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ABSTRACT

Factor Analysis in a New-Keynesian Model*

New-Keynesian models are characterized by the presence of expectations as explanatory variables. To use these models for policy evaluation, the econometrician must estimate the parameters of expectation terms. Standard estimation methods have several drawbacks, including possible lack of identification of the parameters, misspecification of the model due to omitted variables or parameter instability, and the common use of inefficient estimation methods. Several authors have raised concerns over the validity of commonly used instruments to achieve identification. In this paper we analyse the practical relevance of these problems and we propose remedies to weak identification based on recent developments in factor analysis for information extraction from large data sets. Using these techniques, we evaluate the robustness of recent findings on the importance of forward looking components in the equations of the New-Keynesian model.

JEL Classification: E5, E52 and E58
Keywords: determinacy of equilibrium, factor analysis, forward-looking output equation, New-Keynesian Phillips curve, rational expectations and Taylor rule

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Non-technical Summary

This paper is about the estimation of New-Keynesian models of the monetary transmission mechanism. We evaluate a number of recent findings obtained using single equation methods and we develop a system approach that makes use of additional identifying information extracted using factor analysis from large data sets.

A number of authors have used instrumental variable methods to estimate one or more equations of the New-Keynesian model of the monetary transmission mechanism. They used the New-Keynesian paradigm to explain the behavior of U.S. inflation as a function of its lag(s), expected lead(s), and the marginal cost of production or the output gap. This work stimulated considerable debate, much of which has focused on the size and significance of future expected inflation in the New-Keynesian Phillips curve. Similar arguments have been made over the role of expected future variables in other equations of the New-Keynesian model such as Taylor rules in which expected future inflation appears as a regressor or models of the Euler equation for output in which expected future output appears on the right-hand-side. The estimation of models that include future expectations has revived a debate that began in the 1970's with the advent of rational expectations econometrics. The recent empirical literature on the New-Keynesian Model and in particular on estimating the New-Keynesian Phillips curve has highlighted four main problems with the single equation approach to estimation by GMM. First, parameter estimates may be biased due to correlation of the instruments with the error term. Second, an equation of interest could be mis-specified because of omitted variables or parameter instability within the sample. Third, parameters of interest may not be identified. Fourth, parameters may be weakly identified if the correlation of the instruments with the target is low. We

argue, in this paper, that these issues can only be resolved by embedding the individual single equation models in a fully specified structural model. We analyze the practical relevance of these problems, propose remedies for each of them, and evaluate whether the findings on the importance of the forward looking component are robust when obtained within a more general econometric context. First we compare single equation and system methods of estimation for models with forward looking regressors. We then conduct a robustness analysis for a full forward looking system. In extending the information set we analyze the role of information extracted from large data sets to reduce the risk of specification bias and weak instruments problems. Finally we conduct a formal analysis of identification and of issues related to the different characteristics of rational expectations equilibria in the context of determinacy and indeterminacy.

1 Introduction

This paper is about the estimation of New-Keynesian models of the monetary transmission mechanism. We evaluate a number of recent findings obtained using single equation methods and we develop a system approach that makes use of additional identifying information extracted using factor analysis from large data sets.

Following the influential work of Galí and Gertler (1999, GG), a number of authors have used instrumental variable methods to estimate one or more equations of the New-Keynesian model of the monetary transmission mechanism. GG used the New-Keynesian paradigm to explain the behavior of U.S. inflation as a function of its first lag, expected first lead, and the marginal cost of production. Their work stimulated considerable debate, much of which has focused on the size and significance of future expected inflation in the New-Keynesian Phillips curve. Similar arguments have been made over the role of expected future variables in the other equations of the New-Keynesian model: for example Clarida, Galí and Gertler (1998) estimate a Taylor rule in which expected future inflation appears as a regressor and Fuhrer and Rudebusch (2002) have estimated an Euler equation for output in which expected future output appears on the right-hand-side.

The estimation of models that include future expectations has revived a debate that began in the 1970's with the advent of rational expectations econometrics. In this context, a number of authors have raised econometric issues that relate to the specification and estimation of single equations with forward looking variables. For example, Rudd and Whelan (2001, RW) showed that the GG parameter estimates for the coefficient on future inflation may be biased upward if the equation is mis-specified due to the omission of relevant regressors that are instead used as instruments. With regard to the estimation of the coefficients of future variables they pointed out that this problem can yield differences between estimates that are based on the following two alternative estimation methods. The first (direct) method estimates the coefficient directly using GMM; the second (indirect) method computes a partial solution to the complete model that removes the expected future variable from the right-hand-side and substitutes an infinite distributed lag of all future expected forcing variables. RW use their analysis to argue in favor of Phillips curve specifications that favor backward lags of inflation over the New-Keynesian specification that includes only expected future inflation as a regressor.

Galí, Gertler and Lopez-Salido (2003, GGLS) have responded to the RW critique by pointing out that, in spite of the theoretical possibility of omitted variable bias, estimates obtained by direct and indirect methods are fairly close, and when additional lags of inflation are added as regressors in the structural model to proxy for omitted variables, they are not significant. While the Rudd-Whelan argument is convincing, the CGLS response is less so since other (contemporaneous) variables might also be incorrectly omitted from the simple GG inflation equation. Even if additional lags of inflation were found to be insignificant, their inclusion could change the parameters of both the closed form solution and the structural model. We argue, in this paper, that these issues can only be resolved by embedding the single equation New-Keynesian Phillips curve in a fully specified structural model.

Other authors, e.g. Fuhrer and Rudebusch (2002), Lindé (2003) and Jondeau and Le Bihan (2003) have pointed out that the Generalized Method of Moment (GMM) estimation approach followed by GG could be less robust than maximum likelihood estimation (MLE) in the presence of a range of model mis-specifications such as omitted variables and measurement error, typically leading to overestimation of the parameter of future expected inflation. GGLS correctly replied that no general theoretical results are available on the relative merits of GMM and MLE under mis-specification, that the comparison could be biased by the use of an inappropriate GMM estimator, and that other authors such as Ireland (2001) provided evidence in favor of a (pure) forward looking equation for US inflation when using MLE. In this paper we hope to shed additional light on the efficiency and possible bias of GMM estimation by comparing alternative estimation methods on the *same* data set and the same model specification.

A different and potentially more problematic critique of the GG approach comes from Mavroeidis (2002), Bårdsen, Jansen and Nymoen (2003), and Nason and Smith (2003), building upon previous work on rational expectations by Pesaran (1987). Pesaran (1987) stressed that the conditions for identification of the parameters of the forward looking variables in an equation of interest should be carefully checked prior to single equation estimation. To check identification conditions one must specify a model for all of the right-hand-side variables. The articles cited above have shown that in a variety of alternative models, sensible specifications for the right-hand-side variables lead to underidentification of the parameters of forward looking variables. In the presence of underidentification, estimation by GMM yields unreliable results.

A final and related argument against the indiscriminate use of single equation GMM estimation of forward looking equations relates to the quality of the instruments. This issue is distinct from that of underidentification since an equation may be identified, but the instruments may be weakly correlated with the endogenous variables, see in particular Mavroeidis (2002) for an application to the GG case. When the instruments are not particularly useful for forecasting the future expected variable, the resulting GMM estimators suffer from weak identification, which leads to non-standard distributions for the estimators that can yield misleading inference, see e.g. Stock, Wright and Yogo (2002) for a general overview on weak instruments and weak identification.

In summary, the recent literature on the New-Keynesian Phillips curve has highlighted four main problems with the single equation approach to estimation by GMM. First, parameter estimates may be biased due to correlation of the instruments with the error term. Second, an equation of interest could be mis-specified because of omitted variables or parameter instability within the sample. Third, parameters of interest may not be identified. Fourth, parameters may be weakly identified if the correlation of the instruments with the target is low.

In this paper we analyze the practical relevance of these problems, propose remedies for each of them, and evaluate whether the findings on the importance of the forward looking component are robust when obtained within a more general econometric context. In Section 2 we compare single equation and system methods of estimation for models with forward looking regressors. In Section 3 we conduct a robustness analysis for a full forward looking system. In Section 4 we analyze the role of information extracted from large data sets to reduce the risk of specification bias and weak instruments problems. In Section 5 we conduct a formal analysis of identification issues. In Section 6 we summarize the main results of the paper and conclude.

2 Single Equation versus System Approach

We begin this Section with a discussion of the estimation of the New-Keynesian Phillips curve. This will be followed by a discussion of single-equation estimation of the Euler equation and the policy rule. We then contrast the single equation approach to a closed, three-equation, New-Keynesian model. We estimate simultaneously a complete structural model which combines the three previously estimated single-equation models for the Phillips curve, the Euler equation and the policy rule and we compare system estimates of parameters with those of the three single-equation specifications.

Our starting point is a version of the New-Keynesian Phillips curve inspired by the work of Galí and Gertler (GG 1999),

$$\pi_t = \alpha_0 + \alpha_1 \pi^e_{t+1} + \alpha_2 x_t + \alpha_3 \pi_{t-1} + e_t, \tag{1}$$

where π_t is the GDP deflator, π^e_{t+1} is the forecast of π_{t+1} made in period t, x_t is a real forcing variable (e.g. marginal costs as suggested by GG, unemployment - with reference to Okun's law - as in e.g. Beyer and Farmer (2003), or any version of an output gap variable). The error term e_t is assumed to be i.i.d. $(0, \sigma^2_e)$ and is, in general, correlated with the non-predetermined variables (i.e, with π^e_{t+1} and x_t). Since we want to arrive at the specification of a system of forward looking equations, we prefer to use as a real forcing variable the unemployment rate or the output gap[1], measured as the deviation of real GDP from its one-sided HP filtered version as widely used in the literature.

To estimate equation (1) we replace π^e_{t+1} with π_{t+1}, such that (1) becomes

$$\pi_t = \alpha_0 + \alpha_1 \pi_{t+1} + \alpha_2 x_t + \alpha_3 \pi_{t-1} + v_t. \tag{2}$$

Equation (2) can be estimated by GMM, with HAC standard errors to take into account the MA(1) structure of the error term $v_t = e_t + \alpha_1(\pi^e_{t+1} - \pi_{t+1})$.[2]

[1] The forward looking IS curve is usually specified in terms of the output or unemployment gap.

[2] In particular, to compute the GMM estimates we start with an identity weighting matrix, get a first set of coefficients, use these to update the weighting matrix and finally iterate coefficients to convergence. To compute the HAC standard errors, we adopt the Newey West (1997) approach with a Bartlett kernel and fixed bandwidth. These calculations are carried out with Eviews 5.0.

All data is for the US, quarterly, for the period 1970:1-1998:4, where the constraint on the end date is due to the large data set we use in Section 4.

In the first panel of the first column of Tables 1 and 2 we report the single-equation estimation results. In Table 1 x_t represents unemployment and in Table 2 it represents the output gap. As in GG (1999) and Galí et al. (2003), we find a larger coefficient on π^e_{t+1}, about 0.70, than on π_{t-1}, about 0.30. The coefficient on the forcing variable is very small and not statistically significant at the 5% level, again in line with previous results.

There are at least two problems with this single equation approach: first, the validity of the instruments cannot be evaluated and, second, the degree of over, just, or under-identification is undefined.

