DISCUSSION PAPER SERIES

No. 5301

MAKING A DIFFERENCE

Patrick Francois

PUBLIC POLICY

Centre for Economic Policy Research

www.cepr.org

Available online at: **www.cepr.org/pubs/dps/DP5301.asp**

ISSN 0265-8003

MAKING A DIFFERENCE

Patrick Francois, University of British Columbia, CentER, Tilburg University, CIAR, CMPO and CEPR

Discussion Paper No. 5301
October 2005

Centre for Economic Policy Research
90–98 Goswell Rd, London EC1V 7RR, UK
Tel: (44 20) 7878 2900, Fax: (44 20) 7878 2999
Email: cepr@cepr.org, Website: www.cepr.org

CEPR Discussion Paper No. 5301

October 2005

ABSTRACT

Making A Difference*

Despite the potential for free-riding, workers motivated by `making a difference' to the mission or output of an establishment may donate labour to it. When the establishment uses performance related compensation (PRC), these labour donations closely resemble a standard private provision of public goods problem, and are not rational in large labour pools. Without PRC, however, the problem differs significantly from a standard private provision of public goods situation. Specifically, in equilibrium: there need not be free-riding, decisions are non-monotonic in valuations, and contribution incentives are significant even in large populations. When PRC is not used, the establishment tends to favour setting low wages which help to select a labour force driven by concern for the firm's output. Expected output can actually fall with the wage in this situation. For sufficiently high levels of risk aversion, performance related pay can yield less expected output than when compensation is output independent.

JEL Classification: H11, H41, H83 and J45
Keywords: incentive schemes, privately provided public goods, public sector employment and voluntarism

Patrick Francois
Department of Economics
University of British Columbia
#997-1873
East Mall
Vancouver, BC
CANADA, V6T 1Z1
Tel: (1 604) 822 4608
Fax:(1 604) 822 5915
Email: francois@interchange.ubc.ca

* This is a substantially revised version of the previously titled paper “Making a difference: Labor Donations in the Provision of Public Goods”. It has benefited from the comments of participants at the UBC theory workshop, seminars at Rice University, London School of Economics, the Canadian Public Economics Group in Toronto and the CIAR group on Institutions and Growth. Siwan Anderson, Mauricio Drelichman and Ashok Kotwal also provided valuable comments. Errors are mine alone.

Submitted 08 July 2005

1 Introduction

A commonly reported motivation for individuals who donate time and effort to worthy causes is a desire to "make a difference".[1] Donations of labor come in many forms: some are purely voluntary - the amount of labor volunteered in advanced economies is a large and growing phenomenon (44% of the US adult population, or 83.9 million individuals volunteered in 2001); with almost all doing so to not-for-profit institutions;[2] others involve individuals working for wages perceived to be below the market rate for the opportunity to advance causes in which they believe. Estimates of the amount of labor receiving reduced pay are less clear since the estimates require determining precise opportunity costs. But the existence of such donations is difficult to dispute. The non-profit sector is widely perceived as requiring workers to take pay cuts for the privilege of meaningful work, and a number of studies have documented the larger role a Public Service Motivation plays for public sector employees, and its depressing effect on wages.[3] A recent (2002) survey conducted by the Brookings Institution found that nearly half of all paid charity workers believe they could make more money elsewhere but take the work because they are driven by mission not money. The same survey of over 1200 nonprofit workers found that 97% feel they accomplish something worthwhile with their job, and are happy to take the lower pay in order to have a chance to "help people and make a difference". This was the case even though 81% of all workers agreed that it was easy to burn out in their work, 70% agreed they had too much work to do, 75% described their work

[1] There is a substantial non-economic literature on non-pecuniary motivations for employees, particularly in public and non-profit settings. A small sample are Mahoney, Huff and Huff (1992), Kirton (2001), Williams and Windebank (2001), Coyle-Shapiro (2003), Perry (2000), Naff and Crum (1999), Rhoades and McFarland (1999). The general emphasis of these authors, and most others in this field, is that, though pecuniary considerations play an inescapable part, when establishments produce output that is socially valued, altruistic and civic minded considerations are important motivators.

[2] This estimate is taken from the most recent biennial estimate commissioned by the Independent Sector, Independent Sector (2003). See Hodgkinson and Weitzman (1996) for a broad statistical analysis of the non-profit sector, Murnighan and Kim (1993) for a specific focus on non-economic factors motivating people to volunteer, and Menchik and Weisbrod (1987) for an early economic analysis of voluntarism. Segal and Weisbrod (2002) provide a recent investigation of volunteer contributions and their variation with observable individual characteristics.

[3] We survey later the evidence on wages for nonprofit firms, but the perception of a penalty is widespread. For example, a prominent Bay Area non-profit placement agency, "BANJO", states that "As a general rule of thumb, total nonprofit compensation tends to be 25% to 50% lower than similar positions in the private sector". They go on to claim that benefits, and especially bonuses, generally represent a large share of this difference, (see http://www.ynpn.org/banjo/ol_book/app2.htm). Note also that the numbers employed in this sector are large, recent estimates suggest it to be about 9.5% of the paid workforce. There is relatively large body of literature on Public Service Motivation - its prevalence and effect - in the public sector. The first study emphasizing this seems to be Perry and Wise (1990), and a number of authors have tested the implications of such a motivation for performance in the public sector, see for example Alonso and Lewis (2001) and the references therein for a discussion.

as frustrating, and 67% said their pay was low, see Light (2003).[4] This echoes findings of earlier studies administered more broadly, for example the US Quality of Employment Survey analyzed by Mirvis and Hackett (1983), which generally find nonprofit workers reporting higher levels of intrinsic motivation, feelings of accomplishment, and importance of work relative to money in their occupations.

To the extent that this evidence reflects donations of labor, such donations are almost surely not limited to the non-profit sector; for-profit firms may also receive labor donations when involved in activities perceived to be in the social interest (for example pro bono work in law agencies) and a literature has argued for such motivations playing a critical role in working for the public sector; Perry and Wise (1990), Le Grand (1997), Francois (2000), and recently, Besley and Ghatak (2003).

Labor donations are readily understandable if they arise as a warm-glow, in the sense used by Cornes and Sandler (1986) and Andreoni (1990), or for personal investment reasons as in Menchik and Weisbrod (1987). With warm glow giving, the act of donation itself increases the donors utility, independently of the outcome or its effect. However a reported desire to "make a difference" is a distinct motivation from warm glow giving, as it is outcome, not action, oriented. Individuals with this motivation care about the effects their efforts have in bringing about desired social change; a motivation which many like-minded people may share. To the extent that many people share this concern, the benefit generated can have a public good aspect. This contrasts with warm glow benefits which accrue directly to individuals performing the actions themselves.

In economies with numerous workers who may share similar societal goals, outcome oriented giving has the potential to lead to severe free-riding problems. If a single worker does not take an opening in an organization that is widely perceived to affect positive social change, there are potentially many others who will, and the good or service of concern will be provided nonetheless. The problem can be conceived of as a standard private provision of public goods problem, and the insights obtained from analysis of such problems are clear. Equilibria of standard private provision of public goods problems imply: 1) there is free-riding in equilibrium - each individual donates less (often) than they would if there were no others; 2) individuals with high valuations donate more (often) than the low; 3) the extent of free-riding is increasing in population size.

[4] It should be noted however, that econometric studies attempting to estimate this nonprofit wage penalty do not regularly confirm the existence of labor donations. We discuss this literature, and the implications of the present paper for it, after the main results.

In a multi-agent economy, workers wanting to “make a difference” will only be motivated to donate labor to a worthwhile cause if they understand that, were they to withdraw their donation, the cause would be adversely affected. The first part of this paper shows that the unique equilibrium of the labor donation game in such an economy involves less than full provision of the good. The free-riding, and induced expectation of less than full provision, serves an important motivating function for individuals who value the good to donate their effort. Without the free-riding, i.e., if all individuals were ready to donate upto their own personal valuations, any given individual would realize that, were he not to donate, someone else would, but since the benefits are shared by all, this would make it rational not to donate effort. In the limit then, as the potential pool of individuals who value the cause gets large, each individual’s incentive to free-ride also becomes large, precisely as in a standard private provision of public goods problem.

However, the fact that, in reality, individuals do donate effort out of a purported desire to make a difference, even in large economies, suggests that the standard private provision of public goods perspective may be misleading. It is argued here that this is because labor donations are fundamentally different from standard donations, and thus different from the private provision of public goods problem. The main difference is that donations of labor may be subject to moral hazard. When labor effort is not readily supervised or directly contracted, filling a position need not correspond with performing the tasks required of that position. Thus labor donations differ from standard donations, and from the private provision of public goods problem, because the act of claiming to donate labor - that is, filling the job - is seperate from actually doing so, and taking such a position precludes donations from somebody else.

A concrete example of this is the filling of the department head’s position. It is not possible to fully specify an output contingent contract that could correctly align incentives of the university or department with the monetary rewards of the head, moreover it is not possible for the dean (or department members) to fully supervise the activities of the head and ensure these are consistent with the university (or department’s) goals. In most departments, few individuals actively seek the position of department head but instead perform it (often somewhat reluctantly) out of a concern for the welfare of the department. Though many are capable, most would prefer someone else do the task, that is, they would prefer to free-ride on the effort provision of another individual. However, this changes dramatically when individuals who are perceived to be motivated by personal concerns (and not those of the department) actively covet the head's position. In that case, individuals who were previously reluctant to fill administrative positions often step into these in order to avoid the damaging effects of a department head who is not motivated by the collective interest. The very fact that the department head's contract cannot ensure the "right" actions are taken (the moral hazard problem) ensures that individuals who are reluctant to personally undertake the cost will do so (the free-riding problem is lessened).

The difference highlighted here thus arises when firms do not use performance related compensation (PRC) and makes sense of the often reported desire to "make a difference". Firms may not wish to use PRC because the very potential of workers shirking serves to induce participation of workers who would otherwise not donate their effort. Without PRC, individuals with little or no concern for the firm's output are able to take such positions and shirk. The participation of these shirkers induces individuals with a sincere concern for output to also apply for such positions. By doing so, they are "making a difference", and this is rational even in large groups, as it means doing a job better than it is otherwise likely to be done. This motivation can only exist when PRC is not used and can explain why organizations may be willing to tolerate the moral hazard induced when workers are not fully monitored in order to mitigate the free-riding problem inherent to organizations producing socially valued output.

Adding moral hazard to the standard private provision of public goods problem leads to drastically different results. It is shown that: 1) there need not be free-riding in equilibrium; 2) applications for positions are non-monotonic in individual valuations; 3) individuals do donate effort out of a desire to make a difference, and such donations are rational even in large economies.

In the absence of performance related compensation, it is demonstrated that output may not respond positively to higher wages. Increases in wages induce increased participation from workers at both ends of the distribution (shirkers and non-shirkers) and depending on the population's distribution of valuations, can actually lower output. A simple example is constructed where, with a piece-wise uniform distribution, expected output is everywhere decreasing with wages. The example highlights a force which, although not always sufficient to lead expected output to fall with wages, generally dampens the output elasticity of wages, tending to lower overall compensation levels in public good producing firms where workers cannot be fully monitored.

Once again, the department head example makes concrete the forces at play. Most departments register only nominal salary increments for department heads, despite the fact that there is, in general, a reluctance to fill such positions. This problem would surely be ameliorated by offering significant salary increments to accompany the post. According to the present paper, the reason this does not occur is because it induces not only individuals with departmental motivations, but also individuals who are pecuniarily motivated to apply. Given the impossibility of specifying the head's tasks through contract, this can actually lead to a higher probability of the "wrong" candidate getting the job, thus lowering deparmental output.

The power of incentives created by leaving worker tasks uncontracted can actually be strong enough to offset the potential damage that arises from hiring a shirker. Specifically, it can be the case that the use of PRC which fully solves labor's moral hazard problem, can lower expected output relative to when labor is free to simply choose its own level of effort. The model can thus explain why public good producing establishments may eschew the use of performance related compensation even where it can be implemented perfectly and costlessly.

A final section applies the paper's findings to help understand two observations relating to social services workers which have been previously difficult to reconcile. The first is the widespread perception, common amongst workers in sectors providing social services, that working in such sectors requires suffering a wage loss. The second is the evidence from more formal studies that workers in non-profit firms, which are predominately found in such sectors, though earning lower unadjusted wages (which is consistent with that perception), do not seem to suffer lower hourly wages, and may actually receive wage premia.

