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ABSTRACT

Forming Rational Expectations and When it is Right to be 'Wrong'*

In this paper we examine the effects of private agents being less than fully rational. We examine this in the context of monetary policy, where the Central Bank may have uncertain preferences either by choice or by necessity. The new feature is that we allow the public to react in two different ways to this uncertainty. They either form rational expectations and internalize the uncertainty about the Central Bank's preferences in full; or alternatively, and if this process of internalization is costly, it forms a 'best' guess regarding those preferences. This implies a certainty equivalence strategy applied to the preference parameters. As those parameters enter the decisions non-linearly, a systematic error emerges. We examine the magnitude of the resulting error in inflation and output, following the assumption of certainty equivalence. Under all reasonable levels of uncertainty this error turns out to be small but involves trading a deflation bias against the cost of gathering the information needed for the full rational expectations solution.

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Keywords: central bank preference uncertainty, certainty equivalence and rational expectations

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*Views expressed are our own and do not necessarily reflect those of the institutions we are affiliated with.

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1 Introduction

In the process of forming expectations, the assumption of rational expectations allows all agents involved to share from the same information set. However, when the assumption of common information breaks down, either because it is not feasible, or simply because it is too costly to try to acquire all the information that others have, then agents have to resort to second-guessing the reactions of their counterparts. In this paper we exploit how "wrong" agents could be when making educated guesses about the behaviour of others and compare the consequences to the default case of full information associated with rational expectations.

We apply a monetary policy example, in which, from the point of view of the private sector, the Central Bank (CB) has uncertain preferences. This maybe either by choice - because the bank itself chooses not to reveal certain aspects of what it knows - or by necessity - because the Central Bank does not have firm information itself, and is therefore unable to commit to one specific set of parameters for all circumstances. The first case is the traditional one, in which the private sector faces preference uncertainty or control errors and has been studied by many authors [*inter alia*, Faust and Svensson (2001, 2002), Geraats (2002), Muscatelli (1998), Sibert (2002), Walsh (1999)]. The second case can be associated with potentially time inconsistent preferences in which the parameters applied by the Central Bank (CB) are state contingent, randomised or otherwise varied. In that case the Central Bank is unable to announce in advance exactly what values might be taken at any specific time. This kind of model has not been widely studied, but two key examples may be found in the *Rational Inattention* model of Sims (2003) and the *Constructive Ambiguity* approach of Cukierman-Meltzer (1986).

In this paper, we assume that there is uncertainty about what the Central Bank preferences really are, and examine how the private sector might react when it is unsure about the source (or extent) of this uncertainty. We also assume, as in most studies, that the private sector is unable to internalise the full effects of the uncertainty it faces. This may happen either because it does not realise that the Central Bank may be uncertain itself; *or* because it may not have sufficient information to characterise that uncertainty fully; *or* because it is too expensive to do so accurately. Instead it applies a first order certainty equivalent (FOCE) estimate of what the Bank is likely to deliver. Such simplifications, realistic though they may be in terms of how people actually behave, inevitably lead to systematic errors in private sector forecasts of inflation and the output gap. In order to evaluate these errors we compare the resulting inflation rate to that achieved when the private sector is fully rational and internalises the consequences of the CB's preference uncertainty. Note that, unlike the literature on bounded rationality which investigates the *reasons* for not following a full rational expectations solutions, we examine here instead just the *effects*. We find that under all reasonable levels of preference uncertainty, the errors made by following a regime of certainty equivalence are small. If the costs of acquiring

the extra information necessary to form rational expectations are significant, then a regime of certainty equivalence may become the optimal strategy.

The paper is organised as follows. Section 2 provides a standard set-up of monetary policy making. Section 3.1 then presents the Rational Expectations solution when the private sector internalises in full the effects of this uncertainty. Section 3.2 shows what happens to that solution when the private sector makes a simple assumption on the form of this uncertainty. We compare the two solutions in section 4 and discuss how monetary policy might be affected by such an assumption on the way the private sector forms expectations. Section 5 summarises our results and concludes.

