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ABSTRACT

The Impact of Changing Demographics and Pensions on The Demand for Housing and Financial Assets*

Using a calibrated OLG model with several sources of uncertainty we find that the impact of ageing and of reform of social security upon the demand for housing and the level of owner occupation is substantial. The overall structure of household asset holdings – in particular the split between real and financial assets – is sensitive to demographics and to the generosity of state run, pay-as-you-go pensions. The interaction between social security reform and housing market conditions is significant and suggests that any changes in pension rules will have substantial knock on effects on the housing market.

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1 Introduction

This paper explores the link between housing markets and the portfolio allocation of households in a world of changing demographics and shifting pension arrangements. We investigate optimal decisions over saving, portfolio allocation and housing tenure in an environment where there are several assets to choose from, options to rent or buy a house and where there are multiple sources of uncertainty. Several stochastic, calibrated models have studied the impact of uncertainty both in incomes of individual households (which is largely idiosyncratic) and in the rates of returns on some assets on life cycle choices. Very few models, however, allow for the special role of housing in the economy; one exception is Cocco (2000). The main aim of this paper is to introduce housing into households' consumption–savings decisions. We embed the decision making of individual households within a model of overlapping generations to derive aggregate demand for various asset classes, allowing for feedbacks from demand changes onto house prices. We calibrate the model so that it broadly reflects features of the UK economy. We then use the model to assess how changing demographics and their interaction with pension reform can generate changes in savings, portfolio allocation, the overall demand for housing and the choice of housing tenure between ownership and renting.

It is very likely that the links between housing, shifting demographics and changes in the structure of state pensions are of great economic significance. Figure 1 shows a snapshot picture of the balance sheet of UK households at the end of 2003. Direct holdings of residential property (owner-occupied housing) made up just under 50% of a comprehensive measure of wealth that includes an estimate of the value of accrued PAYGO state pensions. Private pension savings makes up about 20% of all wealth. Other net financial assets are about 7% of net worth. The value of accrued claims to future government unfunded pensions (both state pensions and promises to public sector workers) make up about 25% of net household wealth. The scale of knock-on effects on portfolio allocation of demographic changes and shifts in pension arrangements is likely to be large. And because houses are hardly traded internationally - unlike claims on financial assets - the scope for induced impacts upon house values and rents is also large. But little analysis of the scale of these effects exists.

Using a standard OLG model with several sources of uncertainty we find that the impact of ageing and of reform of social security upon the demand for housing and the level of owner occupation is substantial. The overall structure of household asset holdings - in particular the split between real and financial assets - is sensitive to demographics and to the generosity of state run, pay-as-you-go pensions. The interaction between social security reform and housing market conditions is significant and suggests that any changes in pension rules will have substantial knock-on effects on the housing market.

1.1 Related literature

In the wake of the seminal work of Auerbach and Kotlikoff in the 1980's (see Auerbach and Kotlikoff, 1987) several stochastic, calibrated models based on life cycle optimization in an overlapping generations framework have been developed. There is now a substantial literature that uses calibrated models to address issues of risk allocation in a world with less than perfect risk sharing opportunities. Much of that literature focuses on the US economy – Hubbard and Judd (1987); Imrohoroglu and Imrohoroglu and Joines (1995); Storesletten, Telmer and Yaron (1999); and Campbell, Cocco, Gomes and Maenhout (2001) present results from models calibrated to the US economy. De Nardi, Imrohoroglu and Sargent (1999) and Kotlikoff, Smetters and Walliser (1999) look specifically at the transitional and long run impacts of phasing out unfunded, state pensions in the US. Miles and Černý (forthcoming) developed a simulation model of the Japanese economy to analyze the macroeconomic and microeconomic implications of ageing. That model allowed for the impact of various sources of uncertainty and assessed how the level of welfare of cohorts might evolve under different assumptions about the reform of social security.

All these models study the impact of uncertainty both in incomes of individual households (which is largely idiosyncratic) and of the rates of returns on some assets, but they do not allow for the special role of housing in the economy. Housing is an important asset in the portfolio of wealth of most developed economies. We calibrate our model to match features of the UK economy. The UK has a level of owner occupation of just under 70% - a level that is about average across developed economies. The total value of owner occupied housing in the UK economy in 2003 was around three times GDP and greater than the gross value of financial assets held by the household sector. The importance of housing in UK household balance sheets is not unusual - across most economies the value of residential property is greater than the value of net financial assets. Housing not only comprises a very large proportion of the net worth of households it is also a unique asset. Housing is a durable good which yields a substantial flow of real benefits to its owner - the implied value of owner occupation. It can also - and certainly in the UK has fairly consistently in the past forty years - generate real capital gains. Houses and appartments are also one of the few forms of wealth where the scope to borrow against the asset as collateral is significant. Clearly the characteristics of residential property are very different from equity investments and from investments in bonds or in cash.

We introduce housing into a calibrated OLG model in a way that allows for all the peculiar features of housing. We pay particular attention to: the volatility and uncertainty over house prices; the scope to borrow against housing as collateral; and to the role that housing services play in enhancing the utility of owner-occupiers.

We first develop a model designed to allow us to investigate how demographic change and the evolution of social security will impact portfolio holdings and housing choices for different generations. The key features of the calibrated model are that it allows for:

(a) uncertainty over individual incomes,

(b) stochastic rates of return on risky assets and for house prices,

(c) a pay-as-you-go pension system with pensions partly linked to incomes and which are also partly flat rate,

(d) some scope for households to borrow for house purchase,

(e) a rented sector providing an alternative to owner occupation,

(f) a bequest motive.

We calibrate the model so that it generates results which broadly match some (but not all) key features of households' decisions on saving, borrowing and housing choices (renting versus owner-occupation and the size of the house or apartment). We then use it to explore the impact of various policy changes upon portfolio choices, upon the demand for housing and upon the stock of wealth held by households, both within and between generations.

In the next section we describe the model structure. We then begin our description of the simulation results by focusing on results for existing households at a set of parameters that represents a stylized version of the economy. We look at household and aggregate results. We then look at alternative scenarios for the future where two types of pension reform are considered. These are:

1. Reductions in the generosity of the state pension on a scale to keep the budget-balancing contribution rate constant (which requires a gradual fall in the average replacement rate from around 30% in 2004 to under 20% by 2050)

2. A gradual but persistent rise in the retirement age - or the age at which the pay as you go pension is received.

2 The Model of Household Choices

Households enter the model at age 20 and do not survive beyond the age of 90. To simplify the notation $t = 0$ corresponds to the age of 20 and $T = 70$ corresponds to the age of 90 when all households still alive die with certainty. We assume that households are rational, forward-looking and solve a stochastic dynamic programming problem. They receive utility from consumption of goods, and from consumption of housing services. We assume that households make choices in each period of their lives about:

(a) how much to spend on consumption (C_t),

(b) whether to own a property or to rent (α_t^H),

(c) what size of property to live in (H_t),

(d) how to allocate financial wealth between safe assets (cash, denoted by B_t) and risky assets ("equity", denoted by S_t) where the latter are unpredictable but, on average, have a higher return,

(e) how much to borrow (D_t) against ownership of property (which is the only means to acquire debt and then is subject to various limits).

We assume there is a bequest motive. The assumption made about bequests is an important one in any model of saving and portfolio allocation. The impacts of reform to pay-as-you-go pension systems and the holding of residential property are likely to be sensitive to whether households have a bequest motive. How important bequests are, relative to life cycle smoothing motives, in accounting for overall levels of wealth holdings is controversial. Even in economies like Japan - where savings rates have been high and family bonds between generations are seen as strong – the bequest motive may not actually be very strong. Recent work of Horioka (2001) suggests that the bequest motive in Japan may be relatively weak. A survey of motives for saving undertaken by the Central Council for Financial Services (Bank of Japan) showed that under 4% of those surveyed listed bequests as a motive for saving (2003 fiscal year). It is not plausible that in the UK - where the household savings rate has been very much lower - the bequest motive is much higher than in Japan. So there is some justification for assuming that there is a relatively weak bequest motive. (Accidental bequests will, however, naturally arise in a model where annuities markets are less than perfect or where households have an incentive to own their own home while elderly. We find that if we assume that annuities markets are imperfect unintended bequests are important). So while we allow for a bequest motive we set it at a fairly modest level in the main simulations.

Households earn labour income y_t up to an exogenously given retirement age T^R. Labour income consists of a deterministic component $\bar{y}_t$, which is augmented by a stochastic element. The impact of stochastic shocks is persistent. In retirement agents rely upon accumulated savings and a state pension whose value depends partly on the salary earned at the end of the working life. We will set the tax rate levied upon labour earnings to balance the state pension system and will not allow deficit financing. Clearly, this is a very highly simplified version of the tax benefit system.