The paper proceeds as follows. The next section briefly relates strands of the literature relevant for the current research. Section 2 sets up the model and solves for equilibria in the labor donations game both with PRC (Section 2.1) and without it (Section 2.2). Section 3 considers an example which compares optimal wages, output and profits both with and without PRC for a tractable, uniform distribution, case. This section also discusses the paper's main findings in light of current empirical and theoretical literature. Section 4 concludes.

1.1 Previous Literature

Besley and Ghatak (2003) also explore the implications of an employee's concern for outcomes on organizational design. In their framework, this "mission motivation" takes the form of impure altruism, not the pure altruism assumed here.[5] The individual thus only obtains the benefit when working in provision of the good. Treating the motivation in this way removes the free-riding problem so that it plays no role in their analysis. Here, however, the free-riding induced by outcome oriented motivation is a central concern. In reality, worker non-pecuniary motivations are likely to combine both pure and impurely altruistic components. Our exclusive focus on the pure altruism thus complements their exclusively impure altruism focus.

A literature on non-profit firms has argued that, because these do not have a residual claimant, donations are provided to them when they would not be provided to for-profit firms; see for example, Hansmann (1980), Rose-Ackerman (1996), Francois (2003), and Grout and Yong (2003). This happens because for-profit firms are unable to credibly ensure that donated effort will be utilized for intended output, and not merely to enhance profit. This problem of firm commitment is assumed away in the present paper (as in Besley and Ghatak (2003)) so that the nature of the firm's ownership plays no role. Instead, the focus here is on the free-rider problem created even when establishments can credibly commit to convert donated effort to intended causes.[6]

The paper closest to the present one is Engers and Gans (1998) who also examine incentives to provide effort when concern for the output produced is a primary motivation. That paper provides an efficiency rationale for why referees may not be paid. Specifically, upon receipt of a paper to referee, if the accompanying payment for the task is large enough, the referee correctly anticipates that, were he to decline the refereeing assignment, the next person asked would be likely to accept it. If the accompanying payment is low, however, then chances are high that the next referee would not accept the task. In that case, the referee motivated by professional concern accepts the assignment. The main constrast with the present paper is that the free-riding which is central to the private provision of public goods problem is circumvented by the direct targetting that occurs through the editorial process. The participation problem of a referee differs from that of a worker ordinarily deciding on a labor donation because the editor of a journal is able to directly solicit the efforts of the referee, and this is done sequentially. To see this, note that, in Engers and Gans (1998) the arrival of a paper to referee from a non- (or low) paying journal, strictly lowers the referee's utility. A referee would never volunteer to be put into the position of having to decide on whether to accept

[5] Rose-Ackerman (1996) defines pure altruism as altruistic concern which is independent of the provider's identity. An impurely altruistic individual, in contrast, only benefits from the consequences of her own efforts.

[6] That literature is also related to the issue of monetary donations for non-profit firms as explored, for example, by Bilodeau and Slivinski (1998).

an assignment or not. Thus, part of the free-riding problem inherent to the situation is solved by the editor's direct soliciation. This suggests their structure may be of limited applicability to the problem of labor donations in general. Firms are rarely able to directly solicit potential workers, instead, a notice of vacancy is placed with conditions advertised, applicants forward their services, and the firm chooses the required number for the job.

Duncan (1999) is also concerned with donations of worker effort, and specifically on whether such donations will be perfectly crowded out by government provision. However, the model there effectively assumes the use of perfect performance related compensation, as there is no moral hazard in labor supply, and then it is demonstrated that donations of effort are conceptually similar to monetary donations. Here, in contrast, the important results arise when PRC is not used, and the paper further demonstrates reasons for why PRC may not be chosen.

The paper is also related to a relatively large literature on the use of merit pay in the public sector. Many non-economists have been sceptical about the effects of such schemes on employee performance, e.g., Frant (1996), Deckop and Cirka (2000) and IRS (2000). The present paper adds another element of caution, though for more standard economic reasons, and is the first, to the author's knowledge, to establish that the use of performance related compensation, even when costless, perfect and complete (i.e. covers all worker tasks) can actually lower output. Lewin (2003) and Dixit (2002) provided recent reviews of the literature on compensation schemes in the public sector. Prendergast (2003) is also concerned with explaining low powered incentives in public sector organizations. The government chooses how strongly to link bureaucratic compensation to public reports on service provision. He shows that governments may optimally choose weak incentives when bureaucrats are able to distort service in order to gain favourable reports. The results there depend critically on the inability to directly monitor and reward bureaucratic task compliance. Here, in contrast, we allow for perfect monitoring, or contracting, over employee's efforts, and it is shown that the complete absence of effort contingent contracting, or monitoring, can be preferred. We further discuss merit-pay in the public sector after the main results, together with estimates of non-profit pay differentials.

2 The Model

There is a single firm. The firm provides a public good, the amount of which is denoted g. The population comprises $N+1$ heterogeneous individuals varying by their valuations of the good, which are non-observable. Individual $i's$ strength of valuation is denoted by the parameter $\gamma_i \geq 0$, which is private information. For N of these individuals, the parameter γ is independently drawn from a common distribution, $F(\gamma)$, with support $[0, \infty)$.[7] Individuals with high γ_i value the public good relatively more, and those with $\gamma_i = 0$ do not value it at all.[8] The $N+1th$ individual is assumed to have a value of $\gamma_i = 0$. Thus, for any population, there exists at least one individual with zero valuation of the public good, the reason for this assumption will become clear subsequently. The distribution, $F(\cdot)$ is common knowledge and it is continuous. An individual of type $i's$ utility is given by:

$$u_i = \mu(w_i) - c(e_i) + \gamma_i v(g), \tag{1}$$

where w_i denotes $i's$ consumption of a numeraire good and e_i denotes $i's$ effort expended at work, and the functions μ, c and v are strictly increasing and weakly concave and $\mu(0) = c(0) = v(0) = 0$.[9]

We analyze a situation in which the firm requires a worker to participate in its production, and we assume that this worker is to be drawn from the pool of $N+1$ potential applicants whose preferences are characterized by (1). The firm's production function is:[10]

$$g = g(e), \tag{2}$$

where e is the amount of effort exerted by the worker in question. For simplicity, we shall assume

[7] It is not necessary to have an unbounded upper support as all results go through with a finite upper bound. An example developed further in the paper utilizes a finite upper support.

[8] We shall not dwell on the reasons for variation in γ, which seem to be an indisputable feature of reality. These could arise directly from preferences; some individuals may care more for features like environmental quality, public health care, quality of public schooling, etc. Or they may arise from differences in wealth that are orthogoanl to the concerns here; demand for such public goods may have positive income elasticity.

[9] The separability between the sub-components of utility greatly simplifies the analysis but is not strictly necessary. It is possible to obtain qualitatively similar results for more general specifications of preferences. As is standard where wages must serve to elicit effort, restrictions will need to ensure that complementarities between effort and income are not too large. The addition of public goods here simply requires a similar restriction to the complementarity between public goods and the other elements of the utility function.

[10] The nature of the firm, i.e., it's government, non-profit, or for-profit status, is not considered here. When the firm is unable to commit to output, for example if the firm controlled other inputs that could be adjusted in light of donated labor, an individual's desire to donate labor could be affected by its for-profit status. This has been a factor used previously to argue for the existence of non-profit firms, as in Francois (2003), but will not be exploited here, as it is assumed that firms do not have a commitment problem.

this production function takes a binary form - though qualitative results generalize to a more standard smooth production function, and to multiple employees. The production function is:

$$g(e) = \begin{cases} 0 \text{ for } e < \overline{e} \\ 1 \text{ for } e \geq \overline{e}. \end{cases}$$

The sequencing of events is as follows. The firm enters the labor market and advertises the position; i.e., wages and conditions (effort requirements). One of three possible outcomes ensues: (1) the firm is unsuccessful in attracting any applicants for the position, in that case, the firm's output is given by $\overline{g}$; (2) the firm fills the position, but the worker turns out to be a shirker who contributes $e < \overline{e}$, and output is thus 0; (3) the firm fills the position with a worker who contributes the correct level of effort, $e \geq \overline{e}$, and output equals 1.

The type of production we have in mind are servces that employees care about (such as childcare, education, healthcare, care for the aged) and where the person occupying a position within the organization can effect those services. NGOs in developing countries are also organizations that typically provide services about which employees care. In all such instances, being short an employee is problematic. It requires perhaps other resources and effort within the organization to be diverted to make up for the shortfall in the missing worker's expertise and effort, and generally implies that the organization is not optimally configured, and thus suffers an output fall.[11] But, the output decline is not likely to be as severe as that caused by hiring a worker who shirks. In case of a shirker, the organization is not able to accommodate the lost labor, or lack of effort, or perhaps more pernicious damage, that this individual can impose. We have in mind situations like that which would occur in the childcare sector. Being short an employee implies others may have to double up and/or work later, and it is possible that the firm's output (provision of quality child-care) may suffer. However, having a worker that is not concerned for the welfare of the children and is derelict in the performance of duty leads to a serious erosion in the quality of care, and perhaps even danger, so that output (which is weighted by quality here) declines. Another example is the filling of a position in an organization which allows considerable discretion regarding resource allocation, as in the department head example above. Not having a department head is problematic. Many important department functions cannot be performed or must be performed without skilled

[11]In reality, output declines will probably most often correspond with a fall in the quality of services provided. But since we use a unidimensional measure of output here, and abstract from the complications arising from contracting over quality, this simply corresponds to lower output in the model.

oversight - the dean or dean's representatives may chair department meetings, recruiting may be suspended. But the damage which can be done by a department head who is not sufficiently concerned with the welfare of the department is likely to be much worse; hiring poorly qualified friends or co-authors, driving away the department's productive researchers. We will thus assume throughout that failing to fill a position is less detrimental to output than filling a position with a worker who shirks, and naturally that filling a position with a non-shirker is better than not filling it at all. This requires that: $0 < \overline{g} < 1$.

All workers not working at the firm producing the commonly valued output receive a wage that just compensates for the disutitily of work, which is normalized to zero, i.e., for all other workers, $w_i - e_i = 0$. We shall also assume that there is a minimum wage that the firm can set, denoted $\underline{w} \ll \overline{e}$. We impose such a minimum because we are interested in labor donations of paid employees, in contrast with pure volunteers, as for example studied by Menchik and Weisbrod (1987). It also does not make sense to analyze performance related compensation (which amounts to promising payment upon performance) with promised payments that are zero, although nothing in the analysis logically excludes applying the results to a case of $\underline{w} \to 0$.

We proceed by analyzing two distinct cases. In the first, the firm is able to perfectly reward workers for effort supplied. This may be because the worker can be easily monitored, or because it is possible to organize effort contingent compensation through some other means. We shall call this the case of performance related compensation (PRC). The second is a situation where labor cannot be directly compensated for effort, so that a moral hazard problem arises; the non-PRC case. In this case, a non-performing worker can reap a pecuniary gain by taking the job.

2.1 Performance Related Compensation

Under PRC, the sequencing of moves is as follows: The firm calls a wage/effort pair denoted (w, e), where w is the total payment received in return for e units of effort. The wage effort pair is enforceable. All $N + 1$ individuals then simultaneously choose whether to apply for the job or not. If none apply then $g = \overline{g}$. If at least one applies, the firm simply chooses amongst them randomly, selecting one with equal probability from the pool of applicants. All the others remain in the alternative occupation, receiving $w_i - e_i = 0$. Since PRC is used, the successful applicant must contribute effort $\overline{e}$ and output equals 1.

Clearly, any contracted payment, $w : \mu(w) \geq c(\overline{e})$ would induce participation and ensure $g = 1$. With such payments, there is no free-riding problem, but there are also no labor donations, since workers receive more than necessary to compensate for the disutility of effort. Labor donations can only arise if the firm calls a contracted pair with $\mu(w) < c(\overline{e})$. Would anyone participate at such wages? The problem now has a private provision of public goods structure. Individuals with high valuations; $\gamma_i [v(1) - v(\overline{g})] > c(\overline{e}) - \mu(w)$, would strictly prefer to take such a position if they were the only ones able to fill the position, but with others who also value it, individuals can have incentives to free-ride.

These considerations lead to a unique symmetric Nash equilibrium which closely resembles a standard private provision of public goods problem:[12]

Proposition 1: *With PRC, for any payment/effort pair* $(w, \overline{e})$, *with* $\mu(w) < c(\overline{e})$, *there exists a unique symmetric Nash equilibrium of the labor donations game with a cut-off rule* γ^* *solving:*

$$[1 + (N-1)(1 - F(\gamma^*))] F(\gamma^*)^{N-1} \gamma^* [v(1) - v(\overline{g})] = c(\overline{e}) - \mu(w). \tag{3}$$

In equilibrium, all individuals for whom $\gamma_i \geq \gamma^*$ *apply for the job, all individuals for whom* $\gamma^i < \gamma^*$ *do not.*

All proofs are in the appendix.