The issue of identification and the use of valid instruments in rational expectations models is a very subtle one, see e.g. Pesaran (1987), Mavroeidis (2002) or Bårdsen et al. (2003). In linear backward looking models, such as conventional simultaneous equation models, rank and order conditions can be applied in a mechanical way (see e.g. Fisher, 1966). In rational expectation models, however, the conditions for identification depend on the solution of the model, i.e. whether the solution of the model is determinate or indeterminate.

In our case, as it is common in this literature, we have used (three) lags of π_t, x_t and the interest rate, i_t as instruments where i_t is the 3-month US Federal funds interest rate. However, since i_t does not appear in (1), both π^e_{t+1} and x_t may not depend on lags of i_t, which would make i_t useless as an instrument. To evaluate whether or not lagged interest rates are suitable instruments, we estimated the following sub-VAR model:

$$\begin{aligned} x_t &= b_0 + b_1\pi_{t-1} + b_2 x_{t-1} + b_3 i_{t-1} + u_{xt}, \\ i_t &= c_0 + c_1\pi_{t-1} + c_2 x_{t-1} + c_3 i_{t-1} + u_{ct}, \end{aligned} \tag{3}$$

where u_{xt} and u_{ct} are i.i.d. error terms, which are potentially correlated with e_t. If $b_3 = 0$, i.e., i_t does not Granger cause x_t, then lags of i_t are not valid instruments for the endogenous variables in (1).

Whether lagged values of inflation and the real variable beyond order one (i.e., π_{t-2}, π_{t-3}, x_{t-2} and x_{t-3}) are valid instruments for π^e_{t+1} is also questionable. If the solution for π_t only depends on π_{t-1} and x_{t-1}, which is the case when the solution is determinate, then the additional lags are not valid instruments. However, in case of indeterminacy additional lags of π_t and x_t matter, which re-establishes the validity of π_{t-2}, π_{t-3}, x_{t-2} and x_{t-3} as instruments. More details on this issue are provided in Section 5.

In the - for identification - "worst case" scenario of a determinate solution and $b_3 = 0$ in (3), we are left with only x_{t-1} as a valid instrument for π^e_{t+1} and x_t (since π_{t-1} is a regressor in (1)), so that the structural equation is underidentified. With a determinate solution and $b_3 \neq 0$, both x_{t-1} and i_{t-1} are valid instruments for π^e_{t+1} and x_t, which makes the model exactly identified. With an indeterminate solution and $b_3 = 0$, π_{t-2}, π_{t-3}, x_{t-1}, x_{t-2} and x_{t-3} are in general valid instruments because often it is possible to find an equivalent transformation of the rational expectations solution that is free of expectations variables. Instead the solution has a higher order of dynamics, i.e. longer lags in the predetermined variables and moving average errors. (see e.g. Beyer and Farmer (2005)). In that case there are three overidentifying restrictions. Finally, with an indeterminate solution and $b_3 \neq 0$, π_{t-2}, π_{t-3}, x_{t-1}, x_{t-2}, x_{t-3}, i_{t-1}, i_{t-2} and i_{t-3} are in general valid instruments, which leads to six overidentifying restrictions.

As a consequence of the model dependence with respect to the number of valid instruments , the Hansen's J-statistic, a popular measure for the validity of the instruments and overidentifying restrictions that we also present for conformity to the literature, can be potentially uninformative and even misleading when applied in a forward looking context.

Estimating (1) and (3) using only one lag of π, x, and i as instruments, we find that $b_3 \neq 0$ but the null hypothesis $b_3 = 0$ cannot be rejected. In this case, since the instruments are only weakly correlated with their targets, the resulting GMM estimators can suffer from weak identification. This might lead to non-standard distributions for the estimators and can yield misleading inference, see e.g. Stock, Wright and Yogo (2002). Empirically, we find that the size of the standard errors for the estimators of the parameters α_1 and α_2 in (1) matches the estimated values for α_1 and α_2.

However, when we estimate (1) and (3) using three lags of π, x, and i as instruments, we find that $b_3 \neq 0$ but the null hypothesis $b_3 = 0$ is strongly rejected. The estimated parameters for (1) are reported in the first panel in column 2 of Tables 1 and 2. Compared with the corresponding single equation estimates we find that the point estimates of the parameters are basically unaffected (there is a non-significant decrease of about 5% in the coefficient of π^e_{t+1} and a corresponding increase in that of π_{t-1}). Yet, there is a substantial reduction in the standard errors of 30-40%. Similar results are obtained when (3) is substituted for a VAR(3) specification. These findings suggest that the model is identified, but the solution could be indeterminate. Intuitively, indeterminacy arises because the sum of the estimated parameters α_1 and α_3 in (1) is very close to one; a more formal analysis of identification is provided in Section 5.

So far the processes for the forcing variables was assumed to be purely backward looking. As an alternative we consider a forward looking model also for x_t. For example, Fuhrer and Rudebusch (2002) estimated a model for a representative agent's Euler equation (in their notation)

$$x_t = \beta_0 + \beta_1^* x^e_{t+1} + \beta_2^* (\frac{1}{k} \sum_{j=0}^{k-1} (i^e_{t+j} - \pi^e_{t+j+1})) + \beta_3^* x_{t-1} + \beta_4^* x_{t-2} + \eta_t, \quad (4)$$

where x_t is real output (detrended in a variety of ways), x^e_{t+1} is the forecast of x_{t+1} made in period t, $i_t - \pi^e_{t+1}$ is a proxy for the real interest rate at time t, and η_t is an i.i.d. $(0, \sigma^2_\eta)$ error term. In our sample period, the second lag of x is not significant and only the current interest rate matters. Hence, the model becomes

$$x_t = \beta_0 + \beta_1 x^e_{t+1} + \beta_2 (i_t - \pi^e_{t+1}) + \beta_3 x_{t-1} + \eta_t, \tag{5}$$

and for x we use, again, either unemployment, or the GDP gap. Replacing the forecast with its realized value, we get

$$x_t = \beta_0 + \beta_1 x_{t+1} + \beta_2 (i_t - \pi_{t+1}) + \beta_3 x_{t-1} + \mu_t, \tag{6}$$

where $\mu_t = \beta_1 (x^e_{t+1} - x_{t+1}) + \beta_2 (\pi^e_{t+1} - \pi_{t+1})$.

As in the case of the New-Keynesian Phillips curve, this equation can be estimated by GMM, appropriately corrected for the presence of an MA component in the error term μ_t. As in our estimates of the New-Keynesian Phillips curve, we use three lags of x, i and π as instruments. The results are reported in the first column of the second panel of Table 1 (for x_t as the unemployment rate) and Table 2 (for x_t as the output gap). In both cases the coefficient on x^e_{t+1} is slightly larger than 0.5 and significant, and the coefficient on x_{t-1} is also close to 0.5 and significant. These values are in line with those in Fuhrer and Rudebusch (2002), who found lower values when using ML estimation rather than GMM and the positive sign of the real interest in the equation for the output gap is similar to the Fuhrer-Rudebusch results when they used HP de-trending.

As with the New-Keynesian Phillips curve, we estimate Equation (6) simultaneously together with a sub-VAR(1) as in (3), but here for the forcing variables π_t and i_t. Again, the significance of the coefficients in the VAR(1) equations (in particular those for lagged π_t in the i_t equation) lends support to their validity as instruments. The numerical values of the estimated parameters for the Euler equation remain nearly unchanged. However, as in the case of the Phillips curve above, the precision of the estimators increases substantially. These results are reported in the second column of the second panel in Table 1 (for unemployment) and Table 2 (for the GDP gap).

In order to complete our building blocks for a forward looking system we finally also model the interest rate with a Taylor rule as in Clarida, Galí and Gertler (1998, 2000). Our starting point here is the equation

$$i_t^* = \bar{i} + \gamma_1(\pi_{t+1}^e - \pi_t^*) + \gamma_2(x_t - x_t^*), \tag{7}$$

where i_t^* is the target nominal interest rate, $\bar{i}$ is the equilibrium rate, x_t is real output, and π_t^* and x_t^* are the desired levels of inflation and output. The parameter γ_1 indicates whether the target real rate adjusts to stabilize inflation ($\gamma_1 > 1$) or to accommodate it ($\gamma_1 < 1$), while γ_2 measures the concern of the central bank for output stabilization.

Following the literature, we introduce a partial adjustment mechanism of the actual rate to the target rate i^* :

$$i_t = (1 - \gamma_3)i_t^* + \gamma_3 i_{t-1} + v_t, \tag{8}$$

where the smoothing parameter γ_3 satisfies $0 \leq \gamma_3 \leq 1$, and v_t is an i.i.d. $(0, \sigma_v^2)$ error term. Combining (7) and (8), we obtain

$$i_t = \gamma_0 + (1 - \gamma_3)\gamma_1(\pi_{t+1}^e - \pi_t^*) + (1 - \gamma_3)\gamma_2(x_t - x_t^*) + \gamma_3 i_{t-1} + v_t \tag{9}$$

where $\gamma_0 = (1 - \gamma_3)\bar{i}$, which becomes

$$i_t = \gamma_0 + (1 - \gamma_3)\gamma_1(\pi_{t+1} - \pi_t^*) + (1 - \gamma_3)\gamma_2(x_t - x_t^*) + \gamma_3 i_{t-1} + \epsilon_t, \tag{10}$$

with $\epsilon_t = (1 - \gamma_3)\gamma_1(\pi_{t+1}^e - \pi_{t+1}) + v_t$, after replacing the forecasts with their realized values

The results for single equation GMM estimation (with 3 lags as instruments) are reported in the first column of the third panel of Tables 1 and 2. As in Clarida et al. (1998, 2000), the coefficient on future inflation is larger than one. We also found the coefficient on output to be larger than one, although the standard errors around both point estimates are rather large. Again, as in the cases of single equation estimations of the Phillips curve and the Euler equation, we are able to reduce the variance of our point estimates by adding sub-VAR(1) equations for the forcing variables π_t and x_t when estimating the resulting system by GMM (see column 2). As above, for both approaches we have used up to three lags for the intrument variables.

We are now in a position to estimate the full forward looking system, composed of Equations (1), (5) and (9):

$$\begin{aligned} \pi_t &= \alpha_0 + \alpha_1 \pi^e_{t+1} + \alpha_2 x_t + \alpha_3 \pi_{t-1} + e_t, \qquad (11) \\ x_t &= \beta_0 + \beta_1 x^e_{t+1} + \beta_2 (i_t - \pi^e_{t+1}) + \beta_3 x_{t-1} + \eta_t, \\ i_t &= \gamma_0 + (1-\gamma_3)\gamma_1(\pi^e_{t+1} - \pi^*_t) + (1-\gamma_3)\gamma_2(x_t - x^*_t) + \gamma_3 i_{t-1} + v_t \end{aligned}$$

The results are reported in column 3 of Tables 1 and 2. For each of the three equations the estimated parameters are very similar to those obtained either in the single equation case or in the systems completed with VAR equations. Furthermore, the reductions in the standard errors of the estimated parameters are similar to those obtained with sub-VAR(1) specifications. Since the VAR equations can be interpreted as reduced forms of the forward looking equations, this result suggests that completing a single equation of interest with a reduced form may be enough to achieve as much efficiency as within a full system estimation. However, the full forward looking system represents a more coherent choice from an econometric point of view, and the finding that the forward looking variables have large and significant coefficients in all the three equations lends credibility to the complete rational expectations model.

