{v}_1^t \begin{pmatrix} (\hat{\gamma}(a_t = 1, v = 1) - \hat{\gamma}(a_t = 0, v = 1)) \\ -(\hat{\gamma}(a_t = 1, v = 0) - \hat{\gamma}(a_t = 0, v = 0)) \end{pmatrix}.$$

For every $\alpha \in [0, 1]$,

$$\begin{aligned}& (\hat{\gamma}(a_t = 1, v = 1) - \hat{\gamma}(a_t = 0, v = 1)) - (\hat{\gamma}(a_t = 1, v = 0) - \hat{\gamma}(a_t = 0, v = 0)) \\ & \geq \gamma \min_{\alpha} \left(\frac{\sigma_g}{\sigma} - \frac{(1 - \alpha)\sigma_g + 1 - \sigma_g}{(1 - \alpha)\sigma + 1 - \sigma} \right) - \left(\frac{1 - \sigma_g}{1 - \sigma} - \frac{(1 - \alpha)(1 - \sigma_g) + \sigma_g}{(1 - \alpha)(1 - \sigma) + \sigma} \right) \\ & = \gamma \left(\left(\frac{\sigma_g}{\sigma} - 1 \right) - \left(\frac{1 - \sigma_g}{1 - \sigma} - 1 \right) \right) = \gamma \left(\frac{\sigma_g}{\sigma} - \frac{1 - \sigma_g}{1 - \sigma} \right)\end{aligned}$$

is strictly positive. Also

$$\frac{\partial}{\partial p_t} \hat{v}_1^t = \frac{\sigma \Sigma_t - \sigma(2\sigma - 1)p_t}{\Sigma_t^2} = \frac{\sigma(1 - \sigma)}{\Sigma_t^2}$$

But $\max_{p_t \in [0, \frac{1}{2}]} \Sigma_t = \sigma(\frac{1}{2}) + (1 - \sigma)(1 - (\frac{1}{2})) = \frac{1}{2}$. Hence

$$\frac{\partial}{\partial p_t} \hat{v}_1^t > 4\sigma(1 - \sigma) \text{ for all } p_t.$$

Therefore

$$\frac{\partial}{\partial p_t} \Delta r_1(p_t, \alpha) \geq 4\sigma(1-\sigma)\gamma \left(\frac{\sigma_g}{\sigma} - \frac{1-\sigma_g}{1-\sigma} \right) \quad \text{for all } p_t \text{ and } \alpha.$$

It is also easy that $\frac{\partial}{\partial p_t} \Delta \pi_1(p_t, \alpha)$ is bounded below on $p_t \in [0, \frac{1}{2}]$. Hence, given $(\gamma, \sigma_b, \sigma_g, \delta)$, there exists a β low enough such that

$$\beta \frac{\partial}{\partial p_t} \Delta \pi_1(p_t, \alpha) + (1-\beta) \frac{\partial}{\partial p_t} \Delta r_1(p_t, \alpha) > 0$$

for every $p_t \in [0, \frac{1}{2}]$ and α . ■

For the remainder of the proof, consider $\beta < \bar{\beta}_1$. Let \bar{p} to be the unique solution of $\Delta u_1(p_t, 1) = 0$. Clearly, $\Delta u_1(p_t, 1) \geq 0$ for all $p_t \geq \bar{p}$. Note that, for $\beta = 0$

$$\Delta u_1(\bar{p}, 1) = \Delta r_1(\bar{p}, 1) = (2\hat{v}_1(\bar{p}) - 1) \left(\frac{\sigma_g}{\sigma} - \frac{1-\sigma_g}{1-\sigma} \right) \gamma = 0$$

and thus

$$\hat{v}_1(\bar{p}) = \frac{1}{2}$$

We now argue that for $\beta = 0$, $\Delta u_1(\bar{p}, 0) < 0$. When $\beta = 0$

$$\begin{aligned} \Delta u_1(\bar{p}, 0) &= \Delta r_1(\bar{p}, 0) = \hat{v}_1(\bar{p}) \left(\frac{\sigma_g}{\sigma} - 1 \right) \gamma + (1 - \hat{v}_1(\bar{p})) \left(\frac{1-\sigma_g}{1-\sigma} - 1 \right) \gamma \\ &= \frac{1}{2} \left(\frac{\sigma_g}{\sigma} - 1 \right) \gamma + \left(1 - \frac{1}{2} \right) \left(\frac{1-\sigma_g}{1-\sigma} - 1 \right) \gamma = \frac{1}{2} \gamma \left(\frac{\sigma_g}{\sigma} + \frac{1-\sigma_g}{1-\sigma} - 2 \right) < 0 \end{aligned}$$