The donating individual equates her personal cost to providing the effort - the right hand side of (3) $c(\overline{e}) - \mu(w)$, and her personal benefit to providing it, which is that the public good is produced for certain instead of with probability $1 - F(\gamma^*)^{N-1}$ (which occurs when at least one other population members exceeds the cut off). Note that, here, what induces an individual with γ above γ^* to apply is the probability that none of the $N-1$ other individuals will be a type γ above γ^*.

There is free-riding in equilibrium if $\gamma^* > \frac{c(\overline{e}) - \mu(w)}{v(1) - v(\overline{g})}$. Individuals, i, for whom $\gamma^* > \gamma_i > \frac{c(\overline{e}) - \mu(w)}{v(1) - v(\overline{g})}$, do not apply because they conjecture that there is a good enough chance of someone with higher valuation, i.e. $\gamma > \gamma^*$ working instead. Even though these individuals would apply if on their own, they optimally choose to risk provision of the good in a population with $N \geq 2$. In expectation, the probability of any one individual being a free-rider, and thus the expected proportion of free-riders, is given by $F(\gamma^*) - F\left(\frac{c(\overline{e}) - \mu(w)}{v(1) - v(\overline{g})}\right)$. As in standard private provision of public goods problems, as

[12] We focus throughout exclusively on symmetric equilibria.

N increases, the amount of free-riding increases:

Corrollary 1: *(i) For* $N = 1$, $\gamma^* = \frac{c(\overline{e}) - \mu(w)}{v(1) - v(\overline{g})}$. *(ii) The amount of free-riding is strictly increasing in* N*.*

The labor donations problem with contractible labor thus closely resembles a standard private provision of public goods problem: there is free-riding in equilibrium and the extent of free-riding increases with N. Moreover, vacancies are endemic, at least probabilistically, as these provide incentives for labor to donate.

The firm's choice variable, w, simply adjusts the threshold for participation. Increasing w monotonically increases the equilibrium level of provision upto $\overline{e}$. By choosing a high enough w, (limiting at $w : \mu(w) \geq c(\overline{e})$) participation is ensured, and output is produced with probability 1. Lower values of w save on labor costs, but leave open the possibility of non-provision, as the expected number of applicants must be strictly less than one in equilibrium to induce participation.[13]

2.2 Non-performance Related Compensation

Firms do not always use performance related compensation. One reason is simply technological. Contracting for labor effort requires some means of supervising and verifying effort contributions. Contracting on output, as in a piece-rate, is more likely to be feasible. But to do this, one needs the output to be relatively homogeneous and, in order to administer individual piece-rates, individual contributions should be readily discerned. Even where such contracting and/or supervision is technologically feasible, there are often significant costs to doing so.

In standard models, where there is no public good element to the good being produced, or no direct utility gain from provision of effort, workers are motivated to contribute effort only when firms create a pecuniary incentive to doing so; through PRC or some other means. Here we will see that this need not be the case. The set up we use here is similar to that developed by Macleod and Malcomson (1989), elaborated in Malcomson (1999). In this formulation, there is no observable

[13] Our sequencing of the labor market operation assumes the posting of a wage and effort requirement by the firm, with the subsequent participation of workers in response to this. Such a mechanism mirrors the actual functioning of labor markets but is not efficient (i.e. there is always an equilibrium probability of non-provision) and is generally dominated by alternative more complicated mechanisms which can ensure efficiency by eliciting signals from employees. One such mechanism is a type of second price auction where the firm asks potential employees to state the lowest wage at which they would be willing to work, and then commits to hiring the lowest wage worker at a wage equal to that stated by the second lowest. It will be seen that such a mechanism would not solve the problems created when moral hazard also accompanies the position (the next section), and also does not accurately describe job allocation mechanisms that we observe in reality. We thus use the benchmark described in Proposition 1 to compare with the moral hazard case to follow.

signal of effort that is readily available (at feasible cost) on which the establishment can condition remuneration.[14] Specifically, a hired worker is paid an agreed upon wage independent of the firm's performance, and without any possibility of the firm observing the worker's performance. Once employed, the worker simply chooses the effort level she contributes, and this choice has no pecuniary impact. Thus, once the individual has been hired, output either equals 0 or 1 depending on whether $e > \overline{e}$. In the framework developed by Macleod and Malcomson the possibility of repeated interaction serves to maintain incentive compatibility. Here, we will shut down this possibility by analyzing a once off interaction so that, by construction, workers without sufficient valuation of the public good will not contribute the required effort.

The equilibrium of the labor donations game that we now consider will be similar to that of the PRC version already analyzed. Now, however, an employed worker receives the wage independently of whether the correct effort is provided. Thus, two types of workers may potentially fill a position: motivated applicants, for whom $\gamma_i [v(1) - v(0)] \geq c(\overline{e})$, these individuals will provide the correct effort level. But individuals for whom $\gamma_i [v(1) - v(0)] < c(\overline{e})$ can also apply for the job and would not find it worthwhile to provide the correct effort. They would instead provide zero effort, and they are not attracted by the possibility of making a difference, but by the very lack of performance related compensation, which allows them to receive w without effort cost.

Let $n(w, N)$ denote the number of applicants that the single firm receives when offering payment of w, in a population of size N independently drawn from the identical distribution $F(\gamma)$. The variable n is, of course, endogenous and will be determined subsequently. Recall that the total number of potential applicants includes N individuals whose γs are independently drawn from $F(\gamma)$ and the one individual who has $\gamma = 0$ for certain. It is immediately clear that, in the absence of PRC, the individual for whom $\gamma = 0$ will apply at any positive wage, and since we restrict analysis to positive wages only, all N other individuals know that the applicant pool will always be non-empty.

[14] Of course, the present formulation of the problem does have a direct measure of output that could, in principle, be contractible - the level of service, g. But even when this is contractible, there are more fundamental reasons why such output contingent remuneration of labor may not be feasible. Firstly, the firm will usually control other inputs, so that the worker's effort is not as deterministic of service quality as modeled here, secondly, most goods require the contribution of more than one worker, which raises the problem of rewarding individual contributions, thirdly, the writing of such contracts may be extremely costly, or may induce sub-optimal effort allocations (due to multi-tasking concerns). The addition of any of these features would provide a more fundamental reason for worker effort to be non-contractible. For simplicity, none of these are directly modeled here. We take the non-contractibility as given since results would be unchanged no matter what the underlying source.

Consider first the application decision for motivated individuals who have a high valuation of the good, i.e., a $\gamma_i \geq \frac{c(\overline{e})}{[v(1)-v(0)]}$. If obtaining the job, such an individual would contribute effort to good provision since the benefit, exceeds the cost. These individuals apply if they expect that, by doing so, they affect the probability of provision sufficiently much to warrant the effort that they would fill in the position. If applying, since jobs are allocated randomly, the probability of obtaining the position is $\frac{1}{2+n(w,N-1)}$. That is, the applicant pool includes the $n(w, N-1)$ total applicants from the $N-1$ individuals with randomly drawn γ, it includes the individual applicant himself, and it includes the individual with $\gamma = 0$. If not applying, the probability of the good being produced is denoted $\sigma(w, N-1)$, which is also endogenous and determined subsequently.

The following expression compares, at wage w, the expected benefit to applying (the left hand side) with the expected net benefit to not applying (the right hand side):

$$\frac{1}{2+n(w,N-1)}\left(\mu(w)-c(\overline{e})+\gamma_i v(1)\right)+\left(1-\frac{1}{2+n(w,N-1)}\right)\begin{bmatrix}\sigma(w,N-1)\gamma_i v(1)\\ +(1-\sigma(w,N-1))\gamma_i v(0)\end{bmatrix}$$
$$\gtreqless \quad \sigma(w,N-1)\gamma_i v(1)+(1-\sigma(w,N-1))\gamma_i v(0).$$

Re-arranging this expression yields the high values of γ corresponding to individuals who both apply and donate effort to the firm:

$$\gamma_i \geq \frac{c(\overline{e})-\mu(w)}{(1-\sigma(w,N-1))[v(1)-v(0)]}. \tag{4}$$

The intuition for this condition is similar to that for condition (3). Individuals with high valuations are not willing to risk the good not being provided, and are thus willing to volunteer labor effort to ensure it is undertaken.

Now consider those with a low valuation of the good; $\gamma_i < \frac{c(\overline{e})}{[v(1)-v(0)]}$. If employed at the firm, such individuals would never contribute non-contracted effort. Moreover, if their decision to apply were to have no impact on expected output, they would always strictly prefer to take the job at any $w > 0$. The reason they do not all apply is that the level of output provision is affected by their taking the job. Their relative benefit to doing so is given by the two sides of the following expression:

$$\frac{1}{2+n(w,N-1)}\left(\mu(w)+\gamma_i v(0)\right)+\left(1-\frac{1}{2+n(w,N-1)}\right)\begin{bmatrix}\sigma(w,N-1)\gamma_i v(1)\\ +(1-\sigma(w,N-1)\gamma_i v(0))\end{bmatrix}$$
$$\gtreqless \quad \sigma(w,N-1)\gamma_i v(1)+(1-\sigma(w,N-1)\gamma_i v(0))$$

Individuals for whom the left hand side of the expression above is larger than the right, strictly prefer to apply for the job. Rearranging this yields:

$$\gamma_i \leq \frac{\mu(w)}{\sigma(w, N-1)\left[v(1)-v(0)\right]}. \tag{5}$$

Individuals with valuations of γ above the right hand side of (5) but below $\frac{c(\overline{e})}{[v(1)-v(0)]}$, would not apply for positions even though they would obtain a benefit to shirking.[15] The reason is that their valuations, though not high enough to overcome the moral hazard problem, are high enough for them to be better off if the good is provided by someone else. If σ is high enough, and the payment, w, small enough, then by taking the job and shirking, this individual is (with high probability) displacing a worker who would have provided effort and produced the good. Consequently, output, about which the person cares, would fall in expectation, and this fall in expected output is more costly than the benefit obtained by receiving the payment; $\mu(w)$.

We thus obtain two cutoffs for the application decision. From (4), one for those who apply for the position with the intention of truly volunteering the requisite effort; $\gamma_i \geq \frac{c(\overline{e})-\mu(w)}{(1-\sigma(w,N-1))[v(1)-v(0)]}$, and from (5) those lower valuation individuals attracted by the possibility of being paid for doing nothing; $\gamma_i \leq \frac{\mu(w)}{\sigma(w,N-1)[v(1)-v(0)]}$. It will subsequently be shown that, under maintained assumptions, these values are unique, so for now define these cutoffs respectively by

$$\gamma^H \equiv \frac{c(\overline{e})-\mu(w)}{(1-\sigma(w,N-1))\left[v(1)-v(0)\right]} \tag{6}$$

$$\gamma^L \equiv \frac{\mu(w)}{\sigma(w,N-1)\left[v(1)-v(0)\right]}. \tag{7}$$

Using these, we obtain an implicit expression for σ as follows:

$$\sigma(w, N-1) = \frac{(N-1)\int_{\gamma^H}^{\infty} f(\gamma)\,d\gamma}{1+(N-1)\left(\int_{\gamma^H}^{\infty} f(\gamma)\,d\gamma + \int_0^{\gamma^L} f(\gamma)\,d\gamma\right)} \tag{8}$$

$$\text{or equivalently} = \frac{(N-1)\left(1-F\left(\gamma^H\right)\right)}{1+(N-1)\left(1-F\left(\gamma^H\right)+F\left(\gamma^L\right)\right)}. \tag{9}$$

Intuitively, the probability of the good being produced, for given cutoffs γ^H and γ^L depends on the probability that a randomly chosen member of the applicant pool is a non-shirker, the first

[15] Such cases will not exist in all equilibria, but provided that equilibrium $\sigma(w, N-1)\,c(\overline{e}) \geq \mu(w)$, they do.

term.[16] Substituting for $\sigma(w, N-1)$ into (6) and (7), yields two expressions that implicitly define the equilibrium cutoffs, γ^H and γ^L :

$$\gamma^H = \frac{\left(c(\bar{e}) - \mu(w)\right)\left(\frac{1}{(N-1)} + 1 - F\left(\gamma^H\right) + F\left(\gamma^L\right)\right)}{\left[\frac{1}{(N-1)} + F\left(\gamma^L\right)\right]\left[v(1) - v(0)\right]}, \tag{10}$$

$$\gamma^L = \frac{\mu(w)\left(\frac{1}{(N-1)} + 1 - F\left(\gamma^H\right) + F\left(\gamma^L\right)\right)}{\left[1 - F\left(\gamma^H\right)\right]\left[v(1) - v(0)\right]}. \tag{11}$$

