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No. 5262

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*INDUSTRIAL ORGANIZATION and
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Discussion Paper No. 5262
October 2005

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ABSTRACT

Foreign Direct Investment and Exports Dynamics with Demand Learning*

We model a firm's choice between exporting and investing in a foreign country in the presence of possibly persistent demand uncertainty. We allow for demand shocks that, while increasing expected profit, impede learning. The firm learns gradually, in a Bayesian fashion, by observing past demand realizations. We derive the optimal exports and investment paths, examine how they depend on the technology parameters and the structure of uncertainty and show that learning alone may explain the S-shape of these paths. Immediate investment is possible despite the presence of demand uncertainty, if there are significant positive demand shocks and learning is likely to take time.

JEL Classification: D83, D92 and F21

Keywords: exports, foreign direct investment, investment dynamics, learning and uncertainty

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* We are grateful to Monika Schnitzer and participants at the Seminar for Comparative Economics at Ludwig-Maximilians-Universität München, the ASSET 2004 conference at Barcelona, the EEA 2005 conference at Amsterdam and the EARIE 2005 conference at Porto, for helpful suggestions and comments. Kotseva appreciates the hospitality of LMU München during a Marie-Curie visit. Financial support from the European Commission and the Greek Ministry of Education ('Herakleitos' program) is gratefully acknowledged.

Submitted 08 September 2005

1 Introduction

In this paper, we set up a dynamic model to analyze the choice between exports and Foreign Direct Investment (FDI) under demand uncertainty in the foreign market. Our aim is to determine the optimal paths of investments and exports, as well as the timing of FDI when demand is stochastic, with particular emphasis on the presence of persistent uncertainty and the effect of learning. Dynamics, uncertainty and learning are at the core of exports and FDI decisions and, in particular, demand uncertainty is typically significant in foreign markets; however, the literature only provides a partial treatment of these matters in a formal manner.

We consider a firm that has to decide how to serve a foreign market (i.e. how to enter initially and when to switch to a different mode of supply, if at all), as well as what quantity to produce, being uncertain about the level of demand there.¹ More specifically, the firm can either produce at a home-country facility that it owns and then export, or it can establish a production unit and invest in the foreign country. The unit cost of exports is higher than that of installing productive capacity in the foreign country due to, possibly among other reasons, higher transportation costs, tariffs or lower labor costs in the foreign country. However, investing abroad is associated with a fixed entry cost, which becomes sunk once incurred. If demand is high, the firm is better-off by investing, since the fixed entry cost incurred in this case is lower than the accumulated difference between the unit cost of exports and that of capacity installment. On the contrary, if demand is low, the firm is better-off by exporting.

If the firm were certain about its future payoffs, it could easily choose the more profitable mode of serving the market, depending on the demand and cost parameters. However, it has to make the mode-of-entry and the level-of-production decisions without knowing the level of foreign demand. What is more important, demand is stochastic so that uncertainty is resolved with time as the firm learns from past demand realizations. We assume that there is only one channel of learning, which is sales.² Both exports and FDI allow the firm to make sales within a given period and, thus, both allow revenue to be collected and also learning to take place. However, these two options differ in the commitment they represent. The former involves no entry cost, while the latter is associated with a completely sunk entry cost. In addition, the cost of exports applies for only the current period whereas foreign production requires building capacity via (irreversible) investment. The introduction of demand shocks is central to the analysis and captures our basic idea that, if it takes a “long time” to learn the true demand, it may be more profitable for the firm to invest in productive capacity from the very beginning rather than waiting for uncertainty to be resolved while exporting. Thus our setting is complementary to other settings where market uncertainty is viewed as leading to a delay in investment (whether this means supply via exports or no supply at all). In our analysis, learning requires sales in the foreign

¹The choice of entry mode cannot be analyzed separately from the choice of capacity installed and quantities sold; both decisions are endogenously determined and interrelated.

²In other words, the uncertainty we consider is the residual after the firm has gathered information from other sources, like market surveys or learning from similar experiences.

market and investment (FDI) may be the optimal way to make these sales.

We provide an exact solution and characterization for the path of exports and FDI. Our main findings can be summarized around two points, exports and capacity expansion and the timing of the initial FDI. First, related to the dynamics of exports and investment, we find that, if the firm's priors are not centered too close around the true demand (i.e. they are "sufficiently wrong"), the intertemporal paths for both exports and cumulative capacity are S-shaped. This means that sales are increasing initially at an increasing rate, before they enter a decreasing rate range. This results from the fact that the firm enters the foreign market not knowing the true level of demand and then learns gradually from past sales. Second, related to the timing of FDI we establish conditions under which the firm invests immediately despite the presence of uncertainty and the possibility of exporting. More specifically, we show that, for certain values of the probability of a shock occurrence, the firm invests right away irrespective of its prior beliefs about the level of demand and this is due to the prevalence of, what we call, the "expected-demand-shifting effect" of the shock. However, this is not the only way in which the shock affects the entry decision of the firm. The presence of a demand shock impedes learning, i.e. it has an "uncertainty-persistence effect". Still, even when the demand-shifting effect is not strong enough, there are parameter values constellations, for which immediate investment is optimal.³ This result illustrates that uncertainty may not always lead to a delay in investment. Due to a lower unit cost of production in the foreign country, FDI provides a less costly way of learning and, therefore, may be preferable compared to exports if learning is likely to take time. Of course, in other cases entry via exports (and possibly switching to FDI at a later point) is the optimal solution and we identify what technology and demand characteristics favor this behavior.

The problem of market entry under demand uncertainty is central in international investment decisions, especially for developing economies, where foreign investors typically face many aspects of uncertainty; one such aspect is the lack of familiarity with consumer tastes and the level of potential consumption of a certain product. In addition, an important feature of these economies is the volatility of political, social and economic conditions, which means that familiarity with local conditions most probably will not be acquired immediately after entering a certain market. Thus, it appears important from a normative viewpoint to study the dynamics of optimal entry under uncertainty. From a positive viewpoint, the empirical evidence on the way firms choose to serve a foreign market appears mixed, in the sense that there is no single established pattern or sequence of actions. Several studies, however, have examined the entry decisions in various settings and offered a number of insights.⁴

³In this case, whether there will be immediate entry via FDI or switching from exports to capacity installment depends on the probability of a shock occurrence and the firm's prior beliefs.

⁴Many of the studies provide support for the so called incremental approach, where firms first approach a foreign market through exports and, if conditions are favorable, invest at a later point (see e.g. Johanson and Vahlne, 1977, who base their analysis on observations of Swedish firms; also Rhee and Cheng, 2002, on a sample of South Korean firms). Still, there are enough reported cases that do not conform with this pattern. For instance, Millington and Bayliss (1990) examine a sample of UK firms that have established a manufacturing transnational operation in the EC. Ten out of the fifty firms examined had no previous experience in the market before they invested there, that is, they entered the foreign

The subject of the analysis classifies this paper to the literature on Foreign Direct Investment and the Multinational Enterprise (MNE, viewed as the enterprise that engages in FDI). The volume of related research is enormous, due to both the multitude of relative problems and the variety of approaches. The basic idea is to study which firms are most likely to engage in FDI, why they would do it and where they will most probably install productive capacity.⁵ Our focus here is on examining how the FDI vs exports decision (and the timing of the initial investment) depends on demand uncertainty.⁶ While our analysis certainly abstracts away from other important aspects of FDI, we view a dynamic formulation and characterization as part of the core of the problem.

In our analysis, we focus on exports and FDI as representing two extremes in terms of commitment.⁷ We also introduce (possibly persistent) demand uncertainty and explore its impact on the exports and investment decisions of the firm.⁸ Different aspects of this issue have been studied in previous work and we briefly analyze an incomplete sample of the related contributions here. Johanson and Vahlne (1977) present an early informal study of the impact of foreign market uncertainty on the entry decision and argue that many firms minimize the risk associated with this uncertainty by adopting an incremental investment approach. In a static analysis, Das (1983) explores the impact of demand and cost uncertainties on the behavior of the MNE and finds that demand uncertainty is detrimental to both trade and investment. Itagaki (1991) employs a two-step model to examine the investment decision by a MNE under demand uncertainty; in his model, investment and trade decisions are sequential and in the second stage uncertainty has been resolved. McGahan (1993) considers the strategic investment behavior of an incumbent and a follower under demand uncertainty - in the model the choice of the entry mode is not an issue and firms learn immediately after investment. The joint effect of exchange rate and demand volatility on FDI flows is analyzed by Goldberg and Kolstad (1995) and is found to be positive, if demand and exchange rate shocks are nonnegatively correlated and the MNE is risk averse.

market directly via FDI. In addition, in all ten cases this decision was what they call “planned”, i.e. it has been taken on the basis of “systematic appraisal of alternatives and extended search”. It, thus, appears there is significant heterogeneity in firms’ supply decisions in foreign markets, that needs to be explained.

⁵A unifying role is often played by the “classic” OLI (Ownership-Location-Internalization) paradigm introduced by Dunning (1981). For a review of different approaches on FDI see e.g. Dunning (1981, ch. 4) and Norman (1998).

⁶Rivoli and Salorio (1996) explore the link between ownership and internalization advantages and the timing of FDI under uncertainty and show that the former may affect negatively the latter by making FDI more delayable and irreversible.

⁷That we restrict our analysis to exports and FDI and do not study alternative modes of supply (such as licensing to a foreign entity) can be justified by the fact that, under high external uncertainty, the writing and enforcement of contracts becomes more difficult and therefore contractual agreements become less attractive. Of course, by not considering alternative modes of entry we also abstract from the problems specific to them (for instance Horstman and Markusen, 1996, also explore the mode of entry decision but they focus on agency problems as an explanation of the choice between FDI and arm’s-length agreements). Schnitzer (1999) considers a dynamic model of FDI that sheds light on issues of control rights and how investment is affected by the “hold-up” problem.

⁸Of course, there are other possible sources of uncertainty, for example exchange rates. Kogut and Kulatilaka (1994) analyze the value of becoming multinational in a dynamic framework where uncertainty arises from exchange rate fluctuations.

A distinguishing feature of our approach is that the analysis is explicitly dynamic. While investment decisions are in reality dynamic in nature and dynamics have been a central part of the informal studies and discussions about FDI, explicit dynamics are absent to a large extent from formal FDI models. An early exception is Buckley and Casson (1981) who, similarly to us, analyze the foreign market servicing decision. They focus on demand conditions and demand growth and determine the optimal timing of a switch between modes. However, their analysis does not consider uncertainty or learning. Our paper also relates to the real options (or investment under uncertainty) literature (see Dixit and Pindyck, 1994, and Schwartz and Trigeorgis, 2001, for a review). In general, we differ from this approach since we focus on a particular type of investment, namely FDI as opposed to exports, and in this way we concentrate on problems arising from that specific set-up (such as the optimal entry decision and not the investment decision only). More specifically, two distinct real options categories are the option to delay investment waiting for new information, which arrives even if the firm is inactive (i.e. uncertainty is resolved exogenously) and the option to alter the operating scale, which refers to the ability of the firm to expand or contract operating capacity depending on the information it gets.⁹ Our model incorporates both the option to defer investment and the option to alter the operating capacity. In our case there is no information arrival unless the firm actually “puts its product on the shelf”, since sales are the only channel of learning. Still, the option to defer investment is present, since the firm has a second instrument, namely exports, which allows it to learn without making an irreversible investment. Moreover, even if it decides to invest in productive capacity, it does so only gradually.¹⁰ Finally, in contrast to the literature, which focuses mainly on the impact of the “size” of uncertainty on the decisions of the firm, we are primarily interested in the effect of persistent uncertainty and learning.¹¹

A model of explicit FDI and exports dynamics has been provided by Rob and Vettas (2003). That paper also explores dynamic entry into a foreign market and generates the time-paths of exports and FDI. However, demand in that model is growing stochastically over time according to a *known* law of motion (about which of course, there is *no* learning). The focus is on cases where FDI and exports may coexist and, in general, the model then predicts a lagged relationship between exports and FDI.¹² In contrast, the focus of the present paper is on learning about the demand.