2.1 Preferences

The household *utility maximization problem* is given by

$$V_t = \max\left\{(1-\delta)\,U\left(C_t, H_t\right)^{1-\gamma} + \delta \mathrm{E_t}\left[l_t V_{t+1}^{1-\gamma} + (1-l_t)\,B\left(w_{t+1}\right)^{1-\gamma}\right]\right\}^{\frac{1}{1-\gamma}}. \quad (1)$$

The control variables and the budget constraint are detailed in the next subsection. V_t is the value function at time t, δ is a constant time discount factor and l_t is the probability of surviving period t, thus $(1-l_t)$ represents the probability of dying in period t. T is the last possible period of life (or maximum age to which one can live), so that $l_T = 0$. $B(w_{t+1})$ is a *bequest function* – a household receives utility from the knowledge that if they die during period t their heir receives wealth w_{t+1}. $U\left(C_t, H_t\right)$ is a *felicity function* defined by

$$U\left(C_t, H_t\right) = U_t = \left(\varkappa\left(\frac{C_t}{m_t}\right)^{1-\eta} + (1-\varkappa)\left(\frac{\xi H_t}{m_t}\right)^{1-\eta}\right)^{\frac{1}{1-\eta}}. \quad (2)$$

We allow for the preference for consumption to depend on the age of the household – this is reflected by the McClement's equivalence scale m_t. The reciprocal of η is the elasticity of substitution between housing and consumption of goods. This elasticity of substitution is different from the intertemporal elasticity of substitution, γ^{-1}. With these preferences γ also reflects risk aversion. $\varkappa$ and ξ are parameters reflecting the relative value of housing and consumption in the utility function. We will set these parameters so that the simulation results give a broadly plausible pattern of household decisions over the life cycle. With this standard time-separable expected utility model, the value function can be reformulated as

$$V_t^{1-\gamma} = \mathrm{E_t}\left(\sum_{j=0}^{T-t}(1-\delta)\,\delta^j l_t ... l_{t+j-1} U_{t+j}^{1-\gamma} + \sum_{j=0}^{T-t}\delta^{j+1} l_t ... l_{t+j-1}\left(1-l_{t+j}\right)B^{1-\gamma}\left(w_{t+j+1}\right)\right). \quad (3)$$

2.2 The budget constraint

The value of labour income plus net assets at the start of a period t (including ownership of housing from the previous period) is denoted by x_t – this is a comprehensive measure of current net wealth after receipt of labour income for the period ahead. If a households buys a property there is the option to borrow an amount up to a maximum level which we allow to depend on the value of the owned house. If the household rents it pays rent which is the product of the rental yield r_t^H and the value of the house. Denote the price per unit of housing by p_t, then the house value is equal to $p_t H_t$. After deciding about the level of consumption C_t, the size of the house H_t, mortgage D_t and safe investment B_t the amount of risky investment S_t is defined by one of two conditions depending on whether the household owns ($\alpha_t^H = 1$) or rents ($\alpha_t^H = 0$).

$$S_t = x_t + D_t - p_t H_t - C_t - B_t \qquad \text{when buying } (\alpha_t^H = 1), \tag{4}$$

$$S_t = x_t - r_t^H p_t H_t - C_t - B_t \qquad \text{when renting } (\alpha_t^H = 0). \tag{5}$$

All choice variables must be nonnegative ($C_t, H_t, B_t, D_t \geq 0$), so the condition $S_t \geq 0$ naturally forms the *budget constraint*.

Wealth held by a household consists of two parts: financial wealth w_t^F and housing wealth w_t^H

$$w_t = w_t^F + w_t^H. \tag{6}$$

Financial wealth includes both safe assets and risky assets (which we can think of as equity) and is net of debt

$$w_{t+1}^F = S_t \tilde{R}_{t+1} + B_t (1 + r_t) - D_t \left(1 + r_t^B\right), \tag{7}$$

$\tilde{R}$ is the stochastic real rate of return on risky assets (which we call "equities"). In this model we allow for differential real interest rate on borrowing – r_t^B and savings – r_t; specifically we assume that the interest charged on mortgage loans is 5% whereas risk-free deposits earn interest of 2.5%. Because of the difference in rates households will not simultaneously hold debt and safe assets; if $D_t > 0$ for a household it will not hold safe assets ($B_t = 0$) and if $B_t > 0$ it will not have a mortgage ($D_t = 0$). Investment in housing (or housing wealth) is defined as

$$w_{t+1}^H = \alpha_t^H p_{t+1} H_t. \tag{8}$$

Housing wealth equals zero only when the household is renting $\left(\alpha_t^H = 0\right)$. All in all, the dynamics of net worth is determined by the value of risky and non-risky investments, net of debt, plus the value of housing, current labour income and the amount of bequest received by the household

$$x_{t+1} = w^F_{t+1} + w^H_{t+1} + y_{t+1} + \text{bequest received}, \tag{9}$$

$$= \underbrace{S_t\tilde{R}_{t+1}}_{\text{equity}} + \underbrace{B_t\left(1+r_t\right)}_{\text{safe investment}} - \underbrace{D_t\left(1+r^B_t\right)}_{\text{debt service}} + \underbrace{\alpha^H_t p_{t+1} H_t}_{\text{housing wealth}} + \underbrace{\left(1-\tau\right) y_{t+1}}_{\text{after-tax income}} + \underbrace{w^B_t}_{\text{bequest}}, \tag{10}$$

2.3 The availability of credit

The borrowing capacity of a household is based on labour income and the value of collateral (the house or appartment). We assume that debt can be no greater than a fraction (λ_H) of the value of the house *and* debt repayments can be no larger than a given fraction (λ_y) of income. Debt repayments assume that borrowing is in the form of a 25 year fixed term repayment (i.e. amortising) mortgage. Debt must be repaid by retirement; the number of remaining years over which debt is to be repaid is denoted by $T^M_t = \min\left(25, T^R - t\right)$. Thus, the maximum amount the household can borrow is defined as

$$D_t \leq \min\left(\lambda_H p_t H_t,\ \lambda_y y_t \frac{1+r^B}{r^B}\left(1-(1+r^B)^{-(T^M_t+1)}\right)\right).$$

In setting $\lambda_H < 1$ we assume that the fraction of a house financed from saving can be no less than a given percentage. We take that percentage to be 30% until 1981 after which, as a result of financial liberalisation, it falls gradually over a ten year period to 10% (in other words the maximum loan to value ratio, λ_H, rises from 70% to 90% between the start of the 1980's and the end of the 1980's – which is roughly in line with the UK experience). We assume that since 1991 households can spend up to 25% of the annual labour income on mortgage repayment ($\lambda_y = 0.25$); before 1980 we set that ratio lower, at 17.5%, to reflect the rise in the availability of credit which occurred in the 1980's due to mortgage market liberalisation. The interest charged on mortgage loans is set to $r^B = 5\%$.

2.4 Bequest motive

Bequeathable wealth is defined as the sum of financial and housing wealth – w_t. It is always non-negative as the amount of borrowed funds are always collateralized by the value of property. The bequest function is

$$B(w_t) = \bar{\xi}_1 + \bar{\xi}_2 w_t, \tag{12}$$

where the parameters $\bar{\xi}_1$ and $\bar{\xi}_2$ reflect the strength of the bequest motive. When $\bar{\xi}_2 = 0$ there is no bequest motive, i.e. $B(w_t)$ is independent of w_t. We set the bequest motive at a moderate level: $\bar{\xi}_1 = 0.1, \bar{\xi}_2 = 1$. Thus the utility from a bequest depends on $(0.1 + w_{t+1})^{1-\gamma}$. Although we use a model with a relatively weak bequest motive this does not imply that bequests are unimportant. In this model there are no annuities and as a result there can be significant unintended bequests. (In fact we find that a large proportion of bequests are indeed unintended.)

2.5 Labour income and state pension

Pre-tax *labour income* y_t is the product of an underlying deterministic trend $\bar{y}_t$ and a stochastic component. The deterministic part is a function of age and it is calibrated to reflect the average lifetime earnings profile in the UK, peaking at the age of 50 at about twice the salary of a 20-year old (see Miles 1997). The deterministic profile also accounts for productivity growth – productivity is assumed to grow at rate g^P. The random component of income reflects idiosyncratic, persistent income shocks u_t:

$$\ln \frac{y_t}{\bar{y}_t} = u_t = \rho_u u_{t-1} + \varepsilon_t^u, \quad t \leq T^R, \tag{13}$$
$$\{\varepsilon_t^u\}_{t=1}^{T^R} \text{ are i.i.d.,} \qquad \varepsilon^u \sim N(0, \sigma_u^2).$$

We set $\rho_u = 0.98, \sigma_u = 0.11$, numbers which reflect the degree of volatility of UK labour incomes and the degree of persistence of shocks (Miles 1997).

State *pension income* is a combination of a final salary related component and a flat rate component. The flat rate component is defined as a fixed proportion θ of *average* retirement salary, $\bar{y}_{T^R} \exp(\sigma_u^2/2)$. On average it makes up 2/3 of the pension; the rest is the salary related component which is agent specific. For $t > T^R$ pension income y_t is equal to

$$y_t = \left(\underbrace{\frac{2}{3}\theta \bar{y}_{T^R} \exp\left(\sigma_u^2/2\right)}_{\substack{\text{flat rate} \\ \text{component}}} + \underbrace{\frac{1}{3}\theta y_{T^R}}_{\substack{\text{final salary} \\ \text{component}}} \right) \underbrace{\exp((t - T^R) g^P)}_{\substack{\text{productivity} \\ \text{growth}}}, \quad t > T^R, \tag{14}$$

where the average *replacement rate* θ of the pension is set to be around 30%. Every year pension income is adjusted for productivity growth g^P. Pension income is assumed to be tax-free.

2.6 Equity and safe returns

Equity returns are log-normal i.i.d:

$$\ln \tilde{R}_{t+1} \sim N\left(\ln \bar{R} - \sigma_r^2/2, \sigma_r^2\right), \tag{15}$$
$$\bar{R} = 1.065, \sigma_r = 0.175.$$

We take the mean return on risky assets (equities) to be 6.5% and annual volatility to be 17.5%. The real rate of return on a safe asset is 2.5% p.a. We assume these return processes are invariant to domestic saving and portfolio choice. This is in contrast to the assumption about house prices and rental yields, where changes in aggregate demand are allowed to influence the house price process and the rental rate, respectively. This seems a natural set of assumptions to make for a model calibrated to a relatively small open economy like the UK. UK households and financial institutions can access global capital markets; but houses are non-traded goods, specific to the UK and construction is subject to planning restrictions which limit supply.

2.7 House price process and rental yield

We assume that the price of any particular house consists of two components: an overall, aggregate average house price $\bar{p}_t$ and an idiosyncratic (house specific) element. Log house prices p_t, relative to the average price, follow a mean reverting process

$$\ln \frac{p_t}{\bar{p}_t} = \rho_p \ln \frac{p_{t-1}}{\bar{p}_{t-1}} + \varepsilon_t^p, \tag{16}$$
$$\rho_p = 0.95, \quad \sigma_p = 0.156.$$

The shocks ε_t^p are independent, normally distributed with mean zero and standard deviation σ_p.