Uniqueness of these two cutoffs is not generally guaranteed. This is because there arises a complementarity between the actions of those who do not value the good highly enough to provide effort, i.e. the $\gamma_i < \frac{c(\bar{e})}{[v(1)-v(0)]}$, as follows. If most other applicants are true volunteers, that is, individuals who would provide the required effort if employed, then, by taking the job and shirking, an individual with low γ significantly lowers expected output. This is because, were he not to obtain the job, one of the committed others would likely have, and g would equal 1. But suppose there is a large increase in the number of other low γ individuals ($\gamma < \frac{c(\bar{e})}{[v(1)-v(0)]}$) applying, so that these individuals constitute the bulk of the applicant pool. If a given low γ individual applies and shirks now, expected output will not have been greatly affected; g would likely have equalled zero anyway because, with high probability, this worker is simply displacing another shirker from the position. Consequently, the possibility of complementarity in the application decisions of those with low valuations can lead to multiple cut-off levels. Though this multiplicity may be of some interest, it is not the focus here, and can be easily ruled out in the continuous distribution case by the following assumption which ensures that the density at all points is sufficiently "thin", that is:

Assumption: *The density* $f(\gamma)$ *is such that*:

$$\gamma f(\gamma) < 1, \text{ for all } \gamma. \tag{12}$$

The assumption effectively ensures that the direct effect of a higher cut off, γ^L, which is to move the margin to individuals with higher valuations of the good, is not outweighed by the indirect

[16] It is here that the assumption of there being at least one individual with $\gamma = 0$ plays a crucial simplifying role. Dropping this assumption would mean that there would always exist a positive probability of an empty applicant pool, were an individual not to apply. In that case, the existing term for $\sigma(w, N-1)$ would need to be multiplied by the additional term $\left[1 - \left(F\left(\gamma^H\right) - F\left(\gamma^L\right)\right)^{N-1}\right]$, which is the conditional probability of the applicant pool being non-empty. Though conceptually nothing would seem to change, this treatment greatly increases the problem's complexity.

effect, which is that inducing more participation by shirkers, the marginal individual's expectation of output falls. Under this assumption, a unique equilibrium outcome ensues:

Proposition 2: *For given $w : \mu(\underline{w}) \leq \mu(w) < c(\overline{e})$, there exists a unique symmetric Nash equilibrium to the labor donations game, without PRC, which is characterized by a pair of cut-offs $\gamma^H(w), \gamma^L(w)$, with $\gamma^H(w) \geq \frac{c(\overline{e})}{[v(1)-v(0)]} \geq \gamma^L(w)$. All $\gamma_i \geq \gamma^H(w)$ apply at wage w and contribute $\overline{e}$, if receiving the job. All $\gamma_i \leq \gamma^L(w)$ apply at wage w and contribute zero effort if receiving the job. All γ_i with $\gamma^L(w) < \gamma_i < \gamma^H(w)$ do not apply.*

The equilibrium conditions can be more easily understood using the distribution functions rather than the densities:

$$\frac{\frac{1}{(N-1)} + F\left(\gamma^L\right)}{\frac{1}{(N-1)} + 1 - F\left(\gamma^H\right) + F\left(\gamma^L\right)} \gamma^H \left[v(1) - v(0)\right] = c(\overline{e}) - \mu(w) \tag{13}$$

$$\frac{1 - F\left(\gamma^H\right)}{\frac{1}{(N-1)} + 1 - F\left(\gamma^H\right) + F\left(\gamma^L\right)} \gamma^L \left[v(1) - v(0)\right] = \mu(w). \tag{14}$$

Equation (13) is derived from the marginal non-shirker. The right hand side can be interpreted as the cost to obtaining the job, which is the disutility of the effort net of its monetary compensation, $c(\overline{e}) - \mu(w)$. This is equated to the left hand side which is the expected cost of not taking the position; i.e., with probability $\frac{\frac{1}{(N-1)}+F\left(\gamma^L\right)}{\frac{1}{(N-1)}+1-F(\gamma^H)+F(\gamma^L)}$ the good is not produced, and the lost output is valued at $\gamma^H[v(1) - v(0)]$. Similarly, condition (14) is derived from the marginal shirker. A shirker obtains $\mu(w)$ when taking a position, since no effort is expended and no output is produced; this is the right hand side. This is equated to the benefit of not taking the position, which is that, with probability $\frac{1-F\left(\gamma^H\right)}{\frac{1}{(N-1)}+1-F(\gamma^H)+F(\gamma^L)}$ output is produced and valued at $\gamma^L[v(1) - v(0)]$; the left hand side of (14).

Equilibrium actions are non-monotonic in valuations. Though the decision to shirk is monotonic, the decision to apply for work is not. Individuals with high valuations apply and donate labor if employed, individuals with low valuations apply and shirk if hired. Individuals that are in between do not apply. At all $w : \mu(w) < c(\overline{e})$ there are some labor donations, but there may also be free-riding. It is possible, however, for high enough values of the wage that though there remain labor donations, there is no free-riding in equilibrium. Specifically

Proposition 3: *If w satisfies:*

$$\frac{c(\bar{e})}{[v(1)-v(0)]} > \mu(w) > \left(1 - \int_0^{\frac{c(\bar{e})}{[v(1)-v(0)]}} f(\gamma)\, d\gamma\right) \frac{c(\bar{e})}{[v(1)-v(0)]}, \tag{15}$$

then, in equilibrium, labor is donated, but there is no free-riding.

Thus, another unusual feature of this equilibrium is that, for sufficiently high values of the wage, even though individuals would be strictly better off if someone else were to provide effort for the firm, nobody chooses to free ride. The reason free-riding disappears here is that the participation of individuals with low valuations, who will shirk, provides incentives for individuals with higher valuations to apply. With some small, but positive, probability a high γ applicant will be accepted. This probability approaches zero as N gets large. However, if accepted, there is a non-small probability that they will have displaced a shirker. Note that, unlike the selection probability, this probability is invariant with respect to N, and it ensures that, if selected, their application is worthwhile. If not selected for the job, they are no worse off, so they continue to apply independently of N.

Recall that free-riding had to occur under PRC, because it was the possibility of free-riding by others that induced an individual with high enough valuation to apply. Here, however, the inducement comes from the individuals who will take the job and shirk, i.e. it arises directly from the lack of PRC, so that free-riding need not occur in equilibrium.

Both Andreoni (1990) and Vicary (2000) have developed models where individuals' contributions to a public good need not go to zero as the population becomes large, but for entirely different reasons. In Andreoni (1990) the reason is that the good is not a pure public good. Individuals receive personal benefit from the act of participating which persists, and motivates contribution, even in large economies. Vicary's finding depends critically on public good levels being directly affected by consumption as well as individual donations - an example is driving a car (worsening the environment) while simultaneously contributing to Greenpeace. Here, in contrast, worker concern is of the pure public good form, without consumption complementarities and can explain donations even in large economies.

3 Wages

Upto now, wages in both the PRC and non-PRC cases have been taken as given. Here we briefly explore the wage setting decision under each type of compensation scheme.

Performance Related Compensation

As shown in condition (3), the participation cut-off in equilibrium is:

$$\left[1+(N-1)\left(1-F\left(\gamma^{*}\right)\right)\right] F\left(\gamma^{*}\right)^{N-1} \gamma^{*}\left[v(1)-v(\overline{g})\right]=c(\overline{e})-\mu(w).$$

The appendix demonstrates that the left hand side of this expression is monotonically increasing in γ^{*}, so that it follows immediately that for higher values of w, the expected value of output, $1-F\left(\gamma^{*}\right)^{N-1}$,will increase. The firm's optimal wage will thus depend critically on the form of the distribution function, which will determine the slope of the expected output function as a function of the wage, and the price the firm receives for output. For a smooth and concave function, the solution corresponds to the standard marginal condition. But nothing in this problem ensures such a well-behaved solution. In general, the form of the expected output function will depend on the γ distribution, allowing no general conclusions about optimal wages to be drawn.

Without PRC

When setting wages without performance related compensation, the firm faces the following tradeoff. The cost side is obvious - paying a higher wage costs more. The benefit to a higher wage is that the upper cut-off, γ^{H} falls (provided $\gamma^{H}>\frac{c(\overline{e})}{v(1)-v(0)}$) - there will be less free-riding because the amount of donation asked from individuals who value the good is lower. Mitigating this is the corresponding rise in γ^{L}; the higher wage induces more shirkers to apply for the position. The cut-off γ values are both affected by wages according to (10) and (11), so that the expected level of output, $\frac{1-F\left(\gamma^{H}\right)}{\frac{1}{(N-1)}+1-F\left(\gamma^{H}\right)+F\left(\gamma^{L}\right)}$, is effected by the wage through two channels. It turns out that the behavior of these cut-off γ values with variations in the wage is, in general, highly irregular, as they depend critically on the precise form of the distribution of the γs. Specifically, consider the marginal impact of a wage increase for a wage, w_{1}, with two corresponding cut-offs γ_{1}^{H} and γ_{1}^{L}. A marginal increase in the wage induces a relatively large increase in output when $f\left(\gamma_{1}^{H}\right) \gg f\left(\gamma_{1}^{L}\right)$. Intuitively, a small increase in the wage would induce a relatively large influx of good workers, and thus have relatively great impact on output, implying a convex region in the wage output function. The shape of the output function thus depends sensitively on the precise form of $F(\gamma)$ so that, once again, general results are not available.

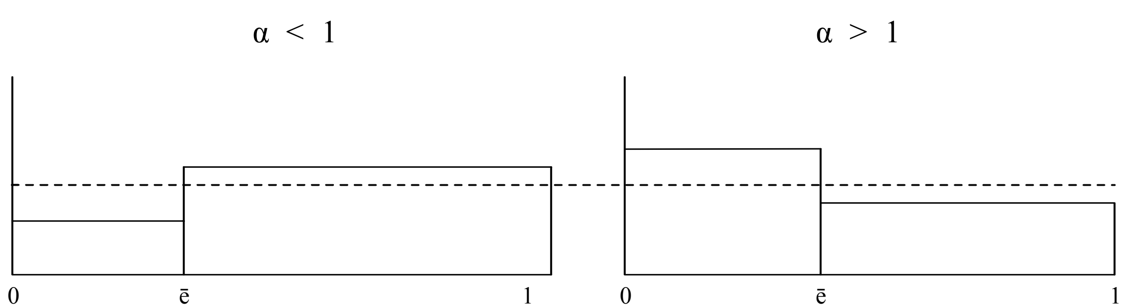
More interestingly, there is no reason why expected output should be increasing in the wage here. In general, this will depend on which margin increases by the greater amount. A simple illustration of this can be provided for the case of a piece-wise uniform distribution for a linear version of the model:

$$u_i = w_i - e_i + \gamma_i g$$

Suppose now that the γs are distributed over the $[0,1]$ support, with the distribution, $F(\gamma)$ being uniform over two parts which may vary in their densities. Specifically, the distribution is uniform upto $\overline{e}$ with the mass of the distribution below $\overline{e}$ denoted by α. The corresponding distribution function is:

$$F(\gamma) = \begin{cases} \alpha \frac{\gamma}{\overline{e}} \text{ for } \gamma < \overline{e} \\ \alpha + (1-\alpha)\left[\frac{\gamma - \overline{e}}{1-\overline{e}}\right] \text{ for } \gamma \geq \overline{e}. \end{cases}$$

If $\alpha = \overline{e}$, this yields the standard uniform distribution, $F(\gamma) = \gamma$ for all γ. Varying the parameter α simply varies the relative mass of the distribution below and above the point $\overline{e}$. As in the figure below, a value of $\alpha < \overline{e}$ implies a distribution where the density at values of γ below $\overline{e}$ is everywhere greater than that above. The converse holds for $\alpha > \overline{e}$.