Saggi (1998) also examines the choice between exports and FDI under demand uncertainty, in a paper related to ours. The model employed there is a two-period one, where uncertainty is resolved

⁹One of the early contributions are McDonald and Siegel (1986) and Pindyck (1988). See Smit and Trigeorgis (2004, pp.108-9) for a categorization of common corporate real options.

¹⁰Demers (1991) shows, in a perfectly competitive framework and using a more general model, that irreversibility of investment, demand uncertainty and information arrival over time may result in a gradual adjustment of the capital stock.

¹¹Learning and experimentation have been studied in more general economic problems (see e.g. the seminal work of Rothchild, 1974). Rob (1991) provides a model of capacity expansion under demand learning and, as in our paper, shows that the capacity expansion is gradual. For an application of Bayesian learning to an international setting see also Hoff (1997).

¹²In that paper, investment and exports play complementary roles: FDI satisfies proven demand, while exports explore uncertain demand.

after the first period and, therefore, the issues of the optimal timing of entry, switching to another mode of supply and the effect of the uncertainty persistence cannot be fully explored. In addition, FDI is associated with a fixed cost which is independent of the scale of investment. In contrast, our dynamic framework allows for gradual expansion of the productive capacity installed abroad and, thus, distinguishes between a fixed entry cost (incurred when the first investment abroad is made) and a per unit capacity installment cost. Finally, in Saggi's model, the flexibility of the investment decision stems from the possibility of reimbursement of the fixed cost associated with FDI, if demand conditions prove to be unfavorable.¹³ In our model, both the entry cost and the capacity installment cost are fully irreversible, however, the possibility of a gradual investment adds flexibility to the FDI decision.

The remainder of the paper is structured as follows. The model, the related assumptions and notation are introduced in Section 2. Section 3 focuses on the dynamics of beliefs. In Section 4 we analyze the role of the demand shock in the model. In Section 5 and 6 we derive respectively the optimal paths of exports and capacity installed. In Section 7 we examine the question of the optimal time of the first investment. Section 8 briefly discusses some possible extensions. We conclude in Section 9. Some derivations are relegated to the Appendix.

2 The model

We set up a dynamic model, where the horizon is infinite and time is discrete.

Costs

There is a single seller (a "firm") that seeks to maximize his expected present value of profits from operating in a foreign market, with one-period discount factor equal to $\delta \in (0, 1)$. Supply for the market can come either from exports or from foreign direct investment (FDI) there. We denote the unit cost of exports by c (this includes all aspects of costs, including transportation, tariffs or other) and the cost of installing one unit of productive capacity in the foreign country (via FDI) by k . We further assume that each unit of capacity installed allows for the production of a unit of the product at a zero marginal cost forever.¹⁴ The entry cost associated with the first time the firm does FDI is denoted by f and becomes sunk once incurred. The product is not storable and therefore quantity that is exported but not sold in a given period has no value.

Demand

We assume that the foreign market demand function is $P(q) = A - q$. Demand can be either high or low, $A \in \{A_L, A_H\}$ with $A_L < A_H$ and the seller does not know the value of A . Let $\rho_1 \in (0, 1)$ be the seller's *prior* belief, at the beginning of period one, that demand is low ($A = A_L$). If the seller could observe the true level of demand after having entered the market, he would be able to deduce the

¹³Therefore, a switch from FDI to exports in case of low true demand is possible.

¹⁴This formulation, thus, includes the simplifying assumption that installed capacity does not depreciate. Introducing depreciation into the analysis is possible at the cost of complicating the model but does not offer some important additional insights.

value of A immediately from his first-period sales.¹⁵ In such a case, all learning would take place just one period after initial entry. However, this is not typically possible in our setting since, depending on the true demand level, there may be a shock, which makes realized demand a *noisy* signal of the true demand.¹⁶ More specifically, we assume that if true demand is low, there is a positive probability that a shock occurs, as a result of which realized demand in that period is high. We model this relation as follows:

$$P_t(q_t; A) = \begin{cases} A_L - q_t + \varepsilon, & \text{if } A = A_L \\ A_H - q_t, & \text{if } A = A_H, \end{cases}$$

where $P_t(\cdot)$ is realized (observed) demand in period t and ε is a random shock, distributed as:

$$\varepsilon = \begin{cases} 0 & \text{with probability } s \\ \Delta A (\equiv A_H - A_L) & \text{with probability } 1 - s. \end{cases}$$

Due to the presence of the shock, the seller cannot always uncover the realization of A from the previous period sales. To be precise, when he observes low demand he can correctly conclude that true demand is low, but when he observes high demand he cannot know whether true demand is indeed high or there has been a (temporary) shock leading to a high current-period realization of the otherwise low demand. This formulation captures the idea that due to demand shocks it takes time for a seller to get convinced that the demand is really high.¹⁷

Allowing for learning, from the seller's point of view, true demand (equivalently A) in period t is a random variable distributed as

$$A = \begin{cases} A_L & \text{with probability } \rho_t \\ A_H & \text{with probability } 1 - \rho_t \end{cases} \quad t = 1, 2, \dots,$$

where ρ_t is the seller's *posterior* belief at time t that $A = A_L$. We discuss next the formation of beliefs.

Beliefs

We model the seller as a rational agent that learns (“updates his beliefs”) according to Bayes’ rule. Before entering the foreign market, the seller has formed some prior beliefs about the level of demand. For instance, he might have done preliminary marketing research, on the basis of which he believes that with probability ρ_1 demand in the foreign country is “low” ($P_L \equiv P(A_L)$), that is, the lowest possible demand he can face. However, there is some positive probability ($1 - \rho_1$) that demand is “high”, $P_H \equiv P(A_H)$. In addition, the seller is aware of the possibility that with probability

¹⁵As e.g. in Saggi (1998). Since $P(q) = A - q$, there is an one-to-one mapping between a given period’s price realization and A .

¹⁶So, to avoid confusion, throughout the paper by “true” demand we refer to the value of A and, by realized (or observed) demand, to the demand realization in a given period (that is, including noise).

¹⁷Other formulations of noisy demand environments are, of course, possible. However, our formulation is a simple one and allows us to solve for the optimal paths while capturing the essence of the learning problem. A formulation with demand shocks in both the high and the low demand state would represent a significant complication and would not allow us a clean characterization of the optimal solution, without necessarily offering additional insights.

$(1 - s)$ a positive (temporary) demand shock may affect the realized demand. Therefore, the firm's first period decisions are based on its prior beliefs, which it subsequently updates in a Bayesian fashion, having observed the realized demand. At this point, it is important to emphasize that the seller learns only if he "does business" in the foreign market, that is, only if he serves the market. He gets *no* additional information on the demand level if he stays out, neither exporting nor investing (in this sense, knowledge is experiential). In addition, since learning about the demand comes from market sales, the seller obtains the same information, irrespective of the way that he serves the market, through exports or FDI.¹⁸

It follows from the above formulation that, if the seller has observed low demand at time $t - 1$, he concludes that true demand is low and updates his beliefs accordingly ($\rho_t = 1$). If he has observed a high demand realization, he cannot conclude that the true demand is high but he still updates his beliefs according to Bayes' rule. Then, the posterior belief of true demand being low (A_L) after a high-demand realization (A_H) in the previous period is:¹⁹

$$\rho_t = \Pr(A_L | A_H \text{ in } t - 1) = \frac{(1 - s)\rho_{t-1}}{(1 - s)\rho_{t-1} + (1 - \rho_{t-1})} = \frac{(1 - s)\rho_{t-1}}{1 - s\rho_{t-1}}. \quad (1)$$

Note that, in each period, given a probability ρ_t that the true demand is low, the probability that the seller expects to *observe* low demand is equal to $\Pr(A_L) = s\rho_t$ (that is, the probability of demand being low and no shock occurrence, which, assuming independence, equals the product of the respective probabilities). The respective probability of a high demand realization is $\Pr(A_H) = 1 - s\rho_t$.

Finally, note that the beliefs' updating rule is time invariant (a given value of ρ_{t-1} will always lead to the same value of ρ_t , irrespective of the period when the firm holds belief ρ_{t-1}) and that it has the Markov property (all relevant information regarding the past is contained in ρ_{t-1}).

Assumptions

To concentrate on the interesting cases for the analysis, we place the following assumptions on the parameters.

1. $A_L > c > k(1 - \delta)$,

where the LHS of the inequality states that, even with low demand, the market is viable and the RHS states that the costs of serving the market forever through exports are higher than the cost of serving it from the foreign-country facility. If the RHS inequality did not hold, FDI would never take place (given that it also has a fixed entry cost).

2. $A_H \leq 2A_L$,

¹⁸Some work has studied scenarios where, under FDI, a firm obtains knowledge about local conditions at an additional cost (e.g. Motta, 1992). Here, our starting point is that learning takes place in the market through sales and these, of course, can be associated either with FDI or with exports.

¹⁹We have $\Pr(A_L | A_H \text{ in } t - 1) = \frac{\Pr(A_H \text{ in } t-1 | A_L) \cdot \Pr(A_L)}{\Pr(A_H \text{ in } t-1 | A_L) \cdot \Pr(A_L) + \Pr(A_H \text{ in } t-1 | A_H) \cdot \Pr(A_H)} = \frac{(1-s)\rho_{t-1}}{(1-s)\rho_{t-1} + (1-\rho_{t-1})}$.

so that the set of feasible quantities in case of high demand are restricted in $[0, A_L]$.²⁰

3. We will restrict attention to cases where, in the absence of demand uncertainty and shocks, the discounted profit from exporting is higher than that from FDI when demand is low, while the opposite is true when demand is high. This assumption restricts the value of the entry cost in case of FDI and serves to focus on the range of entry cost values for which, if true demand is high and there is no uncertainty, the firm will be better-off by investing, while in case of low demand, the firm is better-off by exporting; otherwise the firm's decision would be trivial. We can calculate that there exists such a non trivial trade-off if the fixed cost satisfies $f \in (\underline{f}, \bar{f})$, where:

$$\begin{aligned}\underline{f} &= \frac{(c - k(1 - \delta))(2A_L - c - k(1 - \delta))}{4(1 - \delta)} \text{ and} \\ \bar{f} &= \frac{(c - k(1 - \delta))(2A_H - c - k(1 - \delta))}{4(1 - \delta)}.\end{aligned}$$

In the analysis, we restrict attention to this case, as for $f < \underline{f}$ exports would never take place and for $f > \bar{f}$ FDI would never take place.

Timing of events

At the beginning of the first period, and on the basis of his prior beliefs, the seller decides whether to export or to do FDI abroad and at what level to operate in each case. After serving the market, he receives a noisy signal, the market price. There are, then, two possibilities. Either realized demand is low, so that the seller learns with certainty that this is the true demand and, in the beginning of the next period, he makes his final decision on how to serve the foreign market and what quantity to produce once and for all. Or the realized demand is high, in which case the seller updates his beliefs and the same sequence of actions occurs in the following period. Of course, the mode-of-operation decision depends on whether the seller has already done FDI in the preceding period, in which case the (sunk) entry cost has been paid, or not. Consequently, in every period following a high demand realization the seller has to make two decisions: how to serve the market (FDI or exports) and what quantity to produce. His decisions are based on his current beliefs about the level of true demand or equivalently on his expectations about the future demand.