We set the initial rental yield r^H to 4.0%, a figure consistent with the average net of costs yield to owners of rental property in the UK in recent years. (We also allow for the yield to depend on the demand for rental properties in a way we describe below). $\bar{p}_t$ is the equilibrium price of a unit of housing and we obtain it as the price which clears housing market at given time t. The modelling of aggregate housing demand and supply depends on population structure and on the elasticity of supply . This is described in greater detail in Section 3.3 after we have discussed how the population structure evolves.

3 The Overlapping Generations Model

To analyze the impact of various public pension policies we embed the household decision model within an overlapping generations model. All households that are born at time s belong to the same cohort s. These households have common deterministic labour income profile $\bar{y}_t$ and average house price profile $\bar{p}_t$, but are each subjected to different idiosyncratic income shocks u_t, house price shocks ε_t^p and risky returns $\tilde{R}_t$. Households in different cohorts have different average incomes because of productivity growth[1], and will face a different lifetime profile of average house prices.

We proceed by describing the demographics first, followed by the structure of cohorts and the transfer of bequests amongst them. We then describe how we calculate aggregate housing demand and equilibrium in the housing market.

3.1 Demographics and tax

Let us denote by $N_t(s)$ the number of individuals aged t years at time s. Denote the retirement age at time s by T_s^R. Assuming that productivity (i.e. labour income) in the economy grows at constant rate g^P, the *income adjusted support ratio* $S^y(s)$ is defined as

$$S^y(s) = \frac{\sum_{t=0}^{T_s^R} N_t(s)\,\bar{y}_t}{\sum_{t=T_s^R+1}^{T} N_t(s)\,\bar{y}_{T_s^R}\exp\left(\sigma_u^2/2\right)}. \tag{17}$$

$S^y(s)$ at any time s describes the ratio between the amount of money earned by the working population and the amount of money received by pensioners in a scheme with a 100% average replacement rate. In other words, the denominator represents the pension

[1]The average labour income of cohort $s+1$ is higher than that of cohort s by the factor $\exp(g_p)$.

bill should every pensioner be given an average last-period-at-work salary adjusted for productivity growth, cf. equation (14).

The pension system is balanced by setting the labour income tax rate $\tau(s)$ at the level where tax receipts in a given year exactly match the pension liabilities

$$\tau(s) = \frac{\theta(s)}{S^y(s)}, \tag{18}$$

where $\theta(s)$ is the average replacement rate in period s. This calculation reflects the changing demographic structure, characterized by $N_t(s)$. The number of households of each age are based on UK government actuaries projections of the UK population. In the year 2000 the income adjusted support rate is 3.65, which implies the labour income tax required to support the replacement rate of 30% is 8.2%; this rate increases approximately by a factor of 1.6 by 2050 if pensions remain equally generous relative to income.

3.2 Transfer of wealth among generations

We assume that wealth held by households at the time of their death is inherited by younger generations. The amount of bequeathable wealth w_t is equal to the sum of net financial wealth and the value of owner occupied house. For simplicity, we assume that bequests are passed onto the generation which is born T^B years later than the parent cohort. To eliminate the need for the amount of received bequests to become another state variable we simplify the bequest transfers by assuming that bequests are redistributed evenly across the younger cohort.

The redistribution of bequests works as follows. Let $\bar{w}_t(s)$ be the average bequest left by household aged t in cohort s. Then, the average flow of bequests generated by cohort s over its lifetime is given by

$$\bar{b}_t(s) = l_1 \ldots l_{t-1}(1 - l_t)\,\bar{w}_t(s), \quad 1 \leq t \leq T, \tag{19}$$

where l_t is the probability of surviving one year at age t. To calculate the bequest distribution $\bar{b}_t(s)$ in (19) we run Monte Carlo simulations of income processes, house prices and risky returns and observe the households' holding of wealth. Using this data we calculate the average $\bar{w}_t(s)$ and substitute this result into equation (19).

The bequests left behind by cohort s are redistributed evenly among the members of cohort $s+T^B$. The bequests accumulated before the cohort $s+T^B$ enters the model are added up and passed onto the 20-year-olds when they enter the labour force. We adjust the amount of transferred wealth to account for changes in demographics, therefore the bequest $w_t^B(s)$ received by every household of age t in cohort s equals:

$$w_0^B(s) = \frac{1}{N_0(s)} \sum_{t=0}^{T^B} N_t\left(s - T^B + t\right) \bar{b}_t\left(s - T^B\right), \tag{20}$$

$$w_t^B(s) = \frac{N_{t+T^B}(s+t)}{N_t(s+t)} \bar{b}_{t+T^B}\left(s - T^B\right), \quad 0 < t \leq T - T^B, \tag{21}$$

$$w_t^B(s) = 0 \quad T - T^B < t \leq T. \tag{22}$$

Therefore, at date s the bequests are balanced as follows

$$\underbrace{\sum_{t=0}^{T} N_t(s)\bar{b}_t(s-t)}_{\text{bequests left at time } s} = \underbrace{\sum_{t=0}^{T} N_t(s)\, w_t^B(s-t)}_{\text{bequests received at time } s} + \underbrace{\sum_{t=0}^{T^B-1} N_t(s)\bar{b}_t(s-t) - \sum_{t=0}^{T^B-1} N_t\left(s-T^B+t\right)\bar{b}_t\left(s-T^B\right)}_{\text{change in accumulated bequests}}, \tag{23}$$

where the term before last is the amount of bequests which is added to the stock of accumulated bequests (which are to be distributed later when the appropriate cohort enters the model) and the last term is the amount of bequests distributed among current 20-year-olds but accumulated in previous years[2]. The bequest span T^B is set at 30 years.

3.3 House price equilibrium

To calculate housing demand in a given year s we need to compute the optimal demand for housing services of all cohorts alive in that year, i.e. cohorts $s, s-1, ..., s-T+1$. We use a large number of realizations of income shocks and idiosyncratic house price shocks at the household level to calculate individual demands for housing for many households of different ages at each point in time. Let us denote the average housing demand of

[2] Note that at any time s the total amount of accumulated bequests is $\sum_{u=0}^{T^B-1}\sum_{t=0}^{T^B-u-1} N_t(s-u)\,\bar{b}_t(s-u-t)$.

households of age t in cohort s by $\bar{H}_t(s)$. Then the aggregate housing demand $\bar{H}^D(s)$ at time s is formed by summing demand across all households alive in that period

$$\bar{H}^D(s) = \sum_{t=0}^{T} N_t(s)\,\bar{H}_t(s-t)\,. \tag{24}$$

In the overlapping generations model we calculate the aggregation from year 1990 until 2110. Using the aggregate housing demand we calculate house demand growth as

$$g(s) = \ln \frac{\bar{H}^D(s)}{\bar{H}^D(s-1)}.$$

We then calculate what the aggregate growth in house supply is each period based on an assumed price elasticity of supply of 0.5 – a figure which is the average of all the estimates of UK housing supply elasticity reported by Barker (2004). For the historical periods (up to 1990) we set the growth rate of average house prices equal to the actual average long term real house price growth in the UK (since 1945) of about 2% a year. For the simulation period (1990-2110) we set the average growth rate of real prices each period at a level which matches the change in aggregate demand to the aggregate change in supply, based on the assumed supply elasticity. For the periods after 2110 we approximate the subsequent house price growth by the growth rate observed in 2110.

Searching for equilibrium house price requires an iterative procedure. In every iteration we solve the household optimization problems with the time profile of average house prices that would have cleared the market demand in the previous iteration. We iterate to a fixed point where the growths in aggregate demand and in aggregate supply match, assuming a price elasticity of 0.5 in each period.

We assume that there is a less than perfectly elastic supply of rental properties at any house price level. We assume that a 1% increase in the demand for rental housing raises the rental yield by 1/50 of 1%. So a 50% rise in the demand for rental property would, in addition to any impact on house prices from any associated increase in the overall demand for residential property, also have an effect on the relative price of rental to owner occupied property by raising the rental yield by 1% (from an initial level at the start of the simulation period set to 4%).

3.4 The stylized facts to match with the calibrated model

The key features of the current UK housing market[3] that we aim to replicate with the calibrated OLG model are::

1. **Owner occupation.** The overall owner occupation rate has been very steady at around 68% for the past 10 years. Owner occupation is relatively low amongst people in their 20's and starts to rise strongly amongst people in their 30's. Owner occupation is much higher amongst those in the second half of their working lives. The average age at which people became home owners in 2003 was around 32.

2. **Housing wealth**. By the end of 2004 housing wealth as a proportion of net worth for the household sector, excluding the value of accrued claims to unfunded state pensions, was about 60%. (Net assets here are all financial assets held by households plus the value of tangible assets net of financial liabilities). Housing wealth as a proportion of all net assets - including the value of claims upon future state pensions - was just under 50%. Housing wealth as a proportion of all wealth peaks in the early 30's (which is when many households have recently become home owners and moved out of the rented sector but have yet to accumulate substantial savings).

3. **Rental yield**. Average rental yields in the private rental market for residential property in the UK have been at around 5% in recent years. Rent on public sector housing is lower. Overall rental yield of about 4% is plausible.

4. **Mortgage cost.** In recent years interest rates on mortgages in the UK have been, on average, approximately 1 to 1.5% above the level of short term money market rates (see Miles 2003 and 2004). Rates on deposits are below money market rates; the spread between deposit and mortgage rates is likely to be about 2.5%.

5. **Borrowing limits (down-payment requirement).** In recent years the typical loan to value ratio of first time buyers in the UK is between 85% and 95%. A down

[3]Four useful sources of information on the UK housing market are Barker (2003 and 2004); Miles (2003 and 2004); Turner (2004) and Office for National Statistics (2004).

payment of somewhere between 10% and 15% of the property is now common. At times in the past this down payment requirement has been higher.