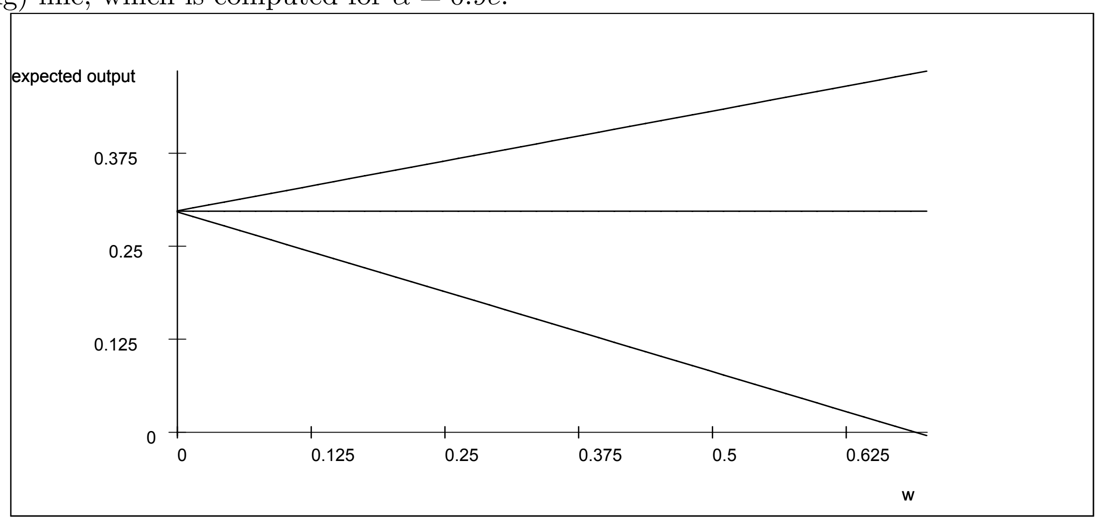


This yields equilibrium values of γ^L and γ^H solving:

$$\frac{\frac{1}{(N-1)}+\alpha\frac{\gamma^L}{\overline{e}}}{\frac{1}{(N-1)}+1-\left(\alpha+(1-\alpha)\left[\frac{\gamma^H-\overline{e}}{1-\overline{e}}\right]\right)+\alpha\frac{\gamma^L}{\overline{e}}}\gamma^H = \overline{e}-w \tag{16}$$

$$\frac{1-\left(\alpha+(1-\alpha)\left[\frac{\gamma^H-\overline{e}}{1-\overline{e}}\right]\right)}{\frac{1}{(N-1)}+1-\left(\alpha+(1-\alpha)\left[\frac{\gamma^H-\overline{e}}{1-\overline{e}}\right]\right)+\alpha\frac{\gamma^L}{\overline{e}}}\gamma^L = w \tag{17}$$

It is possible to numerically solve these two equations for γ^H and γ^L as a function of w, for $w \in [\underline{w}, \overline{e}]$. Using these solutions we obtain an expression for expected output $\frac{1-\left(\alpha+(1-\alpha)\left[\frac{\gamma^H-\overline{e}}{1-\overline{e}}\right]\right)}{\frac{1}{(N-1)}+1-\left(\alpha+(1-\alpha)\left[\frac{\gamma^H-\overline{e}}{1-\overline{e}}\right]\right)+\alpha\frac{\gamma^L}{\overline{e}}}$ as a function of the wage. The figure below sketches this relationship for three different values of α, under specific values, $N = 100$ and $\overline{e} = 0.7$. We assume that the lower bound on wages, $\underline{w}$, approaches zero. The horizontal line corresponds to the case of $\alpha = \overline{e}$. In that case, expected output is constant independent of the wage. This case corresponds to the uniform distribution under which $f(\gamma)$ is a constant for all γ. Consequently, increases in the wage lead to proportionate increases in both γ^H and γ^L leaving expected output unaffected. The solid line (downward sloping) corresponds to the case of $\alpha = 1.1\overline{e}$. In this case, the density below $\overline{e}$ everywhere exceeds that above. Consequently, increases in the wage from $\underline{w}$ lead to monotonically declining output. Increases in the wage in this case always induce more non-shirkers than shirkers to apply. The converse occurs when the density below $\overline{e}$ is less than that above. This is the case drawn with the dashed (upward sloping) line, which is computed for $\alpha = 0.9\overline{e}$.



The firm's optimal wage will thus depend critically on the precise form of the distribution. In the piece-wise uniform cases analyzed above, the decision depends critically on the mass of the distribution below and above $\overline{e}$. Clearly if the mass of the distribution below $\overline{e}$ is high, that is, if it is positively skewed, the firm will optimally choose $w = \underline{w}$. This will also be the case if the distribution is uniform. However, in the converse case, the tradeoff depends on the degree to which the firm trades off higher wages and output. The problem is then similar to that analyzed for the performance related case where, once again, the precise wage paid will depend on the form of the distribution.

3.1 Does Using Performance Related Pay Raise Output?

We now consider the firm's choice of whether to employ performance related compensation. We shall assume that there are no costs to doing so and ask simply whether introducing it will serve to raise output for given values of the wage. The result found here is that PRC will not generally raise output, and especially not when (1) wages are low - i.e., when organizations ask for large donations from labor, and (2) when workers are highly risk averse regarding the level of service provision.

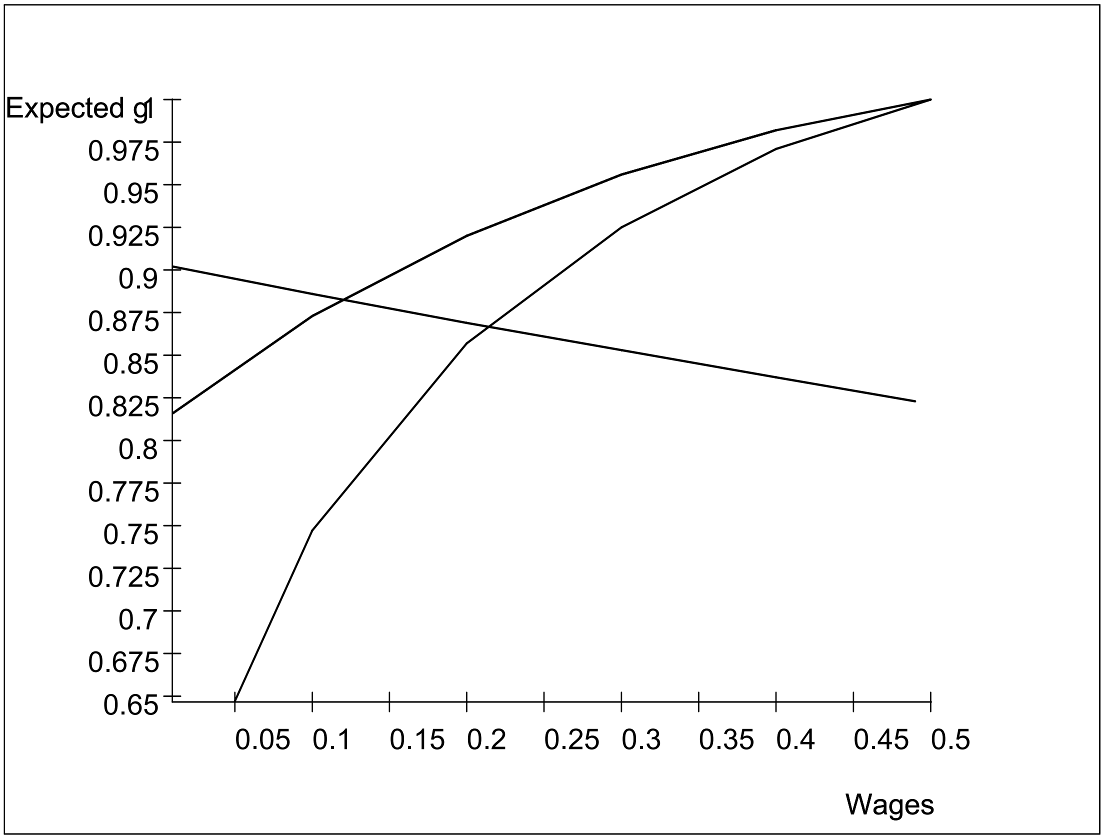
Once again, the uniform distribution example illustrates simply the forces at work. Consider the following representation of preferences similar to the above but where we now allow for some risk aversion: $\mu(w) = w, c(e) = e$ but $v(g) = \frac{g^{1-\sigma}}{\theta}$:

$$u_i = w_i - e_i + \gamma_i \frac{g^{1-\sigma}}{\theta},$$

where increasing the parameter σ increases an individual's degree of risk aversion regarding the level of the public good and θ is a parameter to vary the weighting of the public good.[17]

As before, we have $g(\overline{e}) = 1$ for the correct effort, $g(0) = 0$ with a shirker and we set $\overline{g} = .5$ when the position is left vacant. With a $\sigma = 0.9$, and $\theta = 0.1$ for example, this yields, $v(1) = 10, v(0) = 0$ and $v(\overline{g}) = 9.33$. For an $N = 20$ the equilibrium outcome, in the case of PRC, is determined by the solution to (3) and yields a positive relationship between expected g and the wage as depicted by the upward sloping solid line in the figure below. In these simulations we set $\overline{e} = 0.5$ so that $w < 0.5$ for labor donations to be possible; $w = .5$ fully compensates for the disutility of effort. The dashed upward sloping schedule is the equilibrium expected service level for $\sigma = .93$

[17] Allowing for risk aversion over income has no qualitative effect on the results that will be shown here so that we persist with the linear version for simplicity.

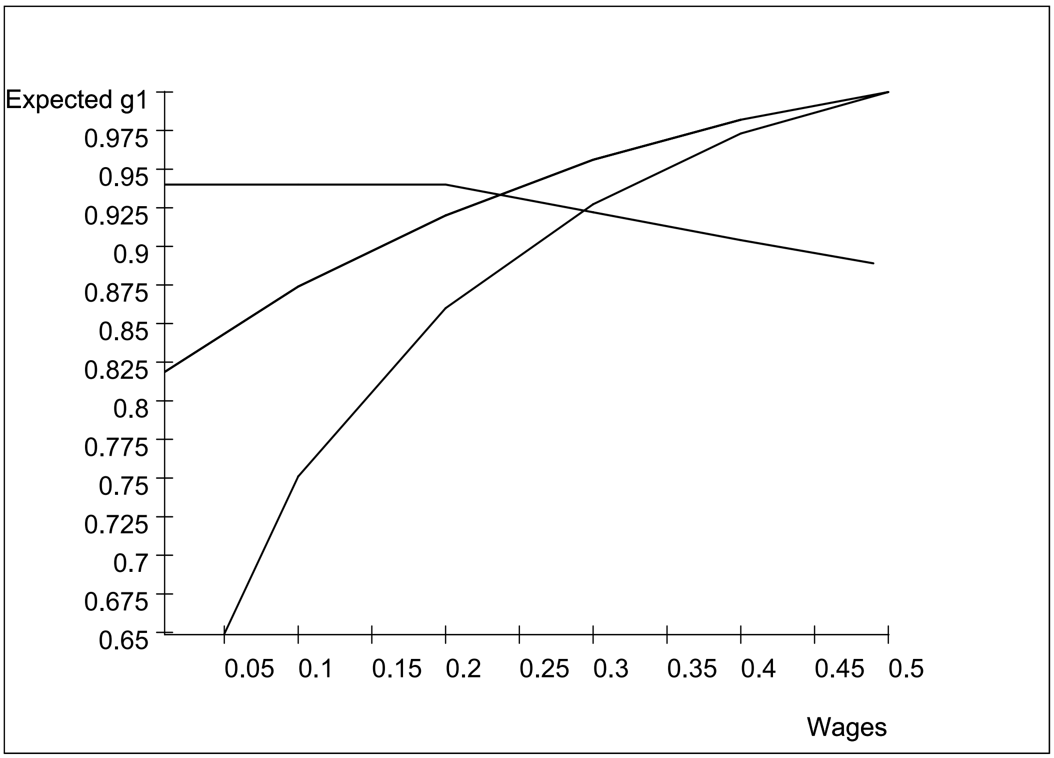


$N = 20, \sigma = .9$ -solid, $\sigma = .93$ -dash

Under PRC, increasing wages, which lowers the size of a worker's donation, increases both participation and expected output. When the firm does not use PRC, however, in this example, since all non-shirkers participate at low wages, increasing the wage only induces greater numbers of shirkers, thus lowering expected output - as shown in the downward sloping schedule. For either degree of risk aversion, wages in excess of 0.2 yield higher expected output under PRC when $N = 20$, but for wages below 0.1 expected output will be higher if the firm does not use PRC no matter what the level of risk aversion.

