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THROUGH CHEAP TALK:
AN ARGUMENT IN FAVOUR OF
PRICE REGULATIONS**

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ABSTRACT

Eliciting Demand Information through Cheap Talk: An Argument in Favour of Price Regulations

A firm must decide whether to launch a new product. A launch implies considerable fixed costs, so the firm would like to assess downstream demand before it decides. We study under which conditions a potential buyer would be willing to reveal his willingness to pay under different pricing regimes. We show that the firm's welfare - as well as consumers' - may be higher with a commitment to linear pricing than when pricing is unrestricted. That is, if informational asymmetries are significant, price regulations such as the Robinson-Patman Act may be endorsed by all parties.

JEL Classification: D82, L11 and L42

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1 Introduction

Whether suppliers' possibility to discriminate between buyers should be regulated by law has been subject to a long-lasting debate among economists. The traditional view has been that if suppliers are allowed to price discriminate, they will favor dominant firms (e.g. chain stores) at the expense of small merchants that have little bargaining power. Such a development was considered undesirable from both an efficiency and equity perspective, and precipitated the adoption of the Robinson-Patman Act in 1936, section 2 of which forbids price differentiation when it has the potential to substantially lessen competition.^{1,2}

Bork (1978), among others, argues that the reasoning behind the Act was flawed: whole categories of buyers may not be served at all unless discrimination is allowed, which may reverse both the efficiency and equity rationale for regulation. That is, if suppliers are forced to use linear pricing, they may (optimally) set prices so high that low-elasticity buyers are completely left out of the market. Consequently, in intermediate markets non-discrimination rules may have a serious effect on downstream competition.³ Today the prevailing view among economists seems to be that, although the welfare effect of price discrimination in general is ambiguous (Schmalensee 1981, Varian 1985), non-discrimination rules probably do more harm than good.⁴

In this paper we bring forth a new argument in support of price regulations in an incomplete-information environment. We show that a uniform pricing rule, which guarantees all (active) buyers a strict surplus, may enable information sharing between buyer and seller. Such communication increases welfare in two ways: it increases the probability of (efficient) production in instances where demand is high, and reduces the probability of (wasteful) production when demand is low. Moreover, the welfare gain may not accrue only to buyers, which means that a pricing restriction may also be preferred by the seller. That is, a price regulation, properly enforced by the judicial system, constitutes a commitment device that sellers may be unable to achieve on their own.

Most closely related to the current paper is Farrell and Gibbons (1995). The

¹For example, in the case of linear demand and constant returns to scale, it has been shown that, if all markets (customers) are served under linear pricing, allowing price discrimination strictly reduces welfare (Schmalensee 1981).

²Similar non-discrimination regulations exist in other countries, see, e.g., Frazer (1988).

³Other arguments raised against the Robinson-Patman Act are that it facilitates collusion, discourages entry, induces artificial product differentiation, and moreover, that enforcement of the Act is very costly (see Martin 1988).

⁴However, Katz (1987) points out that buyers in intermediate good markets often can integrate backwards, i.e., supply the input themselves. When this is the case, Katz shows that price discrimination may lead to higher input prices for *all* buyers. However, see O'Brien and Shaffer (1994) for a result to the opposite effect.

authors consider a producer's problem of eliciting investment-specific information from a buyer. They show that reducing the producer's ex post bargaining power may enhance efficiency as the buyer's incentive to reveal his private information is increased. The authors also show that the gain in communication may outweigh the loss from the increased hold-up problem.

The remainder of the paper is organized as follows. Section 2 sets up the model. Section 3 and 4 derive the firm's profit under a linear pricing restriction and non-regulated pricing, respectively, and section 5 compares the two regimes. Section 6 looks at buyer welfare, section 7 relates the current model to Crawford and Sobel's (1982) cheap-talk model, and section 8 concludes. Appendices A and B contain some proofs and numerical results.