3 The dynamics of beliefs

Time enters into the model through the process of the updating of beliefs and, therefore, exports and investment dynamics are driven by this process. Consequently, we first establish the relationship that determines beliefs at each point in time given a prior ρ_1 and a probability s . We do this by solving recursively equation (1), which represents the beliefs of the firm updated according to Bayes' rule

²⁰This assumption excludes zero sales (at certain capacity levels) after a low demand realization. It simplifies the model without changing qualitatively the results.

after each period of high demand realization. With ρ_t being the posterior probability at period t that demand is low, direct calculations imply the following result:

Proposition 1 *Given a prior ρ_1 and following a sequence of high demand realizations, beliefs in each period evolve according to the rule*

$$\rho_t = \frac{\rho_1(1-s)^{t-1}}{1 - \rho_1(1 - (1-s)^{t-1})} \quad t = 2, 3, \dots \quad (2)$$

We are further interested in the properties of the above solution and how it changes when the primitives of the model change. To be compact in our notation, in what follows we call the time path of a variable z_t ‘concave’ if $z_t - z_{t-1} > z_{t+1} - z_t$ and ‘convex’ if $z_t - z_{t-1} < z_{t+1} - z_t$. We also call it S-shaped if it is increasing, being ‘convex’ up to some time t and ‘concave’ afterwards.²¹ We also denote by $\lceil t \rceil + 1$ the smallest integer larger than t .²² Examining the properties of the beliefs’ path we establish that:

- i) $\rho_t - \rho_{t-1} < 0$.
- ii) $\rho_{t+1} - \rho_t < \rho_t - \rho_{t-1}$, for $t < \lceil \hat{t} \rceil + 1$ and $\rho_{t+1} - \rho_t > \rho_t - \rho_{t-1}$, for $t \geq \lceil \hat{t} \rceil + 1$, where $\hat{t} \equiv \left(\frac{\ln(\frac{1-\rho_1}{\rho_1}) + \ln(1-s)}{\ln(1-s)} \right)$.

These two results, taken together, show that beliefs are decreasing at an increasing rate up to a certain period. Note that the relevant threshold time period, \hat{t} , is a function of the primitives ρ_1 and s . In addition, we need $\hat{t} > 1$ for the initial ‘concave’ part to exist and it is easy to check that this holds for $\rho_1 > 0.5$. Then \hat{t} is the ‘inflection point’ for ρ_t . In the case where $\rho_1 \leq 0.5$ we have that $\hat{t} \leq 1$ and therefore beliefs are everywhere decreasing at a decreasing rate. We summarize the above results as follows:

Remark 1 *The path of beliefs $\{\rho_t\}_{t=1}^{\infty}$ that true demand is low (following a sequence of high demand realizations) is decreasing. For $\rho_1 > \frac{1}{2}$ the beliefs’ path has an inverted S shape. Specifically, there is a threshold time $\hat{t} = \lceil \ln(\frac{1-\rho_1}{\rho_1}) + \ln(1-s) \rceil / \ln(1-s)$ such that ρ_t is ‘concave’ for every $t < \lceil \hat{t} \rceil + 1$ and ‘convex’ for $t \geq \lceil \hat{t} \rceil + 1$.*

In addition, it is useful to examine how the probability s of no shock occurrence affects the firm’s beliefs, as well as the threshold time period \hat{t} . We have:

- iii) $\frac{\partial \rho_t}{\partial s} = -\frac{\rho_1(1-\rho_1)(1-s)^{t-1}(t-1)}{(1-\rho_1 + \rho_1(1-s)^{t-1})^2} \leq 0$, for every $t \geq 1$.
- iv) $\frac{\partial \hat{t}}{\partial s} < 0$.

Both results (iii) and (iv) show that, as the probability of a positive shock $(1-s)$ increases, the updating of beliefs becomes slower. This is captured by the fact that, at each period, ρ_t is higher the lower s is and that \hat{t} , the time period when $\rho_t = 0.5$, is also greater the lower the probability s is. The

²¹Obviously there is a slight abuse of terminology here since the paths are discrete, however this use is standard and no confusion occurs.

²²We employ this notation simply to deal properly with the fact that time is discrete.

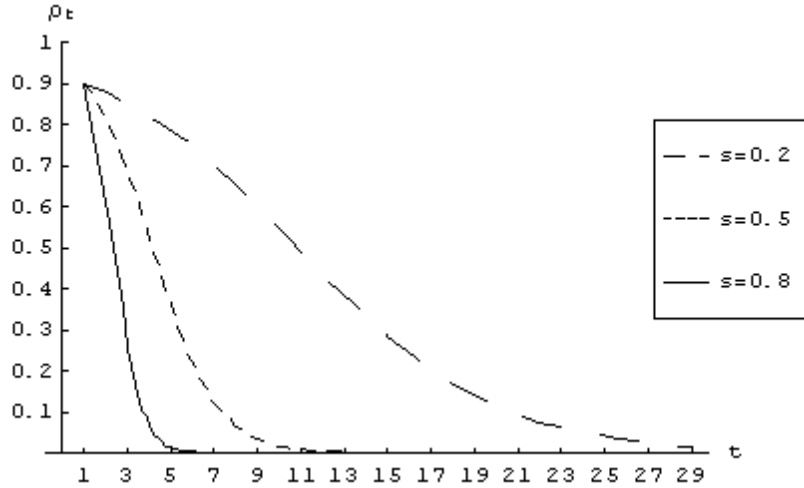


Figure 1: Path of beliefs conditional on s , for $\rho_1 = 0.9$

latter means that for a given ρ_1 it takes longer until beliefs that demand is low reach the level of 0.5. Thus:

Remark 2 *The higher the probability of a positive shock is (that is, the lower the s) the higher the posterior belief, ρ_t , is in every period and the larger the value of \hat{t} is, which shows that learning is slower.*

Figure 1 shows the time path of beliefs for different levels of the probability of a zero shock, assuming that the prior belief ρ_1 is 0.9. We have chosen a value of the prior close to 1 for this Figure in order to illustrate the properties of the beliefs' path for a wider range of the possible prior beliefs.²³ First, we see that for sufficiently high prior belief of demand being low, while demand is actually high, the updating process follows a path that has an inverted S shape. In other words, if the firm has “wrong” enough prior beliefs about demand in the foreign market and the first signals it gets are positive, it will be more eager to “correct these beliefs” (update at an increasing rate). However, after a certain period of time, it will become more cautious since the probability that it assigns to high demand becomes higher than the probability it assigns to low demand, positive signals only confirm these beliefs and do not trigger a large correction. Second, comparing the three graphs that correspond to different values of the probability of a zero-shock, we see that the lower this probability is, i.e. the higher the probability of a positive shock is, the slower the updating of beliefs is and therefore the more slowly the firm learns that true demand is high. This property of the beliefs path plays an important role and we will return to it in the next Section, where we analyze the effect of the demand shock.

²³Any point on the graph of a given path could be interpreted as a *prior* belief as long as we adjust properly the origin of the horizontal axis, so that this point corresponds to $t = 1$. Then the part of the graph from this point and on would represent the path of beliefs for the new prior.

4 The effect of the shock

We study here the role played by the demand shock in the model. In each period, when a sale takes place, the firm observes the demand realization, which may be low or high. A low demand realization reveals that the true demand is low and the effect of the shock in this case is clear; an increase in the probability of a positive shock increases expected demand conditional on the true demand being known to be low. What is more interesting is the effect of the shock on the expected demand following a sequence of high demand realizations, that is, as long as learning is still taking place. In this case, an increase in the probability of a positive shock has an adverse effect on expected demand within a given period. Expected demand in period t , assuming that true demand has not been revealed to be low, can be readily calculated to be

$$\tilde{A}_t \equiv s\rho_t A_L + (1-s)\rho_t A_H + (1-\rho_t)A_H = s\rho_t A_L + (1-s\rho_t)A_H = A_H - s\rho_t(A_H - A_L). \quad (3)$$

Thus, for a high demand realization, either true demand has to be high or true demand can be low but a shock occurs. A low demand realization, on the other hand, is possible only if the true demand is low. We note that a decrease in s has a direct positive effect and an indirect negative effect through the increase of ρ_t (by Remark 2). The positive effect is because, if true demand turns out to be low, a higher probability of a positive shock implies a higher probability that a high demand realization will occur. On the other hand, the indirect effect works through the beliefs. A higher probability of the positive shock makes it less likely that true demand is high, for a given sequence of high demand realizations. The sign of the total effect is determined by the relative magnitude of these two effects. More specifically, for $t > 1$, we find that

$$\frac{d\tilde{A}_t}{ds} = \frac{\partial \tilde{A}_t}{\partial s} + \frac{\partial \tilde{A}_t}{\partial \rho_t} \frac{d\rho_t}{ds} = \frac{\rho_1(1-s)^{t-2}(A_H - A_L)}{(1-\rho_1 + \rho_1(1-s)^{t-1})^2} [(1-\rho_1)(st-1) - \rho_1(1-s)^t].$$

It follows that expected demand in a given period will increase with the probability of a positive shock, if $s < 1/t$, and will decrease, if $s > 1/t$ and $(1-\rho_1)(st-1) > \rho_1(1-s)^t$. Given this, it is clear that a higher probability of a positive shock may lead either to a higher or to a lower conditional expected demand within a given period.²⁴

Now, let us consider the first period. Given a prior belief ρ_1 , only the direct effect of s is present (as the learning has taken place). Thus, expected demand in the beginning always increases with the probability of a positive shock. At the same time, given s , the updating of beliefs in subsequent periods will be slower the higher the probability of a positive shock is. Therefore, we distinguish two effects of the probability s , which play an important role in our analysis.

1. An *expected-demand-shifting* effect: high probability of a positive shock shifts expected first-period demand closer to a high demand realization, P_H , for any given prior belief ρ_1 .

²⁴Again, we emphasize that this result refers to the expected demand path *conditional* on a sequence of high demand realizations and not to the unconditional expected demand, which is always increasing in the probability of a positive shock.

2. An *uncertainty-persistence* effect: high probability of a positive shock impedes learning by slowing down the process of beliefs' updating.

If the expected-demand-shifting effect is strong enough (sufficiently low s), we expect the firm to act *as if* demand is high (even if “true” demand is low), since what really matters for profits in any given period is the realized demand and not whether there has been a shock or not. Consequently, there are values of s , not too close to 1, for which FDI dominates exports even if the firm believes with probability one that demand is low.

We, therefore, proceed the analysis of the optimal paths by first comparing the firm's profits, in case demand is known to be low (i.e. $\rho = 1$). We compare optimal exports and FDI and by doing so we establish conditions under which doing FDI is more profitable even when the beliefs about the demand are as “pessimistic” as possible. Let the per-period profit function in case of exports be Π_L^E . It is obtained from the following expected profit maximization problem:

$$\max_y (sA_L + (1-s)A_H - y - c)y, \quad (4)$$

where we denote by y the level of exporting. Let $E[A] \equiv sA_L + (1-s)A_H$ denote the expected value of the realization of A , assuming the true demand is low. It follows, from solving the maximization problem under (4), that the discounted expected profit in case of exports, when demand is low, is:

$$\frac{\Pi_L^E}{(1-\delta)} = \frac{(E[A] - c)^2}{4(1-\delta)}. \quad (5)$$

Similarly, we obtain the discounted value of gross profits in case of serving the market forever from a foreign facility there (FDI), when demand is low, by solving the following maximization problem:

$$\max_x \frac{(E[A] - k(1-\delta) - x)x}{(1-\delta)}, \quad (6)$$

where we denote by x the level of invested capacity. Solving (6) for x we obtain the discounted profit from FDI (excluding the initial entry cost)

$$\frac{\Pi_L^{FDI}}{(1-\delta)} = \frac{(E[A] - k(1-\delta))^2}{4(1-\delta)}. \quad (7)$$