The nonlinearity of our system of forward looking equations makes the evaluation of global identification impossible. However, if we linearize the model around the estimated parameters and focus on local identification, we can show later on in Section 5 that the model is (at least) exactly identified. Exact identification holds when the point estimates imply a determinate solution. The model would be potentially overidentified in case of an indeterminate equilibrium.

3 Robustness analysis

While system estimation increases efficiency, the full forward looking model in (11) could still suffer from mis-specification problems. To evaluate this possibility, we conducted four types of diagnostic tests. First, we ran an LM test on the residuals of each equation to check for additional serial correlation i.e. serial correlation beyond the one that is due to the MA(1) error structure of the model. Second, we ran the Jarque and Bera normality test on the estimated errors. Although our GMM estimation approach is robust to the presence of non-normal errors,[3] rejection of normality could signal other problems, such as the presence of outliers or parameter instability. Third, we ran an LM test to check for the presence of ARCH effects; rejection of the null of no ARCH effects might more generally be a signal of changes in the variance of the errors. Finally, we checked for parameter constancy by running recursive estimates of the forward looking system.

The results of our mis-specification tests are reported in the bottom lines of each panel in Tables 3 and 4. For convenience, we also present in column 1 again the estimated parameters. When unemployment is used (Table 3) there

[3] Note that this is not the case for maximum likelihood estimation.

are only minor problems of residual correlation in the inflation equation, but normality and no ARCH are strongly rejected in all of the three equations. The outcome of the tests is slightly better with the GDP gap (Table 4), but normality is still strongly rejected for the inflation and interest rate equations, and the interest rate equation also fails the test for absence of serial correlation and absence of ARCH.

The rejection of correct specification could be due to parameter instability in the full sample 1970:3 - 1998:4. Instability might be caused by a variety of sources including external events such as the oil shocks, internal events, such as the reduction in the volatility of output (e.g. McConnell and Perez-Quiros (2000)), or changes in the monetary policy targets. Since we had more faith in the second part of our sample, we implemented a backward recursion by estimating the system first for the subsample 1988:1-1998:4, and recursively reestimating the system by adding one quarter of data to the beginning of the sample, i.e. our second subsample consisted of the quarters 1987:4–1998:4, our third was 1987:3 – 1998:4 and so on until 1970:3-1998:4.

In Figures 1 and 2, we report recursive parameter estimates. These figures confirm that the likely source of the rejection of ARCH, normality and serial correlation tests is the presence of parameter change. Although the parameter estimates are stable back to 1985:1, going further back than this is associated with substantial parameter instability in all three equations, and particularly in the estimated Taylor rule. Although parameter instability is more pronounced when we use unemployment as a measure of economic activity, it is also present in estimates obtained when using the output gap.

Overall, these mis-specification tests cast serious doubts on results obtained for the full sample, and they suggest that a prudent approach would be to restrict our analysis to a more homogeneous sample. For this reason, in the subsequent analysis, we report results only for the subperiod 1985:1-

1998:4.

Our subsample results are presented in the second column of Tables 3 and 4. It is interesting to note that the values of the estimated parameters of the New-Keynesian Phillips curve and the Euler equation are similar to those obtained for the full sample. However, parameter estimates of the coefficients of the Taylor rule differ substantially from the single equation estimates. Table 3 shows that (using unemployment as a measure of economic activity) restricting parameter estimates to the post 1985 subsample caused the estimated coefficient on future inflation to increase substantially. Table 4, (using the output gap) shows instead a marked decrease in the estimated coefficient on the output gap. In the post 1985 subsample we fail to reject the null hypothesis for all four of our diagnostic tests, thereby lending additional credibility to our estimation results.

The final issue we briefly consider is the role of the method of estimation. Fuhrer and Rudebusch (2002), Lindé (2003) and Jondeau and Le Bihan (2003) have suggested that GMM may lead to an upward bias in the parameters associated with the forward looking variables, while maximum likelihood (ML) produces more robust results. In case of exact identification ML coincides with indirect least squares. We compared our estimates with the point estimates from GMM by computing the indirect least squares estimates from the reduced form. Using this approach, we find that our GMM estimates are similar to the ML values.

For the subsample 1985-1998, using unemployment as the activity variable, the estimated coefficient on π^e_{t+1} in the inflation equation is 0.73 and that on u^e_{t+1} in the unemployment equation is 0.64. The corresponding values using GMM are 0.73 and 0.51. Using the output gap, the ML estimates become 0.76 for the coefficient on π^e_{t+1} in the inflation equation and 0.62 for that on future expected output gap in the Euler equation whereas the GMM estimates of these parameters are, respectively, 0.61 and 0.47. The differences are slightly larger for the coefficient on future inflation in the Taylor rule, in the range $2.1 - 2.4$ with ML. Overall we are reassured that our finding of significant coefficients on future expected variables is robust to alternative system estimation methods.

4 Enlarging the information set

The analysis in Sections 2 and 3 supports the use of a system approach to the estimation of forward looking equations. For the 1985:1–1998:4 sample, our estimated system passes a wide range of mis-specification tests. Moreover, the Hansen's J-statistic, reported at the foot of Tables 3 and 4, is unable to reject the null of valid instruments for this period (but it is worth recalling the caveats on the use of the J-test in this context). However, there could still be problems of weak instruments and/or omitted variables which are hard to detect using standard tests, (see e.g. Mavroeidis (2002)). This section proposes a method that can potentially address both of these issues.

Our approach is to augment our data by adding information extracted from a large set of 146 macroeconomic variables as described in Stock and Watson (2002a, 2002b, SW). We assume that these variables are driven by a few common forces, i.e. the factors, plus a set of idiosyncratic shocks. This assumption implies that the factors provide an exhaustive summary of the information in the large dataset, so that they may alleviate omitted variable problems when used as additional regressors in our small system. Moreover, the factors extracted from the Stock and Watson data are known to have good forecasting performance for the macroeconomic variables in our small dataset and they are therefore likely to be useful as additional instruments that may alleviate weak instrument problems, too.

Bernanke and Boivin (2003) and Favero, Marcellino and Neglia (2004) showed that when estimated factors are included in the instrument set for GMM estimation of Taylor rules, the precision of the parameter estimators increases substantially. The economic rationale for inclusion of these variables is that central bankers rely on a large set of indicators in the conduct of monetary policy; our extracted factors may provide a proxy for this additional information. An additional reason for being interested in the inclusion of factors in our analysis is that, the inclusion of factors in small scale VARs has been shown to remove the "price puzzle" suggesting that factors may be used to reduce or eliminate the estimation bias, that arises from the omission of relevant right-hand-side variables.[4]

In the following subsection, we present a brief overview on the specification and estimation of factor models for large datasets. Following this discussion, we evaluate whether the use of the estimated factors changes the size and or the significance of the coefficients of the forward looking components in the New Keynesian model.

4.1 The factor model

Equation (12) represents a general formulation of the dynamic factor model

$$z_t = \Lambda f_t + \xi_t, \tag{12}$$

where z_t is an $N \times 1$ vector of variables and f_t is an $r \times 1$ vector of common factors. We assume that r is much smaller than N, and we represent the effects of f_t on z_t by the $N \times r$ matrix Λ. ξ_{it} is an $N \times 1$ vector of idiosyncratic shocks.

Stock and Watson require the factors, f_t, to be orthogonal although they

[4] For a definition and discussion of this issue the reader is referred to Christiano, Eichenbaum and Evans (1999) pages 97–100.

may be correlated in time and with the idiosyncratic components for each factor.[5] Notice that the factors are not identified since Equation (12) can be rewritten as

$$z_t = \Lambda G G^{-1} f_t + \xi_t = \Psi p_t + \xi_t,$$

where p_t is an alternative set of factors and G is an arbitrary invertible $r \times r$ matrix. This fact makes it difficult to form a structural interpretation of the factors, but it does not prevent their use as a summary of the information contained in z_t.

SW define the estimators $\widehat{f}_t$ as minimizing the objective function

$$V_{N,T}(f, \Lambda) = \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} (z_{it} - \Lambda_i f_t)^2.$$

Under the hypothesis of r common factors, they show that the optimal estimators of the factors are the r eigenvectors corresponding to the r largest eigenvalues of the $T \times T$ matrix $N^{-1} \sum_{i=1}^{N} \underline{z}_i \underline{z}_i'$, where $\underline{z}_i = (z_{i1}, ..., z_{iT})$. Moreover, the r eigenvectors corresponding to the r largest eigenvalues of the $N \times N$ matrix $T^{-1} \sum_{t=1}^{T} z_t z_t'$ are the optimal estimators of Λ. These eigenvectors coincide with the principal components of z_t; they are also the OLS estimators of the coefficients in a regression of z_{it} on the k estimated factors $\widehat{f}_t$, $i = 1, ..., N$.[6] Although there are alternative estimation methods available, we chose the SW approach since there is some evidence to suggest that it dominates the alternatives in this context.[7]

[5] Precise moment conditions on f_t and ξ_t, and requirements on the loading matrix Λ, are given in SW.

[6] SW prove that when r is correctly specified, $\widehat{f}_t$ converges in probability to f_t, up to an arbitrary $r \times r$ transformation matrix, G. When k factors are assumed, with $k > r$, $k - r$ estimated factors are redundant linear combinations of the elements of f_t, while even when $k < r$ consistency for the first k factors is preserved (because of the orthogonality hypothesis). See Bai (2003) for additional inferential results.

[7] Forni, Hallin Lippi and Reichlin (2000) have developed an alternative frequency do-

No statistical test is currently available to determine the optimal number of factors. SW and Bai and Ng (2002) suggested minimizing a particular information criterion, however its small sample properties in the presence of heteroskedastic idiosyncratic errors deserves additional investigation. In their empirical analysis with this data set, SW found that the first 2-3 factors are the most relevant for forecasting key US macroeconomic variables. In the following analysis we however evaluate the role of up to six factors to make sure sufficient information is captured.

4.2 The role of the estimated factors

As we mentioned, the estimated factors can proxy for omitted variables in the specification of the forward looking equations. In particular, we use up to six contemporaneous factors as additional regressors in each of the three structural equations, and retain those which are statistically significant.

Since the factors are potentially endogenous, we use their first lag as additional instruments. These lags are likely to be useful also for the other endogenous variables in each structural equation.

In column 3 of Table 3 we report the results of GMM estimation of the forward looking system over the period 1985-1998 using unemployment as the activity variable, and in column 3 of Table 4 those using the GDP gap.

First, a few factors are strongly significant in the equations for inflation and the real variable. While it is difficult to provide an economic interpretation for this result, it does point to the omission of relevant regressors in the Phillips curve and Euler equation. In contrast, no factors are significant in the Taylor rule, which indicates that output gap and inflation expectations

main estimator. However, Kapetanios and Marcellino (2003) found that SW's estimator performs better in simulation experiments, and Favero et al. (2004) reached the same conclusion when using the estimated factors for the estimation of Taylor rules and VARs.

are indeed the key driving variables of monetary policy over this period.

