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FINANCIAL INTEGRATION AND SYSTEMIC RISK

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ABSTRACT

Financial Integration and Systemic Risk

Recent empirical studies criticize the sluggish financial integration in the euro area and find that only interbank money markets are fully integrated so far. This paper studies the optimal regional and/or sectoral integration of financial systems given that integration is restricted to the interbank market. Based on Allen and Gale's (2000) seminal analysis of financial contagion we derive the interbank market structure that maximizes consumers' *ex ante* expected utility, taking into account the trade-off between the contagion and the diversification effect of financial integration. We analyse the impact of various structural parameters including the underlying stochastic structure on this trade-off. In addition we derive the efficient design of the interbank market that allows for a cross-regional risk sharing between banks. We also provide a measure for the efficiency losses that result if financial integration is limited to an integration of the interbank market.

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NON-TECHNICAL SUMMARY

Several recent empirical studies argue that the most integrated financial market in the euro area is the unsecured money market. Due to legal, regulatory and institutional obstacles the integration of most other financial markets (except for the market for governments bonds) lags far behind the integration of the interbank market.

This paper studies in a first step, in which cases a financial integration that is limited to an integration of the interbank markets, can nevertheless be welfare improving. In doing so this first part assumes that the institutional arrangement in the interbank market allows for a constraint efficient risk sharing. Our study shows that an international integration through the interbank market is preferable, if the expected benefits from the international diversification overcompensate the expected costs from contagion. Thus while an integration of banks from economies with countervailing business cycles is beneficial, the interlinkage of banks that face rather similar risks is welfare reducing. In addition, the benefits of an international integration of the banking sector increases as the return on long-term investments declines. Thus our analysis suggests that an integration through the interbank markets is the more preferable the more mature the economies under considerations are and the lower their marginal product of capital therefore is.

In the second step the paper turns to the institutional arrangement in the interbank market and derives the arrangement required to implement a constraint efficient cross-country risk sharing through the interbank market. Interestingly, the study shows that in order to allow for the most efficient risk sharing interbank deposits must be junior to other liabilities of banks and interbank debt must not be netted at any time. Thus the interbank market can only provide an efficient risk sharing if it also allows for financial contagion between banks through interbank deposits. The risk of financial contagion is a necessary condition to ensure that contributing to the liquidity insurance between banks is ex-post incentive compatible. This institutional arrangement that we prove to be constraint efficient is precisely the arrangement assumed in the first part of the study.

Finally, our paper also shows under which conditions constraint efficient interbank deposits – in contrast to more complex arrangements, i.e. cross-country bank mergers – do not allow to capture all benefits from an international risk sharing.

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Financial Integration and Systemic Risk*

1 Introduction

The most integrated financial market in the Euro area is the interbank or money market and within the money market the most integrated segment is the unsecured interbank market.[1] Besides its importance for a smooth monetary policy transmission across the member states a high degree of integration of this market is also beneficial under risk sharing considerations. An interlinked banking system within the Euro area provides an insurance mechanism against regional liquidity shocks. However, the flip side of the coin is that interbank claims bring about a risk of financial contagion, i.e. increase the systemic risk. The liquidity crisis of a single institute can easily spill-over to other banks if those financial intermediaries hold interbank deposits with the troubled bank.

In the present paper we try to evaluate the obvious trade-off that these two effects bring about and analyze under which conditions the benefits of a cross-regional risk sharing dominate the expected costs of financial contagion. Doing so we also take into account the optimal institutional arrangement of the interbank market. Particularly, we show how the interbank market has to be designed in order to provide a means for a constraint efficient financial integration given the cross-regional distribution of liquidity shocks. Interestingly, for reasonable parameter settings we find an efficient arrangement has to allow for the possibility of interbank bank runs. However, we also show that using simple interbank arrangements does not allow to capture all benefits from a cross regional risk sharing. More complex arrangements, i.e. cross-country mergers, of financial institutions can improve the cross-country insurance against regional liquidity shocks.

Our paper is closely related to Allen and Gale (2000) and Freixas, Parigi, and Rochet (2000). Like our paper both show that regional-specific liquidity shocks provide a rationale for interlinking regionally restricted banks of a Diamond and Dybvig (1983)-type. Interbank deposits in the case of Allen and Gale (2000) and interbank lines of credit in the case of Freixas, Parigi, and Rochet (2000) enable banks to mutually insure against

*The views expressed here are those of the authors and not necessarily those of the Deutsche Bundesbank. We thank seminar participants at Deutsche Bundesbank and in particular Phil Dybvig, Heinz Herrmann, and Julia von Borstel for helpful comments and suggestions.

[1]This results is reported in several studies. It is probably most prominently emphasized in Baelen, Ferrando, Hördahl, Krylova, and Monnet (2004). For a comprehensive institutional analysis of the various segments of the Euro money market see also Hartmann, Manna, and Manzanares (2001) and for a description of the most recent developments in that market see European Central Bank (2005). Barros, Berglöf, Fulghieri, Gual, Mayer, and Vives (2005) also emphasize the disappointing degree of financial integration in most other financial markets in the Euro area particularly the retail financial markets.

negatively correlated regional liquidity shocks.[2] However, when analyzing the structure of the interbank network both do not take the possibility of aggregate liquidity shocks into account. Thus any network that connects directly or indirectly all entities of the financial system provides an efficient mechanism to share the risk of liquidity shocks. In a second step they introduce an aggregate liquidity shock that occurs with probability zero and study its impact on arbitrary interbank networks. In contrast, we analyze the decision of banks to provide each other with interbank deposits in order to share liquidity risk taking the possibility of aggregate liquidity shocks and financial contagion into account. This permits us to derive the optimal integration decision dependent upon (i) the underlying stochastic structure of liquidity needs (ii) individual's risk preferences and (iii) the state of the technology.

Focusing on a two-regional case we also discuss the efficiency of different interbank arrangement in dealing with contagion and in implementing an efficient risk sharing. We show that the rules concerning the seniority of household vs. interbank debt when liquidating a bank are crucial for the performance of the interbank market system. For a wide range of parameter settings the interbank market can only implement a constraint efficient risk sharing if interbank deposits are junior to households claims and interbank debt cannot be netted. But precisely these arrangements bring about the risk of contagion between banks. Thus implementing a constraint efficient risk through an interbank market is necessarily associated with the risk of financial contagion. To that extent our paper is also related to Leitner (2005) who argues that banks put themselves at risk of contagion in order to credibly commit to bail each other out.