Note that the profit functions (5) and (7) depend on the probability that a shock occurs, since shocks persist even after uncertainty with respect to the true demand level has been resolved. Also, from our Assumption 3 above and by continuity, we have that FDI dominates exports for $s \rightarrow 0$, while the opposite holds true for $s \rightarrow 1$. This suggests the existence of a threshold value \tilde{s} (for the probability of a zero shock) such that, if $s \leq \tilde{s}$, doing FDI is more profitable even when true demand is low. We obtain this threshold level \tilde{s} by equating the discounted profit from exports $\Pi_L^E/(1-\delta)$ with the discounted net profit from direct investment $(\Pi_L^{FDI}/(1-\delta)) - f$. Consequently, from (5) and (7) we obtain:

$$\begin{aligned} \tilde{s} &= \frac{(2A_H - k(1-\delta) - c)(c - k(1-\delta)) - 4f(1-\delta)}{2(A_H - A_L)(c - k(1-\delta))} = \\ &= \frac{4(1-\delta)(\bar{f} - f)}{2(A_H - A_L)(c - k(1-\delta))}, \end{aligned} \quad (8)$$

such that, if $s \leq \tilde{s}$, it is more profitable to serve the market via FDI rather than via exports even if true demand is low (simply because positive demand shocks are quite likely to be occurring). We can check that the threshold value \tilde{s} is always positive and, for values of the fixed cost in (\underline{f}, \bar{f}) , smaller than 1. Moreover, for a given s , the beliefs' path and, therefore, the expected demand path (given a sequence of high demand realizations) is everywhere higher the lower the prior belief, ρ_1 , is. Thus, for a given s smaller than \tilde{s} , that is, for a probability of no shock occurrence such that FDI is more profitable even when demand is known to be low ($\rho_1 = 1$), as the prior belief of low true demand decreases, expected demand in every period, following a high demand realization, increases. Consequently, if the probability s is such that FDI is preferred to exports in the worst possible scenario of known low true demand, then it should be preferred in all cases where beliefs of high true demand are positive ($\rho_1 < 1$). We thus have:

Proposition 2 *For every level of entry cost $f \in (\underline{f}, \bar{f})$ and for every prior belief ρ_1 there exists a threshold level of the probability of a zero shock, $\tilde{s} \in (0, 1)$ given by equation (8), such that for every $s \leq \tilde{s}$ the expected-demand-shifting effect of the shock dominates and, therefore, there will be entry via FDI right away.*

Hence we have determined a certain range of the probability that no shock occurs, for which the firm will enter the foreign market by installing capacity, irrespective of its beliefs about the true demand level. This means that, in our setting, a firm may invest abroad right away even in the presence of demand uncertainty. What drives the above result is the presence of a shock that may be misrepresenting the actual demand.²⁵

The case examined above may not be the only one where installing productive capacity from the very beginning is possible, that is, the condition established is only sufficient (as the argument is based on prior beliefs being as pessimistic as possible). Subsequently, we show that immediate entry via FDI is also possible for values of s greater than \tilde{s} , depending on the prior belief ρ_1 . Therefore, we now proceed to the remainder of the analysis and concentrate on the more interesting case of $s > \tilde{s}$, where the expected-demand-shifting effect does not necessarily dominate. Then, the trade-off between incurring the entry cost and saving on variable cost is present, or equivalently, if demand is high, FDI may be preferable but, if demand is low and there are shocks, exports may be more profitable.

Having described the evolution of beliefs and the effect of the demand shock, we next analyze the exports and investment dynamics by considering two cases: either the firm enters the foreign market

²⁵Note that the expected-demand-shifting effect is independent of the way we model uncertainty here, i.e. that the shock is positive. We could easily establish a range for the probability s such that immediate investment takes place also if in the model there was instead, a negative shock. Simply let

$$P_t(q_t; A) = \begin{cases} A_L - q_t & \text{if } A = A_L \\ A_H - q_t - \varepsilon_t & \text{if } A = A_H. \end{cases}$$

Then capacity installment takes place immediately if $s \geq \tilde{s} = \frac{4(1-\delta)(f-\underline{f})}{2(A_H-A_L)(c-k(1-\delta))}$.

via exports or it invests in productive capacity abroad. After these problems have been examined, we will turn to the choice of the entry mode and the optimal timing of the initial FDI.

5 Exports

Let us first assume that the firm starts by exporting in the first period. The expected profit in every period is independent of the quantity exported in previous or future periods (since the product cannot be stored). The only link between periods is the seller's beliefs, summarized by the probability ρ_t of demand being low. Thus, we have period-by-period maximization of the expected profit with respect to each period's quantity exported, y_t :

$$\max_{y_t} \{-cy_t + (s\rho_t A_L + (1 - s\rho_t)A_H - y_t)y_t\}. \quad (9)$$

Solving for y_t , we find the optimal quantity exported in each period following a high demand realization:

$$y_t^*(\rho_t) = (s\rho_t A_L + (1 - s\rho_t)A_H - c)/2 \quad \forall t : \rho_t < 1. \quad (10)$$

Therefore, we can write the optimal quantity exported after either a low or a high demand realization as

$$y_t^*(\rho_t) = \begin{cases} (s\rho_t A_L + (1 - s\rho_t)A_H - c)/2 & \text{for } \rho_t < 1 \\ (sA_L + (1 - s)A_H - c)/2 & \text{for } \rho_t = 1. \end{cases}$$

Note that $y_t^*(\rho_t) \geq (A_L - c)/2$, where the RHS of the inequality is the optimal level of exports when the seller knows that demand is low and there are no shocks (that is when $\rho_t = 1$ and $s = 1$). Actually, for all s and $\rho_t \in [0, 1]$ we have that $y_t^* \geq (E[A] - c)/2$ and therefore $y_1^* \geq (E[A] - c)/2$, which means that the seller will never start by exporting a quantity lower than the monopoly level under low demand and demand shocks. However, exports will drop and stay at that level forever once the seller has observed a low demand realization in some period. Next, we prove the following properties of the optimal exports path:

Proposition 3 *As long as $\rho_t < 1$, the optimal exports path $\{y_t^*\}_{t=1}^\infty$ is increasing in time and satisfies:*

$$y_t^* = \frac{A_H - c}{2} - \frac{s\rho_1(A_H - A_L)(1 - s)^{t-1}}{2(1 - \rho_1 + \rho_1(1 - s)^{t-1})}. \quad (11)$$

In addition, for $\rho_1 < 1/2$ the optimal path is 'concave', while for $\rho_1 > 1/2$ it is S-shaped.

Equation (11) is obtained by substituting (2) into (10) and a little further manipulation. Since the optimal quantity exported in each period is linear in that period's probability of demand being low, the dynamic properties of the optimal exports path follow directly from the properties of the beliefs' updating process. Therefore, the S-shaped path obtained in case of a sufficiently high prior probability, ρ_1 , is a direct consequence of learning about demand. Moreover, the threshold time, after which exports increase at a decreasing rate is \hat{t} , as derived above. These results show that, if the seller initially puts a relatively low probability on high true demand, but the signals he receives from the market are

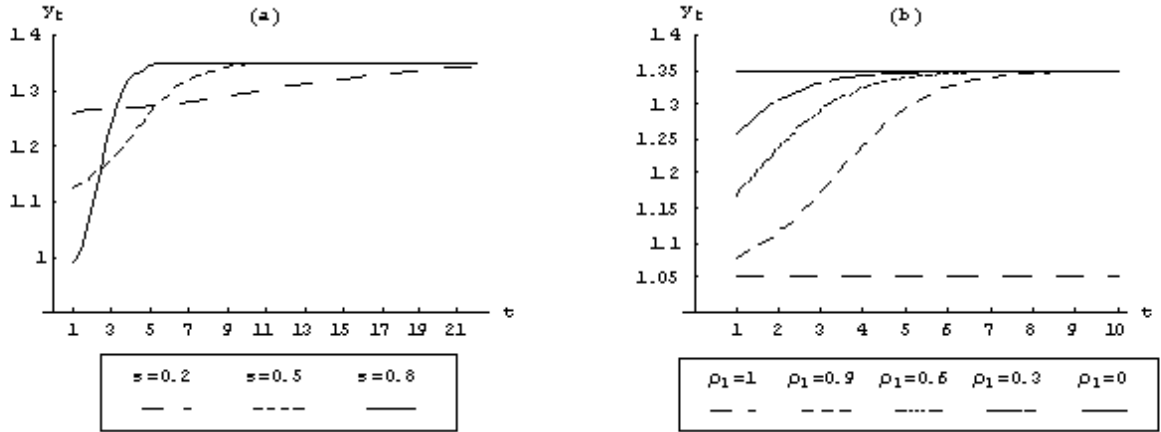


Figure 2: Examples of exports paths for $A_H = 3$; $A_L = 2$; $c = 0, 3$: (a) conditional on s (for $\rho_1 = 0.9$); (b) conditional on ρ_1 (for $s = 0.6$)

positive (high demand realizations), the quantity exported will be increasing at an increasing rate. The reason is that high demand realizations, in this case, provide a strong signal that the beliefs are more probably “wrong”. Therefore, beliefs are updated at an increasing rate, which, in turn, causes the quantity exported to increase at an increasing rate. Once the posterior belief of high demand has exceeded the level of 0.5, the increments to the quantity exported start to decrease. This is because, when the seller’s beliefs of high true demand are already high, high demand realizations do not come at odds with these beliefs (as is the case when the seller believes that demand is more likely to be low) and, therefore, do not trigger a significant correction, that is, do not lead to large increments to the quantity exported.

Finally, we can have a comparative static analysis of the optimal exports path. From (10) we see that, as expected, the quantity exported in every period is higher the higher both the probability of high true demand ($1 - \rho_t$) and the probability of high realized demand ($1 - s\rho_t$) are. However, it is more interesting to see how the prior beliefs and the probability of the shock influence the optimal exports path. Figure 2 illustrates clearly this effect.

Figure 2a represents the exports path for different values of s (given that true demand is high and $\rho_1 = 0.9$). We can see that for low values of s (i.e. high probability of a positive shock) exports are high in the first period but the rate at which they increase in subsequent periods is low and it takes longer till the quantity exported converges to what the optimal quantity would be in case of high demand. In other words, when uncertainty about the level of demand is likely to persist (because of high probability of the occurrence of positive shocks) we observe the impact of the two effects we have already identified: on the one hand, the quantity exported is high initially since for any given ρ_1 the probability of high realized demand ($s\rho_1$) is high (expected-demand-shifting), but on the other hand, the high probability of a positive shock leads to slow learning. The opposite is true for high values of s (low probability of positive shocks): initial exports are low, but since every high-demand realization

is most probably due to high true demand, the prior belief is quickly updated in favor of true demand being high, which leads to a large increase in the quantity exported in subsequent periods and to a fast convergence to the optimal quantity in case of high demand. Figure 2b shows the exports path for different values of the prior belief ρ_1 (given $s = 0.6$). As expected, the lower the ρ_1 is, i.e. the higher the prior belief of demand being high, the higher exports are in all periods and the faster the path converges to the optimal exports level when demand is high. We can also see that the path is S-shaped in the case of a high prior belief (for instance, for $\rho_1 = 0.9$).

Having derived the optimal path assuming that the firm proceeds by exporting, the next Section studies the optimal investment path. After that part is concluded we will examine the optimal mode of entry.

6 FDI

Let us assume now that the firm enters the foreign market by establishing productive capacity there. We denote by x_t the capacity installed in period t and by X_t the accumulated capacity up to that period, that is

$$X_t = X_{t-1} + x_t$$

and we assume that $X_0 = 0$ (i.e. a “greenfield” investment). Note also that assumption (1) along with the fact that the fixed entry cost becomes sunk jointly imply that once the seller has incurred the entry cost *he will never switch back to exports* even if demand turns out to be low. This is because the marginal cost is lower if supply comes from FDI rather than from exports. In this case, given that the firm will continue to serve the market from the foreign facility, the next question we should examine is the possibility of a final adjustment made to the already installed capacity.