The figure makes clear two imporant effects. Firstly, the crossover wage, i.e. the wage at which expected output is higher with PRC than without, is increasing in the degree of aversion to risk. This highlights the role of risk-aversion in generating the higher output without PRC. Risk averse individuals suffer large utility declines when a shirker takes the position and $g = 0$, and relatively small ones when the position remains open, $g = \overline{g}$. Consequently, the lack of PRC provides strong motivation for labor donations, whereas with PRC the only cost can be the position will remain unfilled, so the motivation to donate labor is weak. Consequently, increasing risk aversion, increases the relative motivation of displacing a shirker, and thus favours not using PRC. Secondly, expected output is always bounded to be below 1-$\frac{1}{N}$ without PRC, as it is assumed that at least one individual has $\gamma = 0$. There is no such bound with PRC and wages high enough will ensure output is produced for certain, for instance $w : \mu(w) = c(\overline{e})$ which equals 0.5 in this example, yields $g = 1$.

We now see that increasing the population size increases the relative benefits of not using PRC. This is because of the maintained assumption that $\gamma = 0$, for at least one population member. This individual becomes of less consequence in larger populations. As depicted in the diagram below, which develops the same case for $N = 100$, Now the cross-over wage is approximately 0.3 when $\sigma = .93$.



$N = 100, \sigma = .9$ -solid, $\sigma = .93$ -dash

Depending on the rate at which the firm is rewarded for service, both of these figures show that, even if PRC is costless to implement, a firm's profits may be higher without it.

It is worthwhile reiterating the comparison here, as the result obtained is strikingly at odds with usual intuition. In the case of performance related compensation, all elements relating to the difficulty of contracting over worker effort provision are assumed away; workers are forced to provide effort precisely as contracted. Without performance related compensation, in contrast, the firm has no instruments with which to elicit worker effort. Workers' wages are paid and the worker is free to choose any level of effort in accordance with preferences. Moreover, the firm can never elicit the workers' underlying preferences. Despite the lack of incentives in the latter case without PRC, expected output is higher there for low values of the wage (i.e. the greater the underlying labor donations) and for higher levels of aversion to riskiness in provision of the service.

3.2 Implications

Performance Related Pay

A large literature has explored reasons for why public sector firms may have lower-powered incentives than those in the private sector. It has generally emphasized the difficulties that may arise in implementing PRC. Specifically some have emphasized difficulties arising from multiple principals in the public sector, as developed by Bernheim and Winston (1986), or difficulties of measurement and monitoring when output is multifacted, not traded or not easily observable, as in Holmstrom and Milgrom (1991). Corneo and Rob (2003) incorporate socializing activity into a multi-task model, and show that public firms will have less incentive intensity. Besley and Ghatak (2003), similarly to here, find that organizations producing output that is also valued by their workers will tend to have lower powered incentives. In their framework, successful organizations achieve an alignment between the motivations of workers and principals, and free-riding plays no role. With well aligned objectives, agents have strong personal incentives to provide effort and the need for additional pecuniary motivation is lessened. The ability to condition payment directly upon effort would not lead to reduced performance in their setting. Here, in contrast, the free-riding problem plays a critical role. By not conditioning payment on effort, the firm effectively solves the free-riding problem, though, by doing so, it introduces an adverse selection problem.

To our knowledge, the present paper is the first to show that the use of PRC can actually lower expected output, even if PRC can be perfectly and costlessly installed, and even if it applies to all activities required of workers. In reality, it may well be the case that a primary reason for such firms to not use PRC is that it is, in fact, costly to install and to run. The present analysis shows that, even where these costs are small, firms may still eschew the use of PRC.

The finding that firms may choose to pay low wages (which do not even cover opportunity costs) is similar to that found in Engers and Gans (1998), but arises for entirely different reasons. The reasoning behind their result is tied strongly to the editor/referee context in which their model is set, and unlike here, does not depend on moral hazard. Suppose an editor solicits a referee's report and offers an accompanying payment for timeliness. If the referee shares with the editor a concern for journal quality, then a cost to rejecting the assignment is that refereeing is delayed, lowering the referee's utility. If the accompanying payment is increased, this increases the direct benefit a referee receives to doing the report, but it also increases the willingness of a subequent referee to undertake the assignment, were the current referee to reject it. Consequently, this lowers the cost to the current referee of rejecting the assignment, which mitigates the incentive providing effect of the payment. In the present paper, an entirely different mechanism is at work. A higher monetary payment has the direct effect of increasing the incentive for well motivated non-shirking workers to participate. However, another effect is that this also attracts workers who are not interested in the firm's mission or output, but instead would like to obtain a salary for minimal effort. These workers shirk. Consequently the beneficial impact of raising wages in inducing participation of the "good" workers is mitigated by also inducing shirkers to apply. This is a concern that has been raised by the use of monetary incetives for teachers. Jacobson (1995) surveys the debate on teacher compensation reform in the US and argues the dilemma of monetary incentives leading to the influx of individuals primarily motivated by money is a key issue.

The model predicts less use of performance related pay in the public sector or in non-profit firms, since these sectors are most heavily engaged in production of public goods. Without the public good component, output will not be produced at all in the absence of PRC. When PRC is costly to introduce however, public good producing firms will still obtain positive output without it, and when they do so they will have a tendency to set low wages. Although a comparative reluctance to use PRC in public good firms seems anecdotally supported, formal comparisons of the public sectors' propensity to use performance related pay, relative to the private sector, for similar occupations, are sparse. Burgess and Metcalfe (1999) using cross-sectional establishment data from 1990 find that establishments in the public sector are less likely to operate an incentive scheme than comparable ones in the private sector, and that this difference arises only amongst non-manual workers, which are the workers more likely to be involved in discretionary practices.[18] Roomkin and Weisbrod (1999) report finding greater use of performance related compensation in for-profit than nonprofit hospitals amongst top managerial positions, even though overall earnings were similar. DeVaro and Samuelson (2003) use the US Multi-City Study of Urban Inequality, a cross-sectional employer telephone survey collected from 1992-1995 to analyze, in particular, differences in the use of promotion as an incentive device in non- and for-profit firms. They find a much lower propensity to use promotion in nonprofit firms, and find that promotions are less likely to be based on merit and job performance. They also find that nonprofits are less likely to use incentive contracting - output contingent payment or bonuses. These differences are more pronounced amongst the high skilled worker who are most likely to have significant effects on the firms' missions. Though there is less use of incentives, nonprofit employers do not report differences in worker performance that vary from those reported by for-profit firms. The authors conclude that the evidence supports nonprofit workers being more inherently motivated than their for-profit counterparts which allows non-profit employers to use promotions to optimally allocate workers instead of using them to provide incentives for effort.

Earnings Penalties in Non-profit Firms

Almost all studies comparing non-profit wages with those of for-profits find penalties when considering raw earnings. For example, an early study by Johnston and Rudney (1987) found average annual earnings of nonprofit employees in service industries to be 21.5% lower. Preston (1989) found a nonprofit wage penalty of about 20% for managers and professionals in the US Survey of Job Characteristics. However, findings often change substantially when better controls for individual and workplace characteristics are introduced. Goddeeris (1988) found that public interest lawyers earned 37% less than those in private firms, but that the difference disappeared once characteristics were controlled for. Holtmann and Idson (1993) found a slight wage premium for nurses in non-profit institutions but this became a lower wage when quality was controlled for. Leete (2001) found no systematic non-profit wage differential using the 1990 US census using the finest possible, three digit industry, partition. Mocan and Viola (1997) using extensive controls for human capital and center characteristics found no significant nonprofit wage differential amongst

[18] See also Brugess and Ratto (2003) for an upto date survey of theory and evidence on incentive provision and its relation to the public sector, and Proper and Wilson (2003) for discussion of the effectiveness of mandated PRC schemes introduced in both the US and UK.

child care workers. Ruhm and Borkoski (2003) find, using the CPS, that the raw non profit penalty all but disappears when hours and workplace characteristics are taken into account. Though some studies with controls still find non-profit wage penalties in some sectors, for example Mocan and Tekin (2003) in US childcare, the consensus view seems to be that, though average earnings are clearly lower in NPOs, there does not seem to be a systematic penalty in earnings received per hour worked in non-profit firms, once the differences are controlled for.

A finding of lower average earnings, but no hourly wage penalty in NPOs, is consistent with the model presented here if NPOs are more likely to be engaged in public good producing activities and are thus less likely to use performance related compensation. It is the case that NPOs are overwhelmingly over-represented in the 'care' related sectors which administer to the vulnerable: health-care, child-care, care for the elderly and education. To the extent that citizens have a civic-minded interest in seeing these services well provided, these are sectors producing services which are public goods. The model predicts that public good producing firms in which PRC is difficult to introduce will tend to favor the use of relatively low wages, and low powered incentives. Consequently, average earnings in non-profit firms should tend to be low. However, since these firms will also select some workers who are not attracted by the mission, but by the opportunity to receive pay for little effort, hourly earnings, or earnings measures that appropriately control for effort contributed at work, may be similar or even higher. A testable implication of the present work then is that a type of bi-modal distribution of worker effort should be found in government or not-for-profit organizations. Some, who are driven by concern will excel in performance despite the low pay, but these will co-exist with others with little concern who are driven to work in the sector as it provides an opportunity to slack. Thus, though the model makes no clear predictions about the average cost/quality ratio in such organizations relative to standard private firms, it does predict a higher per worker variation in this ratio for the government and not-for-profit organizations.

4 Conclusions

Firms that are involved in the production of services that have a social value are likely to obtain donations of labor from their workforce. These workers want to make a difference in their working life, by positively affecting society, and do so by working at wages below what would otherwise be required to compensate them for their efforts. The ability to make a difference, however, depends critically on the structure of incentives that operate within the organization. Firms that make heavy use of performance related compensation provide little chance for employees to affect outcomes, because there is little discretion in their behavior. One's motivation to donate labor then arises only when one expects that positions could remain unfilled, and a type of free-riding, which is common to all private provision of public goods problems, arises. In contrast, when firms do not (or are unable to) use performance related compensation, or some other form of direct supervision, increased worker discretion allows for the possibility that poorly motivated workers will shirk and adversely affect outcomes. The possibility of shirking thus provides incentives for genuinely motivated workers to donate labor effort; by working they are making a difference by performing their job better than they expect a replacement employee generally would. Thus, the potential for, and existence of, shirkers provides incentives for the genuinely motivated workers to donate labor and mitigates the free-riding problem. When such motivations are at play, this paper has shown that firms may actually wish to 'engineer' the moral hazard problem by purposely eschewing the use of performance related compensation - even when it is both feasible and costless. Though this is a unique result, it is probably too strong a conclusion to draw in reality. It is likely that many organizations not using performance related compensation do not because it is both costly, difficult to implement and perhaps even infeasible. The conclusion to be drawn from the present work then is that, ceteris parabus, organizations producing socially valued services will tend to use performance related compensation relatively sparingly. It has also been shown that such firms will tend to pay lower wages. This is because, when performance related compensation is not used, increasing wages draws in both highly motivated, and unmotivated (shirking) workers, causing the output elasticity of wages to tend to be low.

The paper's findings square reasonably well with available evidence, and yields a potentially testable and unique implication. The workforce of firms producing such socially valued services will tend to have higher quality variance in performance than comparable workforces in other firms. This implication remains to be tested in future work.

5 References

Alonso, P. and G.B. Lewis (2001) Public service motivation and job performance: Evidence from the federal sector, *American Review of Public Administration*, 31, 4, 363-80.

Andreoni, J. (1990) Impure Altruism and Donations to Public Goods: A Theory of Warm-Glow Giving?, *Economic Journal,* June; 100(401): 464-77.

Bernheim, B, D. and M. D. Whinston (1986) Common Agency, *Econometrica*, July, 54(4): 923-42.

Besley, T. and M. Ghatak (2003) Competition and Incentives with Motivated Agents, Mimeo, London School of Economics, September.

Bilodeau and Slivinski (1998) Rational, non-profit entrepreneurship, *Journal of Economics and Management Strategy*, 7, 551-571.

Burgess, S. and P. Metcalfe (1999) Incentives in Organisations: A Selective Overview of the Literature with Application to the Public Sector, Centre for Market and Public Organisation (CMPO) Working Paper: 99/016.

Burgess, S. and M. Ratto (2003) The role of incentives in the public sector: Issues and evidence, CMPO Working Paper no. 03/071.

Corneo, G. and R. Rob (2003) Working in Public and Private Firms, *Journal of Public Economics*, August 87(7-8): 1335-52.

Cornes, R. and T. Sandler (1986) *The theory of externalities, public goods, and club goods*, Cambridge: Cambridge University Press.

Coyle-Shapiro, J., A.-M. (2003) The employment relationship in the UK public sector: a psychological contract perspective, *Journal of Public Administration Research and Theory*, April, 13, 2, 213-31.