2 A Simple Model

A firm has the opportunity to produce a new good. There is no other firm that can do this, so if the firm produces it becomes a monopolist. A production decision implies a fixed start-up cost F , which is unknown at the outset. There is also a constant marginal cost of production, c , which is normalized to zero. The fixed cost may stem from setting up new machinery or infrastructure, training new staff, etc., and is sunk once incurred. There is a single buyer (or buyer representative). The buyer's utility function is $vq - \frac{1}{2}q^2 - T$, where v is the buyer's type, q is the quantity bought, and T is the total transfer paid to the firm.⁵ Note that with fixed unit price (linear pricing), demand is linearly decreasing in price, $q(p) = v - p$, as long as demand is positive.

The game proceeds as follows. In the first, "constitutional" stage of the game, the supplier chooses whether or not to commit to linear pricing. This may be thought of as a lobbying process where the firm, possibly by spending resources, may convince the legislator to prohibit price discrimination. The buyer's and seller's types (v and F) are then realized. The buyer's type is private information to the buyer, such that $v \in \{v_L, v_H\}$ and $v_H > v_L > 0$. The common prior is that $v = v_H$ with probability μ and $v = v_L$ with probability $1 - \mu$. In turn, the fixed cost is private information to the firm. The common prior distribution is $G(F)$, where G is differentiable and has density $g(F) > 0$ for all $F \in [0, \bar{F}]$, \bar{F} finite. In what follows we shall often consider the uniform case $g(F) = 1/\bar{F}$. We assume that $\bar{F} \geq \frac{v_H^2}{2}$, which implies that the cost density

⁵The quadratic utility function is chosen for analytical simplicity, but we expect our qualitative results to hold for any utility function that exhibits strictly decreasing marginal utility. As long as this holds, linear pricing leaves the consumer with a positive surplus and gives the high type a certain incentive to reveal his type in order to increase the probability of production.

is strictly positive over the entire profit range, which simplifies the exposition.

The buyer then sends a (possibly uninformative) message “Low” or “High”, meaning $v = v_L$ and $v = v_H$, respectively, to the firm. Messages are cheap talk. Given the message and the observed cost, the firm then decides whether to produce or not, and what price or price-quantity bundles to offer. Finally the buyer makes his consumption decision, and payoffs are realized.

To reiterate, the main purpose of the paper is to see whether it may be optimal for the firm to restrict its price setting ex ante, i.e., at the constitutional stage. For brevity we only compare two pricing regimes, linear pricing and unrestricted (second-degree) price discrimination. This is sufficient to illustrate the firm’s trade-off between improved ex ante communication and smaller ex post surplus.

3 Linear Pricing

Consider the situation where the seller commits to use linear prices. We first have to make sure that both buyer types have an incentive to report truthfully; if either type preferred to misreport the firm would gain no information relative to its prior, and would never choose the linear pricing regime.⁶ From the quadratic utility function we have that, conditional on a truthful message v_i , the firm optimally sets price $v_i/2$, sells quantity $v_i/2$, and makes gross profit $v_i^2/4$. This means in turn that, in a truthful equilibrium, the firm produces if and only if $F \leq v_i^2/4$.

The surplus for a type i buyer, if the firm believes he is of type j , is $\frac{1}{2} (\max(v_i - \frac{v_j}{2}, 0))^2$. Therefore, type i will reveal his type truthfully as long as

$$\frac{v_i^2}{8} G\left(\frac{v_i^2}{4}\right) \geq \frac{1}{2} \left(\max\left(v_i - \frac{v_j}{2}, 0\right)\right)^2 G\left(\frac{v_j^2}{4}\right), \quad i \neq j. \quad (1)$$

It should be noted that these incentive constraints are not automatically satisfied. In particular, if $2v_L > v_H$ and the gain in production probability is sufficiently large, the low type might actually prefer to exaggerate his valuation. However, when $G(\cdot)$ is uniformly distributed, (1) reduces to

$$v_i^4 \geq v_j^2 (2v_i - v_j)^2.$$

Taking square roots (both sides are positive) reduces the inequality to $(v_i - v_j)^2 \geq 0$. Hence, in the uniform case the incentive constraints always hold. Since the

