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## ATTITUDE-DEPENDENT ALTRUISM, TURNOUT AND VOTING

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## **ABSTRACT**

### **Attitude-Dependent Altruism, Turnout and Voting\***

This paper presents a goal-oriented model of political participation based on two psychological assumptions. The first is that people are more altruistic towards individuals that agree with them and the second is that people's well being rises when other people share their personal opinions. By conveying credible information on attitudes, votes give pleasure to individuals who agree with them and thereby confer vicarious utility on voters. Substantial equilibrium turnout emerges with nontrivial voting costs and modest altruism. The model can explain higher turnout in close elections as well as higher turnout by more informed and more educated individuals. For certain parameters, the model predicts that third party candidates will lose votes to more popular candidates, a phenomenon often called strategic voting. For other parameters, the model predicts 'vote-stealing' where the addition of a third candidate robs a major candidate of electoral support.

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This paper presents a simple model of turnout and voting that is based on two psychological traits. The first of these is that people have a tendency to like - and to be generous towards - those that agree with them. The second is that people gain both self-esteem and well-being when they find out that others share their opinions. This second trait implies that a person casting a vote for a candidate (or a proposition) is raising the welfare of other people who think highly of this candidate (or proposition), since the vote validates these peoples' opinion. The combination of the two traits then implies that an individual is willing to incur positive costs of voting because this act raises the well-being of people that are targets for his benevolence.

The approach follows the treatment of Downs (1957) in that potential voters are imagined to be rational while pursuing well-defined goals. Unlike some of the research inspired by Downs (1957), the goal of voting is not exclusively to seal the electoral victory an individual's favored candidate.<sup>1</sup> If this were their only motivation, one could use the notation of Riker and Ordeshook (1968) to write people's benefit from voting as  $PB$ , where  $B$  are the benefits from an electoral victory and  $P$  is the probability of casting the decisive vote. Unfortunately for this theory, actual turnout patterns suggest that  $P$  is virtually nil in those elections where people turn out the most.<sup>2</sup>

Take, for example, the 2000 presidential election in Massachusetts. Of the approximately 2.7 million people that voted in this election, about 1.6 million voted for Al Gore, .9 million voted for George Bush and the remaining .2 million split their votes among four additional

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<sup>1</sup>Chapter 2 of Downs (1957) discusses a utility function for voters that depends only on the policies that result from the an election's outcome. In other parts of the book, however, potential voters are given other objectives including the maintenance of democracy itself.

<sup>2</sup>Note that this problem afflicts any model where people vote only to affect the election's outcome, regardless of the reason why they care about this outcome. Thus, while the analysis of Jankowski (2002) is correct that altruistic voters care more about this outcome than selfish ones and should thus turn out more frequently, it remains the case that plausible altruism parameters cannot rationalize having people spend realistic amounts of time voting when the probability of being decisive is so small.

candidates. Ignore for the moment the 200,000 people who voted for these “third” candidates. Following Myerson (2000), suppose that the number of voters for the two main candidates is drawn from a Poisson distribution and that, consistent with the results above, a fraction  $9/25$  of these voters favor Bush while the remaining fraction favors Gore. Myerson’s (2000) formula then implies that, if the expected number of voters were equal to 2500, the probability of being the pivotal voter would be  $3.5e-46$  if one supported Bush and  $2.7e-46$  if one supported Gore. With an expected number of voters equal to 25000, the probabilities would be somewhat smaller than if one squared these numbers, and MATLAB rounds these probabilities to zero. If one expects 2.5 million people to vote, the probabilities of being decisive are lower than if one took these numbers and raised them to the power 1000. The odds of being decisive rise substantially if one considers sub-national elections because there are fewer voters involved. However, as discussed in Blais (2000, p. 40) turnout is considerably lower in these.

That the probability of being pivotal plays a small role in voters’ mind is confirmed by several surveys discussed in Blais (2000). In surveys from the U.S. National Election Studies, for example, between 85 and 90% of respondents disagreed with the statement that “It isn’t so important to vote when your party doesn’t have a chance to win.” (Blais, 2000, p. 94). Blais (2000) also reports that a very small fraction of people who intend to vote would stop voting if “they were absolutely sure that there was no chance their vote could decide which side wins” (Blais, 2000, p. 71). The model developed here is not contradicted by these findings. Since voting is not principally motivated by the desire to affect who gets elected (or whether a proposition passes) turnout can be substantial even in cases where one vote is unlikely to be decisive.

One reason the pivotal voter model retains some appeal is that three observed differences in behavior across voters and elections are broadly consistent with the model’s predictions. The first of these observations is that turnout tends to be somewhat larger in elections with smaller margins of victory.<sup>3</sup>

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<sup>3</sup>A large number of papers explore the empirical connection between turnout and the closeness of elections.

The second is that, as shown in numerous studies starting with Campbell *et al.* (1960, p. 478), more educated individuals are more likely to vote. Feddersen and Pesendorfer (1996) provide an elegant pivotal-voter model where this occurs. The logic of the model is that more educated individuals are more likely to know which candidate would govern better. Less educated individuals are more likely to abstain because they fear that they will be pivotal and thereby cause a less qualified candidate to win the election. This effect also causes “rolloff”, *i.e.* a tendency for people who vote for important offices to leave their ballots blank when it comes to lower offices. This fits with the evidence of Wattenberg *et al.* (2000) that less informed voters are more likely to rolloff parts of their ballot.

The third is that, when their preferred candidate has a low probability of winning, people sometimes vote for a candidate that they like less. Cain (1978) shows that, in British parliamentary elections, survey respondents in close districts were more likely to have voted for candidates that were not the ones they preferred.<sup>4</sup> This survey finding is consistent with Alvarez and Nagler (2000), who show that in British districts where third party candidates have low support, voters whose personal characteristics would predict third party votes are more likely to vote for more popular candidates. This tendency to vote for relatively popular candidates can be interpreted as a desire to have one’s vote be pivotal, and this is indeed the interpretation favored by Cox (1997).

As several researchers have noted, these three findings cast some doubt on alternate models of voting that do tend to predict higher turnout. One of these alternative theories is that people vote because they feel it is their duty to do so (Riker and Ordeshook 1968). At first blush it is not obvious why more educated individuals would feel a higher duty to vote

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Blais (2000) survey concludes that “Yes, turnout is higher when the race is closer, but a gap of ten points between the leading and the second parties seems to reduce electoral participation by only one point” (Blais, 2000, p. 137-8).

<sup>4</sup>Abramson *et al.* (1992) show a similar result for U.S. presidential primaries, where survey respondents are more likely to intend to vote for a candidate that does not score highest on their “feeling thermometers” when they feel that their preferred candidates has little chance of winning.

or why this duty would be larger in closer elections. Ferejohn and Fiorina (1974) proposed an alternative theory of voting, according to which people vote if this has any chance of eliminating the regret caused by not voting at all. As noted by several commentators, this does not predict the observed relation between turnout and victory margin across elections nor the existence of strategic voting (since individuals should always vote for their preferred candidate to avoid regret). More recently, Bendor *et al.* (2003) have proposed a model that predicts substantial turnout as a result of supposing that people who care only about the election's outcome do not reason in a model-consistent way about the causes of this outcome. Bendor *et al.* (2003) consider a population that is homogeneous in the sense that a single formula describes the probability that any particular individual votes as a function of the past election's outcome, the individual's tastes concerning this outcome and whether the individual has voted in the past. If an informed reasoning individual were placed in the environment implied by their model, this individual would abstain since his vote is irrelevant for the election's result. One would thus expect educated and informed individuals, who presumably understand their environment better than their uninformed counterparts, to be less likely to vote.

The model developed here, by contrast, predicts these three observations in a straightforward way. It implies that lopsided elections have lower turnout than close ones because the altruistic benefit from voting is smaller in the former. In lopsided elections, one of the candidates has few supporters and so relatively few people benefit when an additional individual votes for this candidate. Votes for the other candidate, on the other hand, have only a modest impact on the utility of that candidate's supporters because these supporters expect most people, including abstainers, to agree with them.

The model also predicts that uninformed individuals will tend to abstain. By casting the "wrong" vote, these individuals have a chance of hurting rather than helping those that agree with them on matters of substance. They thus are better off not voting.

The reason the model predicts strategic voting is that individuals gain less vicarious utility when their vote raises the utility of only a small number of individuals. Thus, an

individual who prefers a candidate with intrinsically low support may prefer to vote for his second-ranked candidate. Doing so helps people that do not agree completely with the individual in question but which the individual may nonetheless feel somewhat empathetic towards.

On the other hand, the model is also consistent with a phenomenon that appears to be inconsistent with both the pivotal voter model and with some recent attempts to rationalize turnout by appealing to “group preferences”. This phenomenon is that the addition of a third candidate appears capable of reducing the number of votes received by one of two leading candidates. Many Democrats alleged that the addition of Ralph Nader not only reduced the number of votes received by Al Gore in the U.S. presidential race, but even cost Al Gore the election.<sup>5</sup> In the model, such “vote stealing” can arise if some third party supporters feel some altruism towards supporters of a major party (so that they would vote for this party if the third party were not on the ballot) but feel much stronger altruism towards people who are die-hard supporters of a third party.

A recent set of models, which are discussed in Feddersen (2004), seeks to explain turnout by relying on the capacity of groups to mobilize their supporters. These models differ in important respects from each other and they have not explicitly addressed three-candidate elections. It is thus far from obvious whether they can explain such vote stealing or not. However, it seems likely that Ralph Nader supporters would on the whole have preferred an Al Gore to a George Bush presidency. If groups had influence on individuals supporters, the supporters of Nader that preferred Gore to Bush should thus have succeeded in ensuring a

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<sup>5</sup>Ralph Nader’s stance on the issues was to the “left” of Al Gore and thus more distant from George Bush than from Al Gore, so it seems reasonable to suppose that some of his supporters would have voted for the latter if Nader had not been on the ballot. Moreover, Al Gore lost the State of Florida by 537 votes, where Ralph Nader obtained about 97,000 votes. Added to Gore’s victory in other states, a Gore victory in Florida would have led him to become U.S. president. For evidence that many Democrats were angry at Nader, see “A Fading ‘Nader Factor’ ”, Washington Post, October 22, 2004.