Remark 3 *Once the firm has incurred the entry cost of capacity installment, it will never add capacity after observing a low demand realization. Specifically, there are two possibilities along the optimal path. If the capacity already installed, X_t , is lower than the optimal monopoly quantity in case of low demand and zero marginal cost, \hat{X} , then no additional investment will take place and output X_t will be produced from then on. However, if low demand is realized when $X_t > \hat{X}$, then capacity $X_t - \hat{X}$ will be left unutilized and the firm will produce quantity \hat{X} from then on. The latter case occurs if $(A_H - k(1 - \delta))/2 \geq E(A)/2$.*

The intuition behind the above result is as follows. Suppose that the firm has already paid the entry cost and there is a low demand realization in period t . Installed capacity up to that period is X_t and the seller considers a possible final capacity adjustment x_{t+1} . Consequently, x_{t+1}^* , the optimal capacity installed in period $t + 1$ given an already invested capacity X_t , should solve the following maximization problem:

$$\max_{x_{t+1}} \left\{ -kx_{t+1} + \frac{(E(A) - x_{t+1} - X_t)(x_{t+1} + X_t)}{(1 - \delta)} \right\}. \quad (12)$$

By direct maximization of the above equation, it follows that $x_{t+1}^* = (E(A) - k(1 - \delta) - 2X_t)/2$. But from equation (6) we know that, if the seller knew from the very beginning that demand is low, he would have invested at the level $\bar{x}_1 = (E(A) - k(1 - \delta))/2$ and that, if $\rho_1 < 1$, it will always be the case that $x_1^* > \bar{x}_1$. Therefore, x_{t+1}^* will never be positive, that is, there will not be a final period capacity adjustment.

However, there is a possibility that the firm leaves some capacity unutilized. This will be the case if total capacity already installed is higher than the optimal quantity in case the seller is facing low demand, positive shocks and zero marginal cost, which we denote by $\hat{X} = E(A)/2$. Then it is optimal for the firm to leave capacity equal to $X_t - \hat{X}$ idle and produce at the level of \hat{X} at zero marginal cost forever. In order for this case to occur the monopoly quantity in case of high demand, $(A_H - k(1 - \delta))/2$, should be higher than \hat{X} .

Consequently, we have to distinguish two cases, depending on the level of total capacity installed by the time demand is revealed to be low:

1. $X_t < \hat{X}$. In this case, if low demand is observed at time t , the firm will produce X_t forever and the respective discounted profits will equal $(E(A) - X_t)X_t/(1 - \delta)$.
2. $X_t \geq \hat{X}$. After low demand realization at t , capacity $(X_t - \hat{X})$ will be left unutilized and \hat{X} will be produced forever. The respective discounted profit in this case will be $E(A)^2/4(1 - \delta)$.

We next characterize the optimal investment path, if the firm decides to serve the foreign market via FDI, by setting up the corresponding dynamic programming problem. Unlike the case of exports, investment capacity now plays the role of the link between periods. The value function, denoted by $V_t^{FDI}(X_{t-1}; \rho_t)$, is defined as the maximized expected present value of future profits when installed capacity is X_{t-1} and the seller believes with probability ρ_t that demand is low. As usual, this can be separated into two parts, the current period profit and the continuation payoff, which differ between the cases we have distinguished above. The control variable is the incremental capacity (or in other words, investment) x_t , while the state variables are the cumulative capacity X_t and the firm's beliefs ρ_t .

Case 1 ($X_t < \hat{X}$)

The Bellman's equation for the value function in this case is

$$V_t^{FDI}(X_{t-1}, \rho_t) = \max_{x_t} \{ -kx_t + [s\rho_t A_L + (1 - s\rho_t)A_H - (X_{t-1} + x_t)](X_{t-1} + x_t) + \delta s\rho_t \frac{(E(A) - (X_{t-1} + x_t))(X_{t-1} + x_t)}{(1 - \delta)} + \delta(1 - s\rho_t)V_{t+1}^{FDI}(X_t, \rho_{t+1}) \}, \quad (13)$$

where the first term represents the investment cost, the second term represents the current-period profit and the last two terms represent the discounted future payoff in case of both a high and a low current-period demand realization.

Case 2 ($X_t \geq \hat{X}$)

The value function in this case obeys:

$$V_t^{FDI}(X_{t-1}; \rho_t) = \max_{x_t} \{-kx_t + [s\rho_t A_L + (1 - s\rho_t)A_H - (X_{t-1} + x_t)](X_{t-1} + x_t) + \delta s\rho_t \frac{E(A)^2}{4(1-\delta)} + \delta(1 - s\rho_t)V_{t+1}^{FDI}(X_t, \rho_{t+1})\}. \quad (14)$$

Note that the value function differs between the two cases only with respect to the continuation payoff in case demand gets revealed to be low.

Solving for the optimal capacity invested in both cases, we obtain the optimal investment paths. In addition, we can calculate the threshold point in time \bar{t} such that for $t < [\bar{t}] + 1$ the optimal incremental investment level is the solution of (13), while for $t \geq [\bar{t}] + 1$ it solves (14). Since the solutions of (13) and (14) can be characterized along similar lines, we present here only the calculations for Case 1.

By partially differentiating equation (13) with respect to x_t we obtain the following first-order condition:

$$\begin{aligned} \frac{\partial V_t^{FDI}(X_{t-1}, \rho_t)}{\partial x_t} &= -k + \frac{s\rho_t A_L(1 - \delta(1 - s))}{(1 - \delta)} + \frac{A_H(1 - \delta - s\rho_t + 2\delta s\rho_t - \delta\rho_t s^2)}{(1 - \delta)} - \\ &\quad - \frac{2X_{t-1}(1 - \delta(1 - s\rho_t))}{(1 - \delta)} - \frac{2x_t(1 - \delta(1 - s\rho_t))}{(1 - \delta)} + \delta(1 - s\rho_t) \frac{\partial V_{t+1}^{FDI}(X_t, \rho_{t+1})}{\partial x_t}. \end{aligned}$$

Given x_t is positive, we further have that:

$$\frac{\partial V_{t+1}^{FDI}(X_t, \rho_{t+1})}{\partial x_t} = \frac{\partial V_{t+1}^{FDI}(X_t, \rho_{t+1})}{\partial X_t} = \frac{\partial V_{t+1}^{FDI}(X_t, \rho_{t+1})}{\partial x_{t+1}} + k = k,$$

where in writing the second equality we have used the substitution

$$V_{t+1}^{FDI}(X_t, \rho_{t+1}) = V_{t+1}^{FDI}(X_{t-1} + x_t, \rho_{t+1}),$$

while the last equality follows from the envelope condition.

Substituting into the first-order condition above we obtain the optimal values of x_t^1 and X_t^1 , the solution in Case 1 defined above. In a similar way we solve for x_t^2 and X_t^2 , the solution in Case 2. The results are summarized in the following Proposition:

Proposition 4 *As long as the observed demand is high, the optimal investment path in case of FDI is*

$$x_t^*(X_{t-1}, \rho_t) = \begin{cases} x_t^1(X_{t-1}, \rho_t) = \frac{A_H - k(1-\delta)}{2} - \frac{s\rho_t(1-\delta(1-s))(A_H - A_L)}{2(1-\delta(1-s\rho_t))} - X_{t-1} & \text{for } t < [\bar{t}] + 1 \\ x_t^2(X_{t-1}, \rho_t) = \frac{A_H - k(1-\delta)}{2} - \frac{s\rho_t(A_H - A_L + k\delta)}{2} - X_{t-1} & \text{for } t \geq [\bar{t}] + 1 \end{cases}$$

and the corresponding optimal cumulative capacity path is

$$X_t^*(\rho_t) = \begin{cases} X_t^1(\rho_t) = \frac{A_H - k(1-\delta)}{2} - \frac{s\rho_t(1-\delta(1-s))(A_H - A_L)}{2(1-\delta(1-s\rho_t))} & \text{for } t < [\bar{t}] + 1 \\ X_t^2(\rho_t) = \frac{A_H - k(1-\delta)}{2} - \frac{s\rho_t(A_H - A_L + k\delta)}{2} & \text{for } t \geq [\bar{t}] + 1, \end{cases}$$

where $\rho_t = \frac{\rho_1(1-s)^{t-1}}{1-\rho_1(1-(1-s)^{t-1})}$ and $\bar{t} = 1 + \frac{\ln\left(\frac{\bar{\rho}(1-\rho_1)}{\rho_1(1-\bar{\rho})}\right)}{\ln(1-s)}$, with $\bar{\rho} \equiv \frac{(A_H - A_L)s - k(1-\delta)}{(A_H - A_L)s + k\delta s}$.

Note that the time of the optimal switching between Case 1 and 2 depends on all the parameters of the model. We calculate this switching time by first computing a threshold value $\bar{\rho}$ of the posterior beliefs by equating either $X_t^1(\rho_t)$ or $X_t^2(\rho_t)$ to $\hat{X} = E(A)/2$. The threshold probability is

$$\bar{\rho} = \frac{(A_H - A_L)s - k(1 - \delta)}{(A_H - A_L)s + k\delta s},$$

where $\bar{\rho} \in [0, 1]$ holds given that the condition for the existence of Case 2 is satisfied. Having computed $\bar{\rho}$ and knowing that ρ_t is monotonically decreasing in t we can find \bar{t} as a function of the initial parameters of the model. At this point, we should note that, if the prior belief ρ_1 is smaller than $\bar{\rho}$, the capacity will expand according to the solution of (14) (i.e. only Case 2 will be relevant), while for parameter values such that Case 2 does not exist, capacity would evolve according to (13).

As in the case of exports we examined in the previous section, we wish to analyze the dynamic properties of the optimal investment path (conditional on true demand being high). Direct calculations imply that the increments ΔX_t^1 and ΔX_t^2 are positive, which means that there is *gradual capacity expansion* as long as the signals that the firm receives from the market are positive. Also, what may be of greater interest is the speed at which installed capacity expands, in other words, whether the investment rate increases or decreases with time. Examining how the successive increments ΔX_t and ΔX_{t+1} relate in both Case 1 and 2 we establish that, similar to exports, optimal capacity invested may follow an S-shaped path. That is, we can compute a threshold point in time such that cumulative capacity increases at an increasing rate before it and at a decreasing rate afterwards. Similarly to the case of exports, we need a sufficiently high prior probability ρ_1 in order for the above threshold values to exist. More specifically, the threshold points in time and the respective conditions on ρ_1 that ensure their existence are as follows:

$$\text{Case 1: } \tilde{t}^1 = \frac{\ln[(1-\delta)(1-\rho_1)(1-s)/\rho_1(1-\delta(1-s))]}{\ln(1-s)} \quad \text{where } \tilde{t}^1 \geq 1 \quad \text{if } \rho_1 \geq \frac{1-\delta}{2(1-\delta)+\delta s}.$$

$$\text{Case 2: } \tilde{t}^2 = \frac{\ln[(1-\rho_1)/\rho_1] + \ln(1-s)}{\ln(1-s)} \quad \text{where } \tilde{t}^2 \geq 1 \quad \text{if } \rho_1 \geq \frac{1}{2}.$$

Remark 4 *Depending on the value of the prior beliefs ρ_1 , the optimal capacity installed may follow an S-shaped path. More specifically, this is the case if:*

- i) $\rho_1 > (1 - \delta)/(2(1 - \delta) + \delta s)$ for $X_t < \hat{X}$
- ii) $\rho_1 > 1/2$ for $X_t \geq \hat{X}$,

Figures 3 and 4 depict numerical examples of the optimal incremental and cumulative investment paths for Case 1 and 2 respectively.