⁶For simplicity we restrict attention to pure-strategy equilibria.

firm only produces if expected revenues are greater than the realized start-up cost, its ex ante expected profit is, given a truth-telling equilibrium,

$$\begin{aligned} & (1 - \mu) \int_0^{v_L^2/4} \left(\frac{v_L^2}{4} - F \right) dG(F) + \mu \int_0^{v_H^2/4} \left(\frac{v_H^2}{4} - F \right) dG(F) \\ &= (1 - \mu) \int_0^{v_L^2/4} G(F) dF + \mu \int_0^{v_H^2/4} G(F) dF, \end{aligned} \quad (2)$$

where the last step is derived through integration by parts.

4 Price Discrimination

When there are no pricing restrictions, the buyer has clearly no incentive to reveal his type since the firm would extract all surplus. Hence, the firm necessarily faces uncertainty about the buyer's type. There are now two possibilities: either it is optimal for the firm to offer a menu such that both buyer types purchase a positive quantity, or it optimally serves only the high type.

In the latter case it is clearly optimal to offer the high type his first-best quantity, v_H , and charge a lump-sum tariff $\frac{v_H^2}{2}$, thus extracting all surplus. The firm's ex ante expected revenue is in this case

$$\mu \frac{v_H^2}{2},$$

and, analogous to above, the ex ante expected profit is

$$\int_0^{\mu v_H^2/2} G(F) dF. \quad (3)$$

If instead both types are served, two different tariff-quantity menus are offered. Denote these (T_H, q_H) and (T_L, q_L) . The incentive constraint for a buyer type i reads

$$v_i q_i - \frac{q_i^2}{2} - T_i \geq v_i q_j - \frac{q_j^2}{2} - T_j, \quad i \neq j. \quad (4)$$

It is a standard exercise (see Appendix A) to derive the optimal quantities and the associated tariffs

$$T_L^* = \frac{(\mu(v_H - v_L) + v_L(1 - \mu))(v_L - \mu v_H)}{2(1 - \mu)^2}$$

and

$$T_H^* = \frac{v_H^2(1 + \mu) - 2v_H v_L(1 + \mu) + 2v_L^2}{2(1 - \mu)}.$$

This holds as long as $\mu \leq \frac{v_L}{v_H} \equiv \mu^\#$. It is easily shown that if $\mu > \mu^\#$, only the high type should be served. The firm's ex ante expected revenue is

$$\mu T_H^* + (1 - \mu)T_L^* = \frac{1}{2}v_L^2 + \frac{\mu(v_H - v_L)^2}{2(1 - \mu)}.$$

It follows that the firm's ex ante expected profit is

$$\int_0^{\frac{1}{2}v_L^2 + \frac{\mu(v_H - v_L)^2}{2(1 - \mu)}} G(F) dF. \quad (5)$$

5 Committing to Linear Pricing or not

Let us now investigate whether it can be optimal for the firm to commit to linear pricing. Consider first the case when $\mu > \mu^\#$, so that the firm, if price discriminating, only serves the high type. Define the ex ante expected difference in profits between price discrimination and linear pricing (using (2) and (3)) as

$$\Delta_H(\mu) \equiv \int_0^{\mu v_H^2/2} G(F) dF - (1 - \mu) \int_0^{v_L^2/4} G(F) dF - \mu \int_0^{v_H^2/4} G(F) dF. \quad (6)$$