Gore victory. The success of George Bush seems easier to explain if one imagines that Ralph Nader voters acted individually, so that they rationally believed that their individual vote would not have an effect on the elections' outcome.

The paper proceeds as follows. The next section discusses the evidence for the psychological assumptions of the model. Section 2 presents the model's structure and characterizes its equilibria in two-candidate settings with common knowledge about candidates. Its main comparative statics concern the effect of closeness on turnout. Section 3 turns its attention to a setting where some of the voters have weak information about the candidates. This approach is meant to capture the effect of information on turnout as well as the reasons why people who show up to vote often abstain from voting in some individual races. Section 4 studies elections with more than two candidates in order to discuss strategic voting and vote stealing. Section 5 offers some concluding remarks.

## 1 Psychological Foundations

According to the model below, voting is due to two psychological forces. The first is that people's benevolence extends particularly to those whose attitudes are similar to their own and the second is that they gain well being when their own attitudes are echoed by others. This section discusses some evidence for these ideas.

### 1.1 Attitude Similarity, Attraction and Benevolence

In Byrne (1961), subjects reported more liking for people whose attitudes towards issues such as racial integration were more similar to theirs. Byrne (1961) first asked subjects to report their views on both political and lifestyle issues. They were then presented with answers to similar questions that were supposedly given by a bogus subject and were asked for their evaluation of this subject on a variety of scales. In this latter part of the experiment, subjects were also asked the extent to which "they liked" the bogus subject and the extent

to which they would like to “work with” him or her. These latter two scales of interpersonal attraction proved to be strongly correlated with each other while also being correlated with the similarity between the subject’s attitudes and those of the bogus subject.

Numerous variants of this study have been conducted (see Montoya and Horton 2004 for a recent example that discusses some of the earlier literature) and the correlation between measures of interpersonal attraction and measures of attitude similarity has proven to be robust. One variant that is particularly relevant for voting is discussed in Krosnick (1988). In this study, the attitudes of respondents in the American National Election Studied (NES) surveys were compared to the attitudes of presidential candidates (as measured by the NES respondents’ average perception of the candidates attitudes) in the 1968, 1980 and 1984 US presidential elections. Krosnick (1988) shows that the extent to which respondent’s report liking the Democratic candidate relative to the Republican candidate is positively correlated with the extent to which the respondents attitudes mirror those of the Democratic candidate as opposed to those of the Republican one.<sup>6</sup>

While suggestive, the correlation between attitude similarity and the expression of liking is not sufficient to establish that people will act differently towards people who share their beliefs than they will towards people that do not, though the fact that they tend to vote for them is worth keeping in mind. There is, however, evidence of this sort from two different types of studies.

The first type is based on the “lost letter” technique. In Tucker *et al.* (1977) either a 2\$ money order or 2\$ of cash are left on the sidewalk to be picked up by a stranger. Attached to these funds are a contribution form and a stamped and addressed envelope that make it clear that the funds are intended for a medical charity. In at least some of the experiments, there is also a form where the purported contributor filled out an opinion questionnaire that was addressed to a polling organization. These packages were left in a predominantly Jewish

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<sup>6</sup>Brady and Sniderman (1985, p. 1067) also establish, using the NES survey, that people who describe themselves as liberals report warmer and more favorable feelings towards liberals than towards conservatives. Self-described conservatives do the reverse.

neighborhood and the opinion were either favorable to American aid to Israel or opposed such aid. The cash, the money order and the questionnaire were all more likely to be forwarded if the questionnaire contained pro-Israeli views, which is consistent with the idea that people are more inclined to help strangers if they agree with them. Notice in particular that people's desire to help the medical charity is not sufficient to explain this finding, since this would not explain a differential rate of forwarding the funds.

In a related study, Sole *et al.* (1975) used money orders for medical foundations that were attached to questionnaires relating to other political issues (including discrimination and the desirability of war). When the opinions expressed in these questionnaires matched more closely those opinions that were obtained from people chosen randomly in the same neighborhood, a larger fraction of the money orders was forwarded. This effect turned was stronger when the opinions related to important issues such as discrimination than when they referred to less important issues (such as whether groceries should be delivered for free).

Karylowski (1978) carried out a different type of helping study where participants could help a bogus partner earn money by pressing buttons in response to stimulus lights. This helping experiment was carried out both before and after participants saw answers that the bogus partner supposedly completed about their preferred activities. In some cases this information made the bogus partner seem similar to the subject, because the answers corresponded closely to ones the subject had given earlier, while in others it did not. Experimental subjects helped their partners significantly more when the partner's self-description was more similar to their own. Because partner-subject similarity in this experiment concerns preferred activities rather than attitudes, the experiment might be seen as less relevant. However, the questionnaire literature shows that the correlation between similarity and "liking" is fairly robust to varying the dimension along which one measures similarity.

## 1.2 Other's Attitudes and One's Own Well-Being

The model's second key psychological ingredient is that an individual's utility increases when he learns that others share his opinions. Kenworthy and Miller (2001) provide direct evidence for this. They interviewed people in the street and first asked them whether they were for or against the death penalty. Some of the interviewees were then told that their position was held only by a minority of people, others that it was held by a majority. Some were told that the number of people holding the interviewees opinion was shrinking while others were told that it was growing. Interviewees were then asked to state how they felt about these (bogus) poll numbers using a scale from 1 to 4 where 1 represents "very bad" and 4 represents "very good" .

The average response of the subjects that were told they were in a minority was equal to 2.37 and this was below both the neutral value of 2.5, and below 2.62, the mean value reported by those who were told they were in a majority. The average response by those told that their group was shrinking was equal to only 1.81, while those told that their group was growing was equal to 3.21. This suggests both that people's well being is larger when more people agree with them and that their well-being is particularly sensitive to up-to-date news about how other people's opinions have been changing. Elections, of course, are particularly effective at conveying such point-in-time news.<sup>7</sup>

Another study that measures the effect of other people's opinions on well-being is Pool *et al.* (1998). They elicited attitudes towards an issue and then told subjects that a group that the students identified with held opposite views. This led to a measurable drop in reported

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<sup>7</sup>It also suggests that poll responses matter to people. This means that the model can also explain why people choose to respond to polls. This capacity of the model raises the question of why polls elicit less enthusiasm than elections both by those who read polls and those that respond to questionnaires. While detailed survey studies seems necessary to settle this question, one possibility is that individuals fear the use that individual pollsters will make of the information that they provide to them, and have fewer such concerns in polling places.

self-esteem. One difference between the Pool *et al.* (1998) study and Kenworthy and Miller (2001) is that the former suggests only that the opinions of a group one wishes to be part of matter for self-esteem, while the latter suggests that the sheer total level of support matters as well.

One can also find more indirect evidence for this connection between well-being and attitude similarity. One variable that is extremely strongly correlated with subjective measures of well-being is the extent to which individuals are satisfied with themselves, i.e. the extent to which they have high self-esteem. In their survey, Cheng and Furnham (2003, p. 923) report that “self-esteem has been reported to be one of the strongest predictors of well-being.” The literature studying the correlates of self-esteem is, in turn, vast. One of the leading theories of the determinants of self-esteem is the “sociometer theory”. According to this theory, people’s self esteem is a “monitor of social acceptance” (Leary 1999, p. 32) so that the “so-called self-esteem motive functions not to maintain self-esteem per se but rather to avoid social devaluation and rejection.” It would follow from this that being liked by others would provide one with self-esteem, and thereby increase one’s well-being. Leary (1999) also discusses some of the evidence that bears on this theory, though it should be pointed out that this evidence does not directly involve correlations of self-esteem with being liked by others.

Some evidence that seems more directly relevant to voting is provided by Boen *et al.* (2002). They studied the display of political posters and lawn signs around the June 1999 election in Flanders. Their central finding is that posters and signs for parties that did better in these elections than in previous ones tended to be displayed for longer after the election than the posters and signs that supported parties whose performance declined.<sup>8</sup> At a minimum, this is evidence that shows elections have real effects on voter’s actions. It does seem reasonable to infer also that those that keep the posters of parties that did well are

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<sup>8</sup>It should be noted, however, that the earlier study by Sigelman (1986) on Kentucky elections did not obtain these effects. Boen (2002) discusses reasons for these differences, some of which echo the reasons given by Sigelman (1986) for not finding “basking in reflected glory” in his own study.

proud while those who remove the posters of parties that did poorly do not have equally positive feelings. Under this interpretation, individuals gain utility when a larger number of people vote for the candidate or party which they support. This fits closely with the assumption whose consequences are explore below.

## 2 Two-Candidate Equilibria with Common Knowledge

### 2.1 Basic Setting

Individuals have opinions on issues which, as in Davis, Hinich and Ordeshook (1970), can be represented by their ideal positions on these issues. Following their notation, let the vector  $x_i$  denote individual  $i$ 's ideal position. As in their work, individuals are better off when there is a short distance between  $x_i$  and the corresponding vector for their elected representatives  $\bar{x}$ . Let  $d^e(x_i, \bar{x})$  measure this distance where, without loss of generality,  $d^e(x_i, x_i) = 0$ . When confronted with candidates indexed by  $j$  whose preferred positions can be characterized by the vectors  $\theta_j$ , individual  $i$  thus prefers the one whose  $d^e(x_i, \theta_j)$  is the lowest.

Individuals are affected by two additional measures of opinion distance. First, individual  $i$ 's altruism for individual  $j$  depends on the distance he perceives between his own opinions and those of  $j$ . Let  $d(x_i, x_j)$  denote the actual distance between these individuals (where  $d(x_i, x_i) = 0$  and  $d(x_i, x_j) = d(x_j, x_i)$ ), and  $E_i$  be the operator that takes expectations conditional on the information available to  $i$ . A lower value of  $E_i(d(x_i, x_j))$  then leads to a higher level of benevolence. Second, individual  $i$ 's well-being depends on the extent to which he expects others to share his opinions perhaps because a high degree of opinion concordance makes  $i$  feel socially competent and thereby increases his self-esteem. One measurement of this concordance is the total distance  $D_i$

$$D_i \equiv \sum_{j \neq i} d(x_i, x_j) \quad (1)$$

While this total distance appears like a reasonable first step for analyzing the impact of

other people's opinions on a person's well-being, it is important to stress that the available psychological evidence is not sufficient to pin down the details of this dependence. It is possible, for example, that well-being depends on the average rather than the total distance. For a given population size, this would have no effect on the equilibrium, but it would affect the impact of changes in the population size on equilibrium voting. It is also quite possible that individuals care differently about the opinion distance of individuals that are more proximate in certain respects. Future survey evidence might well help clarify these issues and thereby allow for firmer predictions of the effect of demographic variables on turnout and voting.