Second, in general the estimated parameters of the forward looking variables are 10 to 20% lower than those without factors, but they remain strongly statistically significant.

Third, the precision of the estimators systematically increases, as the standard errors of the estimated parameters are 10 to 50% lower than those without the factors. This confirms the usefulness of the additional information contained in the factors.

Fourth, since the highest lag order of the regressors in the structural model is one, it could suffice to include one lag of π_t, x_t, and i_t in the instrument set instead of three lags. In this case, the point estimates are unaffected, as expected, but the standard errors increase substantially. This finding suggests that the solution of the system could be indeterminate, in which case more lags would indeed be required.

Finally, since there is no consensus on the best way to compute robust standard errors in this context, we verified the robustness of our findings based on Newey West (1994) comparing them with those based on Andrews (1991). The latter are in general somewhat lower, but the advantages resulting from the use of factors are still systematically present.

5 An analysis of identification and determinacy

This section analyzes two issues that are related to the internal consistency of the New-Keynesian model studied in Sections 2 - 4; we study the identification of the parameters in our estimated equations and we ask, given our point estimates, if the implied system leads to a determinate economic model. The first is an econometric issue: Are the coefficients in each of our three equations identified? The second is an economic issue: What is the appropriate interpretation of our estimates for the conduct of monetary policy? We turn first, to the question of identification.

5.1 An analysis of identification

Mavroeidis (2002), Bårdsen, Jansen and Nymoen (2003), and Nason and Smith (2003) have criticized the single-equation approach to the estimation of forward looking models based on the earlier work of Pesaran (1987). These authors pointed out that GMM estimates of single-equation rational expectations models only make sense if the equations are identified. In this section we provide a formal analysis of identification within a fully articulated three-equation rational expectations model. As mentioned in Section 2, global identification for this model cannot be tested due to its non-linear specification. But we can demonstrate that, under the given coefficient estimates, the model is locally identified. We introduce a notation for indexing each equation within a matrix representation of the model. To this end, let $Y_t = (\pi_t,\ x_t,\ i_t)'$ be the vector of endogenous variables consisting of inflation, a measure of economic activity (unemployment or the output gap) and the interest rate, respectively, and let $E_t\left(Y_{t+1}\right)$ be the expectation of the realization of Y_{t+1} under the assumption that the model is a correct representation of the time series process for all of the endogenous variables. The models that we estimated in Sections 2 - 4 are three-equation *structural models* with the form:

$$\underset{(3\times3)}{\mathbf{A}}\underset{(3\times1)}{Y_t} + \underset{(3\times3)}{\mathbf{F}}\underset{(3\times1)}{E_t\left[Y_{t+1}\right]} = \underset{(3\times3)}{\mathbf{B}}\underset{(3\times1)}{Y_{t-1}} + \underset{(3\times1)}{\mathbf{\Phi}} C + \underset{(3\times1)}{V_t}, \tag{13}$$

which can be written more compactly as follows

$$\begin{bmatrix} \mathbf{A} & \mathbf{F} \end{bmatrix} \begin{bmatrix} Y_t \\ E_t[Y_{t+1}] \end{bmatrix} = \mathbf{B}Y_{t-1} + \mathbf{\Phi}C + V_t, \tag{14}$$

The matrices $\mathbf{A}$ and $\mathbf{F}$ contain coefficients of the endogenous variables Y_t and $E_t[Y_{t+1}]$ and the matrix $\mathbf{B}$ represents coefficients of the predetermined variables Y_{t-1}. The term C represents a vector of constants. In the following analysis, we drop C and interpret the variables Y_t and Y_{t-1} as deviations from means.

Since $E_t[Y_{t+1}]$ represent expectations formed at date t, they should be considered as distinct endogenous variables. The complete system has six endogenous variables; the three elements of the vector Y_t plus the three expectations of Y_{t+1} at date t. To close the system we need three additional equations which, under the rational expectations assumption, are provided by the forecast equations

$$E_{t-1}[Y_t] - Y_t = W_t,$$

where the W_t are additional non fundamental errors that may or may not be exact functions of the fundamental errors, V_t. Let us now check for local identification in the determinate case. The solution of model (13) is:

$$Y_t = \Pi Y_{t-1} + V_t.$$

In the determinate case the matrix Π is 3-by-3 and is identical to the reduced form. Since

$$Y_{t+1} = \Pi Y_t$$

and

$$\begin{aligned} \mathbf{A}Y_t + \mathbf{F}\Pi Y_t &= \mathbf{B}Y_{t-1} + V_t \\ (\mathbf{A} + \mathbf{F}\Pi)Y_t &= \mathbf{B}Y_{t-1} + V_t \\ (\mathbf{A} + \mathbf{F}\Pi)\Pi Y_{t-1} &= \mathbf{B}Y_{t-1} + V_t \end{aligned}$$

and therefore

$$(\mathbf{A}+\mathbf{F}\Pi)\Pi=\mathbf{B}. \tag{15}$$

To fulfil the order condition for identification of the structural parameters in $\mathbf{A},\mathbf{F}$ and $\mathbf{B}$ the number of free structural parameters must not exceed the number of parameters in Π. We have imposed the following restrictions on the matrices $\mathbf{A},\mathbf{F}$ and $\mathbf{B}$:

$$\overset{[\mathbf{A}\quad\mathbf{F}\quad\mathbf{B}]}{\begin{bmatrix} 1 & a_{12} & a_{13}=0 & f_{11} & f_{12}=0 & f_{13}=0 & b_{11} & b_{12}=0 & b_{13}=0 \\ a_{21}=0 & 1 & a_{23} & f_{21}=-a_{23} & f_{22} & f_{23}=0 & b_{21}=0 & b_{22} & b_{23}=0 \\ a_{31}=0 & a_{32} & 1 & f_{31} & f_{32}=0 & f_{33}=0 & b_{31}=0 & b_{32}=0 & b_{33} \end{bmatrix}}$$

Row 1 of this matrix represents the New-Keynesian Phillips curve. The unit entry in the first column indicates that this equation is normalized on inflation and the zero entry in the third column indicates that the interest rate does not enter the equation. The other rows have similar interpretations. For example, row 2 which represents the Euler equation is normalized by setting the coefficient on x_t to unity. The equality restriction, $f_{21}=-a_{23}$, imposes the same coefficient on the nominal interest rate and the negative of expected future inflation; in words, this restriction means that expected inflation and the nominal interest rate only affect the Euler equation through their effect on the expected real interest rate. Notice that we have imposed exactly six linear restrictions in each equation which implies that, in each case, the order condition is exactly satisfied.

To check the rank condition locally we apply the inverse mapping theorem to equation (15) and take the total differential:

$$\begin{aligned} dA(\Pi)+Ad\Pi+dF(\Pi)(\Pi)+Fd\Pi(\Pi)+F\Pi d\Pi &= dB \\ dA(\Pi)+(A+F\Pi)d\Pi+dF\Pi^2+Fd\Pi(\Pi) &= dB. \end{aligned}$$

We then solve for $d\mathbf{A}$, $d\mathbf{F}$ and $d\mathbf{B}$, given the fixed parameters of the models estimated in Sections 2 - 4. We demonstrate in Appendix A that each $d\mathbf{A}$, $d\mathbf{F}$ and $d\mathbf{B}$ is a function only of fixed $\mathbf{A}, \mathbf{F}, \mathbf{B}$ and $\mathbf{\Pi}$ and of changes in the reduced form $d\mathbf{\Pi}$ but not of changes in the structural parameters.

5.2 An analysis of determinacy and indeterminacy

In this section we check the dynamic properties of the model when theoretical values of the parameters are replaced by their point estimates. This check is important since, if the estimated system is to be useful as a policy guide, the individual pieces must add up to a coherent whole that can be used to provide an economic explanation of the causes of real-monetary interactions over the estimation period. The dominant current explanation provided by Clarida et. al. and substantiated by Boivin and Giannoni (2003, BG) and Lubik and Schorfheide (2004, LS) is that in the period after 1980 the New-Keynesian model was driven by an active monetary policy that led to a determinate equilibrium. We summarize the concept of determinacy briefly below. Following this summary, we study the dynamic properties of New-Keynesian model when we replace theoretical coefficient values by our point estimates and we compare our results with those of CGG, BG and LS.

Since rational expectations have variables that are forward looking, the mapping from the structural to the reduced form is more complicated than in standard Cowles Commission econometrics. The reduced form of the model is a set of equations, one for each endogenous variable, that explains the time paths of each variable as a function of exogenous and predetermined variables. The mapping from the structural model to the reduced form may or may not be unique. It is also possible that no stationary reduced form exists. If the mapping is unique we say that the model is determinate; if it is non-unique the model is indeterminate and if no stationary solution exists the reduced form is non-existent.

The determinacy properties of our model may be analyzed by first writing the system in its *companion* form;

$$\overset{\mathbf{A}_0}{\begin{bmatrix} \mathbf{A} & \mathbf{F} \\ \mathbf{I} & \mathbf{0} \end{bmatrix}} \overset{Y_t^*}{\begin{bmatrix} Y_t \\ E_t\left[Y_{t+1}\right] \end{bmatrix}} = \overset{\mathbf{A}_1}{\begin{bmatrix} \mathbf{B} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}} \overset{Y_{t-1}^*}{\begin{bmatrix} Y_{t-1} \\ E_{t-1}\left[Y_t\right] \end{bmatrix}} + \overset{\mathbf{P}_v}{\begin{bmatrix} \mathbf{I} \\ \mathbf{0} \end{bmatrix}} \left[V_t\right] + \overset{\mathbf{P}_w}{\begin{bmatrix} \mathbf{0} \\ \mathbf{I} \end{bmatrix}} \left[W_t\right], \qquad (16)$$

More compactly,

$$\underset{(6\times6)}{\mathbf{A}_0} \underset{(6\times1)}{Y_t^*} = \underset{(6\times6)}{\mathbf{A}_1} \underset{(6\times1)}{Y_{t-1}^*} + \underset{(6\times3)}{\mathbf{P}_v} \underset{(3\times1)}{V_t} + \underset{(6\times3)}{\mathbf{P}_w} \underset{(3\times1)}{W_t}. \qquad (17)$$

Elements of the 3×1 vector V_t are called fundamental errors and the elements of the 3×1 vector W_t are non-fundamental errors. The non-fundamental errors may be functions of the fundamental errors or, if the model is indeterminate, they may be independently determined. In this case they represent separate 'sunspot' shocks.