6. **House prices**. House prices in the UK have been relatively volatile (See IMF 2004 for an assessment of the relative volatility of house process across developed economies). Over the longer term in the UK house prices have risen by about 2% a year in real terms. Shocks to house prices seem persistent but not all shocks are permanent. (In other words prices do not seem to follow a random walk around a trend. Rather they may swing above and perhaps below trend for sustained periods). Our specification of the process for house prices allows for this feature.

7. **Financial assets and savings.** As well as trying to reflect the nature of household ownership of residential property, we also want to capture other key aspects of household decisions: the saving rate and the split between safe and risky assets. In this model the saving rate should be thought of as the net national saving rate (since neither corporations, who implicitly are assumed to distribute all income, nor government, who run balanced budgets, do any saving).

We chose the three key preference parameters to try to reflect these aspects of behaviour. We set the risk aversion parameter, γ, equal to 4. We set the elasticity of substitution between consumption and housing, $1/\eta$, equal to 2/3. We set the discount factor, δ, to 0.99.

Parameters of the base case model are summarized in Table 1.

4 Simulation Results

We take the calibrated model and use numerical methods to work out optimal decisions at different ages as a function of the state variables (age, current labour income, wealth, house prices). This is a formidable task since we have a four dimensional state space and many sources of uncertainty requiring a large amount of numerical integration. In addition, to solve the first order conditions at each of several thousand nodes requires finding roots of four non-linear simultaneous equations (which are the first order conditions) as a function of four control variables (goods consumption, house size, buy/rent decision, financial portfolio choice).

Having solved the model at a grid of points in the state space we then take 100,000 households and use Monte Carlo methods to follow their optimal decisions over the life cycle. For each household we take a draw from a distribution of the stochastic elements of house prices, incomes and rates of return in every period of the household's life. We use the optimal solution at the points of the grid in the state space to estimate the best decision for each household for any point in the state space. This involves interpolating between points in the four dimensional grid of the state space. Once we have solutions for 100,000 households we can average all the decisions made by households at a particular age to show cohort behaviour, or aggregate across all households to show macroeconomic outcomes.

We present three sets of results. In the first two simulations (scenarios A and B) the retirement age is kept constant at 62; in the third simulation it increases from 62 in 2015 to 70 in 2050. In the first set of results (scenario A) we assume that the generosity of the PAYGO pension remains constant; the replacement rate averages 30% for every cohort. (The replacement rate is not the same for those *within* a cohort because there is a flat rate element to the pension which generates some redistribution within the system). In the second set of simulations (scenario B) we assume that the level of the state pension is gradually reduced so that the contribution rate needed to balance the state pension system is constant at 8.2%. In this scenario the average replacement rate of the state pension falls from 30% to about 19% by 2050. In the third simulation (scenario C) we allow for a continuing rise in the retirement age which helps to reduce the tax burden, but we assume a constant pension generosity (replacement rate of 30% on average).

We begin by focusing on the behavior of a particular cohort (that born in 1960) in scenario A and then describe the macroeconomic aggregates for all three scenarios. The cohort figures report the *average* behavior of those members of the cohort born in 1960 that survive to the relevant age. (The great majority of the cohort born in 1960 survive into their 60's and 70's. By assumption no one lives beyond the age of 90). In looking at the behavior of a particular cohort we focus on the average decisions of the members of that cohort – within any cohort there is considerable heterogeneity because idiosyncratic income shocks play a major role in the model.

In forming the aggregates and tracing their development from 1990 through to 2110 we sum over the decisions of members of every cohort, using the demographic projections to determine the relative numbers of people from each cohort alive at each date.

4.1 Scenario A: Replacement rate at 30%

Figures 2-5 show the average of decisions made by all those households of the 1960 cohort that are still alive at each age[4]. All households begin their adult life (at age 20) by renting. The down payment constraint for buying a house is substantial and so few households buy a house until they reach their 30's. But owner occupation is attractive for households - with real house prices rising on average by about 2% a year and the cost of mortgage debt at 5% the expected effective user cost of housing for an owner occupier borrowing the maximum is around 3%. The rental cost is higher and rises from a level of 4% in 2000 and towards 4.5% by 2030.

Younger households aim to move into owner occupation fairly early in their life cycle and the great majority remain in owner occupation until well into retirement. Because the typical household can expect to stay in owner occupation for many years owning a home gives a form of insurance against unexpected house price shocks. Since these shocks are large there is an element of a precautionary demand for housing. As a result owner occupation is very high within a cohort once the age of its members is above 40; figure 3 shows that owner occupation is close to 80% at age 50 and peaks at just under 90% at retirement. Owner occupation rates fall slightly as households move through their 60' and their 70's. Owner occupation amongst those households in their 80's - and for whom the desired stock of net worth begins to fall significantly - starts to drop rapidly. Initially almost all households borrow as much as they can to finance house purchase. But during their 40's there emerges considerable heterogeneity within the cohort with many households borrowing substantially less than the maximum. All this matches the key features of household choices in the UK.

[4]Behavior among those aged 85 or beyond in this Figure is based on the average across a small and declining number of survivors.

Once households retire there is a significant change in the portfolio of assets. The existence of a bequest motive means that while many households wish to continue holding substantial net assets the composition of portfolios changes. Increasingly households reduce the proportion of their overall wealth in the form of owner occupied housing and increase the proportion of wealth in the form of financial assets. As households go past retirement the composition of their financial assets changes rapidly as people increase sharply the percentage of their assets in safe debt and holdings of risky assets fall sharply. In contrast very few households in their 30's and 40's hold significant amounts of wealth in safe financial assets. When human capital is still the dominant form of lifetime assets, and when holdings of debt (which has a cost in excess of the return on safe assets) tend to be large, holding safe assets is not attractive; where financial assets are held they are therefore overwhelmingly in the form of risky assets. This is a reflection of the assumption that shocks to human capital are largely idiosyncratic and independent of the return on risky assets (which for convenience we call "equities", though which should be interpreted more widely). As we shall see the pace with which the switch from risky to safe financial assets happens is dependent upon the generosity of state pensions.

Overall households move into owner occupation early in the life cycle and only sell the house and move back to rented accommodation if they are still alive in their 80's. The high average real rate of return on property, relatively high cost of renting and the existence of a bequest motive combine to make housing an attractive asset and one which people are content to dominate their portfolios of wealth for much of their lives.

Figures 6-11 and Tables 2-4 show aggregate results for the economy through time under Scenario A (constant generosity of state pensions and rising contribution rates). Several features stand out from these figures. First, if PAYGO pensions remain at the same level (relative to average incomes) it is predicted that the level of owner occupation is fairly steady until about 2040 and then rises gently. Owner occupation starts out at just under 70% in 2000 (close to the actual level of UK owner occupation). By 2050 the rate of owner occupation is 73%. By 2075 it is 74%. House prices rise fairly steadily in real terms and the value of houses bought - relative to incomes - also rises gradually. Real house prices double between 2000 and 2052. Financial assets as a proportion of the gross wealth of the household sector steadily rises - and housing wealth falls as a proportion of gross wealth - as the population ages.

The savings rate in the economy starts at about 4.5% in 2000. This is very close to the UK net national savings rate in recent years which has been slightly above 4%. It is the net national savings rate that corresponds to the household saving rate in this model because the government runs a balanced budget and implicit in the model is the assumption that corporations distribute their income - all savings is done by households. The savings rate starts to decline gradually from about 2010, when it stands at close to 5%. By 2031 it is only just over 1% and only 0.8% by 2050. This change is largely driven by demographics: by 2050 there is a substantially higher proportion of retired households who have, on average, a lower savings rate than working households.

4.2 Scenario B: Constant contribution rate

In this scenario the value of the PAYGO pension declines gradually relative to average earnings. By 2050 the average replacement rate for newly retired households is down to 19%; in scenario A that ratio stays at 30%. This has a significant effect upon the evolution of the macroeconomic aggregates.

The impact on cohorts born later than 1960 is substantial. For example the cohort born in 1990, who start work in 2010, react to significantly lower state pensions by accumulating much more wealth by retirement. On average, the value of net worth of this cohort when it reaches retirement (at around 2050) is about 30% higher as a result of lower state pensions. Owner occupation is also markedly higher for this cohort than when state pensions are kept steady relative to incomes. Demand for financial assets is also greater. The proportionate impact of lower state pensions upon the value of residential property and financial assets held is similar, though demand for financial assets does grow by somewhat more.

As a result the macroeconomic aggregates in an environment of falling PAYGO pensions diverge quite markedly from those where the pension replacement rate is held constant. Figure 6 shows that with falling pensions the owner occupation rate is higher, as is the overall demand for housing; this drives house prices up somewhat faster than in the base case. (The owner occupation and saving rates are actually higher even at the start of the simulation period . This is because we assume in each scenario that agents anticipate the pension policy that is coming. The exact timing of when demand for housing and for financial assets rises under a scenario with falling replacement rates for state pensions will depend on when it becomes clear how policy is evolving). By 2050 the owner occupation is 77% when the pension is cut, but nearer to 72% when the replacement rate is constant. The reason is that falling state pensions substantially increase desired asset holdings among the population. Households are more willing to retain large net holdings of property into retirement. As a result the overall demand for housing is somewhat higher when the value of PAYGO pensions falls - house prices are about 8.5% higher by 2050. The relatively small impact on house prices – but larger impact upon the value of owner occupied residential property (Figure 7) – reflects the fact that a switch from renting to buying (relative to the base case) itself is neutral for the overall demand for housing.

Net worth of the household sector relative to aggregate labour income is substantially higher, and rising, when the value of PAYGO pensions (relative to incomes) is set on a declining path. The overall saving rate is higher in scenario B for several years - in 2010 it is 5.5% compared to 4.7% in the base case. But after 2035 the saving rate is lower under scenario B, and it becomes slightly negative after 2055. It may seem counterintuitive that the savings rate is eventually higher in scenario A with relatively generous state pensions than in scenario B where households need to accumulate more wealth to compensate for falling state pensions. This finding is less surprising if one realizes that in scenario B households keep much more wealth in the form of housing and capital gains on housing, which are significant, are not part of the income that enters the expression for the savings rate. Net worth of households is always higher under scenario B.