People do not know at all times where most other people stand on the issues that matter to them, though elections do provide important information about this. Right after presidential elections, in particular, the media devotes considerable attention to discussing what the election results reveal about peoples' attitudes. It thus seems worth distinguishing between the expectations of  $D_i$  held before and after elections.

Let  $E_j^0 D_i$  denote the expectation held before the election by individual  $j$  about  $D_i$  while  $E_j^1 D_i$  denotes the expectation after the election. The instantaneous utility of each individual presumably depends on their current perception of  $D_i$ . Even right before an election, however, individuals know that their lifetime utility is much more affected by their perception of  $D_i$  after the election since more time will elapse afterwards than before. For simplicity, suppose individual  $i$  is concerned only with  $E_i^1 D_i$  so that his utility from others' opinions is  $S(E_i^1 D_i)$ . To ensure that individuals are happier when others agree with them, the derivative of  $S$  with respect to its argument,  $S'$ , is negative. To simplify further, let the  $S$  function be linear so that  $S = S_0 - S' E_i^1 D_i$  where  $S_0$  and  $S'$  are constants. This means that, by the law of iterated expectations, the expectation right before the election of the utility the individual will get from other people's opinions is  $S_0 - S' E_i^0 D_i$ .<sup>9</sup>

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<sup>9</sup>With a nonlinear  $S$ , to which a linear  $S$  is just an approximation, the expectation before the election of  $S(E_i^1 D_i)$  would also depend on higher order moments including the variance of  $E_i^1 D_i$ . While this would complicate the analysis, it should not materially affect the results.

Given that, as in Downs (1957), individuals are also concerned directly with the outcome of the election, their “direct” payoffs  $y_i$  can then be written as

$$y_i = -d^e(x_i, \bar{x}) + S_0 - S' E_i^1 D_i - c_i \quad (2)$$

where  $c_i$  denotes the cost of voting. Individual  $i$ 's total utility  $u_i$  is then

$$u_i = y_i + \sum_{j \neq i} E_i^1 \lambda(d(x_i, x_j)) y_j \quad (3)$$

where  $\lambda(d(x_i, x_j))$  is the altruism of  $i$  for  $j$ . To capture the idea that people feel more benevolence for those that agree with them, the function  $\lambda$  is decreasing in its argument and reaches a maximum  $\lambda_0$  when this argument equals zero.

At the moment of voting, individual  $i$  maximizes his expectation of  $u_i$  before the election, which is  $E_i^0 u_i$ . An equilibrium is then a mapping from an individuals' tastes and costs of voting to a voting action, where this action can include abstaining. This mapping must ensure that, if individuals expect others to base their actions on this mapping, the action that maximizes  $E_i^0 u_i$  is consistent with this mapping as well.

## 2.2 Symmetric Equilibrium with Two Types

This setup takes a particularly simple form when the vector  $x$  can only take two values  $x^a$  and  $x^b$  and there are two candidates whose positions are known to correspond to these two values of  $x$ . The distances  $d^e$  and  $d$  can then be normalized to equal one when their two arguments differ from each other so that  $S$  depends only on the number of individuals that individual  $i$  expects to agree with him. Suppose that  $\lambda(1) = \lambda_1$  and let  $p$  denote the probability that any given individual has the ideal point  $x^a$ . Suppose for the moment that the equilibrium satisfies the property that those individuals who vote, vote for the candidates that share their preferences.

Let  $z^j$  be the probability that someone who favors  $x^j$  actually votes. Using Bayes rule, the probability that someone who has abstained prefers  $x^a$  is  $\tilde{p}$  where

$$\tilde{p} = \frac{p(1 - z^a)}{1 - pz^a - (1 - p)z^b} \quad (4)$$

Note that  $\tilde{p} = p$  if individuals with both preferences are equally likely to vote. If, instead, people who prefer  $x^a$  are more likely to cast a ballot it follows that those who do not cast a ballot are somewhat more likely to prefer  $x^b$  so that  $\tilde{p} < p$ .

Knowing that all the individuals who cast a ballot for the candidate that prefers  $x^j$  also prefer  $x^j$  themselves,  $E_i^1 D_i$  for those who prefer  $x^a$  is given by

$$D_i^a \equiv E \sum_{j \neq i} d(x^a, x_j) = N^b + [N - 1 - (N^a - J_i) - N^b](1 - \tilde{p}) \quad (5)$$

where  $N$  is the population eligible to vote,  $N^j$  is the total number of people who vote for the candidate who prefers  $x^j$  and  $J_i$  is an indicator variable which equals one if individual  $i$  votes. Similarly,  $E_i^1 D_i$  for those who prefer  $x^b$  is

$$D_i^b \equiv E \sum_{j \neq i} d(x^b, x_j) = N^a + (N - 1 - N^a - (N^b - J_i))\tilde{p} \quad (6)$$

The vote of individual  $i$  raises  $N^a$  by one while also reducing the abstaining population  $N - N^a - N^b$  by one. This act of voting obviously has no effect on the  $D_i^a$  of the voter himself. On the other hand, it lowers the value of  $D_j^a$  for all other individuals  $j$  who support  $x^a$  by  $(1 - \tilde{p})$  while raising the value of  $D_j^b$  by the same amount. Non-voters are expected to support  $x^a$  with probability  $(1 - \tilde{p})$  and, by voting,  $i$  eliminates the uncertainty surrounding his attitudes. The result of  $i$ 's voting is thus that the  $y_j$  of other people that support  $x^a$  rises by  $S'(1 - \tilde{p})$ , which also equals the reduction in the  $y^j$  of the people that support  $x^b$ . Individual  $i$  puts weight  $\lambda_0$  on the former and weight  $\lambda_1$  on the latter.

This means that, by voting,  $i$  raises his expectation of  $\sum_{j \neq i} E_i^1 \lambda(d(x_i, x_j))y_j$  by  $S'(1 - \tilde{p})[\lambda_0 M_i^a - \lambda_1 M_i^b]$  where  $M_i^j$  is the number of individuals (other than himself) that  $i$  expects to support  $x^j$  after the election.  $M_i^a$  and  $M_i^b$  are given by

$$M_i^a = N^a - J_i + (N - 1 - (N^a - J_i) - N^b)\tilde{p} \quad (7)$$

$$M_i^b = N^b + (N - 1 - (N^a - J_i) - N^b)(1 - \tilde{p}) \quad (8)$$

At the moment of voting, individual  $i$  knows that the  $y_j$  of other  $x^a$  supporters will rise by  $S'(1 - \tilde{p})$  as a result of his vote but does not yet know  $M_i^a$ . This means that his expectation at

this point of the effect of his vote on  $\sum_{j \neq i} E_i^1 \lambda(d(x_i, x_j)) y_j$  equals  $S'(1 - \tilde{p}) E_i^0 [\lambda_0 M_i^a - \lambda_1 M_i^b]$ . At the moment of voting, individual  $i$ 's expectation of  $N^a$  equals  $(N - 1)pz^a + J_i$  while his expectation of  $N^b$  equals  $(N - 1)(1 - p)z^b$ . This means that, using (4),

$$E_i^0 M_i^a = (N - 1) \left( pz^a + p(1 - pz^a) \frac{1 - pz^a - (1 - p)z^b}{1 - pz^a - (1 - p)z^b} \right) = p(N - 1)$$

This result makes intuitive sense. The expectation of individual  $i$  before the election is simply that  $x^a$  will be favored by a fraction  $p$  of all individuals other than himself. It is immediately apparent that this expectation is also the number of people that a supporter of  $x^b$  expects to prefer  $x^a$ . A similar calculation shows that  $E_i^0 M_i^b$  equals  $(1 - p)(N - 1)$  for supporters of either candidate.

The model provides two motivations to vote. The first is through the outcome of the election, which affects people both directly and through their altruism for the people who are equally affected by this outcome. The second is that an increase in the number of people voting for a candidate leads equally minded individuals to revise upwards their estimate of the number of people who agree with them. It is well known that the first of these effects is not important if turnout is substantial and the costs of voting are strictly positive. Or, as Palfrey and Rosenthal (1985) show in a closely related model, it leads to voting only by people whose costs of voting are arbitrarily close to zero if  $N$  is large. It is thus easier to start with equilibria where this effect is purposefully ignored. If it turns out that even people with nontrivial voting costs vote at this equilibrium, it follows that an equilibrium where people take into account their effect on outcomes is very close to the one that is computed ignoring this effect.

Ignoring the term  $-d^e(x_i, \bar{x})$  in (2), the increase in  $E_i^0 u_i$  that results from voting for an individual who favors  $x^a$  is

$$\lambda_0 S'(N - 1)(1 - \tilde{p})[\lambda_0 p - \lambda_1(1 - p)] - c_i \tag{9}$$

whereas it equals

$$\lambda_0 S'(N - 1)\tilde{p}[\lambda_0(1 - p) - \lambda_1 p] - c_i \tag{10}$$

for an individual who favors  $x^b$ . In an equilibrium where voting is strictly voluntary those for whom the expressions in (9) or (10) are positive, while others abstain. This means that there is a maximum value for  $c_i$  such that supporters of  $x^i$  vote. Let  $F$  denote the pdf of  $c$  for both types. Then, using using (4), the equilibrium values of the probabilities of voting  $z^a$  and  $z^b$  must satisfy

$$F^{-1}(z^a) = G^a \equiv \frac{S'(N-1)(1-z^b)[\lambda_0 p(1-p) - \lambda_1(1-p)^2]}{1-z^b + p(z^b - z^a)} \quad (11)$$

$$F^{-1}(z^b) = G^b \equiv \frac{S'(N-1)(1-z^a)[\lambda_0 p(1-p) - \lambda_1 p^2]}{1-z^b + p(z^b - z^a)} \quad (12)$$

The first of these equations is a “reaction function” of the supporters of  $x^a$ . It gives the fraction of supporters of  $x^a$  that (weakly) prefers voting to abstaining for a given fraction of voters for  $x^b$ . Similarly, (12) can be interpreted as the reaction function of the supporters of  $x^b$ . An equilibrium is thus a pair of fractions  $z^a$  and  $z^b$  that satisfies these two equations.