Moreover, using a mechanism design approach, we are able to identify further institutional improvements that are not available in a simple interbank market setup where the only decision a bank can make is whether to liquidate its assets with another bank or not. However, we will argue that these more complex mechanisms that use a larger strategy space essentially reflect a merged bank and not what is usually labelled an interbank market.

The remainder of the paper is organized as follows: After specifying the assumptions section 3 derives the utility that separate banks provide depositors. In section 4 we derive the optimal deposit contract and the utility that an integrated financial system generates under the assumption that in order to share regional liquidity risks banks have to use

[2] In Allen and Gale (2000) there exist four regions two of which experience a positive and two of which a negative liquidity shock. Aggregate liquidity demand is constant with probability one. This specification guarantees that there is a need for cross regional liquidity sharing and it permits to study financial contagion which is the main focus of their paper.

interbank deposits that are junior to households deposits. Section 5 analyzes the trade-off between having either a partitioned or an integrated financial systems. In section 2.3 we show under which conditions the optimal interbank deposit contract is indeed junior to households' claims given the optimal deposit contract with households derived in section 4. Finally, we derive in section 7 the optimal mechanism for risk sharing between banks. In section 8 we argue that this mechanism mainly reflects cross-country mergers and show to what extent this mechanism leads to a preferable allocation as compared to the one provided by junior interbank deposits. Section 9 concludes and points out some issues for further research.

2 The model

2.1 Depositors

Consider an economy with three dates ($t = 0, 1, 2$) and two symmetrical regions A and B. Both regions contain a bank and a continuum of mass 1 of households each endowed with 1 unit of the single consumption good. Households are subject to a preference shock that realizes in $t = 1$ and that is not publicly observable. Ex-ante (in $t = 0$) households only know that they will be impatient ($\theta_i = 1$) with probability q. In that case they can only derive utility from consumption in $t = 1$. With probability $(1 - q)$ they know that they will turn out to be patient and only want to consume in $t = 2$. Thus households' ex-ante expected utility is given by the following function

$$E\left[U\left(c_1; c_2\right)\right] = E\left[\theta_i u\left(c_1\right) + \left(1 - \theta_i\right) u\left(c_2\right)\right]$$

with

$$u\left(c_t\right) = \frac{1}{1-\gamma} c_t^{1-\gamma} \quad \text{and} \quad \gamma > 1.$$

Given the law of large numbers the regional fraction of impatient households is

$$\int \theta_i di = q.$$

However, this fraction of patient consumers (i.e. the probability that a given household turns out to be impatient) in each region is itself subject to shocks. It can either turn out to be 0, $\frac{1}{2}$ or 1. The probability distribution of the liquidity shocks in the two regions is given by the following table. Thus the probability that all households in both regions turn out to be patient, for instance, is given by b.

	$q_A = 0$	$q_A = \frac{1}{2}$	$q_A = 1$
$q_B = 0$	b	d	e
$q_B = \frac{1}{2}$	d	a	f
$q_B = 1$	e	f	c

2.2 Investment technology

There is *only one* direct investment technology available in the economy. This technology can be liquidated in $t = 1$ at no costs.[3] However, it only pays a positive interest in $t = 2$:

	$t = 0$	$t = 1$	$t = 2$
finished	-1	0	$R > 1$
liquidated	-1	$+1$	0

In $t = 0$ households can invest in the technology. Because it is not observable whether a particular household is patient or impatient, there is no direct insurance mechanism against liquidity risks available.

Besides direct investment households can deposit their endowment at a bank. Banks offer deposit contracts, that specifies the promised repayment d_1 if deposits are withdrawn in $t = 1$. The residual of banks' assets in $t = 2$ will be repaid to households that keep their deposits until $t = 2$. There is one bank in each region. However, banking markets are contestable. Therefore banks are forced to offer the deposit contract that maximizes the expected utility of depositors.

If a bank cannot serve all withdrawals in $t = 1$ *all* depositors (patient and impatient) receive the same pro-rata repayment. Thus we do not consider any kind of sequential service constraint. Hence we exclude expectation driven bank runs of depositors.

2.3 The interbank market

We analyze under which condition it is reasonable for banks in different regions to provide each other with interbank deposits. Interbank deposit contracts specify a repayment $\left\{d_1^B; d_2^B\right\}$ contingent whether the deposits are withdrawn in $t = 1$ or held until $t = 2$. Interbank deposits are assumed to be subordinated to households' repayments in $t =$

[3]Note that this simplifying assumption compared to the standard Diamond and Dybvig (1983)-model can be seen as a short-cut for the existence of a liquid financial market in $t = 1$ at which agents from other than the considered regions and banks from the region A and B trade liquidity against financial claims promising a $t = 2$-return R at the arbitrage free price $p = 1$ (see Fecht (2004) for a detailed description).

1.[4] However, interbank debt is senior to households' $t = 2$-repayment.[5] The regional liquidity shock is private information to the banks in the respective region. Banks cannot observe the liquidity shock in other regions. Thus interbank deposit contracts cannot be contingent on regional liquidity shocks. In order to provide a means of cross-regional risk sharing interbank deposit contracts have to be incentive compatible.

Note that within each region we assume Bertrand competition among banks. This ensures that banks will offer contracts to depositors and choose interbank market arrangements that maximize depositors' utility.

We make the following general assumptions about the institutional setup of this market:

1. First, we assume that interbank debt is junior to households' deposits. This means that if a bank fails then all households' deposits (those households that wanted to withdraw and those that initially wanted to keep their deposits) are served first before any repayment is made on interbank debt. Note that this requires a gross-settlement of interbank debt–i.e. interbank positions cannot be netted.

2. Next, we assume that if a bank is illiquid then depositors force the bank management to immediately (still within $t = 1$) withdraw deposits from other banks. Thus in case of illiquidity of one bank interbank deposits are withdrawn irrespective of the original decision of the bank management.

3. Finally, we assume that depositors cannot observe whether a bank withdraws interbank deposits or not. Thus the decision of patient depositors in one region cannot depend on the withdrawal decision of banks analyzed here. However, it is important to show that in equilibrium it is preferable for patient depositors to keep their deposits until $t = 2$. This is always true for sufficiently high a.

2.4 Preferences of bank managers

We make the following behavioral assumption about bank managers. Given a consumer deposit contract D and given an interbank market contract D^B bank managers choose a

[4] Note that this is an optimal arrangement, because this gives banks the strongest incentive in this set-up to provide the other bank with liquidity. If interbank deposits were senior to households' deposits then banks would always draw on their interbank deposits first to provide liquidity to other banks or households irrespective of their liquidity shock. See section 6 for a proof of this argument.