The reduced form of the model is found by choosing the expectations variables at date t to eliminate any unstable roots. This procedure may, or may not, eliminate the influence of the endogenous errors, W_t and it leads to a representation of the form

$$Y_t^* = \widetilde{\mathbf{A}} Y_{t-1}^* + \tilde{\mathbf{P}}_v V_t + \tilde{\mathbf{P}}_w W_t.$$

There are three possible cases to consider, all of which may occur in practice. For the case when $\mathbf{A}_0$ is non-singular, these cases may be enumerated by comparing the number of unstable roots of the matrix $\mathbf{A}_0^{-1}\mathbf{A}_1$ with the number of non-predetermined initial conditions. In the singular case, as occurs in our example, they involve a comparison of the generalized eigenvalues of $(\mathbf{A}_0, \mathbf{A}_1)$ with the number of expectations of future endogenous variables.[8] For our example there are three of these. If there are more than three unstable generalized eigenvalues then no stable rational expectations solution exists. If there are exactly three unstable generalized eigenvalues then there is a unique rational expectations equilibrium and the model is said to be determinate. In this case the matrix $\widetilde{\mathbf{A}}$ has rank 3 and $\tilde{\mathbf{P}}_w$ is identically zero. If there are $m < 3$ unstable generalized eigenvalues then $\widetilde{\mathbf{A}}$ has rank $(6 - m)$, $\tilde{\mathbf{P}}_w$ has rank m and the model is said to possess m degrees of indeterminacy. In this last case the model can be closed by specifying a given covariance matrix for $[V_t, W_t]$ and interpreting the W_t as non-fundamental, or 'sunspot' shocks that may be correlated with the fundamentals. Table 5 summarizes the implications of our point estimates for the determinacy properties of the data generating process under alternative estimation schemes and alternative models. Starting with the model without factors, there seems to be either no stationary rational expectations equilibrium or an indeterminate equilibrium, depending on the estimation method. The result is robust to the adoption of either unemployment or the output gap as the scale variable.

The determinacy of a model is a property of the system as a whole and it is sensitive to the model specification.[9] Following Clarida-Galí-Gertler (1998), a number of authors have estimated systems or partial systems of equations similar to those in this paper and they have used these estimates

[8]For a description of how the Schur decomposition can be used to solve linear rational expectations models, the reader is referred to Sims (2001). For a survey of indeterminacy and sunspots in macroeconomics, see Benhabib and Farmer (1999).

[9]Beyer and Farmer (2003a) conduct a systematic search of the parameter space in a model closely related to the one studied in this paper. They sample from the asymptotic parameter distribution of the GMM estimates and find, for typical identification schemes, that point estimates lie in the indeterminate region, but anywhere from 5% to 20% of the parameter region may lie in the non-existence or determinate region.

to infer the determinacy properties of the U.S. data. A consistent conclusion that arises in this literature is that the data before 1980 appears consistent with an indeterminate equilibrium partly driven by sunspots and the data after 1980 is well characterized by a determinate equilibrium in which only fundamental shocks influence the data-generating-process.

Clarida-Galí Gertler (1998) estimate a policy rule and embed it into a calibrated model. Later, full system estimates by Boivin and Giannoni (2003) and Lubik and Schorfheide (2004) confirm these determinacy findings. All of these authors either impose the values of some of the key parameters or they use Bayesian estimators in which parameter values are strongly influenced by priors. A likely source of divergence of our results from theirs, is that we allow all of the parameters of the model to be freely estimated. Since our results contradict the received wisdom, we conducted a sensitivity analysis to investigate this conjecture.

One of the main differences of our estimates from those of previous literature is the low estimated value of β_2, the real interest rate coefficient in the Euler Equation. Using the output gap as the scale variable led to an estimated value for β_2 of -0.021. However, this estimate is highly imprecise with a standard error of 0.015. Since β_2 is a key parameter for the determinacy properties of the model, we checked these properties for values of the real interest rate parameter that were larger but still within two standard errors of the point estimate. When we increased the absolute value of β_2 to -0.03 (preserving the negative sign) we found that the model has a unique determinate rational expectations equilibrium. For the case of unemployment as a scale variable our findings were similar. In this case, the estimated parameter has the wrong sign and although it is small, -0.006, the estimates are, in this case, more precise.[10] However, by imposing a value for α_2 of

[10]When unemployment is the scale variable, the sign of the real interest coefficient in

+0.011, in line with economic theory, we were able to restore determinacy of equilibrium.[11]

As an alternative to a priori restricting the parameters, the use of factors as additional regressors and instruments can yield a determinate solution, at least in the case where unemployment is used as the real variable in the system, see the last column of Table 5.

We conclude from our study that evaluating the determinacy properties of our model is difficult since minor changes in the parameter values can move the solution from the unstable region to the determinate or even indeterminate regions. However, with few simple and reasonable constraints on the parameters, or using the factors, data after 1985 is not inconsistent with the New-Keynesian interpretation of a determinate equilibrium driven by three fundamental shocks. The estimated parameters of the complete model form a consistent picture which coincides with New-Keynesian economic theory.[12]

6 Conclusions

In this paper we provided a general econometric framework for the analysis of models with rational expectations, focusing in particular on the hybrid version of the New-Keynesian Phillips curve that has attracted considerable attention in the recent period.

First, we showed that system estimation methods where the New-Keynesian

the Euler equation is predicted to be positive rather than negative.

[11] Another way to achieve determinacy is to constraint the parameters of x^e_{t+1} and x_{t-1} in the output equation to sum to a value sufficiently smaller than one.

[12] We should note however, that the New-Keynesian explanation is one of many interpretations of the same data set that are consistent with the time-series properties of the data. Beyer and Farmer (2003b) show that there are alternative exactly identified models that can explain the data equally well as the New-Keynesian model, but which have different policy implications.

Phillips curve is complemented with equations for the interest rate and either unemployment or the output gap yield more efficient parameter estimates than traditional single equation estimation, while there are only minor changes in the point estimates and the expected future variables play an important role in all the three equations. The latter result remains valid even if MLE is used rather than system GMM.

Second, we stressed that it is important to evaluate the correct specification of the model, and we showed that our systems provide a proper statistical framework for the variables over the 1985-1998 period, while during the '70s there is evidence of parameter changes, in particular in the interest rate equation.

Third, we analyzed the role of factors that summarize the information contained in a large data set of U.S. macroeconomic variables. Some factors were found to be significant as additional regressors in the New-Keynesian Phillips curve and in the Euler equation, alleviating potential omitted variable problems. Moreover, using lags of the factors as additional instruments in our small New-Keynesian system, the standard errors of the GMM estimates systematically decrease for all the estimated parameters; the gains are particularly large for the coefficients of forward looking variables. In addition, the use of factors can influence the characteristics of the equilibrium.

Fourth, we demonstrated that our GMM procedures were well defined and the equations we estimated are identified. The point estimates of our system form a coherent whole that has dynamic properties that are similar to the systems estimated (and calibrated) by Clarida et. al., Boivin and Giannoni, and Lubik and Schorfheide. If we impose prior information, as to these earlier studies, we find that the system after 1980 is associated with a unique determinate equilibrium driven solely by shocks to fundamentals. However, we detected substantial uncertainty on the characteristics of the equilibrium, which suggests that existing interpretations of the data are fragile, and are sensitive to the priors of the researcher.

In conclusion, we should note that while our results support the relevance of forward looking variables in our estimated equations there is a large variety of alternative models compatible with the observed data which can have very different properties both in terms of the relevance of the forward looking variables and of the characteristics of their dynamic evolution. A more detailed analysis of this issue represents an interesting topic for further research in this field.

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Appendix A

In this Appendix we demonstrate that each parameter of the structual model is at least exactly (locally) identified. For fixed $\mathbf{A}, \mathbf{F}$ and $\mathbf{B}$ each $d\mathbf{A}$, $d\mathbf{F}$ and $d\mathbf{B}$ is a function only of the fixed $\mathbf{A}, \mathbf{F}, \mathbf{B}$ and $\mathbf{\Pi}$ and of $d\mathbf{\Pi}$. Here we present the functions for $d\mathbf{A}$ and $d\mathbf{F}$, the more cumbersome expressions for $d\mathbf{B}$are available from the authors upon request.

$$dF_{22}=\frac{1}{\Pi_{11}\Pi_{21}\Pi_{33}-\Pi_{21}\Pi_{13}\Pi_{31}+\Pi_{21}\Pi_{22}\Pi_{33}-\Pi_{22}\Pi_{31}\Pi_{23}}\ x$$

$$\left(\begin{array}{l}
[d\Pi_{23}]\Pi_{31}+a_{23}[d\Pi_{31}]\Pi_{33}+a_{23}[d\Pi_{33}]\Pi_{31}-a_{23}[d\Pi_{11}]\Pi_{13}\Pi_{31} \\
+a_{23}[d\Pi_{13}](-\Pi_{11}\Pi_{31}-\Pi_{31}\Pi_{33})-[d\Pi_{21}][d\Pi_{23}]\Pi_{33} \\
-a_{23}[d\Pi_{12}]\Pi_{31}\Pi_{23}-a_{23}[d\Pi_{23}]\Pi_{12}\Pi_{31}-a_{23}[d\Pi_{31}]\Pi_{13}\Pi_{33} \\
-a_{23}[d\Pi_{33}]\Pi_{13}\Pi_{31}+f_{22}[d\Pi_{21}]\Pi_{13}\Pi_{31}+f_{22}[d\Pi_{13}]\Pi_{21}\Pi_{31} \\
+f_{22}[d\Pi_{22}]\Pi_{31}\Pi_{23} \\
+f_{22}[d\Pi_{23}]\Pi_{22}\Pi_{31}+f_{22}[d\Pi_{31}]\Pi_{23}\Pi_{33}+f_{22}[d\Pi_{33}]\Pi_{31}\Pi_{23} \\
+2a_{23}[d\Pi_{11}][d\Pi_{23}]\Pi_{11}\Pi_{33}+a_{23}[d\Pi_{21}][d\Pi_{23}]\Pi_{12}\Pi_{33} \\
+a_{23}[d\Pi_{12}][d\Pi_{23}]\Pi_{21}\Pi_{33}-f_{22}[d\Pi_{22}][d\Pi_{23}]\Pi_{21}\Pi_{33} \\
+a_{23}[d\Pi_{13}][d\Pi_{23}]\Pi_{31}\Pi_{33}-f_{22}[d\Pi_{11}][d\Pi_{23}]\Pi_{21}\Pi_{33} \\
+f_{22}[d\Pi_{21}][d\Pi_{23}](-\Pi_{22}\Pi_{33}-\Pi_{11}\Pi_{33})
\end{array}\right)$$

$$dF_{31}=\frac{1}{\Pi_{13}\Pi_{22}\Pi_{31}-\Pi_{21}\Pi_{13}\Pi_{32}-\Pi_{11}\Pi_{12}\Pi_{21}+\Pi_{11}^2\Pi_{22}}\ x$$

$$\left(\begin{array}{l}
[d\Pi_{32}]\Pi_{21}-[d\Pi_{31}]\Pi_{22}-a_{32}[d\Pi_{21}]\Pi_{22} \\
+a_{32}[d\Pi_{22}]\Pi_{21}-2f_{31}[d\Pi_{11}]\Pi_{11}\Pi_{22} \\
+f_{31}[d\Pi_{11}]\Pi_{12}\Pi_{21}+f_{31}[d\Pi_{12}]\Pi_{11}\Pi_{21} \\
-f_{31}[d\Pi_{21}]\Pi_{12}\Pi_{22}+f_{31}[d\Pi_{22}]\Pi_{12}\Pi_{21} \\
+f_{31}[d\Pi_{13}]\Pi_{21}\Pi_{32}-f_{31}[d\Pi_{13}]\Pi_{22}\Pi_{31} \\
-f_{31}[d\Pi_{31}]\Pi_{13}\Pi_{22}+f_{31}[d\Pi_{32}]\Pi_{21}\Pi_{13}
\end{array}\right)$$