Consider first the baseline case where  $\lambda_1 = 0$  so that supporters of  $x^a$  feel neither benevolence nor animosity towards supporters of  $x^b$ , and viceversa. It is immediately apparent from these equations that there then exists a (unique) symmetric equilibrium where  $z^a = z^b = z^*$  and  $z^*$  satisfies

$$F^{-1}(z^*) = G^* \equiv \lambda_0 S'(N-1)p(1-p) \quad (13)$$

It follows from this equation that the model can account for large and realistic turnout rates for modest (and thus plausible) degrees of voter altruism. Suppose that  $\lambda_0$  is equal to .05 so that each individual puts .05 as much weight on the utility of like-minded people as he does on his own. Suppose that  $S'$  equals .001 of a penny so that an individual gains a penny when he discovers that another 1000 people agree with him and that, analogously to the US case,  $N$  equals 150 million. Now consider a close election with  $p = .5$ . Then, in a symmetric equilibrium,  $\tilde{p}$  is equal to .5 as well. Equations (9) and (10) then imply that  $G^*$ ,  $G^a$  and  $G^b$  equal \$18.75. This implies that all those for whom  $c_i$  is lower than \$18.75 should vote in national elections. Turnout should thus be substantial if, as argued by Blais (2000 p. 84-87), voting costs are fairly modest.

Given that the model predicts large turnout even when voters ignore their effect on election outcomes, letting them take this effect into account does not have an important effect on the equilibrium. Once turnout is large, individual votes have only a negligible effect on the outcome. Individuals who were just indifferent between voting and not voting because their costs of voting were just equal to  $G^*$ , would strictly prefer voting once they take into account their potential effect on the outcome. But individuals with even slightly higher costs would remain on the sidelines.

The left hand side of (13) is increasing in  $z^*$  and the right hand side of this equation reaches a maximum for  $p = .5$ . Thus turnout is increasing in the closeness of elections. This result is somewhat unexpected because lopsidedness has two effects that work in opposite directions and the relative strength of these effects may not be clear *a priori*. On the one hand, lopsidedness increases the expected number of supporters of the front-runner while reducing those of the underdog. This means that more people benefit when a vote is cast for the front-runner than when one is cast for the underdog and so might suggest that turnout for the front-runner rises while that for the underdog falls. On the other hand, lopsidedness changes the informational content of an additional vote. It reduces the extent to which an additional vote for the front runner constitutes good news for her supporters while raising the extent to which an additional vote for the underdog raises their happiness. This effect tends to increase turnout for the underdog while lowering that for the front-runner.

The simplest intuition for why close elections lead to higher turnout for both candidates is to focus on the extreme case of lopsidedness and to notice that votes in this case have very little value to potential voters. If essentially everyone favors the candidate that supports  $x^a$ , so that  $p$  is close to one, additional votes for the front-runner are essentially worthless for her supporters because these supporters only reduce their subjective probability that the new voter agrees with them from  $(1 - \tilde{p})$  to zero. At the same time,  $\tilde{p}$  is close to one in this case regardless of turnout rates because the fact that  $p$  is near one implies that most people support  $x^a$  whether they have voted or not. Thus, supporters of  $x^a$  gain essentially nothing from such a vote and turnout by them should be quite low. Similarly, if  $p$  is close to one,

there are almost no supporters of  $x^j$  so that essentially no one gains from an additional vote for the candidate that supports  $x^j$ . Thus, turnout by these supporters should be quite low as well.

For realistic variations in the closeness of elections, this effects should be quite small however. If  $p = .55$ , the odds facing the candidate favoring  $x^b$  are essentially insurmountable if the number of voters is substantial. Still, keeping the previous parameters, all individuals whose costs of voting is lower than \$18.56 should still vote. The exact fall in turnout thus depends on the fraction of people whose voting costs fall between \$18.56 and \$18.75. Still, the predicted falls in turnout are probably not dramatic for plausible choices of the pdf  $F$ . This fits with the conclusion of Blais (2000, p. 137-8) that “a gap of ten points between the leading and the second parties seems to reduce electoral participation by only one point.”

As written equation (13) implies that turnout should be increasing in the number of eligible voters  $N$ . For a given  $S'$ , that is for a given increase in the utility of voters when there is one additional person that agrees with them, a larger  $N$  implies that more people benefit from this additional vote so that voters derive more vicarious utility from voting. This result hinges crucially on the supposition that people care about the total distance  $D$  as opposed to caring about other functions of the  $d$ 's. If, for example, people cared about the *average* distance between themselves and other voters in their jurisdiction, the analysis above would remain valid but  $S'$  would be proportional to  $1/(N - 1)$ . Predicted turnout rates would then be independent of  $N$ . Thus, as discussed above, the model's implications regarding the effect of changes in the population depend on aspects of preferences about which more information is needed.

A more robust implication of the model would seem to be prediction that turnout should be larger in elections where a single individual is elected in the whole country than in elections where only representatives for sub-national districts are chosen. The reason is that, in the former, more people receive a self-relevant message when a vote is cast. Blais (2000, p. 40) shows that, indeed, turnout in (sub-national) legislative elections is generally lower than in presidential elections.

The presidential election of the United States provides another type of evidence that seems robustly relevant for testing the model. In this election, the President is elected indirectly with voters choosing state-wide representatives to the electoral college, which then vote for the President. Because electoral college members are pledged to presidential candidates, votes in any state send a national message.  $N$  ought thus to represent the national electorate and  $p$  ought to represent the nation-wide average popularity of the candidate that prefers  $x^a$ . Thus, the closeness that affects turnout is the closeness of the national election itself, rather than the closeness of the state-wide election for members of the electoral college. This ought to be testable with US data, where there exists at least a weak association between national turnout rates and the closeness of presidential elections.<sup>10</sup> Blais (2000, p. 76) shows a result of this type based on correlating the intention to vote and the perceived closeness of the 1996 British Columbia parliamentary election. Individuals who expected this election to be closer were more likely to say they intended to vote. However, perceived closeness at the provincial level appears to be a better predictor of the intention to vote in this election than perceived closeness at the level of the constituency.

Consider now the effect of letting  $\lambda_1$  differ from zero. The simplest case to consider is where  $p = .5$  so that the outcome remains symmetric. The solution to (11) and (12) is then

$$F^{-1}(z) = (\lambda_0 - \lambda_1)S'(N - 1)/4$$

This shows that animus towards people who support the other candidate (*i.e.* a negative  $\lambda_1$ ) increases turnout in exactly the same way as does altruism towards people that support one's own candidate. When  $p$  is not equal to .5, the effect of a negative  $\lambda_1$  is not the same on the turnout of both candidates. Still, it is straightforward to show that, when the system

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<sup>10</sup>While a much more thorough study is needed to check this implication, anecdotal evidence suggests it may be valid. From the presidential election of 1996 to the presidential election of 2004, the total number of voters rose in Massachusetts and New York by 12% and 9% respectively even though the populations in these two states were stagnant and even though the electoral college results in all four of these elections were a foregone conclusion at the time.

consisting of (11) and (12) is stable, increases in the animosity of the supporters of  $x^i$  towards the supporters of  $x^j$  raises the turnout of the supporters of  $x^i$ .<sup>11</sup>

It is worth noting that while the model is one where costs of voting vary in the population while the altruism parameter is constant, one could obtain an essentially identical equilibrium relation by supposing that this parameter varies across people as well. Setting (9) and (10) would still determine the boundary between voters and non-voters but this boundary would refer to a ratio of  $c$  to  $\lambda_0$  rather than simply to a maximal value of  $c$ . One attraction of this alternative interpretation is that individuals with large values of  $\lambda_0$  should not only be willing to vote but should also be willing to incur other costs to express their preferences. By, for example, wearing political buttons or putting signs on their lawns they would provide further support for individuals that share their beliefs. This fits with the finding of Copeland and Laband (2002) that people who carry out such activities are more likely to vote.<sup>12</sup>

The fact that the election's outcome matters so little for voting suggests that the relevance of a candidate's attitudes might be muted as well. When ignoring  $d^e(x_i, \bar{x})$  in  $y_i$ , there is also an equilibrium where all the people who prefer  $x^a$  vote for the candidate that prefers  $x^b$  and viceversa. At this equilibrium, again,  $G^*$  is the cutoff for voting costs so that those with higher costs abstain and the others vote. Even if one lets  $d^e(x_i, \bar{x})$  affect  $y_i$  in (2) there is an essentially identical equilibrium. At this equilibrium, people with costs exactly equal to  $G^*$  abstain because they dislike the outcome that their vote might induce. However, because the probability of affecting the outcome is so small given the large turnout, people with even

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<sup>11</sup>Stability is here defined by supposing that  $z^i$  tends to rise when  $G^i > F^{-1}(z^i)$ . It thus requires that the matrix obtained by differentiating  $G^a - F^{-1}(z^a)$  and  $G^b - F^{-1}(z^b)$  with respect to  $z^a$  and  $z^b$  be negative definite.

<sup>12</sup>This raises the general question of how this model relates to the “expressive voter” model that Copeland and Laband (2002) see as being supported by their evidence. The model of this paper shares with expressive voter models such as Brennan and Hamlin (1998) the idea that voters vote to express an opinion (rather than to affect the election outcome). Where the current model differs is in supposing that this desire to express oneself is the result of seeking to help others, as opposed to being directly useful to the self.

slightly lower  $c_i$  do vote. While it is true that they help to bring about a disliked outcome by doing so, their effect on the outcome is so small that this effect is swamped by their desire to reduce the  $D$  of the people they feel altruism towards.