2 The Set Up

We apply a monetary policy example to demonstrate that although preferable, rational expectations are not very different to scenarios where simple assumptions are imposed on the uncertain variables. We adopt a standard framework for examining the society's welfare losses:

$$L = \frac{1}{2} E\left[\pi^2 + b\left(y - k\right)^2\right] \tag{1}$$

constrained by a simple Lucas supply function:

$$y = \pi - \pi^e + \varepsilon \tag{2}$$

where following standard notation, y and π are measured as deviations from their steady state paths[1]. Note that we depart from Rogoff's (1985) conservative central banker, in that the CB's objective function and that of the society as a whole are identical. This is done as we are only interested in the way private sector forms expectations, and not in the CB's incentives as such. The model produces the following solutions for the policy variables in question:

$$\begin{aligned} \pi &= bk - \frac{b}{1+b}\varepsilon \\ \pi^e &= bk \\ y &= \frac{1}{1+b}\varepsilon \end{aligned} \tag{3}$$

[1] This system of objectives in inflation and output gaps is based on the utility-based approach by Rotemberg and Woodford (1998). The microfoundations of this model were originally derived by Lucas, (1972, 1973). Extensions to his model, that incorporate fiscal policy, appear in Hughes Hallett and Viegi (2003). To justify the presence of k, a target deviation from the steady state, see the discussions by Rogoff (1985), Blinder (1997). Furthermore, as the literature remains ambiguous about the use of k, Demertzis and Hughes Hallett (2002) recognise it explicitly as one common source of uncertainty. For our purposes however, it is sufficient to note that k might be *either* positive *or* zero.

For simplicity, we assume that there is only one policy authority in the game and no uncertainty about the policy transmission mechanism. However, we impose the fact that the Bank picks a value for b from a given distribution, either because it is unsure about the precise value of b itself, or because it is unable to commit to one particular value of b for all circumstances. We model this uncertainty in the Central Bank's true preference for output stabilisation as follows:

$$b = \beta + u \text{ with } E(u) = 0 \text{ and } V(u) = \sigma_u^2 \tag{4}$$

No preference uncertainty *ex ante* requires both $E(u) = 0$ as well as $\sigma_u^2 = 0$. The Central Bank is fully aware of (4) and can correctly identify the regime that the private sector follows. Appendix A summarises alternative models that motivate uncertainty in b.

3 Forming Expectations

We examine two cases. First, the default position is when the private sector is aware of the values of (4) either because the Central Bank reveals them voluntarily, or because it makes the necessary effort to evaluate (the higher moments of) the full b distribution. This enables the private sector to evaluate the Bank's decision rules accurately. As the information set is common to both parties[2], we call this the Rational Expectations case. In the second case, the private sector considers the mean of that distribution, $E(b) = \beta$, to be the actual preference parameter. This will be the private sector's best guess or 'consensus estimate' if it lacks accurate information on the remaining characteristics of the distribution used by the Central Bank to generate b. Equally, this estimate can be used to generate *first order certainty equivalent* approximations to the Bank's best decisions when the higher order moments of the b distribution are unknown or are considered difficult or too expensive to estimate accurately. This will be identified as the Certainty Equivalence case.

3.1 The Rational Expectations Solution

We assume first that the private sector is aware of the properties of the Central Bank's preferences. Given information on the distribution of b, as supplied by the Bank or obtained from its own research, the private sector knows the Bank will solve (1) subject to (2) and (4) to obtain

$$\pi = \frac{b}{1+b}(\pi^e - \varepsilon + k) \tag{5}$$

[2] Note that so long as we deal with the linear-quadratic case as specified here, we need to know only the first two moments of the b distribution. A non-linear set-up would require the private sector to have access to all the higher order moments of the distribution of b (Hughes Hallett, 1984, p39).

Taking expectations through (5) and solving for the value of π^e, yields the rationally expected inflation rate, as:

$$\pi^e_{RE} = \left[\frac{(1+\beta)^2 - \sigma_u^2/\beta}{(1+\beta)^2 + \sigma_u^2}\right]\beta k = \theta\beta k \tag{6}$$

Notice that $\lim \theta = 1$ when $\sigma_u^2 \to 0$ and/or $\beta \to \infty$, and $\theta < 1$ if $\sigma_u^2 \neq 0$. But θ may turn negative if β becomes very small ($\beta \to 0$) and provided $\sigma_u^2 \neq 0$[3]. To obtain the actual inflation outcome in this case, we substitute (6) into (5)

$$\pi^*_{RE} = \frac{b(1+\theta\beta)k}{1+b} - \frac{b\varepsilon}{1+b} \tag{7}$$

and hence obtain

$$E(\pi^*_{RE}) = \left[\beta - \frac{\sigma_u^2}{(1+\beta)^2}\right]\left[\frac{1+\theta\beta}{1+\beta}\right]k \tag{8}$$

It is straight forward to show that (8) is equal to (6), demonstrating that expectations are indeed consistent with the rational outcome. As regards output, using (7) and (2), we have

$$y^*_{RE} = \frac{uk + (1-\theta)\beta k + \varepsilon}{1+b} \tag{9}$$

and hence,

$$E(y^*_{RE}) = 0 \tag{10}$$

This shows that any output bias is eliminated even with preference uncertainty.