4.3 Scenario C: Replacement rate at 30% but increasing retirement age

The difference between scenario C and scenario A stems from an assumed gradual increase in the retirement age from 62 in 2015 to 70 in 2050. When the retirement age begins to rise the demand for assets held into retirement falls since that wealth sustains consumption over a shorter period. So the saving rate is significantly lower - it is only 3% in 2010, 2.7% in 2030 and just under 1% in 2050. The demand for housing rises rapidly. Higher demand for housing is driven by the larger lifetime flow of labour income that is a by-product of later retirement. House prices now double by 2044. Demand for financial assets, not surprisingly, peaks later in life when retirement is pushed back. Although the saving *rate* out of income is lower when retirement is significantly later, the overall accumulation of wealth (net worth) by cohorts who work longer is higher. The fall in the rate of saving out of income is more than offset by the impact of higher lifetime labour income.

4.4 Portfolios of wealth and pension reform

The impact of different pension rules upon the structure of aggregate household balance sheets of assets is illustrated in Figures 9 to 11. In all scenarios, aggregate gross financial assets rise relative to the value of owner occupied housing over time. This is because as households age and pay off their mortgage they accumulate financial assets and, if they live into their 80's and beyond, eventually tend to move out of owner occupation. In contrast younger households - particularly those in their 30's - hold the great majority of their wealth in the form of newly acquired homes and have relatively few financial assets. Over time the proportion of households in the later stages of their lives, when financial assets rise in importance relative to housing, increases. But the scale of this aggregate portfolio shift varies depending on the state pension system. When state pensions become less generous (scenario B relative to scenario A) the switch towards financial assets in household portfolios over time is more marked. By 2050 financial assets comprise about 67% of gross assets in the economy relative to the base case (constant generosity of state pensions) of around 62%. But when the age at which households receive the state pension is significantly later (scenario C) the switch towards financial assets in gross wealth is slower than in the base case.

While we are able to say something fairly clear about the evolution of the demand for different types of assets under different pension systems it is harder to draw clear welfare conclusions. Figure 12 and Table 5 show that while in the long run a move towards less generous pay-as-you-go state pensions does increase average welfare for future generations, households who are middle aged (the cohorts born around 1960) tend to lose as the value of the state pension is scaled back. We are not able to find a Pareto improving change in the structure of state pensions. In a model where the average rate of return on assets is greater than the rate of growth of the economy it is not surprising that the long run impact of reducing the generosity of unfunded pensions is to raise average welfare. But nor is it surprising that on a transition there are likely to be losers (see Breyer 1989). The scale of the gains of the future winner generations from less generous unfunded pensions is, however, very significantly greater than the small welfare losses of the current middle aged. The welfare gain for a cohort born in 1990 of lower pensions (scenario B), relative to unchanged pension replacement rates, is the equivalent of just under 50% of initial annual average salary, or about 0.7% of lifetime income. The loss of this reform for the cohort born 30 years earlier (the 1960 cohort) is just above 4% of their average initial income, and only about 0.06% of lifetime income.

5 Conclusions

The aim of this paper has been to explore the role of housing in a model of life cycle choices for UK households that face various forms of uncertainty, receive PAYGO pensions of various levels of generosity and have a limited bequest motive. Households also face borrowing constraints. We developed a model calibrated to reflect key features of the UK economy. We derived results on the behavior of cohorts over their life cycle and on the evolution of macroeconomic aggregates in an environment of rapidly changing demographics. We solved the model under various assumptions about how pensions and contribution rates will evolve.

We find that the demand for owner occupied housing is sensitive to demographic change and to reform of the PAYGO pension system. If PAYGO pension replacement rates remain high as the population ages the level of owner occupation may rise only marginally over time. But if pension generosity is scaled back sharply, the rate of owner occupation and directly held housing wealth may rise significantly faster. When the pension age is raised significantly the demand for housing rises most strongly and the demand for financial assets grows relatively less strongly. As a result there is a marked change in the aggregate pattern of household wealth relative to other pension scenarios. But one thing all our scenarios have in common is that the importance of owner occupied property in overall portfolios declines over time. This is despite the fact that owner occupation rates continue to rise as do real house prices.

The links between the property market, demographic change and pension reform seem to be significant. The key policy conclusion from this paper is that the knock-on effect upon the housing market of demographic shifts and reform of the pension system is likely to be substantial.

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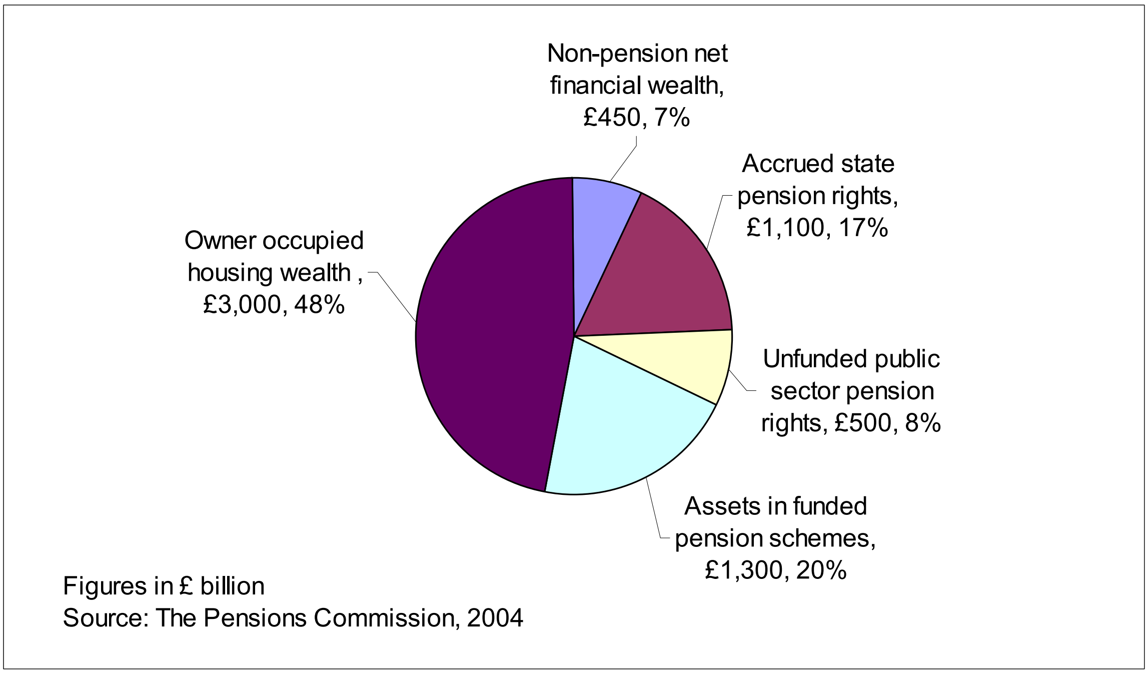

Figure 1: Balance sheet of UK households at the end of 2003

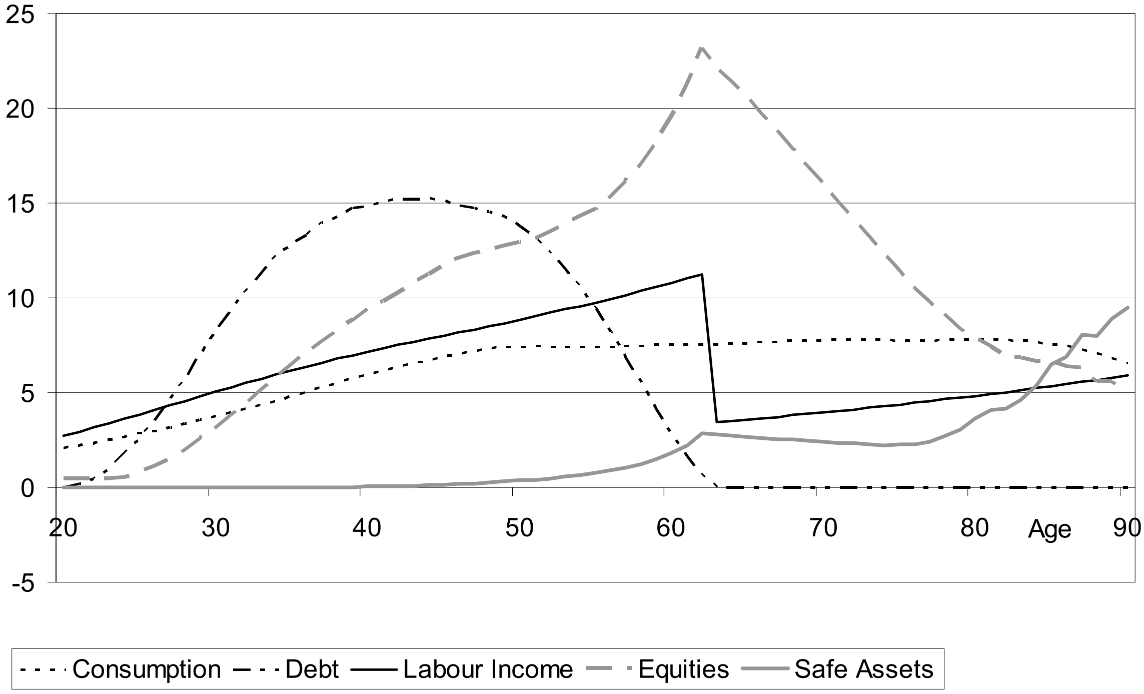

Figure 2: **Life-cycle portfolios, cohort born in 1960, scenario A.** This chart shows five important characteristics of life cycle decision making – consumption of goods, labour income, amount of borrowed funds and holding of safe assets and equities. These profiles present average (or expected) values obtained by simulating optimal lifetime decisions of 100,000 households. Each household is born at age 20 and starts with no wealth. We assume that if any household is still alive at age 90, it dies then with certainty. Households receive random persistent income shocks, house price shocks and equity return shocks, which are idiosyncratic across all households.