since $\frac{\sigma_g}{\sigma} + \frac{1-\sigma_g}{1-\sigma} < 2$. Then, by continuity in β , for a given $(\gamma, \sigma_b, \sigma_g, \delta)$, there exists $\bar{\beta}_2 > 0$ such that for all $\beta < \bar{\beta}_2$, $\Delta u_1(\bar{p}, 0) \leq 0$. But then, by Lemma 13, for such β , $\Delta u_1(p, 0) \leq 0$ for all $p < \bar{p}$. Now set $\bar{\beta} = \min(\bar{\beta}_1, \bar{\beta}_2)$ and the proof of existence is complete. For the non-existence of “more informative equilibria, we need the following additional result:

Lemma 14 *In the limit as $\beta \rightarrow 0$, for every $\alpha \in (0, 1)$ and for all $p < \bar{p}$, $\Delta u_1(\bar{p}, 1) > \Delta u_1(p, \alpha)$.*

Proof. Consider

$$\frac{d}{d\alpha} \Delta r_1(p, \alpha)|_{p=\bar{p}} = -\hat{v}_1(\bar{p}) \frac{d}{d\alpha} \hat{\gamma}(a_t = 0, v = 1) - (1 - \hat{v}_1(\bar{p})) \frac{d}{d\alpha} \hat{\gamma}(a_t = 0, v = 0)$$

But note that

$$\begin{aligned} \frac{d}{d\alpha} \frac{(1-\alpha)\sigma_g + 1 - \sigma_g}{(1-\alpha)\sigma + 1 - \sigma} &= -\frac{\sigma_g - \sigma}{((1-\alpha)\sigma + 1 - \sigma)^2} \\ \frac{d}{d\alpha} \frac{(1-\alpha)(1-\sigma_g) + \sigma_g}{(1-\alpha)(1-\sigma) + \sigma} &= \frac{\sigma_g - \sigma}{((1-\alpha)(1-\sigma) + \sigma)^2} \end{aligned}$$

But $(1 - \alpha)\sigma + 1 - \sigma < (1 - \alpha)(1 - \sigma) + \sigma$ for $\alpha \in (0, 1)$ implies that

$$-\frac{d}{d\alpha}\hat{\gamma}(a_t = 0, v = 1) > \frac{d}{d\alpha}\hat{\gamma}(a_t = 0, v = 0)$$

If $\beta \rightarrow 0$, $\hat{v}_1(\bar{p}) \rightarrow \frac{1}{2}$ and

$$\frac{d}{d\alpha}\Delta r_1(p, \alpha)|_{p=\bar{p}} > 0.$$

But then $\Delta u_1(\bar{p}, 1) > \Delta u_1(\bar{p}, \alpha)$ for every $\alpha \in (0, 1)$. By lemma 13, for every α and $p < \bar{p}$, $\Delta u_1(\bar{p}, \alpha) \geq \Delta u_1(p, \alpha)$. ■

Suppose that $p_t < \bar{p}$ and that there exists an informative nonperverse equilibrium. We know that in such a case a manager with $s_t = 0$ must sell for sure. But lemma 14 also says that $\Delta u_1(p, \alpha) < 0$ for all α and $p < \bar{p}$. Then, a manager with $s_t = 1$ would sell for sure as well. But then the equilibrium is not informative. This completes the proof of the result. ■

8.3 Proof of Proposition 6

First, it is easy to check that in a non-perverse equilibrium, given h_t , there are at most two realizations of s_t for which the agent randomizes among actions. A non-perverse equilibrium is characterized by a partition of the interval $[s_{\min}, s_{\max}]$ into three regions. Let s_L denote the threshold below which the fund manager sells for sure and above which he does not trade. Let s_H be the equivalent threshold between not trading and buying. It is clear that if this non-perverse equilibrium is to be informative (that is, allow for prices to converge to true value in the long run), either $s_H < s_{\max}$ or $s_L > s_{\min}$ or both. We assume that $s_L > s_{\min}$ and show that in this equilibrium, it is impossible for $p_t \rightarrow v_{\min}$. The case for $s_H < s_{\max}$ and $p_t \rightarrow v_{\max}$ is symmetric.