[5] Since we assume below that regional shocks are non-verifiable to non-regional banks, this is also an efficient arrangement.

state contingent plan for their behavior on the interbank market that maximizes expected utility of their non-bank customers.

This assumption needs some further justification because, at the stage where bank managers observe their respective liquidity shock, they cannot contractually be forced to act in a certain state dependent way (i.e. withdraw deposits only in certain states). Contractual arrangements that align bankers' and depositors' interests do however exist. Consider e.g. a contract that guarantees the manager a transfer which is an increasing function $f(\cdot)$ of the sum actually paid out to an individual depositor in $t = 1$. In particular let

$$f(\cdot) = \varepsilon \cdot \frac{1}{1-\gamma} d^{1-\gamma},$$

where d denote the sum paid out and ε is arbitrarily small. This contract obviously aligns the interests of the banker and those of the depositors from the ex-ante point of view.

2.5 Equilibrium concept

An interbank market with a given contract D^B induces a Bayesian game among bank managers. In this game a strategy is a state contingent plan for withdrawal decisions

$$w(q) : \{0, 1/2, 1\} \rightarrow \{0, 1\}.$$

An equilibrium is a combination of deposit contracts and bank strategies that are mutually consistent and compatible with competitive banking bahavior.

Definition 1 *An equilibrium consists of*

(i) A deposit contract $D = (d_1, d_2)$,

(ii) An interbank deposit contract $D^B = \left(d_1^B, d_2^B\right)$ *and*

(iii) bank strategies $w(q) : \{0, 1/2, 1\} \rightarrow \{0, 1\}$,

such that

(i) Bank strategies are a Bayesian equilibrium of the interbank market game given the interbank contract D^B.

(ii) The deposit contract D *maximizes consumer utility given* D^B *and the withdrawal behavior of all banks.*

2.6 Institutional choice

Whether or not financial institutions engage on the interbank market will depend on whether the equilibrium outcome yields higher utility to consumers than under financial separation.

3 Welfare implications of separated banks

Because of the contestability of the banking market a separated bank will offer the deposit contract d_1^S that maximizes the expected utility subject to the per capita budget constraint. Given $d_1^S > 1$ the bank is always illiquid if $q = 1$. In that case the bank can only repay 1 to each depositor. If $q = 0$ the bank can finish all its projects and pay R in $t = 2$ to all depositors. Thus the contractual repayment is only important if some households want to withdraw while others want to keep their deposits until $t = 2$. Only for $q = \frac{1}{2}$ the bank actually has to pay the contracted amount d_1 to its impatient depositors. It immediately follows from the per capita budget constraint that the patient depositors receive a repayment $2R - Rd_1$ in $t = 2$. Thus the optimal deposit contract that the bank can offer maximizes the expected utility function

$$E\left[U^S\left(d_1^S\right)\right] = (a+d+f)\left[\frac{1}{2}u\left(d_1^S\right) + \frac{1}{2}u\left(2R - Rd_1\right)\right] + (b+d+e)u\left(R\right) + (c+e+f)u\left(1\right).$$

Since $u'\left(c_t\right) = c_t^{-\gamma}$ it is easy to see that the optimal deposit contract solves

$$\frac{1}{2}\left(d_1^S\right)^{-\gamma} = \frac{R}{2}\left(2R - Rd_1^S\right)^{-\gamma}, \tag{1}$$

and is given by

$$d_1^S = \frac{2}{R^{(1-\gamma)/\gamma} + 1}. \tag{2}$$

4 Welfare implications of an integrated financial system

As already emphasized in the introduction we focus on cross-regional risk-sharing over the interbank market in this paper, because this seems to be particularly within the Euro area the most integrated part of the financial systems. The interbank market allows banks to provide each other with interbank deposits that specify the provision of liquidity at date 1.

In this section we consider a potential equilibrium where (i) deposit contracts are given by $D = (d_1, d_2)$ and (ii) interbank contracts specify that each bank has the right to either withdraw $d_1/2$ at date 1 or $d_2/2$ at date 2 from the other bank.[6] Since banks' liquidity needs are not observable for other players, the contract leads to a Bayesian game among banks. In what follows we will consider the strategy profile where each bank withdraws its deposits at date 1 if and only if the early liquidity shock has realized. In section 6 we will then show that this is indeed a Bayesian Nash equilibrium of the interbank market game.

Depositors' equilibrium payoffs can be characterized as follows: When both regions are not hit by any shock, depositors realize their desired consumption levels d_1^* at date 1 or d_2^*. When both regions experience a late shock, consumers get R at date 2. When shocks in different directions occur the levels d_1^* or d_2^* are realized. A liquidity crises occurs in case of a liquidity shortage in both regions. Moreover, if in addition $d \geq 4/3$ both banks end up being illiquid in those cases where only one region is hit by a negative liquidity. Finally, no cross-regional risk-sharing is possible in cases in which one region has no impatient depositors while a fraction of 1/2 is impatient in the other region.

Thus, households' expected utility in an integrated financial systems is given by

$$E\left[U^M\left(d_1^M\right)\right] = (a+d+2e)\left[\frac{1}{2}u\left(d_1^M\right)+\frac{1}{2}u\left(2R-Rd_1^M\right)\right] \\ +(b+d)\,u\,(R)+(c+2f)\,u\,(1)\,.$$

It is easy to see that the deposit contract that maximizes households' expected utility in a financial system that only implements cross-regional risk sharing over the interbank market is the same as in a financial system with separated banks

$$d_1^M = \frac{2}{R^{(1-\gamma)/\gamma}+1}.$$

Note, however, that this is only the optimal deposit contract if $d_1^M > \frac{4}{3}$ or put differently if

$$R > 2^{\frac{\gamma}{\gamma-1}} \tag{3}$$

Only for these parameter settings contagion will occur in the integrated financial system. If (3) does not hold then an optimally integrated financial system also provides a means for risk-sharing for those cases in which one region is hit by a negative liquidity shock while in the other region half of the households turn out to be patient. Consequently,

[6] Other potential equilibria (with different levels of interbank deposits) can be ruled out since they would imply that withdrawals on the interbank market are either (i) not sufficient to finance additional liquidity needs or (ii) too large so that a withdrawal even leads to the liquidation of a bank with excess liquidity.

under these conditions linking the two regions does not bring about any costs. However, if (3) holds then the decision to integrate the two regional financial systems is non-trivial and is discussed in the following section.