Similarly to exports, if the prior belief of low true demand exceeds a certain threshold level, the cumulative investment path increases initially at an increasing rate and, after a number of positive signals (high demand realizations), the incremental investments start to fall. Note, that in the present model the S-shaped investment path is the result of gradual learning about the demand when true demand is high and the seller has sufficiently “wrong” priors.²⁶ Having in mind that investment

²⁶Vettas (1998, 2000) obtains an S-shaped path under learning, where the shape of the diffusion path is determined by the relative magnitude of two effects: learning about the product by the consumers and learning about demand by

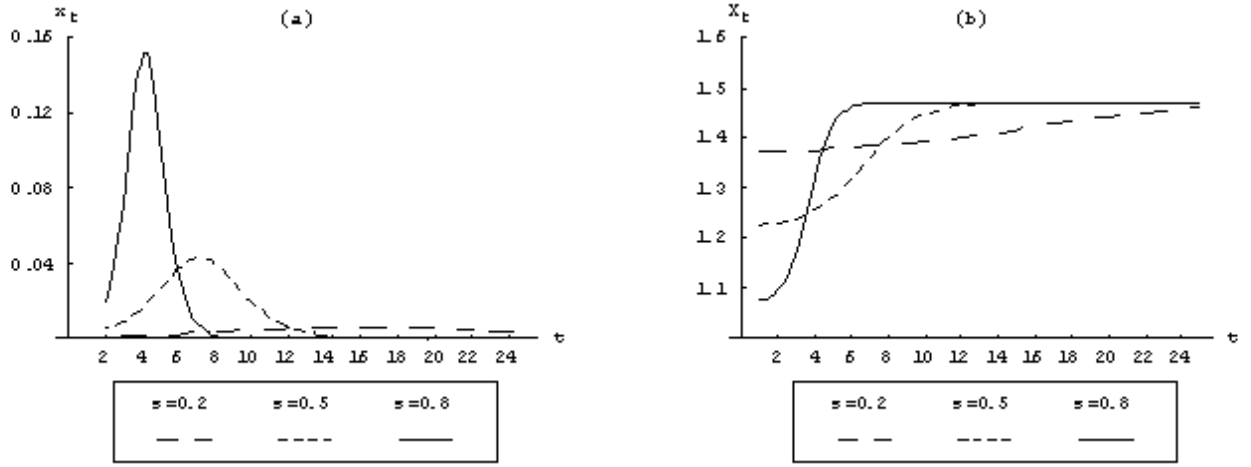


Figure 3: Case 1 with $A_H = 3, A_L = 2, c = 0.3, k = 0.65, \delta = 0.9, \rho_1 = 0.9$: (a) Optimal incremental investment x_t ; (b) Optimal cumulative investment X_t

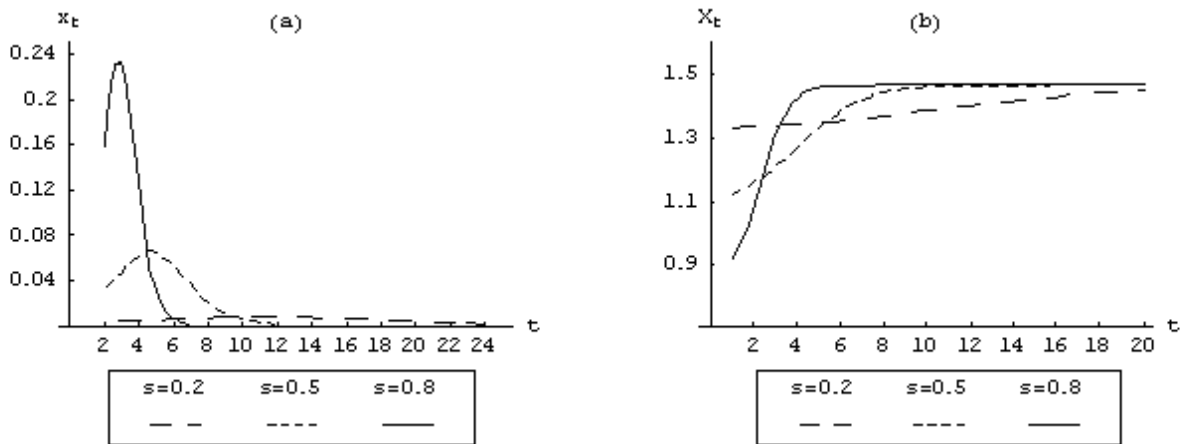


Figure 4: Case 2 with $A_H = 3, A_L = 2, c = 0.3, k = 0.65, \delta = 0.9, \rho_1 = 0.9$: (a) Optimal incremental investment x_t ; (b) Optimal cumulative investment X_t

is increasing over time and that there is a threshold \bar{t} denoting the point of time when cumulative capacity has reached the level \hat{X} , we can draw some conclusions regarding the effect of uncertainty persistence on the process of capacity expansion. More specifically, we find that $\partial\bar{t}/\partial s < 0$, which implies that the higher the probability of a positive shock is, i.e. the slower the learning process is, the more time is needed in order that installed capacity reaches the level \hat{X} . Expressed in terms of incremental capacity, this means that as long as $X_t < \hat{X}$, capacity added in each period x_t^* will be smaller the lower the s is. This captures the idea that *the possibility of a greater uncertainty persistence leads to the seller choosing a more cautious investment path.*

Finally, we compare the sequences of the optimal quantity exported and that produced at the foreign country facility (with FDI). Although there is no general relationship that holds in every time period and for all parameter values, some observations are worth mentioning. First, for values of the posterior belief lower than $\bar{\rho}$, capacity installed abroad (and quantity produced) will be higher than the quantity exported if $c > k(1 - \delta(1 - s\rho_t))$. Given that ρ_t is decreasing in time, we can see that for $\rho_t < \bar{\rho}$ the exports path will be everywhere below the optimal cumulative investment path if $c > k(1 - \delta(1 - s\bar{\rho}))$. Furthermore, we know that, if $\rho_1 > \bar{\rho}$, cumulative capacity will evolve (for a number of periods) according to the solution of (14) and for some t will increase at an increasing rate if $\rho_1 > (1 - \delta)/(2(1 - \delta) + \delta s)$. We have also established that exports increase at an increasing rate if $\rho_1 > 1/2$ and until the posterior belief has been reduced to $1/2$. Since $(1 - \delta)/(2(1 - \delta) + \delta s) < 1/2$, we can conclude that, for values of the prior belief, ρ_1 , between $(1 - \delta)/(2(1 - \delta) + \delta s)$ and $1/2$, there will be a period of time such that, if the firm enters the market via exports the quantity exported will increase at a decreasing rate, while if it enters via FDI, the quantity produced will increase at an increasing rate. This is independent of whether capacity installed in the first period is higher or lower than the quantity exported.²⁷

7 Choice of entry mode: the timing of FDI

7.1 Derivations

As our analysis has shown, the quantity exported increases with time as long as the seller observes high demand and, therefore, as he updates his beliefs in favor of true demand being high. It is natural to suggest then that, after receiving positive signals for a number of periods, the seller would be “convinced” enough that demand is high and would switch to FDI. In fact, in terms of the level of the demand that will be observed in the market, a small probability ρ_t plays a similar role as a small

the producers. A strong enough demand-shifting effect leads to an initially convex competitive diffusion path. However, obtaining an S-shaped path in the monopoly case (as the one we are studying here) is not easy in such learning models since the monopolist would internalize the demand-shifting effect. Such a path would only be possible either for values of the discount factor that are very low or for intermediate values of the responsiveness of demand to past sales.

²⁷Both situations are possible. In general, for $\rho_1 > \bar{\rho}_t$, capacity installed in the first period will be higher than the quantity exported if $c - k(1 - \delta) > \frac{\delta s^2 \rho_1 (1 - \rho_1)(A_H - A_L)}{1 - \delta(1 - s\rho_1)}$, while for $\rho_1 < \bar{\rho}_t$, this holds if $c - k(1 - \delta) > \delta k s \rho_1$.

s . Since after each positive signal the posterior probability ρ_t decreases, for any given s , expected demand gets closer to P_H . In addition, we know from Proposition 1 that if $s > \tilde{s}$ there is a trade-off between incurring the entry cost and saving on variable costs. Equivalently, this trade-off means that for $\rho_t \rightarrow 1$ exports dominate FDI, while the opposite holds for $\rho_t \rightarrow 0$. This suggests the existence of a threshold value for ρ_t such that, if the firm has started exporting initially, it will find it profitable to install capacity abroad once beliefs cross this threshold. Further, if the prior belief ρ_1 is lower than the threshold belief, the firm would enter the foreign market directly by installing productive capacity there. This is because, if FDI becomes more profitable than exports once beliefs have exceeded a certain threshold level, then it will also be more profitable for all levels of the prior beliefs, ρ_1 , lower than this threshold level (since such priors attach higher probability to high true demand and, therefore, increase expected demand).

In this Section we establish conditions for the existence of threshold beliefs that, depending on the prior, ρ_1 , would indicate either a switch from exports to FDI or immediate investment. We are looking, therefore, for a threshold value of ρ_t which makes the seller indifferent between exporting and investing abroad. Then, for more favorable beliefs, the seller would choose to invest and, for less favorable beliefs, would choose to export. The indifference condition can be written as follows:

$$\begin{aligned} & \frac{(s\rho_t A_L + (1-s\rho_t)A_H - c)^2}{4} + \delta s\rho_t \left[\frac{\Pi_L^E(s)}{1-\delta} \right] + \delta(1-s\rho_t) [V_{t+1}^{FDI}(x_{t+1}^*(0, \rho_{t+1}), \rho_{t+1}) - f] \\ & = V_t^{FDI}(x_t^*(0, \rho_t), \rho_t) - f, \end{aligned} \quad (15)$$

where x_t^* is the maximizer of $V_t^{FDI}(0, \rho_t)$. The first term on the LHS expresses the current profit from exports, while the second and the third terms represent the continuation payoff in case of low and high demand. Since we are looking for the probability, ρ_t^* , for which the seller is indifferent between exports and investment, the RHS of (15) represents the expected payoff in case of FDI from t^* (the time period corresponding to ρ_t^*) onwards. Since at ρ_t^* the seller is indifferent between exporting and investing, at ρ_{t+1} he will strictly prefer investing (having received one more positive signal). Therefore the continuation payoff in case of high demand realization (the third term on the LHS) is the expected payoff of investment from period $t+1$ and on. Note that the solution of the indifference equation denotes the level of beliefs that, if exceeded, will trigger an investment abroad. Once the firm has invested, it will never switch back to exports (this is incorporated in the indifference equation indirectly, through the value function). Moreover, interpreted in terms of prior beliefs, the solution of the indifference equation gives the highest level of the prior belief of true low demand (equivalently, the lowest level of the prior belief of high true demand) such that investment is done from the very beginning. Therefore, since expected demand increases with the prior belief of high true demand, for all ρ_1 lower than the solution of the indifference equation, the firm will enter with FDI in the first period.