3.2 A Certainty Equivalence Solution

We examine next the case when the private sector does not have accurate information on the moments of the distribution of b. This maybe either because it cannot access it (i.e. the CB withholds it for reasons of exploiting elements of 'constructive ambiguity'), or it finds it too expensive to obtain compared to the potential benefits in terms of improved decision making. In either case, it takes the mean of that distribution, $E(b) = \beta$, as its certainty equivalent ('best estimate') and using (3), the private sector expects the Central Bank to implement the following policy rule:

$$\pi = \beta k - \frac{\beta\varepsilon}{1+\beta} \tag{11}$$

which implies $\pi^e_{CE} = \beta k$ is the inflation expected in the markets. In fact, in view of (13) and (14) below, we can see that π^e_{CE} is the **first order** certainty

[3] In fact, $\theta < 0$ only if $\sigma_u^2 > \beta(1+\beta)^2$. However, if this happens because β is small, instead of σ_u^2 being large, then $\pi^e_{CE} \to \pi^e_{RE} \to 0$.

equivalent estimate of the optimal policy rule when b and ε are uncorrelated. But it is also a biased estimate, as (15) will show. The Central Bank now optimises the following loss function

$$\min_{\pi} L = \frac{1}{2} E\left[\pi^2 + b\left(\pi - \beta k + \varepsilon - k\right)^2\right] \tag{12}$$

Since π is a choice entirely within the Bank's control, the result is:

$$\pi^*_{CE} = \frac{b\left(1+\beta\right)k}{1+b} - \frac{b\varepsilon}{1+b} \tag{13}$$

But, notice that $E(\pi^*_{CE})$ does not equal bk or βk, since

$$E\left(\frac{b}{1+b}\right) \neq \frac{\beta}{1+\beta} \tag{14}$$

In fact, taking expectations, we get (approximately),

$$E(\pi^*_{CE}) = \beta k - \frac{\sigma_u^2 k}{\left(1+\beta\right)^2} < \beta k = \pi^e_{CE} \tag{15}$$

assuming, b and ε to be independent. Equation (15) has **two** immediate implications: a) greater preference uncertainty *increases* average inflation; b) the private sector will consistently overestimate inflation. Regarding the effects on output, we substitute (13) into (2) where $\pi^e_{CE} = \beta k$:

$$y^*_{CE} = \frac{b(1+\beta)k}{1+b} - \frac{b\varepsilon}{1+b} - \beta k + \varepsilon = \frac{uk+\varepsilon}{1+b} \tag{16}$$

This implies, that this time the average level of output will also be affected by a lack of clarity at the CB:

$$E(y^*_{CE}) = -\frac{k\sigma_u^2}{\left(1+\beta\right)^2} \tag{17}$$

As $E(y^*_{CE}) < 0$ there is a deflation bias here, which *decreases* with greater transparency. Adopting Certainty Equivalence is a sensible strategy if the expectational errors created ($\pi^e_{CE} \neq E\left(\pi^*_{CE}\right)$) are small in relation to the cost of gathering the information necessary to support the full rational expectations equilibrium. With the aid of numerical examples, we identify next when this is likely to happen for different levels of uncertainty. This will give an idea of the importance of the difference between the two solutions.