It is also worth noting that the equilibrium where people vote for the candidate they dislike may be attractive to those in the minority. Members of the minority might be quite happy to vote for the candidate they like the least if this somehow induces the members of the majority to vote for the candidate whose preferences match those of the minority. Nonetheless, equilibria where people all vote for the candidate they dislike are unattractive. They probably arise in this model because it neglects two important real-world phenomena. The first is the process by which candidates get selected, which usually requires that like-minded people make a consistent effort in favor of a candidate. The second, which is closely related, is the opportunity people have to communicate their intentions to vote before the election. One might wonder whether this communication is credible in the presence of a secret ballot. However, even with this institution, voting can be credibly revealed if lying is detectable (which it often is).

A modification of the model that incorporates elements of these two phenomena does not have these these unappealing equilibria. Suppose that there exist two individuals, one that supports  $x^a$  and one that supports  $x^b$ , which “nominate” two distinct candidates in sequence. In other words, the supporter of  $x^a$  first nominates a candidate and this is followed by the nomination of a different candidate by the supporter of  $x^b$ . Individuals nominating a candidate vote first. Following Farrell and Saloner’s (1985) suggestion, communication is modelled by allowing agents, in this case voters, to take actions in sequence and letting these actions be observable. The assumption of full observability is obviously quite strong, and is relaxed below. This assumption does, however, serve the useful purpose of highlighting the importance of credible communication.

Suppose without loss of generality that the majority prefers  $x^a$ . Suppose further that, at the nominating stage, the person favoring  $x^a$  has chosen a candidate that supports  $x^a$  as well. With sequential and observable voting, the unique symmetric subgame perfect

equilibrium for supporters of  $x^a$  has all these supporters vote for this candidate. To see why the equilibrium is unique in this case, notice that any supporter of  $x^a$  would vote for this candidate if all previous supporters of  $x^a$  did so and if he expects that, after his vote for this candidate, all subsequent supporters of  $x^a$  will do so as well. Since this voter knows that subsequent voters will reason analogously, he expects them to vote for this candidate if he does. He thus votes for this candidate and the candidate wins the election even if all the voters that support  $x^b$  vote for their own candidate.

Now consider the supporters of the minority position  $x^b$ . Since voters that support  $x^a$  vote for their own candidate, any vote for the candidate nominated by the supporter of  $x^b$  raises people's estimate of the support for  $x^b$ . Supporters of  $x^b$  thus vote for this candidate.

In the nominating stage, the supporter of  $x^a$  can guarantee that her nominee will win the election if he nominates a person that prefers  $x^a$ . Since he would like the elected official to have this preference, he nominates such an individual regardless of whether he nominates a candidate before or after the supporter of  $x^b$  nominates his own. Since the candidate supported by those who prefer  $x^b$  cannot win, the supporter of  $x^b$  that nominates a candidate ought to be indifferent about the preferences of the candidate he nominates. However, if individuals derive even a small amount of utility from being a candidate, he will nominate a supporter of  $x^b$  since he derives more indirect utility from the candidate's own utility.

### 3 Poorly Informed Voters

Campbell *et al.* (1960) show that many individuals are poorly informed about candidates. In their sample of respondents, only 40 to 60 percent of the people who had an opinion on an issue “perceive party differences and hence can locate one or the other party as closer to their ‘own’ position” (Campbell *et al.*, 1960, p. 180). At the same time, there appears to exist a correlation between being informed and voting. More educated individuals are more likely to vote, as are individuals who describe themselves as “paying attention to political campaigns” (Campbell *et al.* 1960 p. 103). This section shows that there exists a simple

form of informational symmetry such that, indeed, individuals that are more informed about candidates end up having a higher turnout rate.

Suppose once again that individuals prefer either  $x^a$  or  $x^b$  and that there are two candidates  $m$  and  $n$ . The difference between this model and the one considered earlier is that there is now some uncertainty regarding which candidate is more attractive to people with either preference. While consistent with the Campbell *et al.* (1960) evidence discussed above, it may seem strange to suppose that people know their ideal point while not knowing which candidate comes closer to it. This would not be the case if the ideal point reflected positions on specific issues and candidates could costlessly transmit their own positions.

There are three sources of uncertainty that might justify this modelling approach. In the first, it is costly for people to interpret the messages sent by candidates.<sup>13</sup> In the second, people know their preference for policy consequences, but are uncertain about the consequences of the policies that are being discussed during election campaigns. People thus have difficulty converting campaign proposals into matches with their own preferences. In the third, people differ in the personal qualities that they want in an elected official and these qualities are difficult to discern because both candidates seek to demonstrate that they rate highly on all the qualities that are of interest to voters.<sup>14</sup>

Suppose that it is not clear in advance whether candidate  $m$  or candidate  $n$  is preferred by people whose ideal point is  $x^i$ . Right before the election, a fraction  $\gamma$  of individuals learns which candidate they prefer. At this point there are two states of nature. In state 0, the informed individuals who prefer  $x^a$  are better off with  $m$  while the informed individuals who prefer  $x^b$  would like to be governed by  $n$ . In state 1, these preferences are reversed. Suppose that these two states are equally likely.<sup>15</sup>

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<sup>13</sup>See Popkin (1994, especially pages 96-114) for a discussion of how people obtain information from both intended and unintended campaign messages.

<sup>14</sup>See Popkin (1994, especially 61-63) for examples in which changes in the perceived competence of candidates along various dimensions affected election outcomes.

<sup>15</sup>An obvious extension of this analysis would cover the case where one group of people is more likely to

A fraction  $(1 - \gamma)$  of individuals remains relatively uninformed. This means that, rather than knowing which state of nature has been realized, they only observe an imperfect signal  $s$ . This signal equals 0 or 1 as well, and is identically and independently distributed across individuals. When the true state is 0, the signal equals 0 with probability  $\beta$ . Similarly, it equals 1 with probability  $\beta$  when the true state is 1. Because the two states are equally likely *ex ante*, Bayes rule implies that the conditional probability that the true state is 0 when the signal is 0 equals  $\beta$  as well. By appropriate choice of signal labels,  $\beta > .5$  so that observing a signal of 0 makes it more likely that the true state is 0 and that informed supporters of  $x^a$  prefer candidate  $m$ .

Suppose that the group of individuals who know which candidate they prefer vote for that candidate, if they vote at all.<sup>16</sup> After the election, the actions of these voters become known so that the true state is revealed. This means that the people who voted under uncertainty learn whether they cast the same vote as that of the informed voters that agree with them. To maximize the probability that these votes are indeed the same, an individual who prefers  $x^a$  and observes  $s = 0$  votes for  $m$  if he votes at all. Analogously, supporters of  $x^a$  that observe  $s = 1$  vote for  $n$  if they vote at all. The votes of uninformed supporters of  $x^b$  are, as a function of  $s$ , the mirror image of those of  $x^a$  supporters.

Individual  $i$  cares about voting in this model because his vote affects the expected distance  $E_i^1 D_j$  of those who agree with him. As we saw above, the exact value of  $E_i^1 D_i$  as a function of the number of votes cast depends on whether individual  $i$  has voted (because a vote by individual  $i$  does not change his estimate of the views of other people). This dependence is not important for measuring the effect of other peoples' votes on an individual's welfare so that it suffices to focus only on the  $E_i^1 D_i$  of an abstainer. The  $E_i^1 D_i$ 's of voters behave 

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prefer one candidate, say because this candidate is associated with a political party. A thorough analysis of parties, however, would have to cover situations where party loyalty differs across individuals. Such an analysis is well beyond the scope of this paper.

<sup>16</sup>As discussed earlier, groups that have a chance of winning an election want to do this and we would expect them to achieve this goal if they can nominate candidates and communicate with one another.

similarly. Either depends on  $i$ 's subjective probability assessment that he agrees with people who have abstained, as well as on his assessment that he agrees with people who have voted for  $m$  or  $n$ .

Consider first state 0 and use  $P(j|m, 0)$ ,  $P(j|n, 0)$  and  $P(j|A, 0)$  to denote, respectively, the posterior probability in state 0 that someone who has voted for  $m$  prefers  $x^j$ , that someone who has voted for  $n$  does so and that an abstainer chosen at random does so. There are three types of voters for  $m$ , namely the informed supporters of  $x^a$ , the uninformed supporters of  $x^a$  who drew the “correct” signal  $s = 0$  and the uninformed supporters of  $x^b$  which drew the “incorrect” signal  $s = 1$ . This means that, using Bayes rule

$$P(a|m, 0) = \frac{p(\gamma z^a + (1 - \gamma)\beta w^a)}{p(\gamma z^a + (1 - \gamma)\beta w^a) + (1 - p)(1 - \gamma)(1 - \beta)w^b} \quad (14)$$

where  $z^i$  is the turnout rate of informed people that support  $x^i$  while  $w^i$  denotes the turnout rate of their partially informed counterparts.

Using the same logic, the probabilities that voters for  $n$  and abstainers support  $x^a$  are

$$P(a|n, 0) = \frac{p(1 - \gamma)(1 - \beta)w^a}{(1 - p)(\gamma z^b + (1 - \gamma)\beta w^b) + p(1 - \gamma)(1 - \beta)w^a} \quad (15)$$

$$P(a|A, 0) = \frac{p(1 - \gamma z^a - (1 - \gamma)w^a)}{1 - \gamma(z^b + p(z^a - z^b)) - (1 - \gamma)(w^b + p(w^a - w^b))} \quad (16)$$

In state 0, the change in  $E_i^1 D_i$  for a supporter of  $x^a$  when the number of votes for  $m$  rises by one unit while the number of abstentions declines by one unit is  $\delta_0^a$ ,

$$\delta_0^a \equiv -P(a|m, 0) + P(a|A, 0) \quad (17)$$

while the corresponding change when there is an additional vote for  $n$  and one less abstention is  $\delta_1^a$

$$\delta_1^a \equiv -P(a|n, 0) + P(a|A, 0) \quad (18)$$

In these definitions of the  $\delta$ 's, a 0 subscript indicates that there has been an additional vote that is compatible with those of informed voters (*i.e.* a “favorable” vote) while a 1 indicates the addition of an incompatible one. The gains in  $y_i$  for supporters of  $x^a$  when

they see a single abstention replaced by either a favorable or by an unfavorable vote are then, respectively,  $-S'\delta_0^a$  and  $-S'\delta_1^a$ .