The proof is by contradiction. Suppose that $v = v_{\min}$ (the proof for a high v is analogous) and that there exists an equilibrium in which $p_t \rightarrow v_{\min}$. It is clear that in a non-perverse equilibrium

$$\lim_{p_t \rightarrow v_{\min}} E[r(\hat{\gamma}(\text{buy}, v))] = r(\hat{\gamma}(\text{buy}, v_{\min})),$$

that is, as prices converge to v_{\min} all fund managers expectation of the reputational return from buying converges to the ex post reputational return when the posterior is evaluated at v_{\min} .

Consider a fund manager with signal $s_t \in (s_H, 0)$. The equilibrium dictates that he buys.

When $p_t \rightarrow v_{\min}$, the expected reputation of a manager who buys is

$$\begin{aligned}
\Pr(\theta = g | s > s_H, v_{\min}) &= \frac{\gamma \Pr(s > s_H | \theta = g, v_{\min})}{\Pr(s > s_H | v_{\min})} \\
&= \frac{\gamma \int_{s_H}^{s_{\max}} f_g(s | v_{\min}) ds}{\gamma \int_{s_H}^{s_{\max}} f_g(s | v_{\min}) ds + (1 - \gamma) \int_{s_H}^{s_{\max}} f_b(s | v_{\min}) ds} \\
&= \frac{\gamma \int_{s_H}^{s_{\max}} f_g(s | v_{\min}) ds}{(\gamma + \tau(1 - \gamma)) \int_{s_H}^{s_{\max}} f_g(s | v_{\min}) ds + (1 - \gamma)(1 - \tau) \int_{s_H}^{s_{\max}} \bar{f}(s) ds}
\end{aligned}$$

and similarly, the reputation of a manager who sells is

$$\Pr(\theta = g | s < s_L, v_{\min}) = \frac{\gamma \int_{s_{\min}}^{s_L} f_g(s | v_{\min}) ds}{(\gamma + \tau(1 - \gamma)) \int_{s_{\min}}^{s_L} f_g(s | v_{\min}) ds + (1 - \gamma)(1 - \tau) \int_{s_{\min}}^{s_L} \bar{f}(s) ds}$$

We shall show that $\Pr(\theta = g | s > s_H, v_{\min}) < \Pr(\theta = g | s < s_L, v_{\min})$. For this, it suffices to show that

$$\frac{\int_{s_H}^{s_{\max}} f_g(s | v_{\min}) ds}{\int_{s_{\min}}^{s_L} f_g(s | v_{\min}) ds} < \frac{\int_{s_H}^{s_{\max}} \bar{f}(s) ds}{\int_{s_{\min}}^{s_L} \bar{f}(s) ds}$$

But

$$\frac{\int_{s_H}^{s_{\max}} \bar{f}(s) ds}{\int_{s_{\min}}^{s_L} \bar{f}(s) ds} = \frac{\sum_v k(v) \int_{s_H}^{s_{\max}} f_g(s | v) ds}{\sum_v k(v) \int_{s_{\min}}^{s_L} f_g(s | v) ds}$$

where for all $s'' > s'$ and all $v > v_{\min}$,

$$\frac{f_g(s'' | v) ds}{f_g(s' | v) ds} > \frac{f_g(s'' | v_{\min}) ds}{f_g(s' | v_{\min}) ds}$$

by the MLRP. This then implies that the above is true for all $s'' \in (s_H, s_{\max}]$ and $s' \in [s_{\min}, s_L)$ and thus $\Pr(\theta = g | s > s_H, v_{\min}) < \Pr(\theta = g | s < s_L, v_{\min})$. Note that this analysis could have been just as well carried out even if $s_H = s_{\max}$. In this case, we would have considered a trader whose equilibrium strategies for $s > s_L$ would have been not to trade. An argument identical to the above would establish that as $p_t \rightarrow v_{\min}$, this trader would enjoy a strict reputational gain by defecting and selling.

Finally, from our earlier arguments, it is clear that as $p_t \rightarrow v_{\min}$, expected trading profits become infinitesimal. Thus, traders whose equilibrium strategies require buying or not trading would prefer to deviate and sell, and thus this cannot be an equilibrium. ■

8.4 Proof of Proposition 7

Suppose there exists a non-perverse equilibrium in which a fund manager who observes $z_t = g$ always plays $a_t = s_t$.