Deckop, J.R. and C.C. Cirka (2000) The risk and reward of a double-edged sword: effects of a merit pay program on intrinsic motivation, *Nonprofit and Voluntary Sector Quarterly,* September, 29, 3, 400-418.

DeVaro, J and D. Samuelson (2003) Why are promotions less likely in nonprofit firms? mimeo, Cornell University, Ithaca, NY.

Dixit, A. (2002) Incentives and organizations in the public sector: an interpretative review, *Journal of Human Resources*, Fall, 37(4): 696-727.

Duncan, B. (1999) Modeling Charitable Contributions of Time and Money, *Journal of Public Economics*, May; 72(2): 213-42.

Engers and Gans (1998) Why Referees Are Not Paid (Enough), *American Economic Review*, December, 88(5): 1341-49.

Francois, P. (2000) 'Public Service Motivation' as an Argument for Government Provision, *Journal of Public Economics*, November, 78(3): 275-99.

Francois, P. (2003) Not-for-Profit Provision of Public Services, *Economic Journal*, March, 113(486): C53-61.

Frant, H. (1996) High-powered and low-powered incentives in the public sector, *Journal of Public Administration Research and Theory*, July, 6, 3, 365-82.

Goddeeris, J.H., (1988), Compensating Differentials and Self-selection: An Application to Lawyers, *Journal of Political Economy*, April; 96(2): 411-28.

Grout, P. A. and M. J. Yong (2003) The role of donated labour and not for profit at the public/private interface, CMPO Working Paper No. 03/074.

Hansmann, H. B., (1996) The Role of Nonprofit Enterprise, *Yale Law Journal*, April; 89(5): 835-901.

Hodgkinson, V. A. and M. S. Weitzman (1996) *Dimensions of the Independent Sector: A Statistical Profile.* Second Edition, Independent Sector, Washington, DC.

Holmstrom, B. and P. Milgrom (1991) Multitask Principal-Agent Analyses: Incentive Contracts, Asset Ownership, and Job Design, *Journal of Law, Economics, and Organization* 7(0): 24-52.

Holtmann, A.G. and T.L. Idson (1993) Wage Determination of Registered Nurses in Proprietary and Nonprofit Nursing Homes, *Journal of Human Resources*, Winter, 28(1): 55-79.

Independent Sector (2003) *Giving and volunteering in the United States, 2001*, available at http://www.independentsector.org/programs/research/gv01main.html

IRS (2000), Merit Malaise: performance pay in the public sector, *IRS Employment Review,* Pay and Benefits Bulletin, April, 701, 5-11.

Jacobson, S. L. (1995) Monetary incentives and the reform of teacher compensation: a persistent organizational dilemma, *International Journal of Educational Reform,* 4, 1, 29-35.

Johnston, D. and G. Rudney (1987) Characteristics of Workers in Nonprofit Organizations, *Monthly Labor Review,* July,110(7): 28-33.

Kirton, D (2001) Love and Money: Payment, motivation and the fostering task, *Child and Family Social Work*, 6, 3, August: 199-208.

Leete, L. (2001) Whither the Nonprofit Wage Differential? Estimates from the 1990 Census, *Journal of Labor Economics*, January,19(1): 136-70.

Le Grand, J. (1997) Knights, Knaves or pawns? Human behaviour and social policy, *Journal of Social Policy,* 26, 2, 149-169.

Lewin, D. (2003) Incentive compensation in the public sector: evidence and potential, *Journal of Labor Research*, Fall, 24, 4, 597-620.

Light, P.C. (2003) Health of the human services workforce, Center for Public Service, Brookings Institution, available at: http://www.openminds.com/indres/HumanServicesBrookings.pdf

Macleod W.B. and J.M. Malcomson (1989) Implicit Contracts, Incentive Compatibility, and Involuntary Unemployment, *Econometrica*,1989; 57(2): 447-80

Mahoney, J.T., A.S. Huff and J. O. Huff (1992) Towards a new social contract theory in organization science, *Journal of Management Inquiry,* December, 1, 4, 153-69.

Malcomson, J. (1999) Individual employment contracts, in *Handbook of Labor Economics, Volume 3* Edited by O. Ashenfelter and D. Card, North Holland.

Menchik, P. L; Weisbrod, B. A (1987) Volunteer Labor Supply, *Journal of Public Economics* March 32(2): 159-83.

Mocan, H. N.. and E. Tekin (2000) Nonprofit Sector and Part-Time Work: An Analysis of Employer-Employee Matched Data on Child Care Workers, *Review of Economics and Statistics*, February, 85(1): 38-50.

Mocan, H. N. and D. Viola (1997) The Determinants of Child Care Workers' Wages and Compensation: Sectoral Difference, Human Capital, Race, Insiders and Outsiders, National Bureau of Economic Research Working Paper: 6328.

Murnighan, J.K. and J-W. Kim (1993) The volunteer dilemma, *Administrative Science Quarterly*, December, 38, 4, 515-39.

Naff, K.C. and J. Crum (1999) Working for America, *Review of Public Administration*, Fall, 19, 4, 5-17.

Perry, J.L. (2000) Bringing society in: toward a theory of public service motivation, *Journal of Public Administration Research and Theory,* April, 10, 2, 471-89.

Perry, J.L. and L.R. Wise (1990) The motivational bases of public service, *Journal of Public Administration Research and Theory,* 50, 3, 367-73.

Prendergast, C. (2003) The Limits of Bureaucratic Efficiency, *Journal of Political Economy*, 111(5) 929-58.

Preston, A. E. (1989) The Nonprofit Worker in a For-Profit World, *Journal of Labor Economics*, October; 7(4): 438-63.

Proper, C. and D. Wilson (2003) The use and usefulness of performance measures in the public sector, CMPO Working Paper no. 03/073.

Rhoades, D.R., and K. F. McFarland (1999) Caregiver meaning: a study of caregivers of individuals with mental illness, *Health and Social Work*, 24, 4, Nov. 291-98.

Roomkin, M.J. and B.A. Weisbrod (1999) Managerial Compensation and Incentives in For-Profit and Nonprofit Hospitals, *Journal of Law, Economics, and Organization*, October; 15(3): 750-81.

Rose-Ackerman, S. (1996) Altruism, Nonprofits, and Economic Theory, *Journal of Economic Literature*, 34(2): 701-28.

Ruhm, C. and C. Borkoski (2003) Compensation in the nonprofit sector, *Journal of Human Resources*, Fall, 38, 992-122.

Segal, L. M; and B.A. Weisbrod (2002) Volunteer Labor Sorting across Industries, *Journal of Policy Analysis* and Management, Summer, 21(3): 427-47.

Vicary, S. (2000), Donations to a Public Good in a Large Economy, *European Economic Review,* March, 44(3): 609-18.

Williams, C.C., and J. Windebank (2001), Beyond profit-motivated exchange: some lessons from the study of paid informal work, *European Urban and Regional Studies*, 8, 1, 49-61.

6 Appendix

Proof of Proposition 1: Consider the utility of an individual i applying for a job, given a cut-off rule, $\widehat{\gamma}$. Linearity of the preferences implies that expected values can be considered. The individual applies, if and only if:

$$\begin{aligned}
& \frac{1}{1+(N-1)\int_{\widehat{\gamma}}^{\infty} f(\gamma)\, d\gamma}\left(\mu(w)-c(\overline{e})+\gamma_i v(1)\right)+\left(1-\frac{1}{1+(N-1)\int_{\widehat{\gamma}}^{\infty} f(\gamma)\, d\gamma}\right)\gamma_i v(1) \\
\geq\; & \left(1-F(\widehat{\gamma})^{N-1}\right)\gamma_i v(1)+F(\widehat{\gamma})^{N-1}\gamma_i v(\overline{g}) \\
\Rightarrow\; & \left(1+(N-1)\int_{\widehat{\gamma}}^{\infty} f(\gamma)\, d\gamma\right) F(\widehat{\gamma})^{N-1}\gamma_i\left[v(1)-v(\overline{g})\right] \geq c(\overline{e})-\mu(w). \qquad (18)
\end{aligned}$$

With N being the total number of individuals drawn from the distribution $F(\cdot)$, from the perspective of a single individual there are $N-1$ other potential applicants. The left hand side of the first expression above is the expected utility of an applicant. The first term is the expected utility if employed weighted by the probability of receiving the position. The second is the utility if not employed (which is $\gamma_i v(1)$, since if not employed it implies someone else filled the position) weighted by its probability. A symmetric equilibrium is a common cut off value of γ^* such that the induced optimal decision under (18) yields only individuals with $\gamma_i \geq \gamma^*$ applying for the job. That is, an equilibrium is a fixed point solving:

$$(1+(N-1)(1-F(\gamma^*)))\, F(\gamma^*)^{N-1}\gamma^*\left[v(1)-v(\overline{g})\right]=c(\overline{e})-\mu(w), \qquad (19)$$

which is equation (3) in the text. Differentiating the left hand side of (19) with respect to γ^* yields:

$$\begin{aligned}
& \left[1+(N-1)(1-F(\gamma^*))\right] F(\gamma^*)^{N-1}\left[v(1)-v(\overline{g})\right]-(N-1)\, F(\gamma^*)^{N-1}\gamma^* f(\gamma^*)\left[v(1)-v(\overline{g})\right] \\
& +f(\gamma^*)(N-1)\, F(\gamma^*)^{N-2}\gamma^*\left[1+(N-1)(1-F(\gamma^*))\right]\left[v(1)-v(\overline{g})\right].
\end{aligned}$$

Since $F(\gamma^*) < 1$, the first term is positive and the absolute value of the second term is strictly smaller than the third, so that this expression is positive. Thus the left hand side of (19) is monotonically increasing in γ^* and, given continuity of F, is continuous in γ^*. Also, using L'hopital's rule, it can be shown that $\lim_{\gamma\rightarrow\infty} LHS$ (19) $\rightarrow \infty > c(\overline{e})-\mu(w)$, and $\lim_{\gamma\rightarrow 0} LHS$ (19) $\rightarrow 0 < c(\overline{e})-\mu(w)$ for any, $N > 1$, $w \geq 0$. Thus a point solving (19) exists. The monotonicity of the left hand side implies that such a fixed point is unique. Note finally that, given an equilibrium cut off rule, γ^*, each individual's best response is uniquely determined by their own γ_i according to (18).

■

Proof of Corrollary 1: Part (i) immediate by setting $N = 1$ in (19).

Part (ii) Consider the impact of an increase in N while holding fixed γ^* on the left hand side of (19). For given γ^*, and $F(\gamma^*)$ denoted by F below, the relevant part of the derivative is:

$$\begin{aligned} & \frac{d}{dN}\left((1+(N-1)(1-F))F^{N-1}\right) \\ = \;& (\ln F)F^{N-1}(F+N-FN)+F^{N-1}(1-F) \\ = \;& F^{N-1}\left[(\ln F)(F+N(1-F))+(1-F)\right]. \end{aligned}$$

Since $F < 1$ then $\ln F < 0$, and the term in square brackets is decreasing in N. Consider then, the case of $N = 1$, in which the term in square brackets becomes: $(\ln F) + (1 - F) < 0$. Consequently the left hand side of (19) is decreasing in N, and clearly the right hand side is unchanged. Since, in the proof of Proposition 1, it was already demonstrated that the left hand side of (19) is increasing in γ^*, for equilibrium to be restored following an increase in N, necessarily γ^* increases. Thus, the amount of free-riding $\gamma^* - c(\overline{e}) + \mu(w)$ also increases.■

Proof of Proposition 2: Conditions (10) and (11) can be expressed as:

$$\frac{\frac{1}{(N-1)}+\int_0^{\gamma^L} f(\gamma)\,d\gamma}{\frac{1}{(N-1)}+\int_{\gamma^H}^{\infty} f(\gamma)\,d\gamma+\int_0^{\gamma^L} f(\gamma)\,d\gamma}\gamma^H = \frac{c(\overline{e})-\mu(w)}{[v(1)-v(0)]} \tag{20}$$

$$\frac{\int_{\gamma^H}^{\infty} f(\gamma)\,d\gamma}{\frac{1}{(N-1)}+\int_{\gamma^H}^{\infty} f(\gamma)\,d\gamma+\int_0^{\gamma^L} f(\gamma)\,d\gamma}\gamma^L = \frac{\mu(w)}{[v(1)-v(0)]}. \tag{21}$$