The conditional probability that, in state 0, a voter prefers  $x^b$  given that he votes for  $m$ ,  $n$  or that he abstains are given respectively by the probabilities that are complementary to  $P(a|m, 0)$ ,  $P(a|n, 0)$  and  $P(a|A, 0)$ . This means that the change in  $E_i^1 D_i$  for a supporter of  $x^b$  when the number of voters for  $m$  rises by one unit while the number of abstainers declines by one unit is  $\delta_1^b$ ,

$$\delta_1^b = -\delta_0^a$$

while the corresponding change when there is an additional vote for  $n$  and one less abstention is  $\delta_0^b$

$$\delta_0^b = -\delta_1^a$$

Supporters of  $x^b$  thus see their  $y_i$  rise by  $S'\delta_0^a$  or  $S'\delta_1^a$  respectively when a single abstention is replaced by either a favorable or by an unfavorable vote.

In the special case where  $\beta = 1$  (so that everyone has full information about the connection between the  $x^j$ 's and the candidates) one would expect the turnout rates  $z^j$  to equal the corresponding turnout rates  $w^j$ . This leads to  $\delta_1^a = \tilde{p} = \delta_0^a - 1$  where  $\tilde{p}$  is given by (4). The gains in utility by supporters of  $x^j$  from seeing additional votes that agree with their own are thus the same as in Section 1.

State 1 differs from state 0 only in that the roles of candidates  $m$  and  $n$  are reversed. This means that

$$P(j|m, r) = P(j|n, 1 - r), \quad P(j|A, 0) = P(j|A, 1) \quad j = a, b; r = 0, 1$$

This implies that the gains to a supporter of  $x^a$  from seeing an additional favorable vote continue to equal  $\delta_0^a$  except that a favorable vote now corresponds to a vote for  $n$  rather a vote for  $m$ . With an analogous reinterpretation, the utility gains that stem from favorable and unfavorable votes are the same as before for both supporters of  $x^a$  and for supporters of  $x^b$ .

At the moment of voting, people have just as much information about the expected number of people that prefer  $x^a$  as they did in section 1. This means that a supporter of  $x^a$  expects  $p(N - 1)$  individuals to agree with him and this is the value of  $E_i^0 M_i^a$ . Importantly, individuals who only observe  $s$  and are otherwise uninformed about the true state, do not expect  $E_i^1 M_i^a$  to depend on whether the true state turns out to be 1 or 0. In other words, the number of people that they expect to agree with them is independent of whether they drew a correct or an incorrect signal of the state. Similarly,  $E_i^0 M_i^b$  continues to equal  $(1 - p)(N - 1)$  and uninformed individuals do not imagine that this expectation depends on the correctness of the signal they observed.

Consider the changes in  $E_i^0 u_i$  that result from voting where, as before, the term  $-d^e(x_i, \bar{x})$  in (2) is ignored. These gains differ depending on whether the individual is informed, so that his vote is sure to be seen as favorable by those that agree with him, or whether he is not. In the case where a supporter of  $x^a$  is informed, his vote is sure to be seen as favorable regardless of the state. This means that, by voting, he increases his  $E_i^0 u_i$  by

$$\lambda_0 S'(N - 1)p\delta_0^a - c_i. \quad (19)$$

Meanwhile, voting increases the  $E_i^0 u_i$  of an informed individual that support  $x^b$  by

$$\lambda_0 - S'(N - 1)(1 - p)\delta_1^a - c_i. \quad (20)$$

Uninformed individuals have a probability  $\beta$  of observing a signal that is identical to the true state. This means that their expected gains from voting equal  $\beta$  times the increase in  $E_i^0 u_i$  that results when their signal is correct plus  $(1 - \beta)$  times the increase in  $E_i^0 u_i$  that comes about when their signal is incorrect. Thus, the expected utility gains from voting for an uninformed supporter of  $x^a$  are

$$\lambda_0 S'(N - 1)p[\beta\delta_0^a + (1 - \beta)\delta_1^a] - c_i. \quad (21)$$

Similarly, the effect of voting on the  $E_i^0 u_i$  of an uninformed individual that supports  $x^b$  is

$$\lambda_0 S'(N - 1)(1 - p)[\beta\delta_1^a + (1 - \beta)\delta_0^a] - c_i. \quad (22)$$

An equilibrium is a set of four maximum values of  $c_i$ , one each for informed and uninformed individuals that support both  $x^j$ 's such the people whose voting costs are below these maximum values vote and the rest abstains. These maximum values must ensure that equations (19), (20), (21) and (22) equal zero when the turnout rates  $z$  and  $w$  used to compute  $\delta_0^a$  and  $\delta_1^a$  correspond to these maximal voting costs. Using (17) and (18), the resulting equilibrium conditions are

$$F^{-1}(z^a) = \lambda_0 S'(N-1)p[P(a|m, 0) - P(a|A, 0)] \quad (23)$$

$$F^{-1}(z^b) = \lambda_0 S'(N-1)(1-p)[P(a|A, 0) - P(a|n, 0)] \quad (24)$$

$$F^{-1}(w^a) = \lambda_0 S'(N-1)p[\beta P(a|m, 0) + (1-\beta)P(a|n, 0) - P(a|A, 0)] \quad (25)$$

$$F^{-1}(w^b) = \lambda_0 S'(N-1)(1-p)[P(a|A, 0) - \beta P(a|n, 0) - (1-\beta)P(a|m, 0)] \quad (26)$$

As before, substantial turnout at the solution to these equations ensures that these solutions are essentially indistinguishable from those that result from including the  $-d^e(x_i, \bar{x})$  terms in (2). Moreover, by continuity, the equilibria that satisfy these four equations continue to have substantial turnout by both informed and uninformed individuals as long as  $\beta$  is close to 1.

Unfortunately, computing and characterizing these equilibria is more complicated for the case  $\beta < 1$  because, unless  $p = .5$ , there do not exist equilibria with  $z^a = z^b$  and  $w^a = w^b$ . To see this, suppose  $z^a = z^b = z$  and  $w^a = w^b = w$  and divide the left and right hand sides of 23 by the corresponding sides of (24). Taking into account that the equality of turnout rates implies that  $P(a|A, 0) = p$  and rearranging yields

$$\frac{F^{-1}(z^a)}{F^{-1}(z^b)} = \frac{p(1-p)(\gamma z + (1-\gamma)\beta w) + p^2(1-\gamma)(1-\beta)w}{p(1-p)(\gamma z + (1-\gamma)\beta w) + (1-p)^2(1-\gamma)(1-\beta)w}$$

This equation clearly cannot hold with  $z^a = z^b = z$  unless  $p = .5$ . It is easy to provide intuition for this impossibility. With equal turnout rates, the probability that an abstainer prefers  $x^a$  is the same as the unconditional probability that he prefers  $x^a$ , namely  $p$ . Even with equal turnout rates, lack of full information reduces the conditional probability that someone who votes for the candidate favored by informed supporters of  $x^j$  actually supports

$x^j$ . This reduction in conditional probability is the same if  $p = 1 - p$  because, then, the fraction of the people who voted for a candidate by mistake is the same for both candidates. However, if  $p > 1 - p$  and turnouts are the same for both supporters, there are more supporters of  $x^a$  making mistakes than supporters of  $x^b$  doing so, and this means supporters of  $x^b$  ought to find voting less attractive (since it conveys less information about the support of  $x^b$ ).

To simplify the analysis, let  $p = .5$  and consider the symmetric equilibrium where  $z^a = z^b = z$  and  $w^a = w^b = w$ . By continuity, the qualitative conclusions obtained for this case ought to carry over to cases where  $p$  is slightly different from  $1 - p$ . The main objective of this analysis is to show that the turnout rate of less informed voters,  $w$  is smaller than the turnout rate of the more informed ones.

In the symmetric case, (23) and (25) become

$$F^{-1}(z) = \lambda_0 S'(N-1)p \left[ \frac{\gamma z + (1-\gamma)\beta w}{\gamma z + (1-\gamma)w} - .5 \right] \quad (27)$$

$$F^{-1}(w) = \lambda_0 S'(N-1)p \left[ \frac{\gamma\beta z + (1-\gamma)(\beta^2 + (\beta-1)^2)w}{\gamma z + (1-\gamma)w} - .5 \right] \quad (28)$$

For  $.5 < \beta < 1$ ,  $\beta^2 + (\beta-1)^2$  is less than  $\beta$  so the right hand side of the second equation is smaller than the right hand side of the first one for any positive  $z$  and  $w$ . This means that any positive  $w$  and  $z$  combination that solves this equation has  $w < z$ . Given that  $w < z$ , the first of these equations also implies that the equilibrium value of  $z$  with  $\beta < 1$  is lower than that which results when everyone is fully informed about the relationship between the  $x^j$ 's and the candidates.

The intuition for this second result is that the existence of individuals whose votes do not reveal their stance on  $x^j$  implies that voting is less useful even for informed individuals as a mechanism for communicating political attitudes. This effect also tends to reduce the turnout of uninformed individuals because their voting message is garbled even when their signal  $s$  leads them to vote for the same candidate as the informed individuals that agree with them. Uninformed individuals reduce their turnout further because they also know that they have a chance of voting for a candidate that is not the one that is chosen by

the informed people who agree with them and that, in this case, their vote is particularly uninformative.

It is worth noting that, if uncertainty is sufficiently acute that  $\beta = .5$ , the model predicts that individuals with even a trivial cost of voting would abstain. To see this, note that the right hand side of (28) is zero when  $\beta = .5$ . In this extreme case, the model can thus account for rolloff, where an individual who is already voting for a major candidate decides to abstain in a different race so as to avoid the cost of filling in the appropriate ballot entry.<sup>17</sup> This fits with the evidence of Wattenberg *et al.* (2000) that uninformed individuals are more likely to rolloff and that rolloff is smaller in states like Indiana where voters can vote for straight “party tickets” with a single punch or pull of a lever.