5 Optimal financial integration

Obviously, it is efficient to integrate the two regions in one financial systems if for the optimal deposit contract d_1^* given by (2)

$$E\left[U^M\left(d_1^*\right)\right] > E\left[U^S\left(d_1^*\right)\right]. \tag{4}$$

Defining

$$E\left[U^G\left(d_1^*\right)\right] = \frac{1}{2\left(1-\gamma\right)}\left(\left(\frac{2}{R^{(1-\gamma)/\gamma}+1}\right)^{1-\gamma} + \left(\frac{2R^{1/\gamma}}{R^{(1-\gamma)/\gamma}+1}\right)^{1-\gamma}\right), \tag{5}$$

it is easy to see that (4) requires that

$$e\left(2E\left[U^G\left(d_1^*\right)\right] - u\left(R\right) - u\left(1\right)\right) > f\left(E\left[U^G\left(d_1^*\right)\right] - u\left(1\right)\right). \tag{6}$$

The LHS of (6) are the benefits from diversification and the RHS are the costs from contagion. Thus if the benefits overcompensate the costs an integration of the two regions in one financial system is preferable.

Obviously, (6) can be transformed in

$$e\left(u\left(R\right) - E\left[U^G\left(d_1^*\right)\right]\right) < \left(e-f\right)\left(E\left[U^G\left(d_1^*\right)\right] - u\left(1\right)\right).$$

We have:

Proposition 2 *A separated financial system is always preferable if $e \leq f$. For $e > f$ an integrated financial system for the two regions is preferable if*

$$\left(2e-f\right)\left(\frac{R^{(1-\gamma)/\gamma}+1}{2}\right)^{\gamma} > \left(e-f\right) + eR^{1-\gamma}. \tag{7}$$

Proof. Reinserting (5) in (6) and rearranging yields (7). ■

More details about the optimality of different arrangements can be derived from the following example.

Example 3 *For $\gamma = 2$ (7) can be reduced to*

$$\left(4e-2f\right)R^{-1/2} > \left(2e-3f\right) + \left(2e+f\right)R^{-1}.$$

This holds for all

$$R \in \left[1, \frac{(2e+f)^2}{(2e-3f)^2}\right].$$

Thus taking (3) into account an integrated financial system – even though it brings about a risk of financial contagion – is preferable, if

$$R \in \left[4, \frac{(2e+f)^2}{(2e-3f)^2}\right],$$

and this is a non-empty set. In contrast, for

$$R > \frac{(2e+f)^2}{(2e-3f)^2}$$

separation is preferable if $e > f$.

Accordingly an increase in the long run rate of return R makes financial separation more desirable. A higher rate of return R raises the cost of financial contagion because contagion always reduced payoffs in the regions concerned to 1.

6 Interbank market and the implementation of the constraint efficient risk-sharing

6.1 Payoffs of the interbank market game

In this section we analyze in detail the equilibrium behavior of banks on the interbank market. Given the institutional arrangement of the interbank market the state contingent pay-offs of both banks given their decisions to either withdraw or keep their deposits with the other bank are summarized in table 1 and 2.

Because of the assumed gross settlement of interbank liabilities, both banks have to liquidate some of their assets in order to repay their interbank deposits even if they withdraw their deposits in the same period. Assume that both banks promise a repayment $\{d_1^{IB}; d_2^{IB}\} = \{\frac{1}{2}d_1; \frac{1}{2}d_2\}$ on interbank deposits. Thus in order to repay the deposits of bank B if bank B withdraws in $t = 1$ bank A has to liquidate $\frac{1}{2}d_1$ of its assets. At the same time bank B has to liquidate $\frac{1}{2}d_1$ of its deposits if bank A also withdraws in $t = 1$. If both banks are hit by a positive liquidity shock ($\{q_A; q_B\} = \{0; 0\}$) they are both able liquidate this amount without collapsing. Thus given that both banks withdraw

Table 1: State contingent repayments to depositors given banks' withdrawal decision

$\{q_A; q_B\}$	$Prob$	$\{s_A; s_B\} = \{1;1\}$ [1)]	$\{s_A; s_B\} = \{0;1\}$
$\{0;0\}$	b	$\{0; \frac{1}{2}(d_1+d_2)\}; \{0; \frac{1}{2}(d_1+d_2)\}$	$\{0; d_2\}, \{0; R - \frac{1}{2}(d_2-d_1)\}$
$\{0;\frac{1}{2}\}$	d	$\{0; \frac{1}{2}d_2 + \varpi\}; \{d_1; d_1\}$	$\{0; d_2\}, \{d_1; R - \frac{1}{2}d_2\}$
$\{0;1\}$	e	$\{0; \frac{1}{2}d_2 + \varpi\}; \{d_1; 0\}$	$\{0; d_2\}, \{d_1; 0\}$
$\{\frac{1}{2};0\}$	d	$\{d_1; d_1\}; \{0; \frac{1}{2}d_2 + \varpi\}$	$\{d_1; d_1\}, \{0; \frac{1}{2}d_2 + \varpi\}$
$\{\frac{1}{2};\frac{1}{2}\}$	a	$\{1;1\}; \{1;1\}$	$\{1;1\}; \{1;1\}$
$\{\frac{1}{2};1\}$	f	$\{1;1\}; \{1;0\}$	$\{1;1\}; \{1;0\}$
$\{1;0\}$	e	$\{d_1; 0\}; \{0; \frac{1}{2}d_2 + \varpi\}$	$\{d_1; 0\}; \{0; \frac{1}{2}d_2 + \varpi\}$
$\{1;\frac{1}{2}\}$	f	$\{1;0\}; \{1;1\}$	$\{1;0\}; \{1;1\}$
$\{1;1\}$	c	$\{1;0\}; \{1;0\}$	$\{1;0\}; \{1;0\}$ [5)]

[1)] $s_i = 1$: Bank i withdraws

($\{s_A; s_B\} = \{1;1\}$) in this state the repayment $\{d_1; d_2\}$ to depositors of bank A and bank B, respectively, is given by $\{0; \frac{1}{2}(d_1 + d_2)\}$.[7]

In contrast, in those states where $\{q_A; q_B\} = \{0; \frac{1}{2}\}$ only bank A can fulfill its payment obligation if bank B withdraws. Because $d_1 > 1$ bank B is illiquid and has to serve households deposits first. After receiving the payment from bank A the overall liquidity available to bank B is given by: $1 + \frac{1}{2}d_1$. Thus the fractional repayment ϖ that bank A will receive from bank B follows from $d_1 + \varpi = 1 + \frac{1}{2}d_1$ and is (for the optimal deposit contract with $d_1 = \frac{2R}{R+R^{1/\alpha}}$) given by $\varpi = \frac{R^{1/\alpha}}{R+R^{1/\alpha}} < \frac{1}{2}d_1$. Thus bank A can only repay $R(1 - \frac{1}{2}d_1) + \varpi = \frac{1}{2}d_2 + \varpi$ to its depositors.[8]

Similarly, if both banks withdraw in those states with $\{q_A; q_B\} = \{0; 1\}$ bank A has to liquidate $\frac{1}{2}d_1$ in order to fulfill its payment obligation. Bank B is illiquid has to serve households deposits first. Just like in the above described case only the remaining liquidity ϖ can be paid to bank A.