Figure 3: **Owner occupation rate, cohort born in 1960, scenario A.** This plot illustrates the average lifetime owner occupation rate of households born in 1960.

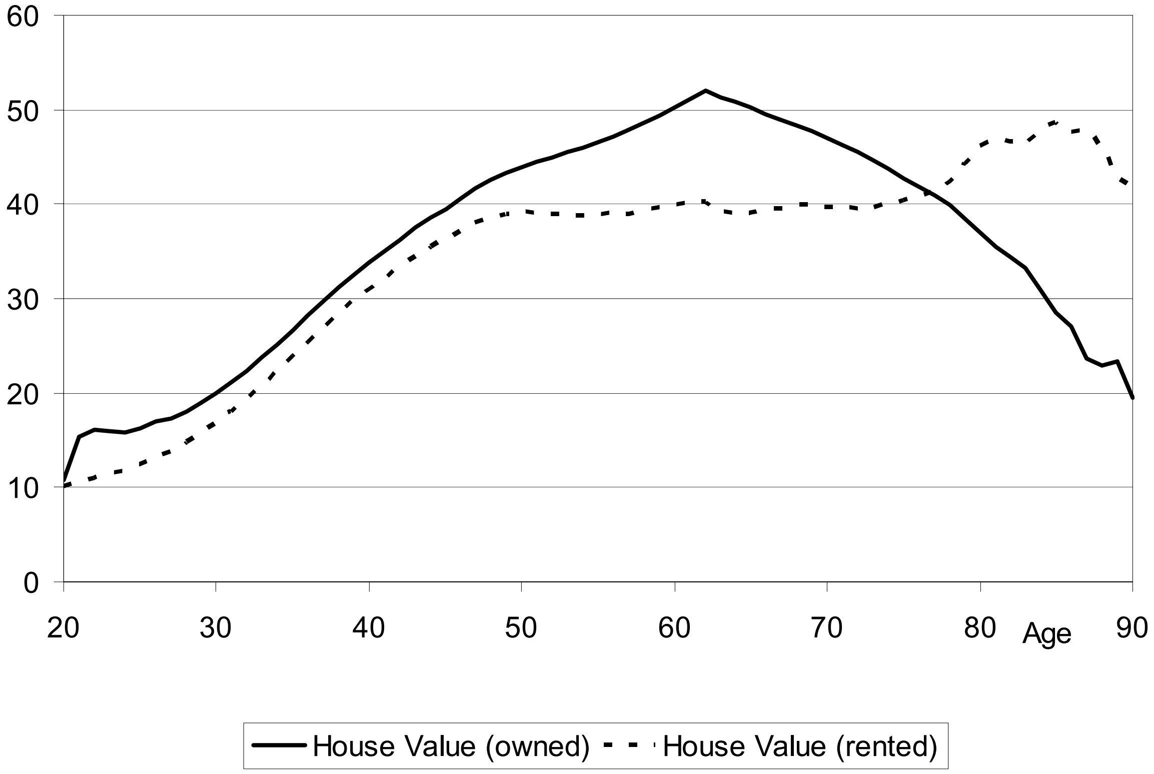

Figure 4: **House value, cohort born in 1960, scenario A.** At every age we separate the population of the 1960 cohort into two groups – owner occupiers and renters. For each group we trace the value of houses the members of that group live in and report the average for both groups.

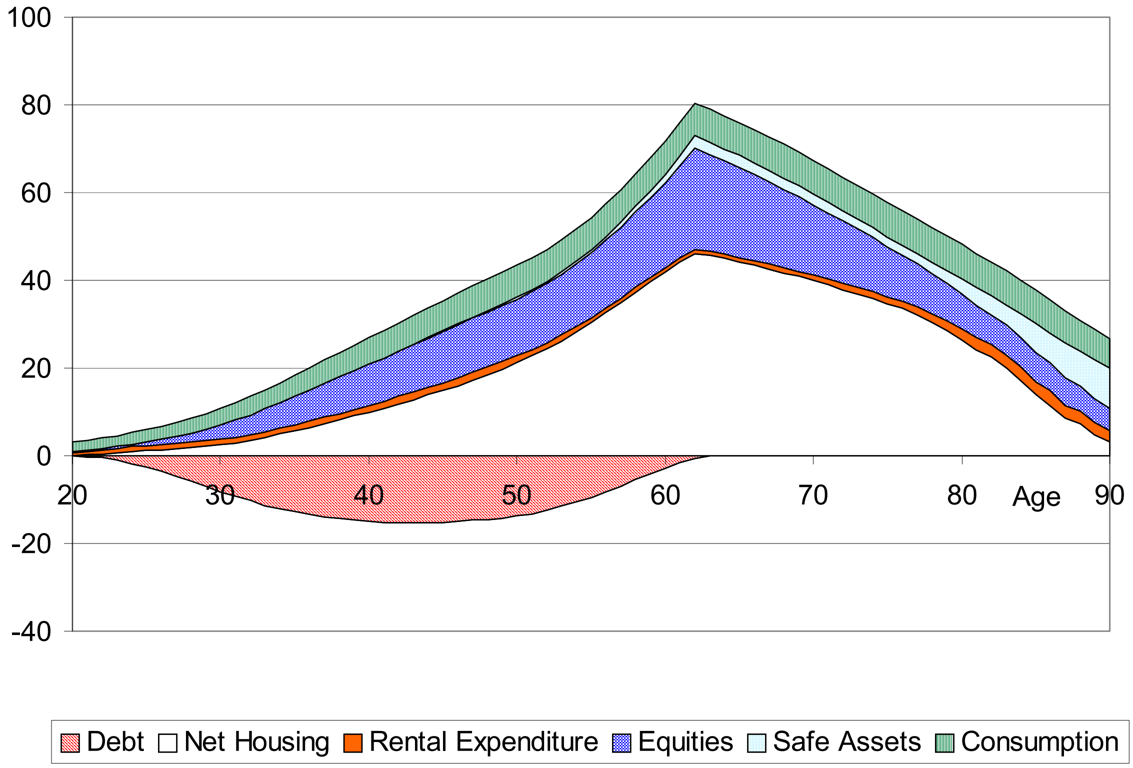

Figure 5: **Composition of net worth, cohort born in 1960, scenario A.** This layered chart shows the average lifetime composition of net worth. Net worth is defined as wealth available to a household, including the value of an owner-occupied house net of any mortgage. Net worth is represented by the topmost line. Net housing wealth represents the value of owner occupied property net of mortgage debt.

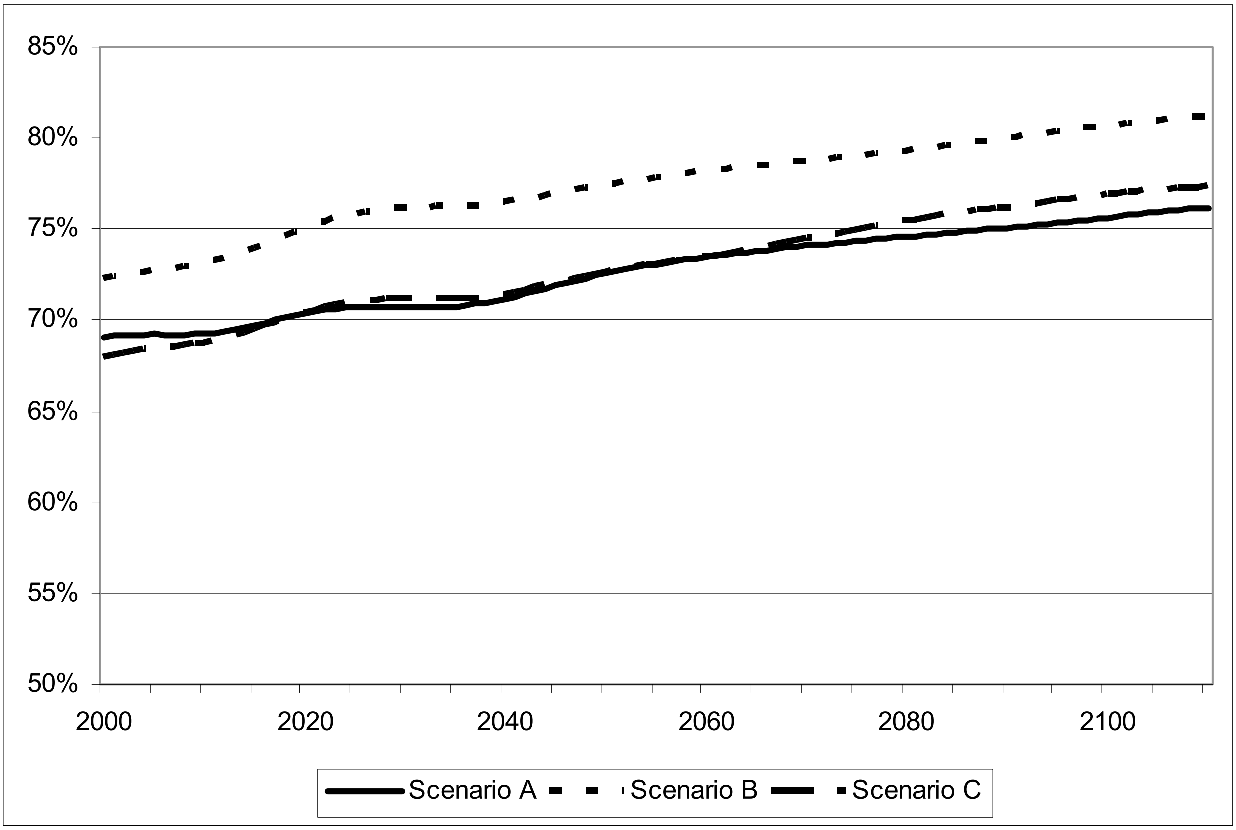

Figure 6: **Aggregate owner occupation.** This figure shows aggregate owner occupation rates for the three pension policy scenarios – A) fixed replacement rate at 30%, increasing contribution rate, B) fixed contribution rate at 8.2%, decreasing replacement rate, C) fixed replacement rate, increasing retirement age.