4 CE versus RE: A comparison

We now show how the errors caused by taking a certainty equivalence approach vary with the parameters of the model:

Proposition 1 *Rational Expectations always result in lower actual and expected inflation.*

Proof 1: The proportional error in expected inflation[4] that the private sector would obtain in the certainty equivalence case is

$$\text{Expectations Error: } \frac{(1-\theta)\beta k}{\beta k} = \frac{\sigma_u^2 (1+\beta)}{\beta \left[(1+\beta)^2 + \sigma_u^2\right]} > 0 \tag{18}$$

which vanishes if $\sigma_u^2 \to 0$ but increases if $\beta \to 0$. This implies that $\pi_{RE}^e < \pi_{CE}^e$. Similarly, given (15), the proportional error in actual inflation in the certainty equivalence solution, compared to the rational expectations equilibrium, is

$$\text{Inflation Error: } \frac{(1-\theta)\beta}{1+\beta} = \frac{\sigma_u^2}{(1+\beta)^2 + \sigma_u^2} > 0 \tag{19}$$

on average. Hence $E(\pi_{RE}^*) < E(\pi_{CE}^*)$[5]. Notice that this error also vanishes as $\sigma_u^2 \to 0$ or as β increases, but rises if β becomes smaller. Finally, and most important, the fact that (19) and (18) are both positive means that the **proportional** forecasting error - and hence the incentive for the private sector to change from the certainty equivalent solution to rational expectations - is the difference between the two:

$$\text{Forecast Error: } \frac{\sigma_u^2}{\beta \left[(1+\beta)^2 + \sigma_u^2\right]} \tag{20}$$

Proposition 2 *Rational Expectations eliminates the deflation bias that arises under the assumption of Certainty Equivalence.*

Proof 2: It is straight forward to demonstrate this by comparing (10) to (17). We find that $E(y_{CE}^*) < E(y_{RE}^*) = 0$ and therefore rational expectations prevent any deflation bias from arising.

4.1 The Relevance of Errors

Systematic errors in expectations are the key determinant of which solution would be chosen in practice, since the private sector has no interest beyond being able to forecast the Central Bank's decisions accurately. As the private sector leads, once it has decided which expectations solution to follow, the Central Bank is locked in - except in so far as the Bank may decide to supply the private sector with enough information to get an equilibrium which , on its own estimate, it thinks to be superior or more stable. Assuming $k = 1$, we evaluate the errors for two standard probability distributions for b.

[4] This is calculated as $\frac{\pi_{CE}^e - \pi_{RE}^e}{\pi_{CE}^e}$; similarly $\frac{E(\pi_{CE}^e) - E(\pi_{RE}^e)}{E(\pi_{CE}^e)}$ for the average outcomes.

[5] Note that the errors in the expectations go in the same direction as the 'errors' in the outcomes, but are bigger.

Table 1. Inflation, Expectations and Errors

	Uniform ($\beta = 0.5$, $\sigma_u^2 = \frac{1}{12}$)	Normal ($\beta = 0.5$, $\sigma_u^2 = 0.028$)
$\pi_{CE}^e =$	0.500	0.500
$E(\pi_{CE}^*) =$	0.463	0.487
$\pi_{RE}^e =$	0.446	0.482
$E(\pi_{RE}^*) =$	0.446	0.482
Expectations Error:	0.107	0.037
Inflation Error:	0.036	0.012
Forecast Error:	0.071	0.025
$E(y_{CE}^*) =$	-0.037	-0.012
$E(y_{RE}^*) =$	0	0
$L_{CE} =$	0.64	0.63
$L_{RE} =$	0.59	0.62

Suppose b is distributed uniformly in the unit interval. This implies a certain degree of conservatism, with $\beta = 0.5$ and $\sigma_u^2 = 1/12$. Compared to the correct rational expectation (π_{RE}^e), π_{CE}^e will be too high by 10.7 *per cent*. As a result, the Central Bank will end up choosing inflation rates that are higher, on average, than those in the full rational expectations solution by 3.6 *per cent*[6]. Consequently the private sector will find itself making systematic errors in its forecasts of inflation of just 7.1 *per cent* on average. More realistically, we also examine the case where the Central Bank chooses b from a normal distribution such that it can be 99 *per cent* certain to remain inflation averse (conservative) at all times. Assuming the same mean, this implies $b \sim N(0.5, 0.028)$, with 99 *per cent* of the distribution in the unit interval. In this case, private sector expectations, π_{CE}^e, will be in error (too high) by 3.7 *per cent* and $E(\pi_{CE}^*)$ higher than the rational equilibrium solution by 1.2 *per cent*. The private sector then faces systematic errors in the inflation forecasts of just 2.5 *per cent*. For low levels of inflation these numbers appear to be rather small. For example, if underlying inflation is 2 *per cent*, the private sector will find it has forecast inflation at 2.14 *per cent* and the CB will find it has chosen inflation (on average) at 2.07 *per cent* instead of 2, in the uniform distribution case. If b is instead assumed to be normally distributed, these numbers are 2.05 *per cent* and 2.02, respectively.