If neither bank has a liquidity shock ($\{q_A; q_B\} = \{\frac{1}{2}; \frac{1}{2}\}$) and both banks withdraw their interbank deposits then both banks fail. They cannot raise sufficient funds in $t = 1$ by liquidating their projects to repay the impatient depositors and honor the interbank deposits. In that case all projects are liquidated and households deposits are served

[7]Note that this follows from the fact that both banks can only store the repayment on interbank deposits until $t = 2$ when the funds are needed to repay the depositors and therefore $R(1 - \frac{1}{2}d_1) + \frac{1}{2}d_1 = \frac{1}{2}(d_1 + d_2)$

[8]Note that if households could observe the interbank payments and the regional liquidity shock depositors in region A would run if $d_1 > \frac{1}{2}d_2 + \varpi$. Thus depositors in region A would end up with $\{1; 1\}$ just like depositors in region B in that case.

pro-rata–i.e. each depositor receives a repayment of 1.

The same happens if both banks withdraw their interbank deposits in all those states in which at least one bank has a negative liquidity shock. Both banks turn out to be illiquid and can only repay a pro-rata repayment of 1 to households. However, in those cases both banks will always fail irrespective of their decision to withdraw interbank deposits or not. For instance, even if in the state $\{q_A; q_B\} = \{1; \frac{1}{2}\}$ bank A would not decide to withdraw it would turn out to be illiquid because of the withdrawals of households. Depositors will therefore still within $t = 1$ force bank A to withdraw its interbank deposits from bank B in order to raise some additional liquidity. As soon as bank A withdraws its interbank deposits bank B will also turn out to be illiquid. Thus in those states both banks will be forced to withdraw irrespective of their initial decision. Consequently, both banks will finally fail and repay only the pro-rata repayment 1 to each depositor.

It is easy to see that in all those states with $q_A = \frac{1}{2}$ and $q_A = 1$ bank A will always be illiquid if bank B withdraws. Therefore, its own decision to withdraw or not does not matter for the repayment to its depositors.

In contrast, if both banks are hit by a positive liquidity shock and bank B withdraws while bank A keeps its interbank deposits bank A has to liquidate $\frac{1}{2}d_1$ of its investment to repay bank B. But because bank B is solvent bank A receives $\frac{1}{2}d_2$ in $t = 2$ from bank B. Thus bank A can repay $R(1 - \frac{1}{2}d_1) + \frac{1}{2}d_2 = d_2$. In contrast, bank B receives $\frac{1}{2}d_1$ and can only store this liquidity. It does not have to liquidate any investments but has to pay $\frac{1}{2}d_2$ to bank A in $t = 2$. Thus bank B can repay $R - \frac{1}{2}d_2 + \frac{1}{2}d_1$. In general, it is easy to see that if bank A has a liquidity surplus and does not withdraw its interbank deposits while bank B withdraws bank B will always be liquid and able to repay $\frac{1}{2}d_2$ in $t = 2$ to bank A.

If both banks do not withdraw their interbank deposits in $t = 1$ ($\{s_A; s_B\} = \{0; 0\}$) then no funds are transferred between the two banks if both stay liquid. If either bank fails to repay its depositors then depositors can force the bank to collect its interbank deposits. This happens, for instance, if $\{q_A; q_B\} = \{0; 1\}$. Because bank B is illiquid it will be forced to withdraw its interbank deposits. In the liquidation of bank B bank A can only recoup a repayment of ϖ on its interbank deposits.

Finally, it is important to note that bank A can only survive $\{q_A; q_B\} = \{\frac{1}{2}; \frac{1}{2}\}$ if neither bank A nor bank B withdraws.

6.2 Equilibrium

Due to the fact that liquidity needs are not observable among banks, the interbank market induces a Bayesian game among banks at $t = 1$. A bank's strategy maps the realized

Table 2: State contingent repayments to depositors given banks' withdrawal decision (cont.)

$\{q_A; q_B\}$	*Prob*	$\{s_A; s_B\} = \{1; 0\}$	$\{s_A; s_B\} = \{0; 0\}$
$\{0; 0\}$	b	$\{0; R - \frac{1}{2}(d_2 - d_1)\}, \{0; d_2\}$	$\{0; R\}; \{0; R\}$
$\{0; \frac{1}{2}\}$	d	$\{0; \frac{1}{2}d_2 + \varpi\}, \{d_1; d_1\}$	$\{0; R\}; \{d_1; d_2\}$
$\{0; 1\}$	e	$\{0; \frac{1}{2}d_2 + \varpi\}; \{d_1; 0\}$	$\{0; \frac{1}{2}d_2 + \varpi\}; \{d_1; 0\}$ [6])
$\{\frac{1}{2}; 0\}$	d	$\{d_1; R - \frac{1}{2}d_2\}, \{0; d_2\}$	$\{d_1; d_2\}; \{0; R\}$
$\{\frac{1}{2}; \frac{1}{2}\}$	a	$\{1; 1\}; \{1; 1\}$	$\{d_1; d_2\}; \{d_1; d_2\}$
$\{\frac{1}{2}; 1\}$	f	$\{1; 1\}; \{1; 0\}$	$\{1; 1\}; \{1; 0\}$
$\{1; 0\}$	e	$\{d_1; 0\}, \{0; d_2\}$	$\{d_1; 0\}; \{0; \frac{1}{2}d_2 + \varpi\}$
$\{1; \frac{1}{2}\}$	f	$\{1; 0\}; \{1; 1\}$	$\{1; 0\}; \{1; 1\}$
$\{1; 1\}$	c	$\{1; 0\}; \{1; 0\}$	$\{1; 0\}; \{1; 0\}$

liquidity demand (the fraction of early withdrawing depositors) at $t = 1$ into a decision to withdraw or to keep the interbank deposit.

Our first main result is that indeed an interbank market with the above institutional features may allow for a limited extend of risk sharing. This means that negative liquidity shocks can be compensated by positive shocks, however, excess liquidity is not equally distributed among regions.