Rearranging (15) we obtain:

$$\begin{aligned} & \frac{(s\rho_t A_L + (1-s\rho_t)A_H - c)^2}{4} + \delta s\rho_t \left[\frac{\Pi_L^E(s)}{(1-\delta)} \right] = \\ & = V_t^{FDI}(x_t^*(0, \rho_t), \rho_t) - \delta(1-s\rho_t)V_{t+1}^{FDI}(x_{t+1}^*(0, \rho_{t+1}), \rho_{t+1}) - f(1-\delta(1-s\rho_t)). \end{aligned} \quad (16)$$

It is useful to provide some intuition for the above condition. The LHS of (16) represents the gain from exporting for one more period, while the RHS represents the net gain from investing today compared to waiting for one more period. The first two terms of the RHS show the gain in terms of unit cost savings, while the third term captures the loss in terms of incurred entry cost. Obviously the firm will export if the LHS of (16) is higher than the RHS and will invest in productive capacity if the opposite holds. Therefore, we proceed to establishing conditions under which the net benefit from investing today rather than tomorrow is greater than the benefit from exporting for one more period. In doing so we consider two cases depending on whether beliefs, ρ_t , belong to $[0, \bar{\rho}]$ or to $[\bar{\rho}, 1]$.²⁸ Then we show that whether the level of beliefs that triggers the first investment is higher or lower than $\bar{\rho}$ depends on how large the fixed entry cost is. The computational details are presented in the Appendix, while the results are summarized in the following Proposition:

Proposition 5 *The firm will enter the market either with FDI right away or will export initially and will switch to FDI at a later point. More specifically, if :*

1. *the entry cost is relatively low, $f \in [\underline{f}, f^*]$, the firm will serve the foreign market via FDI from the beginning,*
2. *the entry cost is at intermediate levels, $f \in (f^*, f^{**}]$, there exists a threshold belief $\rho^* \in [\bar{\rho}, 1)$ such that, if $\rho_1 \in (\rho^*, 1]$, switching from exports to FDI will occur, while, if $\rho_1 \in [0, \rho^*]$, there will be entry via FDI right away,*
3. *the entry cost is relatively high, $f \in (f^{**}, \bar{f}]$, there exists a threshold belief $\rho^{**} \in [0, \bar{\rho})$ such that, if $\rho_1 \in (\rho^{**}, 1]$, switching from exports to FDI will occur, while, if $\rho_1 \in [0, \rho^{**}]$, there will be entry via FDI right away.*

The above Proposition characterizes the firm's entry behavior when demand uncertainty in the foreign market is likely to persist and becomes resolved with learning from sales. It shows that immediate investment is plausible for the whole range of possible values of the fixed entry cost and despite the fact that all investment costs are completely irreversible. This finding complements the results of Saggi's (1998) model, where there are no demand shocks and uncertainty is resolved after one period. In this model, the choice between FDI and exports depends crucially on the degree of irreversibility of the fixed cost associated with the FDI decision. If the fixed cost is fully reversible, there is always immediate investment. If the fixed cost is highly irreversible, the firm would generally export and switch to FDI in the second period only if demand turns out to be high. First-period FDI is possible only for low values of the fixed cost. In our model, uncertainty is not resolved after the initial sales and learning takes time. We have to distinguish between three ranges of the level of the fixed cost. In the first case of Proposition 5, the fixed entry cost is relatively low and the firm does FDI right away, due to the favorable effect that a positive shock has on expected demand. The second and the third cases of the same Proposition show that, even if this effect does not dominate, the firm will switch from exports to

²⁸These cases correspond to Case 1 and Case 2 of the analysis in Section 6.

FDI or, depending on its prior beliefs, will enter with FDI in the first period. The conditions established in Proposition 5 are very intuitive. For given parameter values, if the fixed entry cost is relatively low there will be immediate investment for any value of the prior belief ρ_1 . But as the fixed entry cost increases, possible entry with FDI depends also on the prior belief of demand being low, namely this prior belief should be lower than some threshold value. From Case 2 and Case 3 of the Proposition we can see that the threshold level is lower for higher values of the fixed entry cost ($\rho^{**} < \rho^*$), which means that the higher the f is, the higher the probability of high demand should be for the firm to invest right away.

7.2 Comparative statics

One of the main purposes of our analysis is to show that uncertainty persistence may, under certain conditions, favor capacity installment despite the possibility of exporting, because FDI provides a “cheaper” way of establishing the possibility of sales (and sales lead both to revenue and to learning). Therefore, we are interested in how changes in the costs associated with exports and capacity installment influence the threshold beliefs ρ^* and ρ^{**} and consequently the timing of the seller’s initial investment.

a) One would expect that an increase of the unit cost of exports c would result in an increase of ρ^* and ρ^{**} , meaning that the firm will do FDI for lower levels of its belief that true demand is high. Comparative statics results show that this is indeed the case. (formally this can be shown by employing equations (A3) and (A5) in the Appendix - it can be shown that an increase of c leads to a decrease of the value of the LHS of both equations and to an increase of the RHS of the second equation). In particular, the threshold probabilities ρ^* and ρ^{**} will be higher the higher the unit cost of exports, c , is.

b) The effect of an increase of the unit cost of capacity installment k will be exactly the opposite to that of an increase in the unit cost of exports. It is straightforward to show that a higher k will result in a lower level of capacity installed in each period, which, in turn, will result in a decrease of the value of the RHS of both equations (A3) and (A5) in the Appendix. Consequently, ρ^* and ρ^{**} will be lower, which means that immediate investment will take place for a higher prior belief of high true demand or that a switch from exports to FDI will occur at a later point.

7.3 A numerical example

In this Subsection, we present a numerical example, which illustrates the insights generated by our analysis. We assume the following parameter values: $A_H = 20$, $A_L = 12$, $f = 300$, $k = 8$, $c = 5$, $\delta = 0.9$, $s = 0.5$. Note that the fixed entry cost satisfies Assumption 3, since $\bar{f} = 359.1$ and $\underline{f} = 191.1$. Also, the value of s is higher than the threshold value \tilde{s} , which equals 0.35. From Proposition 1 we know that, for values of the probability of a zero shock lower than \tilde{s} , the firm will always enter the market by investing in productive capacity. Therefore, by considering $s = 0.5 > 0.35$ we concentrate on a case where the expected-demand-shifting effect does not dominate and, therefore, there will not be investment in the first period driven purely by this effect. The threshold value of the posterior belief

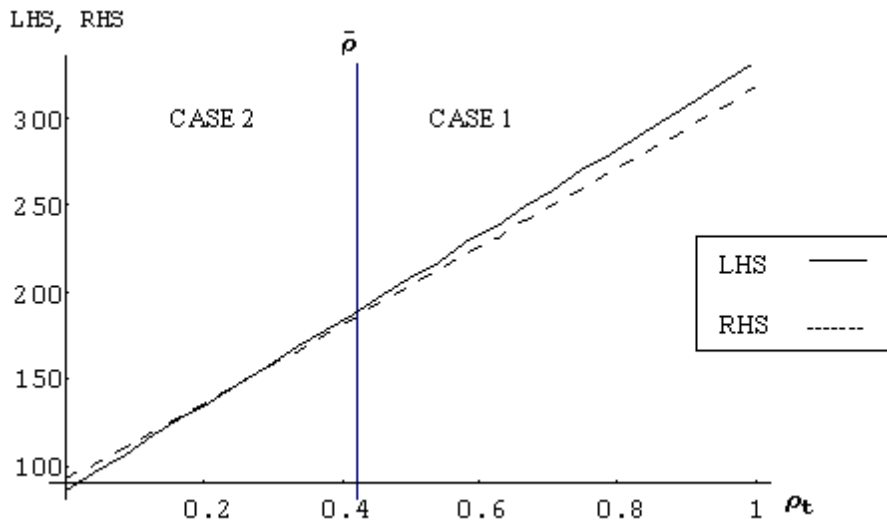


Figure 5: Example where switching between exports and FDI occurs for $\rho_t = 0.25$.

$\bar{\rho}$ equals 0.42 (for $\rho_1 = 0.9$, the respective time period is $\bar{t} = 4.63$, meaning that for $t < 5$ the relevant case is Case 1, while for $t \geq 5$, Case 2).

We now proceed to evaluating equation (A1) in the Appendix, the equation that indicates the value of the posterior belief for which the firm is indifferent between exporting and investing, based on equation (16). This threshold value is $\rho^{**} = 0.25$, which, depending on the level of the prior belief, denotes either switching from exporting to investing in productive capacity or immediate investment in the first period. Given that $f > f^{**} = 288.87$, these results comply with the predictions of Proposition 5, Case 3. We also illustrate the results at Figure 5, where we have plotted the LHS and the RHS of equation (A1), for $\rho_1 = 0.9$. Since $\rho_1 > \rho^{**}$, the diagram depicts the case of a switch from exports to FDI when $\rho_t = 0.25$ (equivalently, at time $t^{**} = 5.74$, the threshold point in time corresponding to ρ^{**}).

To sum up, in our numerical example we find that, for $s < 0.35$, there will be immediate investment. For $s = 0.5$ and $\rho_1 = 0.9$, the firm will enter the foreign market by exporting, with first-period quantity exported $y^* = 5.7$, which, conditional on high demand, will evolve according to (11) for five periods. After that time, the firm will switch to FDI by investing $x^* = 74.7$. For $s = 0.5$ and $\rho_1 < 0.25$, there will be immediate investment.

7.4 Comparison to the case of zero demand shocks

Since the purpose of our analysis is to examine the effect of uncertainty persistence and learning on the investment and exports behavior of the firm, it is useful to discuss briefly the benchmark case where there are no demand shocks and, therefore, uncertainty is resolved after the first-period sales.²⁹

²⁹This formulation is similar to the Saggi's (1998) model. However, we stick to the structure of our model, in order to make results directly comparable with those of the previous analysis.

Note that our model incorporates this possibility as an extreme case. By setting the probability of no shock occurrence equal to 1, we obtain exactly the case of immediate learning (that is, uncertainty resolution after the first-period sales). Using the same parameter values as in the numerical example in the previous Subsection, we find that, when there are no shocks ($s = 1$), the firm will always enter with exports and will switch to FDI, in the second period, only if demand is revealed to be high. Thus, irrespective of the prior beliefs about demand in the foreign market, there will never be immediate investment (except for the case of ρ_1 close to zero, of course). In contrast, in the previous Subsection we showed that, if the probability of a positive shock is $s = 0.5$, the investment decision of the firm depends on its prior beliefs. Specifically, if the prior belief of low true demand is lower than $\rho^{**} = 0.25$, the firm will invest in the first period. It is clear that, when there are shocks and uncertainty is resolved with time, it is possible to obtain immediate investment for lower levels of the prior belief of high demand, compared to the case of immediate learning.

8 Extensions

A number of non-trivial extensions of this model may be desirable and would generate additional insights into the problem of investing in a foreign market with learning. We briefly discuss some of them here. This may provide some intuition for cases that do not completely fall into our model.

1. *Competition.* In the present model, we do not consider the possibility of other firms willing to enter the specific market but have focused on a seller's behavior given his learning about the demand. This seems to be a reasonable approach to the problem when strategic competition considerations are not as important as demand uncertainty. This may be the case at early stages of entering a new foreign market. In an oligopoly context, the observability of the competitors' sales will play an important role. We expect that, if firms cannot learn from the sales of other firms, the presence of (potential) competitors may reinforce the argument in favor of doing FDI in spite of the presence of demand uncertainty, because of an additional "preemption effect" that will be in work (since investment has a commitment value).³⁰ If there is a possibility of learning by observing the competitors' sales, then there may be an incentive to postpone investment.
2. *Partial irreversibility of the capacity installment cost.* We have assumed that the unit cost of capacity installment is completely irreversible, which, together with the assumption that the fixed entry cost becomes sunk once incurred, makes the investment decision one of full commitment. Relaxing the assumption of complete irreversibility will affect the analysis in two ways. First, a switch from FDI to exports, after a low demand realization, may become profitable (as in Saggi, 1998). Second, even in the case where the firm continues to operate its foreign facility, perhaps part of the cost of the capacity which is left unutilized (given that $X_t > \widehat{X}$) could be reimbursed.

³⁰See e.g. Maggi (1996) and references therein, on the commitment power of investment.

We expect that, in both cases, partial irreversibility would favor earlier investment and would lead to higher optimal incremental capacity compared to the optimal level under full irreversibility.

3. *A second informative signal.* In our model, what leads to learning are sales, which allows the firm to obtain the same information about the demand, even if it does not own a production unit in the foreign country. It could be possible that capacity installment has an additional information advantage, in the sense that the seller may learn more if he not only sells in the foreign market but also produces there. Adding a second informative signal, which the firm receives only if it does FDI will favor early investment and therefore will only reinforce our results.
4. *Negative shocks.* The way we model the shock is important for some of the results. More specifically, the fact that the shock is positive determines the gradual expansion of both exports and cumulative capacity. This would not be the case if the shocks were modelled as negative, that is, if by observing a high demand realization the firm would learn with certainty that true demand is high but by observing a low demand realization it would not know whether demand is really low or there has been a temporary negative demand shock. With such an alternative formulation, the exports' path would have the same properties as in the present model, but it would be decreasing. The investment problem would become significantly more complicated since one would have to separate the capacity-installment from the production decision in each period. In general, we expect a one-time capacity installment in the first period, followed by production at that level for a few periods. After having received negative signals (that is, observed low demand realizations) for a number of periods, the firm would start reducing production gradually (leaving capacity unutilized), until it reaches the optimal production level in case of low demand. In this case, entry via FDI from the beginning would occur if the sum of the entry cost and the capacity installment cost (both incurred in the first period) is lower than the present value of the differential unit cost incurred (in every period) in case of exports. Finally, with a negative shock, a switching from exports to FDI would occur only after a high-demand realization.