$$dF_{11}=\frac{1}{\Pi_{11}\Pi_{13}\Pi_{22}-\Pi_{11}\Pi_{12}\Pi_{23}+\Pi_{13}\Pi_{22}\Pi_{33}-\Pi_{13}\Pi_{23}\Pi_{32}}\ x$$

$$\left(\begin{array}{l}
[d\Pi_{12}][d\Pi_{32}]\Pi_{23}-a_{12}[d\Pi_{23}]\Pi_{22} \\
-[d\Pi_{13}]\Pi_{22}+a_{12}[d\Pi_{22}][d\Pi_{32}]\Pi_{23} \\
-f_{11}[d\Pi_{11}]\Pi_{13}\Pi_{22}-f_{11}[d\Pi_{13}]\Pi_{11}\Pi_{22} \\
-f_{11}[d\Pi_{12}]\Pi_{22}\Pi_{23} \\
-f_{11}[d\Pi_{23}]\Pi_{12}\Pi_{22} \\
-f_{11}[d\Pi_{13}]\Pi_{22}\Pi_{33} \\
+f_{11}[d\Pi_{32}]\Pi_{13}\Pi_{23}-f_{11}[d\Pi_{33}]\Pi_{13}\Pi_{22} \\
+f_{11}[d\Pi_{11}][d\Pi_{32}]\Pi_{12}\Pi_{23}+f_{11}[d\Pi_{12}][d\Pi_{32}]\Pi_{11}\Pi_{23} \\
+f_{11}[d\Pi_{12}][d\Pi_{32}]\Pi_{22}\Pi_{23}+f_{11}[d\Pi_{22}][d\Pi_{32}]\Pi_{12}\Pi_{23} \\
+f_{11}[d\Pi_{13}][d\Pi_{32}]\Pi_{23}\Pi_{32}
\end{array}\right)$$

$$
\begin{aligned}
dA_{12}=&\frac{1}{\Pi_{11}\Pi_{13}\Pi_{22}-\Pi_{11}\Pi_{12}\Pi_{23}+\Pi_{13}\Pi_{22}\Pi_{33}-\Pi_{13}\Pi_{23}\Pi_{32}}\ x\\
\Big(&[d\Pi_{13}]\Pi_{11}\Pi_{12}+[d\Pi_{13}]\Pi_{12}\Pi_{22}+[d\Pi_{13}]\Pi_{13}\Pi_{32}\\
&+a_{12}[d\Pi_{23}](\Pi_{11}\Pi_{12}+\Pi_{12}\Pi_{22}+\Pi_{13}\Pi_{32})\\
&+[d\Pi_{12}][d\Pi_{32}](-\Pi_{11}\Pi_{13}-\Pi_{12}\Pi_{23}-\Pi_{13}\Pi_{33})\\
&-a_{12}[d\Pi_{22}][d\Pi_{32}]\Pi_{11}\Pi_{13}\\
&-a_{12}[d\Pi_{22}][d\Pi_{32}]\Pi_{12}\Pi_{23}-a_{12}[d\Pi_{22}][d\Pi_{32}]\Pi_{13}\Pi_{33}+f_{11}[d\Pi_{11}]\Pi_{11}\Pi_{12}\Pi_{13}\\
&+f_{11}[d\Pi_{11}]\Pi_{12}\Pi_{13}\Pi_{22}-f_{11}[d\Pi_{32}]\Pi_{12}\Pi_{13}\Pi_{23}\\
&+f_{11}[d\Pi_{12}]\Pi_{11}\Pi_{12}\Pi_{23}\\
&+f_{11}[d\Pi_{13}](\Pi_{11}\Pi_{12}\Pi_{22}+\Pi_{11}\Pi_{12}\Pi_{33}+\Pi_{11}\Pi_{13}\Pi_{32}+\Pi_{12}\Pi_{22}\Pi_{33})\\
&+f_{11}[d\Pi_{23}](\Pi_{12}\Pi_{13}\Pi_{32}+\Pi_{12}^2\Pi_{22})\\
&+f_{11}[d\Pi_{12}]\Pi_{12}\Pi_{22}\Pi_{23}+f_{11}[d\Pi_{33}]\Pi_{11}\Pi_{12}\Pi_{13}\\
&+f_{11}[d\Pi_{12}]\Pi_{13}\Pi_{23}\Pi_{32}\\
&+f_{11}[d\Pi_{33}]\Pi_{12}\Pi_{13}\Pi_{22}+f_{11}[d\Pi_{13}]\Pi_{13}\Pi_{32}\Pi_{33}\\
&+f_{11}[d\Pi_{12}][d\Pi_{32}](-\Pi_{11}\Pi_{13}\Pi_{22}-\Pi_{11}\Pi_{13}\Pi_{33}-\Pi_{13}\Pi_{22}\Pi_{33})\\
&-f_{11}[d\Pi_{11}][d\Pi_{32}]\Pi_{11}\Pi_{12}\Pi_{13}-f_{11}[d\Pi_{12}][d\Pi_{32}]\Pi_{11}\Pi_{12}\Pi_{23}\\
&-f_{11}[d\Pi_{22}][d\Pi_{32}]\Pi_{11}\Pi_{12}\Pi_{13}-f_{11}[d\Pi_{11}][d\Pi_{32}]\Pi_{12}\Pi_{13}\Pi_{33}\\
&-f_{11}[d\Pi_{12}][d\Pi_{32}]\Pi_{12}\Pi_{22}\Pi_{23}-f_{11}[d\Pi_{22}][d\Pi_{32}]\Pi_{12}\Pi_{13}\Pi_{33}\\
&-f_{11}[d\Pi_{13}][d\Pi_{32}](\Pi_{12}\Pi_{23}\Pi_{32}-\Pi_{13}\Pi_{32}\Pi_{33}-\Pi_{11}\Pi_{13}\Pi_{32})\\
&+f_{11}[d\Pi_{13}]\Pi_{11}^2\Pi_{12}+f_{11}[d\Pi_{23}]\Pi_{11}\Pi_{12}^2+f_{11}[d\Pi_{11}]\Pi_{13}^2\Pi_{32}\\
&+f_{11}[d\Pi_{32}](-\Pi_{11}\Pi_{13}^2-\Pi_{13}^2\Pi_{33})\\
&+f_{11}[d\Pi_{33}]\Pi_{13}^2\Pi_{32}-f_{11}[d\Pi_{12}][d\Pi_{32}]\Pi_{11}^2\Pi_{13}\\
&-f_{11}[d\Pi_{11}][d\Pi_{32}]\Pi_{12}^2\Pi_{23}\\
&-f_{11}[d\Pi_{22}][d\Pi_{32}]\Pi_{12}^2\Pi_{23}\Big)
\end{aligned}
$$

$$
dA_{23}=\frac{1}{\Pi_{11}\Pi_{21}\Pi_{33}-\Pi_{21}\Pi_{13}\Pi_{31}+\Pi_{21}\Pi_{22}\Pi_{33}-\Pi_{22}\Pi_{31}\Pi_{23}}\ x
$$

$$
\left(\begin{array}{l}
[d\Pi_{21}][d\Pi_{23}]\Pi_{21}\Pi_{13}-[d\Pi_{23}]\Pi_{21}\Pi_{22}-[d\Pi_{23}]\Pi_{31}\Pi_{23}-a_{23}[d\Pi_{31}]\Pi_{21}\Pi_{13}\\
+a_{23}[d\Pi_{31}](-\Pi_{22}\Pi_{23}-\Pi_{23}\Pi_{33})-[d\Pi_{23}]\Pi_{11}\Pi_{21}\\
+[d\Pi_{21}][d\Pi_{23}](\Pi_{22}\Pi_{23}+\Pi_{23}\Pi_{33})+a_{23}[d\Pi_{31}](\Pi_{13}\Pi_{22}\Pi_{23}+\Pi_{13}\Pi_{23}\Pi_{33})\\
+a_{23}[d\Pi_{33}](-\Pi_{21}\Pi_{22}-\Pi_{11}\Pi_{21}-\Pi_{31}\Pi_{23}+\Pi_{11}\Pi_{21}\Pi_{13})\\
+a_{23}[d\Pi_{12}]\Pi_{11}\Pi_{21}\Pi_{23}+a_{23}[d\Pi_{13}]\Pi_{11}\Pi_{21}\Pi_{22}\\
+a_{23}[d\Pi_{23}](\Pi_{11}\Pi_{12}\Pi_{21}+\Pi_{12}\Pi_{21}\Pi_{22}+\Pi_{12}\Pi_{31}\Pi_{23})\\
+a_{23}[d\Pi_{12}]\Pi_{21}\Pi_{22}\Pi_{23}+a_{23}[d\Pi_{13}]\Pi_{11}\Pi_{21}\Pi_{33}\\
+a_{23}[d\Pi_{13}](\Pi_{11}\Pi_{31}\Pi_{23}\Pi_{21}\Pi_{22}\Pi_{33}+\Pi_{31}\Pi_{23}\Pi_{33})\\
+a_{23}[d\Pi_{11}](\Pi_{13}\Pi_{31}\Pi_{23}+\Pi_{11}\Pi_{21}\Pi_{13}+\Pi_{21}\Pi_{13}\Pi_{22})\\
+f_{22}[d\Pi_{21}](-\Pi_{21}\Pi_{13}\Pi_{22}-\Pi_{11}\Pi_{21}\Pi_{13}-\Pi_{13}\Pi_{31}\Pi_{23})\\
+f_{22}[d\Pi_{22}](\Pi_{11}\Pi_{21}\Pi_{23}-\Pi_{21}\Pi_{22}\Pi_{23})\\
+f_{22}[d\Pi_{13}](-\Pi_{11}\Pi_{21}^{2}-\Pi_{21}^{2}\Pi_{22}-\Pi_{21}\Pi_{31}\Pi_{23})-f_{22}[d\Pi_{31}]\Pi_{21}\Pi_{13}\Pi_{23}\\
+a_{23}[d\Pi_{33}](\Pi_{21}\Pi_{13}\Pi_{22}+\Pi_{13}\Pi_{31}\Pi_{23})\\
-f_{22}[d\Pi_{23}](\Pi_{11}\Pi_{21}\Pi_{33}+\Pi_{21}\Pi_{13}\Pi_{31}+\Pi_{11}\Pi_{21}\Pi_{22}-\Pi_{21}\Pi_{22}\Pi_{33})\\
+f_{22}[d\Pi_{33}](-\Pi_{11}\Pi_{21}\Pi_{23}-\Pi_{21}\Pi_{22}\Pi_{23})\\
+2a_{23}[d\Pi_{11}][d\Pi_{23}](-\Pi_{11}\Pi_{21}\Pi_{13}-\Pi_{11}\Pi_{22}\Pi_{23}-\Pi_{11}\Pi_{23}\Pi_{33}-\Pi_{12}\Pi_{21}\Pi_{13})\\
+a_{23}[d\Pi_{12}][d\Pi_{23}](\Pi_{21}\Pi_{22}\Pi_{23}-\Pi_{21}\Pi_{23}\Pi_{33}-\Pi_{21}^{2}\Pi_{13})\\
-a_{23}[d\Pi_{21}][d\Pi_{23}]\Pi_{12}\Pi_{22}\Pi_{23}-a_{23}[d\Pi_{13}][d\Pi_{23}]\Pi_{21}\Pi_{13}\Pi_{31}\\
-a_{23}[d\Pi_{21}][d\Pi_{23}]\Pi_{12}\Pi_{23}\Pi_{33}-a_{23}[d\Pi_{13}][d\Pi_{23}]\Pi_{22}\Pi_{31}\Pi_{23}\\
+f_{22}[d\Pi_{11}][d\Pi_{23}]\Pi_{21}\Pi_{22}\Pi_{23}+f_{22}[d\Pi_{21}][d\Pi_{23}]\Pi_{11}\Pi_{22}\Pi_{23}\\
-a_{23}[d\Pi_{13}][d\Pi_{23}]\Pi_{31}\Pi_{23}\Pi_{33}+f_{22}[d\Pi_{11}][d\Pi_{23}]\Pi_{21}\Pi_{23}\Pi_{33}\\
+f_{22}[d\Pi_{22}][d\Pi_{23}]\Pi_{21}\Pi_{22}\Pi_{23}+f_{22}[d\Pi_{22}][d\Pi_{23}]\Pi_{21}\Pi_{23}\Pi_{33}\\
+a_{23}[d\Pi_{13}]\Pi_{11}^{2}\Pi_{21}+a_{23}[d\Pi_{31}]\Pi_{21}\Pi_{13}^{2}+a_{23}[d\Pi_{12}]\Pi_{31}\Pi_{23}^{2}\\
-f_{22}[d\Pi_{23}]\Pi_{21}\Pi_{22}^{2}-f_{22}[d\Pi_{22}]\Pi_{31}\Pi_{23}^{2}-f_{22}[d\Pi_{31}]\Pi_{22}\Pi_{23}^{2}\\
-f_{22}[d\Pi_{31}]\Pi_{23}^{2}\Pi_{33}-f_{22}[d\Pi_{33}]\Pi_{31}\Pi_{23}^{2}+f_{22}[d\Pi_{11}][d\Pi_{23}]\Pi_{21}^{2}\Pi_{13}\\
+f_{22}[d\Pi_{22}][d\Pi_{23}]\Pi_{21}^{2}\Pi_{13}+f_{22}[d\Pi_{21}][d\Pi_{23}]\Pi_{11}\Pi_{21}\Pi_{13}\\
+f_{22}[d\Pi_{21}][d\Pi_{23}](\Pi_{21}\Pi_{13}\Pi_{22}+\Pi_{11}\Pi_{23}\Pi_{33}+\Pi_{22}\Pi_{23}\Pi_{33}+\Pi_{22}^{2}\Pi_{23})
\end{array}\right)
$$