## 4 The Effects of a Third Candidate on Turnout and Voting

Even though most of this paper has focused on just two candidates, it is important to recognize that some of the most interesting empirical observations concerning voting arise in situations with more candidates. Lacy and Burden (1999) show that, in the 1992 U.S. presidential race, some of the people who voted for Ross Perot were highly likely to abstain if Perot had not been running. Other Perot voters, by contrast, would have voted for Bill Clinton if Perot had not been an official candidate. This leads Lacy and Burden (1999) to view Ross Perot’s entry into the presidential race as having led to both increased turnout and to “vote-stealing” from candidates with more votes. As discussed in the introduction, many Democrats claimed that Ralph Nader engaged in much more dramatic vote stealing in the 2000 U.S. presidential race.

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<sup>17</sup>One obvious question raised by this analysis is whether rolloff becomes more common as the election becomes less close because  $p$  differs from .5.

Vote stealing from viable candidates by non-viable candidates is in some sense the opposite of “strategic voting,” which refers to a situation where candidates with some chance of getting elected receive votes from individuals who actually prefer a less viable candidate. Strategic voting has been widely studied in the empirical literature, and evidence for this phenomenon has sometimes been treated as evidence for models where people vote in the hope of being pivotal. One aim of this section is to show that certain observations that resembles strategic voting can arise in models where the motivation for voting is quite different. This leaves the question of whether other observations involving third candidates are or are not consistent with the model.

The most stripped down model where these issues can be discussed has three potential candidates, which are labelled  $m$ ,  $n$  and  $\ell$  and four types of individuals, which are labelled  $a$ ,  $b$ ,  $g$  and  $h$ . As before, the favorite positions of individuals of types  $a$  and  $b$  are  $x^a = x^m$  and  $x^b = x^n$  respectively. Individuals of type  $g$  and  $h$  both prefer the position of candidate  $\ell$ ,  $x^\ell$  to either  $x^m$  or  $x^n$ . Where  $g$  and  $h$  differ is in the extent to which they see  $x^a$  as being far from their own position. Individuals of type  $g$  dislike  $x^a$  enough to derive no utility from the existence of individuals who prefer  $x^m$  to  $x^n$ . By contrast, individuals of type  $h$  derive self-satisfaction from knowing that the number of individuals who prefer  $x^a$  to  $x^b$  is large. By the same token, individuals of type  $h$  feel some altruism towards individuals of type  $a$  whereas individuals of type  $g$  do not.

This means that, in an election with only  $m$  and  $n$  running for office, individuals of type  $g$  abstain whereas some individuals of type  $h$  vote for  $m$ . The issue is then how these individuals vote when  $\ell$  runs as well. A vote for  $m$  can be interpreted as “strategic voting” since people are not voting for their preferred candidate. A vote for  $\ell$ , by contrast, can be interpreted as “vote stealing”. As a matter of logic, it is possible for both strategic voting and vote stealing to take place simultaneously if some individuals of type  $h$  vote for  $\ell$  while others vote for  $m$ .

Let  $p^j$  represent the probability that an individual chosen at random is of type  $j$  and let  $z^j$  represent the turnout rate of this type. Let  $d_{ij}$  be the distance  $d(x_i, x_j)$  of individuals of type

$i$  and  $j$  and suppose again that  $d_{ii} = 0$ . The disagreements among  $a$ ,  $b$  and  $g$  can be captured by supposing that  $d_{ab} = d_{ag} = d_{bg} = 1$ . Types  $h$  and  $b$  are equally in disagreement so that  $d_{bh} = 1$ . Given that  $g$  and  $h$  do not agree on everything, one might suppose that there is some distance between them, but this needlessly complicates the analysis, so  $d_{gh} = 0$ . Lastly, the partial agreement between  $a$  and  $h$  can be captured by letting  $0 < d_{ah} < 1$ . Suppose, as in the earlier baseline with no animus that  $\lambda(1) = 0$ , and denote  $\lambda(0)$  by  $\lambda_0$  while  $\lambda_{ag}$  denotes  $\lambda(d_{ag})$ . The parameters  $d_{ag}$  and  $\lambda_{ag}$  determine whether there is strategic voting and/or vote stealing in equilibrium.

Before carrying out this analysis, it is worth stating the benefits of voting for the various types in some generality. Consider only pure strategies where, if any individual of a type votes for candidate  $j$ , then all individuals of this type that vote also vote for  $j$ .<sup>18</sup> With pure strategies, there is a set  $V_j$  of types that vote for candidate  $j$ , and each type belongs to only one such set. If the set of types that votes for a particular candidate  $j$  consists of a single type  $i$ , then the probability that an individual is of type  $i$  given that he has voted for candidate  $j$ ,  $P(i|j)$  is equal to one, and  $P(k|j) = 0$  for all  $k \neq i$ . More generally

$$P(i|j) = \frac{p^i z^i}{\sum_{k \in V_j} p^k z^k}$$

For an individual who abstains, the probability that he is of type  $i$  is

$$P(i|A) = \frac{p^i (1 - z^i)}{\sum_k p^k (1 - z^k)}$$

The change in  $E_i^1 D_i$  for a member of type  $j$  when someone other than  $i$  himself moves from abstaining to voting for candidate  $k$  is  $\delta_k^j$  where

$$\delta_k^j = \sum_i d_{ij} [P(i|k) - P(i|A)]$$

The effect on the total utility of an individual of type  $i$  of voting for candidate  $k$  is thus

$$-(N-1)S' \sum_j \lambda(d_{ij}) p^j \delta_k^j - c \tag{29}$$

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<sup>18</sup>The extension to mixed strategies is straightforward.

where  $c$  is the individual's cost of voting. This expression takes on different values for different candidates  $k$ . For given turnouts of others', each individual of type  $i$  would then pick the candidate that maximizes (29). Define  $G^i$  as the quantity which ensures that  $G^i - c$  is equal to the maximized value of (29). This ensures that, as before,  $G^i$  is the benefit of voting for an individual of type  $i$ .

These formulas can easily be applied to the particular parameters that are of interest in this section. The sets  $V_i$  depend on whether there are two or three candidates running. In the case of two candidates  $n$  and  $m$ ,  $V_n$  includes just type  $b$  while  $V_m$  includes both types  $a$  and type  $h$ . Using the formulae above, the benefits of voting are thus

$$G^a = S'(N-1) \left( \lambda_0 p^a \left[ \frac{p^b(1-z^b) + p^g(1-z^g) + d_{ag}p^h(1-z^h)}{1 - \sum p^j z^j} - \frac{d_{ag}p^h z^h}{p^a z^a + p^h z^h} \right] + \lambda_{ag} p^h \left[ \frac{p^b(1-z^b) + d_{ag}p^a(1-z^a)}{1 - \sum p^j z^j} - \frac{d_{ag}p^a z^a}{p^a z^a + p^h z^h} \right] \right) \quad (30)$$

$$G^b = S'(N-1) \lambda_0 p^b \left( 1 - \frac{p^b(1-z^b)}{1 - \sum p^j z^j} \right) \quad (31)$$

$$G^g = 0 \quad (32)$$

$$G^h = S'(N-1) \left( \lambda_{ag} p^a \left[ \frac{p^b(1-z^b) + p^g(1-z^g) + d_{ag}p^h(1-z^h)}{1 - \sum p^j z^j} - \frac{d_{ag}p^h z^h}{p^a z^a + p^h z^h} \right] + \lambda_0 \left[ \frac{(p^g + p^h)p^b(1-z^b) + (p^g + d_{ag}p^h)p^a(1-z^a)}{1 - \sum p^j z^j} - \frac{(p^g + d_{ag}p^h)p^a z^a}{p^a z^a + p^h z^h} \right] \right) \quad (33)$$

In equilibrium, these benefits must equal the maximum costs of voting  $F^{-1}(z^a)$ ,  $F^{-1}(z^b)$ ,  $F^{-1}(z^g)$  and  $F^{-1}(z^h)$  respectively. One obvious property of the equilibrium is that, with even trivial costs of voting,  $z^g = 0$  and no individuals of type  $g$  votes.

Equations (30) and (33) have important implications for coalitions because they concern the voting behavior of individuals that agree on their preferred candidate but do not fully agree with each other. This lack of agreement tends to reduce turnout, even holding constant the altruism that the groups have for each other. This can be seen by noting that, for sufficiently high  $z^a$  and  $z^h$ , increases in  $d_{ag}$  reduce  $G^a$  and  $G^h$ . Insofar as these groups see each other as having attitudes that are some distance from each other, an additional vote for candidate  $m$  contains a smaller element of good news about  $D$  for either supporters of  $x^a$  or of  $x^h$ .

In addition, turnout among voters for  $m$  depends on the extent to which supporters of  $x^a$  and  $x^h$  like each other. Consider the case where  $d_{ag}$  is sufficiently small that the terms in square brackets in both (30) and (33) are positive. It then follows that reductions in  $\lambda_{ag}$ , which indicate that the coalition members like each other less, reduce  $G^a$  and  $G^h$ .

When there are three candidates running, the formulae giving the gains from voting depend on whether people of type  $h$  vote for  $m$  or for  $\ell$ . In the former case, the formulae above for  $G^a$ ,  $G^b$  and  $G^h$  remain valid but (32) is replaced by

$$G^g = S'(N-1)\lambda_0(p^g + p^h) \left( 1 - \frac{p^g(1-z^g) + p^h(1-z^h)}{1 - \sum p^j z^j} \right) \quad (34)$$

This leads  $z^g$  to be positive. The introduction of a candidate that agrees somewhat with type  $g$  leads some individuals of this type to vote. This fits with the finding of Lacy and Burden (1999) that, in the 1992 elections, some voters would have abstained if Ross Perot had not been a candidate.

One interesting implication of the analysis is that this does not exhaust the turnout consequences of entry by a third candidate. To see this, note that even if all other types keep their turnout constant, the increased voting by type  $g$  lowers  $1 - \sum p^j z^j$ , so that the right hand side of (31) falls. By itself, this effect tends to reduce the turnout of individuals of type  $b$ . The intuition for this effect is that increased turnout by type  $g$  individuals leads the abstention pool to contain fewer  $g$ 's so that the share of type  $b$  individuals in this pool rises. Thus, the move by a type  $b$  individual from abstention to voting for  $n$  constitutes a smaller dose of good news for other type  $b$  individuals and so this move becomes less attractive.