Proposition 4 *The interbank market game has a Bayesian Nash equilibrium where both banks withdraw at t=1 if and only if they experience an early liquidity shock. In this equilibrium the interbank market (i) provides an optimal risk sharing for states with opposed regional liquidity shocks (ii) does not implement a cross regional risk sharing for states with excess aggregate liquidity (iii) causes contagion in states with aggregate liquidity shortages.*

Proof. Given that bank B only withdraws its interbank deposits if it is hit by a negative liquidity shock ($q_B = 1$) we have to show that it is optimal for bank A to withdraw its interbank deposits in $t = 1$ if and only if the bank is hit by a negative liquidity shock ($q_A = 1$). From table 1 and 2 it is easy to derive the following incentive compatibility constraints and verify under which conditions they hold:

First consider $q_A = 0$. Expecting that bank B only withdraws its interbank deposits if $q_B = 0$ bank A keeps its interbank deposits until $t = 2$ iff

$$\begin{aligned}&(b+d)\,U\,(0;R) + eU\,(0;d_2) \geq \\ &bU\left(0; R - \frac{1}{2}\,(d_2 - d_1)\right) + (d+e)\,U\left(0; \frac{1}{2}d_2 + \varpi\right).\end{aligned} \tag{IC1}$$

(IC1) always hold because $R > R - \frac{1}{2}\,(d_2 - d_1)$ and $R > d_2 > \frac{1}{2}d_2 + \varpi$.

For $q_A = \frac{1}{2}$ bank A will not withdraw its interbank deposits in $t = 1$ iff

$$\begin{aligned}&(d+a)\,U\,(d_1;d_2) + fU\,(1;1) \geq \\ &dU\left(d_1; R - \frac{1}{2}d_2\right) + (a+f)\,U\,(1;1)\,.\end{aligned} \tag{IC2}$$

Since $d_2 > d_1 > 1$ (IC2) always holds for the optimal deposit contract that banks offer to households[9] if $R < 2^{\gamma/(\gamma-1)}$. Even if $R > 2^{\gamma/(\gamma-1)}$ and therefore $U\left(d_1; R - \frac{1}{2}d_2\right) > U\,(d_1;d_2)$ (IC2) always holds if a is relative to d sufficiently large that

$$a\,[U\,(d_1;d_2) - U\,(1;1)] \geq d\left[U\left(d_1; R - \frac{1}{2}d_2\right) - U\,(d_1;d_2))\right]$$

Finally consider $q_A = 1$. Given that bank B only withdraws if $q_B = 1$ the repayment bank A's depositors receive is independent of the decision of bank A to withdraw its interbank deposits or not because if the bank does not withdraw its interbank deposits the bank fails and will be forced by the depositors to withdraw. Thus the incentive compatibility constraint is given by

$$eU\,(d_1;0) + (f+c)\,U\,(1;0) \geq eU\,(d_1;0) + (f+c)\,U\,(1;0)\,,$$

and holds with an equality. ■

It is important to note that for reasonable parameter settings banks would not be able to implement any cross regional risk sharing using interbank deposits if those deposits were senior to households claims.

If interbank deposits were senior both banks would have an incentive to withdraw their deposits irrespective whether or not they are actually in need for liquidity–i.e. independent of their respective q_i.

To see this note that if interbank deposits were senior bank A's incentive compatibility constraint to keep deposits until $t = 2$ if $q_A = \frac{1}{2}$ would change from (IC2) to (IC2')

$$(d+a)\,U\,(d_1;d_2) + fU\,(1;1) \geq dU\left(d_1; R - \frac{1}{2}d_2\right) + (a+f)\,U\,(d_1;d_2) \tag{IC2'}$$

[9]Remember that the optimal deposit contract that banks offer to households always implies a risk sharing $\frac{d_2}{d_1} = R^{1/\gamma}$ for integrated as well as separated financial systems given that banks cannot implement a cross regional risk sharing in cases of positive aggregate liquidity shocks.

which obviously does not hold for very large a and not even for $R > 2^{\gamma/(\gamma-1)}$ iff

$$d\left[U\left(d_1; R-\frac{1}{2}d_2\right)-U\left(d_1; d_2\right)\right] < f\left[U\left(d_1; d_2\right)-U\left(1; 1\right)\right]$$

However, taking into account that it is preferable for both banks to withdraw their interbank deposits for $q_i = \frac{1}{2}$ the incentive compatibility constraint of bank A to keep their interbank deposits until $t = 2$ if $q_A = 0$ changes to (IC1')

$$bU\left(0; R\right)+\left(d+e\right)U\left(0; d_2\right) \geq bU\left(0; R-\frac{1}{2}\left(d_2-d_1\right)\right)+\left(d+e\right)U\left(0; R\right) \qquad \text{(IC1')}$$

Obviously, for $(d+e)$ being sufficiently high relative to b it is also for banks that are hit by a positive liquidity shock preferable to withdraw their interbank deposits in $t = 1$.

In contrast, if (IC2') does not hold but (IC1') then an interbank market with senior interbank deposits can implement some cross regional risk sharing. In that case the interbank market would allow for the cross regional risk sharing without bringing about the risk of contagion. Because if interbank deposits are senior and banks only keep their deposits until $t = 2$ if they are hit by a positive liquidity shock then there are no spill-over of the illiquidity of one bank to the other in those cases with $\{q_A; q_B\} = \left\{\frac{1}{2}; 1\right\}$ and $\left\{1; \frac{1}{2}\right\}$.

But if banks always withdraw their interbank deposits in $t = 1$ irrespective of their particular liquidity shock–because (IC2') and (IC1') do both not hold–then the interbank market with senior interbank debt cannot provide any cross regional risk sharing.

7 Alternative Market Mechanisms

So far, we took the organization of the interbank market as exogenously given. In this section we study whether there is an alternative interbank market mechanism that yields a superior outcome. We start out by assuming that consumer deposit contracts of the sort derived in section 4 already exist. In our setup the revelation principle holds. Hence, we may restrict our analysis to direct revelation mechanisms. A direct interbank market mechanism asks both banks for a report on their realized liquidity parameter and maps the tuple of reports $(\hat{q}_A, \hat{q}_B)$ into a decision x, i.e.

$$x = f\left(\hat{q}_A, \hat{q}_B\right).$$

The decision consists of four elements:

- A transfer t_1 from bank 1 to bank 2 in period 1.
- A transfer t_2 from bank 1 to bank 2 in period 2.

- A decision on the closure of each bank in period 1.