Consider a fund manager with $z_t = g$ and $s_t = 0$ and suppose that the current price is p_t .
Let

$$\hat{v}_t^{s_t, z_t} = E[v | s_t, z_t, h_t].$$

It is easy to see that

$$\lim_{p_t \rightarrow 1} \hat{v}_t^{s_t, z_t} = 1 \quad \text{for all } s_t, z_t.$$

Hence, the expected benefit in terms of trading profit of playing $a_t = 0$ instead of $a_t = 1$ for a fund manager with $z_t = g$ and $s_t = 0$ goes to zero as price approaches 1:

$$\lim_{p_t \rightarrow 0} \Delta\pi = 0.$$

The expected reputation benefit/cost of playing $a_t = 0$ instead $a_t = 1$ for a fund manager with $z_t = g$ and $s_t = 0$ is

$$\begin{aligned} \Delta r &= \hat{v}_t^{0, g} ((\hat{\gamma}_t(a_t = 0, v = 1) - \hat{\gamma}_t(a_t = 1, v = 1))) \\ &\quad + (1 - \hat{v}_t^{0, g}) (\hat{\gamma}_t(a_t = 0, v = 0) - \hat{\gamma}_t(a_t = 1, v = 0)). \end{aligned}$$

Thus,

$$\lim_{p_t \rightarrow 1} \Delta r = \hat{\gamma}_t(a_t = 0, v = 1) - \hat{\gamma}_t(a_t = 1, v = 1).$$

As $p_t \rightarrow 1$, a fund manager with $z_t = g$ and $s_t = 0$ plays $a_t = 0$ only if

$$\hat{\gamma}_t(a_t = 0, v = 1) \geq \hat{\gamma}_t(a_t = 1, v = 1). \quad (1)$$

In a non-perverse equilibrium in which a fund manager with $z_t = g$ plays $a_t = s_t$, as $p_t \rightarrow 1$, beliefs have the following bounds (based on the assumption that all agents with $z_t = b$ play $a_t = 1$):

$$\begin{aligned} \hat{\gamma}^t(a_t = 1, v = 1) &\geq \Pr(\theta = g | \text{not } (z_t = g \text{ and } s_t = 0), v = 1) \\ \hat{\gamma}^t(a_t = 0, v = 1) &\leq \Pr(\theta = g | z_t = g, s_t = 0, v = 1). \end{aligned}$$

It is easy to see that

$$\begin{aligned} &\Pr(\theta = g | \text{not } (z_t = g \text{ and } s_t = 0), v = 1) \\ &> \Pr(\theta = g | z_t = b) = \frac{(1 - \rho)\gamma}{(1 - \rho)\gamma + \rho(1 - \gamma)}, \end{aligned}$$

and

$$\Pr(\theta = g | z_t = g, s_t = 0, v = 1) = \frac{(1 - \sigma_g)\rho\gamma}{(1 - \sigma_g)\rho\gamma + (1 - \sigma_b)(1 - \rho)(1 - \gamma)}.$$

Inequality (1) is satisfied only if

$$\frac{(1 - \sigma_g)\rho\gamma}{(1 - \sigma_g)\rho\gamma + (1 - \sigma_b)(1 - \rho)(1 - \gamma)} \geq \frac{(1 - \rho)\gamma}{(1 - \rho)\gamma + \rho(1 - \gamma)}.$$

If $\rho \rightarrow \frac{1}{2}$, the inequality reduces to

$$\frac{(1 - \sigma_g) \gamma}{(1 - \sigma_g) \gamma + (1 - \sigma_b)(1 - \gamma)} \geq \gamma,$$

which is false. ■

8.5 An Example of Mispricing

In this section we provide an example of how the reputational concerns of traders can lead to systematic mispricing of assets when the supply of the asset is not infinitely elastic. The model underlying this example is described in Section 5. We use the parameter values used earlier in the paper: $\gamma = 0.5$, $\beta = 0.4$, $\sigma_g = 1$ and $\sigma_b = 0.5$, but we now let $\delta = 0$ (rather than $\delta = 0.25$).

When the monopolistic seller sets the price, she faces three options: sell to the high type only ($s_t = 1$), always sell, or never sell. If she never sells, her expected payoff is simply p_t . If she sells to the high-type only, she sets the price p^a such that a manager with $s_t = 1$ is indifferent between buying or selling, which implies (note that the equilibrium is separating) the condition:

$$\beta (2\hat{v}_1 - p^a - \hat{v}_0) + (1 - \beta) (2\hat{v}_1 - 1) \left(\frac{\sigma_g}{\sigma} - \frac{1 - \sigma_g}{1 - \sigma} \right) \gamma = 0,$$

or

$$\begin{aligned} p^a &= 2\hat{v}_1 - \hat{v}_0 + \frac{1 - \beta}{\beta} (2\hat{v}_1 - 1) \left(\frac{\sigma_g}{\sigma} - \frac{1 - \sigma_g}{1 - \sigma} \right) \gamma \\ &= 12 \frac{p}{2p + 1} - \frac{p}{3 - 2p} - 1 \end{aligned}$$