The introduction of candidate  $\ell$  has additional effects, since changes in the turnout of type  $b$  individuals affect the behavior of other types and this, in turn, feeds back into  $z^b$ . What the analysis does show is a more general phenomenon, namely that the model predicts that the turnout of individuals who do not support the third candidate is also affected by the entry of this candidate into the race. Whether this is important in actual elections remains a question for empirical research.<sup>19</sup>

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<sup>19</sup>It is straightforward to show that the model is consistent with having this effect be arbitrarily small if

When there are three candidates running, it is possible that all type  $h$  individuals vote for  $\ell$ . When this happens, the benefits to type  $g$  individuals from voting still satisfy (34) while those to type  $b$  individuals are still given by (31). For type  $g$  individuals the benefits are different because they are pooling with type  $h$  individuals rather than with those of type  $a$ . This also affects the benefits of type  $a$  individuals because their vote for  $m$  now conveys a less ambiguous message.

$$G^a = S'(N-1) \left[ \lambda_0 p^a \frac{p^b(1-z^b) + p^g(1-z^g) + \hat{\delta} p^h(1-z^h)}{1 - \sum p^j z^j} + \lambda_{ag} p^h \left( \frac{p^b(1-z^b) + \hat{\delta} p^a(1-z^a)}{1 - \sum p^j z^j} - d_{ag} \right) \right] \quad (35)$$

$$G^h = S'(N-1) \left[ \lambda_0 \frac{(p^g + p^h) p^b(1-z^b) + (p^g + \hat{\delta} p^h) p^a(1-z^a)}{1 - \sum p^j z^j} + \lambda_{ag} p^a \left( \frac{p^b(1-z^b) + p^g(1-z^g) + \hat{\delta} p^h(1-z^h)}{1 - \sum p^j z^j} - \frac{p^g z^g + d_{ag} p^h z^h}{p^g z^g + p^h z^h} \right) \right] \quad (36)$$

Solving the equilibrium conditions for these three settings analytically is difficult because four nonlinear equations are involved. To keep the analysis manageable, it focuses on the conditions under which no equilibrium exists where all type  $h$  individuals who vote cast their ballot for  $m$  and on the conditions under which no equilibrium exists where all type  $h$  individuals who vote do so in favor of  $\ell$ .

Consider first a proposed equilibrium where all voters of type  $h$  vote for  $m$ . The gain to a single such individual from changing his vote equals the  $G^h$  in (36) with  $z^h$  replaced by zero (because we start at a proposed equilibrium where voters of type  $h$  vote for  $m$ ) minus the  $G^h$  in (33). This gain is thus proportional to

$$\frac{\lambda_0 (p^g + d_{ag} p^h) p^a z^a + \lambda_{ag} p^a d_{ag} p^h z^h}{p^a z^a + p^h z^h} - \lambda_{ag} p^a = \frac{p^a z^a}{p^a z^a + p^h z^h} \left[ \lambda_0 (p^g + d_{ag} p^h) - \lambda_{ag} p^a \left( 1 + \frac{(1 - d_{ag}) p^h z^h}{p^a z^a} \right) \right] \quad (37)$$

$f$  is arbitrarily small at  $z^*$ , where  $z^*$  is defined in (??symz). When  $f$  is small in this neighborhood, changes in the right hand side of (30), (31) or (34) induced by changes in  $z^g$  are consistent with  $z^a$  and  $z^b$  remaining near  $z^*$ .

The first term on the LHS is the cost of casting a vote for  $m$  knowing that this will be interpreted by both  $g$  and  $h$  supporters as possibly indicating the individual is of type  $a$  while the second is the cost of casting a vote for  $\ell$  knowing that this will be interpreted by people of type  $a$  as proving that the person is of type  $g$ . The expression on the RHS is obtained by rearranging terms.

Now consider a proposed equilibrium where all voters of type  $h$  vote for  $\ell$ . An individual who deviates by voting for  $m$  gains the  $G^h$  in (30) evaluated at  $z^h = 0$  and loses the  $G^h$  in (36). His net gain is thus proportional to

$$\lambda_{ag} p^a \frac{p^g z^g + d_{ag} p^h z^h}{p^g z^g + p^h z^h} - \lambda_0 (p^g + d_{ag} p^h) \quad (38)$$

The first term on the LHS is now the cost of casting a vote for  $\ell$  given that this might be interpreted as coming from type  $g$  while the second is the cost of having one's vote for  $m$  interpreted as coming from type  $a$ .

The analysis is simplified in the limit where  $d_{ag} = 1$ . In this limit, if  $\lambda_{ag} p^a > \lambda_0 (p^g + p^h)$ , (38) is positive while (37) is negative. The first inequality implies that there is no equilibrium with complete “vote stealing”, agents of type  $h$  are sufficiently fond of agents of type  $a$ , and the latter are sufficiently numerous relative to the supporters of  $\ell$  that an individual  $h$  deviates from voting for  $\ell$ . At the same time, the second inequality implies that individuals of type  $h$  do not deviate from an equilibrium where they all vote for  $m$  so that there is an equilibrium with complete strategic voting.

If, in the limit where  $d_{ag} = 1$ ,  $\lambda_{ag} p^a < \lambda_0 (p^g + p^h)$  instead, (38) is negative while (37) is positive. There is then no equilibrium where all members of type  $h$  engage in “strategic voting” in the sense of voting for the “major” candidate  $m$ . On the other hand, there is now no reason to deviate from an equilibrium where the third candidate  $\ell$  has “stolen” all the  $h$  votes that would have gone to  $m$  if there had only been two candidates. Since no voter can plausibly think of himself as pivotal in this setting, this can cost  $m$  the election.

The case where  $d_{ag}$  is strictly less than 1 is more complicated. One immediate implication, however, is that the equality of  $\lambda_{ag} p^a$  and  $\lambda_0 (p^g + d_{ag} p^h)$  now implies that both (37) and

(38) are negative. Indeed, for given  $\lambda_0$ ,  $p^g$  and  $p^h$ , there is now a nontrivial region of values of  $\lambda_{ag}p^a$  such that both expressions are negative so that type  $h$  individuals do not have an incentive to deviate from either an equilibrium where they all vote for  $m$  or from an equilibrium where they all vote for  $\ell$ . For larger values of  $\lambda_{ag}p^a$ , the equilibrium where all  $h$  vote for  $\ell$  ceases to exist, while the equilibrium with pure strategic voting no longer exists when  $\lambda_{ag}p^a$  is sufficiently low. The reason for the multiplicity of equilibria in the intermediate region is, once again, that an increase in the number of agents of a type that take an action makes this action more palatable to other agents. As more type  $h$  individuals vote for  $m$ , for example, such a vote looks more attractive both to type  $h$  and to type  $g$  individuals.

Broadly speaking, these results imply that votes by  $h$  for  $m$  are more likely the larger is the support for  $a$  ( $p^a$ ), the smaller is the support for  $\ell$  ( $p^g + d_{ag}p^h$ ) and the less agents of type  $h$  care for  $g$  rather than for  $a$  ( $\lambda_0/\lambda_{ag}$ ). The first two effects are broadly consistent with results in the empirical literature on strategic voting. Abramson *et al.* (1992) run a regression explaining the likelihood that individuals plan to vote for their preferred candidate and show that this is increasing in the extent to which respondents perceive this candidate to have a chance to win. Naturally, candidates have a higher chance to win if their support is higher so this is tantamount to saying that type  $h$  respondents are more likely to vote for  $\ell$  the higher are  $p^g$  and  $p^h$  relative to  $p^a$ .

Alvarez and Nagler (2000) run a regression that is similar in spirit, though they do not use survey responses either about voting intentions or about feelings for candidates. Rather, they ascribe preferred candidates to individuals based on their observable characteristics and look at actual votes in individual districts. They show that, conditional on a voter's characteristics, an individual is more likely to vote for a party if it has a chance of winning in the sense that its own popularity is not low. This fits with the idea that  $\ell$  is more likely to lose votes if  $p^g$  and  $p^h$  are low.

Conditional on a party having a low popularity (so that it is unlikely to win), Alvarez and Nagler (2000) show that its vote total declines further if the gap between the leading parties is small. This second finding is related but not identical to the model's prediction

that  $m$  tends to receive votes from  $h$  when  $p^a$  is substantial.<sup>20</sup> It is thus worth noting that the model's predictions concerning when individuals will fail to vote for their favorite candidate are not identical to those of the pivotal voter, though they are related in the sense that voters abandon weak candidates and support strong ones. Whether the specific implications of this model are borne out in constituencies with more than two candidates is a topic deserving further research.

## 5 Conclusion

The model of voting in this paper is both derived from assumptions about human psychology that have some empirical support and predicts patterns of voting that fit with some existing empirical evidence. It is important to stress, however, that the model's assumptions and predictions could both be subject to much sharper tests than those that have already been carried out in the literature. Indeed, one of the principal strengths of the model is that seems to be possible to check not only its qualitative predictions but also some of its quantitative ones.

This would be particularly true if estimates could be obtained of the distribution of voting costs as well as of the parameters concerning the interdependence of preferences. In the absence of independent information about these parameters, it might still be possible to analyze the quantitative implications of the model across elections in which these parameters remain constant. Even for a given election, the model often implies that turnout rates of

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<sup>20</sup>The difference is that the gap between the leading parties would also be large if  $p^a$  were large and  $p^b$  were small, and the model would not predict low votes for  $m$  in this case. Since the districts in question are ones where the support for the party in question is small, however, it seems unlikely that these are frequently districts where the support for the party that is relatively similar to the party in question is extremely large. Thus, lopsided districts where the party in question receives low votes are likely to be ones where the similar party can expect low support as well.

individuals ought to differ as a function of individual characteristics and this seems directly amenable to study.

Specifically, the model implies that turnout rates of individuals depend not only on how much they like a particular candidate but also on how much they like the other supporters of the candidate. Since voting is geared at increasing the utility of other supporters, an individual ought to be more likely to vote for his favorite candidate the more he likes the candidate's other supporters. It remains to be studied whether this means that candidates for office can receive more total votes when they receive less support from groups that other supporters dislike.

A related area that deserves more attention is the effect of institutional arrangements on voting patterns. Cox (1997) displays some remarkable differences in voting institutions and shows that these institutions do seem related both to turnout and to the viability of third parties. It would thus be worthwhile to inquire whether a model of the sort considered here can account for these differences.

## 6 References

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