Linear pricing is thus preferred when $\Delta_H(\mu) < 0$. Note first that $\Delta_H(1) > 0$; if the firm knows that it faces high demand it strictly prefers to price discriminate. Differentiating (6) twice with respect to μ gives

$$\Delta_H''(\mu) = \frac{v_H^4}{4} g\left(\mu \frac{v_H^2}{2}\right) > 0,$$

so that $\Delta_H(\mu)$ is a convex function. It follows that there exists a unique $\mu = \mu^{\min}$ where $\Delta_H(\mu)$ attains its minimum value. Intuitively, the more certain is the firm that it faces either high *or* low demand, the less there is to gain from communication. Whether μ^{\min} is larger than $\mu^\#$ and whether $\Delta_H(\mu^{\min}) < 0$ depend on parameters. In general, the higher is the probability that serving the low type is profitable, $\int_0^{v_L^2/4} G(F) dF$, the more valuable becomes information about the buyer's type.

The analogous argument holds for the case $\mu < \mu^\#$. Using (2) and (5), let

$$\Delta_{LH}(\mu) \equiv \int_0^{\frac{1}{2}v_L^2 + \frac{\mu(v_H - v_L)^2}{2(1 - \mu)}} G(F) dF - (1 - \mu) \int_0^{v_L^2/4} G(F) dF - \mu \int_0^{v_H^2/4} G(F) dF$$

denote the ex ante expected difference in profits between price discrimination and linear pricing when $\mu < \mu^\#$. The firm strictly prefers to price discriminate

if it knows that demand is low, i.e., $\Delta_{LH}(0) > 0$. Linear pricing becomes more profitable the larger is uncertainty over the buyer's type (i.e., $\Delta_{LH}(\mu)$ is also convex), and, as opposed to above, the lower is the probability that serving the low type is profitable. The intuition is that, given that the firm serves both types under price discrimination, there is a larger value in discovering the buyer's type if low demand is likely to be unprofitable.

Proposition 1. *For any distribution function G , there exist values of μ , v_L , and v_H such that (i) the firm's ex ante expected profit is strictly larger under linear pricing than under price discrimination, and (ii) the revelation constraints (1) hold. In particular, this happens for μ 's close to $\mu^\#$ and for v_L 's close to zero.*

Proof. First note that for v_L sufficiently close to zero, (1) holds for any distribution G and any $v_H > 0$. Let $\Delta^\#(v_L, v_H) \equiv \Delta_H(\mu^\#)$ ($\equiv \Delta_{LH}(\mu^\#)$). We thus have

$$\Delta^\#(v_L, v_H) = \int_0^{v_L v_H / 2} G(F) dF - \left(1 - \frac{v_L}{v_H}\right) \int_0^{v_L^2 / 4} G(F) dF - \frac{v_L}{v_H} \int_0^{v_H^2 / 4} G(F) dF.$$

Clearly, $\Delta^\#(0, v_H) = 0$. Therefore, to prove the claim, it suffices to show that $\partial \Delta^\#(v_L, v_H) / \partial v_L < 0$ for v_L 's close to zero. Differentiating yields

$$\begin{aligned} \frac{\partial \Delta^\#(v_L, v_H)}{\partial v_L} &= \frac{v_H}{2} G\left(\frac{v_L v_H}{2}\right) + \frac{1}{v_H} \int_0^{v_L^2 / 4} G(F) dF \\ &\quad - \frac{v_L}{2} \left(1 - \frac{v_L}{v_H}\right) G\left(\frac{v_L^2}{4}\right) - \frac{1}{v_H} \int_0^{v_H^2 / 4} G(F) dF. \end{aligned}$$

Evaluating this expression at $v_L = 0$ yields

$$\frac{\partial \Delta^\#(v_L, v_H)}{\partial v_L} \Big|_{v_L=0} = -\frac{1}{v_H} \int_0^{v_H^2 / 4} G(F) dF,$$

which is strictly negative. By continuity, it must be strictly negative also for some $v_L > 0$. \square

Numerically it is easy to show that linear pricing also is preferred for other parameter values than the ones considered in Proposition 1. Let $\mu'(v_L, v_H)$ denote the set of μ 's such that $\Delta_H(\mu) = 0$, and $\mu''(v_L, v_H)$ denote the set of μ 's such that $\Delta_{LH}(\mu) = 0$. Figure 1 illustrates the firm's trade-off between linear pricing and price discrimination with a uniform cost distribution.⁷

⁷The data used to generate Figure 1 is provided in Appendix B.