With respect to output, the potential deflation bias (output loss) in the certainty equivalence solution, as a proportion of the target level for stabilising output, is

$$\frac{E(y_{RE}^*) - E(y_{CE}^*)}{k} = \frac{\sigma_u^2}{(1+\beta)^2} > 0$$

which shows that the bias is eliminated in the rational expectations solution. Evaluating this expression for the case of uniformly distributed b values, we find the certainty equivalent solution leads to a deflation bias of 3.7 *per cent* in

[6] These figures are derived from (18) and (19) respectively and reflect proportional errors - *not* percentage points.

output target units k. If preferences are normally distributed, the bias would be 1.2 *per cent* of k. For $k = 1$ and if we pick the units of y as percentage deviations from trend output, then the deflation biases are 0.037 *per cent* and 0.012 *per cent* of trend output itself.

Loss function (1) implies that the optimal level of inflation (inflation target) is zero. However, the very existence of preference uncertainty, implies that the private sector will form inflation expectations equal to, at best, 0.446 and face the same level of inflation if it has the same knowledge as the central bank; or it will form expectations equal to 0.5 and face an inflation rate of 0.463, if it applies certainty equivalence under the assumption a uniform distribution for b. We can then evaluate what this would imply for monetary policy, by looking at the effects of such divergences on the interest rate. We apply a standard Taylor rule (Woodford, 2003):

$$\hat{\imath}_t = \bar{\imath}_t + \phi_\pi \left(E_t \pi_{t+1} - \bar{\pi}\right) + \phi_y y_t \tag{21}$$

where $\bar{\imath}_t$ represents the natural rate of interest, $\bar{\pi}$ the inflation target and ϕ_π, ϕ_y the weights on inflation and the output gap, respectively. We approximate the inflation forecast term $E_t \pi_{t+1}$, with π^e for the two cases separately, and assume no further shocks. The change in the interest rate $(\hat{\imath}_t - \bar{\imath}_t)$ as a result of preference uncertainty when $\bar{\pi} = 0$, is shown in Table 2 for conventional values of ϕ_π and ϕ_y.

Table 2. Taylor rule effect

	Uniform ($\beta = 0.5$, $\sigma_u^2 = \frac{1}{12}$)	Normal ($\beta = 0.5$, $\sigma_u^2 = 0.028$)
$\pi^e_{CE} =$	0.500	0.500
$\pi^e_{RE} =$	0.446	0.482
$E\left(y^*_{CE}\right) =$	-0.037	-0.012
$E\left(y^*_{RE}\right) =$	0	0
$(\hat{\imath}_t - \bar{\imath}_t)_{CE}$	0.731	0.744
$(\hat{\imath}_t - \bar{\imath}_t)_{RE}$	0.670	0.722
	ϕ_π : 1.5	ϕ_y : 0.5

Table 2 shows that the interest rate will increase by 0.67 (0.722) if agents are rational, and by 0.731 (0.744) if they follow certainty equivalence, under the assumption of a uniform (normal) distribution. The difference between the two expectations regimes amounts to 6.1 or 2.2 basis points for the two assumed distributions, respectively[7]. As interest rate changes are rarely less than 25 basis points, the adoption of CE is unlikely to affect the interest rate outcome. Additionally, although Table 1 shows that $L_{CE} > L_{RE}$, the actual losses attained under the two regimes hardly differ (0.64 versus 0.59 and 0.63 versus 0.62 for the two distributions respectively). This amounts to having made an

[7] For weights $\phi_\pi = 1.9$ and $\phi_y = 0.3$ the numbers are 9.4 and 3.2 basis points, respectively (based on estimates by Gerdesmeier and Roffia, 2004).