If $p^a < \hat{v}_1$, the seller prefers not selling. This occurs when $p < 0.1535$. If the price computed above were greater than 1, the seller would be constrained to setting $p^a = 1$. This happens when $p > 0.27129$. The expected profit of a seller who sells to the high-type only is

$$\Pr(s_t = 1) p^a + \Pr(s_t = 0) \hat{v}_0 = (p\sigma + (1 - \sigma)(1 - p)) p^a + (1 - \sigma) p$$

Suppose instead that the seller sells to the low type as well. The price p_a is then given by the maximum price that a manager with $s_t = 0$ is willing to pay to buy in a pooling equilibrium:

$$\beta (\hat{v}_0 - p^a) + (1 - \beta) \left(\hat{v}_0 \left(1 - \frac{1 - \sigma_g}{1 - \sigma} \right) + (1 - \hat{v}_0) \left(1 - \frac{\sigma_g}{\sigma} \right) \right) \gamma = 0,$$

that is

$$\begin{aligned} p_{\text{pool}}^a &= \hat{v}_0 + \frac{1-\beta}{\beta} \left(\hat{v}_0 \left(1 - \frac{1-\sigma_g}{1-\sigma} \right) + (1-\hat{v}_0) \left(1 - \frac{\sigma_g}{\sigma} \right) \right) \gamma \\ &= 2 \frac{p}{3-2p} - \frac{1}{4} \end{aligned}$$

This price would be greater than 1 when $p > 0.8333$.

The seller prefers selling to the both types only if

$$(p\sigma + (1-\sigma)(1-p)) + (1-\sigma)p \leq p_{\text{pool}}^a,$$

which is true if $p \leq 0.78078$.

Note that it is easy to check that for $0.27129 < p < 0.78078$, when the market maker would charge $p^a = 1$, and wish to sell only to the high type, it is not beneficial for the low type to deviate from the equilibrium strategies and buy. Thus the equilibrium is indeed sincere in the range $0.1535 < p < 0.78078$.

The average price \bar{P}_t is now given by

$$\bar{P} = \begin{cases} p & \text{if } 0 \leq p < 0.1535 \\ (\sigma p + (1-\sigma)(1-p))p^a + (1-\sigma)p & \text{if } 0.1535 \leq p < 0.27129 \\ (\sigma p + (1-\sigma)(1-p)) + (1-\sigma)p & \text{if } 0.27129 < p < 0.78078 \\ p_{\text{pool}}^a & \text{if } 0.78078 < p < 0.8333 \\ 1 & \text{if } 0.8333 < p \leq 1 \end{cases}$$

This function is plotted in Figure 3. The thick line represents \bar{P}_t while the thin line represents the true expected value p_t . Notice that there is overpricing at all prices, except when p is so low that there is a cascade in which the manager never buys.

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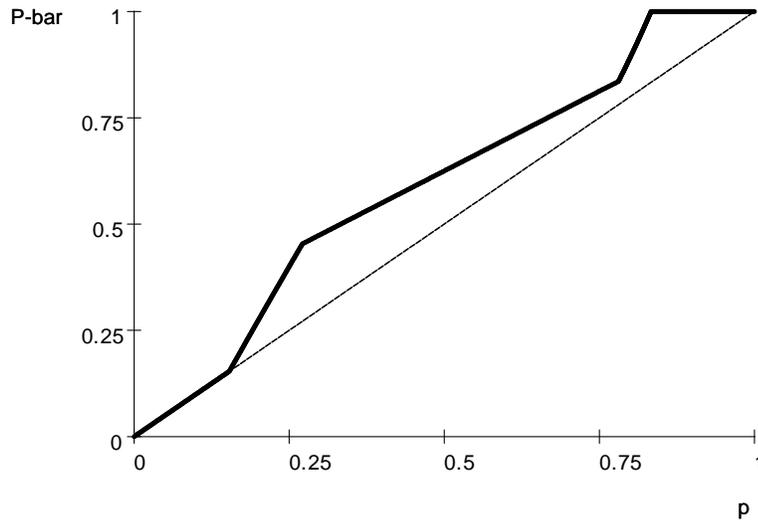


Figure 3: Reputational Mispricing

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