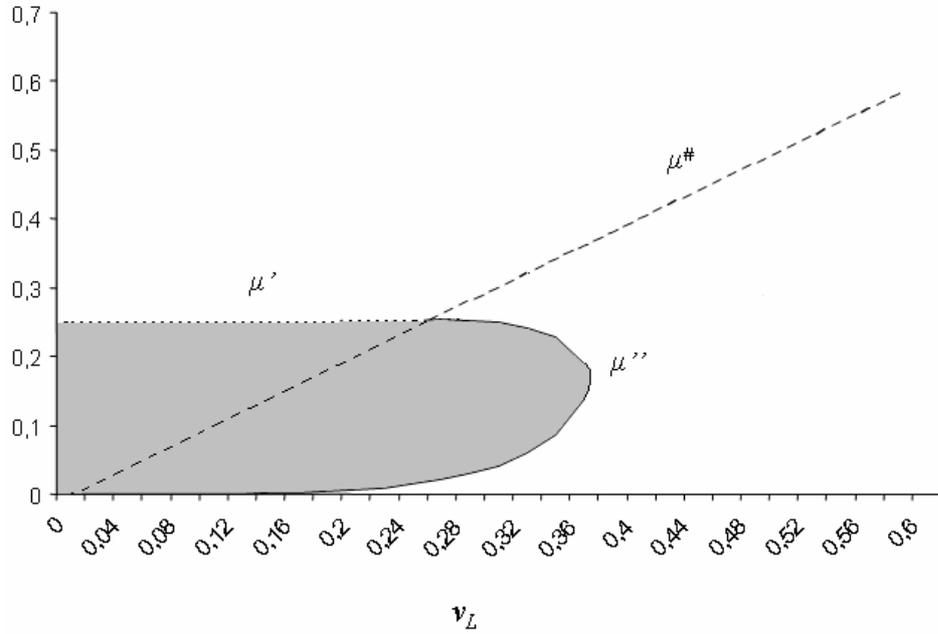


Figure 1. Price discrimination vs. linear pricing with information transmission ($G(F) = F$, $v_H = 1$). Linear pricing is preferred by the firm in the shaded area.

To summarize, transmission of demand information increases the firm's profit in two ways: it generates additional sales when demand is unexpectedly high (and production was unprofitable ex ante), and saves the firm from production costs when demand is unexpectedly low (and production was profitable ex ante). The drawback is that linear pricing leads to lower quantities consumed (due to decreasing marginal utility) and thus lower ex post profit. Hence, the firm prefers linear pricing when uncertainty over the buyer's type is high (μ is neither very high nor very low) and when the profitability of production varies a lot depending on the contingency ($v_H \gg v_L$).

6 Buyer's Welfare

Finally we should check whether linear pricing is also in the buyer's interest. The buyer's ex ante expected surplus from linear pricing is

$$(1 - \mu) \frac{v_L^2}{8} G\left(\frac{v_L^2}{4}\right) + \mu \frac{v_H^2}{8} G\left(\frac{v_H^2}{4}\right) > 0. \quad (7)$$

This should be compared to his expected surplus under price discrimination. First, if $\mu > \mu^\#$ the firm would only serve the high type, which implies that the buyer gets zero surplus regardless of type. Hence, in this case the buyer obviously prefers linear pricing. If $\mu \leq \mu^\#$, both types are served and the buyer's ex ante expected surplus is (the low type still gets zero surplus)

$$\mu \left(\frac{v_H^2}{2} - T_H^* \right) G\left(\frac{1}{2} v_L^2 + \frac{\mu (v_H - v_L)^2}{2(1 - \mu)} \right). \quad (8)$$

The difference (7) – (8) is ambiguous but seems to be positive in most cases when $\mu \leq \mu^\#$.⁸ Intuitively, the firm affords the high type a positive surplus under price discrimination only in order to keep him from switching to the low type's bundle. If this rent were too large (μ or v_H large), the firm would prefer to only serve the high type.