$$dA_{32}=\frac{1}{\Pi_{13}\Pi_{22}\Pi_{31}-\Pi_{21}\Pi_{13}\Pi_{32}-\Pi_{11}\Pi_{12}\Pi_{21}+\Pi_{11}^2\Pi_{22}}\ x$$

$$\left(\begin{array}{l}
[d\Pi_{31}]\Pi_{11}\Pi_{12}+[d\Pi_{31}]\Pi_{12}\Pi_{22}-[d\Pi_{32}]\Pi_{12}\Pi_{21}+[d\Pi_{31}]\Pi_{13}\Pi_{32}-[d\Pi_{32}]\Pi_{13}\Pi_{31} \\
+a_{32}[d\Pi_{21}]\Pi_{11}\Pi_{12}+a_{32}[d\Pi_{21}]\Pi_{12}\Pi_{22}-a_{32}[d\Pi_{22}]\Pi_{12}\Pi_{21}+a_{32}[d\Pi_{21}]\Pi_{13}\Pi_{32} \\
-a_{32}[d\Pi_{22}]\Pi_{13}\Pi_{31}+2f_{31}[d\Pi_{11}]\Pi_{11}\Pi_{12}\Pi_{22} \\
+2f_{31}[d\Pi_{11}]\Pi_{11}\Pi_{13}\Pi_{32}-f_{31}[d\Pi_{11}]\Pi_{12}\Pi_{13}\Pi_{31} \\
-f_{31}[d\Pi_{12}]\Pi_{11}\Pi_{13}\Pi_{31}+f_{31}[d\Pi_{13}]\Pi_{11}\Pi_{12}\Pi_{31} \\
+f_{31}[d\Pi_{31}]\Pi_{11}\Pi_{12}\Pi_{13}+f_{31}[d\Pi_{12}]\Pi_{21}\Pi_{13}\Pi_{32}-f_{31}[d\Pi_{12}]\Pi_{13}\Pi_{22}\Pi_{31} \\
+f_{31}[d\Pi_{21}]\Pi_{12}\Pi_{13}\Pi_{32}-f_{31}[d\Pi_{13}]\Pi_{12}\Pi_{21}\Pi_{32}+f_{31}[d\Pi_{13}]\Pi_{12}\Pi_{22}\Pi_{31} \\
-f_{31}[d\Pi_{22}]\Pi_{12}\Pi_{13}\Pi_{31}+f_{31}[d\Pi_{31}]\Pi_{12}\Pi_{13}\Pi_{22} \\
-f_{31}[d\Pi_{32}]\Pi_{12}\Pi_{21}\Pi_{13}-[d\Pi_{32}]\Pi_{11}^2-a_{32}[d\Pi_{22}]\Pi_{11}^2 \\
-f_{31}[d\Pi_{12}]\Pi_{11}^3+f_{31}[d\Pi_{11}]\Pi_{11}^2\Pi_{12}-f_{31}[d\Pi_{11}]\Pi_{12}^2\Pi_{21} \\
+f_{31}[d\Pi_{21}]\Pi_{11}\Pi_{12}^2-f_{31}[d\Pi_{12}]\Pi_{11}^2\Pi_{22}-f_{31}[d\Pi_{22}]\Pi_{11}^2\Pi_{12} \\
+f_{31}[d\Pi_{21}]\Pi_{12}^2\Pi_{22}-f_{31}[d\Pi_{22}]\Pi_{12}^2\Pi_{21}-f_{31}[d\Pi_{13}]\Pi_{11}^2\Pi_{32} \\
-f_{31}[d\Pi_{32}]\Pi_{11}^2\Pi_{13}+f_{31}[d\Pi_{31}]\Pi_{13}^2\Pi_{32}-f_{31}[d\Pi_{32}]\Pi_{13}^2\Pi_{31}
\end{array}\right)$$

Appendix B Tables and Figures

Table 1. Single equation vs sytem estimation, unemployment

Eqn.	Var. (coeff.)	Estimation method Single	Sub-VAR	System
		[1]	[2]	[3]
π_t	π^e_{t+1} (α_1)	0.701***	0.675***	0.658***
		(0.139)	(0.103)	(0.092)
	u_t (α_2)	0.063	0.053	0.045
		(0.059)	(0.043)	(0.038)
	π_{t-1} (α_3)	0.296**	0.327***	0.342***
		(0.137)	(0.098)	(0.084)
	$Adj.\ R^2$	0.866	0.869	0.870
	$J - stat$	6.085 (6)	16.696 (18)	15.024 (18)
$x_t = u_t$	u^e_{t+1} (β_1)	0.566***	0.558***	0.544***
		(0.033)	(0.020)	(0.024)
	ri_t (β_2)	−0.014***	−0.014***	−0.011***
		(0.005)	(0.003)	(0.004)
	u_{t-1} (β_3)	0.462***	0.468***	0.479***
		(0.029)	(0.020)	(0.022)
	$Adj.\ R^2$	0.987	0.987	0.988
	$J - stat(p)$	9.826 (6)	20.908 (18)	–
i_t	π^e_{t+1} (γ_1)	1.532***	1.265***	1.316***
		(0.503)	(0.327)	(0.312)
	u_t (γ_2)	−1.767**	−1.406**	−1.337**
		(0.847)	(0.549)	(0.530)
	i_{t-1} (γ_3)	0.907***	0.894***	0.895***
		(0.024)	(0.019)	(0.019)
	$Adj.\ R^2$	0.882	0.882	0.882
	$J - stat\ (p)$	10.169 (6)	16.458 (18)	–

Note: The instrument set includes the constant and three lags of u, π, i. Sample is 1970:1-1998:4.
The columns report results for single equation estimation (Single), system estimation where
the completing equations are Sub-VARs Sub-VAR), and full forward looking system (System).
HAC s.e. in (). *, **, and *** indicate significance at 10%, 5% and 1%.
J-stat is $\chi^2(p)$ under the null hypothesis of p valid over-identifying restrictions.

Table 2. Single equation vs sytem estimation, GDP gap

Eqn.	Var. (coeff.)	Estimation method Single	 Sub-VAR	 System
		[1]	[2]	[3]
π_t	π^e_{t+1} (α_1)	0.778^{***}	0.726^{***}	0.672^{***}
		(0.139)	(0.090)	(0.084)
	gap_t (α_2)	-0.067^{*}	-0.051^{**}	-0.038^{*}
		(0.034)	(0.025)	(0.023)
	π_{t-1} (α_3)	0.231^{*}	0.281^{***}	0.334^{***}
		(0.128)	(0.082)	(0.072)
	$Adj.\ R^2$	0.860	0.899	0.870
	$J - stat$	4.809 (6)	17.993 (18)	13.780 (18)
$x_t = gap_t$	gap^e_{t+1} (β_1)	0.544^{***}	0.538^{***}	0.540^{***}
		(0.046)	(0.033)	(0.034)
	ri_t (β_2)	0.017	0.016	0.015
		(0.013)	(0.011)	(0.011)
	gap_{t-1} (β_3)	0.487^{***}	0.486^{***}	0.484^{***}
		(0.040)	(0.030)	(0.029)
	$Adj.\ R^2$	0.954	0.954	0.954
	$J - stat$	5.778 (6)	18.453 (18)	–
i_t	π^e_{t+1} (γ_1)	1.103^{**}	1.072^{***}	1.186^{***}
		(0.458)	(0.362)	(0.308)
	gap_t (γ_2)	1.675^{*}	1.659^{**}	1.476^{**}
		(0.922)	(0.691)	(0.704)
	i_{t-1} (γ_3)	0.921^{***}	0.922^{***}	0.920^{***}
		(0.027)	(0.020)	(0.022)
	$Adj.\ R^2$	0.885	0.885	0.885
	$J - stat\ (p)$	10.702^{*} (6)	15.574 (18)	–

Note: The instrument set includes the constant and three lags of gap, π, i. Sample is 1970:1-1998:4. The columns report results for single equation estimation (Single), system estimation where the completing equations are Sub-VARs (Sub-VAR), and full forward looking system (System). HAC s.e. in (). *, **, and *** indicate significance at 10%, 5% and 1%.
J-stat is $\chi^2(p)$ under the null hypothesis of p valid over-identifying restrictionns