estimation error of 0.23 *per cent* of GDP in the output gap (say, due to an unexpected shock); or alternatively, 0.32 percentage points in the inflation rate. These numbers are well within the errors made in a typical forecasting exercise (see den Reijer and Vlaar, 2003) and therefore, unlikely to influence the private sector's actions. Furthermore, if one was to allow for the fact that the process of acquiring the information necessary to allow private agents to form rational expectations is not without cost, then it is not long before the certainty equivalence solution becomes the best solution. Demertzis and Hoeberichts (2005) in fact show that the existence of such costs, with a linear functional form $C = c\alpha$ where $\alpha = 1/\sigma_u^2$, implies that there exists a maximum level of information precision α, beyond which the costs of increasing it any further, outweigh the benefits of doing so. If forming Rational Expectation requires information that can be obtained short of that point, naturally agents will seek to do so; otherwise they will rationally prefer to form a best guess about what b is[8]. As the difference in losses shown in Table 1 are rather small, it is not long before adding the costs of acquiring extra information to the RE losses tips the balance in favour of the CE choice. For greater degrees of nonlinearity in the cost function - as assumed by Svensson (2005) for instance, where $C = c\alpha^2$ - this reversal can come even earlier.

5 Conclusions

We have investigated whether the fact that private agents often choose to make simplifying assumptions about issues where they are uncertain, causes substantial losses in welfare. Rational expectations are undoubtedly the right solution from a theoretical perspective but the extra gains are typically very small. Moreover, if in practice private agents find it difficult to acquire the precision of information required to formulate the RE solution, then the Certainty Equivalence approximation is going to be the likely outcome. Over and above its higher likelihood however, if there are costs associated with acquiring that information, then the benefits of RE may be altogether overturned. The CE outcome therefore, in which agents just form their best guess about the uncertain parameters, is then not only the most likely outcome but also their optimal choice. This is the sense in which the private sector is then right to be "wrong".

[8] In fact Demertzis and Hoeberichts (2005) also show that, ideally, the private sector would rather have the Central Bank incur all the costs required to increase the precision of information and then communicate that to the public, such that it reaps the benefits of better quality information without having to incur any of the cost - a free ride in other words.

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APPENDIX

A Possible Sources of Preference Uncertainty

The are a number of alternative models that motivate the inclusion of (4) as an extra constraint in the model. The best known model of preference uncertainty is one in which policy makers deliberately retain 'randomised' preferences in order to exploit the effects of ambiguity (Cukierman and Meltzer, 1986). This is modelled as a process of control errors in monetary policy whereby policy makers create shifts in their relative preferences for output stabilisation and inflation in a manner which they perceive to be favourable and with suitable timing. As that shift is random but persistent, policy makers manage to affect the speed with which the private sector becomes aware of it. That then allows the Central Bank to choose its moment for monetary policy changes. For example, it may plan a positive surprise to stimulate the economy when output is down; but with expectations lagging and being uncertain what the Bank's true intentions are, the private sector will not anticipate a rise in inflation. The stimulus can therefore, be achieved at lower cost in terms of extra inflation. Conversely, negative surprises can be timed to reduce inflation without the private sector anticipating a loss in output. But to gain these advantages, the Bank must allow the precise value of b to remain uncertain.

Alternative explanations stem from uncertainty in the underlying model parameters or from the fact that a simple quadratic loss function maybe just an approximation to the true preferences which are more complicated when seen over the entire policy space. In that case, (1) represents a local approximation for the current position of the economy and, being dependent on the uncertain values of π and y, the marginal rate of transformation between them (and hence the desired value of b becomes a random variable Hughes Hallett 1979). The same happens if policy makers wish to retain an element of risk aversion in their preferences since the strict linear-quadratic framework adopted here generates risk neutral decisions. There is also a possibility that policy makers cannot always specify their relative priorities in advance, but have to uncover them iteratively by revealed preference (van Eijck and Sandee, 1959).

A second version of this idea is to note that the 'robust control' approach used by Hansen and Sargent (2003), Hansen *et al* (1999), Basar and Bernhard (1995) is equivalent to minimising (1) minus a term in the variance in the state variable y. Again, if the degree of risk sensitivity (the Lagrange multiplier attached to that extra variance term) is state dependent, the implied change to the value of b will be uncertain *ex ante*. But that, as Kasa (2002) points out, is also identical to the model of 'rational inattention' introduced by Sims (2003). In the latter model, the problem is one in which policy makers have a limited capacity to monitor all the variables in the economy. They will rationally reduce the effort made to forecast and control the most volatile of them in order to concentrate on those that can be controlled effectively. As Sims shows, that is a problem

which can be solved by minimising (1), less a term in the variance of the state variable which is subject to a monitoring constraint. That implies a one-to-one correspondence with our robust control problem.