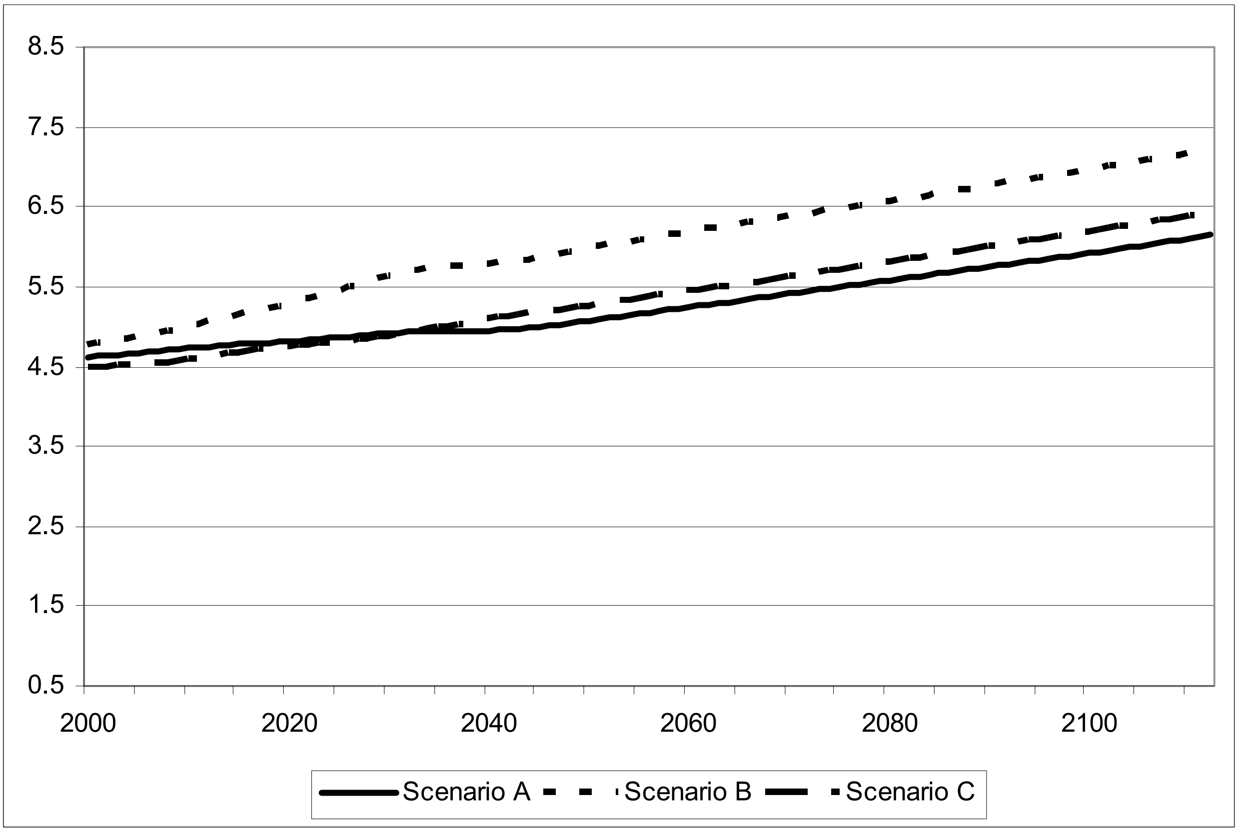

Figure 7: **Aggregate value of owner occupied housing.** This plot shows the aggregate value of owner occupied houses over aggregate labour income.

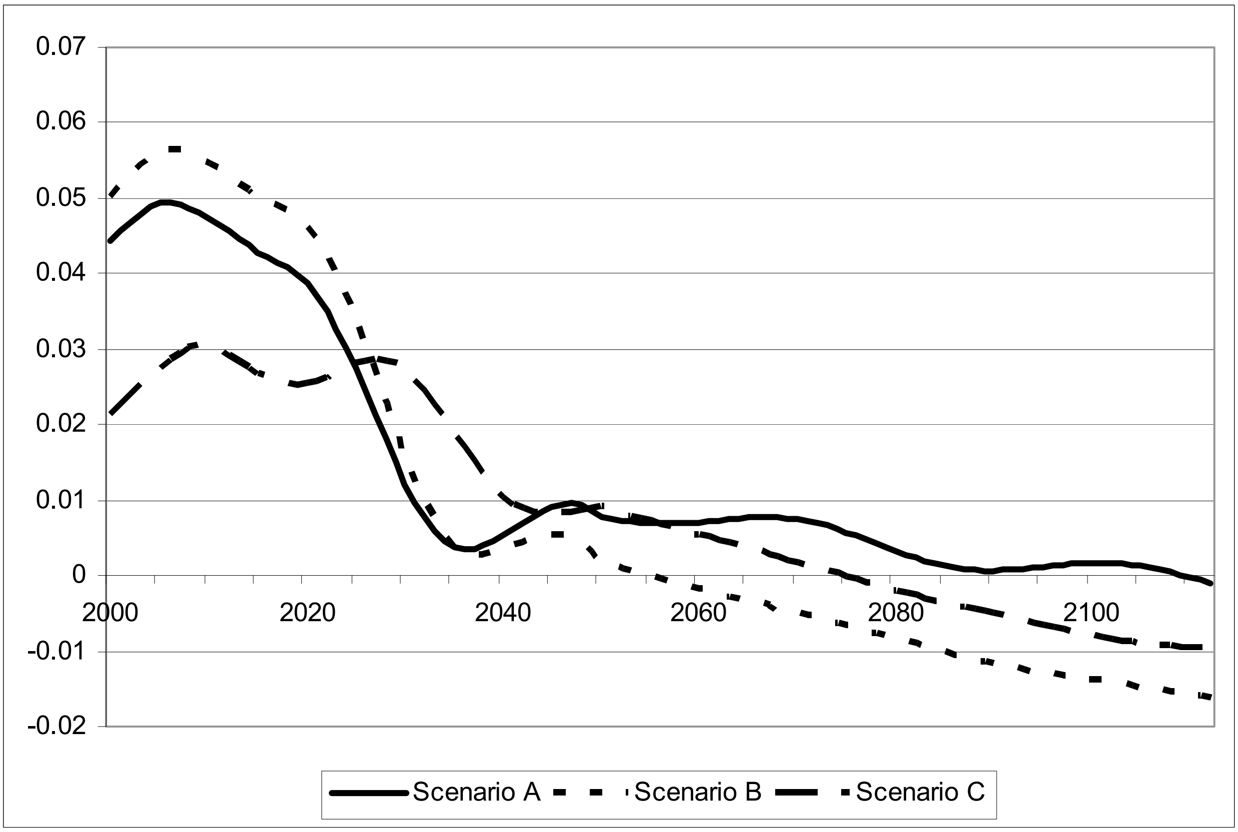

Figure 8: **Aggregate national savings rate.** The national savings rate has been calculated as $1 - \frac{\text{aggregate consumption}}{\text{labour income + net financial income}}$.

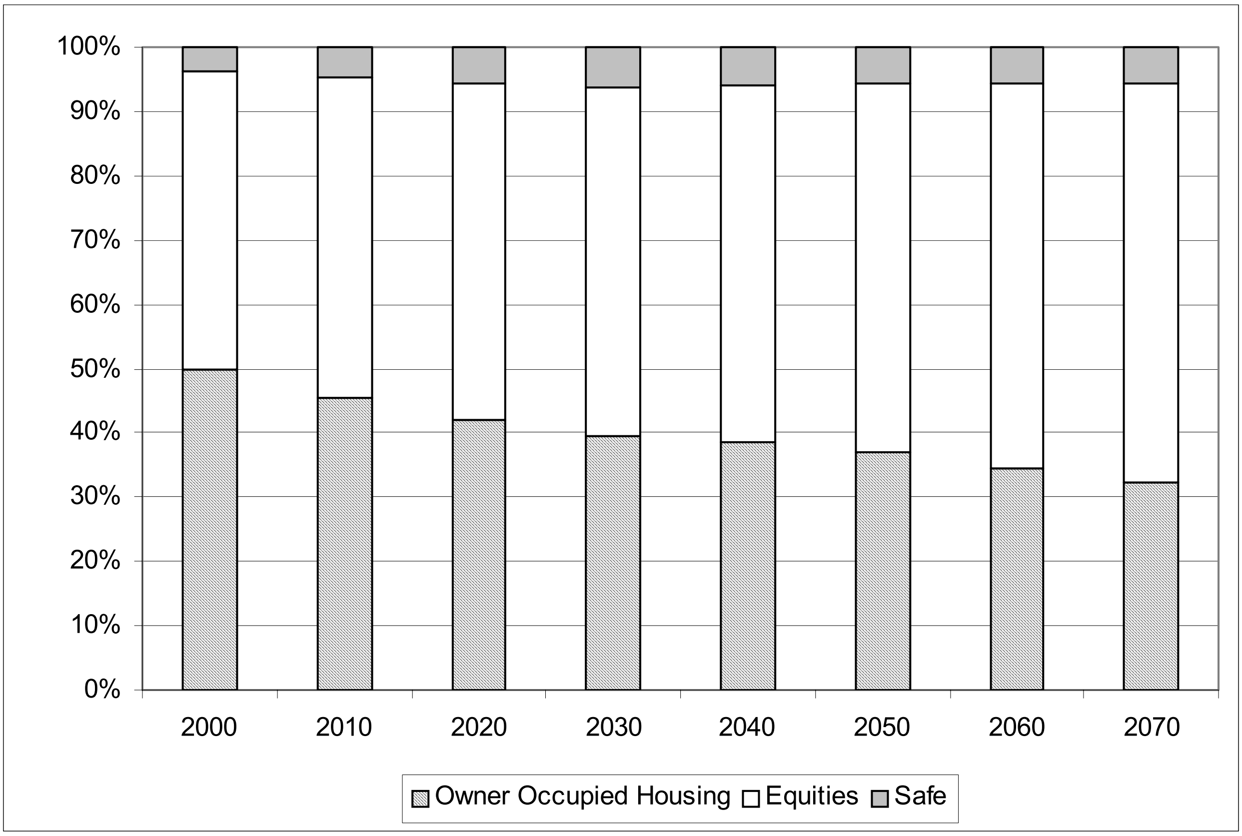

Figure 9: Aggregate Portfolios, Scenario A.