Table 3. Alternative forward looking sytems, unemployment

		1970-1998	1985-1998	
Eqn.	Var. (coeff.)	No factors	No factors	Significant factors as regressors
		[1]	[2]	[3]
π_t	π^e_{t+1} (α_1)	0.658***	0.727***	0.516***
		(0.092)	(0.092)	(0.043)
	u_t (α_2)	0.045	0.042	0.0323
		(0.038)	(0.038)	(0.035)
	π_{t-1} (α_3)	0.342***	0.250***	0.332***
		(0.084)	(0.053)	(0.041)
	Adj. R^2	0.870	0.441	0.513
	No corr (4)	2.096*	1.820	2.658*
	Norm	7.243**	2.022	0.459
	No ARCH (4)	4.605***	0.489	3.166**
$x_t = u_t$	u^e_{t+1} (β_1)	0.544***	0.508***	0.436***
		(0.024)	(0.024)	(0.016)
	ri_t (β_2)	−0.011***	−0.006*	−0.006
		(0.004)	(0.003)	(0.004)
	u_{t-1} (β_3)	0.479***	0.501***	0.543***
		(0.022)	(0.025)	(0.013)
	Adj. R^2	0.987	0.990	0.99
	No corr (4)	1.117	0.748	0.589
	Norm	40.88***	1.081	0.817
	No ARCH (4)	4.810***	0.438	4.161***
i_t	π^e_{t+1} (γ_1)	1.316***	1.774***	1.508***
		(0.312)	(0.346)	(0.36)
	u_t (γ_2)	−1.337**	−1.345***	−1.844***
		(0.530)	(0.295)	(0.221)
	i_{t-1} (γ_3)	0.895***	0.850***	0.881***
		(0.019)	(0.024)	(0.010)
	Adj. R^2	0.882	0.944	0.948
	No corr (4)	1.845	0.126	0.155
	Norm	512.7***	2.041	1.896
	No ARCH (4)	2.678**	0.732	0.758
	$J-stat$ (p)	15.024 (18)	12.611 (18)	14.187 (30)

Note: The instrument set includes the constant and three lags of u, π, i (no factors) plus the first lag of the six estimated factors (other cases).
The regressors are either as in Table 1 (no factors) or include some contemporaneous factors (see text for details)
HAC s.e. in (). *, **, and *** indicate significance at 10%, 5% and 1%; The mis-specification tests (No corr, Norm, No ARCH) are conducted on the residuals of an MA(1) model for the estimated errors.
No corr is LM(4) test for no serial correlation,Norm is Jarque-Bera statistic for normality, and ARCH in LM(4) test for no ARCH effects.
J-stat is $\chi^2(p)$ under the null hypothesis of p valid over-identifying restrictionns

Table 4. Alternative forward looking sytems, GDP gap

		1970-1998	1985-1998	
Eqn.	Var. (coeff.)	No factors	No factors	Significant Factors as regressors
		[1]	[2]	[3]
π_t	π^e_{t+1} (α_1)	0.672***	0.605***	0.435***
		(0.083)	(0.093)	(0.081)
	gap_t (α_2)	−0.038*	−0.012	0.067
		(0.023)	(0.018)	(0.049)
	π_{t-1} (α_3)	0.334***	0.319***	0.343***
		(0.072)	(0.067)	(0.042)
	Adj. R^2	0.870	0.481	0.531
	No corr (4)	1.599	2.007	2.302*
	Norm	6.907**	1.877	0.065
	No ARCH (4)	2.990	0.565	1.505
$x_t = gap_t$	gap^e_{t+1} (β_1)	0.534***	0.466***	0.623***
		(0.034)	(0.044)	(0.033)
	ri_t (β_2)	0.015	−0.021	0.057***
		(0.011)	(0.015)	(0.010)
	gap_{t-1} (β_3)	0.484***	0.558***	0.491***
		(0.029)	(0.047)	(0.040)
	Adj. R^2	0.954	0.966	0.953
	No corr (4)	0.177	0.884	0.453
	Norm	2.721	2.198	0.440
	No ARCH (4)	1.510	0.896	1.270
i_t	π^e_{t+1} (γ_1)	1.186***	1.123**	1.098***
		(0.308)	(0.458)	(0.178)
	gap_t (γ_2)	1.476**	0.771***	0.846***
		(0.704)	(0.160)	(0.012)
	i_{t-1} (γ_3)	0.920***	0.867***	0.841***
		(0.022)	(0.024)	(0.090)
	Adj. R^2	0.885	0.945	0.947
	No corr (4)	2.172*	0.237	0.227
	Norm	525.0***	1.833	3.257
	No ARCH (4)	2.743**	0.765	0.583
	$J-stat$ (p)	13.780 (18)	12.824 (18)	12.942 (29)

Note: The instrument set includes the constant and three lags of gap, π, i (no factors) plus the first lag of the six estimated factors (other case).
The regressors are either as in Table 1 (no factors) or include some contemporaneous factors (see text for details)
HAC s.e. in (). *, **, and *** indicate significance at 10%, 5% and 1%; The mis-specification tests (No corr, Norm, No ARCH) are conducted on the residuals of an MA(1) model for the estimated errors.
No corr is LM(4) test for no serial correlation,Norm is Jarque-Bera statistic for normality, and ARCH in LM(4) test for no ARCH effects.
J-stat is $\chi^2(p)$ under the null hypothesis of p valid over-identifying restrictionns

Table 5: Determinacy properties of the forward looking system

	No Factors GMM	Indirect Least Squares (MLE)	Factors GMM
Unemployment	No Stable-Equilibrium	Indeterminacy	Determinacy
Output Gap	No Stable-Equilibrium	Indeterminacy	Indeterminacy

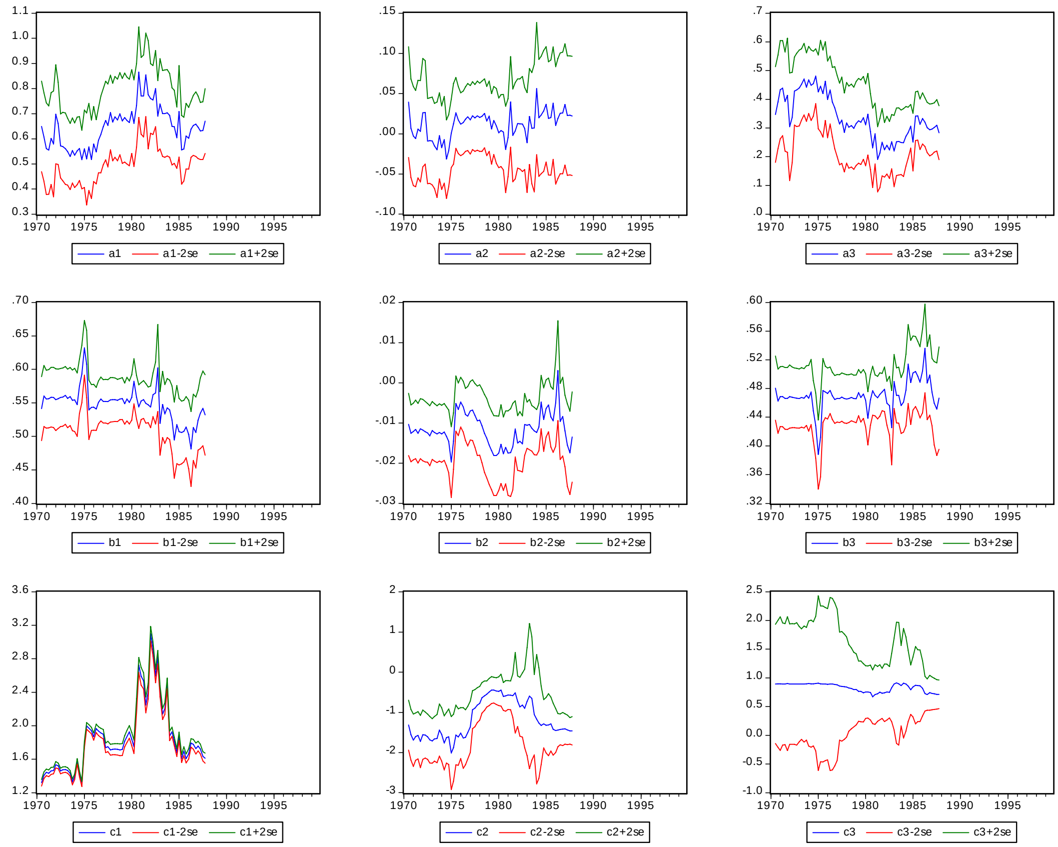

Figure 1: Backward recursive estimation, 1988:1 - 1970:1, system with unemployment

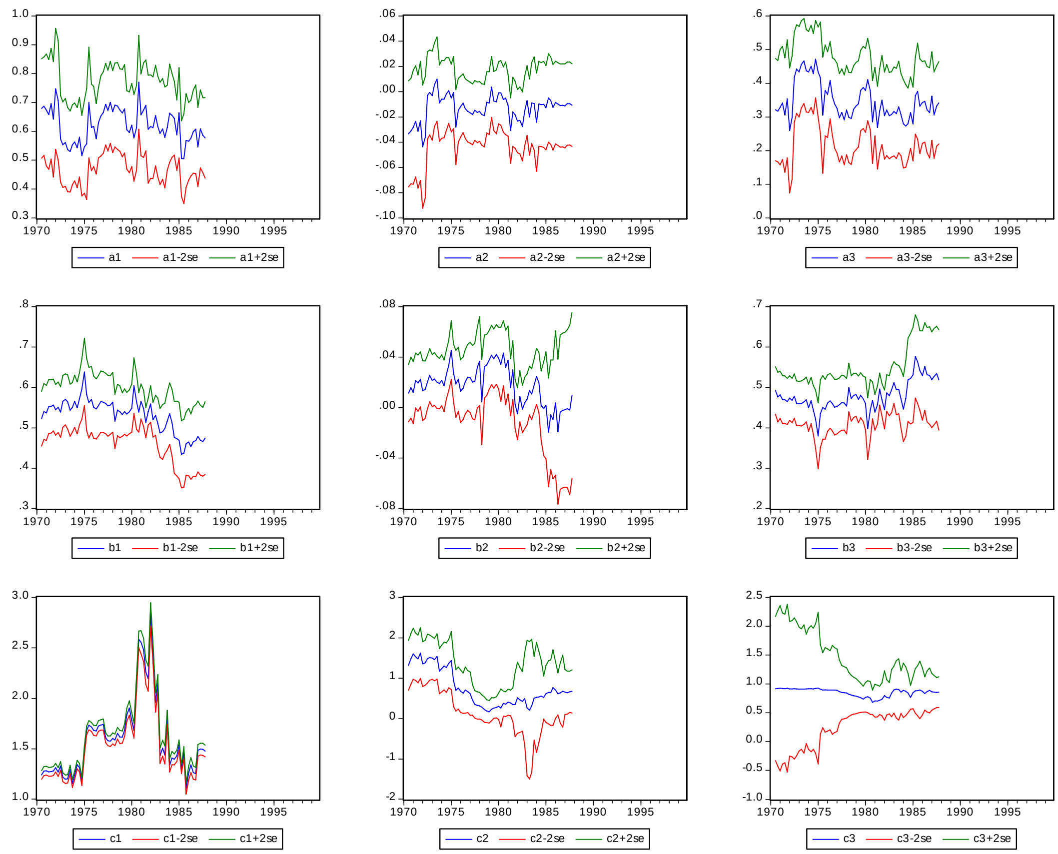

Figure 2: Backward recursive estimation, 1988:1 - 1970:1, system with GDP gap