Is is possible to improve the efficiency of the risk sharing implemented by the interbank market mechanism? The interbank market studied so far does not provide an efficient risk sharing in case of a positive liquidity shock in one region and no liquidity shock ($q_i = \frac{1}{2}$) in the other region. Is it possible to improve upon that outcome? Consider some alternative mechanism that has the feature that (i) no bank closes when announced aggregate liquidity would be sufficient to honor the withdrawals in period 1 (ii) with opposing liquidity needs there is a sufficient transfer to the bank with high liquidity needs in period 1 (iii) provides efficient risk sharing in cases of a positive liquidity shock in one region and no liquidity shock in the other. Such a mechanism would have to fix contingent transfers as described in table 3.

It is useful to highlight the fundamental role of bankruptcy in providing incentives to managers.

Lemma 5 *Suppose that a bank's date* 2 *value is constant. In order to ensure that self-revelation is incentive compatibility the probability of a bank closure has to increase weakly in the announced liquidity need for period* 1*.*

Proof. This follows immediately from the specification of the bank manager's preferences. ■

Table 3: Transfers implementing unconstraint efficient risk sharing

$(\hat{q}_A, \hat{q}_B)$	0	$\frac{1}{2}$	1
0	$t_1 = t_2 = 0$	$0 < t_1 < \frac{1}{2}d_1^*$ $0 > t_2 > -\frac{1}{2}d_2^*$	$t_1 = \frac{1}{2}d_1^*$ $t_2 = -\frac{1}{2}d_2^*$
$\frac{1}{2}$	$0 > t_1 > -\frac{1}{2}d_1^*$ $0 < t_2 < \frac{1}{2}d_2^*$	$t_1 = t_2 = 0$	$-$
1	$t_1 = -\frac{1}{2}d_1^*$ $t_2 = \frac{1}{2}d_2^*$	$-$	$-$

The following proposition derives the necessary properties of bankruptcy in a social choice function $f(\cdot)$ that guarantees an efficient risk sharing in case of a positive liquidity shock in one region and no shock in the other. It shows that the social choice function has to implement a closure of all banks in those cases where the reported liquidity needs of both banks indicate an aggregate negative liquidity shock in period 1: $\hat{q_A} + \hat{q_B} \leq \frac{1}{2}$.

Proposition 6 *There is no mechanism that simultaneously satisfies the following conditions: (i) Bayesian incentive compatibility, (ii) efficient risk sharing in case of a positive liquidity shock in one region and no shock in the other (iii) the survival of one bank in case of a negative liquidity shock in one region and no shock in the other region.*

Proof. Consider without loss of generality bank A. In order to implement the efficient risk sharing bank A would have to pay a transfer $\frac{1}{2}d_1^*$ in states with $(\hat{q}_A, \hat{q}_B) = (0, 1)$. However, to make self-revelation incentive compatible for a bank with zero liquidity needs ($q_A = 0$) one has to implement a transfer larger than $\frac{1}{2}d_1^*$ in case that announcements are $(\hat{q}_A, \hat{q}_B) = (\frac{1}{2}, 1)$. This implies the closure of a bank with $q_A = \frac{1}{2}$ for $(\hat{q}_A, \hat{q}_B) = (\frac{1}{2}, 1)$.

On the other hand, to avoid that bank A reports a high liquidity need ($\hat{q_A} = 1$) but actually has $q_A = \frac{1}{2}$ one has to close that bank for announced liquidity needs $(\hat{q}_1, \hat{q}_2) = (1, \frac{1}{2})$.[10] ■

Moreover, one can show that the above necessary conditions are also sufficient.

Proposition 7 *There is a mechanism that simultaneously satisfies Bayesian incentive compatibility and the efficient risk sharing in case of one positive liquidity shock. Under this mechanism no bank survives in case of an excessive aggregate liquidity need at date 1.*

Proof. It is straightforward to verify that all IC constraints hold. ■

This shows that the interbank market mechanism from section 6 is not the optimal mechanism. Risk sharing with an aggregate positive liquidity shock can be improved. However, the mechanism described in Proposition 7 cannot be implemented by an interbank market in which only contracts can be traded that prespecify a certain payment d_1^B on the funds withdrawn in $t = 1$ and d_2^B for those kept until $t = 2$ with some other bank. Even if funds can be withdrawn fractionally such a market mechanism can never implement the optimal mechanism, i.e. it can never implement any risk sharing in cases of positive liquidity shocks.

A mechanism such as the one described in Proposition 7 would be quite complex in situations with more than two realizations of liquidity shocks. It would actually require both banks to choose a signal from a set that is as large as the set of possible realizations of the shock and map the tuple of announcements into an outcome. Such a mechanism would establish a tight relationship among two banks that we will call a merger in what follows.

[10]Note that one other option would be to make bank A pay in cases of announced liquidity needs $(\hat{q}_A, \hat{q}_B) = (1, \frac{1}{2})$ and close it only for announcements $(\hat{q}_A, \hat{q}_B) = (1, 1)$. However, given the budget constraint of banks A if it really has $q_A = 1$ this means to the closure bank A at $(\hat{q}_A, \hat{q}_B) = (1, \frac{1}{2})$.

8 Merger

The maximum degree of risk sharing can be reached by a bank operating in both regions or by a complex mechanism such as the one described in the previous section. Such a contract can be viewed as a multi-regional bank that can use its assets to serve the withdrawal of depositors from both regions. However, given that the deposit contract promises $d_1^M > 1$ the bank is illiquid if the fraction of impatient depositors is 1 in both regions, i.e. if the aggregate fraction of early withdrawals is 1. If in addition $d_1^M > \frac{4}{3}$ the bank is also illiquid in those cases where only one region is hit by a negative liquidity shock. Thus assuming $d_1^M > \frac{4}{3}$ the bank can also only pay 1 to all depositors if the aggregate fraction of impatient depositors is $\frac{4}{3}$. Therefore, from the perspective of one region the costs of a multi-regional bank are that with probability f a state occurs in which the multi-regional bank provides a channel for contagion of a negative liquidity shock from the other region.

Obviously, similar to banks that use the interbank market described in section 6 a multi-regional bank is beneficial in those states where the regional liquidity shocks compensate each other. In these cases the bank is not illiquid because it can repay the promised amount d_1^M to the aggregate fraction of $\frac{1}{2}$ of impatient depositors. Therefore, in those states that occur with probability $2e$ multi-regional banks provide a way for an inter-regional risk-sharing.