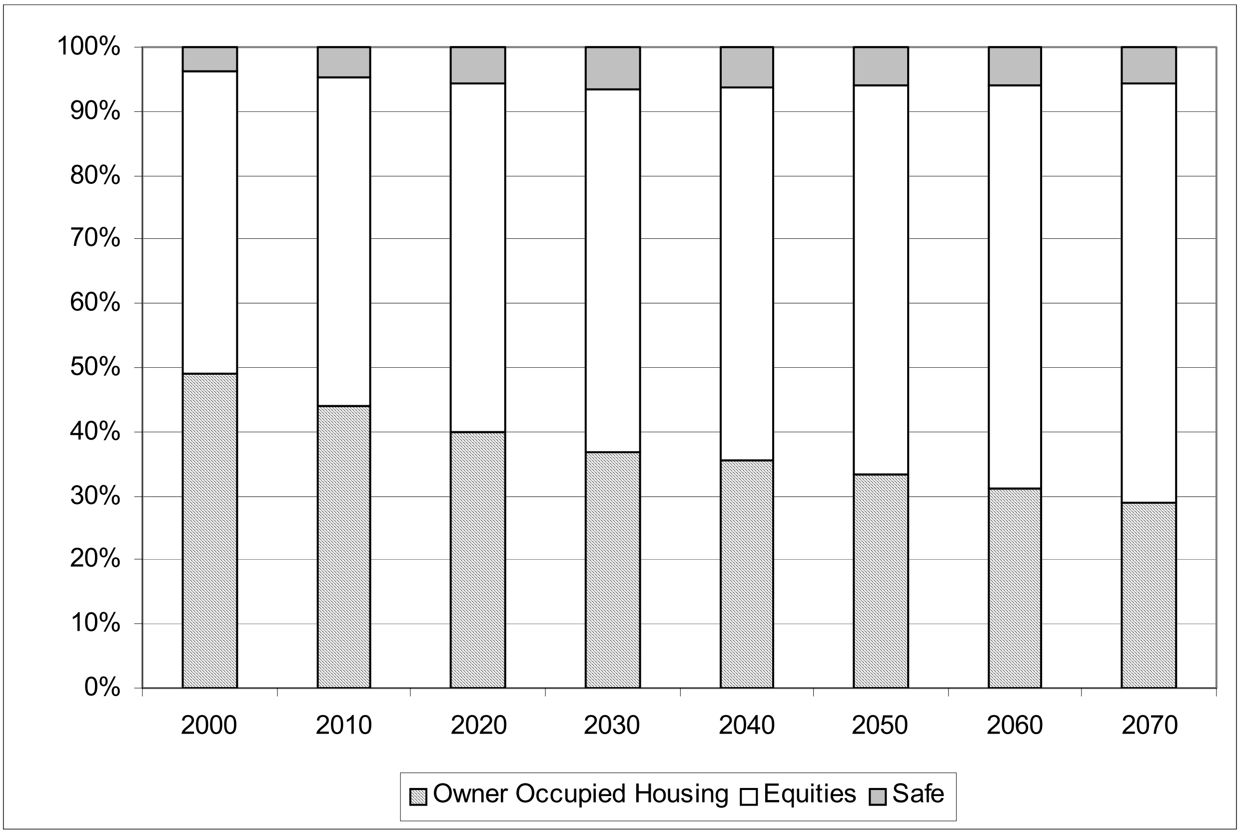

Figure 10: Aggregate Portfolios, Scenario B

Figure 11: Aggregate Portfolios, Scenario C

Figure 12: **Compensating variation** of moving from scenario A to scenario B as a proportion of labour income at age 20. The horizontal axis depicts the year when each cohort is born.

Symbol	Description	Value
γ	risk aversion, inverse of elasticity of intertemporal substitution	4
$1/\eta$	elasticity of substitution between consumption and housing	2/3
λ_H	mortgage to house value ratio	0.7 to 0.9
λ_y	mortgage repayment	0.175 to 0.25
r	safe rate ($R_t = 1 + r$)	0.025
r^B	borrowing rate	0.05
$\varkappa$	weight between consumption and housing	0.8
ξ	weight of housing in utility function	1
δ	time discount factor	$1.01^{-1} \approx 0.99$
$\tilde{R}$	risky return	–
$\bar{R}$	expected risky return	1.065
σ_r	volatility of risky assets returns	0.175
σ_u	volatility of persistent income shocks	0.11
σ_p	volatility of house price returns	0.156
ρ_u	persistence of income shocks	0.98
ρ_p	persistence of house price shocks	0.95
g^P	productivity growth	0.02

Table 1: Parameter values

Year	1 Gross Fin. Assets	2 Saving rate	3 Mortgage debt	4 Owner occup. rate	5 Value of Owner Occ. Housing	6 House price level	7 Rental yield
2000	1.54	4.4%	1.33	69.0%	4.62	1.00	4.14%
2010	1.61	4.7%	1.27	69.3%	4.73	1.15	4.26%
2020	1.62	3.9%	1.23	70.4%	4.82	1.31	4.36%
2030	1.60	1.2%	1.23	70.7%	4.92	1.48	4.49%
2040	1.44	0.5%	1.30	71.2%	4.95	1.68	4.61%
2050	1.35	0.8%	1.37	72.6%	5.08	1.94	4.70%
2060	1.31	0.7%	1.43	73.5%	5.24	2.26	4.81%
2070	1.29	0.7%	1.48	74.1%	5.41	2.63	4.93%
2080	1.25	0.3%	1.53	74.6%	5.58	3.06	5.04%
2090	1.20	0.1%	1.58	75.1%	5.75	3.55	5.16%
2100	1.15	0.2%	1.62	75.6%	5.92	4.13	5.28%
2110	1.12	0.0%	1.67	76.2%	6.11	4.80	5.40%

Table 2: Aggregate results, scenario A. Columns 1, 3 and 5 show values relative to average pre-tax labour income

Year	1 Gross Fin. Assets	2 Saving rate	3 Mortgage debt	4 Owner occup. rate	5 Value of Owner Occ. Housing	6 House price level	7 Rental yield
2000	1.63	5.0%	1.37	72.2%	4.78	1.00	4.12%
2010	1.79	5.5%	1.32	73.1%	5.00	1.16	4.25%
2020	1.92	4.6%	1.32	75.0%	5.28	1.34	4.38%
2030	2.03	1.5%	1.38	76.2%	5.62	1.54	4.52%
2040	1.90	0.3%	1.44	76.5%	5.78	1.77	4.62%
2050	1.82	0.2%	1.50	77.4%	5.98	2.06	4.69%
2060	1.77	-0.2%	1.55	78.1%	6.18	2.40	4.80%
2070	1.73	-0.5%	1.59	78.7%	6.37	2.79	4.92%
2080	1.67	-0.8%	1.64	79.3%	6.56	3.24	5.04%
2090	1.60	-1.2%	1.70	80.0%	6.76	3.77	5.16%
2100	1.53	-1.4%	1.75	80.6%	6.96	4.37	5.29%
2110	1.47	-1.6%	1.80	81.2%	7.17	5.07	5.43%

Table 3: Aggregate results, scenario B. Columns 1, 3 and 5 show values relative to average pre-tax labour income

Year	1 Gross Fin. Assets	2 Saving rate	3 Mortgage debt	4 Owner occup. rate	5 Value of Owner Occ. Housing	6 House price level	7 Rental yield
2000	1.37	2.1%	1.41	67.9%	4.48	1.00	4.16%
2010	1.33	3.1%	1.46	68.8%	4.58	1.17	4.30%
2020	1.32	2.5%	1.52	70.4%	4.75	1.37	4.41%
2030	1.31	2.7%	1.50	71.2%	4.88	1.60	4.55%
2040	1.29	1.0%	1.57	71.4%	5.09	1.85	4.68%
2050	1.23	0.9%	1.65	72.6%	5.26	2.15	4.79%
2060	1.21	0.5%	1.71	73.5%	5.44	2.51	4.91%
2070	1.16	0.2%	1.78	74.4%	5.61	2.92	5.03%
2080	1.11	-0.2%	1.84	75.4%	5.80	3.40	5.13%
2090	1.06	-0.5%	1.91	76.2%	5.99	3.96	5.25%
2100	1.02	-0.8%	1.96	76.9%	6.18	4.60	5.36%
2110	0.97	-0.9%	2.01	77.3%	6.38	5.35	5.49%

Table 4: Aggregate results, scenario C. Columns 1, 3 and 5 show values relative to average pre-tax labour income

Cohort born in	1960	1975	1990	2005	2020	2035	2050
$\frac{\text{Compensating variation}}{\text{Income at age 20}}$	-4.1%	15.1%	46.5%	74.4%	74.2%	66.1%	60.5%
$\frac{\text{Compensating variation}}{\text{Expected lifetime income}}$	-0.06%	0.21%	0.65%	1.05%	1.04%	0.93%	0.85%

Table 5: Compensating variation of moving from scenario A to scenario B for cohorts born at different dates. The expected lifetime income represents the expected present discounted value of lifetime labour income (discount factor equal to the safe deposit rate of 2.5% p.a.)