7 Relation to Crawford and Sobel (1982)

It may be instructive to contrast our results with the abstract cheap talk game in Crawford and Sobel (1982, CS). In CS there is one perfectly informed agent and one uninformed principal. The state space is the unit interval and the agent's and principal's preferred decision (as a function of the state) differ by a known amount, $b \neq 0$. The agent sends a costless message to the principal, whose subsequent decision affects the welfare of both parties. The authors show that the signalling equilibrium takes the form of a partitioning of the type space into intervals, and that full information revelation never occurs.

The situation we study in the current paper does not, as a matter of fact, apply to the CS setting. On the one hand, with a commitment to linear pricing the conflict of interest is not as severe as in CS, in the sense that full information revelation may be possible. With unrestricted pricing, on the other, *no* information transmission is possible. Although we only study the binary type case, this impossibility also holds for the continuous case (see Riley and Zeckhauser 1983 for a formal proof). To see the intuition, suppose that there were a signalling equilibrium with more than one kind of message, which partitioned the buyer type space into intervals. Clearly, the firm's optimal pricing scheme

⁸For example, with G linear or exponential we have not been able to find a case where the buyer prefers non-linear pricing.

would leave the lowest buyer type in each interval with zero surplus, so this type would strictly prefer to switch to a “lower” message. Hence, such a partitioning could not be an equilibrium. However, the CS conditions apply to the current setting (in particular, utility and profit functions are concave) which means that the signalling equilibrium *must* take the form of a partitioning into intervals. Hence, the only equilibrium is the non-informative one.

8 Conclusion

This paper puts forward a new argument in favor of pricing restrictions, namely that such restrictions, by giving buyers a larger share of the surplus from transactions, may enable the communication of demand information from buyers to sellers. We show that such a commitment may also be in the seller’s interest, which means that regulation Pareto dominates non-regulation. An alternative conclusion is that, in the absence of pricing restrictions, firms will not spend resources on preventing agents from reselling, since such agents provide a commitment against non-linear pricing.

Truthful revelation is possible only if buyers face a sufficient degree of uncertainty over the prospects of production. Since high-demand buyers suffer more from the absence of production, a separating signalling equilibrium may exist. Although the current model is simplistic, nothing suggests that the basic intuition would fail to apply to more complex environments, e.g., with several competing sellers. However, the beneficial effect of communication must then be traded off against other welfare costs of pricing restrictions, in particular, the potentially increased risk of price coordination. Such extensions are postponed for future research.

Appendix A

By standard arguments (see, e.g., Tirole 1988), in optimum the high type is exactly indifferent between his own bundle and the low type’s, the high type is served his first-best quantity, $q_H = v_H$, and the low type gets exactly zero surplus. Setting (4) to equality gives

$$T_H^* = \frac{1}{2}(v_H - q_L)^2 + T_L^*. \tag{A1}$$

The low type’s participation constraint reads

$$v_L q_L - \frac{q_L^2}{2} - T_L \geq 0,$$

which gives that, in optimum,

$$T_L^* = q_L(v_L - \frac{1}{2}q_L). \quad (\text{A2})$$

Using (A1) and (A2), the firm's unconstrained problem reads

$$\begin{aligned} & \underset{q_L}{\text{Max}} \quad \mu T_H^* + (1 - \mu)T_L^* \\ &= \underset{q_L}{\text{Max}} \quad \mu \left(\frac{1}{2}(v_H - q_L)^2 + q \left(v_L - \frac{1}{2}q_L \right) \right) + (1 - \mu) q_L \left(v_L - \frac{1}{2}q_L \right), \end{aligned}$$

which has the unique optimum

$$q_L^* = \frac{v_L - \mu v_H}{1 - \mu}. \quad (\text{A3})$$

Using (A3) gives the expression for T_L^* and T_H^* in the text.