In addition, a multi-regional bank also provides a means for cross-regional risk-sharing in those states in which in one region $\frac{1}{2}$ of the depositors turn out to be impatient while in the other region all households want to consume in $t = 2$. In these states the patient depositors from the first region benefit at the expense of the patient depositors from the other region, because the bank can finish more projects and can therefore generate the higher per capita $t = 2$-repayment: $\frac{4}{3} - \frac{R}{3} d_1^M$.

In this section we ask whether the possibility of a bank merger changes the main result from section 5 that financial decentralization may be superior to financial integration. In general the optimal deposit contract is different under a merger. Given $d_1^M > \frac{4}{3}$ the expected utility of depositors is given by

$$
\begin{aligned}
E\left[U^M\left(d_1^M\right)\right] &= (a+2e)\left[\frac{1}{2}u\left(d_1^M\right) + \frac{1}{2}u\left(2R - Rd_1^M\right)\right] \\
&+ 2d\left[\frac{1}{4}u\left(d_1^M\right) + \frac{3}{4}u\left(\frac{4}{3}R - \frac{1}{3}Rd_1^M\right)\right] \\
&+ bu\left(R\right) + \left(c+2f\right)u\left(1\right)
\end{aligned}
$$

Taking $u'\left(c_t\right) = c_t^{-\gamma}$ into account it follows from the first order conditions that the optimal deposit contract solves

$$(a+2e+d)\frac{1}{2}\left(d_1^M\right)^{-\gamma} = d\frac{R}{2}\left(\frac{4}{3}R - \frac{1}{3}Rd_1^M\right)^{-\gamma} + (a+2e)\frac{R}{2}\left(2R - Rd_1^M\right)^{-\gamma} \quad (8)$$

Comparing (8) and (1) it is easy to see that the optimality condition for the deposit contract offered by a multi-regional bank is identical to the one for the deposit contract offered by regionally separated banks if $d = 0$. Consequently, in that case a multi-regional bank will offer the same deposit contract as regionally separated banks.[11] Hence we have

Proposition 8 *For $d > 0$ integration over the interbank market is less efficient than cross-regional bank mergers. However, there are risk structures (a, b, c, d, e, f) such that a bank merger delivers lower utility to consumers than a separated financial system.*

9 Conclusion

In this paper we have derived the trade-off that emerges between keeping regional banking systems separated and integrating regional financial systems given that an integration is only possible using interbank deposits. We found that for most parameter values a cross-regional risk sharing using interbank deposits cannot be implemented without incurring the risk of financial contagion. Moreover, our results also show that not all benefits from financial integration can be realized using only an integrated interbank market. Cross-country mergers of banks provide a more efficient cross regional risk sharing mechanism.

In analyzing the trade-off we have shown that an integrated financial system is more preferable the lower the rate of return on long-term investment. Thus our results suggest that it is particularly more preferable for advanced economies with a lower marginal productivity of capital to establish a common financial system. However, to analyze this issue in-depth within a growth model is one important point for further research.

Another issue that we have to leave for further research is to extent the model to a multi-regional setting and analyze which properties of the cross-regional distribution of liquidity shocks might lead to an imperfectly integrated financial system and which implications this has for systemic risk.

[11] As is shown in the appendix for $d > 0$ the deposit contract offered by a multi-regional bank will promise a higher repayment in $t = 1$. Thus for $d > 0$ it follows that $d_1^M > d_1^S$.

Appendix

Optimal deposit contract of a multi-regional bank The optimal deposit contract solves

$$U\left(d_1^M; d_2^M\right) = (a+2e)\left[\frac{1}{2}u\left(d_1^M\right) + \frac{1}{2}u\left(2R - Rd_1^M\right)\right] \\ + 2d\left[\frac{1}{4}u\left(d_1^M\right) + \frac{3}{4}u\left(\frac{4}{3}R - \frac{1}{3}Rd_1^M\right)\right] \\ + bu(R) + (c+2f)\,u(1)$$

$$\frac{\partial U\left(d_1^N; d_2^N\right)}{\partial d_1^M} = 0$$

$$(a+2e)\left[\frac{1}{2}u'\left(d_1^M\right) - \frac{R}{2}u'\left(2R - Rd_1^M\right)\right] + 2d\left[\frac{1}{4}u'\left(d_1^M\right) - \frac{R}{4}u'\left(\frac{4}{3}R - \frac{1}{3}Rd_1^M\right)\right] = 0$$

$$(a+2e)\left[\frac{1}{2}u'\left(d_1^M\right) - \frac{R}{2}u'\left(2R - Rd_1^M\right)\right] + d\left[\frac{1}{2}u'\left(d_1^M\right) - \frac{R}{2}u'\left(\frac{4}{3}R - \frac{1}{3}Rd_1^M\right)\right] = 0$$

$$(a+2e+d)\,\frac{1}{2}u'\left(d_1^M\right) = d\frac{R}{2}u'\left(\frac{4}{3}R - \frac{1}{3}Rd_1^M\right) + (a+2e)\,\frac{R}{2}u'\left(2R - Rd_1^M\right)$$

For $u'(c_t) = c_t^{-\gamma}$ the optimality condition can be rewritten yielding

$$(a+2e+d)\,\frac{1}{2}\left(d_1^M\right)^{-\gamma} = d\frac{R}{2}\left(\frac{4}{3}R - \frac{1}{3}Rd_1^M\right)^{-\gamma} + (a+2e)\,\frac{R}{2}\left(2R - Rd_1^M\right)^{-\gamma}$$

Proof that $d_1^M > d_1^S$ for $d > 0$

$$d_1^M > d_1^S$$

if

$$(a+2e+d)\,\frac{1}{2}\left(d_1^S\right)^{-\gamma} > d\frac{R}{2}\left(\frac{4}{3}R - \frac{1}{3}Rd_1^S\right)^{-\gamma} + (a+2e)\,\frac{R}{2}\left(2R - Rd_1^S\right)^{-\gamma}$$

Given that

$$(a+2e)\,\frac{1}{2}\left(d_1^S\right)^{-\gamma} = (a+2e)\,\frac{R}{2}\left(2R - Rd_1^S\right)^{-\gamma} \tag{9}$$

this holds if

$$\left(d_1^S\right)^{-\gamma} > R\left(\frac{4}{3}R - \frac{1}{3}Rd_1^S\right)^{-\gamma} \tag{10}$$

From (9) follows that

$$\left(d_1^S\right)^{-\gamma} = R\left(2R - Rd_1^S\right)^{-\gamma}$$

Since $x^{-\gamma}$ is a strictly decreasing function in x (10) holds if

$$2R - Rd_1^S < \frac{4}{3}R - \frac{1}{3}Rd_1^S$$

which is obviously true for all $d_1^S > 1$.

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