9 Conclusions

This paper examines optimal entry into a foreign market and derives the time paths of optimal exports and capacity installment under demand uncertainty. A central feature of the model is the introduction of a demand shock that has two effects: on the one hand, it increases expected first-period demand (the expected-demand-shifting effect) and, on the other, it prolongs the time until uncertainty about the true demand can be resolved (the uncertainty-persistence effect). We examine how this uncertainty and its resolution over time affect the dynamics of exports and FDI. We find that beliefs formed before entry play a crucial role. Specifically, if the firm believes that it is more likely that demand is low, while it is actually high, exports and capacity paths will be S-shaped, that is, there will be an increase in sales initially at an increasing rate, until this rate becomes decreasing. Further, we calculate the

exact time when the increments to exports and capacity, respectively, will start to decrease. Finally, a central result of the analysis is related to the timing of FDI. We show that there are cases where the firm would enter the market by installing capacity right away, and therefore would incur the sunk entry cost, despite the presence of uncertainty and the firm's ability to alternatively serve the market via exports. Part of the intuition is that not only the presence of uncertainty matters but also its 'structure'. If learning is expected to take a long time, an investment (FDI) strategy may dominate exports as it allows the firm to supply the market for a number of periods at a lower unit cost. Uncertainty, if it is persistent and may get resolved after a number of periods with sales, would not tend to favor exports relative to FDI, as the sequence of exports required for learning would have a higher expected total cost than an FDI decision.

Appendix

Proof of Proposition 5. In order to establish the existence of beliefs that trigger investment in the foreign country, we need to analyze separately the cases where invested capacity, X_t , is lower than the optimal level with low demand and zero marginal cost, \widehat{X} , and where X_t exceeds \widehat{X} . The former case corresponds to levels of beliefs $\rho_t \in (\bar{\rho}, 1]$, while the latter to $\rho_t \in [0, \bar{\rho})$. In order to obtain the relevant indifference equations for each case, we further simplify equation (16) by employing the assumption of constant unit cost of capacity installment, from which it follows that

$$V_{t+1}(x_t, \rho_{t+1}) = V_{t+1}(0, \rho_{t+1}) + kx_t.$$

Then (16) becomes:

$$\begin{aligned} & \frac{(s\rho_t A_L + (1 - s\rho_t)A_H - c)^2}{4} + \delta s\rho_t \left[\frac{(sA_L + (1 - s)A_H - c)^2}{4(1 - \delta)} \right] + f(1 - \delta(1 - s\rho_t)) = \quad (A1) \\ & = U(0, \rho_t) + \delta(1 - s\rho_t)kx_t^*, \end{aligned}$$

where x_t^* is the optimal incremental capacity and $U(0, \rho_t)$ incorporates the current period payoff and the continuation payoff in case of low demand realization under FDI. As we know, the continuation payoff after a low demand realization differs, depending on whether the total capacity already installed exceed the optimal level of investment with low demand and zero marginal cost, \widehat{X} .³¹ Therefore,

$$U(0, \rho_t) = \begin{cases} -kx_t^* + (s\rho_t A_L + (1 - s\rho_t)A_H - x_t^*)x_t^* + \delta s\rho_t \frac{(E(A) - x_t^*)x_t^*}{(1 - \delta)} & \text{for } x_t^* < \widehat{X} \\ -kx_t^* + (s\rho_t A_L + (1 - s\rho_t)A_H - x_t^*)x_t^* + \delta s\rho_t \frac{E(A)^2}{4(1 - \delta)} & \text{for } x_t^* \geq \widehat{X}, \end{cases}$$

where, in case $x_t^* < \widehat{X}$, the optimal capacity x_t^* equals $x_t^1(0, \rho_t)$, while, for $x_t^* \geq \widehat{X}$, the optimal capacity x_t^* is given by $x_t^2(0, \rho_t)$ (see Proposition 4). After some manipulation, the RHS of (A1) can be written as:

$$U(0, \rho_t) + \delta(1 - s\rho_t)kx_t^* = \begin{cases} \frac{(x_t^1)^2(1 - \delta(1 - s\rho_t))}{1 - \delta} & \text{for } \rho_t \in (\bar{\rho}, 1) \\ (x_t^2)^2 + \frac{\delta s\rho_t E(A)^2}{4(1 - \delta)} & \text{for } \rho_t \in (0, \bar{\rho}). \end{cases} \quad (A2)$$

Note that at $\rho_t = \bar{\rho}$ we have that $x_t^1 = x_t^2 = E(A)/2$ and, therefore, the RHS of (A1) becomes

$$U(0, \bar{\rho}) + \delta(1 - s\bar{\rho})kx_t^* = \frac{E(A)^2(1 - \delta(1 - s\bar{\rho}))}{4(1 - \delta)}$$

Substituting the relevant expression from (A2) into (A1), the indifference equation for $X_t > \widehat{X}$ (equivalently for $\rho_t \in [0, \bar{\rho}]$) becomes

$$\frac{(s\rho_t A_L + (1 - s\rho_t)A_H - c)^2}{4} + \delta s\rho_t \left[\frac{(sA_L + (1 - s)A_H - c)^2}{4(1 - \delta)} \right] + f(1 - \delta(1 - s\rho_t)) = (x_t^2)^2 + \frac{\delta s\rho_t E(A)^2}{4(1 - \delta)},$$

and, after further simplification, we have

³¹Here we consider the first investment made and, therefore, cumulative (X_t) and incremental (x_t) capacity are equal and will be used interchangeably.

$$\frac{(s\rho_t A_L + (1 - s\rho_t)A_H - c)^2}{4} - c\delta s\rho_t \frac{(2sA_L + 2A_H(1 - s) - c)}{4(1 - \delta)} = (x_t^{2*})^2 - f(1 - \delta(1 - s\rho_t)). \quad (\text{A3})$$

It is easy to show that both the LHS and the RHS of (A3) are decreasing and ‘convex’ functions of ρ_t . In addition, as $\rho_t \rightarrow 0$, the RHS is always higher than the LHS (by Assumption 3). Consequently, it suffices to establish conditions such that, as $\rho_t \rightarrow \bar{\rho}$, the LHS is higher than RHS, in order to show that there is a unique ρ_t such that exports are more profitable for all higher levels of beliefs, while investment is preferable for all smaller levels of beliefs. Examining (A3) as $\rho_t \rightarrow \bar{\rho}$, we find that LHS $\begin{smallmatrix} \geq \\ \leq \end{smallmatrix}$ RHS whenever

$$f \begin{smallmatrix} \geq \\ \leq \end{smallmatrix} f^{**} = \frac{Z(sA_L + (1-s)A_H)^2}{4s(A_H - A_L)(1-\delta(1-s))} - \frac{((A_H - c)Z - s(A_H - A_L)W)^2}{4sZ(A_H - A_L)(1-\delta(1-s))} + \frac{c\delta sW(2sA_L + 2(1-s)A_H - c)}{4s(A_H - A_L)(1-\delta)(1-\delta(1-s))}, \quad (\text{A4})$$

where $Z \equiv s(A_H - A_L) + k\delta s$ and $W \equiv s(A_H - A_L) - k(1 - \delta)$.

Now we turn to the case of $X_t < \hat{X}$ (equivalently $\rho_t \in [\bar{\rho}, 1]$), where the indifference equation becomes

$$\frac{(s\rho_t A_L + (1 - s\rho_t)A_H - c)^2}{4} + \delta s\rho_t \left[\frac{(sA_L + (1 - s)A_H - c)^2}{4(1 - \delta)} \right] + f(1 - \delta(1 - s\rho_t)) = \frac{(x_t^1)^2(1 - \delta(1 - s\rho_t))}{1 - \delta},$$

which can be further simplified to

$$\frac{(s\rho_t A_L + (1 - s\rho_t)A_H - c)^2}{4(1 - \delta(1 - s\rho_t))} + f = \frac{(x_t^1)^2}{1 - \delta} - \delta s\rho_t \left[\frac{(sA_L + (1 - s)A_H - c)^2}{4(1 - \delta)(1 - \delta(1 - s\rho_t))} \right]. \quad (\text{A5})$$

As in the previous case we want to establish conditions for the existence of beliefs that satisfy equation (A5) and, therefore, indicate the time of the first investment. Again, we can show that both the LHS and the RHS of (A5) are decreasing and ‘convex’ functions of ρ_t . In addition, we know that for $\rho_t \rightarrow \bar{\rho}$ equations (A3) and (A5) are equivalent and therefore condition (A4) holds in this case too. So we have to examine how the LHS and the RHS of (A5) relate as $\rho_t \rightarrow 1$. We find that LHS $\begin{smallmatrix} \geq \\ \leq \end{smallmatrix}$ RHS if:

$$f \begin{smallmatrix} \geq \\ \leq \end{smallmatrix} f^* = \frac{(c - k(1 - \delta))(2sA_L + 2(1 - s)A_H - c - k(1 - \delta))}{4(1 - \delta)} = \bar{f} - \frac{2s(A_H - A_L)(c - k(1 - \delta))}{4(1 - \delta)}. \quad (\text{A6})$$

In order to exclude the possibility that investment is more profitable for $\rho_t \rightarrow 1$ but exports become preferable at some $\rho_t < 1$, we require that $f^* < f^{**}$, a sufficient condition for which is $c > k(1 - \delta) + k\delta s/2$.

Conditions (A4) and (A6) result in the partitioning of the range of possible values of the fixed entry cost into three relevant intervals. For $f \leq f^*$, the LHS of (A5) is lower than the RHS and, thus, investment is always preferable (since FDI is more profitable even when the firm believes with probability one that true demand is low). Note that by rearranging the inequality $f \leq f^*$ and expressing it with respect to s we obtain exactly $s \leq \tilde{s}$, which is the condition that isolates the demand-shifting effect of the shock. As we have already shown, this effect leads to immediate investment in productive capacity irrespective of the level of the prior beliefs ρ_1 . For $f \in (f^*, f^{**}]$, the LHS of (A5) is higher

than the RHS as $\rho_t \rightarrow 1$, but the opposite holds as $\rho_t \rightarrow \bar{\rho}$. Therefore, there is a unique level of beliefs, $\rho^* \in [\bar{\rho}, 1)$, that satisfies the indifference equation and, therefore, if $\rho_1 \in (\rho^*, 1]$ the firm will start by exporting but will switch to FDI once the posterior belief of low true demand has reached ρ^* , and will enter with FDI, if $\rho_1 \leq \rho^*$. Finally, for $f > f^{**}$, the LHS of (A3) is higher than the RHS as $\rho_t \rightarrow \bar{\rho}$, while the RHS is always higher as $\rho_t \rightarrow 0$. Consequently, there is a unique level of beliefs, $\rho^{**} \in [0, \bar{\rho})$, satisfying the indifference equation, meaning that, if $\rho_1 \in (\rho^{**}, 1]$ the firm will export initially but will switch to FDI as long as the level of the posterior belief reaches ρ^{**} , or will enter via FDI, if $\rho_1 \leq \rho^{**}$.³²

³²It cannot be the case that FDI is more profitable for some $\rho_t > \bar{\rho}$ but is not for $\bar{\rho}$, where expected demand is higher. Therefore, if the firm is better off by exporting, both for $\rho_t \rightarrow 1$ and for $\rho_t \rightarrow \bar{\rho}$, then exports should be preferable for all $\rho_t \in [\bar{\rho}, 1]$.

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