The right hand side of both expressions is constant, given w. For given γ^L, the left hand side of (20) is monotonic in γ^H. For given γ^H, the left hand side of (21) is not necessarily monotonic in γ^L, but if

$$\begin{aligned} & \int_{\gamma^H}^{\infty} f(\gamma)\,d\gamma+\int_0^{\gamma^L} f(\gamma)\,d\gamma-\gamma^L\int_{\gamma^H}^{\infty} f(\gamma)\,d\gamma f\left(\gamma^L\right)>0 \\ & or \\ & 1-F\left(\gamma^H\right)+F\left(\gamma^L\right)-\gamma^L f\left(\gamma^L\right)\left(1-F\left(\gamma^H\right)\right)>0 \\ \Leftrightarrow\; & F\left(\gamma^L\right)+\left[1-\gamma^L f\left(\gamma^L\right)\right]\left(1-F\left(\gamma^H\right)\right)>0 \end{aligned}$$

then the left hand side of (21) is monotonically increasing in γ^L. A sufficient condition for this is condition (12). Consequently, the LHS of (21) is monotonic. Thus since the Left hand side is increasing in γ^L, define $\gamma^L\left(\gamma^H\right)$ as the value of γ^L that solves (21) given γ^H and w. Note that $\frac{\partial\gamma^L(\gamma^H)}{\partial\gamma^H} > 0$. Substitute the function $\gamma^L\left(\gamma^H\right)$ for γ^L into the left hand side of (20) to obtain the expression $\frac{\frac{1}{(N-1)}+\int_0^{\gamma^L(\gamma^H)} f(\gamma)d\gamma}{\frac{1}{(N-1)}+\int_{\gamma^H}^{\infty} f(\gamma)d\gamma+\int_0^{\gamma^L(\gamma^H)} f(\gamma)d\gamma}\gamma^H$. Now use this to evaluate the expression (20):

$$\frac{\frac{1}{(N-1)}+\int_0^{\gamma^L(\gamma^H)} f(\gamma)\,d\gamma}{\frac{1}{(N-1)}+\int_{\gamma^H}^{\infty} f(\gamma)\,d\gamma+\int_0^{\gamma^L(\gamma^H)} f(\gamma)\,d\gamma}\gamma^H \gtreqless \frac{c(\overline{e})-\mu(w)}{[v(1)-v(0)]}. \tag{22}$$

Note that the derivative of the LHS of this function in γ^H is positive, i.e.

$$\begin{aligned}
&\left(\frac{1}{(N-1)}+\int_{\gamma^H}^{\infty} f(\gamma)\,d\gamma+\int_0^{\gamma^L(\gamma^H)} f(\gamma)\,d\gamma\right)\left(\frac{1}{(N-1)}+\int_0^{\gamma^L(\gamma^H)} f(\gamma)\,d\gamma+\gamma^H\frac{\partial\gamma^L\left(\gamma^H\right)}{\partial\gamma^H}f\left(\gamma^L\right)\right)- \\
&\left(\frac{1}{(N-1)}+\int_0^{\gamma^L(\gamma^H)} f(\gamma)\,d\gamma\right)\gamma^H\left(-f\left(\gamma^H\right)+\frac{\partial\gamma^L\left(\gamma^H\right)}{\partial\gamma^H}f\left(\gamma^L\right)\right) \\
\equiv\;& \left(\frac{1}{(N-1)}+\int_{\gamma^H}^{\infty} f(\gamma)\,d\gamma+\int_0^{\gamma^L(\gamma^H)} f(\gamma)\,d\gamma\right)\left(\frac{1}{(N-1)}+\int_0^{\gamma^L(\gamma^H)} f(\gamma)\,d\gamma\right) \\
&+\int_{\gamma^H}^{\infty} f(\gamma)\,d\gamma\gamma^H\frac{\partial\gamma^L\left(\gamma^H\right)}{\partial\gamma^H}f\left(\gamma^L\right)+\left(\frac{1}{(N-1)}+\int_0^{\gamma^L(\gamma^H)} f(\gamma)\,d\gamma\right)\gamma^H f\left(\gamma^H\right) \\
>\;& 0
\end{aligned}$$

Thus the value of γ^H solving (22), if it exists, is unique, and therefore also is γ^L. We now show that, either a γ^H solving (22) exists, in which case the equilibrium cut-offs are $\left(\gamma^H,\gamma^L\left(\gamma^H\right)\right)$ with $\gamma \leq \gamma^L\left(\gamma^H\right)$ applying and shirking if obtaining work, γ such that $\gamma^L\left(\gamma^H\right) < \gamma < \gamma^H$ not applying, and $\gamma \geq \gamma^H$ applying and donating labor if hired. If a γ^H solving (22) does not exist, then the corresponding equilibrium cut-offs are $\gamma^H = \frac{c(\overline{e})}{[v(1)-v(0)]}$ and $\gamma^L\left(\frac{c(\overline{e})}{[v(1)-v(0)]}\right)$.

Clearly for $\gamma^H \to \infty$, LHS > RHS of (22). For $\gamma^H = \frac{c(\overline{e})}{[v(1)-v(0)]}$, either (i) LHS < RHS of (22) or (ii) LHS $\geq$ RHS of (22). Case (i): If LHS < RHS of (22), then by the continuity and monotonicity of LHS (22) $\exists\ \gamma^H > \frac{c(\overline{e})}{[v(1)-v(0)]}$ which solves (22) with equality, and monotonicity implies this value is unique. The lower cut-off is then given by $\gamma^L = \gamma^L(\gamma^H)$ from (21), provided that a γ^L can be found to solve (21) with equality. If not, then $\gamma^L = \frac{c(\overline{e})}{[v(1)-v(0)]}$. In the case where an equality exists, the two cut-offs γ^H and $\gamma^L\left(\gamma^H\right)$ correspond to the unique fixed point pair of the system given by (20) and (21). Given these cut-offs $\left(\gamma^H,\gamma^L\right)$, the induced optimal individual decisions yield aggregate probabilities which coincide with these cut-offs. In the case where $\gamma^L = \frac{c(\overline{e})}{[v(1)-v(0)]}$, given that all $\gamma \leq \frac{c(\overline{e})}{[v(1)-v(0)]}$ apply, γ^H is the unique solution to (20) when setting $\gamma^L = \frac{c(\overline{e})}{[v(1)-v(0)]}$, so that the upper cut off for individuals is generated by individual decisions consistent with that optimum. The left hand side of (21) strictly exceeds the right hand side for this value of γ^H and $\gamma^L = \frac{c(\overline{e})}{[v(1)-v(0)]}$, implying that all individuals with $\gamma \leq \frac{c(\overline{e})}{[v(1)-v(0)]}$ strictly prefer to apply.

Case (ii) For $\gamma^H = \frac{c(\overline{e})}{[v(1)-v(0)]}$, and LHS $\geq$ RHS of (22) then set $\gamma^H = \frac{c(\overline{e})}{[v(1)-v(0)]}$ and $\gamma^L = \gamma^L\left(\frac{c(\overline{e})}{[v(1)-v(0)]}\right) > 0$ using equation (21). The inequality can be seen directly from (21). Consider γ^L more precisely. The two sides of expression (21) in this situation with $\gamma^H = \frac{c(\overline{e})}{[v(1)-v(0)]}$ yield:

$$\frac{\int_{\frac{c(\overline{e})}{[v(1)-v(0)]}}^{\infty} f(\gamma)\, d\gamma}{\frac{1}{(N-1)} + \int_{\frac{c(\overline{e})}{[v(1)-v(0)]}}^{\infty} f(\gamma)\, d\gamma + \int_0^{\gamma^L} f(\gamma)\, d\gamma}\gamma^L \gtreqless \frac{\mu(w)}{[v(1)-v(0)]}. \tag{23}$$

There are two possibilities at $\gamma^L = \frac{c(\overline{e})}{[v(1)-v(0)]}$, either: (a) LHS $>$ RHS of (23) or (b) LHS $\leq$ RHS of (23). Case (a), if for $\gamma^L = \frac{c(\overline{e})}{[v(1)-v(0)]}$ and LHS $>$ RHS of (23), then by the continuity and monotonicity of LHS (23) $\exists\ \gamma^L < \frac{c(\overline{e})}{[v(1)-v(0)]}$ which uniquely solves (23) with equality. The equilibrium cut-offs in this case are given by $\gamma^H = \frac{c(\overline{e})}{[v(1)-v(0)]}$ and $\gamma^L < \frac{c(\overline{e})}{[v(1)-v(0)]}$. Condition (21) holds, and the solution is unique, but the left hand side of (20) always exceeds the right. In this equilibrium, all motivated individuals apply, but not all shirkers do. Finally case (b) where for $\gamma^L = \frac{c(\overline{e})}{[v(1)-v(0)]}$, LHS $\leq$ RHS of (23). In that case $\gamma^L = \frac{c(\overline{e})}{[v(1)-v(0)]} = \gamma^H$, all N individuals apply for the job, and neither (20) nor (21) hold as $\left(1 - \int_0^{\frac{c(\overline{e})}{[v(1)-v(0)]}} f(\gamma)\, d\gamma\right) c(\overline{e}) < \mu(w)$. Given all individuals are applying, both high and low γ individuals strictly prefer to apply for the job.

The uniqueness of the symmetric equilibrium cut-offs is guaranteed as it has been demonstrated that the unique fixed points induced by such a problem correspond to $\gamma^H(w), \gamma^L(w)$ as characterized above.

■

Proof of Proposition 3: We demonstrate that when the wage satisfies the second inequality in (15), all apply for the job, thus there is no free-riding. A sufficient condition for all $\gamma \geq \frac{c(\overline{e})}{[v(1)-v(0)]}$ to apply, given that all $\gamma < \frac{c(\overline{e})}{[v(1)-v(0)]}$ are applying, is that the left hand side of condition (22) strictly exceeds the right hand side at $\gamma^H = \gamma^L = \frac{c(\overline{e})}{[v(1)-v(0)]}$. That is:

$$\begin{aligned}
\frac{\frac{1}{(N-1)} + \int_0^{\frac{c(\overline{e})}{[v(1)-v(0)]}} f(\gamma)\, d\gamma}{\frac{1}{(N-1)} + \int_{\frac{c(\overline{e})}{[v(1)-v(0)]}}^{\infty} f(\gamma)\, d\gamma + \int_0^{\frac{c(\overline{e})}{[v(1)-v(0)]}} f(\gamma)\, d\gamma} c(\overline{e}) &> c(\overline{e}) - \mu(w) \\
\Leftrightarrow \left[\frac{1}{(N-1)} + \int_0^{\frac{c(\overline{e})}{[v(1)-v(0)]}} f(\gamma)\, d\gamma\right] c(\overline{e}) &> \left(\frac{1}{(N-1)} + 1\right)(c(\overline{e}) - \mu(w)) \quad (24) \\
\Leftrightarrow \left(\frac{1}{(N-1)} + 1\right)\mu(w) &> c(\overline{e})\left[1 - \int_0^{\frac{c(\overline{e})}{[v(1)-v(0)]}} f(\gamma)\, d\gamma\right]. \quad (25)
\end{aligned}$$

A sufficient condition for all $\gamma < \frac{c(\overline{e})}{[v(1)-v(0)]}$ to apply given that all $\gamma \geq \frac{c(\overline{e})}{[v(1)-v(0)]}$ are applying is that the left hand side of (23) is strictly less than its right hand side under $\gamma^H = \gamma^L = \frac{c(\overline{e})}{[v(1)-v(0)]}$. That is:

$$\begin{aligned}
\frac{\int_{\frac{c(\overline{e})}{[v(1)-v(0)]}}^{\infty} f(\gamma)\, d\gamma}{\frac{1}{(N-1)} + \int_{\frac{c(\overline{e})}{[v(1)-v(0)]}}^{\infty} f(\gamma)\, d\gamma + \int_0^{\frac{c(\overline{e})}{[v(1)-v(0)]}} f(\gamma)\, d\gamma} \frac{c(\overline{e})}{[v(1) - v(0)]} &< \frac{\mu(w)}{[v(1) - v(0)]} \\
\Longleftrightarrow \left(\int_{\frac{c(\overline{e})}{[v(1)-v(0)]}}^{\infty} f(\gamma)\, d\gamma\right) c(\overline{e}) &< \left(\frac{1}{(N-1)} + 1\right)\mu(w) \\
\Longleftrightarrow \left(1 - \int_0^{\frac{c(\overline{e})}{[v(1)-v(0)]}} f(\gamma)\, d\gamma\right) c(\overline{e}) &< \left(\frac{1}{(N-1)} + 1\right)\mu(w)
\end{aligned}$$

which is identical to (25) and identical to the second inequality in (15). Thus, under this condition, all apply. The first inequality in (15) is necessary and sufficient to ensure that labor is being donated, as workers are not fully compensated for the disutility of effort.■