It is easily shown that the condition $\mu \leq \mu^\#$ also is necessary for the firm to prefer serving both buyer types. Serving both types gives

$$\mu T_H^* + (1 - \mu)T_L^* = \frac{1}{2}\mu v_H^2 - \mu v_H q_L^* + q_L^* v_L - \frac{1}{2}(1 - \mu)q_L^{*2}, \quad (\text{A4})$$

whereas serving only the high type gives

$$\mu \frac{v_H^2}{2}. \quad (\text{A5})$$

The difference (A4) - (A5) reads

$$\frac{1}{2}q_L^* (q_L^* (\mu - 1) + 2(v_L - \mu v_H)).$$

This is positive if and only if

$$2(v_L - \mu v_H) - q_L^* (1 - \mu) \geq 0.$$

Using (A3) this reduces to

$$\mu \leq \frac{v_L}{v_H} = \mu^\#.$$

Appendix B

Setting $v_H = 1$, $\bar{F} = 1$, and $G(F) = F$ gives

$$\Delta_H(\mu) = \frac{1}{32} (4\mu^2 - \mu - v_L^4 + \mu v_L^4).$$

Setting $\Delta_H(\mu) = 0$ and solving for $\mu' = \{\mu \mid \Delta_H(\mu) = 0\}$ gives the (relevant) solution

$$\mu' = \frac{1}{8} \sqrt{16v_L^4 + (v_L^4 - 1)^2} - \frac{1}{8} v_L^4 + \frac{1}{8}.$$

Likewise,

$$\Delta_{LH}(\mu) = \frac{3v_L^4 + \mu^3(v_L^4 - 1) + \mu(8v_L^2 - 16v_L^3 + 3v_L^4 - 1) + \mu^2(16v_L^2 - 16v_L - 3v_L^4 + 6)}{32(1 - \mu)^2}.$$

The correspondence $\mu'' = \{\mu \mid \Delta_{LH}(\mu) = 0\}$ can be solved for numerically by inserting specific values for v_L first, and then solve for the two (relevant) roots of $\mu''(v_L)$. The numerical results are displayed in the table below.

v_L	v_L/v_H	μ'	μ''_1	μ''_2
0	0	0,25	0	0,17157
0,02	0,02	0,25	0	0,18108
0,04	0,04	0,25	0	0,1902
0,06	0,06	0,25	0	0,19892
0,08	0,08	0,25003	0,000129	0,20721
0,1	0,1	0,25007	0,000321	0,21505
0,12	0,12	0,25016	0,000684	0,2224
0,14	0,14	0,25029	0,001309	0,22921
0,16	0,16	0,25049	0,002313	0,23542
0,18	0,18	0,25078	0,003855	0,24094
0,2	0,2	0,25119	0,006137	0,24567
0,22	0,22	0,25174	0,009425	0,24945
0,24	0,24	0,25246	0,01407	0,25206
0,26	0,26	0,25337	0,020542	0,25325
0,28	0,28	0,2545	0,029496	0,25252
0,3	0,3	0,25589	0,04191	0,24917
0,32	0,32	0,25756	0,059427	0,24184
0,34	0,34	0,25953	0,085588	0,22729
0,36	0,36	0,26184	0,13715	0,18914
0,38	0,38	0,2645		
0,4	0,4	0,26752		
0,42	0,42	0,27093		
0,44	0,44	0,27474		
0,46	0,46	0,27894		
0,48	0,48	0,28353		
0,5	0,5	0,28853		
0,52	0,52	0,29391		
0,54	0,54	0,3		
0,56	0,56	0,30581		
0,58	0,58	0,3123		
0,6	0,6	0,31913		

Note: μ''_1 and μ''_2 meet approximately at $v_L = 0,3625$.

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